SCATTERING PARAMETERS FROM PRODUCTION REACTIONS

November 7, 2024 | Christoph Hanhart | IKP/IAS Forschungszentrum Jülich

MOTIVATION

A systematic study of final state interaction effects allows one to

- investigate the interaction of unstable particles for very small relative momenta.

  E.g. hyperon-nucleon and hyperon-hyperon scattering lengths, addressing

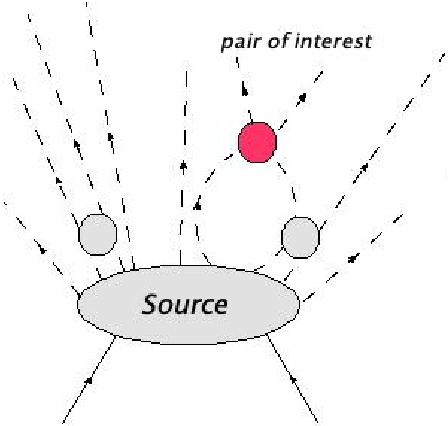
  $\rightarrow$ Flavor–$SU(3)$ breaking pattern?
  $\rightarrow$ Structure of neutron stars?

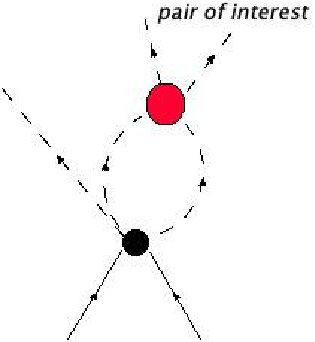
- nature of particles: bound system vs. elementary state

  $\rightarrow$ encoded in effective range parameters

- study meson–nucleus interactions: Bound states or not?

DIFFERENT OPTIONS





High energies $\Longrightarrow$
(near) elastic scattering

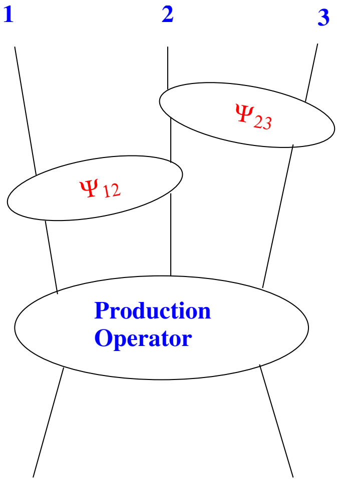
Near Threshold $\Longrightarrow$
production from point source

DIFFERENT OPTIONS

Production from	
small momentum transfer (femtoscopy)	large momentum transfer (this talk)
e.g. heavy ion or *pp* collisions	e.g. meson production in *pp* coll.
weak dependence from production	sizeable dep. from production
uncertainty difficult to quantify	controllable uncertainties
spin states with known weights	admixture of spin states unknown

In any case: Two methods with very different systematics

ABOUT LARGE MOMENTUM TRANSFER



Final state interaction

strongly energy dependent

sensitive to interactions of all subsystems; for more than 2 final particles: Dalitz plot analysis!

Production operator

weakly energy dependent;

selection rules!

(isospin, parity, Pauli principle ...)

$$\rightarrow \; d\sigma \propto |fM|^2, \text{ where } M \simeq const. \text{ and } f = f[\Psi_{ij}].$$

SCALES

Variation of M controlled by typical momentum transfer p_t

Variation of f relevant momentum range of subsystem p'

For $p' \ll p_t$ FSI minimally distorted

Use initial momentum at threshold p for p_t; $1/a$ for p'

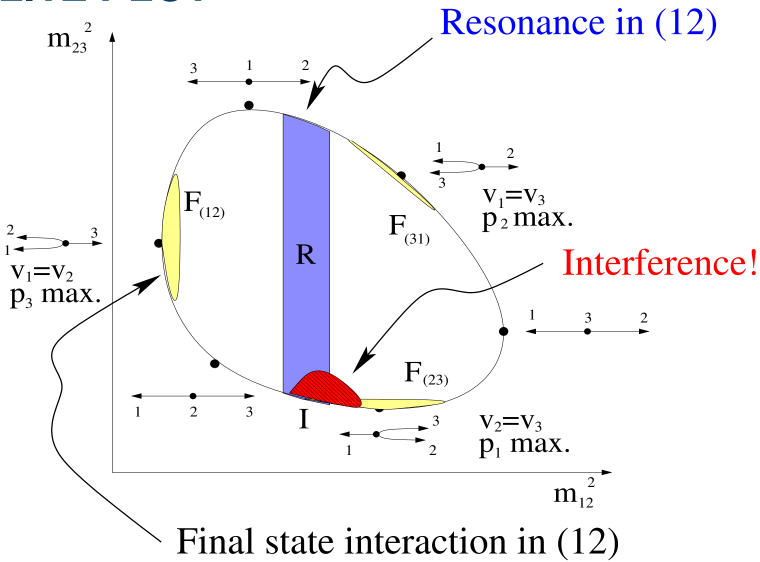
Examples: for $a \sim 1$ fm in proton-proton collisions:

$$|\vec{p}\,| = \sqrt{M_p m_x + m_x^2/4} \quad (m_x = \text{mass produced} = (\textstyle\sum_f M_f) - 2M_p)$$

- $pp \to pK^+\Lambda \quad \Rightarrow \; |\vec{p}\,| \sim 860 \text{ MeV} \Rightarrow (p'/p)^2 \sim 0.05$
- $pp \to K^0 K^0 \Sigma^+ \Sigma^+ \quad \Rightarrow \; |\vec{p}\,| \sim 1400 \text{ MeV} \Rightarrow (p'/p)^2 \sim 0.02$

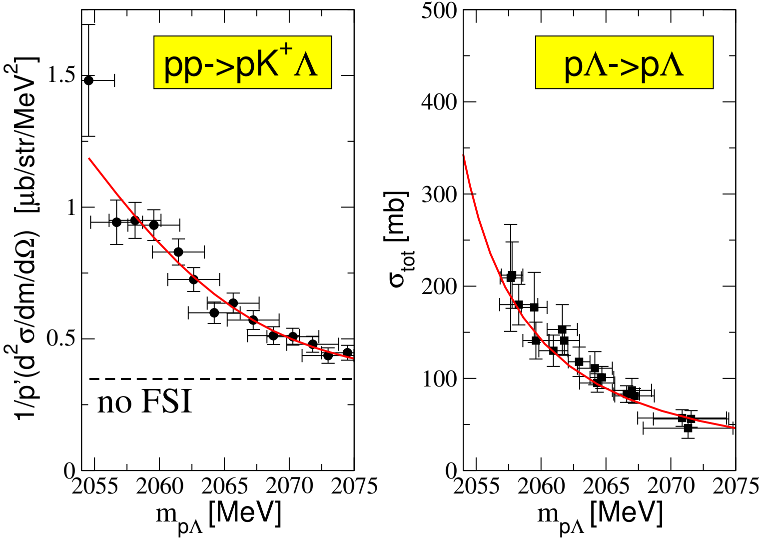
Expansion parameter decreases as system produced gets heavier!

DALITZ PLOT



m_{23}^2
Resonance in (12)
$F_{(12)}$
R
$F_{(31)}$
$F_{(23)}$
I
Interference!
$v_1=v_3$
p_2 max.
$v_1=v_2$
p_3 max.
$v_2=v_3$
p_1 max.
m_{12}^2
Final state interaction in (12)

GENERALITIES II: PROD. VS. SCATTERING

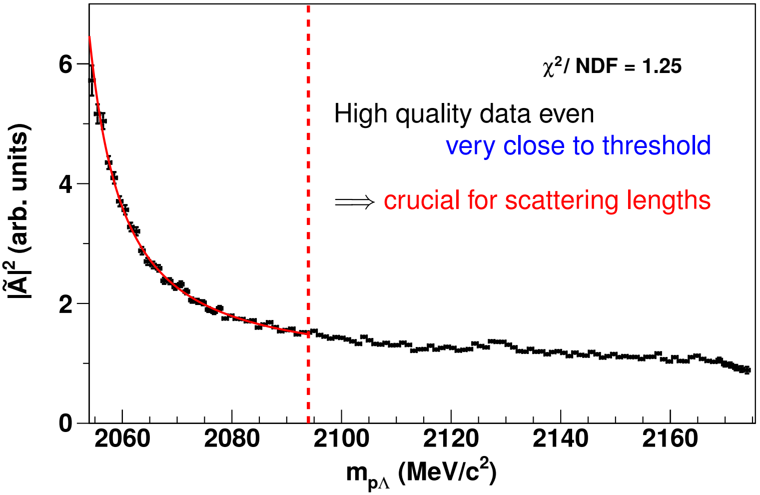


R. Siebert et al. (1994); G. Alexander et al. (1968)

GENERALITIES II: PROD. VS. SCATTERING

With more modern experiment

F. Hauenstein et al. [COSY-TOF], PRC95(2017)034001



ELASTIC INTERACTIONS

We see that production and scattering are related, but not equal

Goal:

reliable extraction of scattering parameters for unstable particles
particles from production reactions with large momentum transfer

This demands:

- Error estimation
  - Residual effect of production operator
  - Effects of resonances
  - Effects of inelastic channels
- Proper theoretical framework

$\Longrightarrow$ Use dispersion relations!

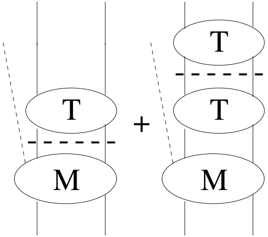
DISPERSION INTEGRAL I

$$A(s,t,m^2) = \underbrace{\frac{1}{\pi}\int_{-\infty}^{\tilde{m}^2} \frac{D(s,t,m'^2)}{m'^2 - m^2} dm'^2}_{left-hand\ cut: production} + \underbrace{\frac{1}{\pi}\int_{m_0^2}^{\infty} \frac{D(s,t,m'^2)}{m'^2 - m^2} dm'^2}_{right-hand\ cut: FSI},$$

where m_0 =production threshold, $\tilde{m}$=start of left-hand cut and

$D(s,t,m^2) = \text{Disc}(A(s,t,m^2))$ with

$$D(s,t,m^2>m_0^2) =$$



$$= ip' A T^*_{on-shell}$$

Muskhelishvili (1953), Omnes (1958), ...

DISPERSION INTEGRAL II

For this equation a solution exists:

$$A(s,t,m^2) = \exp\left[\frac{1}{\pi}\int_{m_0^2}^{\infty}\frac{\delta(m'^{\,2})}{m'^{\,2}-m^2-i0}dm'^{\,2}\right]\Phi(s,t,m^2),$$

- large momentum transfer $\rightarrow$ Φ is at most weakly m^2 dependent.

  $\Longrightarrow$ included into uncertainty estimate

- The FSI effect in terms of the (elastic) scattering phaseshift.

  The factor can be interpreted as wavevfunction at the origin (inverse Jost function).

  $\Longrightarrow$ large m' region into uncertainty estimate

FSI enhancement $\leftrightarrow$ elastic phase-shift

THREE STRATEGIES

Three different strategies to proceed:

1. Assume that phaseshifts are given by effective range expansion;
   $p' \operatorname{ctg}(\delta(m^2)) = 1/a + (1/2)rp'^{\,2}$ (sign!) $\Rightarrow$
   $$A(m^2) = \frac{(p'^{\,2} + \alpha^2)r/2}{1/a + (r/2)p'^{\,2} - ip'}\Phi(m^2)\ ,$$
   $\alpha = 1/r(1 + \sqrt{1 + 2r/a})$ (*Jost–function method*)
   Sibirtsev et al. (1996, 2004), Shyam et al. (2001), ...
2. Ignore numerator (*Watson method*) Goldberger, Watson 1964
3. Invert the equation and express phaseshifts through observables Geshkenbein (1969)
   and restrict integration range! (*Integral*) Gasparyan et al. (2004)ff

This is the most systematic approach in line with goal

SCATTERING LENGTH

It is possible to invert the Omnes-function: Geshkenbein (1969), Gasparyan et al. (2004)

$$\delta_S(m^2) = -\frac{1}{2\pi}\int_{m_0^2}^{m_{max}^2}\frac{dm'^{\,2}}{m'^{\,2}-m^2}\sqrt{\frac{(m_{max}^2-m^2)(m^2-m_0^2)}{(m_{max}^2-m'^{\,2})(m'^{\,2}-m_0^2)}}\log\left\{\frac{1}{p'}\left(\frac{d^2\sigma_S}{dm'^{\,2}dt}\right)\right\}$$

with $\lim_{m^2\to m_0^2}\delta_S(s) = a_S p(s)$ and S denoting a specified spin state

we chose: $\epsilon_{max} = m_{max} - m_0 \simeq 1/(2\mu a^2) \approx 40$ MeV for $a \sim 1$ fm

Estimates for uncertainties: $\delta a^{(th)} = \delta a^{(lhc)} + \delta a^{m_{max}} \sim 0.3$ fm since

$$|\delta a^{m_{max}}| = \frac{2}{\pi p'_{max}}\left|\int_0^\infty \frac{\delta(y)dy}{(1+y^2)^{(3/2)}}\right| \leq \frac{2}{\pi p'_{max}}|\delta_{max}| \sim 0.2 \text{ fm}$$

$$\delta a^{(lhc)} \sim (p'_{max}/p^2) \sim 0.05 \text{ fm}$$

using $\delta_{max} = 0.4$ rad (for ΛN - see next pages) and $p' \sim 1/a$

TESTING THE METHOD

Gasparyan, Haidenbauer, CH PRC72(2005)034006

We want to test the dispersive method

$\Longrightarrow$ Can only be done if we know true parameters

Our strategy:

- Generate pseudo data for $d\sigma/dm^2$ for various models for ΛN (and NN)
- Extract scattering length.
- Compare to exact value.

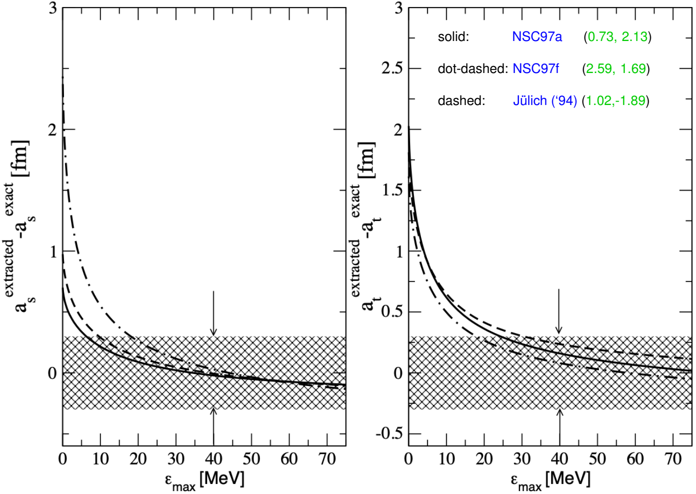
Note: any working method should work for any realistic model

We use $S = 0$ & $S = 1$ for YN from (where in green: scattering lengths in fm)

NSC97a (0.73,2.13), NSC97f (2.59,1.69), Jülich ('94) (1.02,-1.89)

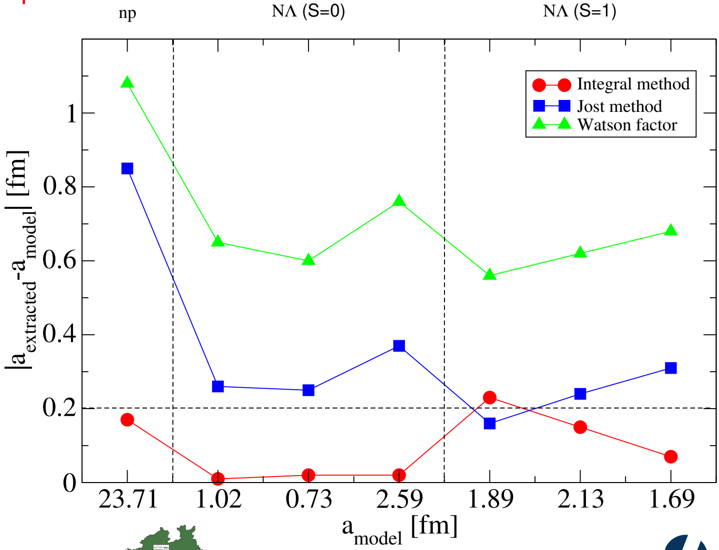
TESTING THE DISPERSIVE METHOD

Calculations for production operator with π and K exchange!



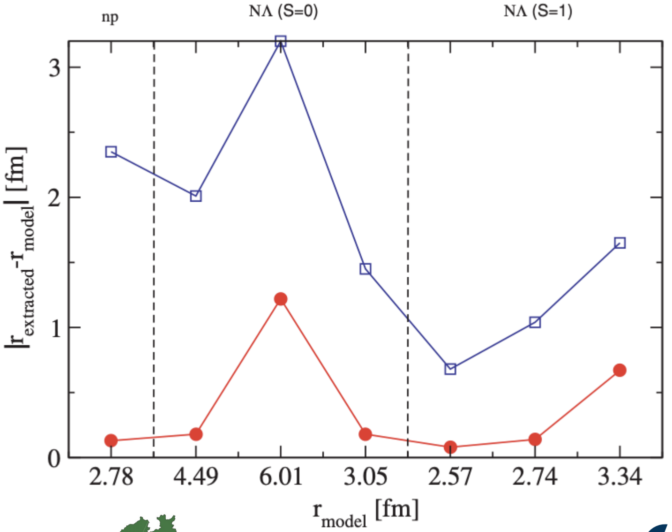
COMPARISON OF METHODS

Pointlike production



EFFECTIVE RANGE

$\Longrightarrow$ enters as $a^2((2/3)a - r_e)$



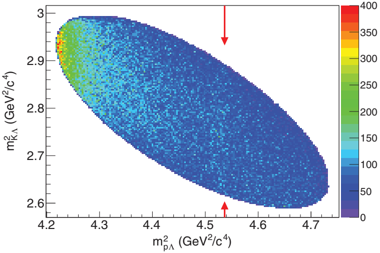
$a_{\Lambda p}$ FROM $pp \to pK\Lambda$

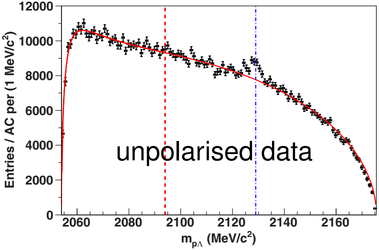
F. Hauenstein et al. [COSY-TOF], PRC95(2017)034001

S=1 and S=0 possible in final state with unknown relative weight

Unpolarised data give access only to effective scattering length a_{eff}

Gasparyan, Haidenbauer, CH PRC72(2005)034006





Procedure: Fit $m_{p\Lambda}$ spectrum with $\exp\{C_0 + C_1/(m^2_{p\Lambda} - C_2)\}$, then

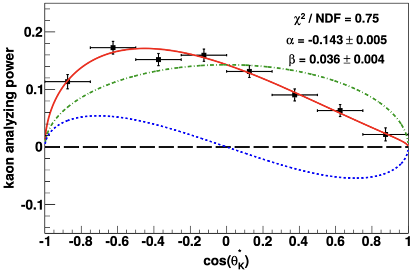
$$a_{\text{eff}} = \frac{C_1}{2}\sqrt{\frac{m_0^2(m_{max}^2 - m_0^2)}{m_p m_\Lambda (m_{max}^2 - C_2)(m_0^2 - C_2)^3}} \Longrightarrow -1.38^{+0.04}_{-0.05}\,{}_{\text{stat.}} \pm 0.22_{\text{syst.}} \pm 0.3_{\text{theo.}}\ \text{fm}$$

SPIN TRIPLET $a_{\Lambda p}$ FROM $\vec{p}p \to pK\Lambda$

F. Hauenstein et al. [COSY-TOF], PRC95(2017)034001

Spin=1 can be isolated from analysing power

Gasparyan, Haidenbauer, CH PRC72(2005)034006



$$A_{0y}\sigma_0 = -\frac{1}{4}k^2\beta\sin(2\theta)\cos(\phi) + \sin(\theta)\cos(\phi)(\text{spin triplet only}) ,$$

Needs for each $m_{p\Lambda}$ bin angular dist.

Procedure: Fit $m_{p\Lambda}$ spectrum with $\exp\{C_0 + C_1/(m_{p\Lambda}^2 - C_2)\}$, then

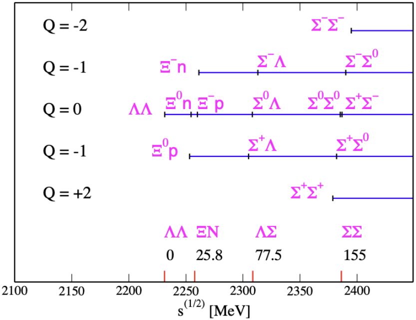
$$a_t = \frac{C_1}{2}\sqrt{\frac{m_0^2(m_{max}^2 - m_0^2)}{m_p m_\Lambda (m_{max}^2 - C_2)(m_0^2 - C_2)^3}} \Longrightarrow -2.55^{+0.72}_{-1.39}{}_{\text{ stat.}} \pm 0.6_{\text{syst.}} \pm 0.3_{\text{theo.}} \text{ fm}$$

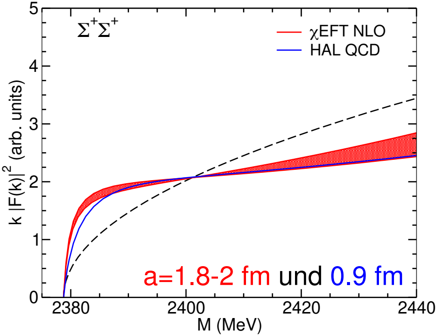
Combined analysis of femtoscopy and scattering: $-1.4 \pm 0.2 \pm 0.?_{\text{theo.}}$ fm

see Mihaylov, Haidenbauer, Sarti PLB850(2024)138550

THE $S = -2$ SYSTEMS

Moving to $S = -2$





J.Haidenbauer, private communication

- Needs high statistics and high resolution
- $\Sigma^{\pm}\Sigma^{\pm}$ and $\Lambda\Lambda$ are identical fermions $\Longrightarrow$ *S*-wave must be spin zero

  Note: repulsive Coulomb interaction can be accounted for

  Gasparyan, CH, Haidenbauer, Phys.Rev.C 72 (2005) 034006
- Threshold difference $\Lambda\Lambda$-ΞN rather small

OPPORTUNITIES WITH PROTONS AT SIS100

The high initial energy ($\sqrt{s_{\max}} = 7.5$ GeV) promises access to

- $pp \to ppJ/\psi$ and the pJ/ψ interaction $\sqrt{s} > 5$ GeV

  $\Longrightarrow$ discovery channel of $\bar{c}c$ pentaquarks & role of $\Lambda_c D^{(*)}$ channels

- $pp \to p\Sigma_c^{(*)}\bar{D}^{(*)}$ and the $\Sigma_c^{(*)}\bar{D}^{(*)}$ interaction $\sqrt{s} > 5.6$ GeV

  $\Longrightarrow$ formation of $\bar{c}c$ pentaquarks

- $pp \to \bar{K}^0\bar{K}^0\Sigma^+\Sigma^+$ and the $\Sigma^+\Sigma^+$ interaction (S=0 only!) $\sqrt{s} > 3.4$ GeV

  $\Longrightarrow$ closely SU(3) related to pp scattering

- .... certainly many more

Note: Measurements need to be well above threshold, but not too high ...

Challenge: Needs high resolution for small relative momenta and
high statistics

SUMMARY

For elastic interactions dispersion integrals allow one to connect

scattering data to FSI effects.

Method allows for error estimates.

Comparison with Femtoscopy!

Important to keep in mind:

- Dalitz plot required to control crossed channels.
- High statistics and high resolution needed
- employ polarization observables to project on spin states;
  e.g. in case of $pp \to pK\Lambda$ and $\gamma d \to K^+\Lambda n$ single spin observables sufficient to isolate spin triplet.