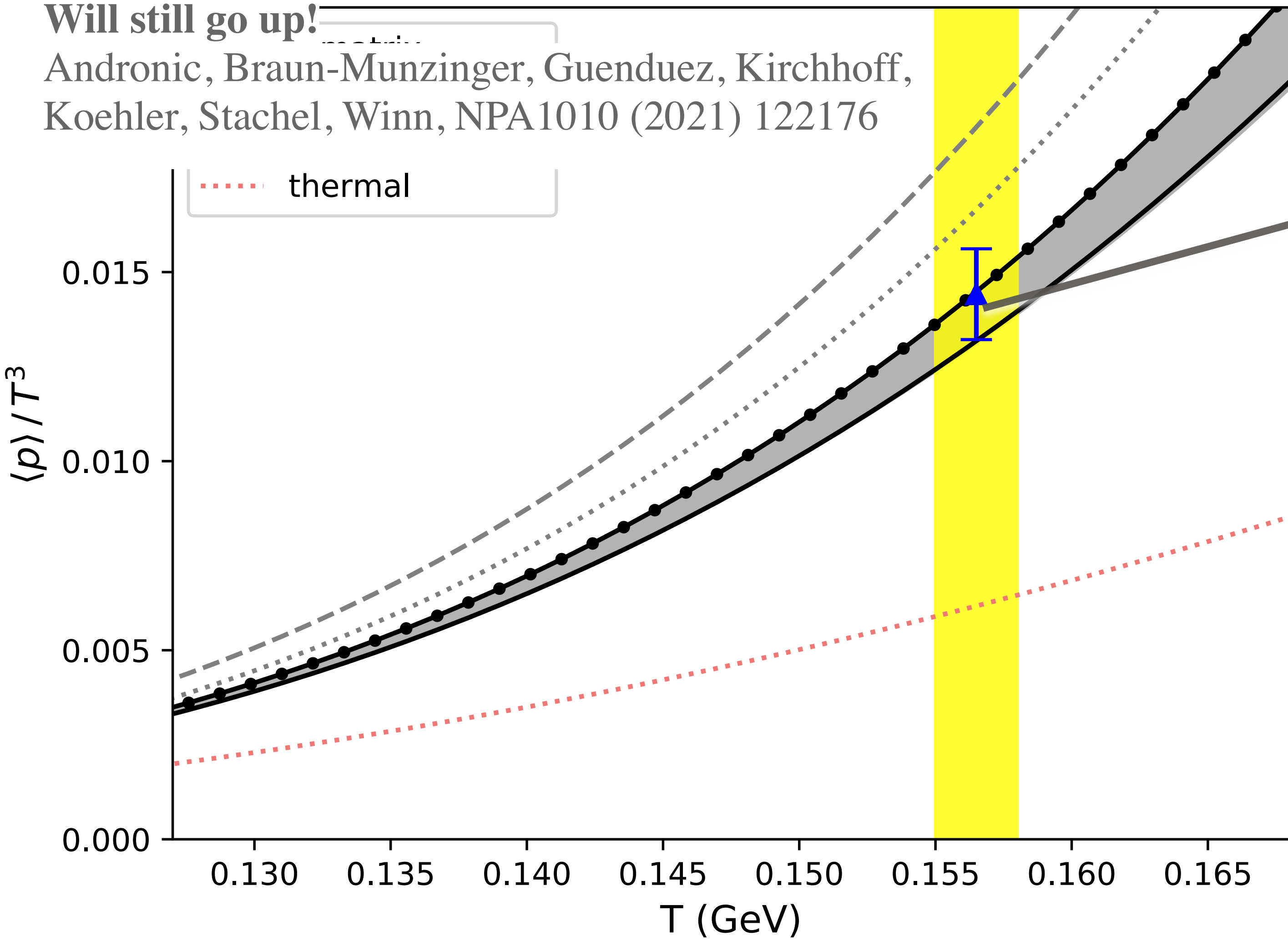
IMPROVING THE HRG MODELS: THE S-MATRIX PERSPECTIVE

POK MAN LO (盧博文)

University of Wroclaw

NEVER AT REST: A LIFETIME INQUIRY OF QGP
11.02.2025 (BAD HONNEF)

Will still go up!
Andronic, Braun-Munzinger, Guenduez, Kirchhoff, Koehler, Stachel, Winn, NPA1010 (2021) 122176

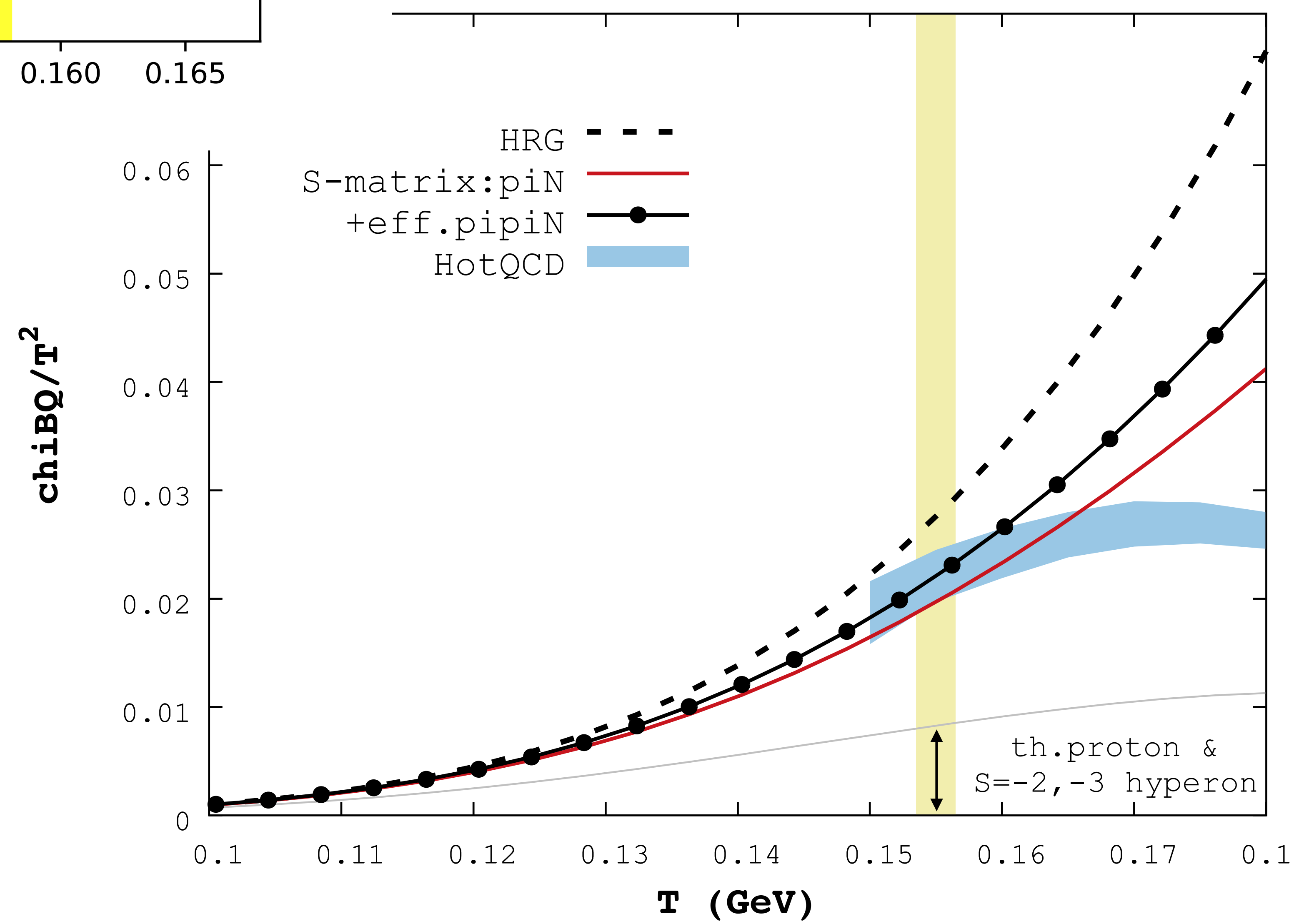


ALICE proton yield
Pb-Pb @ 2.76 TeV
thermal model est.

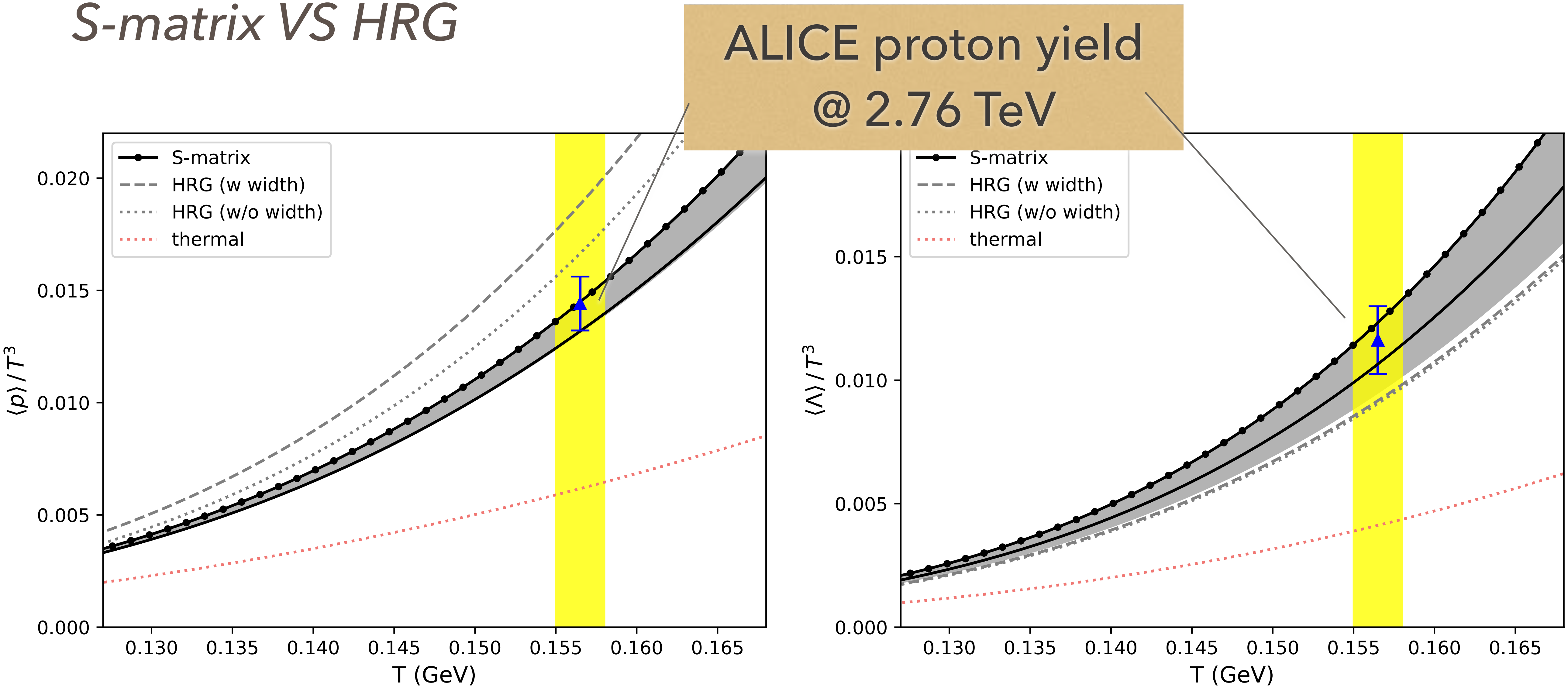
LQCD result on chiBQ

A. Bazavov, et al.,
Phys. Rev. D 86 (2012) 034509.

see also
Bellwied et al.
Phys. Rev. D 101, 034506 (2020)



S-matrix VS HRG



piN phase shifts
pipiN BGs
hyperons

consistent treatment of res and non-res. int.

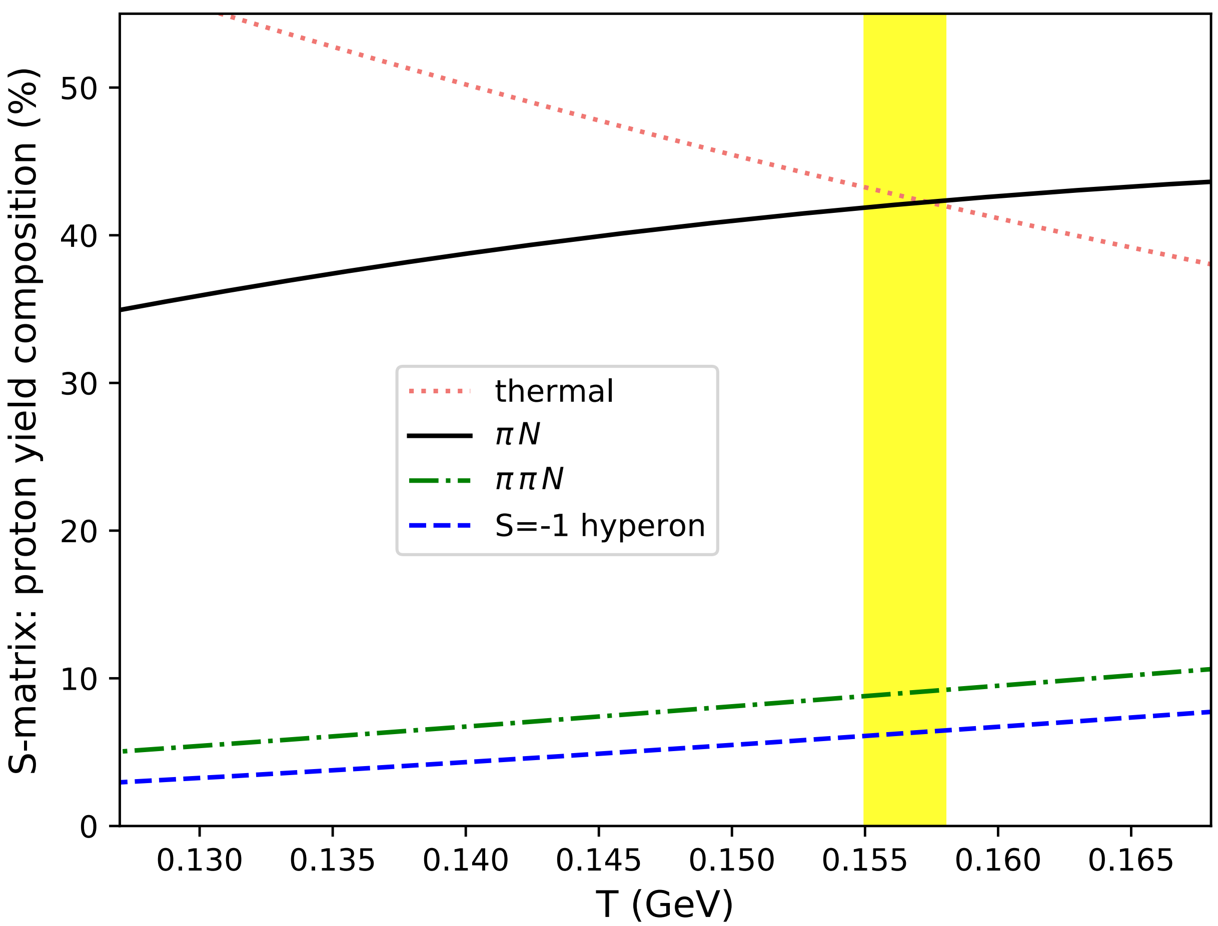
Coupled-Channel model:

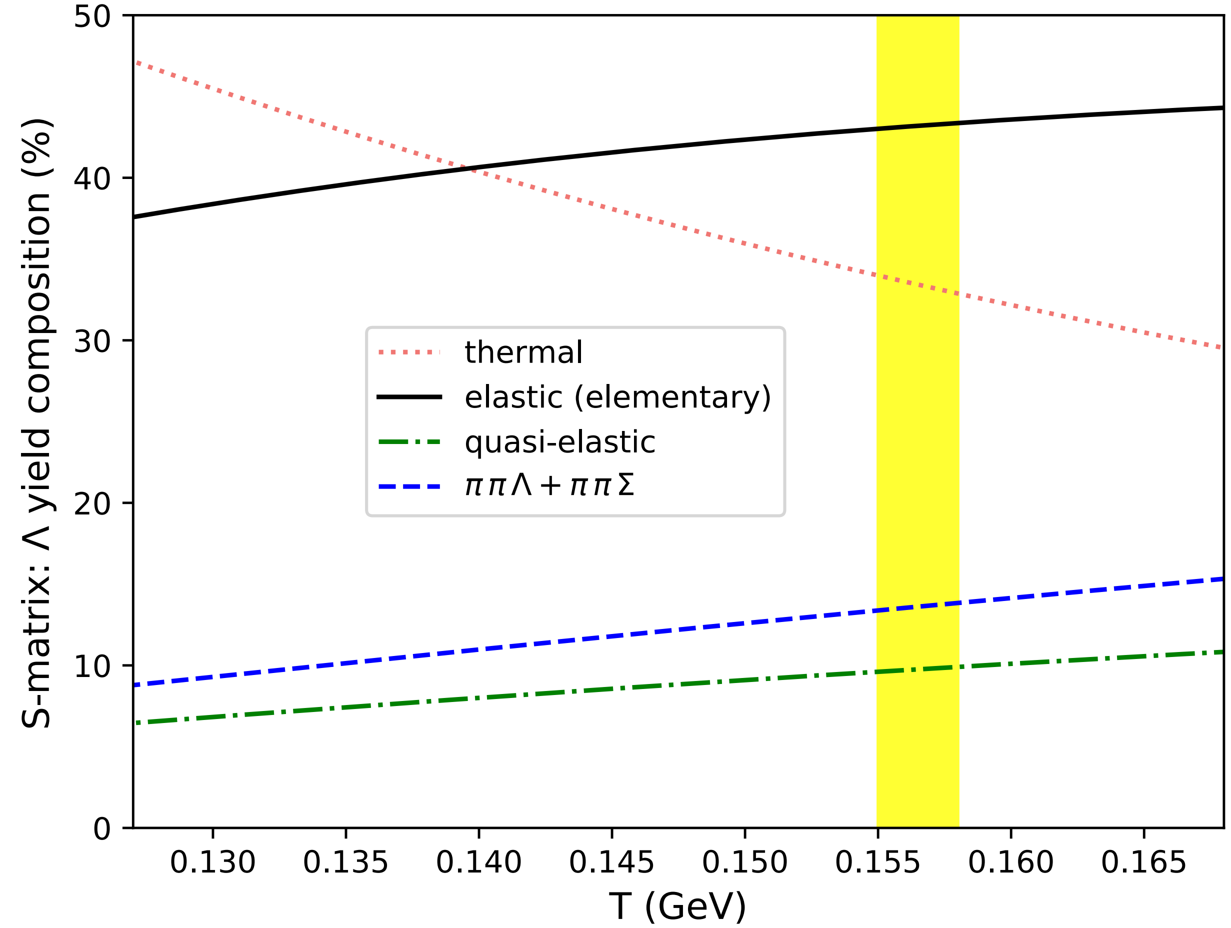
$\bar{k}N, \pi\Lambda, \pi\Sigma, \ldots$

extra hyperon states

beyond PDG

unitarity BGs





SAID GWU

piN phase shifts
pipiN BGs
hyperons

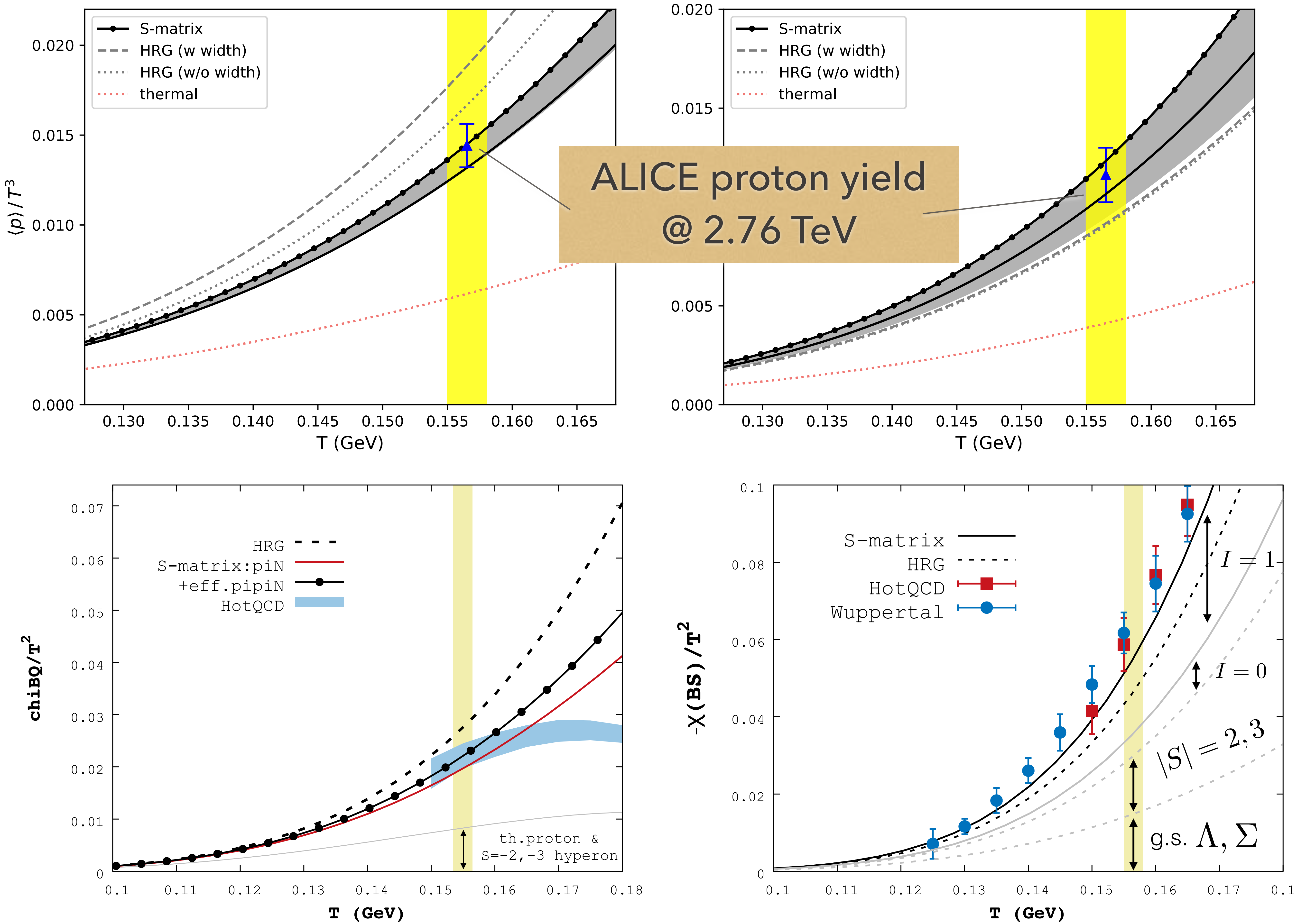
JPAC

Coupled-Channel system:

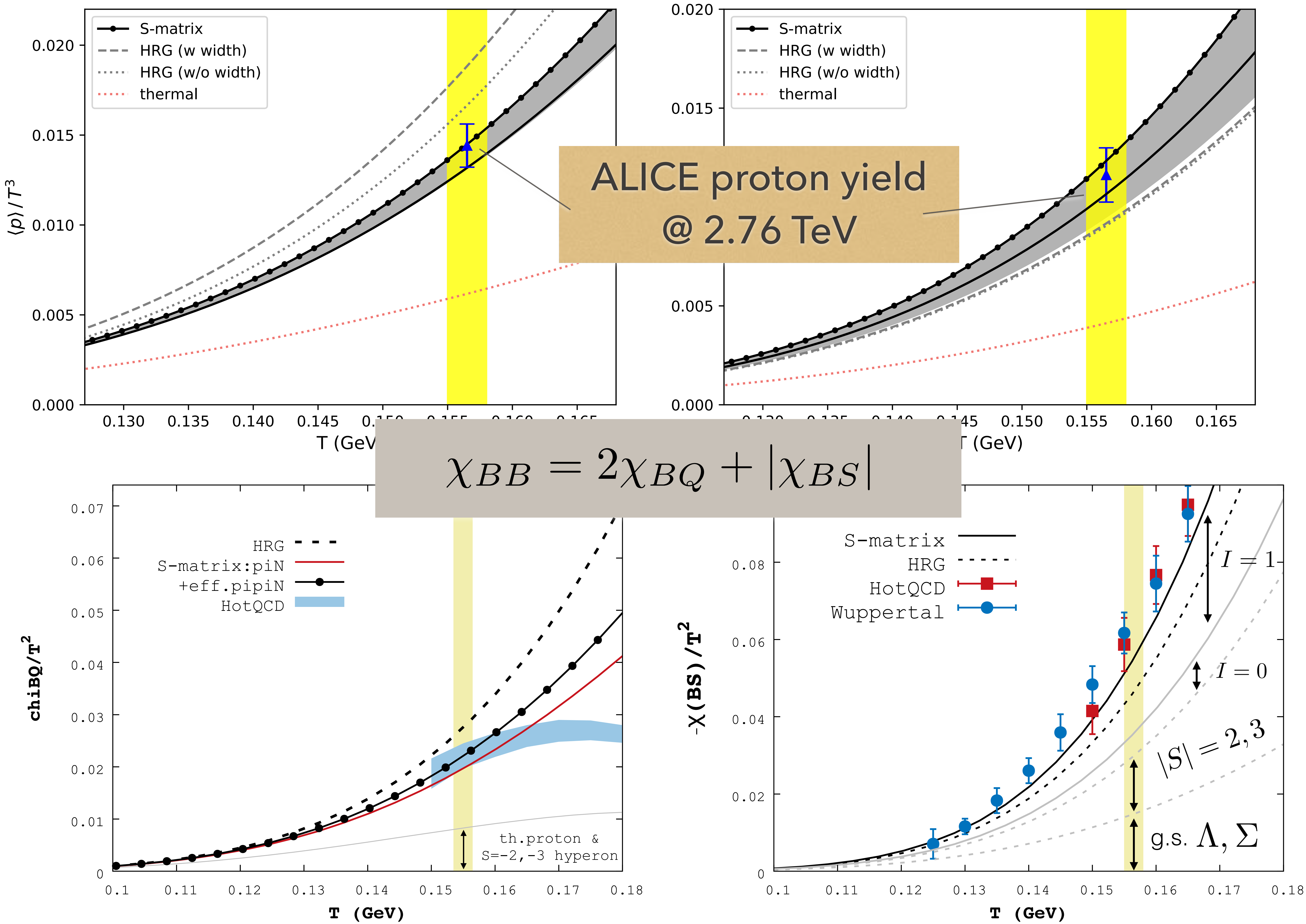
$$\bar{k}N, \pi\Lambda, \pi\Sigma, ...$$

extra hyperon states
beyond PDG
unitarity BGs

consistent treatment of res and non-res. int.



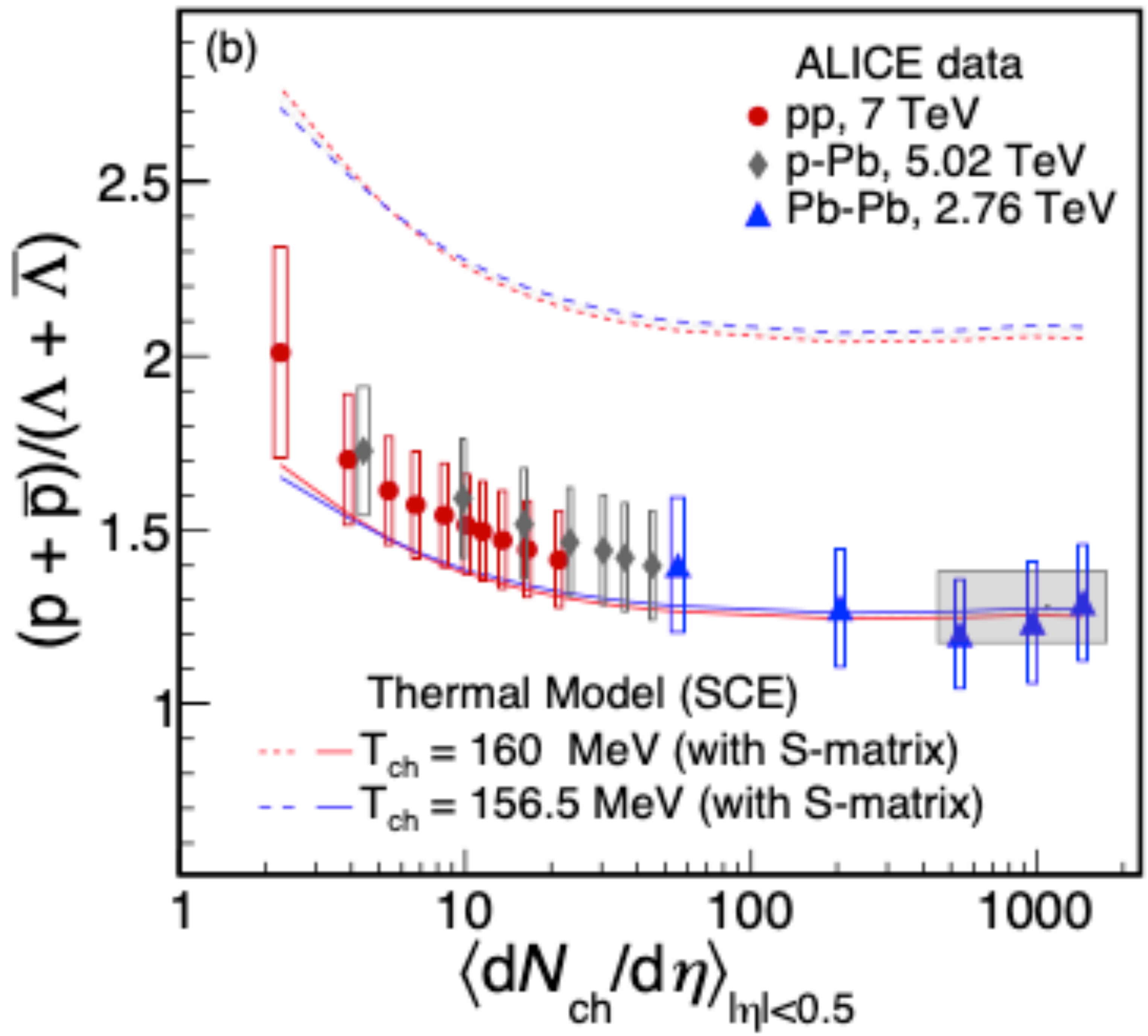
S-matrix
HRG (w width)
HRG (w/o width)
thermal
⟨p⟩/T^3
T (GeV)
ALICE proton yield
@ 2.76 TeV
HRG
S-matrix:piN
+eff.pipiN
HotQCD
chiBQ/T^2
th.proton &
S=-2,-3 hyperon
T (GeV)
S-matrix
HRG
HotQCD
Wuppertal
-χ(BS)/T^2
I = 1
I = 0
|S| = 2, 3
g.s. Λ, Σ
T (GeV)



S-matrix
HRG (w width)
HRG (w/o width)
thermal
⟨p⟩ / T^3
0.000
0.005
0.010
0.015
0.020
0.130
0.135
0.140
0.145
0.165
T (GeV)
ALICE proton yield
@ 2.76 TeV
0.150
0.155
0.160
$\chi_{BB} = 2\chi_{BQ} + |\chi_{BS}|$
HRG
S-matrix:piN
+eff.pipiN
HotQCD
chiBQ/T^2
th.proton &
S=-2,-3 hyperon
T (GeV)
HRG
HotQCD
Wuppertal
-χ(BS)/T^2
$I = 1$
$I = 0$
$|S| = 2, 3$
g.s. Λ, Σ

less protons

more lambdas

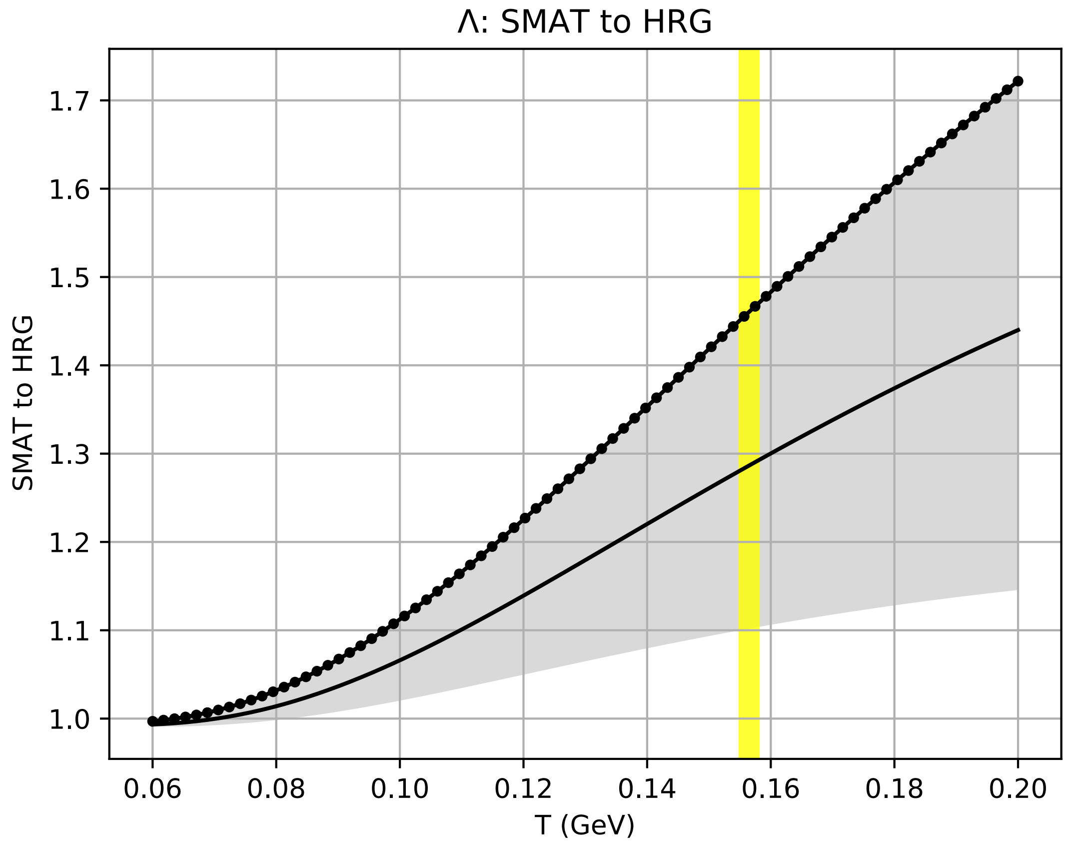


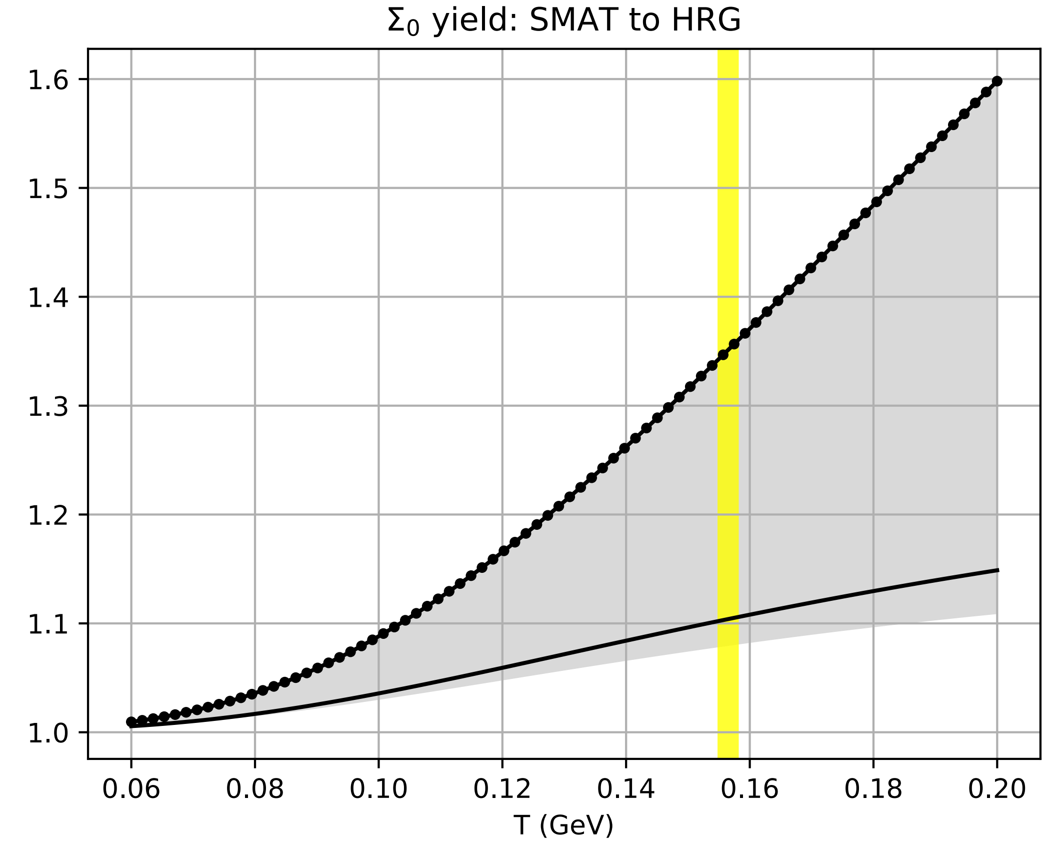
hrg

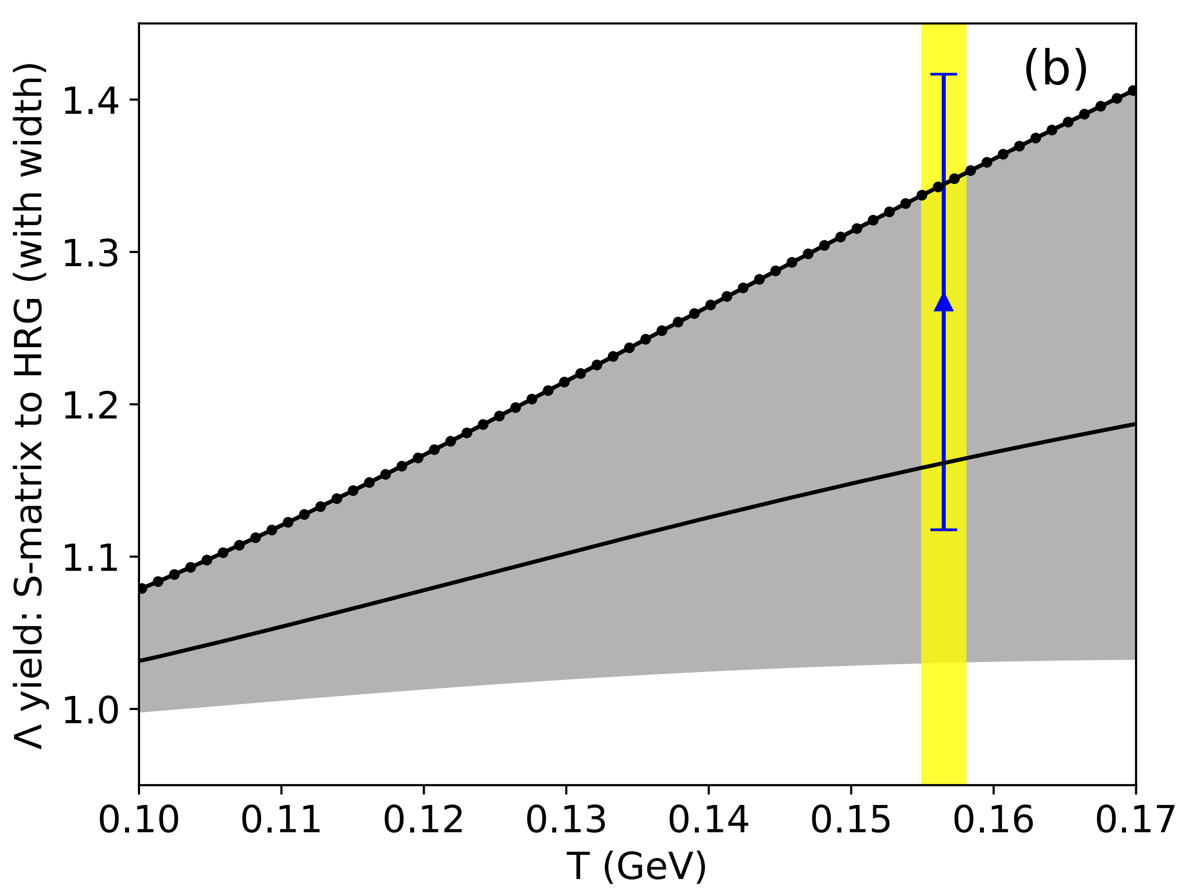
smat

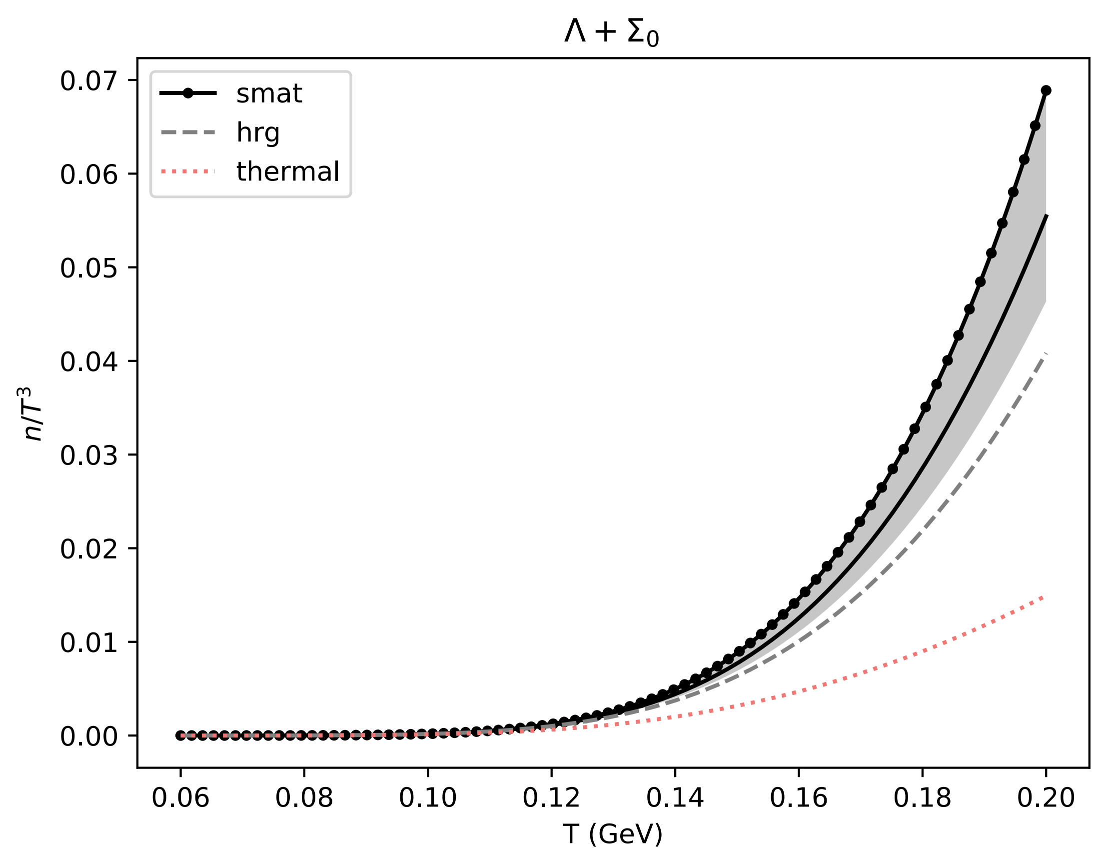
Phys. Rev. C 103, 014904 (2021).

Phys. Lett. B 792, 304 (2019).


Λ: SMAT to HRG
SMAT to HRG
T (GeV)


Σ_0 yield: SMAT to HRG
T (GeV)


(b)
Λ yield: S-matrix to HRG (with width)
T (GeV)


$\Lambda + \Sigma_0$
smat
hrg
thermal
n/T^3
T (GeV)

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187, 345 (1969).

R. Venugopalan and M. Prakash,
Nucl. Phys. A546, 718 (1992).

(study notes)

PML, EPJC **77** no.8 533 (2017)

PML PRD **102**, 034038 (2020)

HADRON RESONANCE GAS MODEL

- Confinement

physical quantities → hadronic states representation

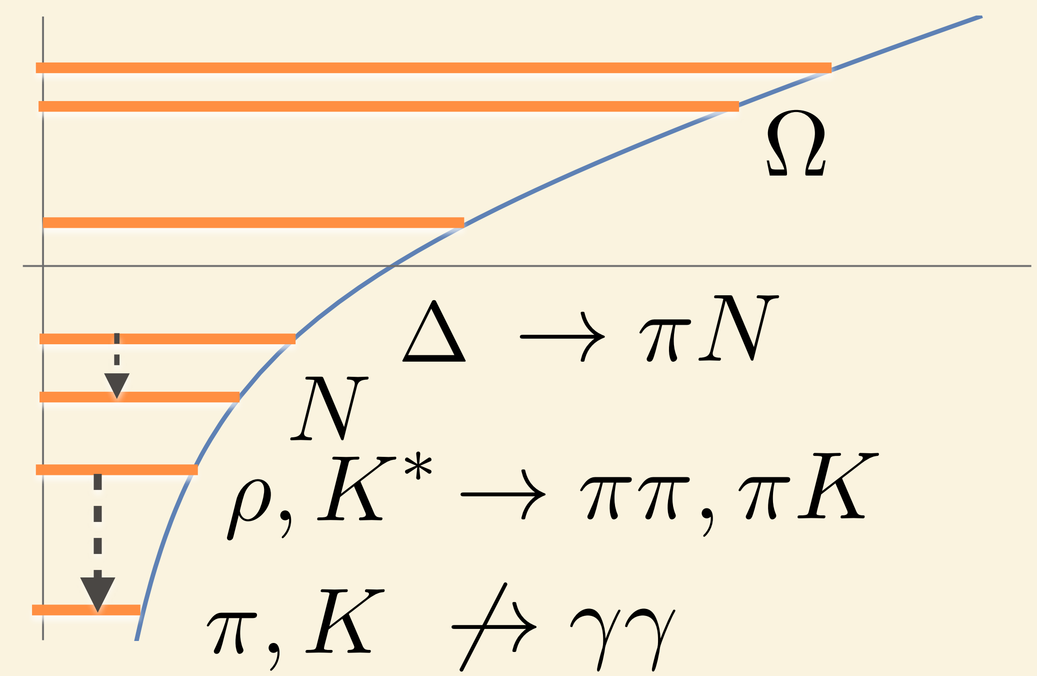
$$Z = \sum_{\alpha=B,M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

HADRON RESON
MODEL

- Confinement

physical quantities

QCD spectrum

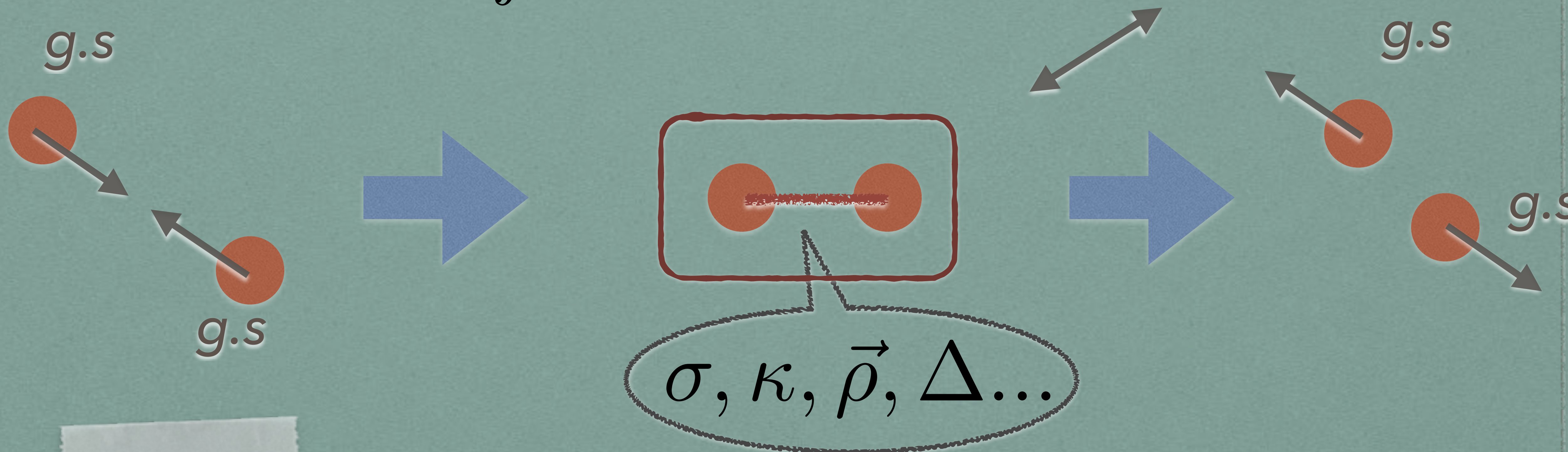


$$Z = \sum_{\alpha=B,M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

confinement +
spontaneous chiral symmetry breaking

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

$$\Delta \ln Z = \int dE \, e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \mathrm{tr} \, (\delta_E) \, .$$



+ repulsions

PWA

X

S-matrix thermo.

$$\delta \longrightarrow \mathcal{Q}(M) \equiv \frac{1}{2} \, \mathrm{Im} \, (\mathrm{tr} \ln \mathrm{S})$$

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE \, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr}\left\{ S_E^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S_E \right\}_c$$

QM *N-body* *QFT*

$$\operatorname{tr}\{...\} \iff \int d^3q \, \langle q| \ldots |q\rangle \longrightarrow \int (d\,k)\, \langle k_1 k_2| \ \ldots \ |k_1 k_2\rangle$$

Fock Space Expansion

$$\int (dk)\,(\cdots) \to \int \frac{d^3p_1}{(2\pi)^3}\frac{1}{2E_1}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2}(\cdots)$$

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \mathrm{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

completeness assumption: via asymptotic states

$$b_{\pi\pi}\xi_\pi^2 + b_{\pi K}\xi_\pi\xi_K + b_{\pi N}\xi_\pi\xi_N + b_{\pi\eta}\xi_\pi\xi_\eta + b_{K\bar{K}}\xi_K\xi_{\bar{K}} + \ldots$$

$$b_{\pi N} = 2 \times b_{\pi N}^{I=1/2} + 4 \times b_{\pi N}^{I=3/2}$$

orbital L: S, P, D, F, etc..

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

thermo-statistical ***dynamical***

$$\Delta \ln Z = \int dE\, e^{-\beta E} \frac{1}{4\pi i} \operatorname{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

Schleiching Notes

$a_S = 20\, fm$

$r \approx 0.0727$ LHC

$r \approx 0.36$ HADES

$r \approx 1.92$

$T = 60\, MeV$

$\mu_B = 700, 800\, MeV$

CONVERGENCE?

$$\Delta P = T\xi_N^2 \int \frac{d^3P}{(2\pi)^3}\frac{d\epsilon}{2\pi} e^{-\beta(\frac{P^2}{2m_{\rm tot}}+\epsilon)} \times 2\frac{\partial}{\partial\epsilon}(-q a_S)$$

$$= -a_S \frac{2\pi}{m_{\rm red}} \xi_N^2 n_N^2$$

$$P^{(0)} = n_N T \xi_N \qquad n_N = (\lambda_T^{-1})^3 = (\frac{m_N T}{2\pi})^{3/2}$$

$$r = \frac{\Delta P}{P^{(0)}} = -2 \times (a_S/\lambda_T) \times \xi_N$$

$$= -2a_S\,(\frac{m_N T}{2\pi})^{1/2}\, e^{(\mu - m_N)/T}$$

adding other channels

$$a_S \to a_S + 3\,a_P\, m_{\rm red} T + \ldots$$

CONVERGENCE?

zero T, finite density

$$\Delta P = T\xi_N^2 \int \frac{d^3P}{(2\pi)^3}\frac{d\epsilon}{2\pi} e^{-\beta(\frac{P^2}{2m_{\text{tot}}}+\epsilon)} \times 2\frac{\partial}{\partial\epsilon}(-qa_S)$$

$$= -a_S\frac{2\pi}{m_{\text{red}}}\xi_N^2 n_N^2 \longrightarrow -\frac{4\pi a_S}{2m_{\text{red}}}n_N^2$$

$$P^{(0)} = n_N T\xi_N \qquad n_N = (\lambda_T^{-1})^3 = (\frac{m_N T}{2\pi})^{3/2}$$

$$P^{(0)} \longrightarrow \frac{1}{6\pi^2}\frac{k_F^5}{5m_N} \qquad n_N \longrightarrow \frac{k_F^3}{6\pi^2}$$

$$r = \frac{\Delta P}{P^{(0)}} = -2\times(a_S/\lambda_T)\times\xi_N$$

$$= -2a_S(\frac{m_N T}{2\pi})^{1/2} e^{(\mu-m_N)/T}$$

$$\longrightarrow -\frac{10}{3\pi}\times(k_F a_S)$$

CONVERGENCE?

PART of the 3rd virial coefficients

$$\Delta P = T\xi_N^2 \int \frac{d^3P}{(2\pi)^3}\frac{d\epsilon}{2\pi} e^{-\beta(\frac{P^2}{2m_{\rm tot}}+\epsilon}$$

$$= -a_S \frac{2\pi}{m_{\rm red}} \xi_N^2 n_N^2$$

$$+ \propto \xi_N^3 n_N^3 (\frac{a_S}{m_{\rm red}})^2 \frac{1}{T} + \ldots$$

$$P^{(0)} = n_N T \xi_N \qquad n_N = (\lambda_T^{-1})^3 = (\frac{m_N T}{2\pi})^{3/2}$$

$$r = \frac{\Delta P}{P^{(0)}} = -2 \times (a_S/\lambda_T) \times \xi_N$$

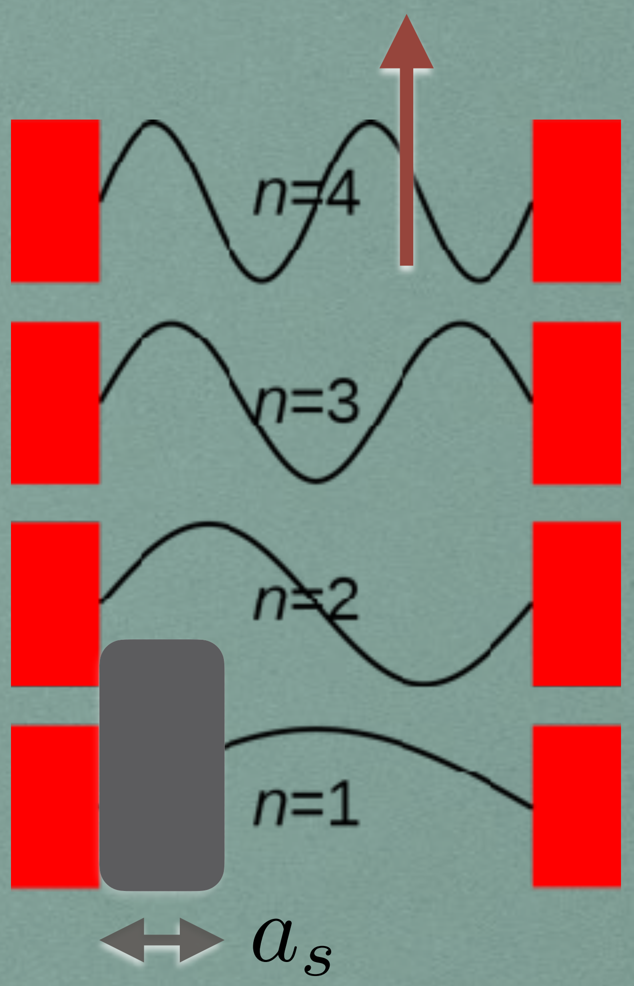
$$= -2a_S\,(\frac{m_N T}{2\pi})^{1/2}\, e^{(\mu - m_N)/T}$$

REPULSION AND RESONANCES

PHASE SHIFT AND DENSITY OF STATES

particle in a box

$$\psi \sim \sin(k^{(0)}x) \qquad k^{(0)} = \frac{n\pi}{L}$$



in the presence of a scattering potential

$$\psi \sim \sin(kx + \delta(k))$$

density of states

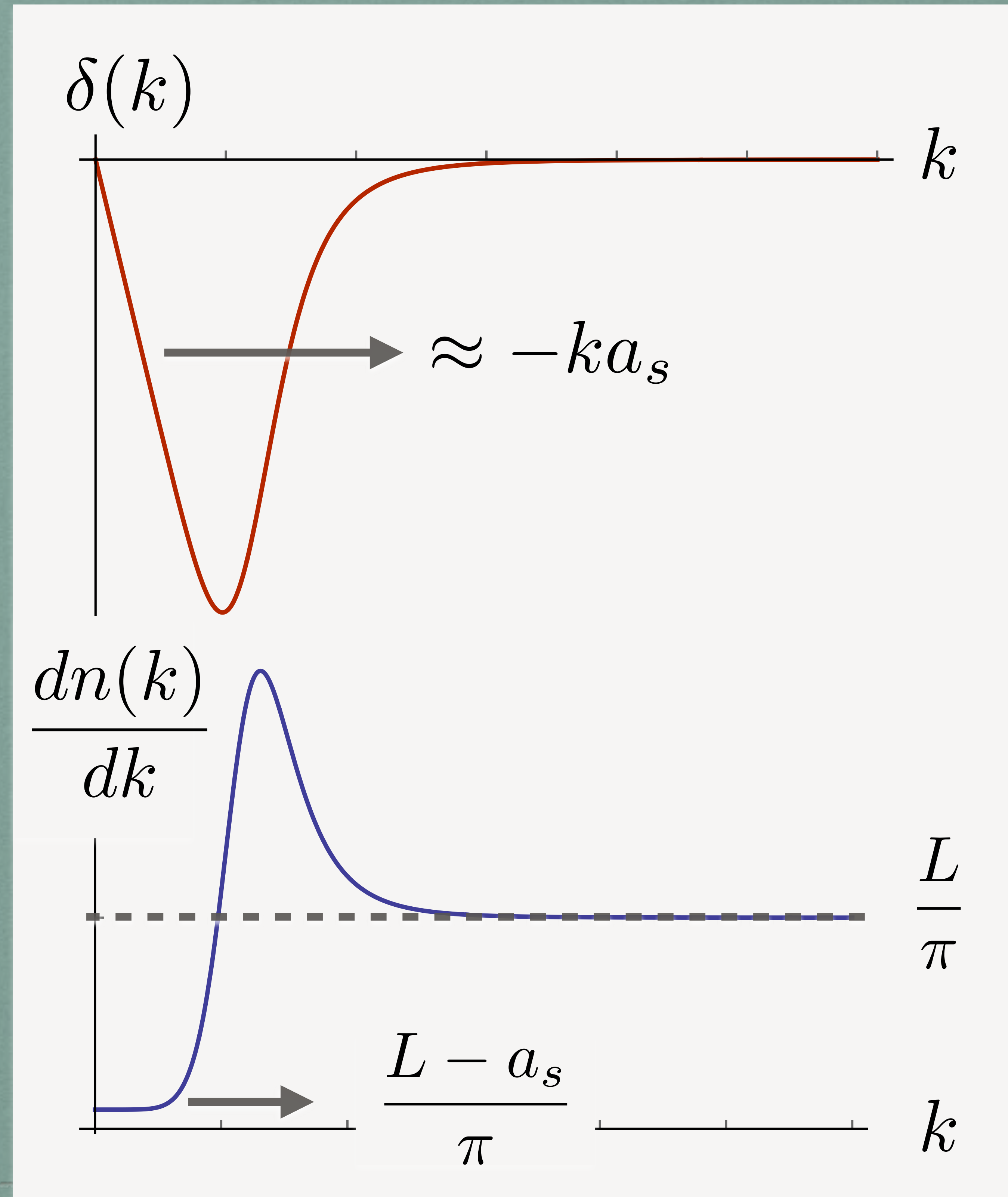
$$kL + \delta(k) = n\pi \quad \rightarrow \quad \frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

PHASE SHIFT AND DENSITY OF STATES

$$\frac{dn(k)}{dk} = \frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}$$

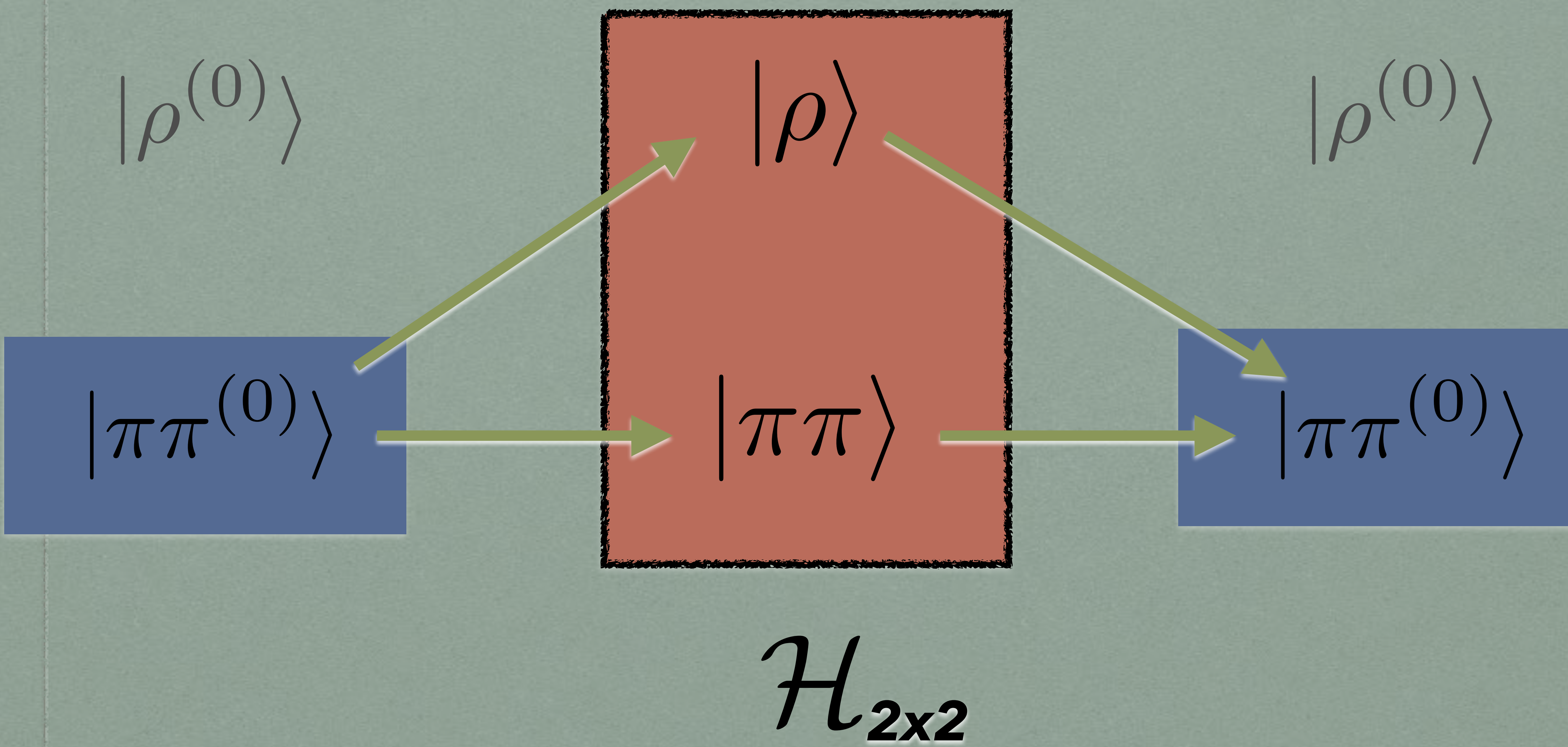
change in d.o.s. due to int.

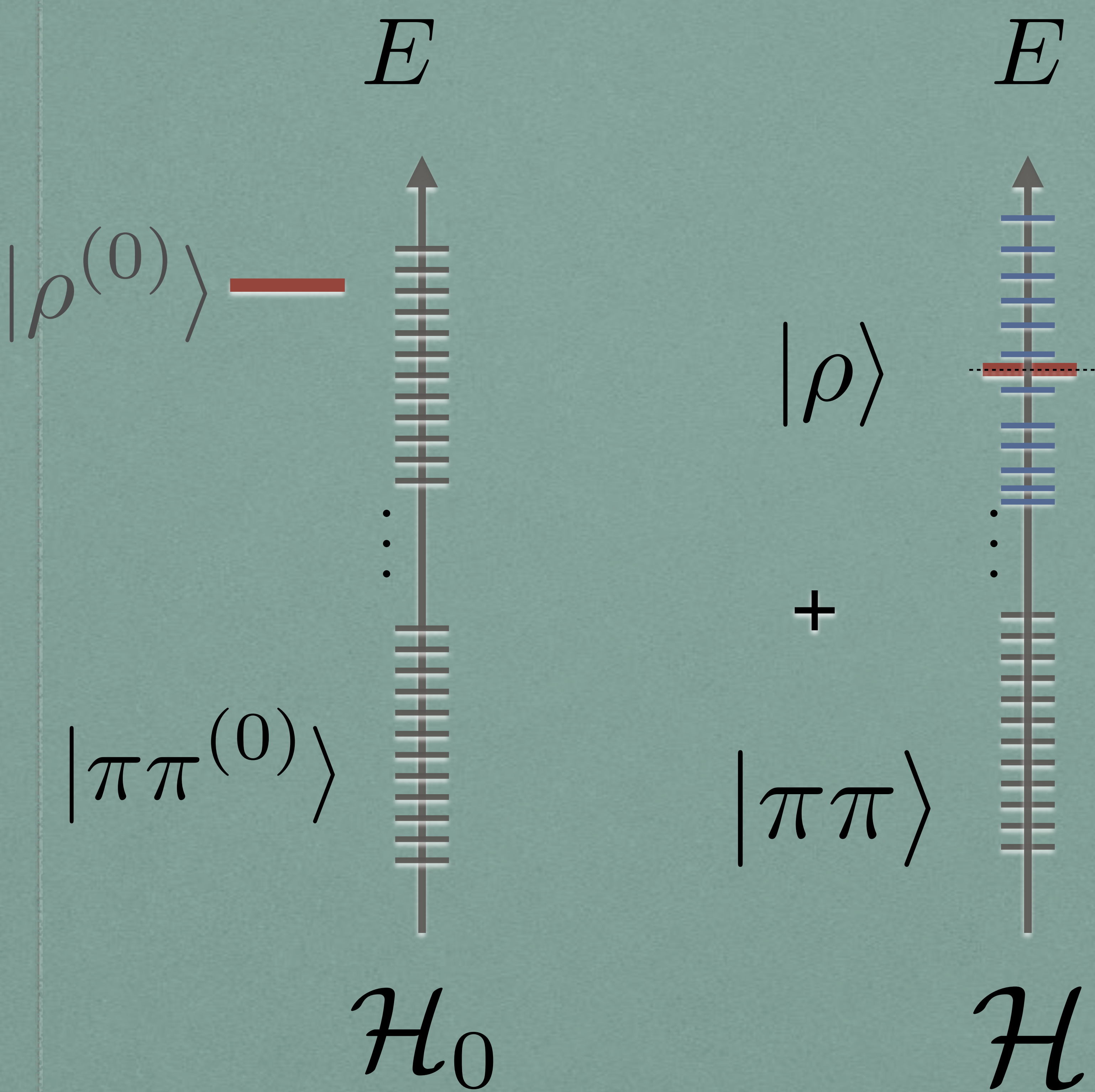
Effect of repulsive interaction: pushing states from low k to high k

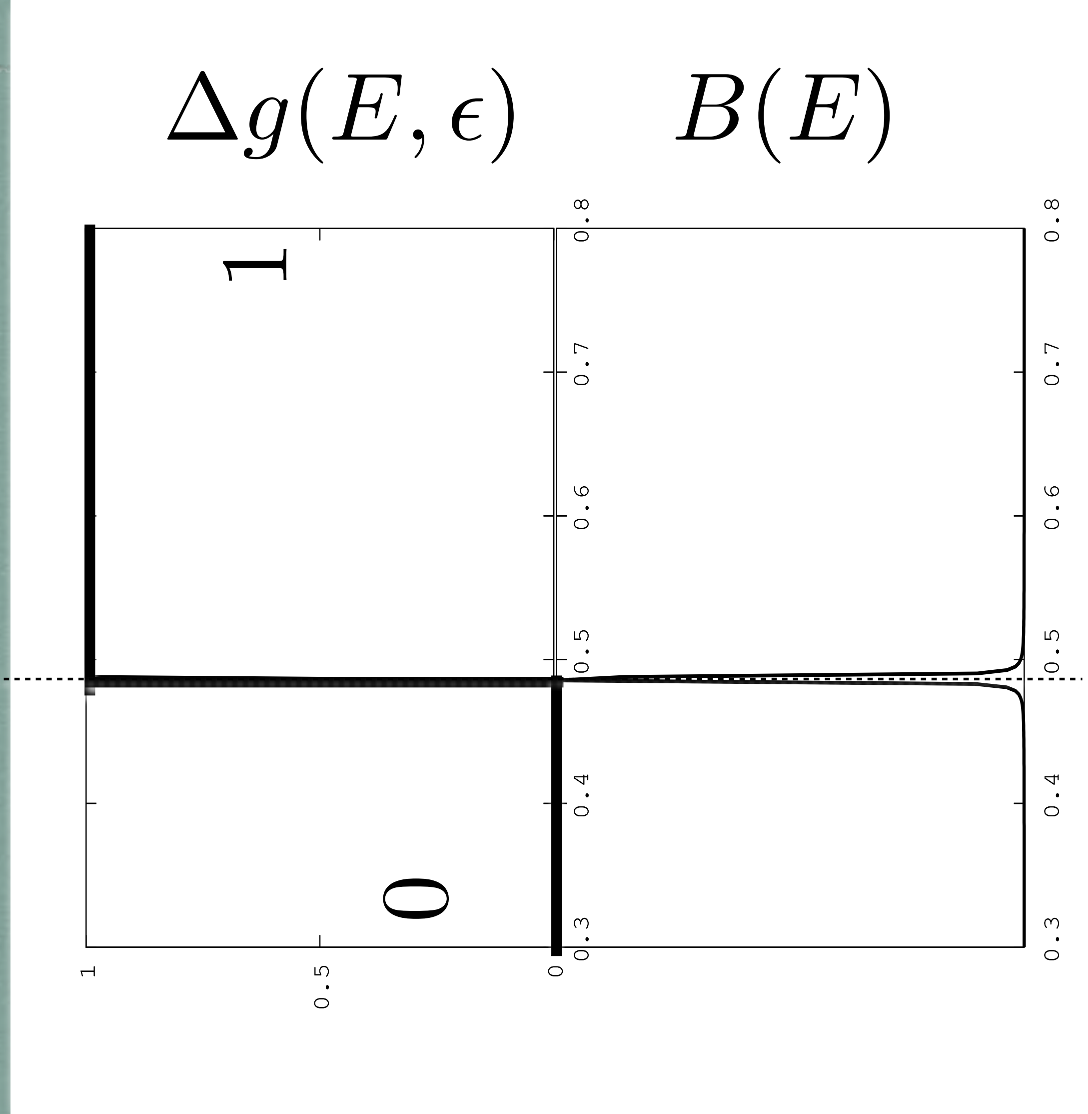


phase shift and d.o.s. (schematics)

SCATTERING THEORY VS HAMILTONIAN (LEE MODEL)

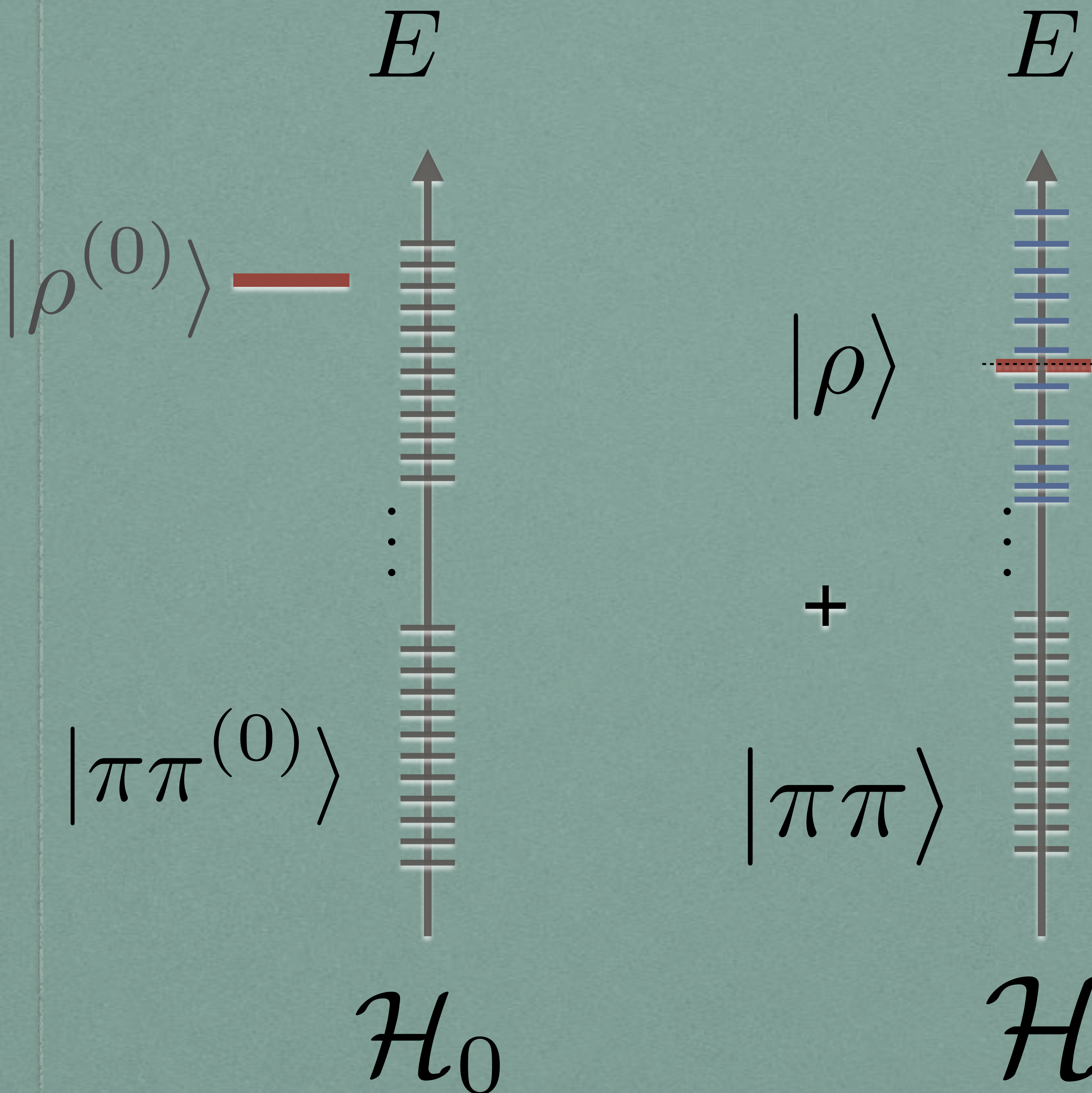


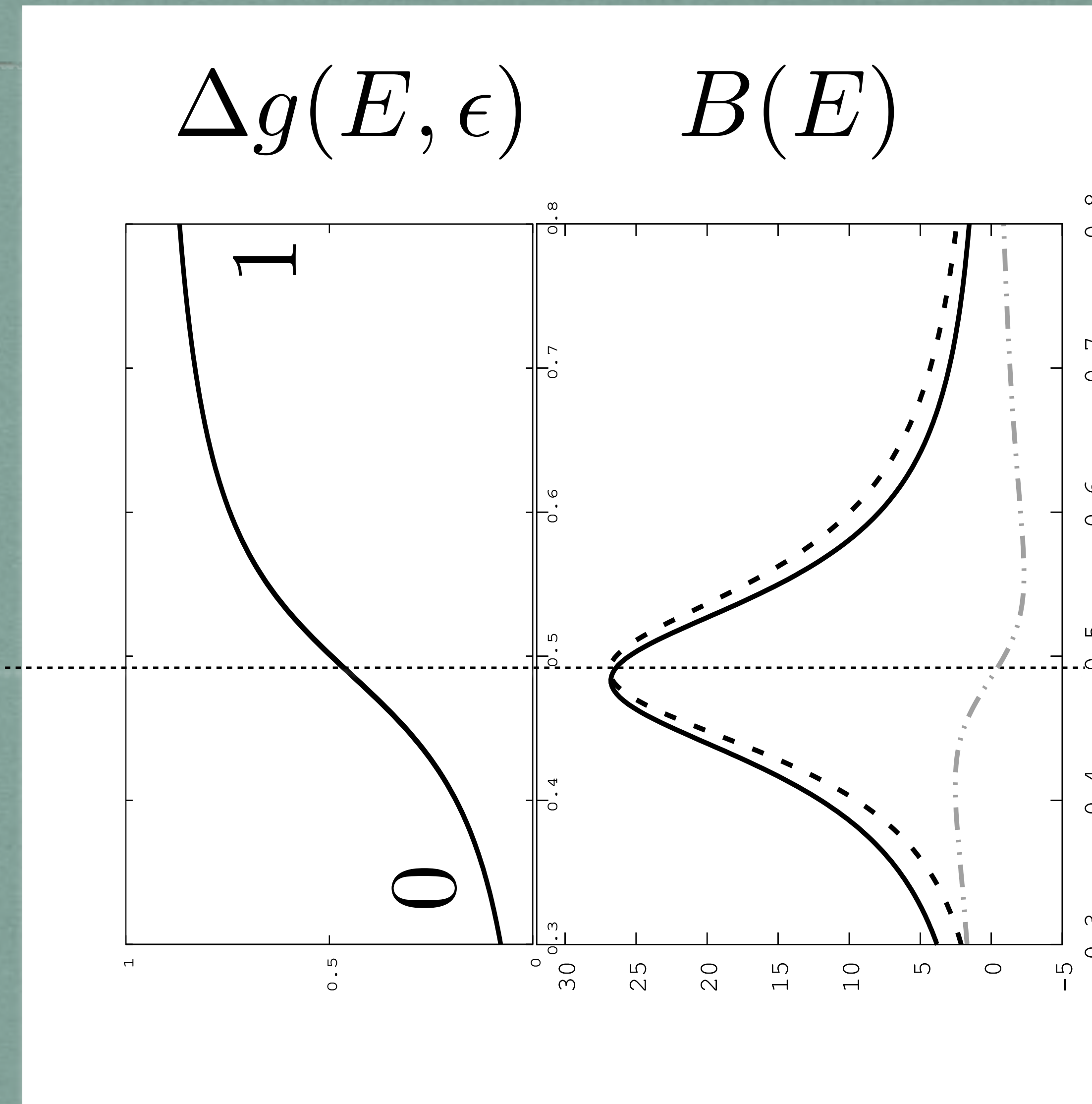




$$g(E,\epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE} \Delta g(E,\epsilon)$$





$$g(E,\epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE} \Delta g(E,\epsilon)$$

$$= A_\rho + \Delta A_{\pi\pi}$$

$\mathrm{Tr}\, e^{-\beta \mathcal{H}_0}$ *vs* $\mathrm{Tr}\, e^{-\beta \mathcal{H}}$

PML, Francesco Giocosa EPJC **79** (2019)

W. Weinhold & B. Friman, Phys. Lett. B 433, 236 (1998).

PHYSICS OF B

$$\delta = -\mathrm{Im}\,\mathrm{Tr}\ln G_\rho^{-1}$$

physical interpretation:

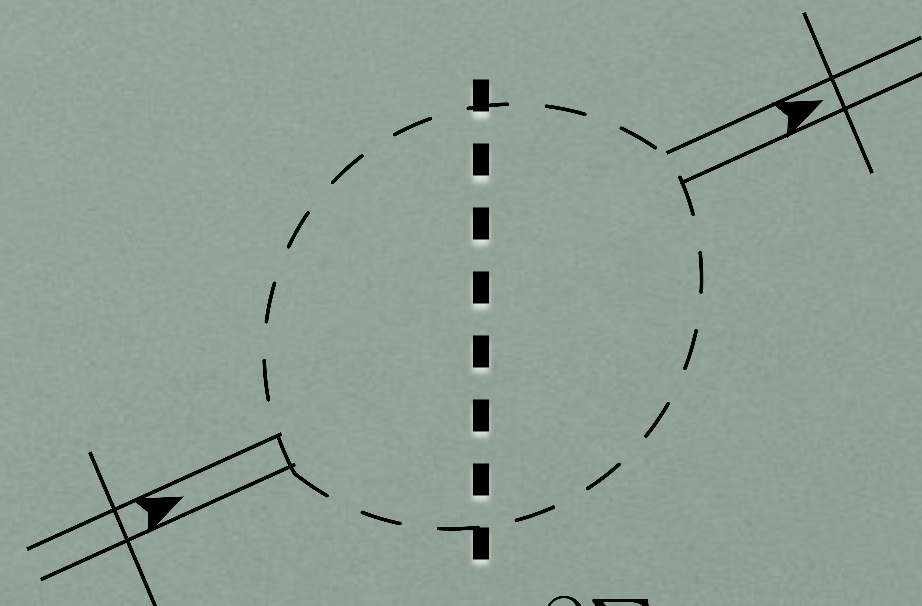
contribution from correlated pi pi pair

$$B = 2\,\frac{\partial}{\partial E}\delta$$

$$= -2\,\mathrm{Im}\frac{\partial}{\partial E}\ln G_\rho^{-1}$$

$$= -2\,\mathrm{Im}[G_\rho](2E) + 2\,\mathrm{Im}[\frac{\partial \Sigma_\rho}{\partial E}G_\rho]$$

$$= A_\rho(E) + \Delta A_{\pi\pi}$$



$$\frac{\partial \Sigma_\rho}{\partial E}$$

$$-\frac{\partial}{\partial E}\int d\phi_E\, T_{\mathrm{re}}$$ pipi -> pipi

PHYSICS OF B

to rho or not to rho?
that's OUT of the question!

resonance's picture:

$$B(E) = A_\rho(E) + \Delta A_{\pi\pi}$$

rho

scattering picture:

$$B_1 = \frac{\partial}{\partial E} \mathrm{Tr}\, \hat{t}_{\mathrm{re}}$$

$$B_2 = \frac{1}{2} \mathrm{Im}\, \mathrm{Tr}\, \hat{t}^\dagger \overleftrightarrow{\partial_E} \hat{t}$$

pipi -> pipi

$\delta = -\mathrm{Im\,T}$

on:

n

air

$B = 2\,\frac{\partial}{\partial E}\delta$

$= -2\,\mathrm{Im}\frac{\partial}{\partial E}$

$= -2\,\mathrm{Im}[G$

$= A_\rho(E) + \Delta A_{\pi\pi}$

$\frac{\partial \Sigma_\rho}{\partial E}$

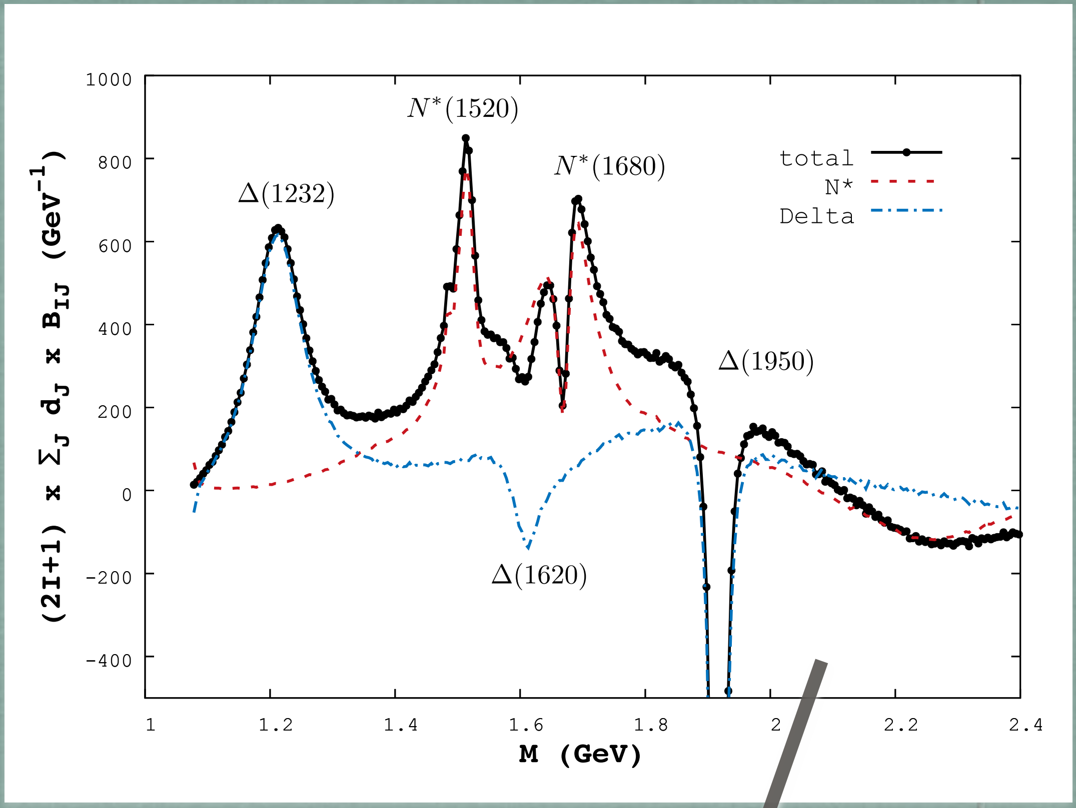
$-\frac{\partial}{\partial E}\int d\phi_E\, T_{\mathrm{re}}$ pipi -> pipi

THE S-MATRIX PROGRAM

step 1: build a model for S-matrix

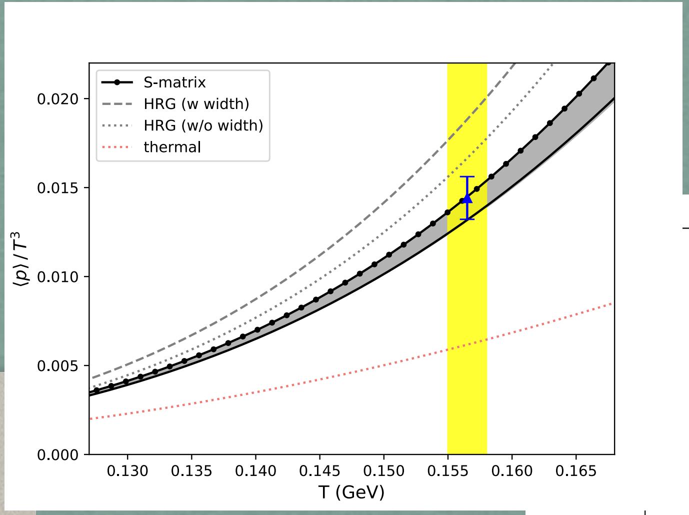
step 2: adjust model parameters to match scattering experiments

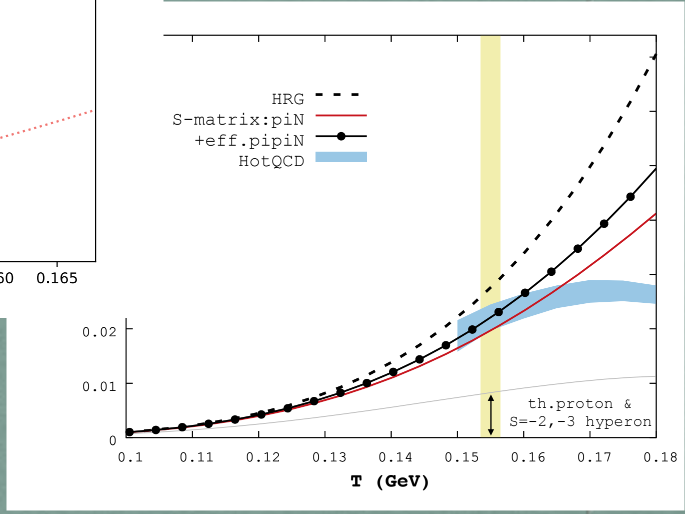
board resonances & thresholds
coupled channel effects
energy dependent branchings



step 3: compute thermodynamics

HICs & LQCD



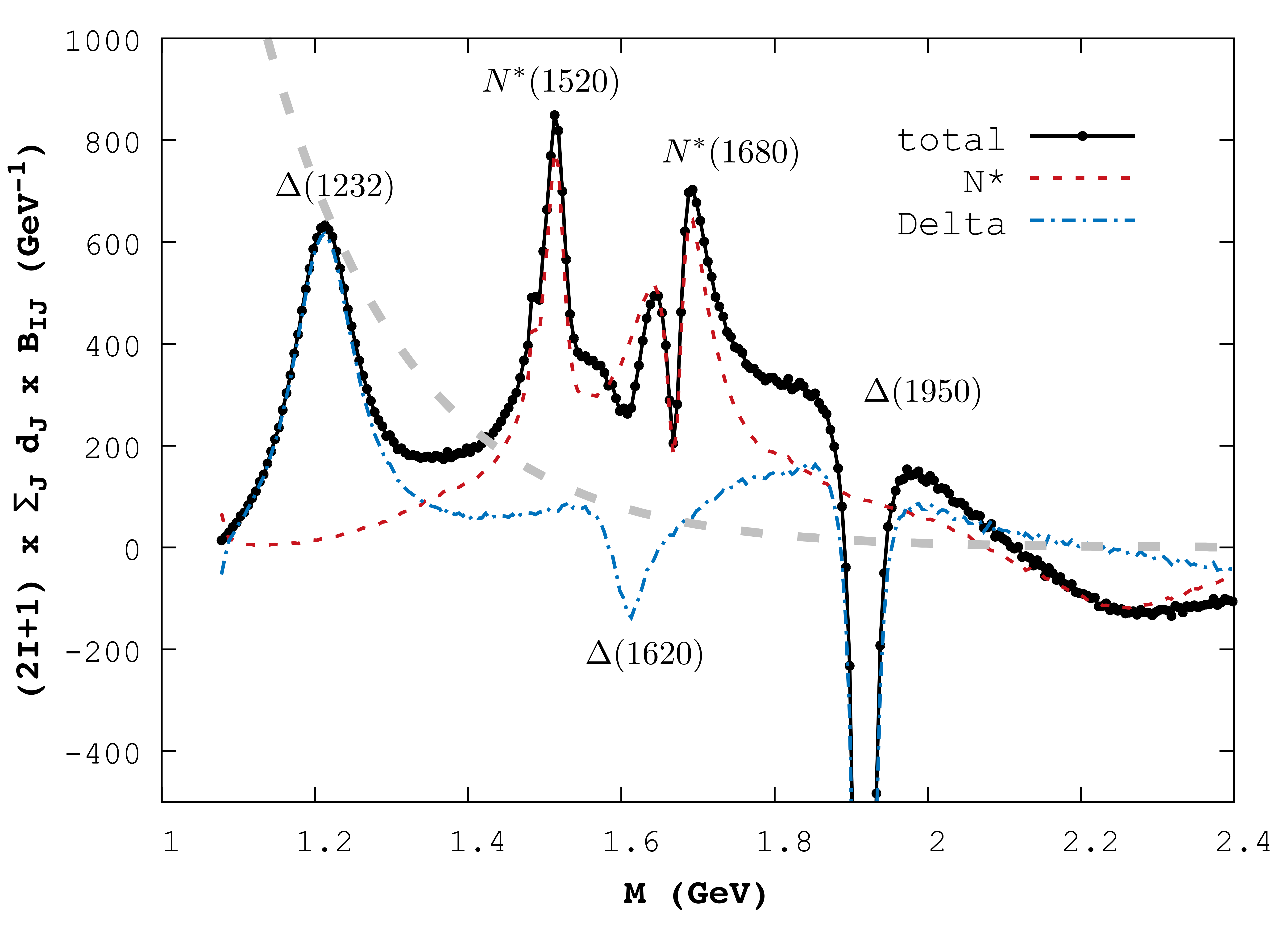


S = -1 HYPERONS COUPLED CHANNEL SYSTEM

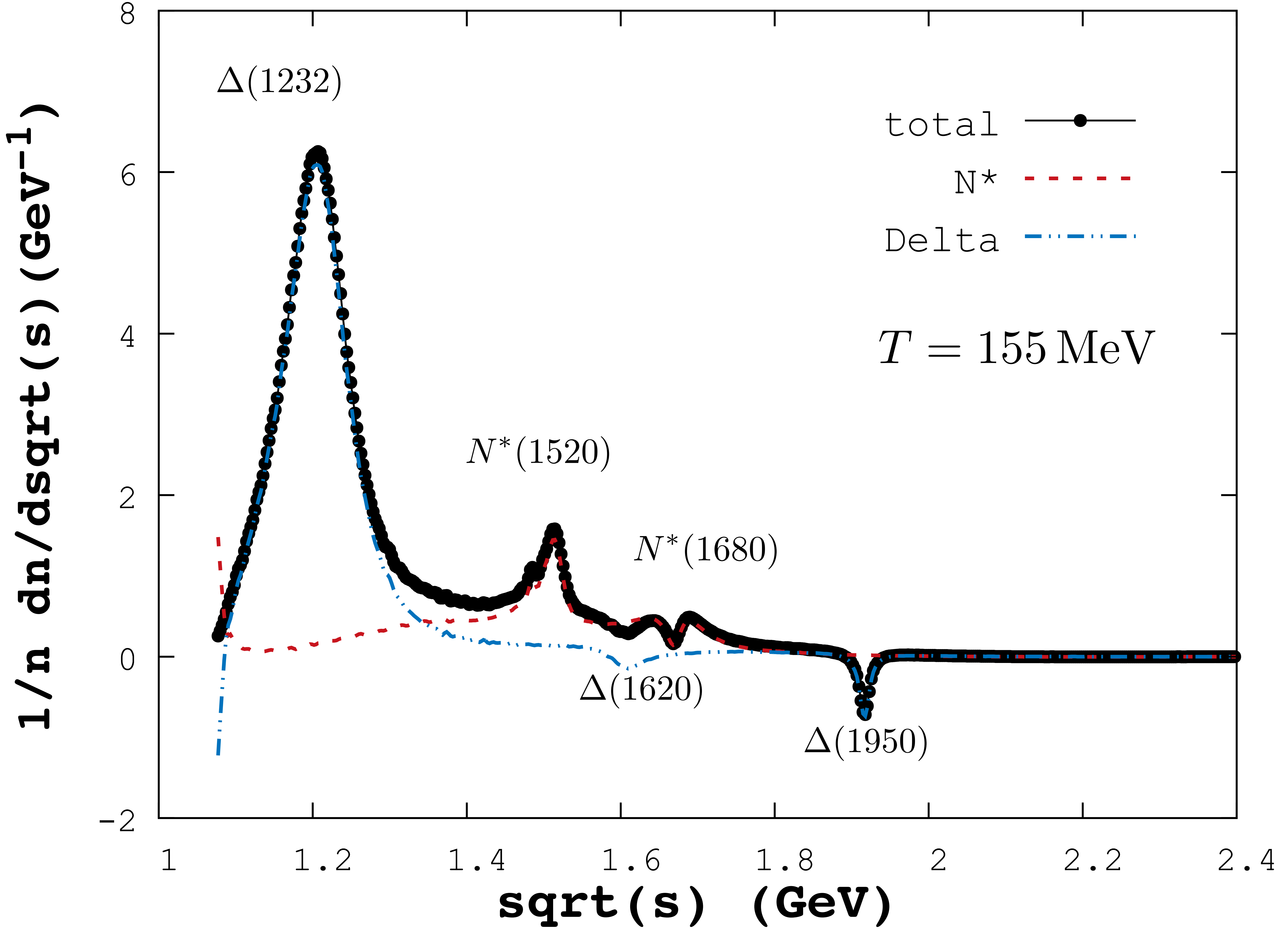
JPAC, PRD **93**, 034029 (2016)

C. Fernandez-Ramirez, PML, and P. Petreczky,
PRC **98**, 044910 (2018)

J. Cleymans, PML, K. Redlich, and N. Sharma
PRC **103**, 014904 (2021)



1000
800
600
400
200
0
-200
-400
(2I+1) x $\sum_J$ d_J x B_{IJ} (GeV^{-1})
1
1.2
1.4
1.6
1.8
2
2.2
2.4
M (GeV)
total
N*
Delta
$\Delta(1232)$
$N^*(1520)$
$N^*(1680)$
$\Delta(1950)$
$\Delta(1620)$



Δ(1232)
N*(1520)
N*(1680)
Δ(1620)
Δ(1950)
total
N*
Delta
T = 155 MeV
8
6
4
2
0
-2
1
1.2
1.4
1.6
1.8
2
2.2
2.4
1/n dn/dsqrt(s) (GeV-1)
sqrt(s) (GeV)

JPAC, PRD **93**, 034029 (2016)

PHASE SHIFT FROM PWA

```
-----------------------------------------------------------------------

Coupled Channels partial wave calculator for KN scattering

by the Joint Physics Analysis Center (JPAC)

Version: September 1, 2015

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-----------------------------------------------------------------------

Authors:

Cesar Fernandez-Ramirez (Jefferson Lab)
Igor V. Danilkin (Jefferson Lab)
Vincent Mathieu (Indiana University)
Adam P. Szczepaniak (Indiana University and Jefferson Lab)

Citation: Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

First version: Cesar Fernandez-Ramirez (Jefferson Lab)
This version: Cesar Fernandez-Ramirez (Jefferson Lab)

Contact: cefera@gmail.com (Cesar Fernandez-Ramirez)

Disclaimers:

1 - This code follows the 'garbage in, garbage out' philosphy. If your
parameters do not make sense, the output will not make sense either.
2 - You can use, share and modify this code under your own responsability.
3 - This code is distributed in the hope that it will be useful,
but WITHOUT ANY WARRANTY; without even the implied warranty of
MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.
4 - No PhD students or postdocs were severely damaged during the development
of this project.
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channel	elastic	channel	quasi-elastic	channel	unitarity
1	$\bar{K}N$	6	$\bar{K}_1^* N$	15	$\pi\pi\Lambda$
2	$\pi\Sigma$	7	$[\bar{K}_3^* N]_-$	16	$\pi\pi\Sigma$
3	$\pi\Lambda$	8	$[\bar{K}_3^* N]_+$		
4	$\eta\Lambda$	9	$[\pi\Sigma(1385)]_-$		
5	$\eta\Sigma$	10	$[\pi\Sigma(1385)]_+$		
		11	$[\bar{K}\Delta(1232)]_-$		
		12	$[\bar{K}\Delta(1232)]_+$		
		13	$[\pi\Lambda(1520)]_-$		
		14	$[\pi\Lambda(1520)]_+$		

elastic scatterings (elementary)

quasi elastic scatterings

unitarity background

STRANGENESS CONTENT IN A HADRON GAS

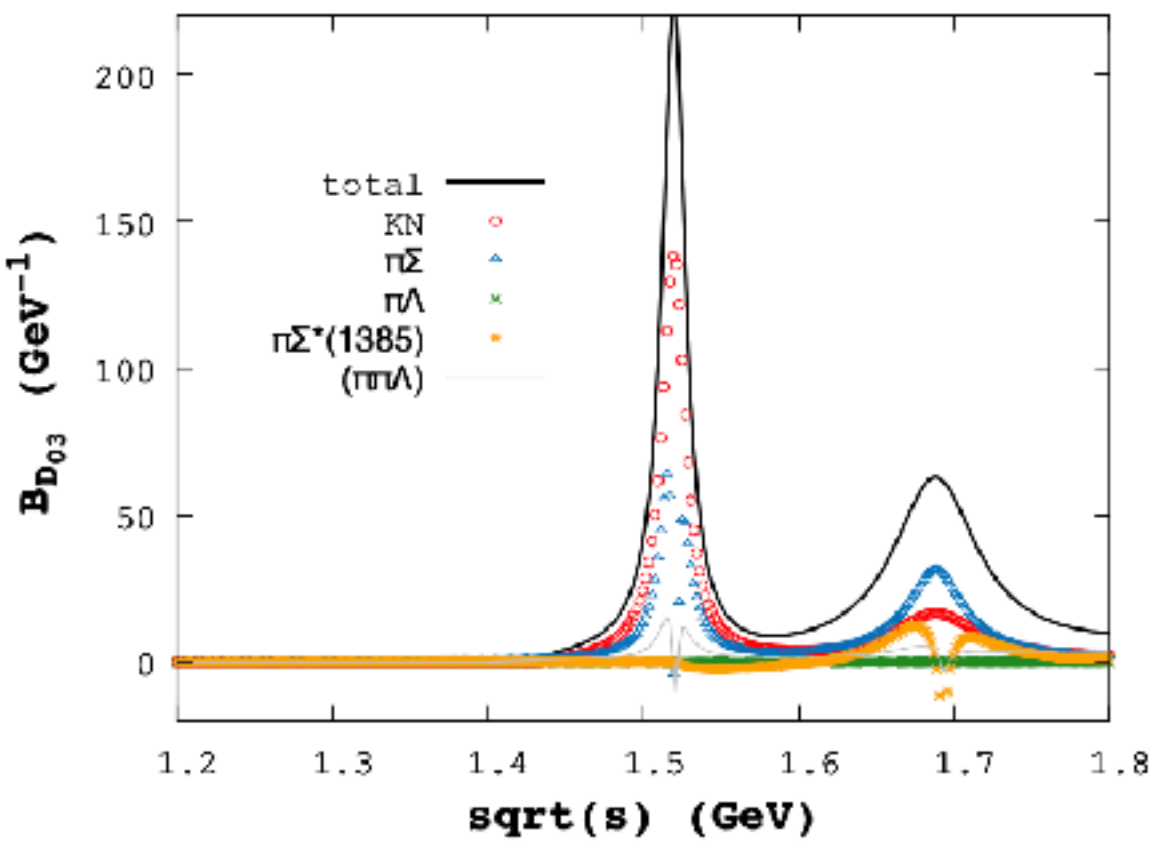
- K-N system requires a coupled channel analysis

$|\bar{K}N\rangle, |\pi\Sigma\rangle, |\pi\Lambda\rangle, |\eta\Lambda\rangle, \ldots$ *16 basis states*

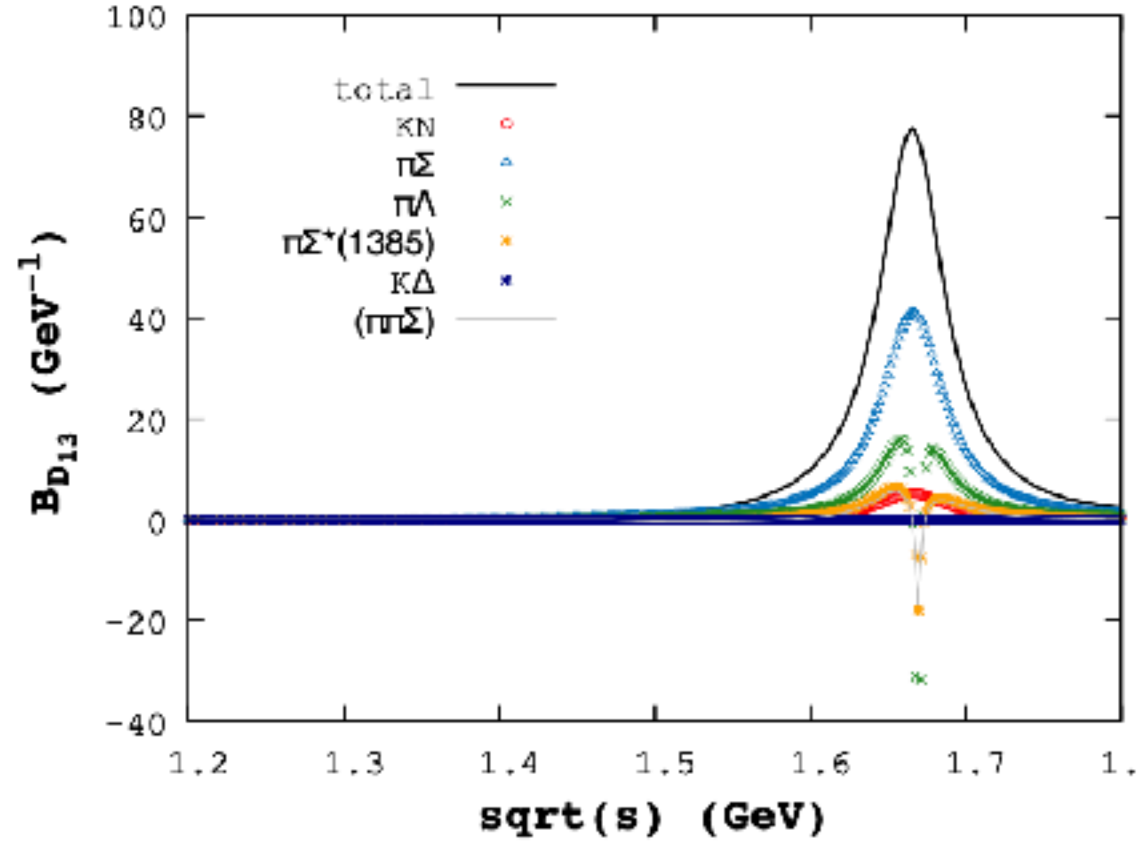
$$\mathcal{Q}(M) \equiv \frac{1}{2}\,\mathrm{Im}\,(\mathrm{tr}\ln\,\mathrm{S}) = \frac{1}{2}\,\mathrm{Im}\,(\ln\,\det\,[\mathrm{S}]) = \delta_{\bar{K}N} + \delta_{\pi\Sigma} + \delta_{\pi\Lambda} + \ldots$$

Compute det S for each sqrt(s) for each channel isospin conserving

1520, 1690



1670



Λ(1520) 3/2⁻ $I(J^P) = 0(\frac{3}{2}^-)$

Mass $m = 1519.5 \pm 1.0$ MeV [d]
Full width $\Gamma = 15.6 \pm 1.0$ MeV [d]
$p_{beam} = 0.39$ GeV/c $4\pi\lambda\!\!\!^-{}^2 = 82.8$ mb

Λ(1520) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	$45 \pm 1\%$	243
$\Sigma\pi$	$42 \pm 1\%$	268
$\Lambda\pi\pi$	$10 \pm 1\%$	259
$\Sigma\pi\pi$	$0.9 \pm 0.1\%$	169
$\Lambda\gamma$	$0.85 \pm 0.15\%$	350

Λ(1690) 3/2⁻ $I(J^P) = 0(\frac{3}{2}^-)$

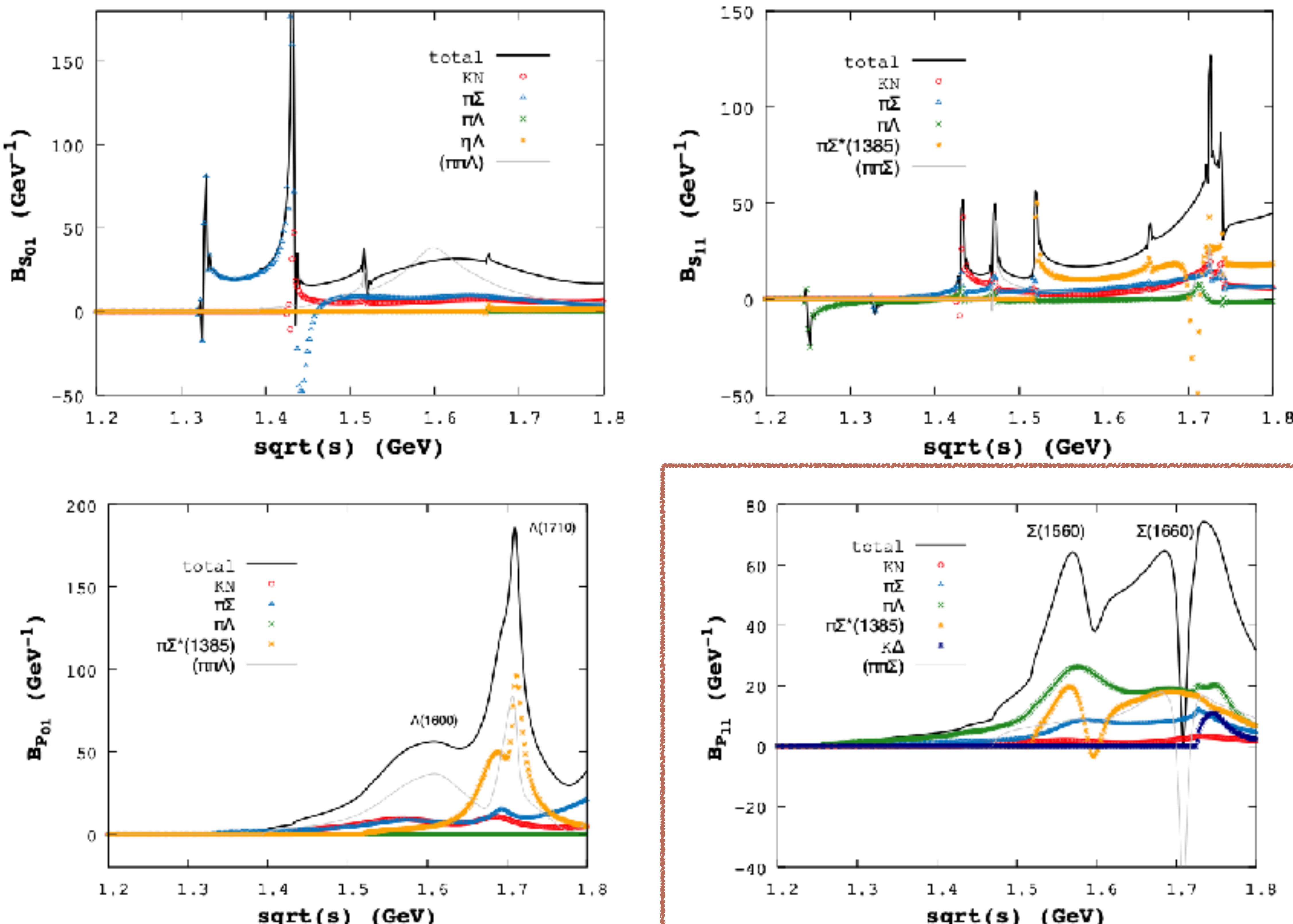
Mass $m = 1685$ to 1695 (≈ 1690) MeV
Full width $\Gamma = 50$ to 70 (≈ 60) MeV
$p_{beam} = 0.78$ GeV/c $4\pi\lambda\!\!\!^-{}^2 = 26.1$ mb

Λ(1690) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	20–30 %	433
$\Sigma\pi$	20–40 %	410
$\Lambda\pi\pi$	$\sim$ 25 %	419
$\Sigma\pi\pi$	$\sim$ 20 %	358

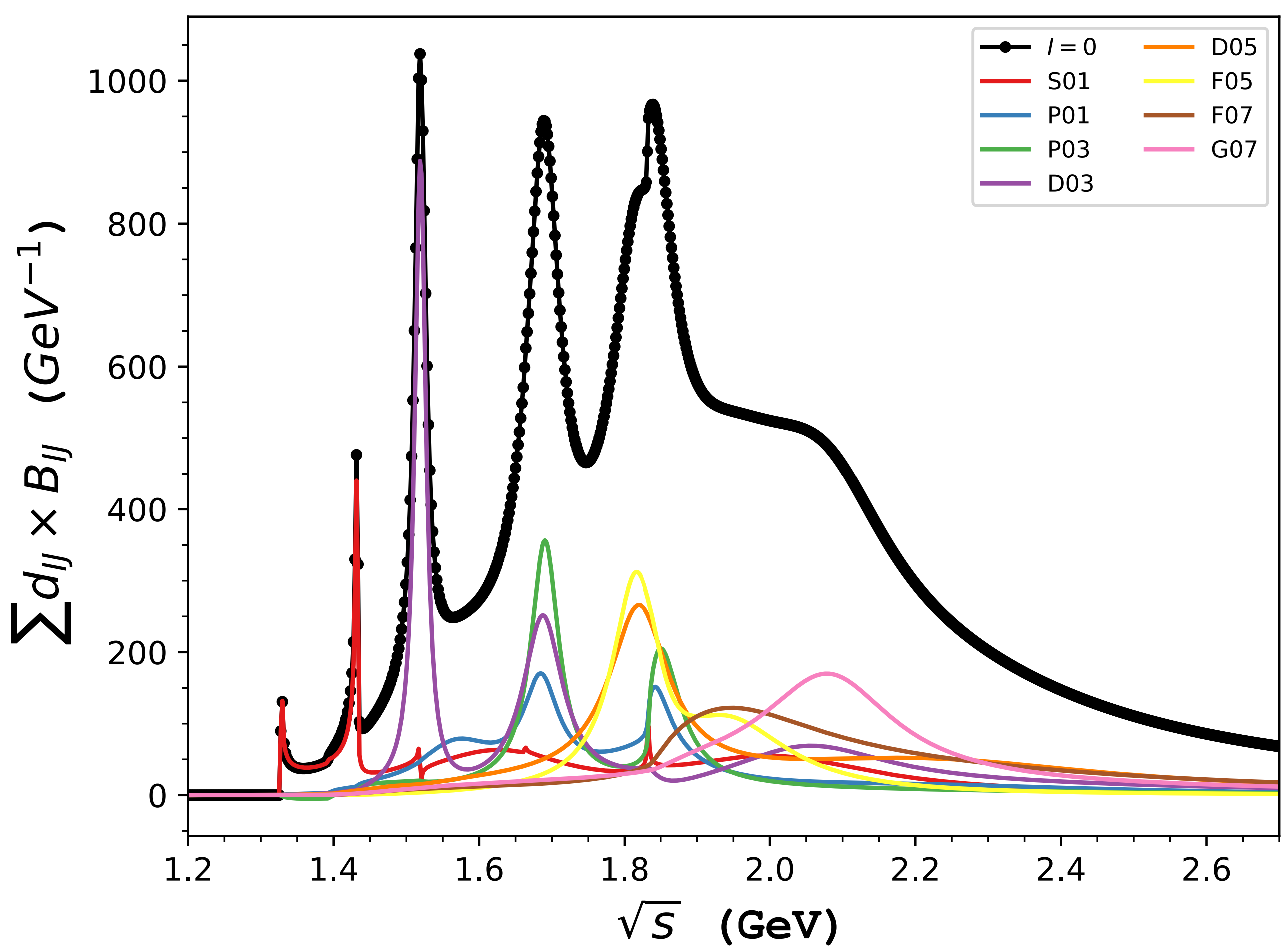
Σ(1670) 3/2⁻ $I(J^P) = 1(\frac{3}{2}^-)$

Mass $m = 1665$ to 1685 (≈ 1670) MeV
Full width $\Gamma = 40$ to 80 (≈ 60) MeV
$p_{beam} = 0.74$ GeV/c $4\pi\lambda\!\!\!^-{}^2 = 28.5$ mb

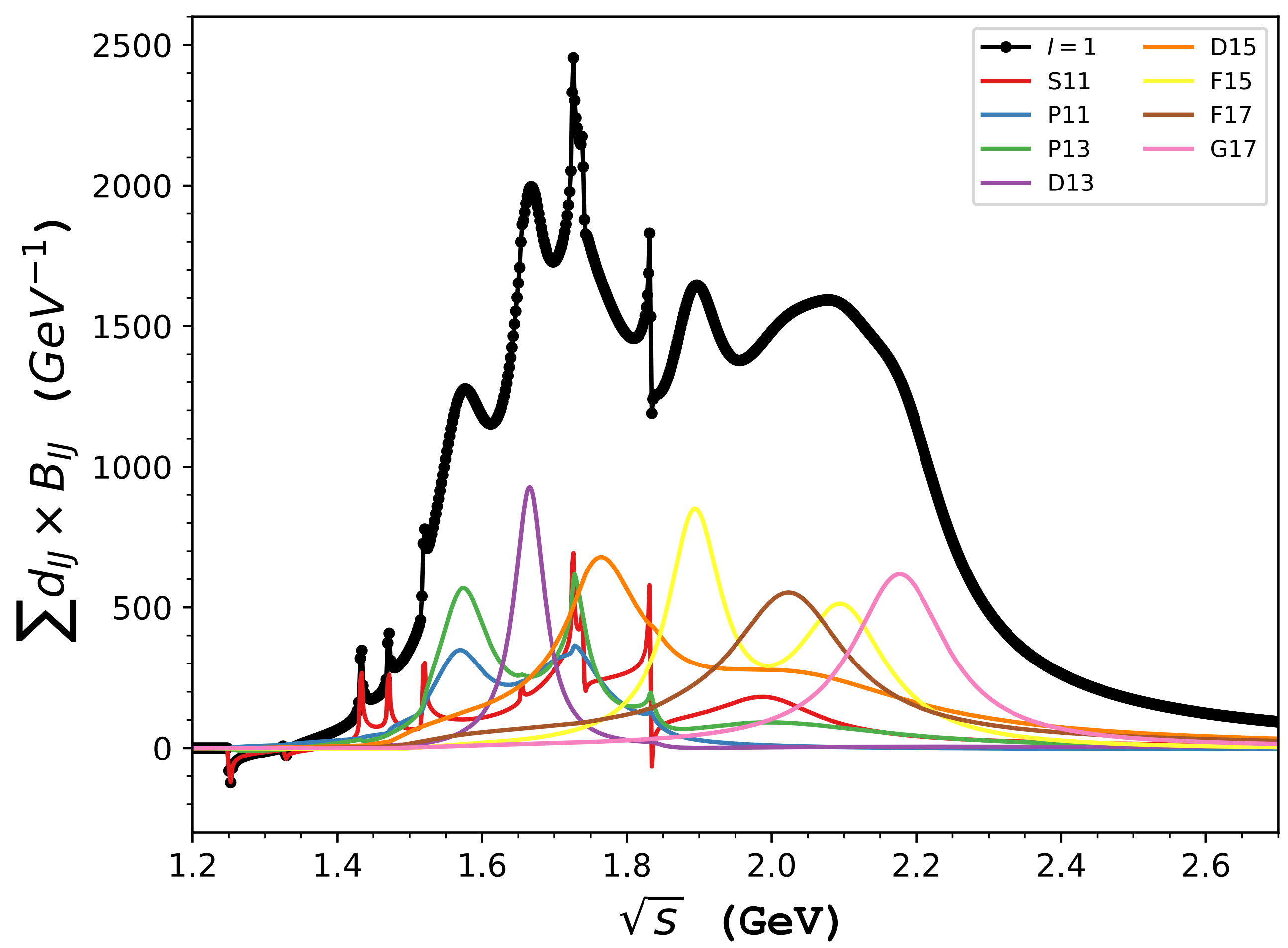
Σ(1670) DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\overline{K}$	7–13 %	414
$\Lambda\pi$	5–15 %	448
$\Sigma\pi$	30–60 %	394



branching ratio?



$I=0$
S01
P01
P03
D03
D05
F05
F07
G07
$\sum d_{IJ} \times B_{IJ}$ (GeV^{-1})
$\sqrt{S}$ (GeV)



$I=1$
S11
P11
P13
D13
D15
F15
F17
G17
$\sum d_{IJ} \times B_{IJ}$ (GeV^{-1})
$\sqrt{S}$ (GeV)

HOW TO PROPERLY ADD STATES

DYNAMICAL GENERATION OF BS / RESONANCES

- dynamical generation of bound states / resonances:
  f(980) close to K Kbar threshold
  f(500) dynamically generated

- coupling of open channels: pipi, kkbar with a $|q\bar{q}\rangle$ state

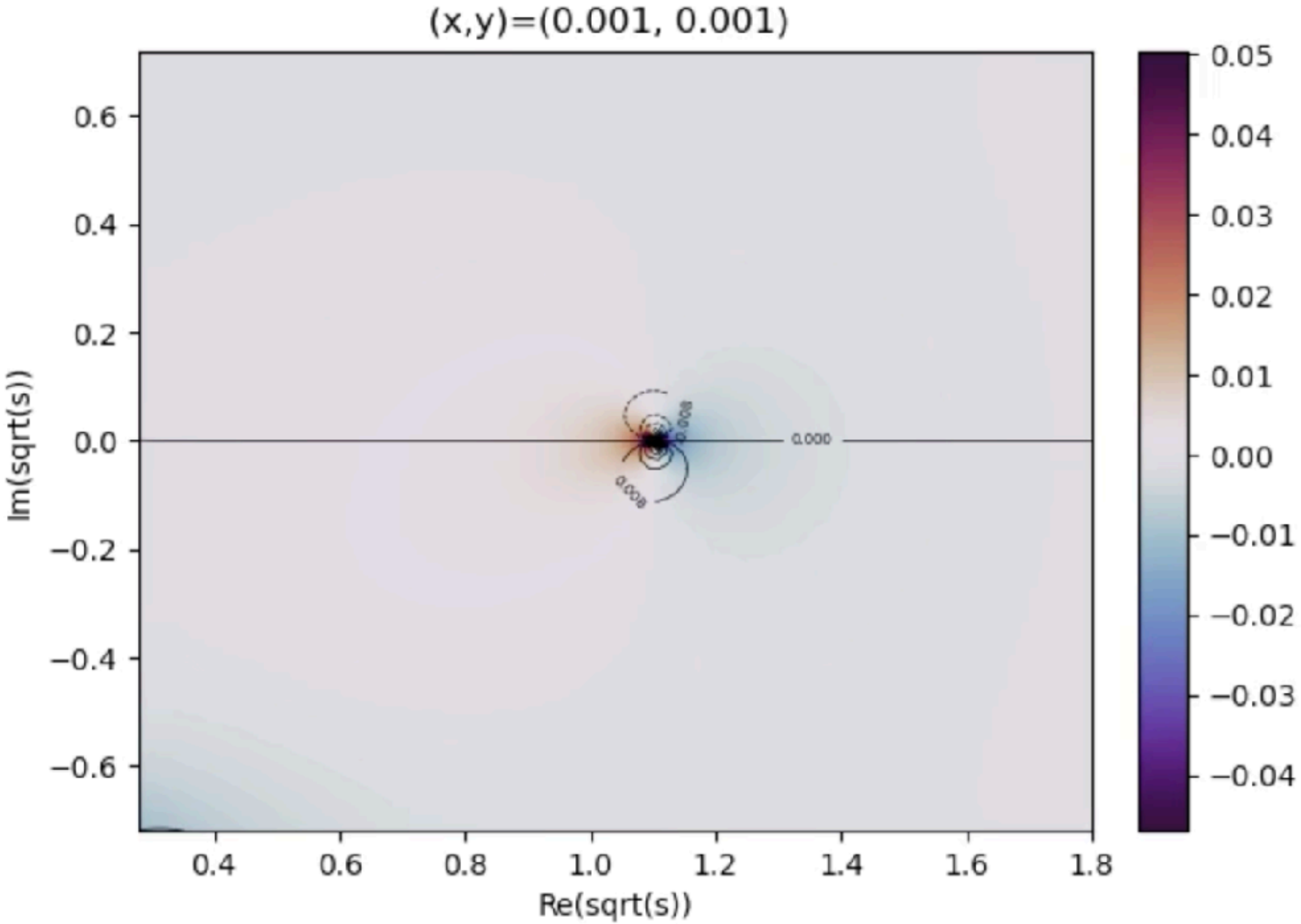
Locher, Markushin, Zheng, EPJC 4 (1998)
Kaminski, Lesniak, Loiseau, EPJC 9 (1999)

what you give $\neq$ *what you get*   *1 in 5 out!*

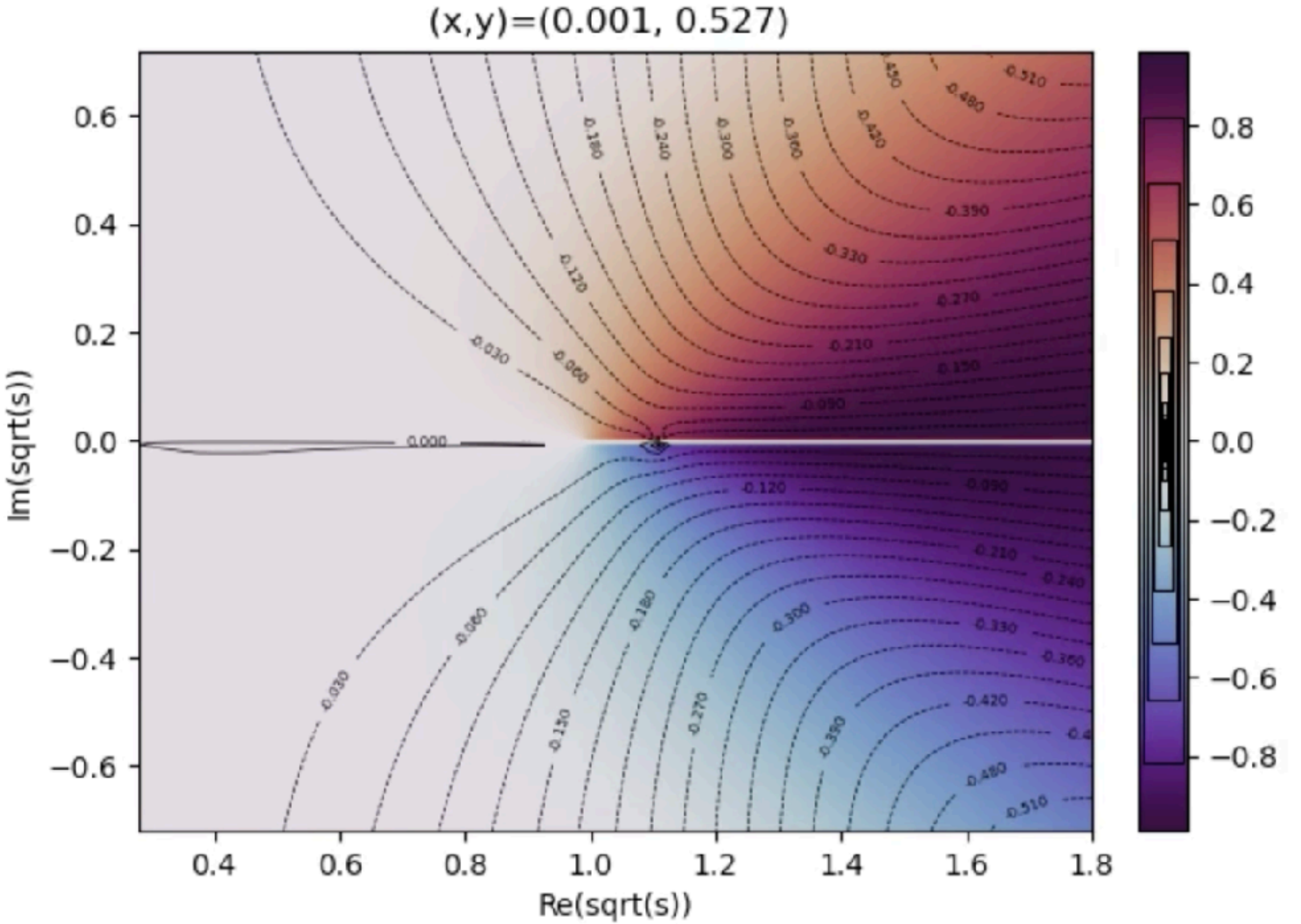
$$\frac{1}{E-\mathcal{H}_0} = \begin{array}{r} |\pi\pi\rangle \\ |K\bar{K}\rangle \\ |R^0\rangle \;\; (|q\bar{q}\rangle) \end{array} \begin{bmatrix} \Pi_{\pi\pi}(E) & & \\ & \Pi_{K\bar{K}}(E) & \\ & & \frac{1}{E-m^0_{res}} \end{bmatrix}$$

$$V_{int} = \begin{bmatrix} g_{\pi\pi} & g_{\pi K} & g_{\pi R} \\ g_{\pi K} & g_{KK} & g_{KR} \\ g_{\pi R} & g_{KR} & \end{bmatrix}$$

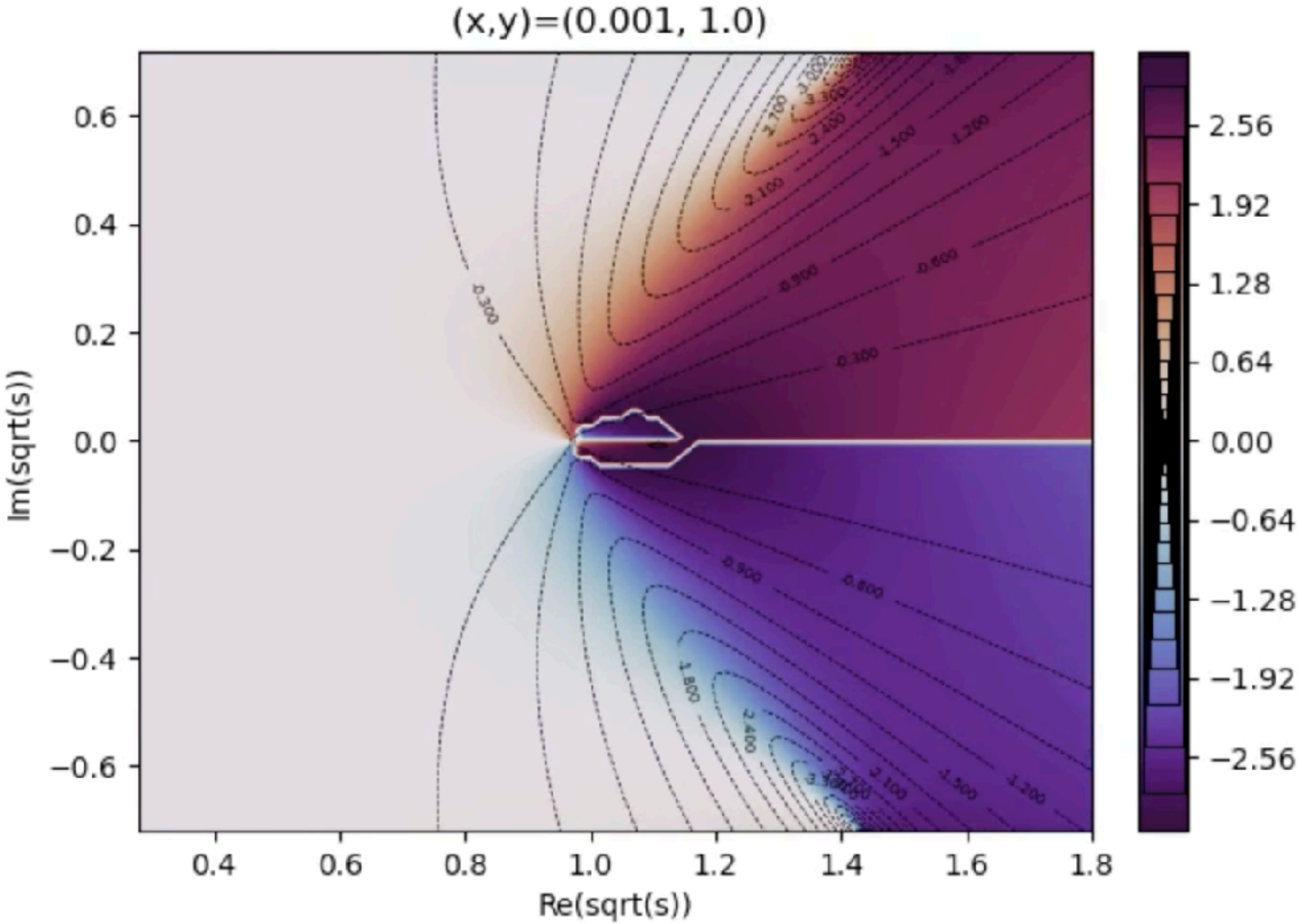
$$G = G_0 + G_0 V_{int} G$$

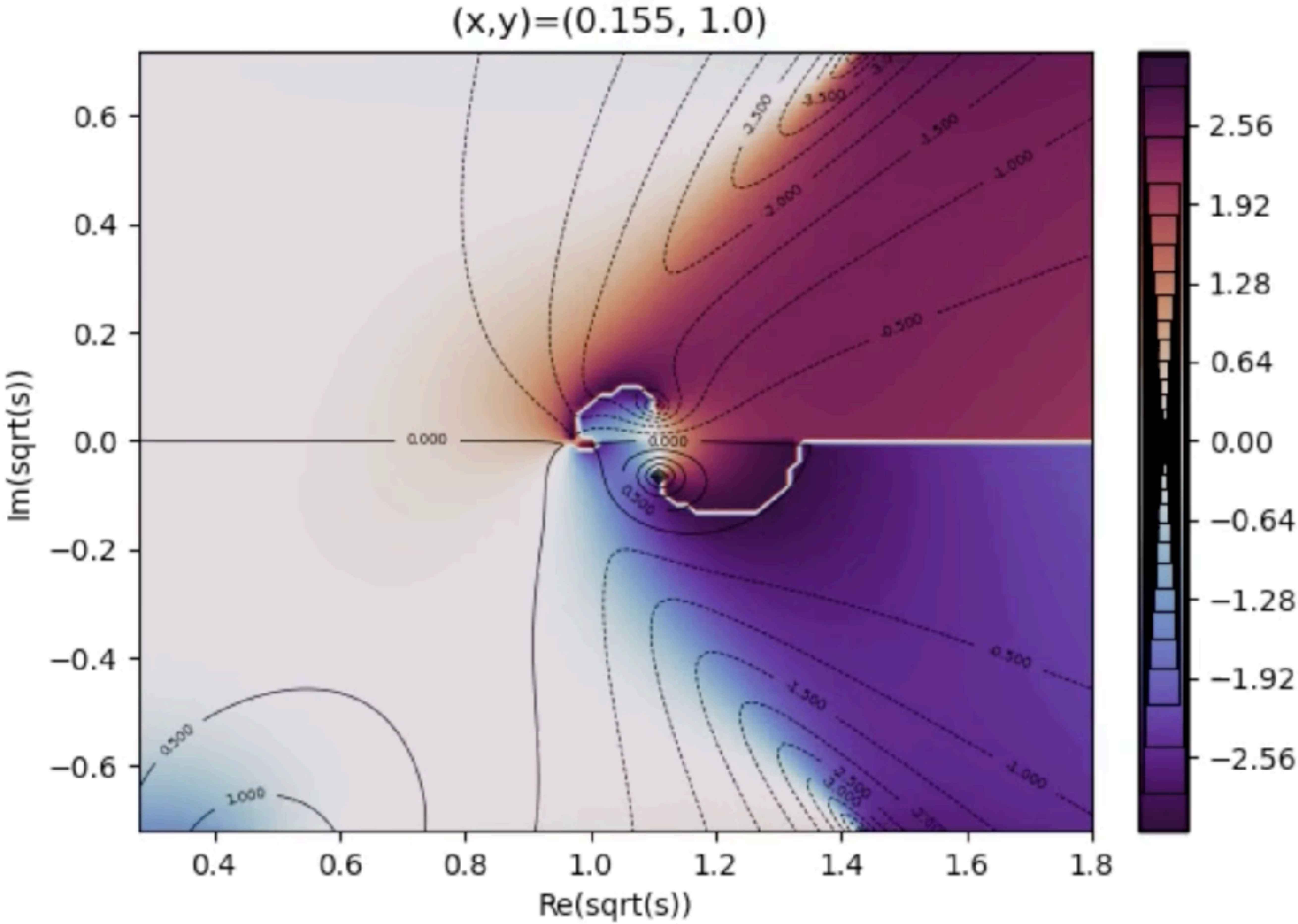


(x,y)=(0.001, 0.001)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
0.05
0.04
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
−0.04

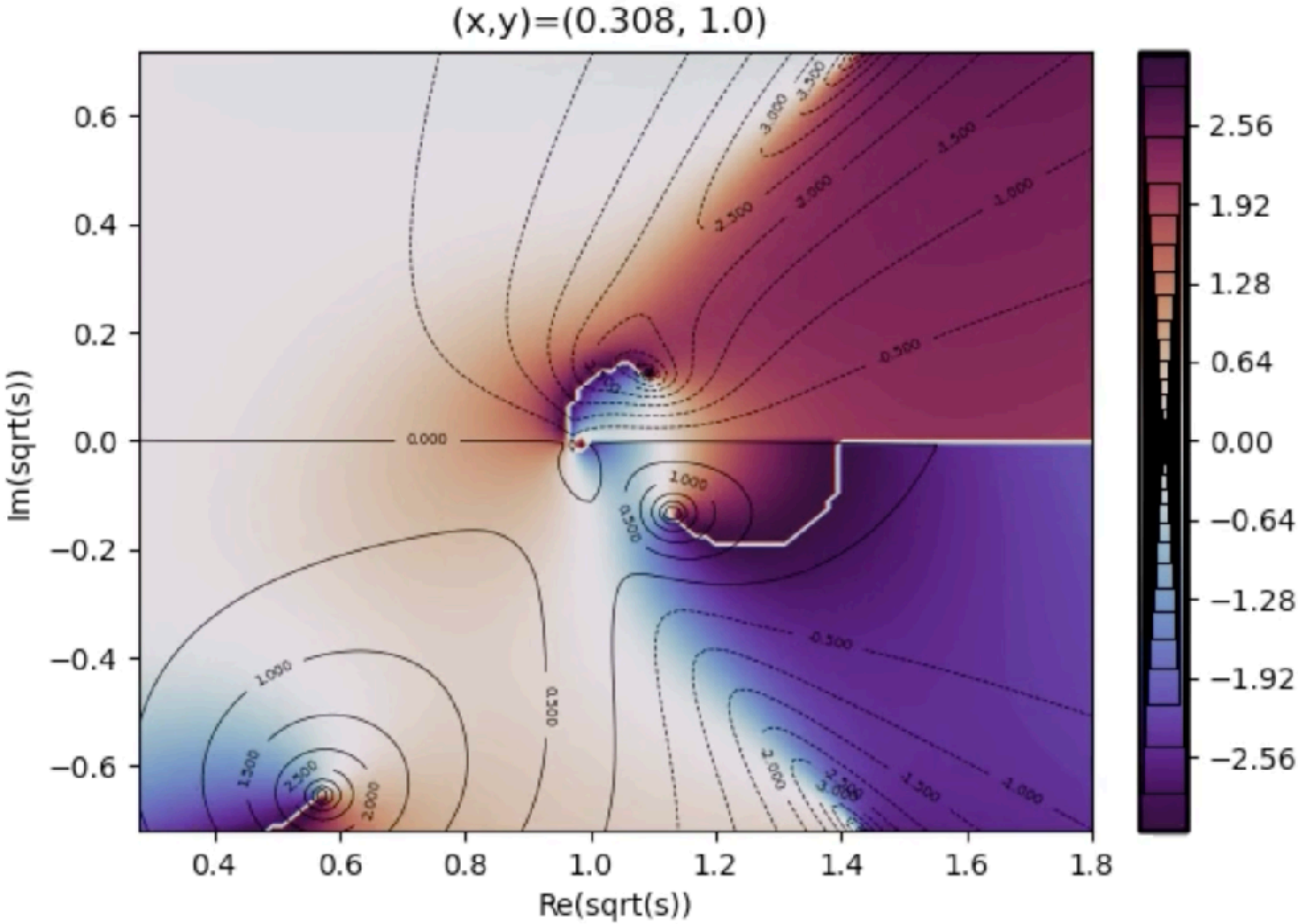


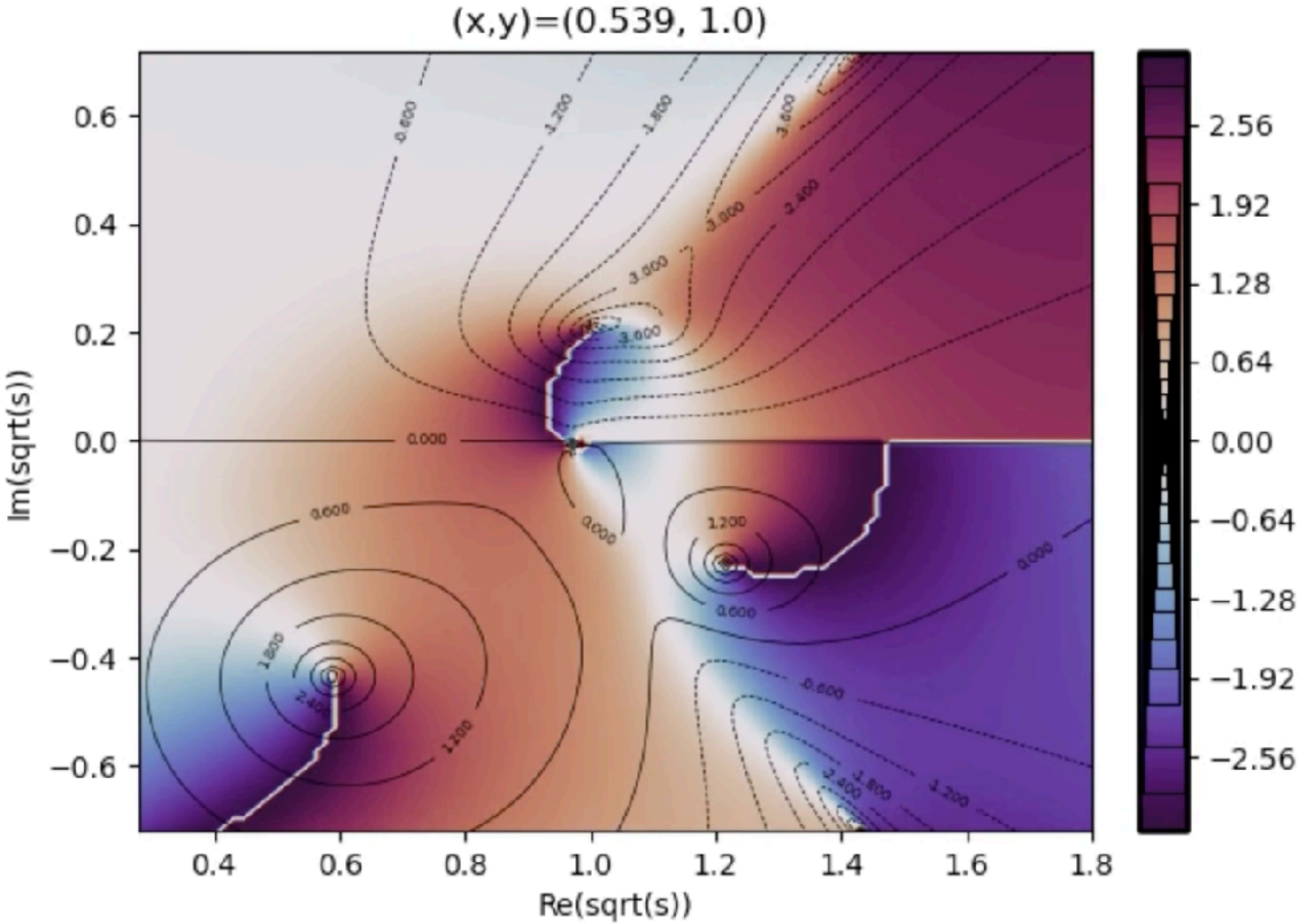
(x,y)=(0.001, 0.527)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8

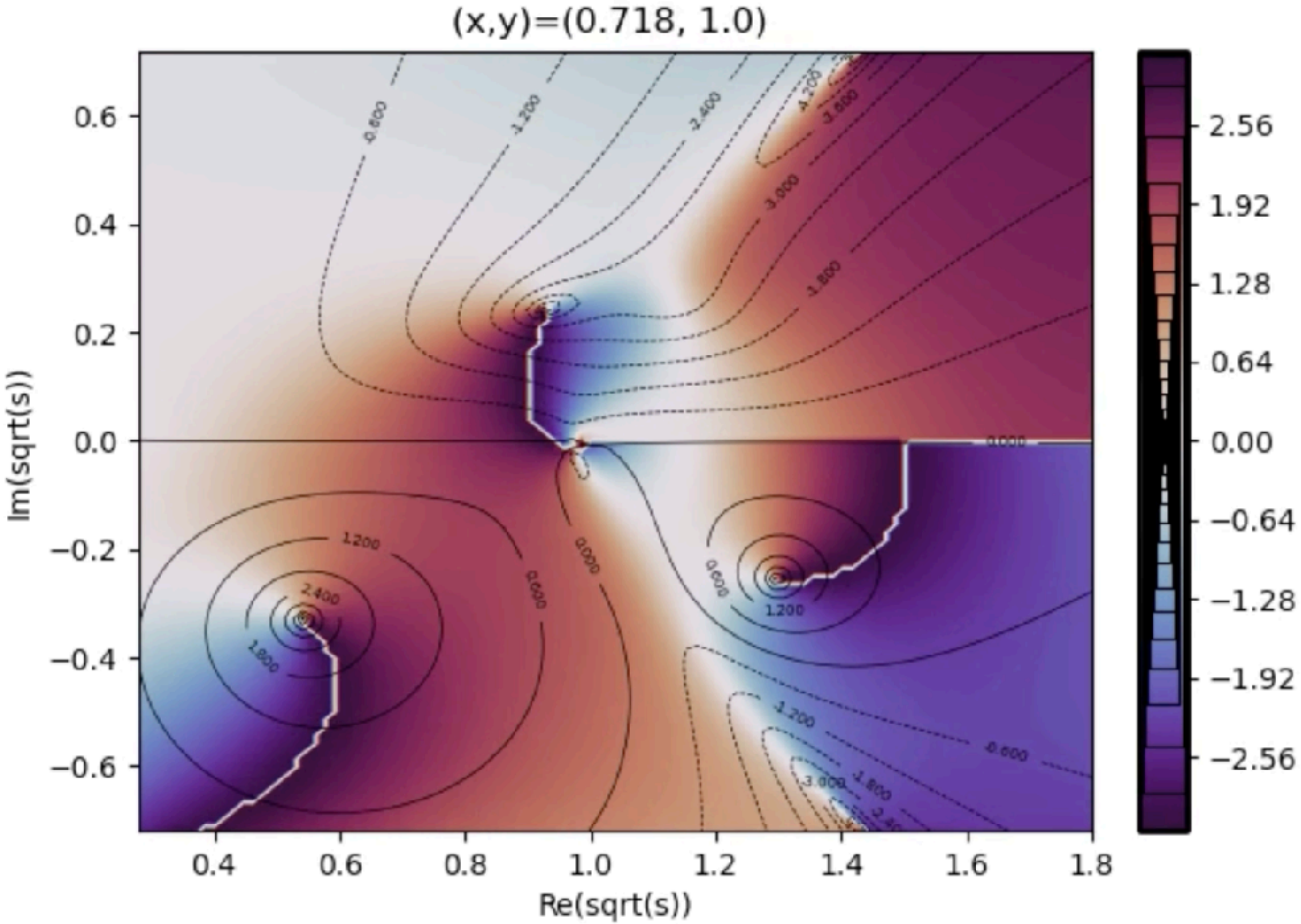

(x,y)=(0.001, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56



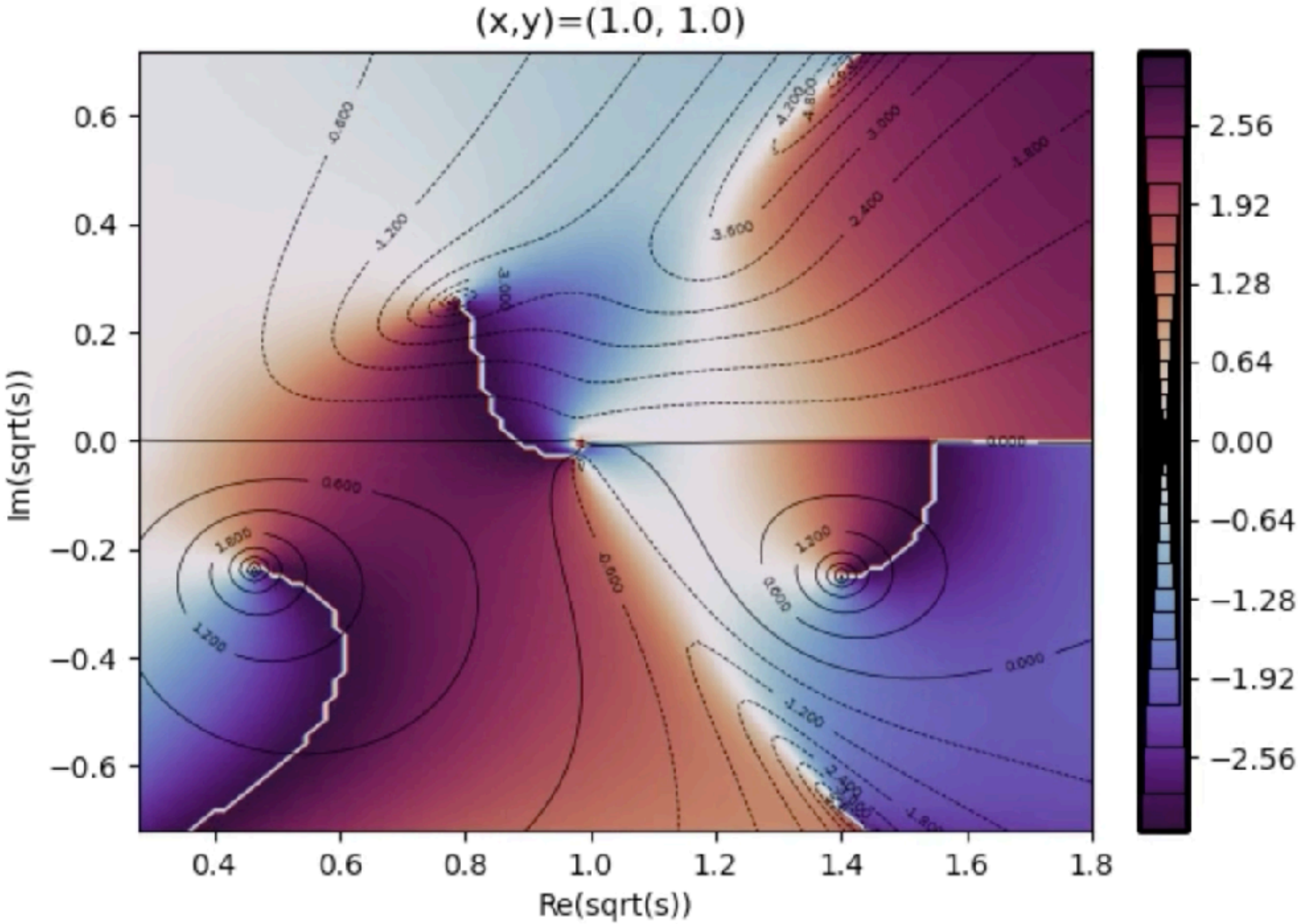
(x,y)=(0.155, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56


(x,y)=(0.308, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56
0.000

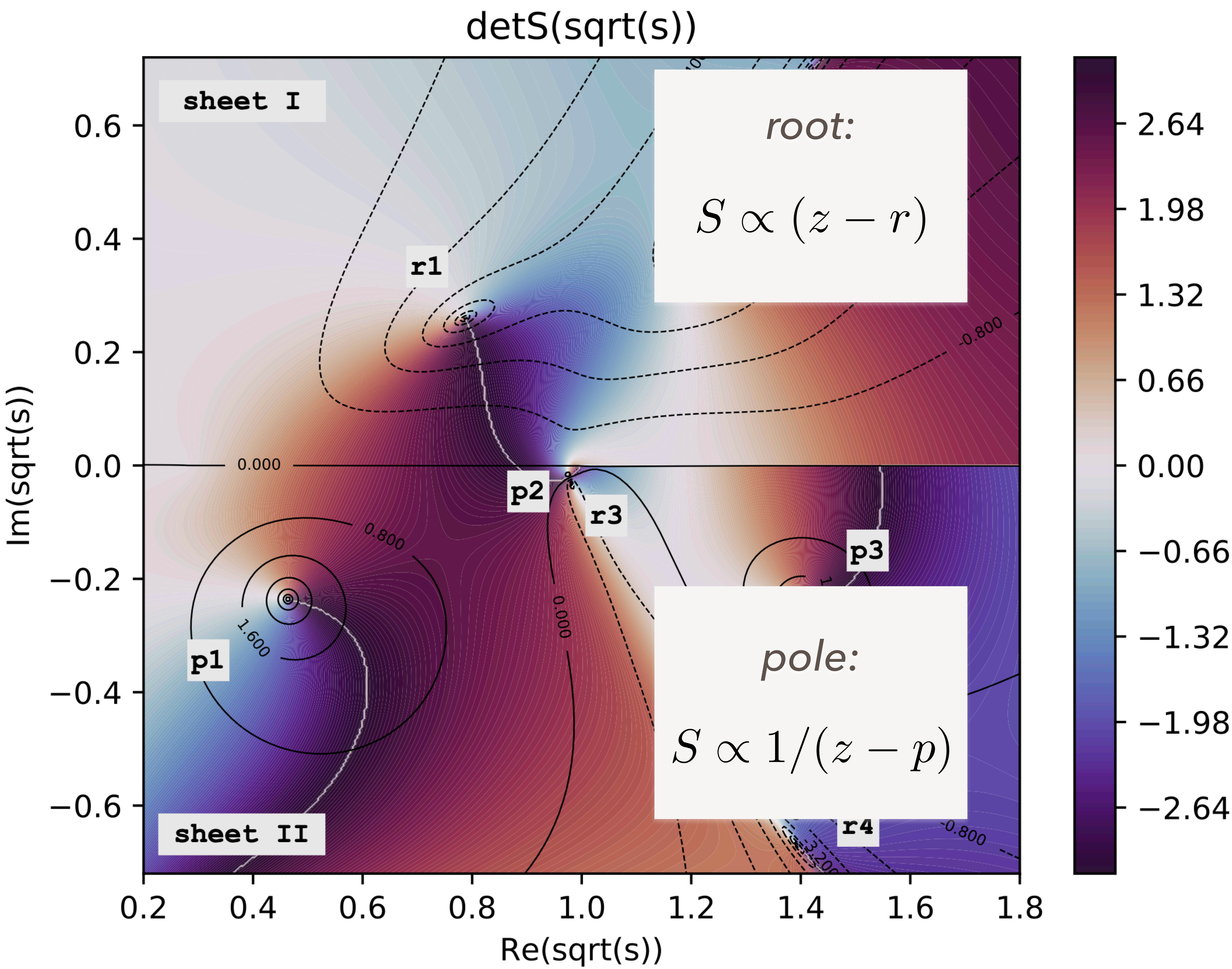

(x,y)=(0.539, 1.0)
Im(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
Re(sqrt(s))
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56

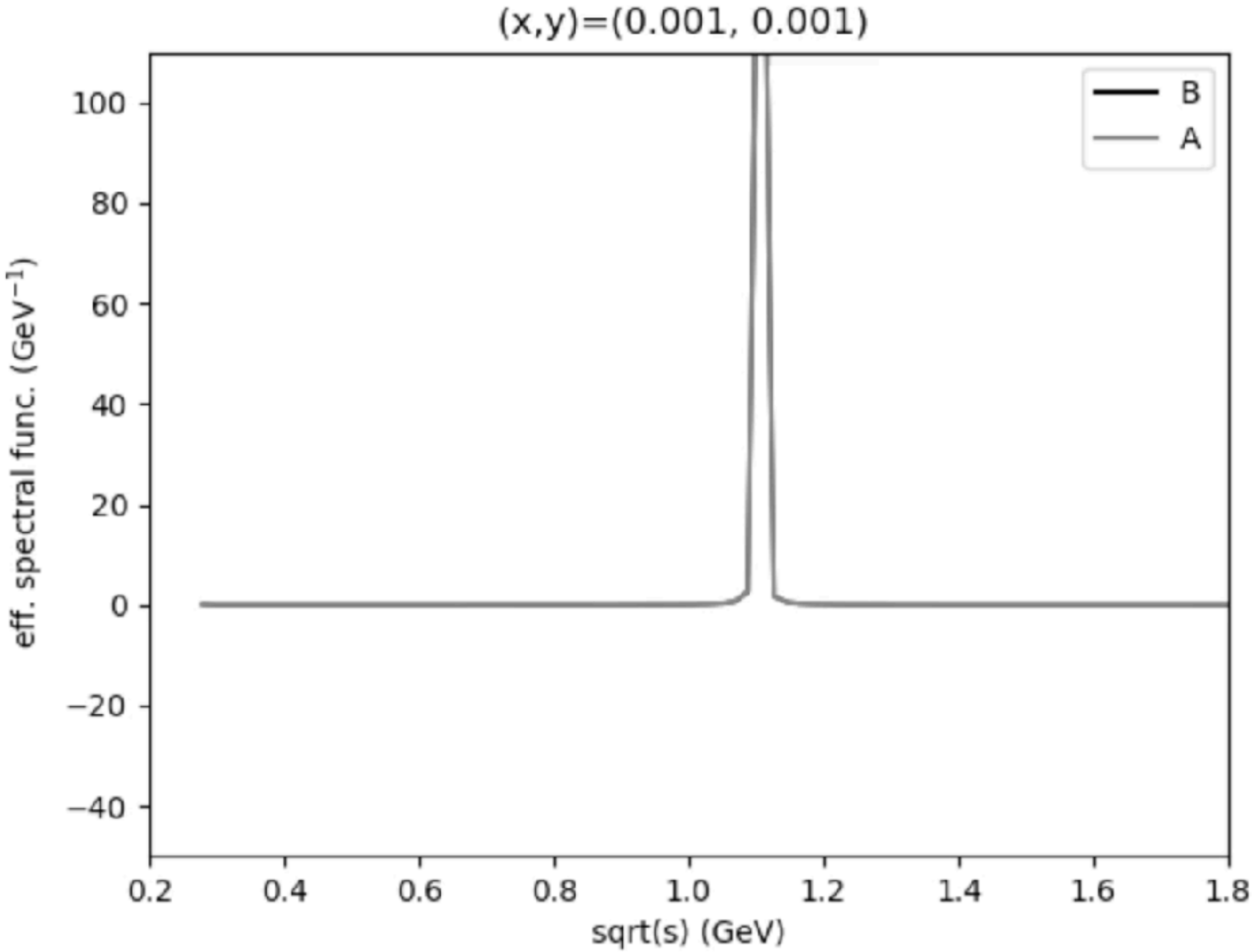


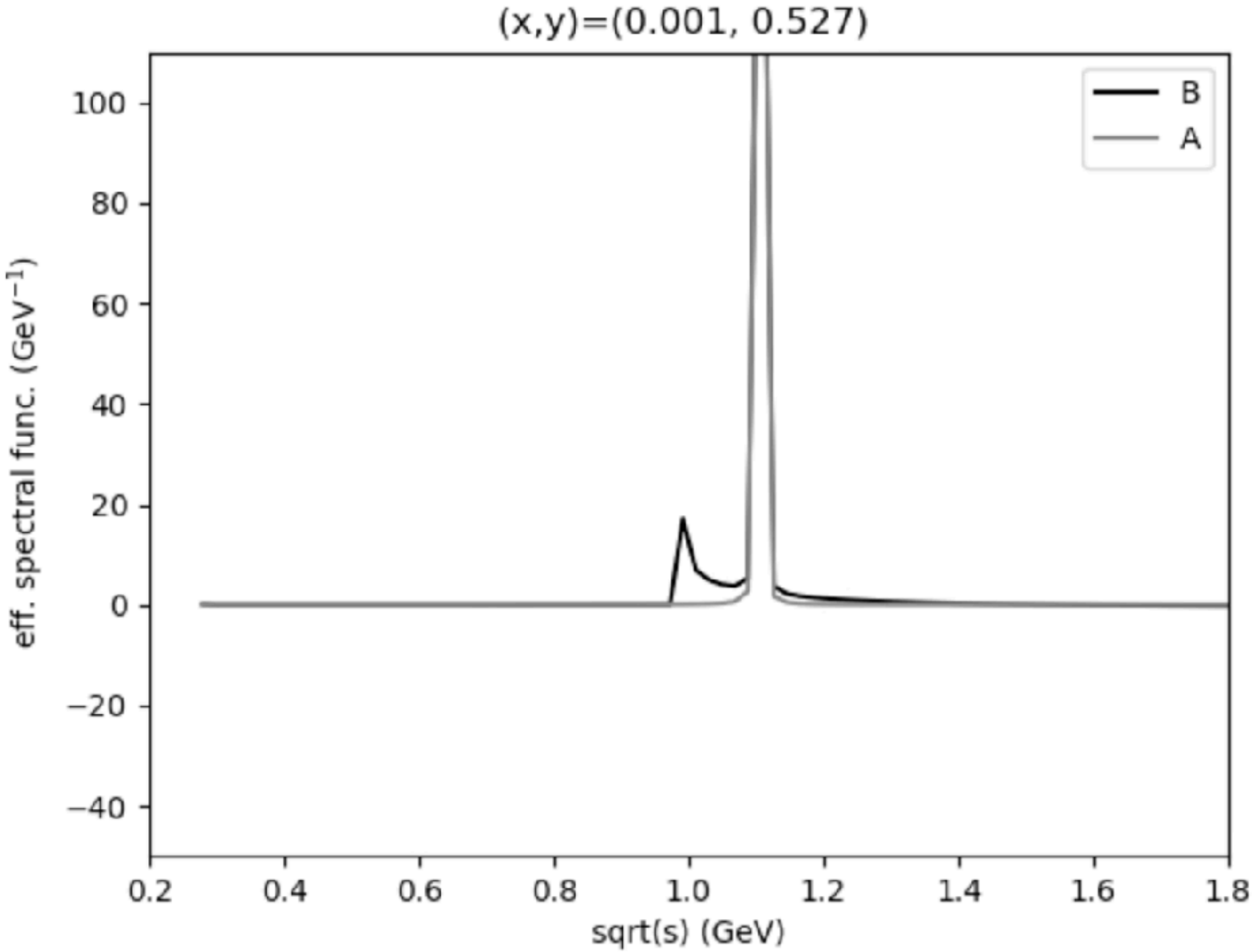
(x,y)=(0.718, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56



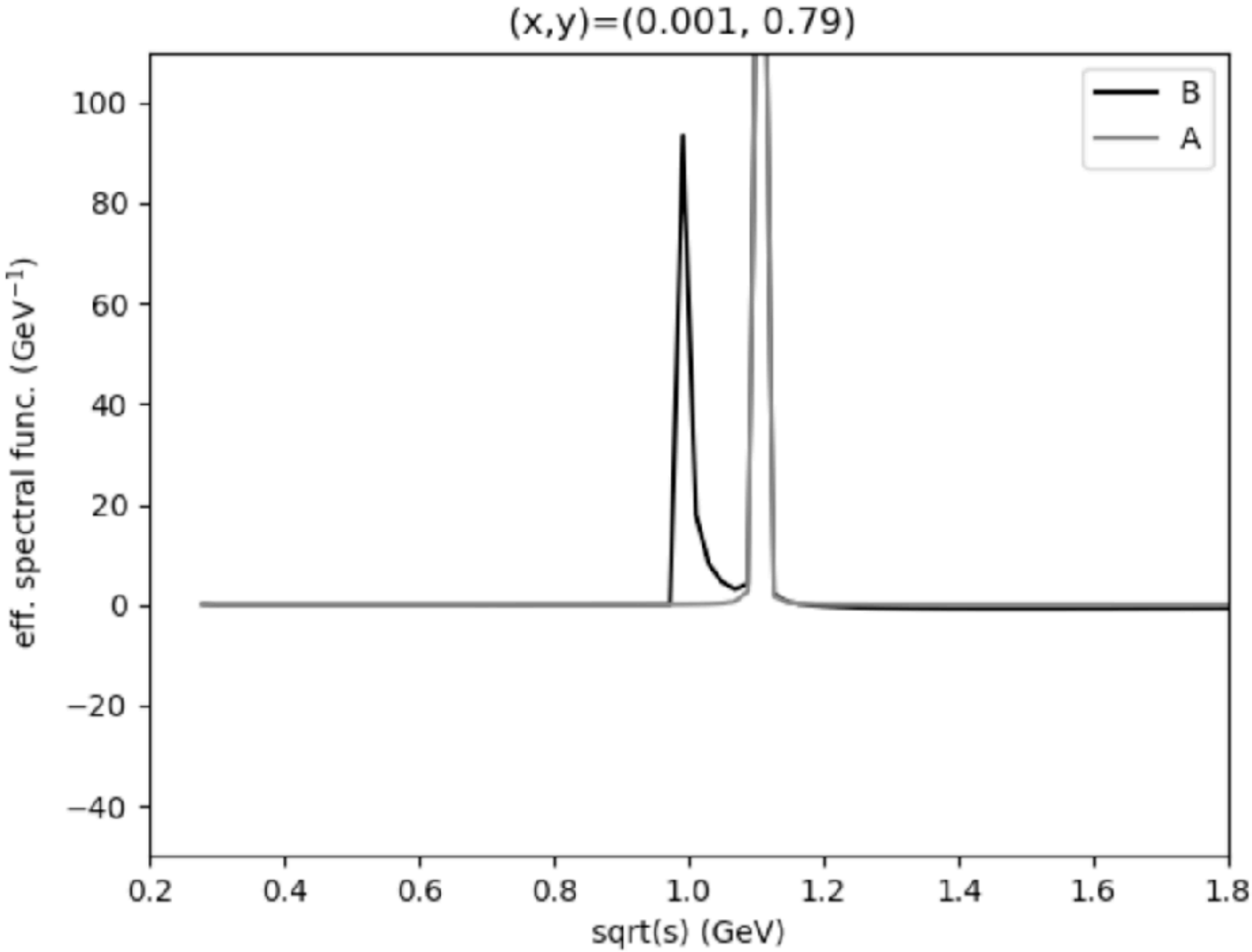
(x,y)=(1.0, 1.0)
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.56
1.92
1.28
0.64
0.00
−0.64
−1.28
−1.92
−2.56

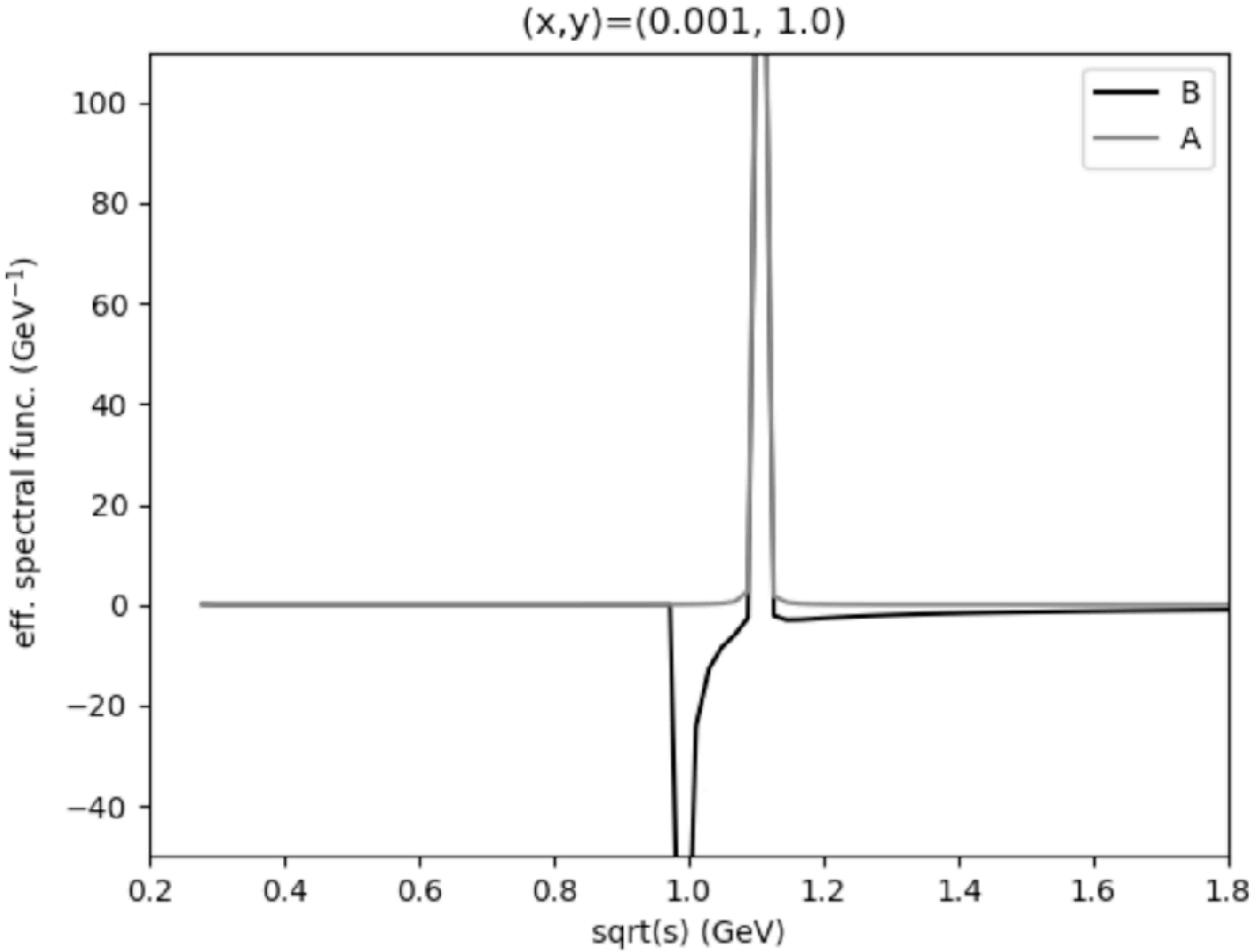

detS(sqrt(s))
sheet I
sheet II
root:
$S \propto (z - r)$
pole:
$S \propto 1/(z - p)$
r1
r3
r4
p1
p2
p3
0.000
0.800
1.600
-0.800
Im(sqrt(s))
Re(sqrt(s))
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.64
1.98
1.32
0.66
0.00
−0.66
−1.32
−1.98
−2.64

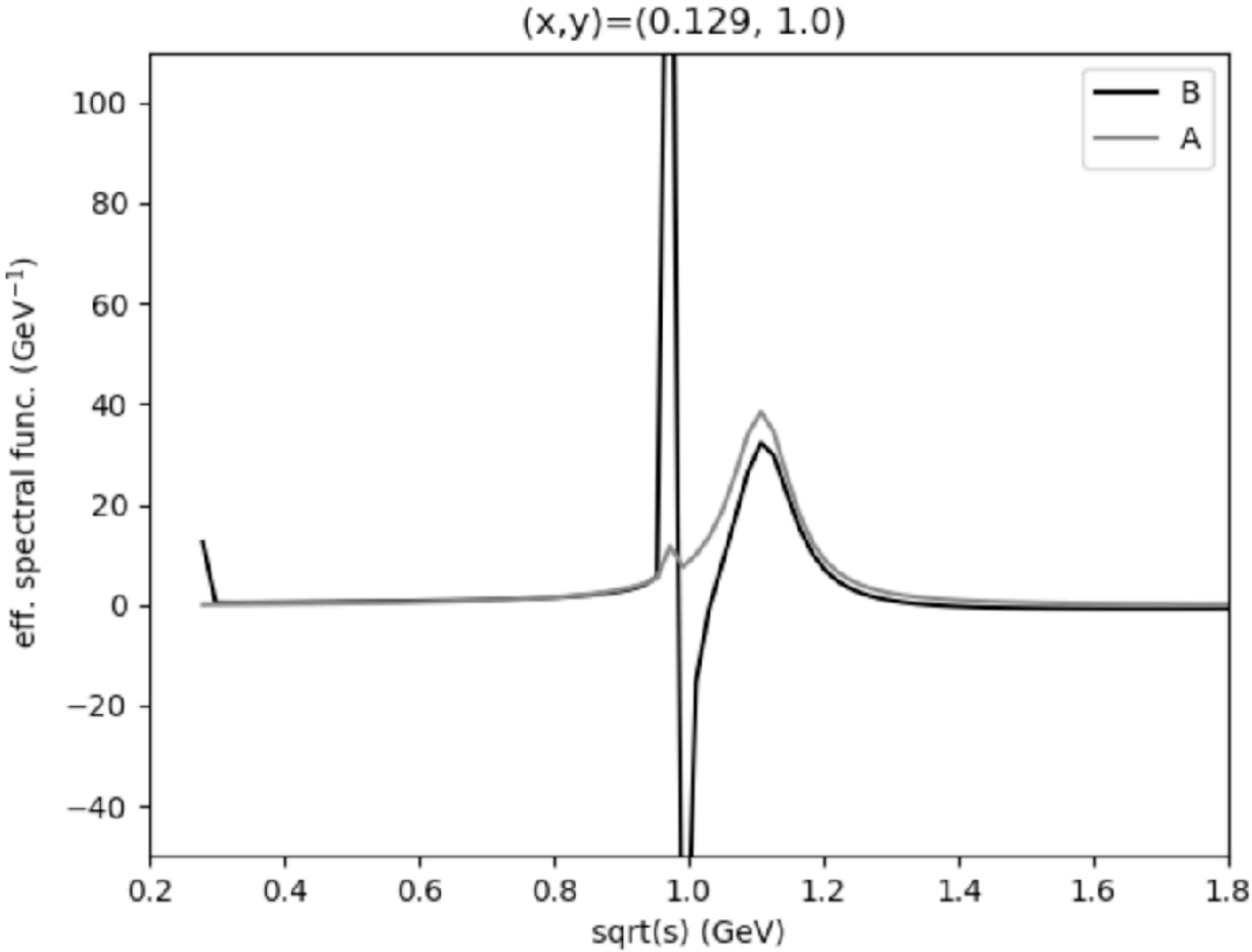

(x,y)=(0.001, 0.001)
B
A
eff. spectral func. (GeV⁻¹)
sqrt(s) (GeV)
100
80
60
40
20
0
−20
−40
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8

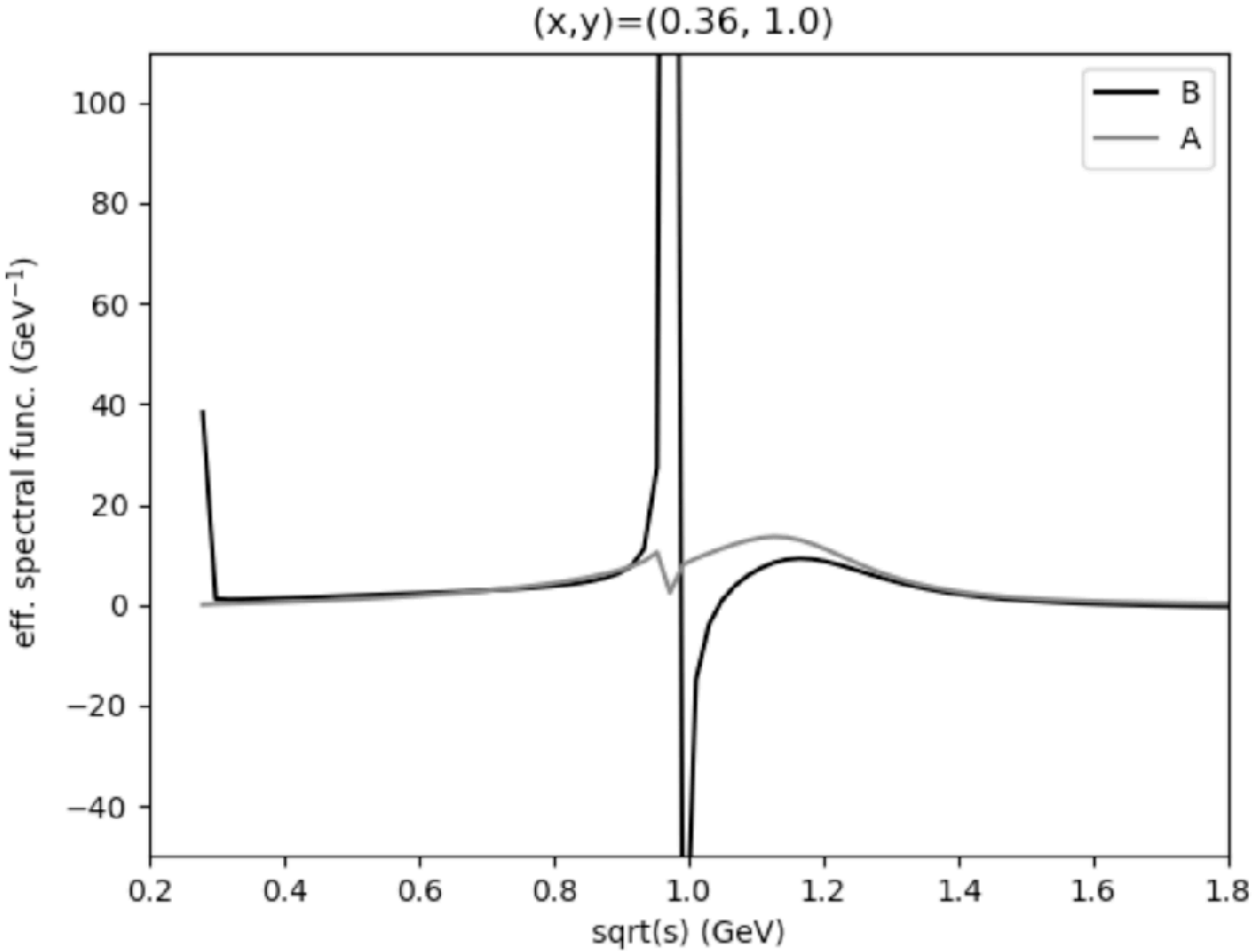


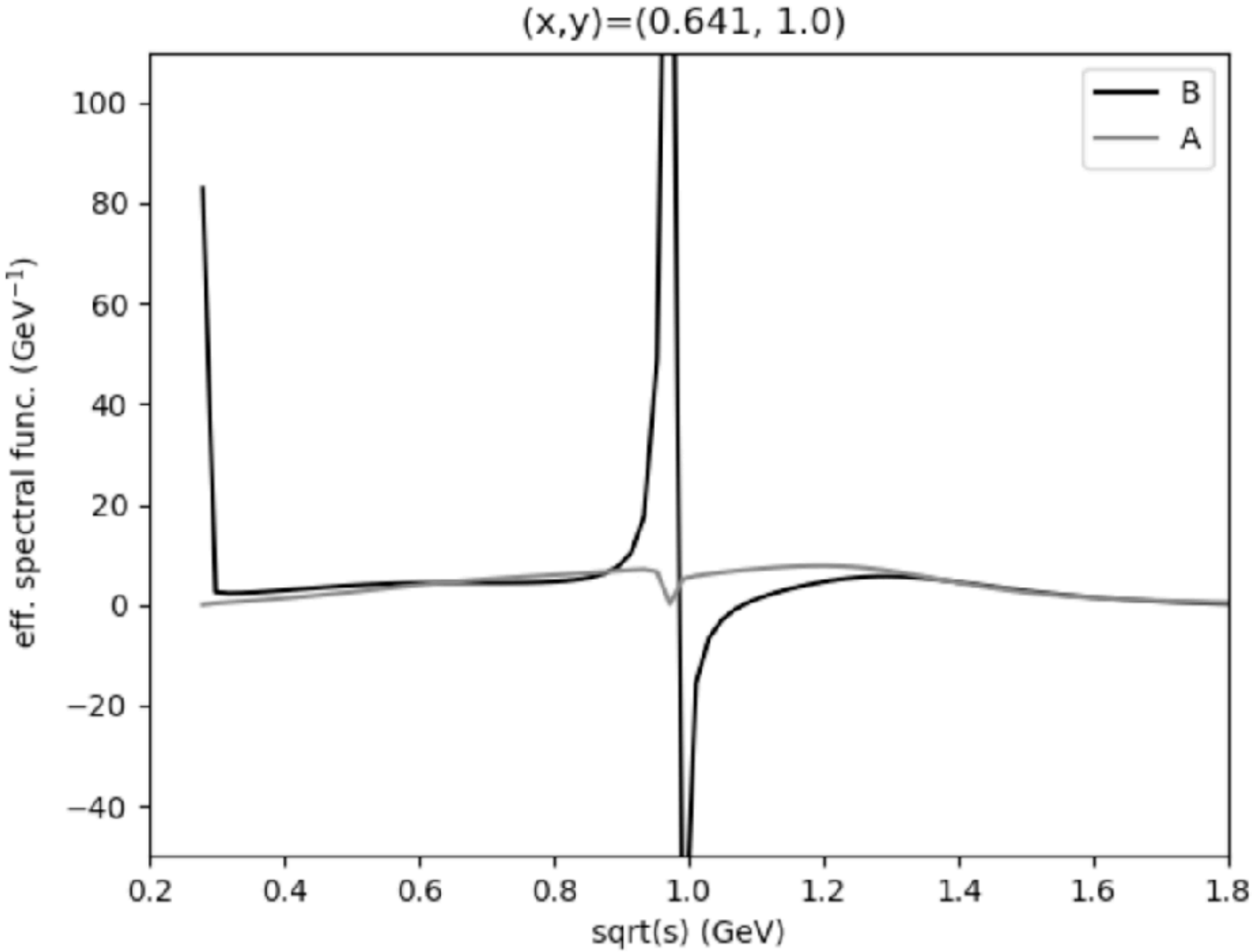
(x,y)=(0.001, 0.527)
B
A
100
80
60
40
20
0
−20
−40
eff. spectral func. (GeV^{-1})
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)

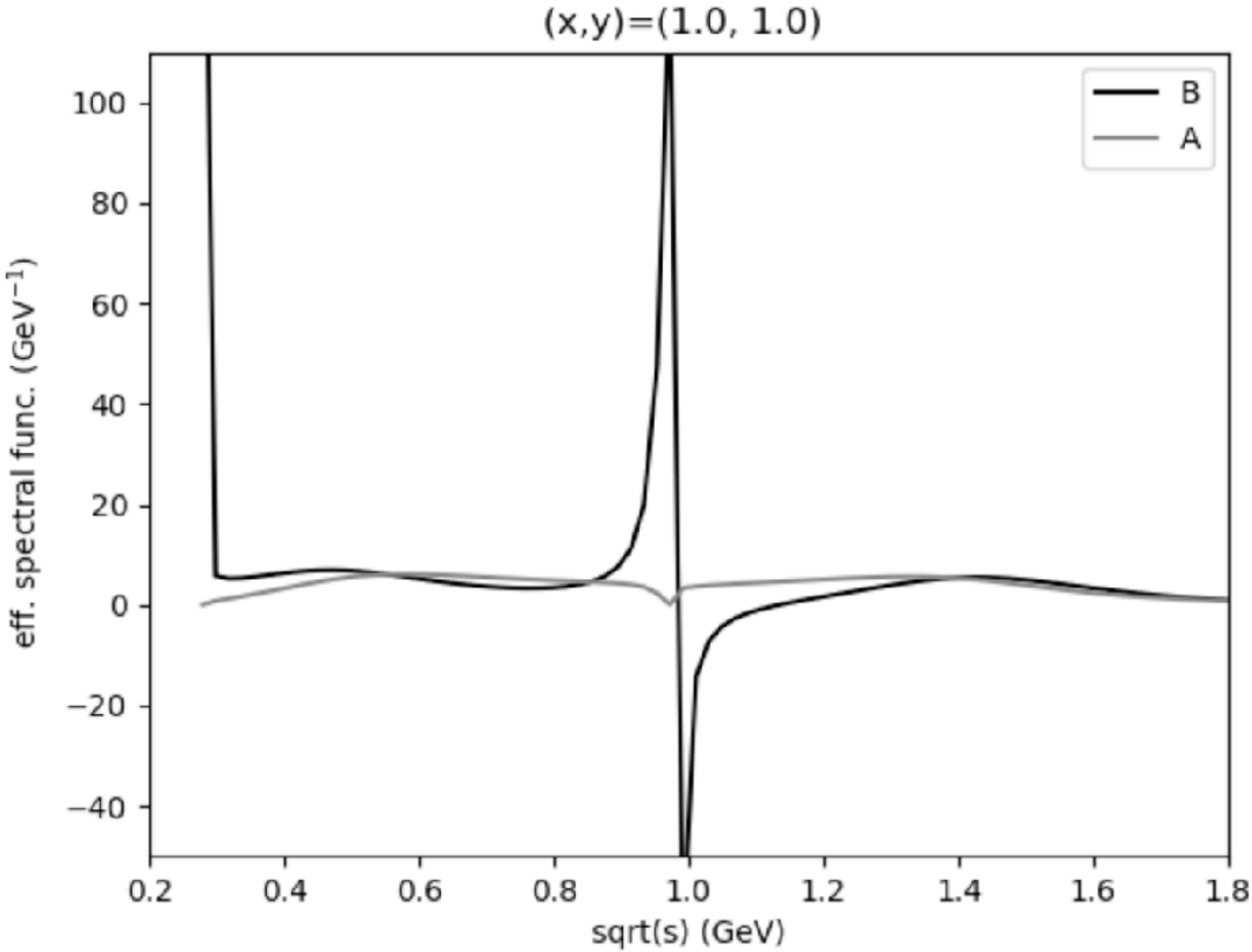

(x,y)=(0.001, 0.79)
B
A
100
80
60
40
20
0
−20
−40
eff. spectral func. (GeV^{-1})
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)


(x,y)=(0.001, 1.0)
B
A
eff. spectral func. (GeV^{-1})
100
80
60
40
20
0
−20
−40
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)

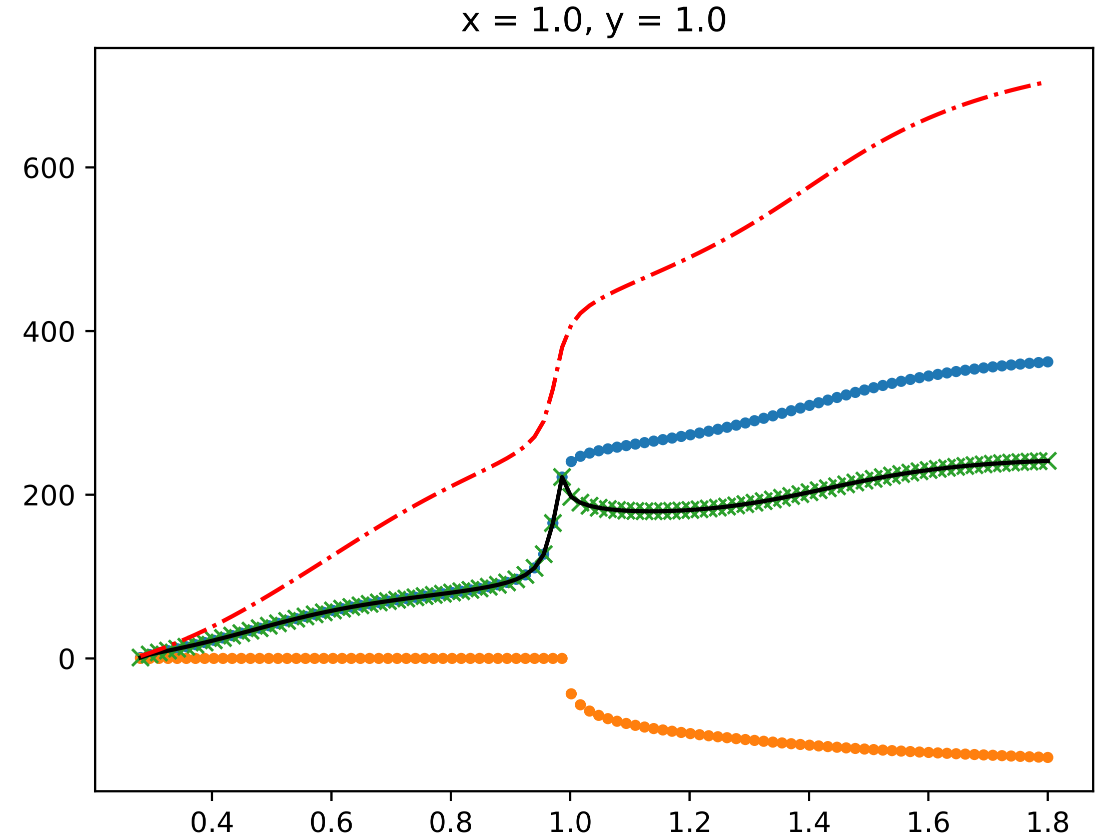

(x,y)=(0.129, 1.0)
B
A
100
80
60
40
20
0
−20
−40
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
eff. spectral func. (GeV^{-1})
sqrt(s) (GeV)


(x,y)=(0.36, 1.0)
B
A
eff. spectral func. (GeV^{-1})
sqrt(s) (GeV)
100
80
60
40
20
0
−20
−40
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8


(x,y)=(0.641, 1.0)
B
A
100
80
60
40
20
0
−20
−40
eff. spectral func. (GeV^{-1})
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)



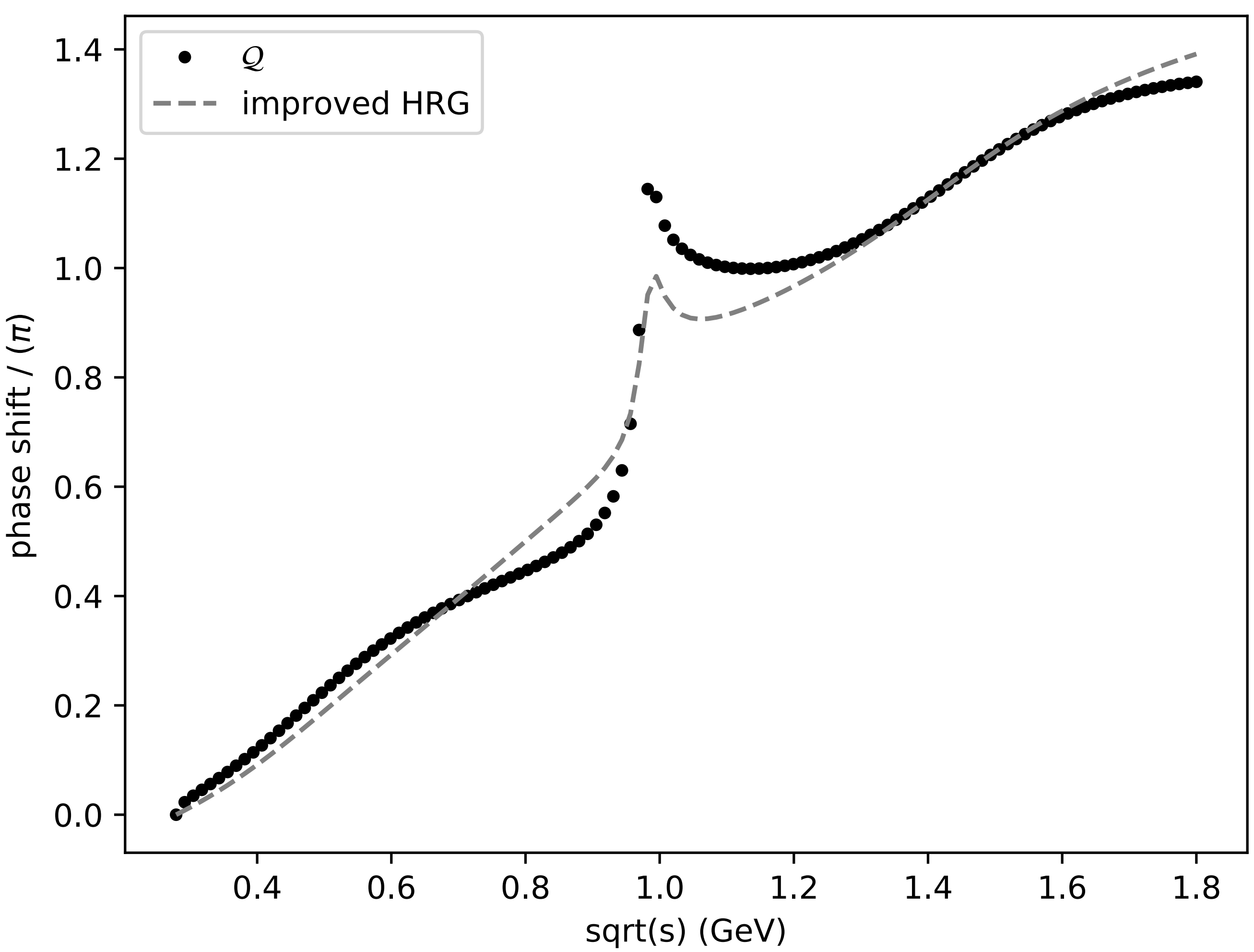
(x,y)=(1.0, 1.0)
B
A
100
80
60
40
20
0
−20
−40
eff. spectral func. (GeV^{-1})
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
sqrt(s) (GeV)



f. [54, 55]

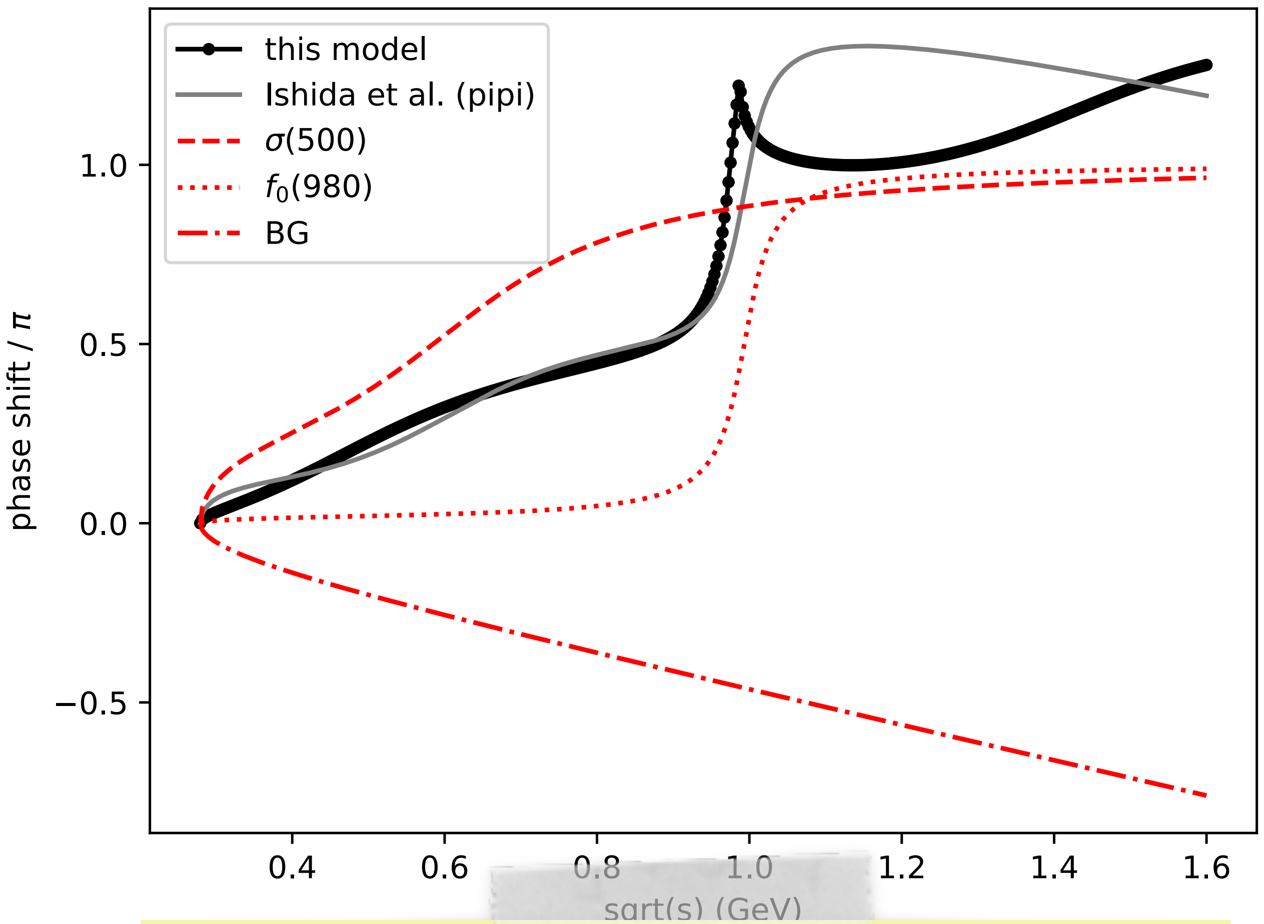
	Re $\sqrt{s}$	Im $\sqrt{s}$	sheet
p1	0.4637	-0.2357	II
p2	0.975	-0.0164	II
p3	1.401	-0.249	II
p4	0.6654	-0.2263	III
p5	1.4176	-0.2640	III
r1	0.787	+0.259	I
r2	1.410	+0.691	I
r3	0.981	-0.032	II
r4	1.393	-0.669	II
r5	0.918	+0.248	IV

II. Location of resonance poles (p_i) and roots (r_i) in the model.



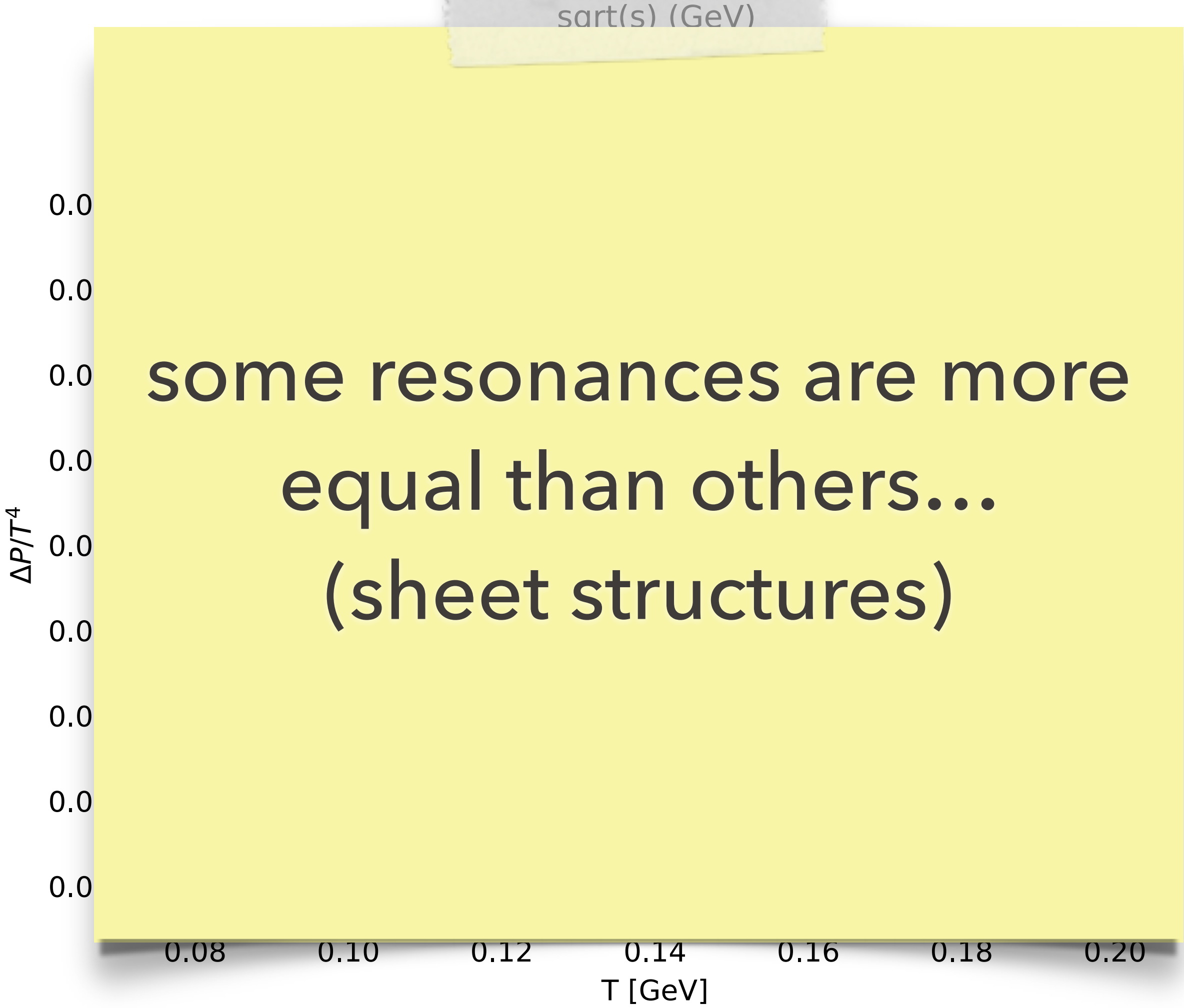
repulsive corrections in HRG-like scheme: via roots

	$\mathrm{Re}\sqrt{s}$	$\mathrm{Im}\sqrt{s}$	sheet

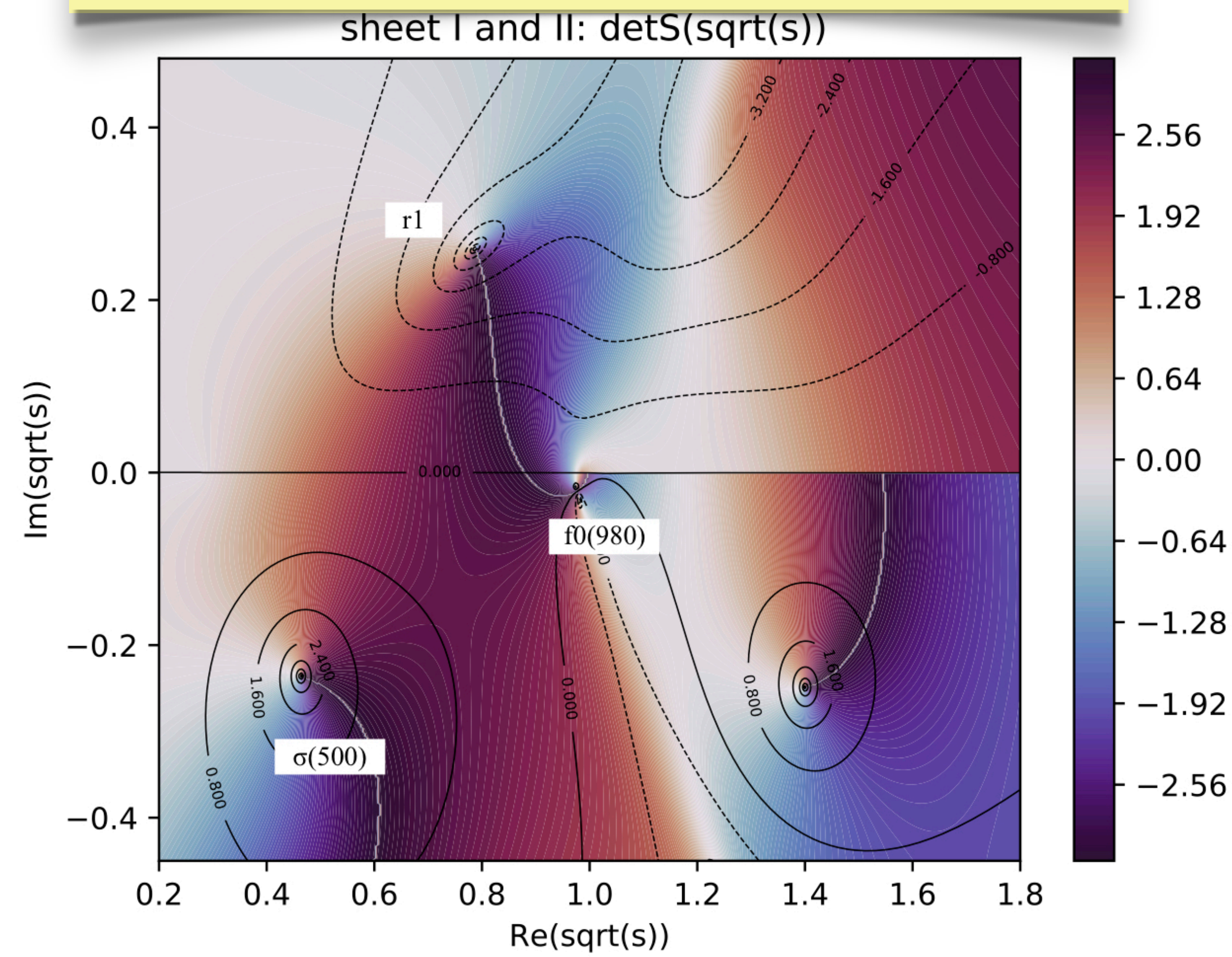


non-Briet-Wigner
=> poles and roots
distribution

subtractive corrections

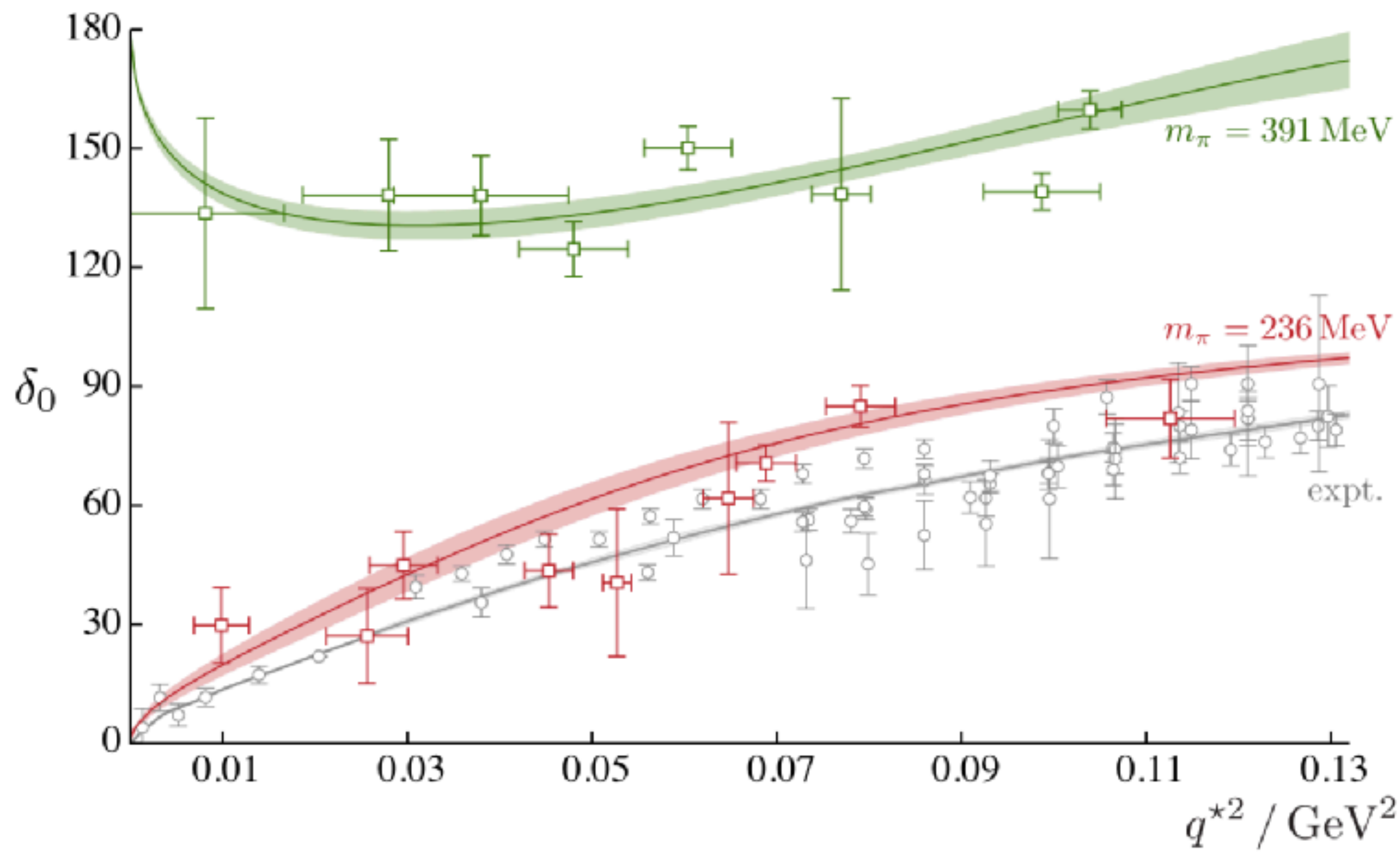


some resonances are more
equal than others...
(sheet structures)



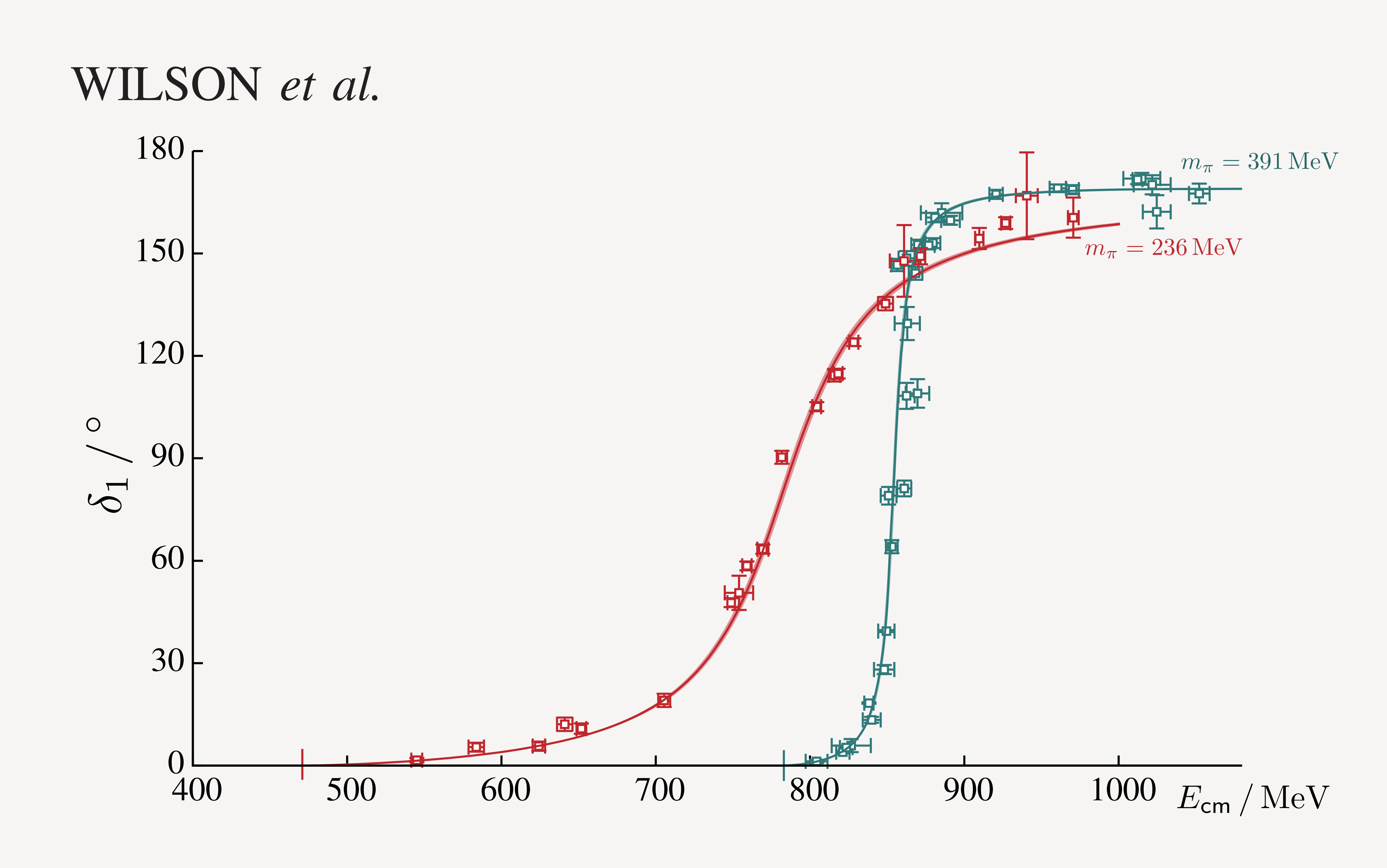
LATTICE COMPUTATIONS ON PHASE SHIFT

deuteron physics?



R. A. Briceno, J. J. Dudek and R. D. Young, arXiv:1706.06223 [hep-lat].

LATTICE COMPUTATIONS ON PHASE SHIFT



NO SERIOUS MESON SPECTROSCOPY
WITHOUT SCATTERING*

George Rupp

CFIF, Instituto Superior Técnico, Universidade de Lisboa, 1049-001, Portugal

Eef van Beveren

CFC, Departamento de Física, Universidade de Coimbra, 3004-516, Portugal

Susana Coito

Institute of Modern Physics, CAS, Lanzhou 730000, China

(Received January 25, 2015)

CONTINUUM

$$Z = \sum_{\alpha=B,M} \langle\alpha|e^{-\beta H}|\alpha\rangle$$

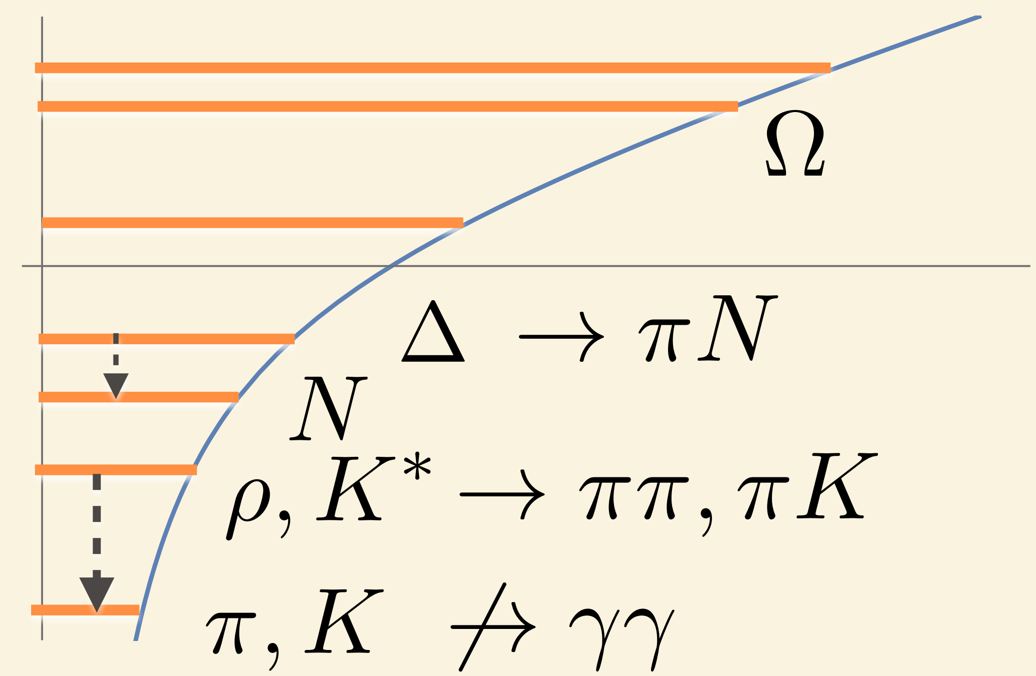
meson loops effects:

shift in hadron masses

Quark Model States:

mixing with continuum

QCD spectrum



S-MATRIX INTERPRETATION OF IN-MEDIUM EFFECTS

VACUUM PHYSICS?

Quantum statistical mechanics of gases in terms of dynamical filling fractions and scattering amplitudes

André LeClair

Newman Laboratory, Cornell University, Ithaca, NY, USA

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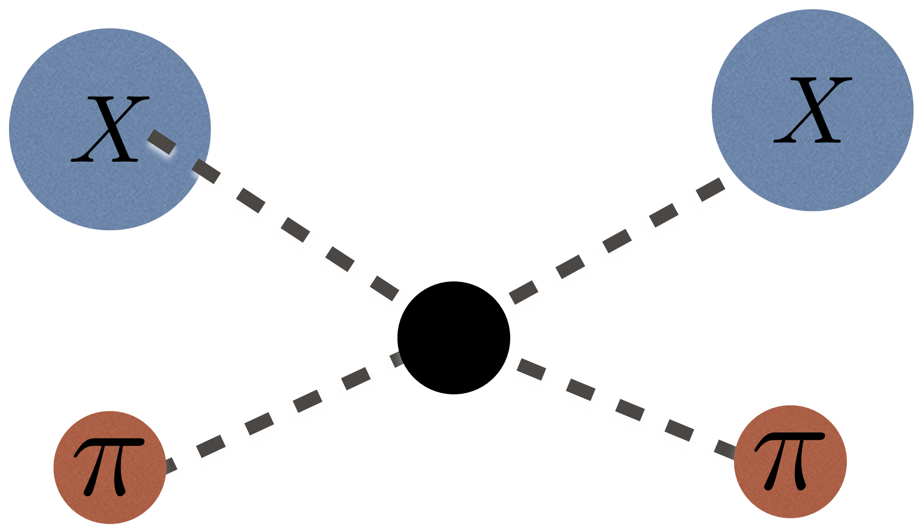
Published 19 July 2007
Online at stacks.iop.org/JPhysA/40/9655

It helps to realize that at least in principle it is possible to decouple the zero temperature dynamics and the quantum statistical sums. The argument is simple: the computation of the partition function $Z = \mathrm{Tr}(\mathrm{e}^{-\beta \mathrm{H}})$ is in principle possible from the complete knowledge of the zero temperature eigenstates of the Hamiltonian H. In practice this is rather difficult and one resorts to perturbative methods such as the Matsubara method, which unfortunately entangles the zero temperature dynamics from the quantum statistical mechanics. However,



$$T_{\rm nr} \approx -\frac{4\pi f}{2m_{\rm red}}.$$

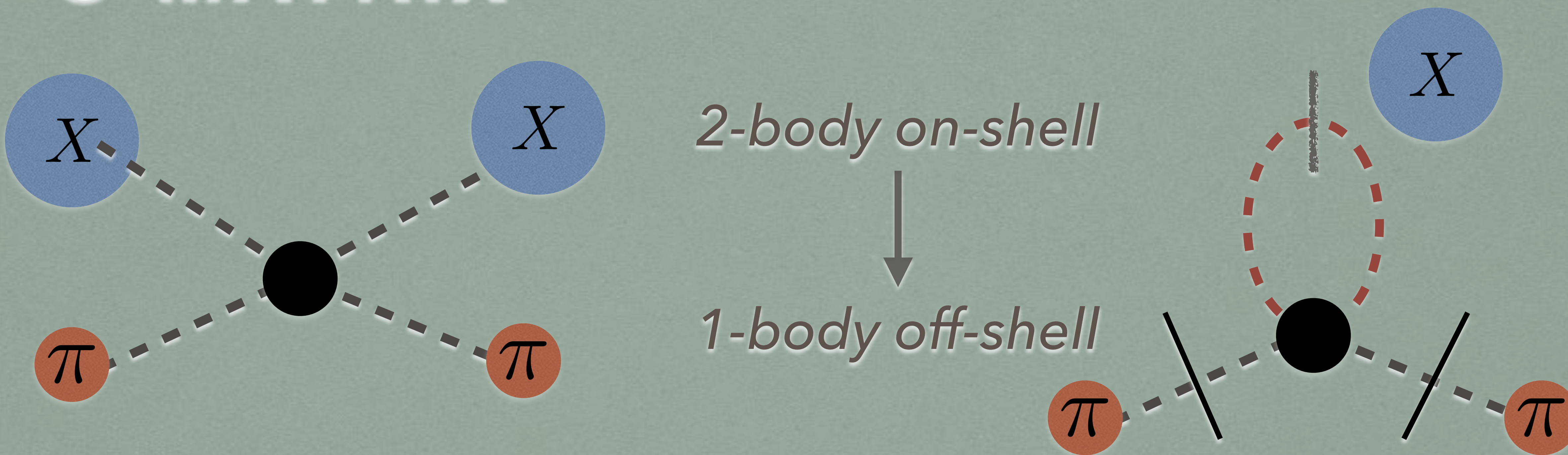
$$\begin{aligned}\Delta P &\approx \int \frac{d^3P}{(2\pi)^3}\frac{dE'}{(2\pi)}\, e^{-\beta(m_{\rm tot}+\frac{P^2}{2m_{\rm tot}}+E')} 2Q(E') \\ &= \int \frac{d^3P}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\, e^{-\beta(m_{\rm tot}+\frac{P^2}{2m_{\rm tot}}+\frac{q^2}{2m_{\rm red}})}(-T_{\rm nr}) \\ &\approx N_{\rm th}^A N_{\rm th}^B \times (-T_{\rm nr}).\end{aligned}$$

Change of pressure to due "Dressed mass"

$$\begin{aligned}\Delta P &\approx T\int \frac{d^3p_A}{(2\pi)^3}\, e^{-\beta(m_A+\frac{p_A^2}{2m_A})}(-\beta\Delta m_A) \\ &= -\Delta m_A N_{\rm th}^A \\ &= N_{\rm th}^A N_{\rm th}^B \times \frac{4\pi f}{2m_{\rm red}}.\end{aligned}$$

$$\begin{aligned}\Delta m_A &= \frac{1}{2E_A}\,{\rm Re}\,\Sigma_A(p) \\ &\approx N_{\rm th}^B \times \frac{-4\pi f}{2m_{\rm red}}.\end{aligned}$$

IN-MEDIUM EFFECTS FROM S-MATRIX



$$\Delta P = N_{\rm th}^{A}\, N_{\rm th}^{B} \times \frac{4\pi f}{2m_{\rm red}}.$$

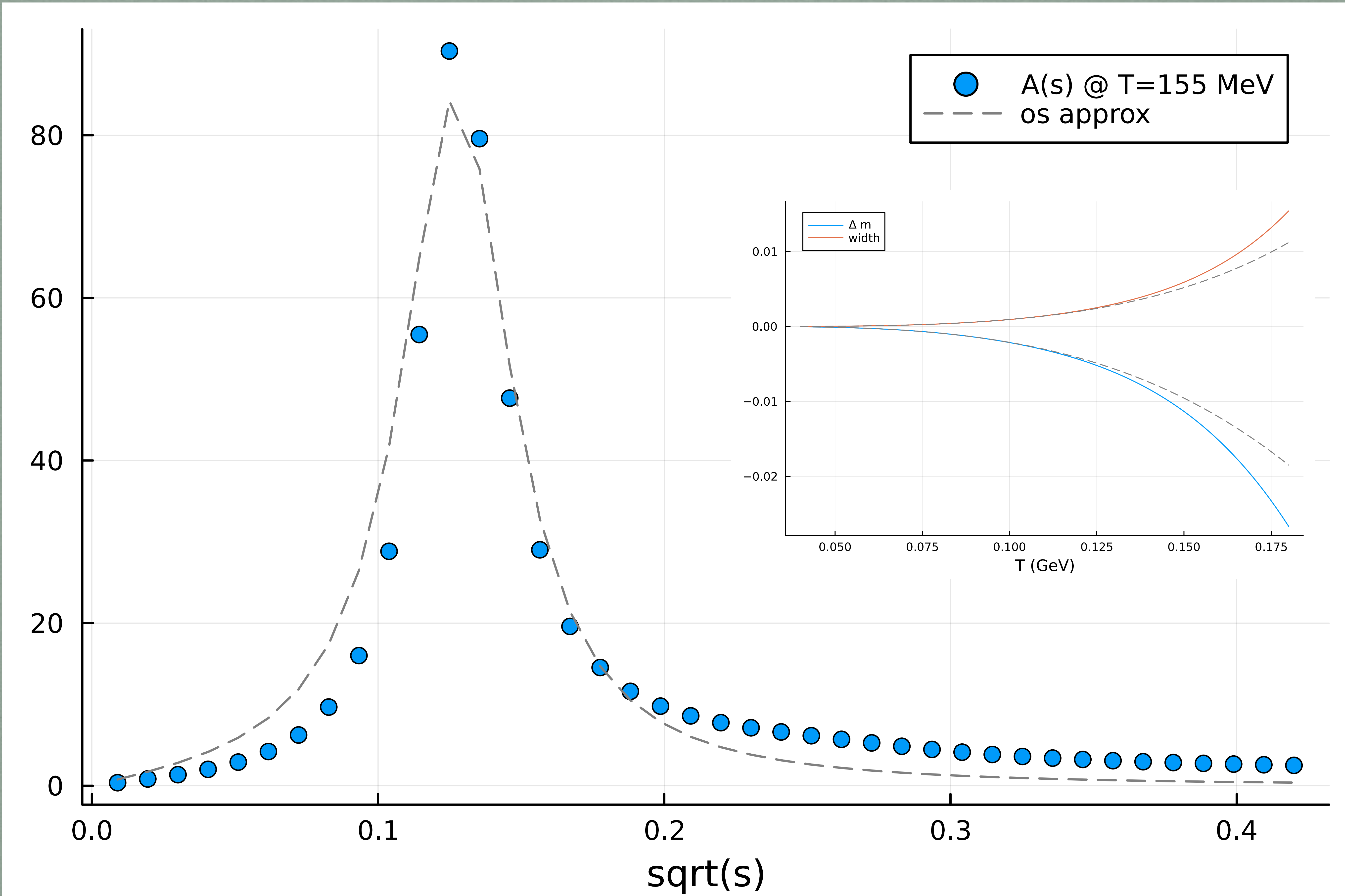
$$\Sigma_A(E_A) = \int \frac{d^3k_B}{(2\pi)^3}\frac{1}{2E_B}\, n_{\rm th}(E_B)\, T(AB \to AB).$$

$$\Delta m_A = \frac{1}{2E_A}\,{\rm Re}\,\Sigma_A(p)$$

$$\approx N_{\rm th}^{B} \times \frac{-4\pi f}{2m_{\rm red}}.$$

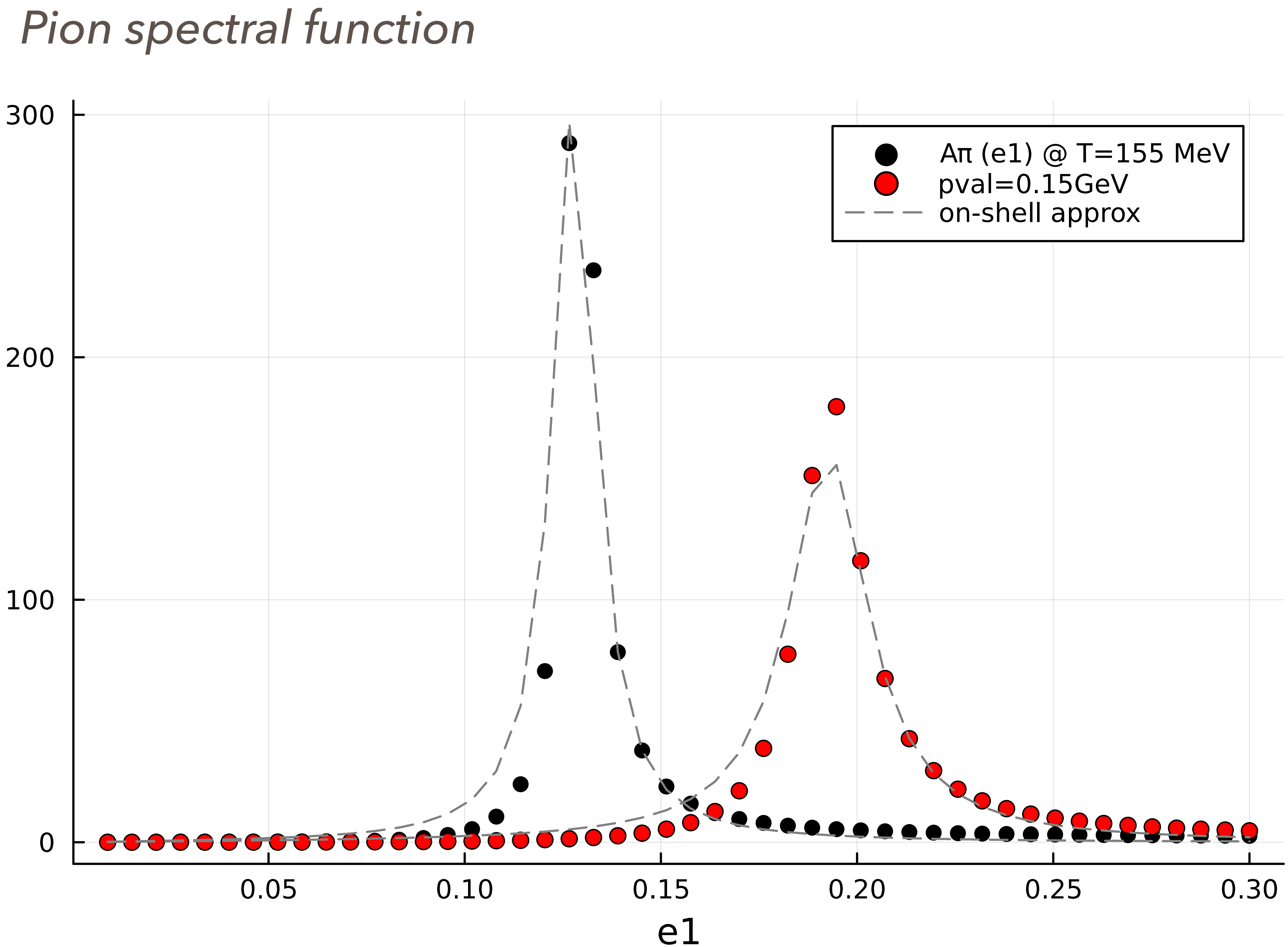
A. Schenk NPB 363(1991)

S. Jeon and P. J. Ellis PRD 58 045013 (1998)

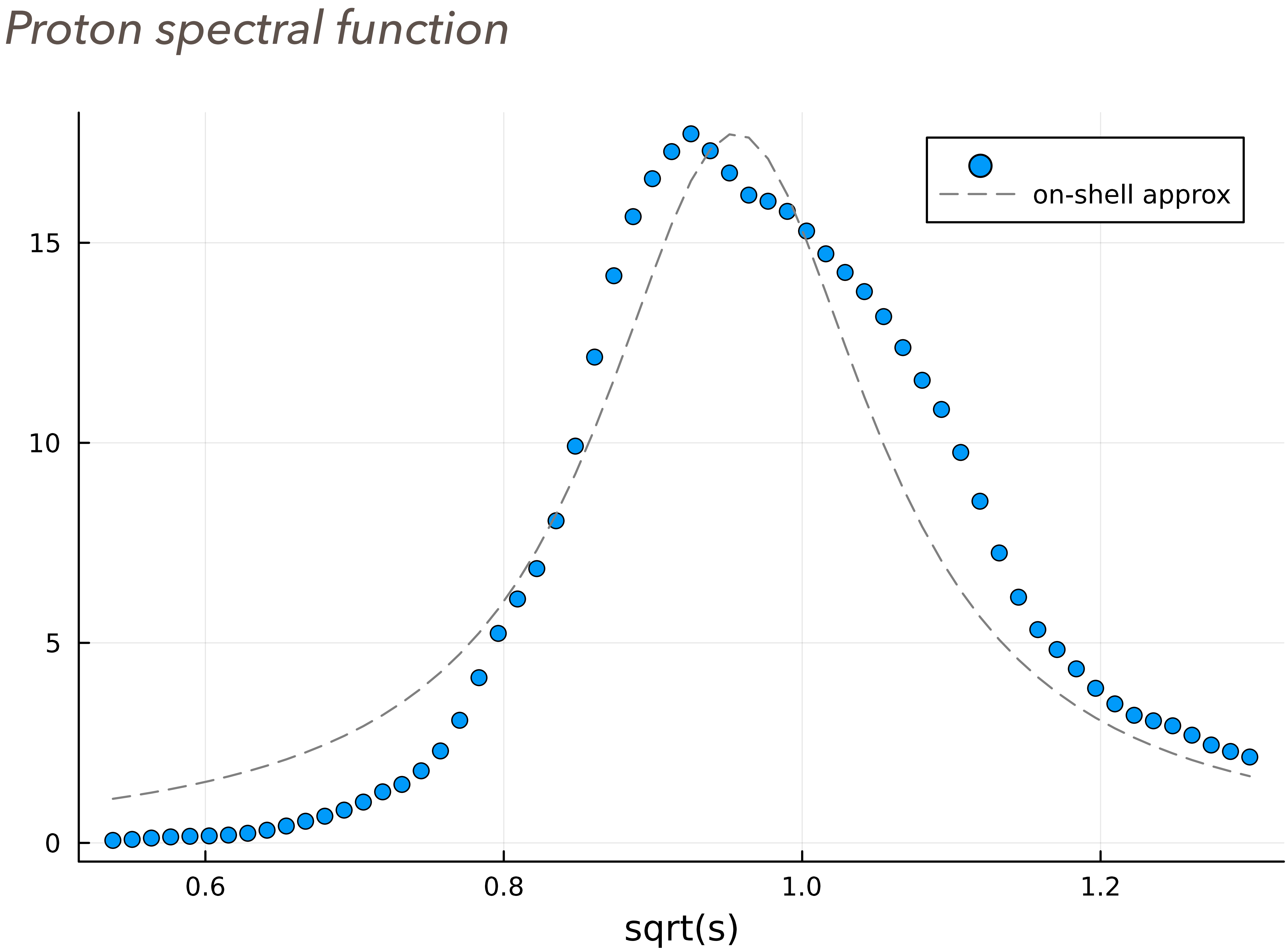


A. Schenk NPB 363(1991)

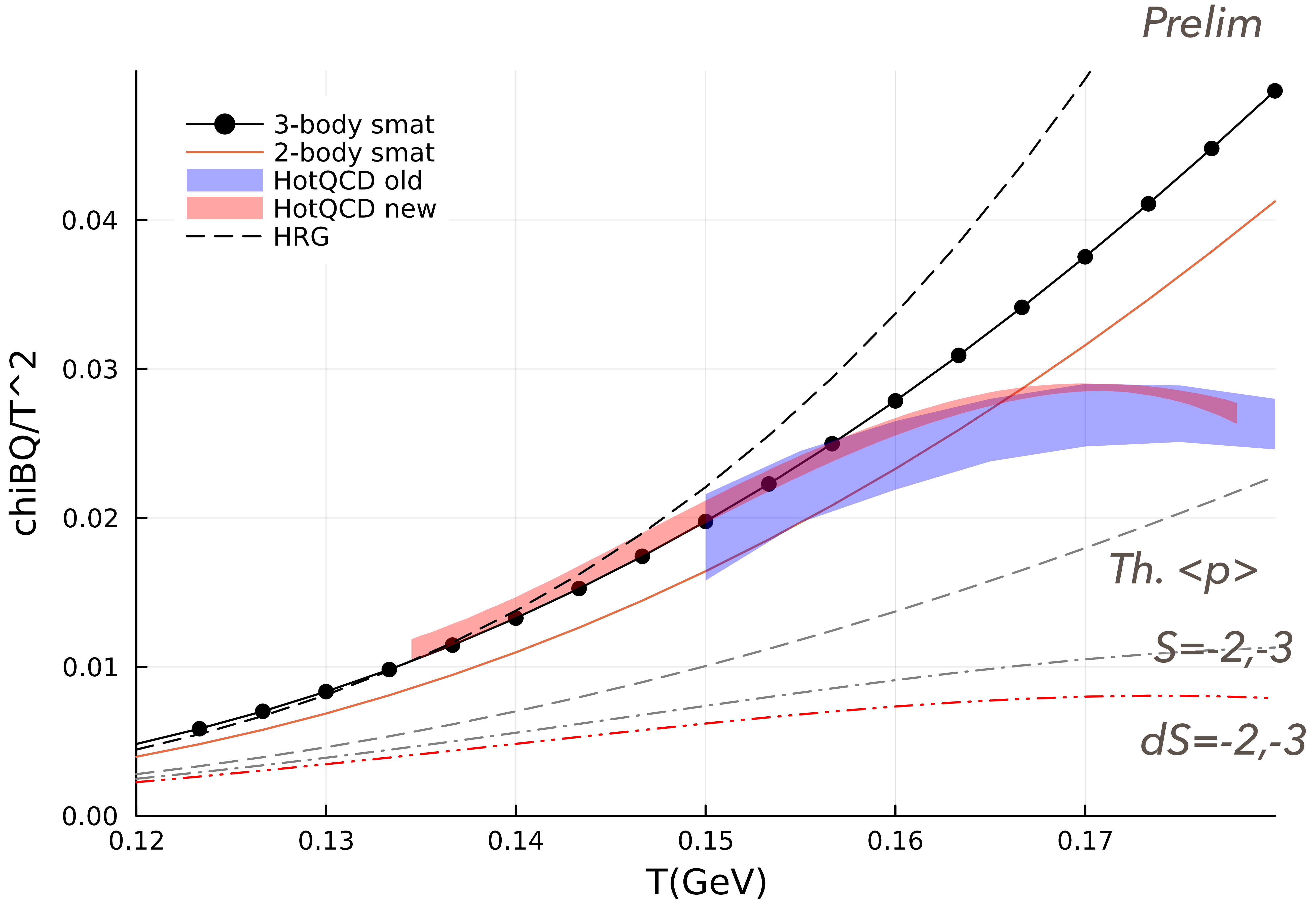
S. Jeon and P. J. Ellis PRD 58 045013 (1998)

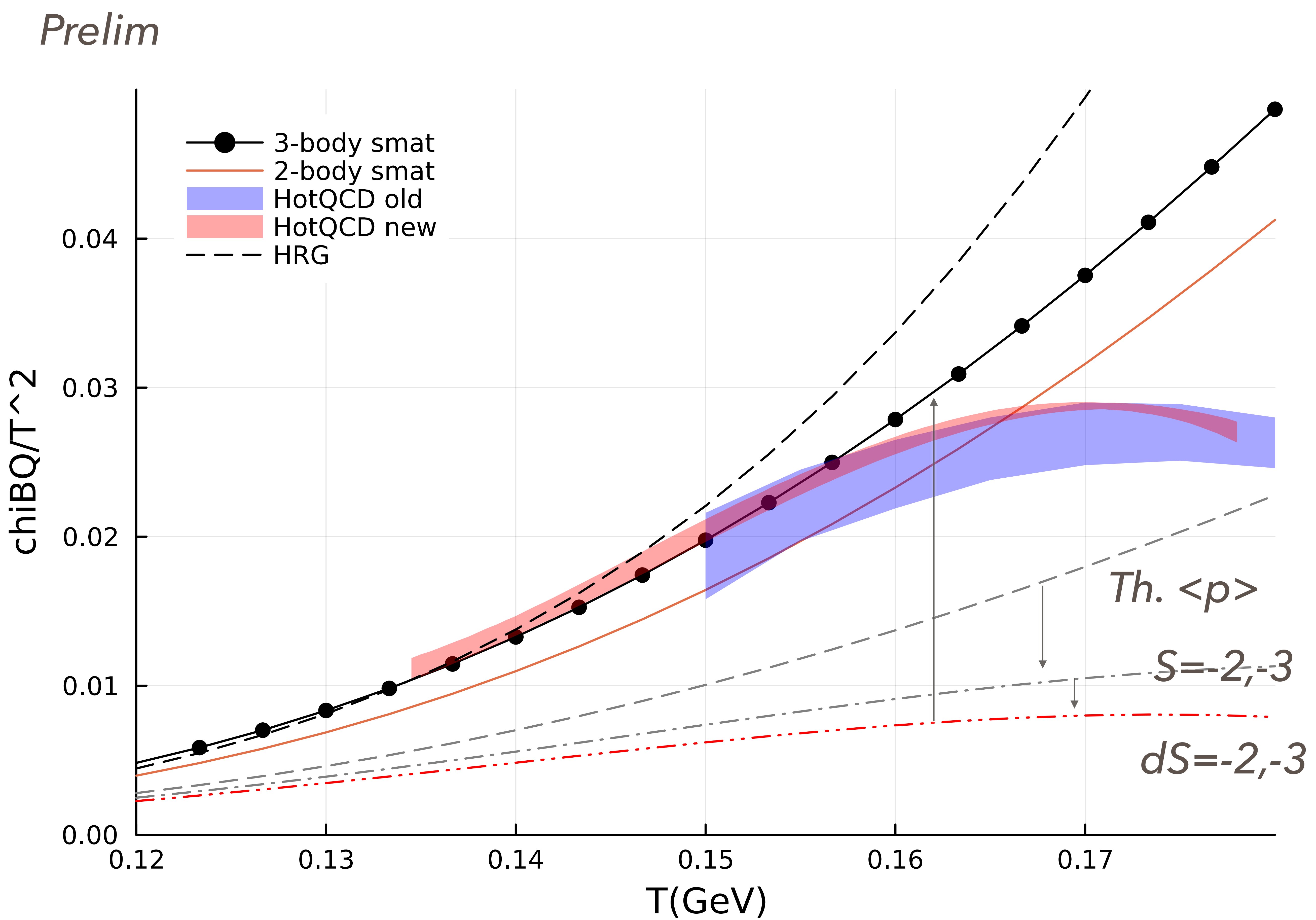


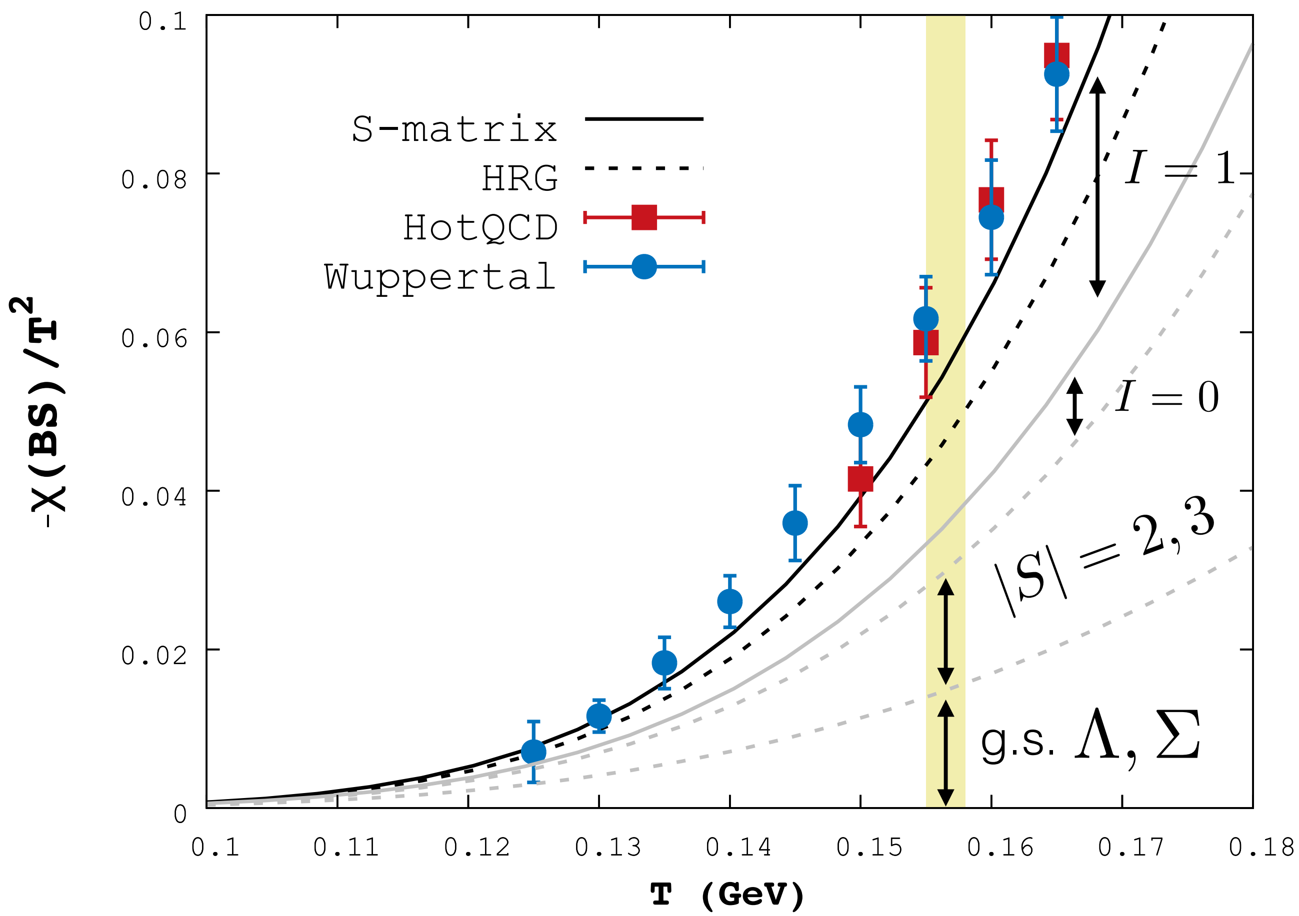
Pion spectral function
Aπ (e1) @ T=155 MeV
pval=0.15GeV
on-shell approx
300
200
100
0
0.05
0.10
0.15
0.20
0.25
0.30
e1


Proton spectral function
on-shell approx
15
10
5
0
0.6
0.8
1.0
1.2
sqrt(s)

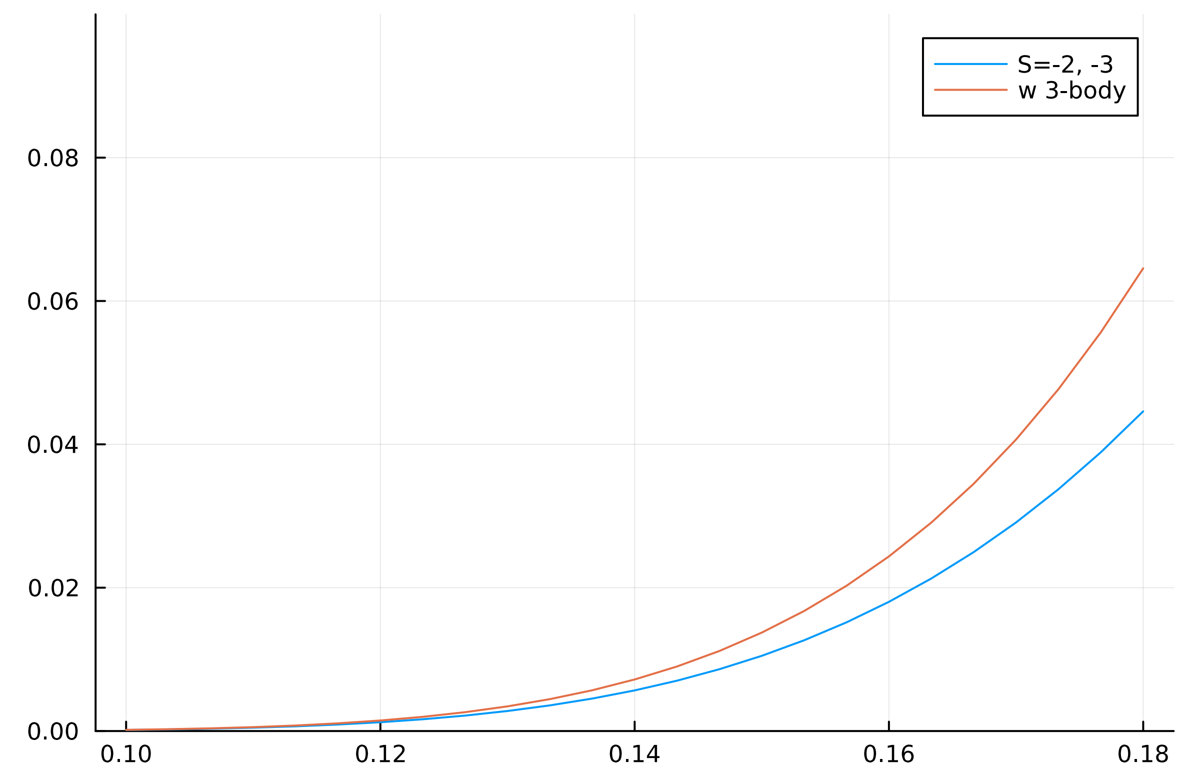
A SMALL UPDATE


Prelim
3-body smat
2-body smat
HotQCD old
HotQCD new
HRG
0.04
0.03
0.02
0.01
0.00
chiBQ/T^2
0.12
0.13
0.14
0.15
0.16
0.17
T(GeV)
Th. <p>
S=-2,-3
dS=-2,-3


Prelim
3-body smat
2-body smat
HotQCD old
HotQCD new
HRG
Th. <p>
S=-2,-3
dS=-2,-3
chiBQ/T^2
T(GeV)
0.00
0.01
0.02
0.03
0.04
0.12
0.13
0.14
0.15
0.16
0.17



-χ(BS)/T^2
T (GeV)
S-matrix
HRG
HotQCD
Wuppertal
$I = 1$
$I = 0$
$|S| = 2, 3$
g.s. Λ, Σ
0.1
0.08
0.06
0.04
0.02
0
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18



S=-2, -3
w 3-body
0.08
0.06
0.04
0.02
0.00
0.10
0.12
0.14
0.16
0.18

THANK YOU!

What about Levinsen's theorem

it must also decrease gradually away from resonances by 180° times the total number of resonances and bound states.

This is a remarkable result, but not a very useful one. It holds only for elastic scattering due to a non-relativistic central potential, but it refers to the phase shift at infinite energy, where inelastic channels are open and relativistic effects are important. There have been many attempts to generalize this theorem to models that are realistic at all energies, but so far without success.