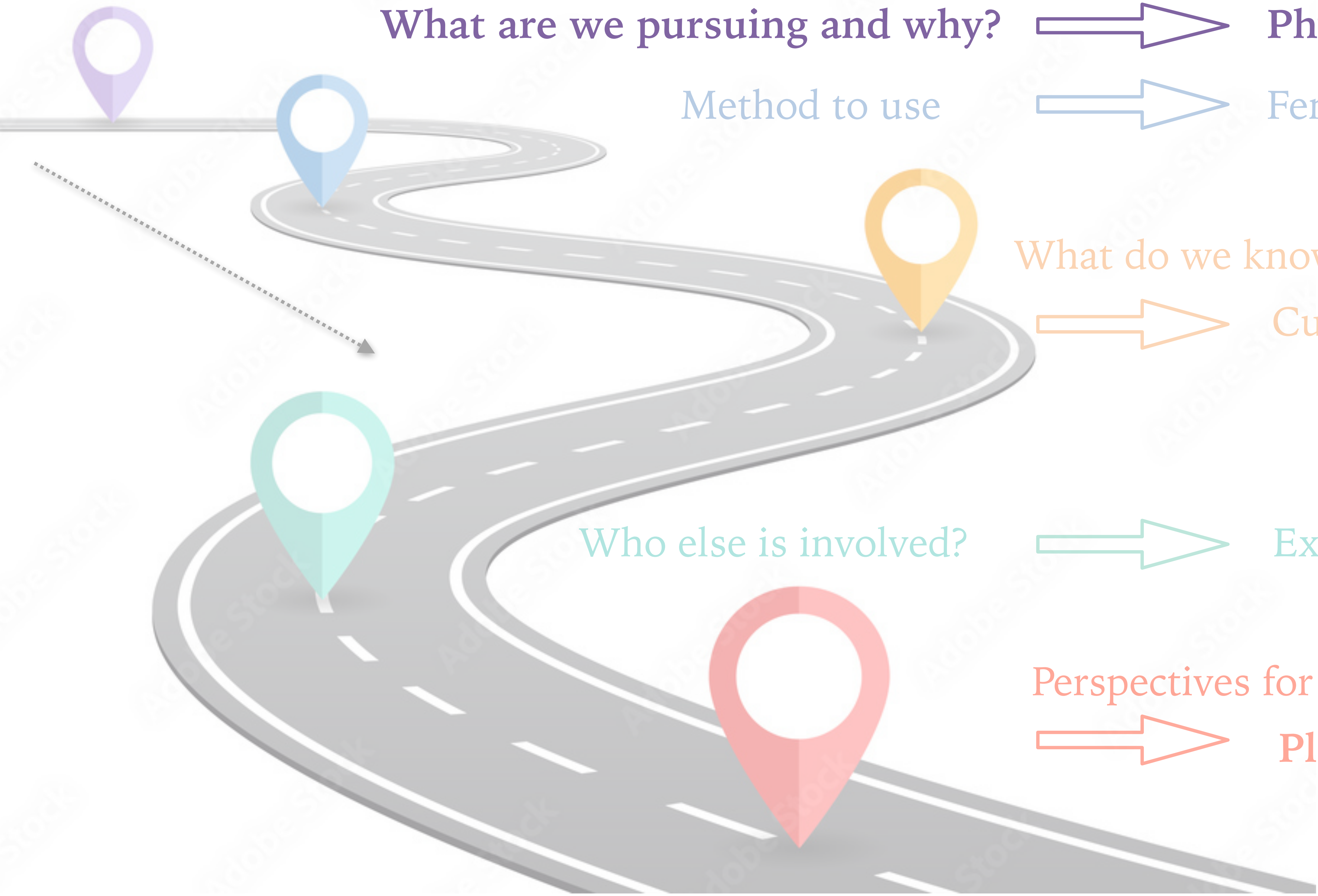
Probing final state interactions with femtoscopy

Hanna Zbroszczyk

Warsaw University of Technology

QCD at FAIR workshop, Darmstadt, November 11-14, 2024

Road map



What are we pursuing and why? ⇒ **Physics motivation**

Method to use ⇒ Femtoscopy

What do we know now?
⇒ Current status

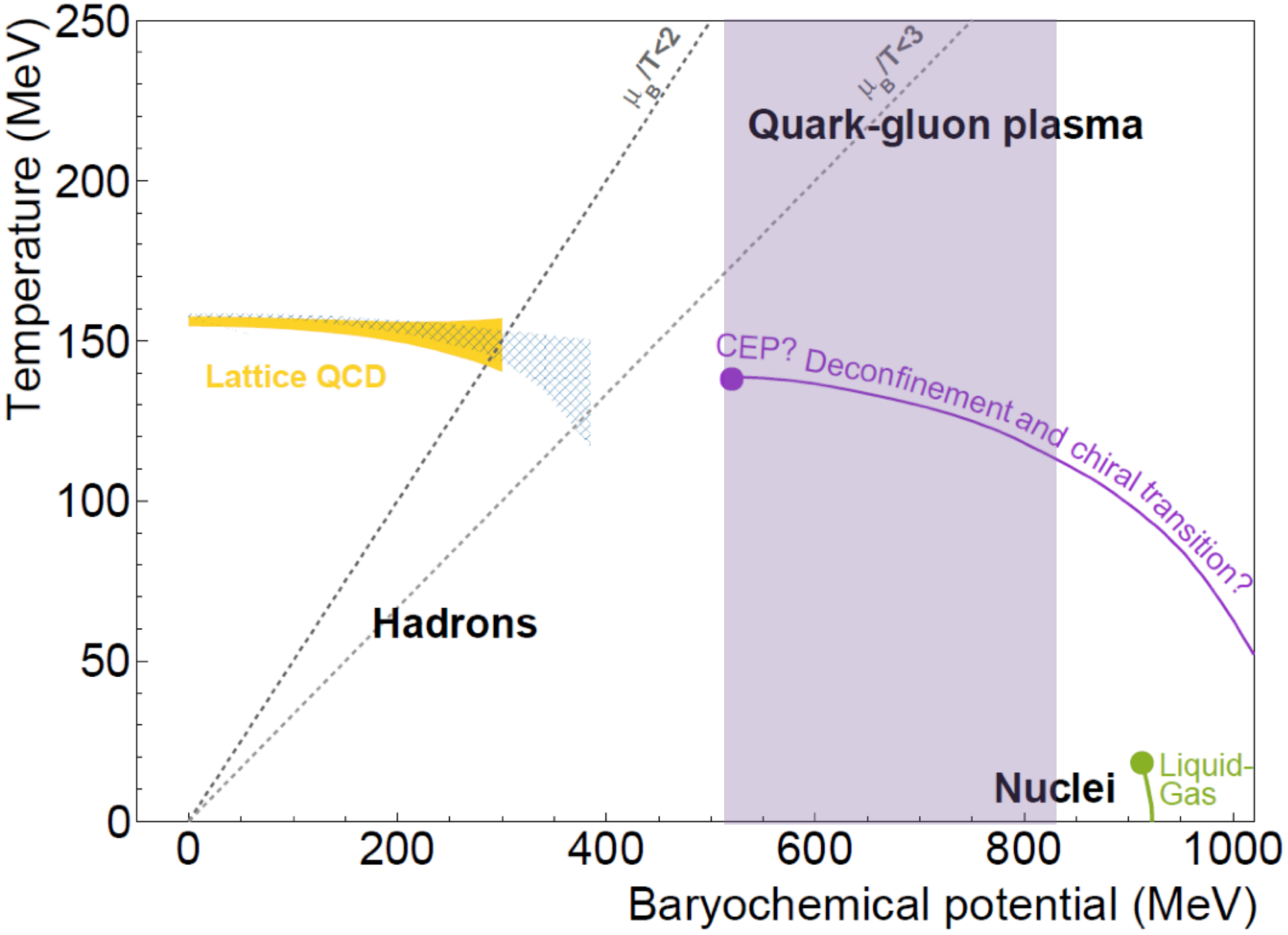
Who else is involved? ⇒ Experimental facilities

Perspectives for future
⇒ **Plans**

QCD phase diagram

Low μ_B, hight T:

- **Cross-over** transition from hadronic to quark matter - comprehensive studies of **QGP** properties
- No **critical point** anticipated for $\mu_B/T < 3$ (LQCD)

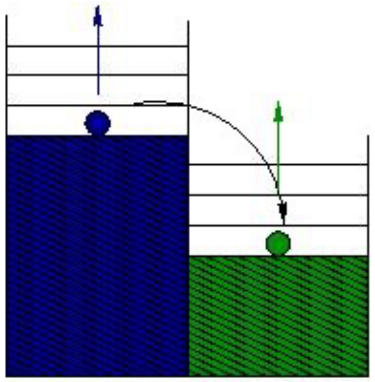


High μ_B, low T:

- Unknown **phase structure** (first-order phase transition, critical point possible, mixed phases, new phases, …)
- Properties of matter to determine
- Characteristics of hadrons
- Equation of State (**EoS**) to establish
- Neutron Star (**NS**)

Bazavovet al.[HotQCD], PLB 795 (2019) 15-21
Dinget al., [HotQCD], PRL 123 (2019) 6, 062002
Borsanyiet al., PRL125(2020)5,052001
Isserstedt et al. PRD 100 (2019) 074011
Gao, Pawlowski, PLB 820 (2021) 136584

NS puzzle



$M_{NS} \approx 1 \div 2\ M_{\odot}$

$R \approx$ 10-12 km

$\rho \approx 3 \div 5\ \rho_0$

$\rho_0 \approx 2.8 \times 10^{14}$ g/cm^3

- Observation of **NS** indicates their **mass** $\sim 2M_{\odot}$

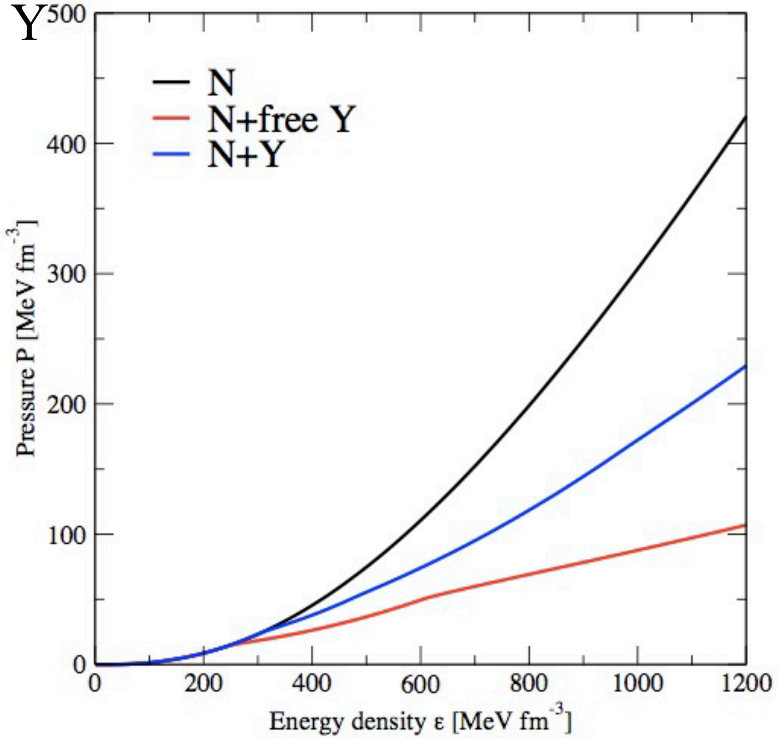
  (Shapiro-delay: Post-Keplerian parameters of orbits)
- **Hyperons:** Expected in core of NS, the conversion of N into Y is energetically favorable
- **Appearance of Hyperons:** The presence of Y alleviates Fermi pressure, resulting in a EoS and a reduction in NS mass (inconsistent with observations)

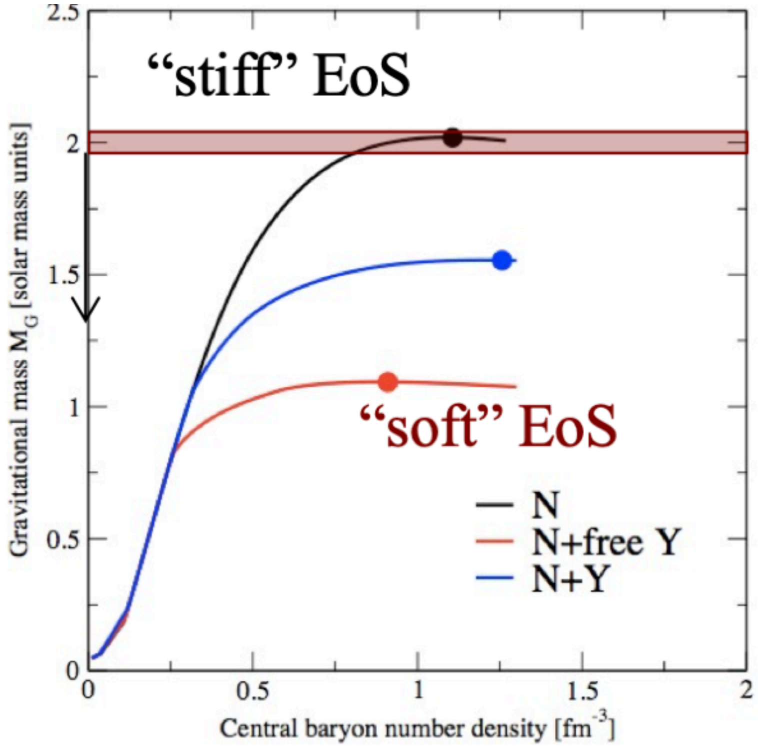
  Can they still be considered as components of NS?

- **Proposed Solution:** A mechanism that provides additional pressure to ensure a stiffer EoS

  One emergent mechanism involves many-body interactions, such as YN, YY, NNY, NYY

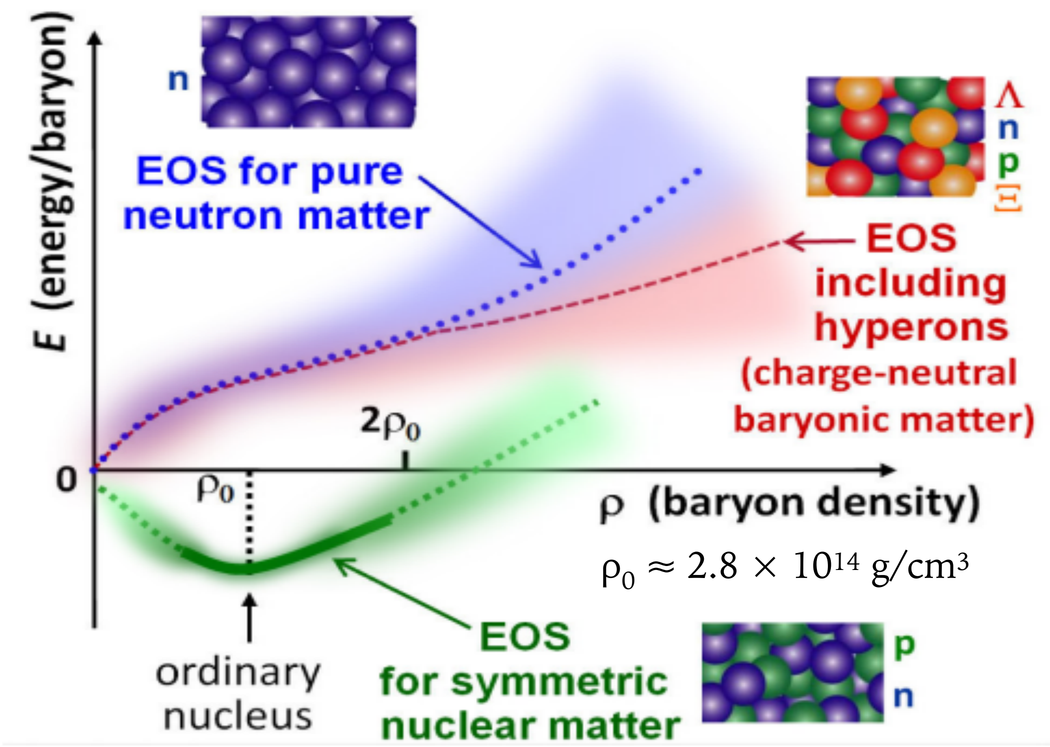
  (Other: hypersonic three-body forces, Quark Matter Core - a transition to deconfined phase below hyperon threshold in density)





Neutron star (NS) puzzle

H.Tamura, JPS Conf. Proc. , 011003 (2014)



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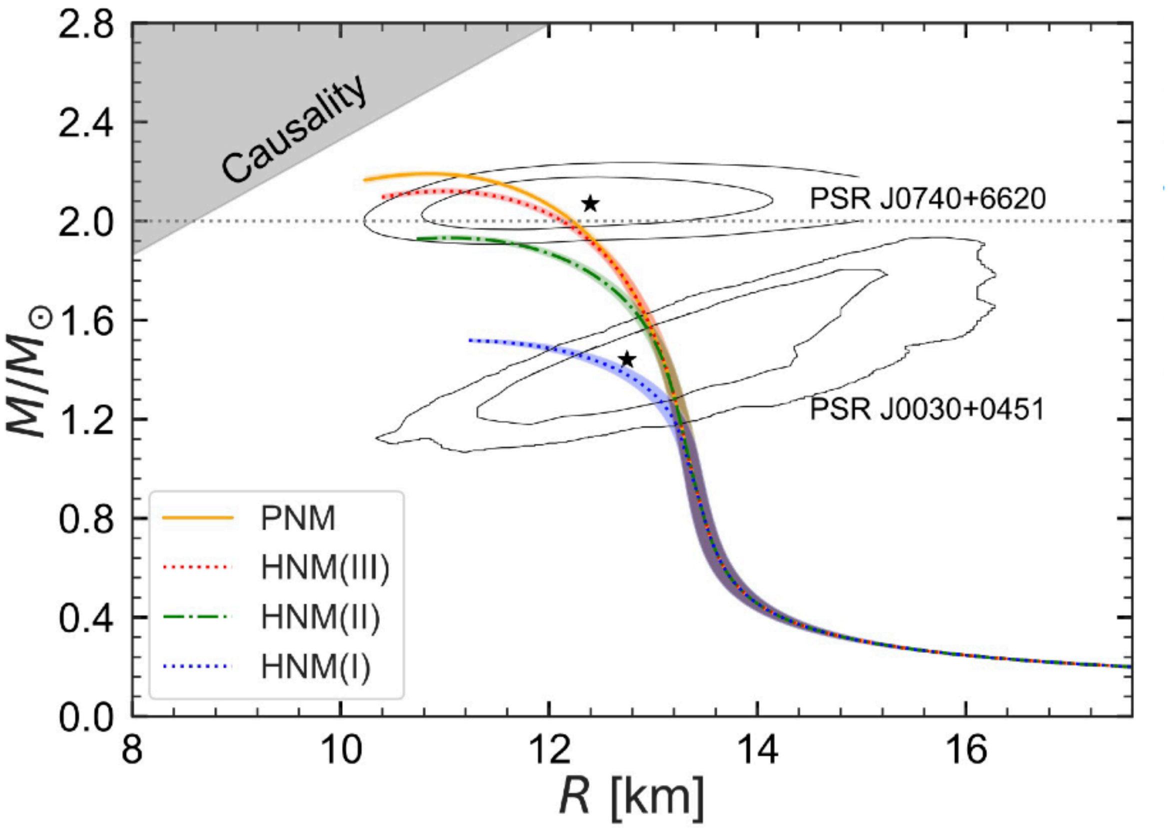

„To establish the EoS applicable to the neutron star has been one of the most important subjects in nuclear physics for a long time but has not been achieved yet." T. Hamura

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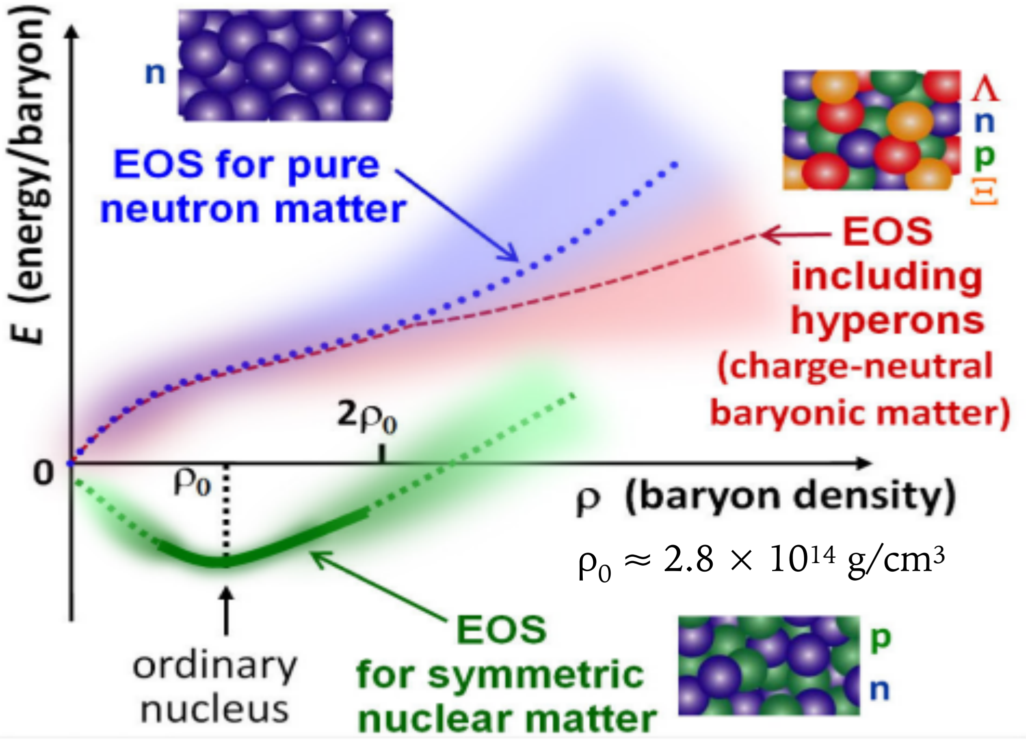

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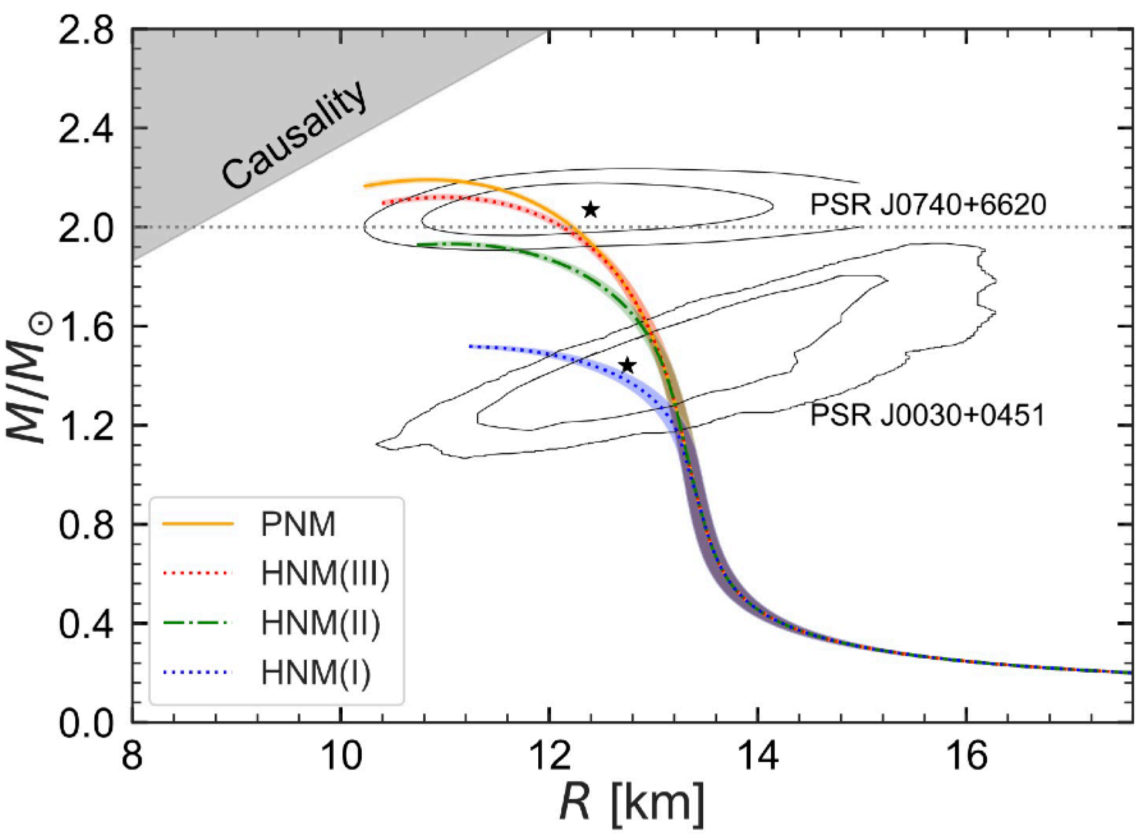

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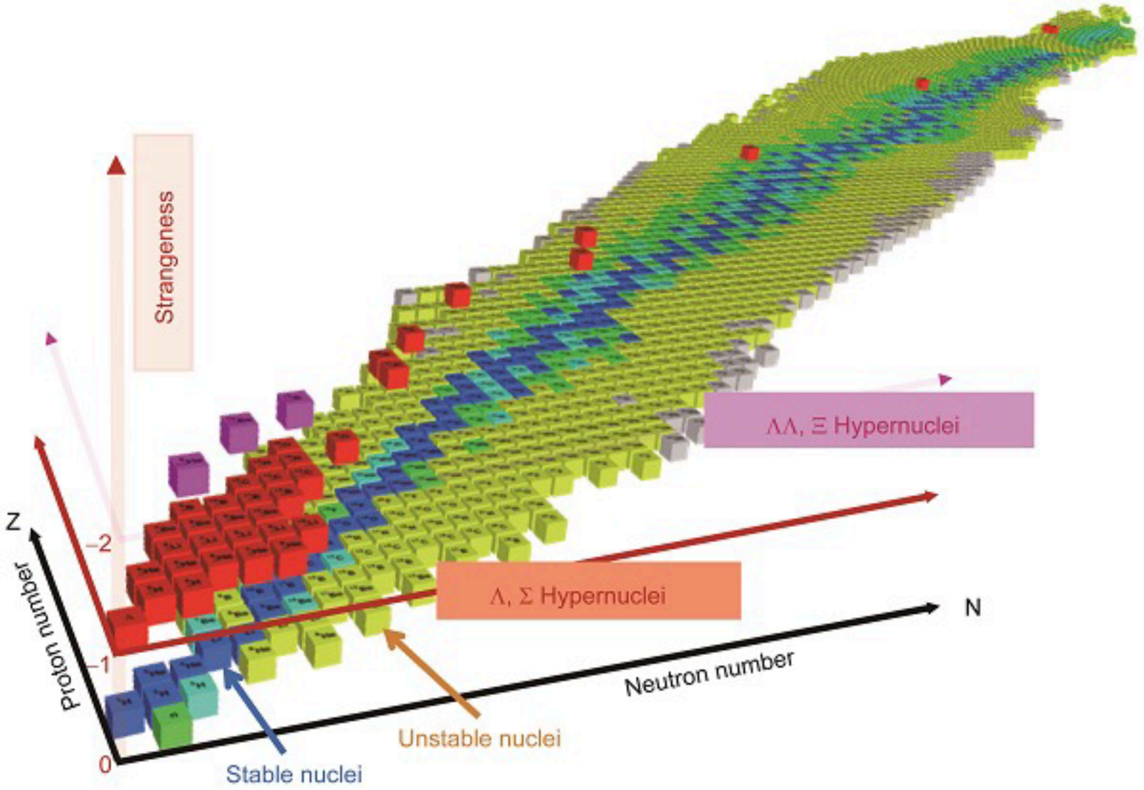


M. Kaneta, Department of Physics, Tohoku University, Japan

Hypernuclei are pivotal for the EoS of the NS

- How do nuclei and hyper-nuclei form?
- What are their characteristics?
- How do nuclei (N) and hyperons (Y) interact?

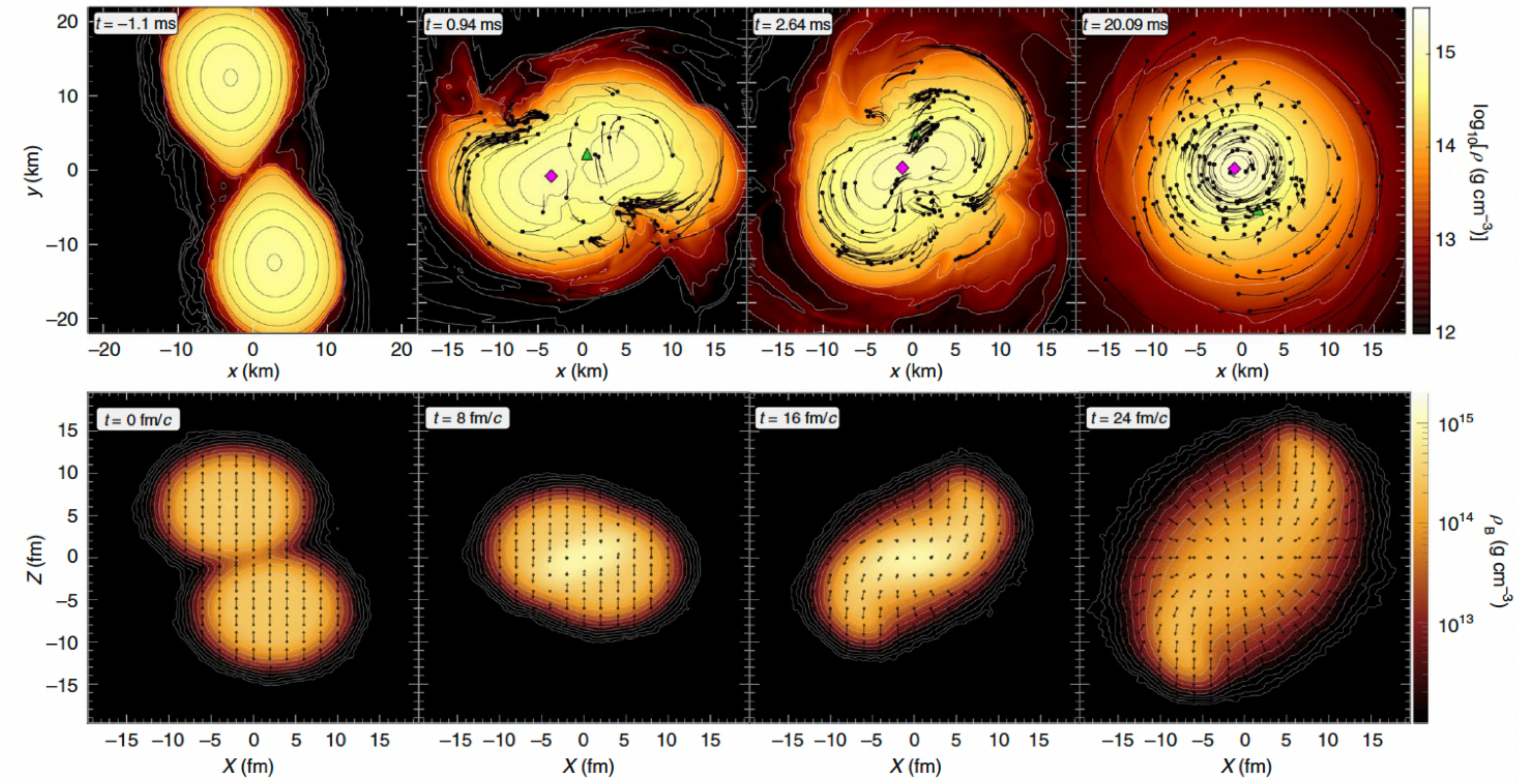
NSM and HIC

Top row: simulation of **NS mergers (NSM)**
2 NSs of 1.35 M⊙ each,
merging into a single object (2R ~ 10 km, $n \sim 5n_0$, $T \leq 20$ MeV).
Overlap region: $t \sim 20$ ms, $n \sim 2n_0$, $T \sim 75$ MeV
- max. temperature
- max. density

Bottom row: non-central **Au+Au collision** at $\sqrt{s_{NN}} = 2.42$ GeV
$n \simeq 3n_0$, $T \simeq 80$ MeV

Artist's depiction of a neutron star collision after inspiral, NASA/Swift/Dana Berry

HADES, Nature Phys. 15, 1040–1045 (2019)

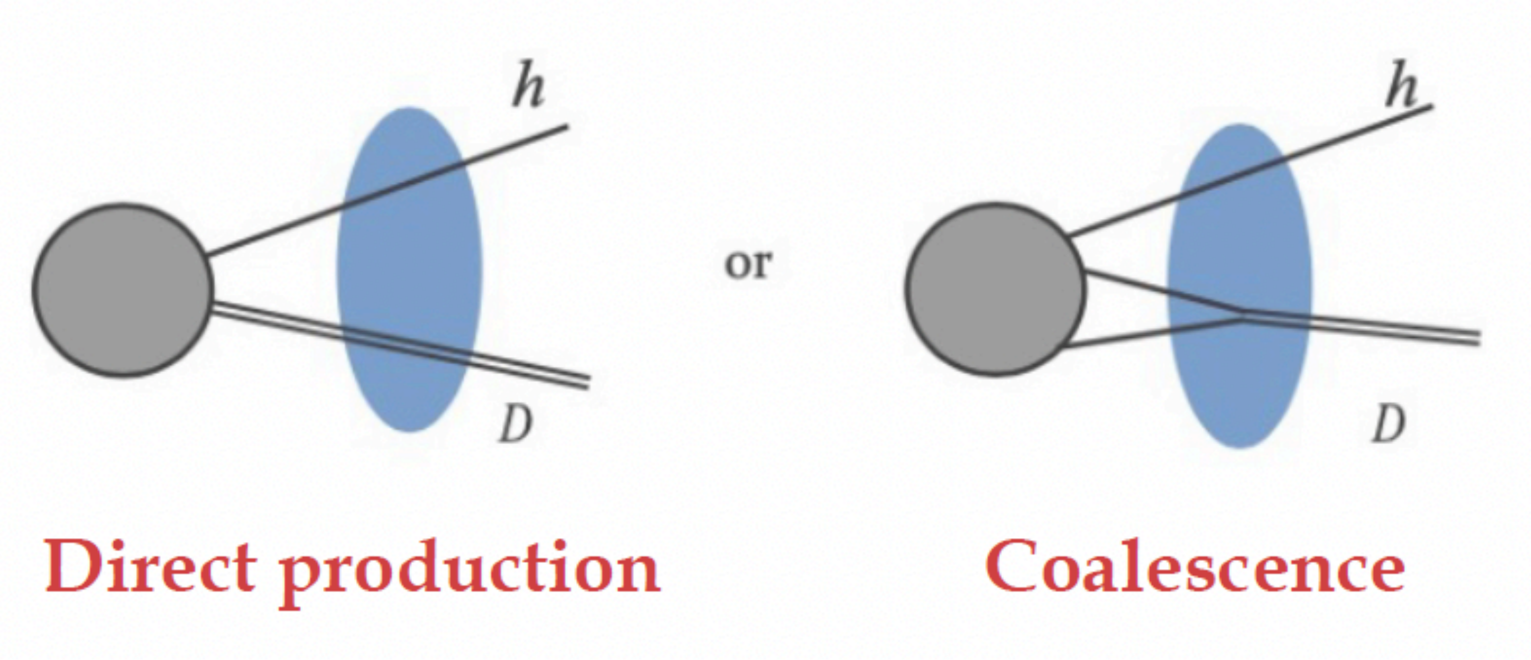


Space and **time** scales vastly contrasting (**km**-NS / **fm**-HIC - 18 orders of magnitude; **duration** - 20 orders of magnitude)

Similar **densities** and **temperatures** achieved

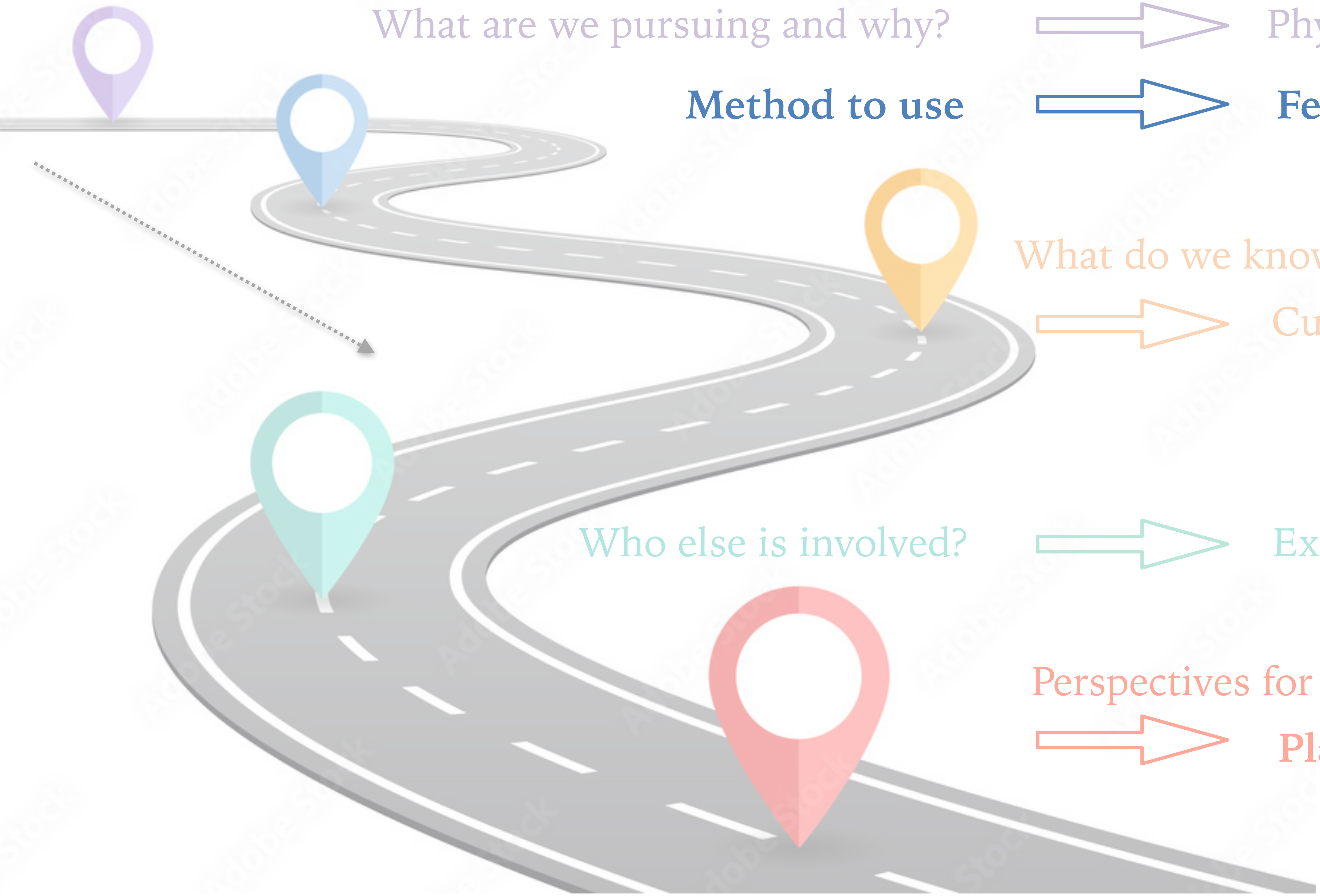
Light nuclei production mechanisms

1) A systematic measurement of **p-p**, **p-d**, and **d-d** correlations may tell us whether **deuterons** are directly emitted from the fireball or formed due to final-state interactions
2) It is important to understand if we can consider two- or more-body interactions
3) Learning more about deuteron formation, we can move towards other light nuclei



S. Mrówczyński and P. Słoń, Acta Physica Polonica B 51, 1739 (2020)
S. Mrówczyński and P. Słoń, Physical Review C 104, 024909 (2021)

Road map



What are we pursuing and why? ⟹ Physics motivation

Method to use ⟹ **Femtoscopy**

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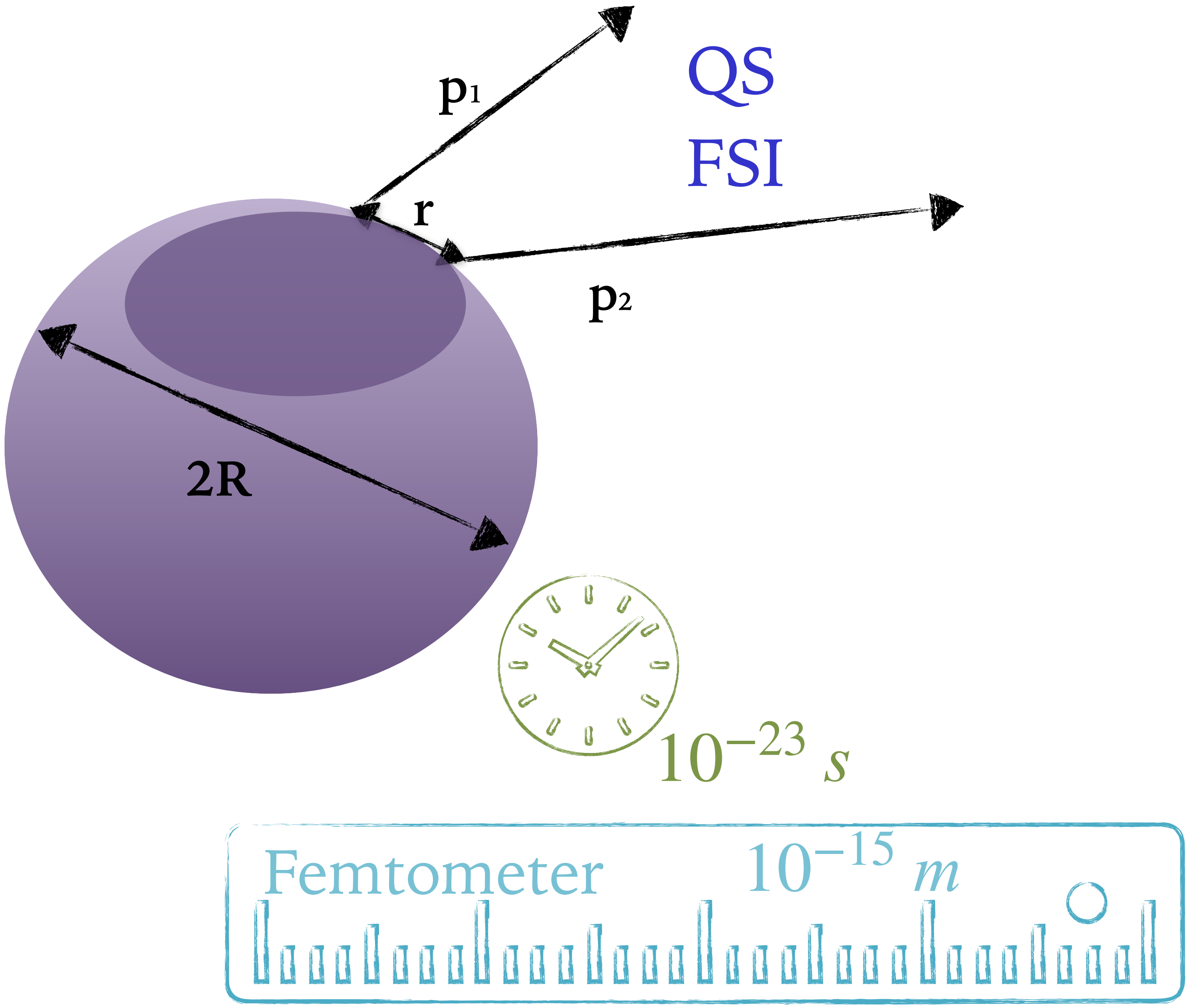
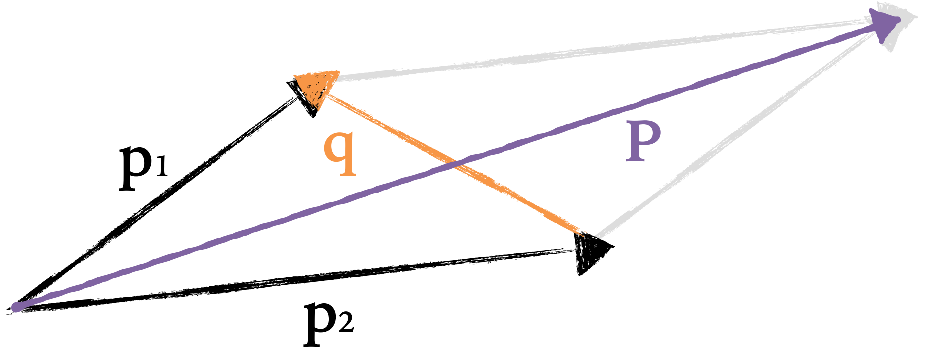
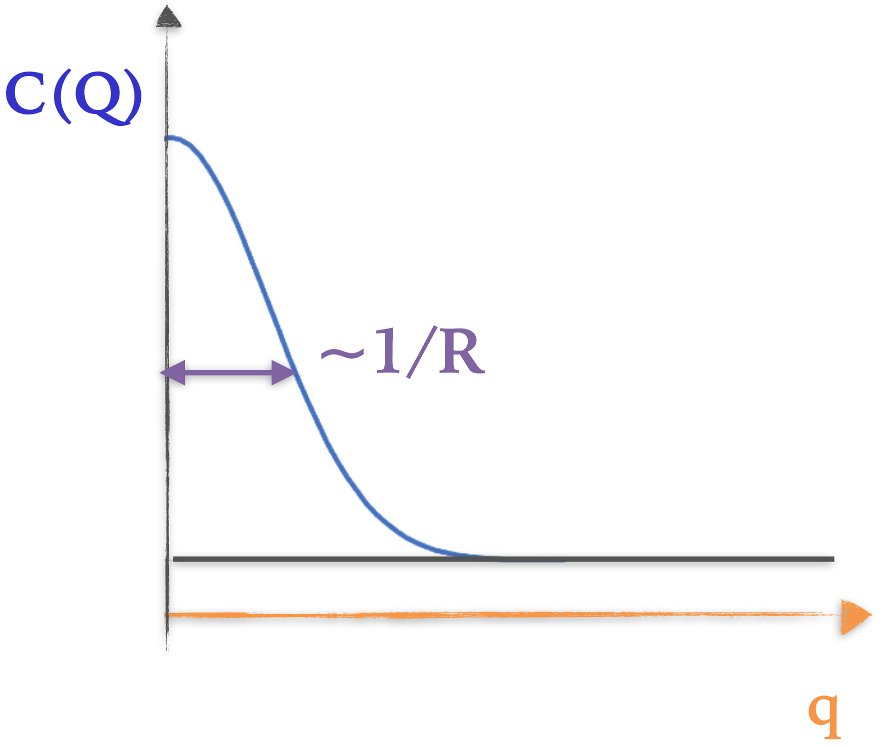






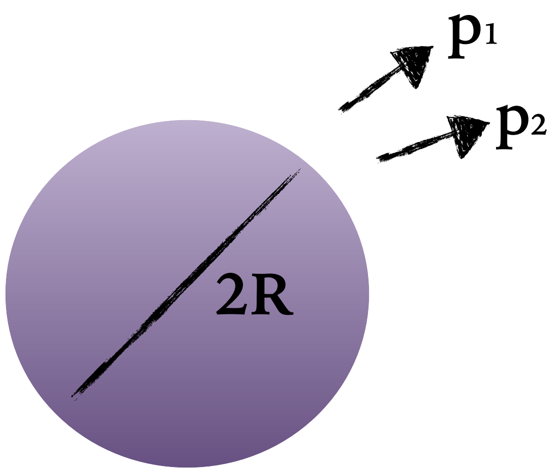
Femtoscopy

… the method to probe **geometric** and **dynamic** properties of the source (emission region, range of correlations-interactions, phase-space cloud, …)

Femtoscopy does not measure the whole source, but homogeneity length.

Classic femtoscopy

Femtoscopy (originating from HBT):
the method to probe **geometric** and **dynamic** properties of the source



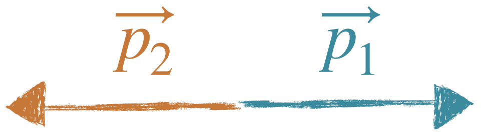
Space-time properties ($10^{-15}m$, $10^{-23}s$) determined thanks to two-particle correlations:

Quantum Statistics (Fermi-Dirac, Bose-Einstein);

Final State Interactions (Coulomb, strong)

$$C(k^*, r^*) = \int \overset{\text{determined}}{S(r^*)}\, \overset{\text{assumed}}{|\Psi(k^*, r^*)|^2}\, d^3r^* = \overset{\text{measured}}{\frac{Sgnl(k^*)}{Bckg(k^*)}}$$

k^* - momentum of the first particle in the Pair Rest Frame reference



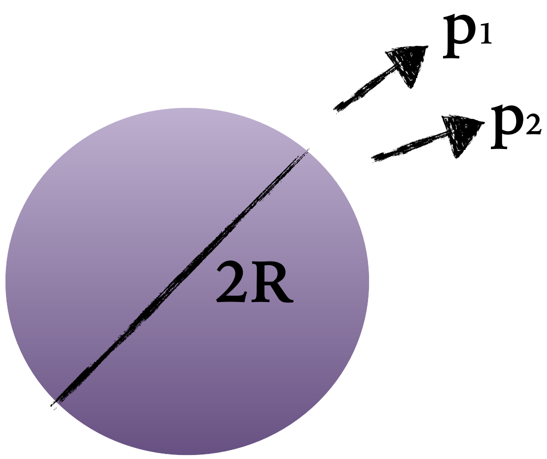
$S(r^*)$ - source function

$\Psi(k^*, r^*)$ - two-particle wave function (includes e.g. FSI interactions)

$\frac{Sgnl(k^*)}{Bckg(k^*)}$ - correlation function

Gateway to study interactions

If we assume we know the **source function**, measured **correlations** are used to determine **interactions in the final state.**



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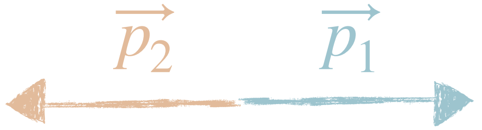

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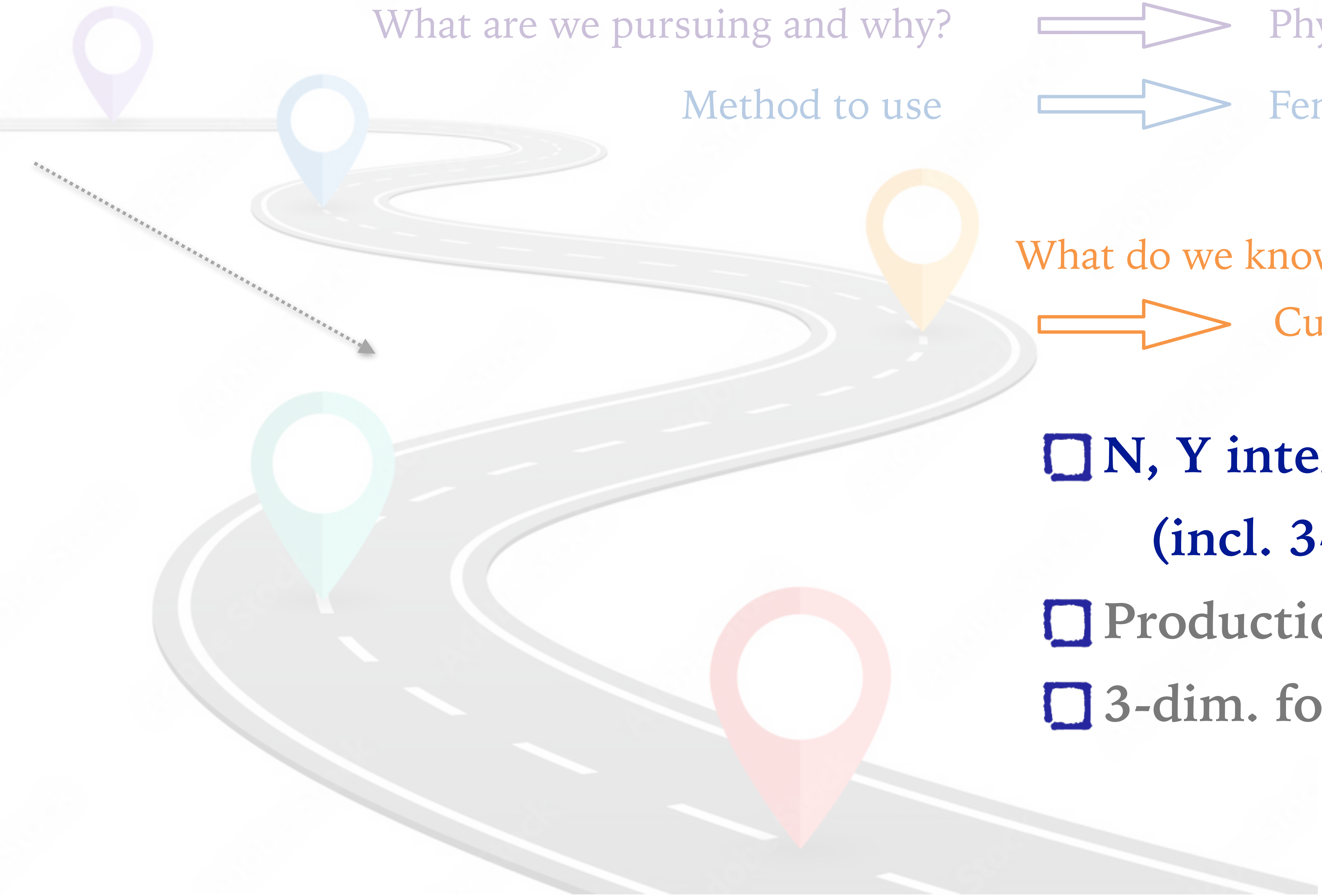

What are we pursuing and why? ⇨ Physics motivation

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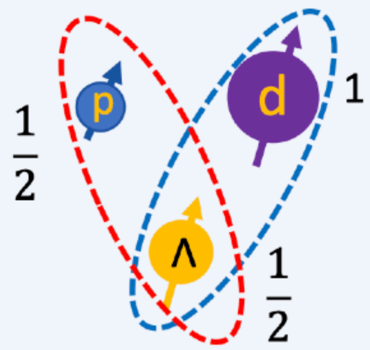
- ☐ **N, Y interactions (incl. 3-body approach)**
- ☐ **Production mechanism**
- ☐ **3-dim. form**

NY ($p-\Lambda$, $d-\Lambda$) interactions at STAR



Singlet State $^1S_0$ (S)
Triplet State $^3S_1$ (T)

Doublet State $^2S_{1/2}$ (D)
Quartet State $^4S_{3/2}$ (Q)

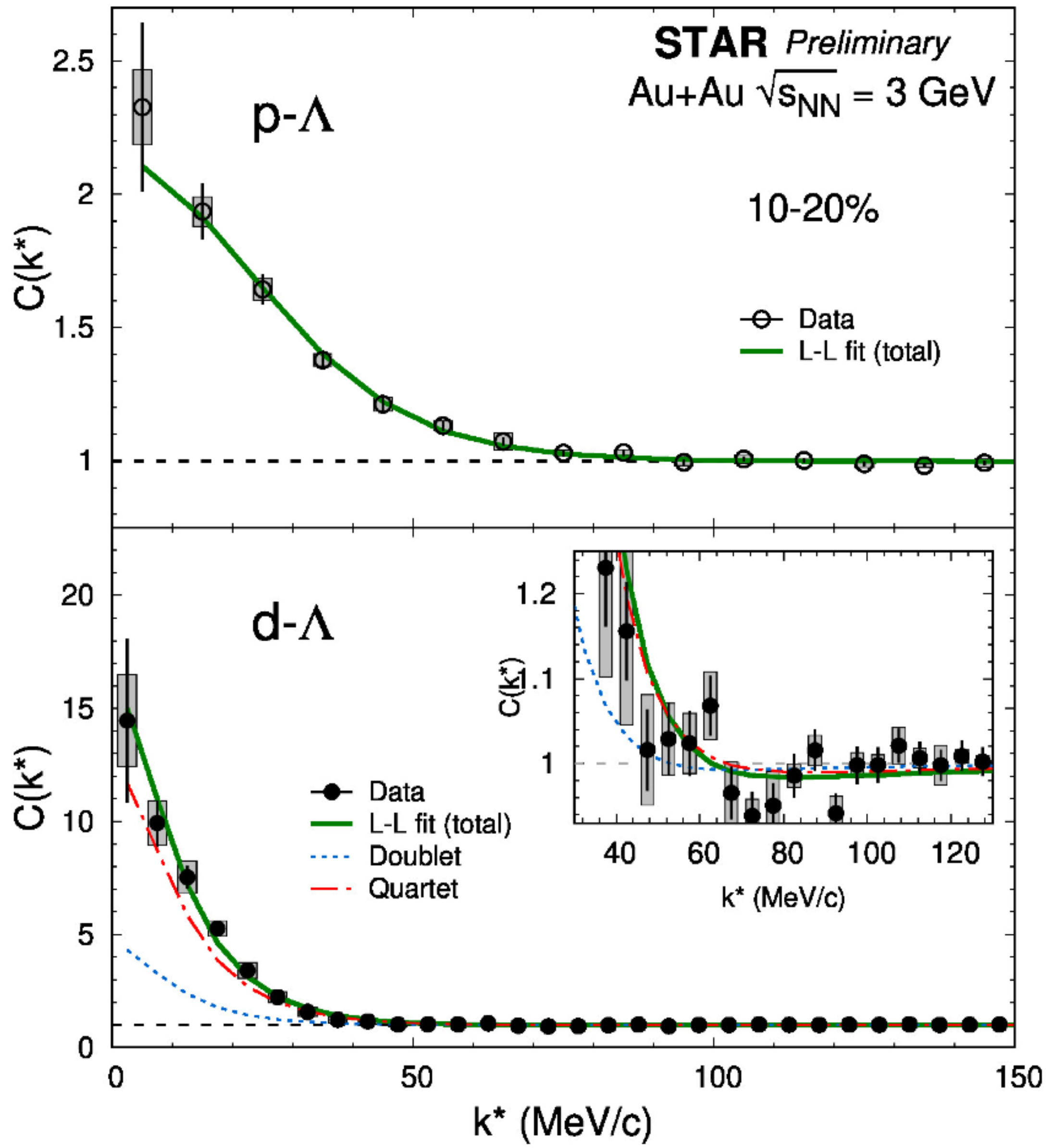
p-Λ: $|\psi(r,k)|^2 \rightarrow \frac{1}{4}|\psi_0(r,k)|^2 + \frac{3}{4}|\psi_1(r,k)|^2$

d-Λ: $|\psi(r,k)|^2 \rightarrow \frac{1}{3}\left|\psi_{1/2}(r,k)\right|^2 + \frac{2}{3}\left|\psi_{3/2}(r,k)\right|^2$

❖ Different spin states with different f_0 and d_0 parameters

❖ **p-Λ correlation:** current statistics is not enough to separate two spin states → spin-averaged fit

❖ **d-Λ correlation:** very different f_0 for (D) and (Q) are predicted → **Spin-separated fit**

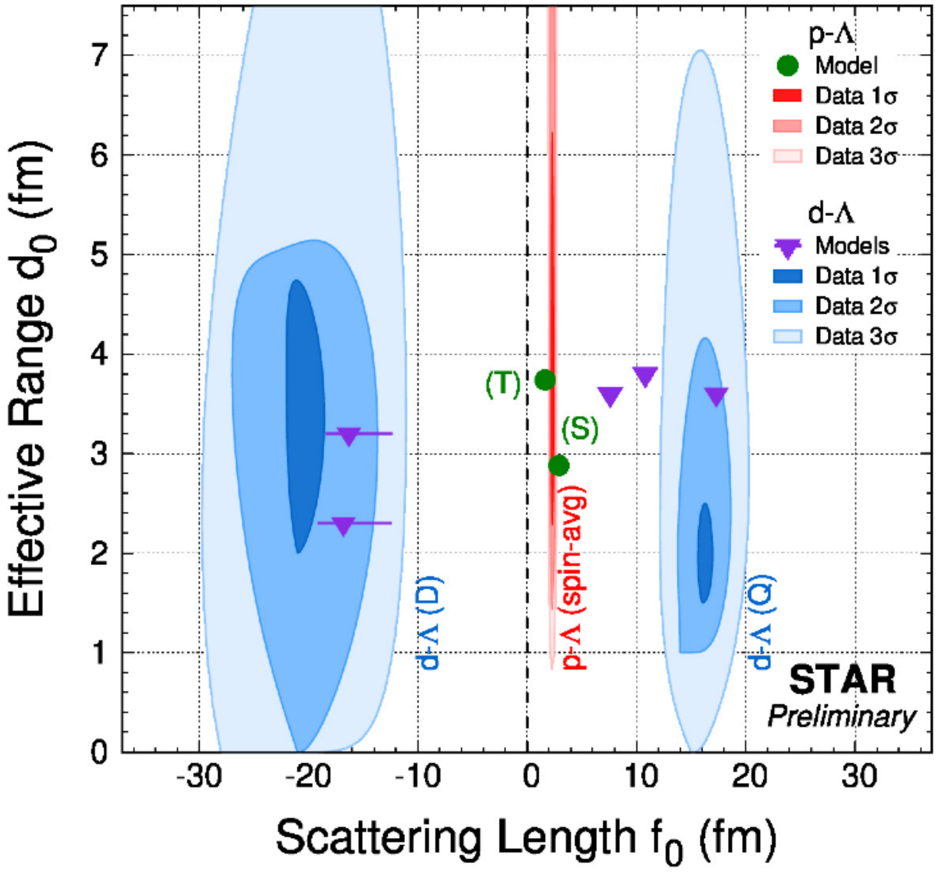


Different spin states with different FSI parameters

p-Λ correlation: currently spin-averaged fit

d-Λ correlation: spin-separated fit

NY ($p - \Lambda$, $d - \Lambda$) interactions at STAR



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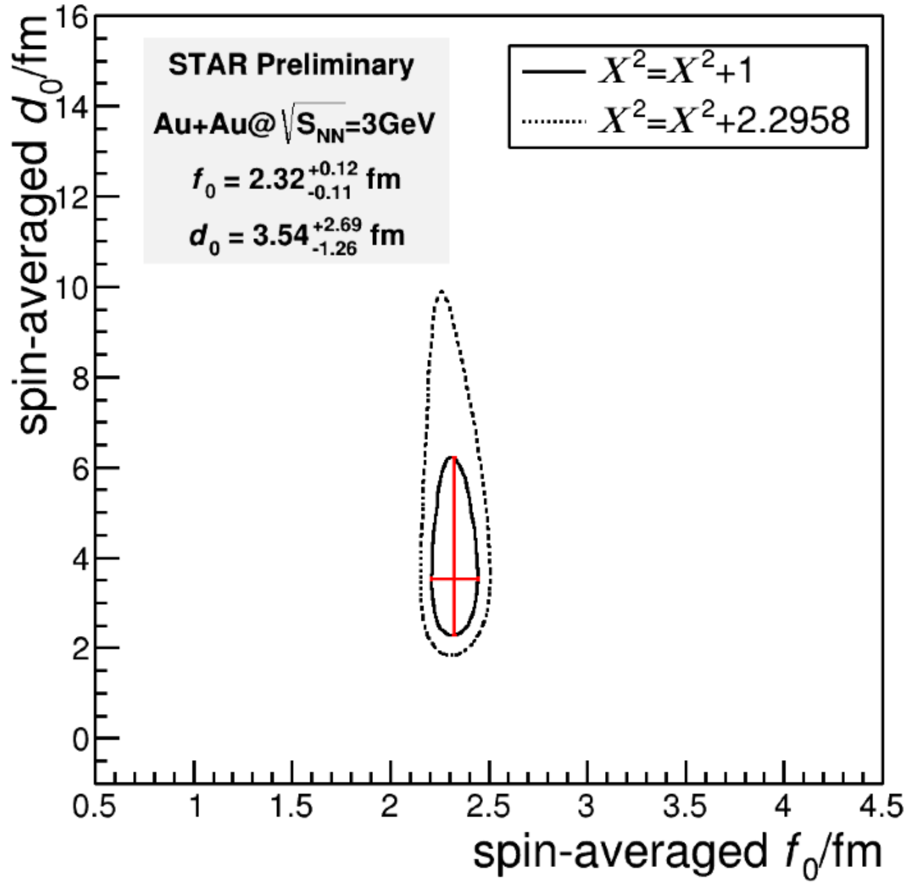
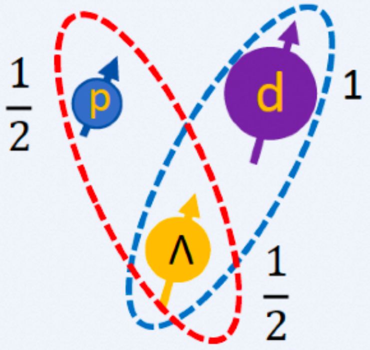




$$\frac{1}{f(k)} \approx \frac{1}{f_0} + \boxed{\frac{d_0 k^2}{2}} - ik$$

- The constraint of the effective range (d_0) is weaker

- The measurement is done at freeze-out
- Spin-avg for f_0 & d_0 p-Λ system

  $f_0 = 2.32^{+0.12}_{-0.11}$ fm $\quad d_0 = 3.5^{+2.7}_{-1.3}$ fm
- Successfully separate two spin states in d-Λ

  $f_0(\mathrm{D}) = -20^{+3}_{-3}$ fm $\quad d_0(\mathrm{D}) = 3^{+2}_{-1}$ fm

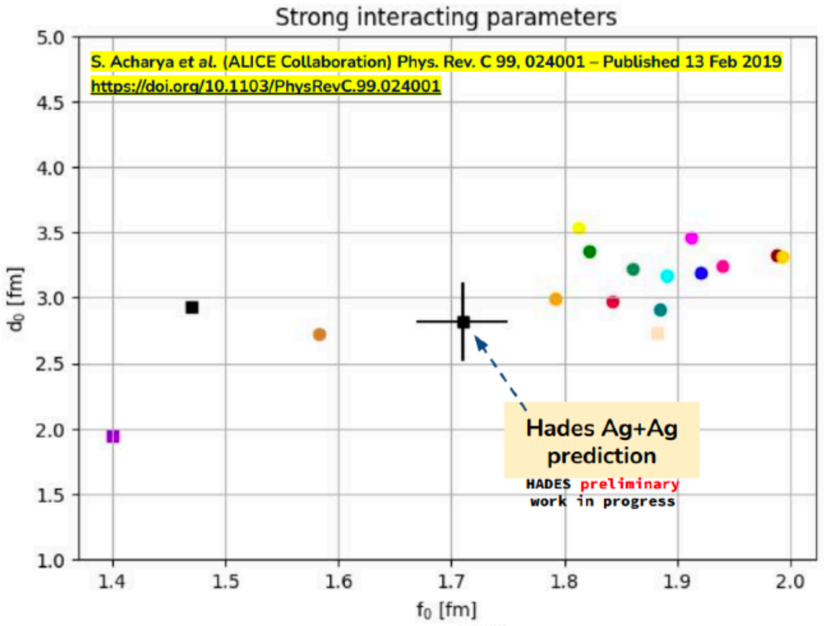
  $f_0(\mathrm{Q}) = 16^{+2}_{-1}$ fm $\quad d_0(\mathrm{Q}) = 2^{+1}_{-1}$ fm

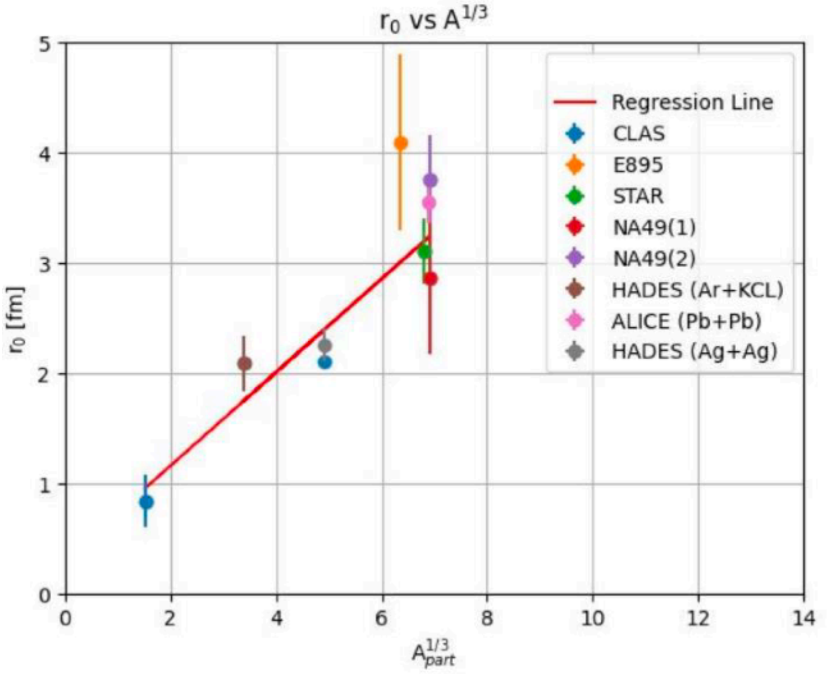
Source size extracted from the source assuming Gaussian shape;

Separation of emission source from the parameters of the final state interaction;

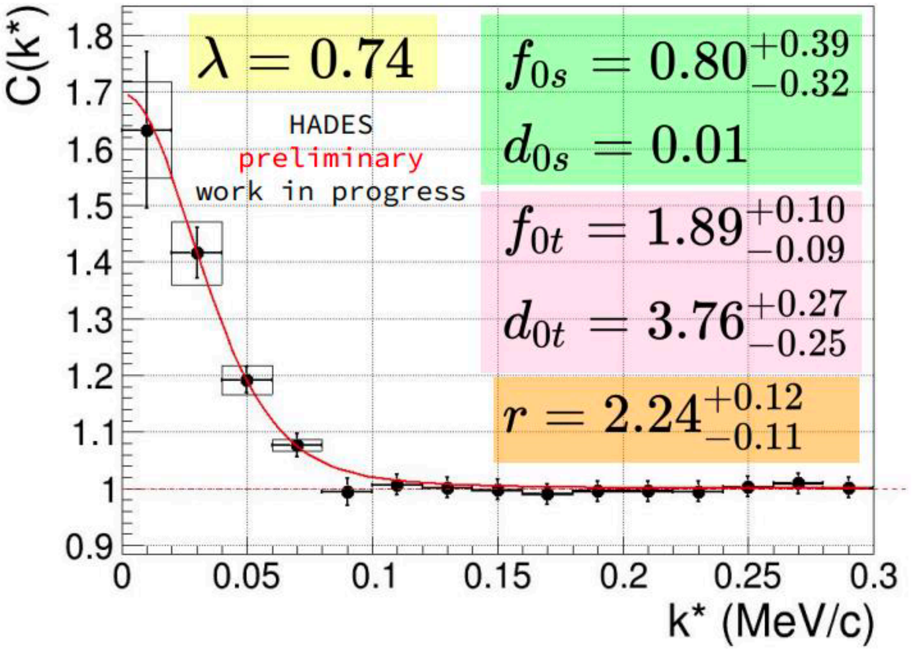
H. W. Hammer, Nucl. Phys. A 705 (2002) 173
A. Cobis, et al. J. Phys. G 23 (1997) 401
J. Haidenbauer, Phys.Rev.C 102 (2020) 3, 034001
M. Schäfer, et al. Phys.Lett.B 808 (2020) 135614
G. Alexander, et al. Phys. Rev. 173 (1968) 1452
J. Haidenbauer, et al. Nucl. Phys. A 915 (2013) 24
F. Wang, et al. Phys.Rev.Lett. 83 (1999) 3138

NY ($p - \Lambda$) interactions at HADES





Spin separation performed



$\lambda = 0.74$

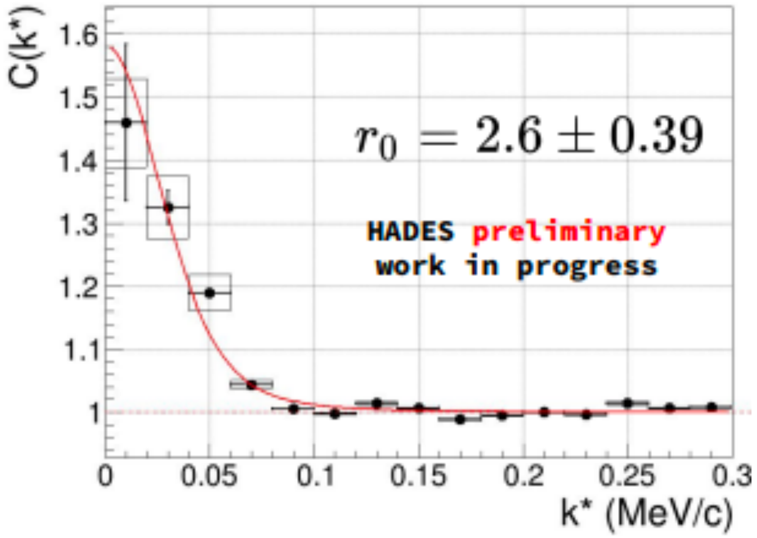
$f_{0s} = 0.80^{+0.39}_{-0.32}$

$d_{0s} = 0.01$

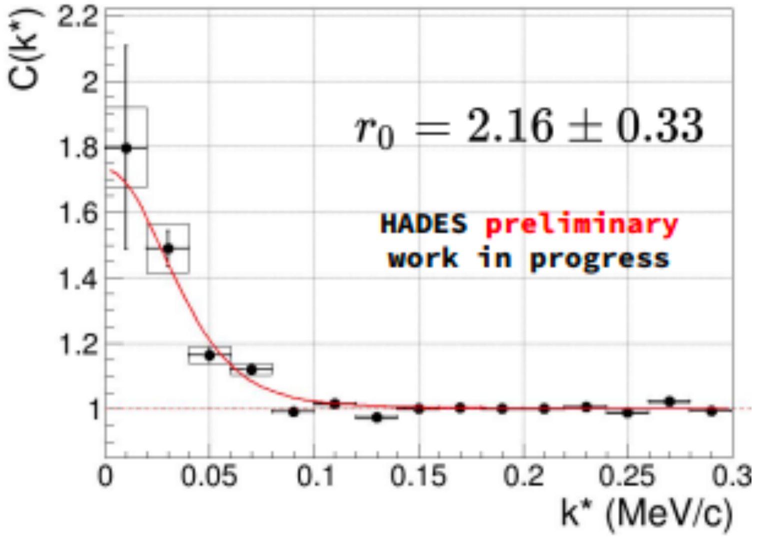
$f_{0t} = 1.89^{+0.10}_{-0.09}$

$d_{0t} = 3.76^{+0.27}_{-0.25}$

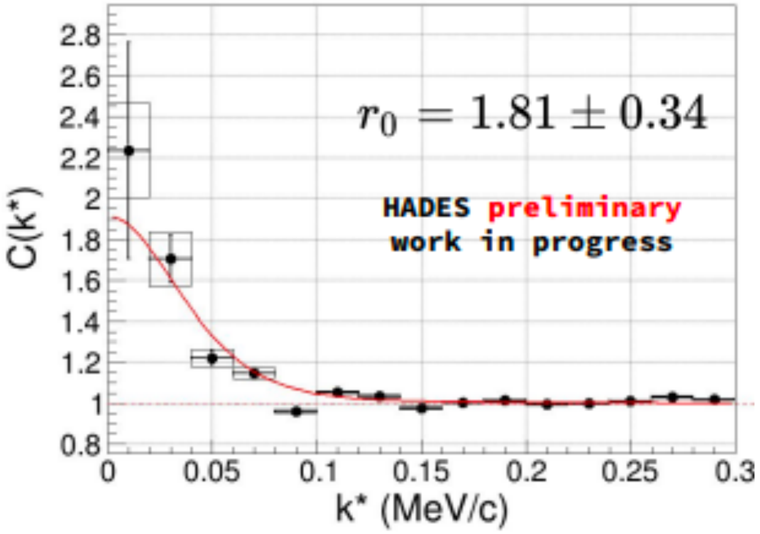
$r = 2.24^{+0.12}_{-0.11}$



$r_0 = 2.6 \pm 0.39$



$r_0 = 2.16 \pm 0.33$



$r_0 = 1.81 \pm 0.34$

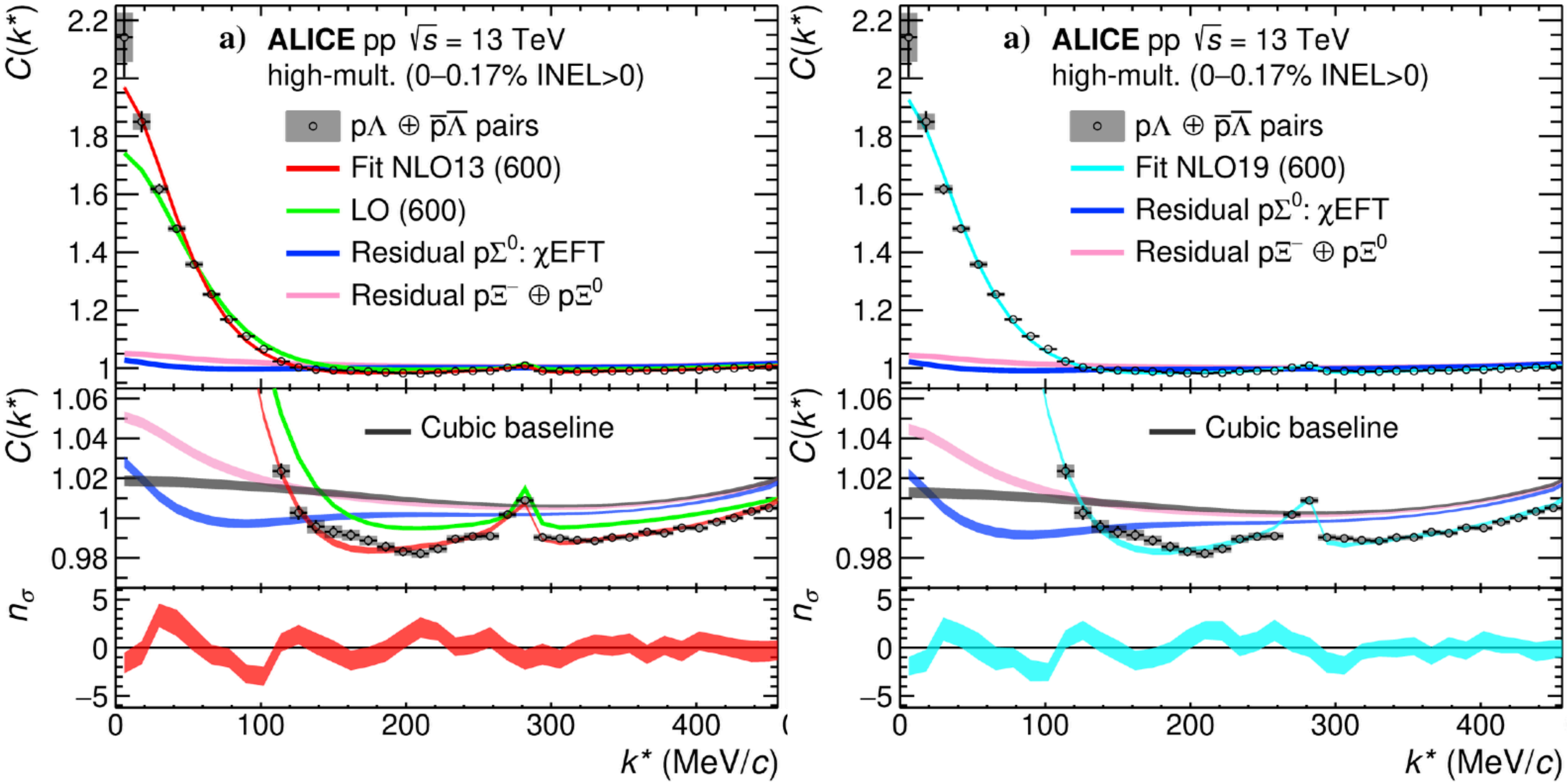
$$C(k^*) = 1 + \sum_S \rho_s \left[\frac{1}{2} \left| \frac{f^S(k^*)}{r_0} \right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi} r_0}\right) + \frac{2\Re f^S(k^*)}{\sqrt{\pi} r_0} F_1(Q r_0) - \frac{\Im f^S(k^*)}{r_0} F_2(Q r_0) \right]$$

with $F_1(z) = \int_0^z dx e^{x^2 - z^2}/z$ and $F_2(z) = (1 - e^{-z^2})/z$

Decomposition for spin channels :

$$C(k^*) = \frac{1}{4}(1 + \lambda C(k^*, s = 0)) + \frac{3}{4}(1 + \lambda C(k^*, s = 1))$$

NY ($p - \Lambda$) interactions at ALICE



$n_\sigma \sim 4.5$ for k*<110 MeV/c

$n_\sigma \sim 3.2$ for k*<110 MeV/c

New insights into $N\Lambda - \Sigma N$ interactions

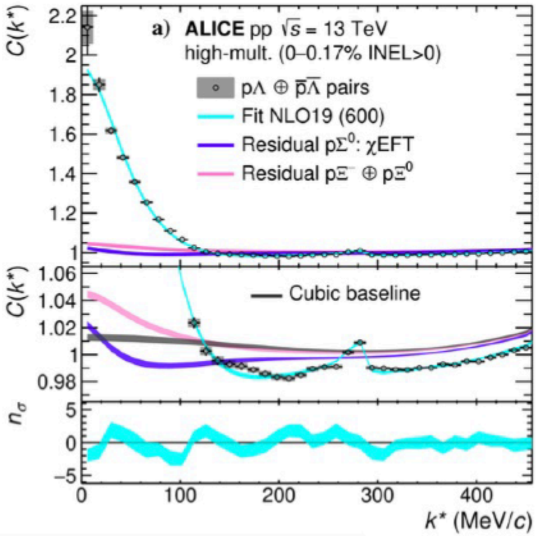
NLO19 potentials favored

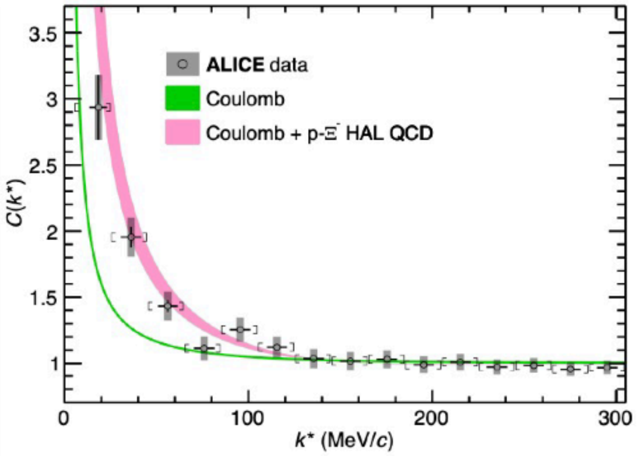
(e.g.: weaker ΛN – ΣN coupling,
significant attraction of Λ at high densities)

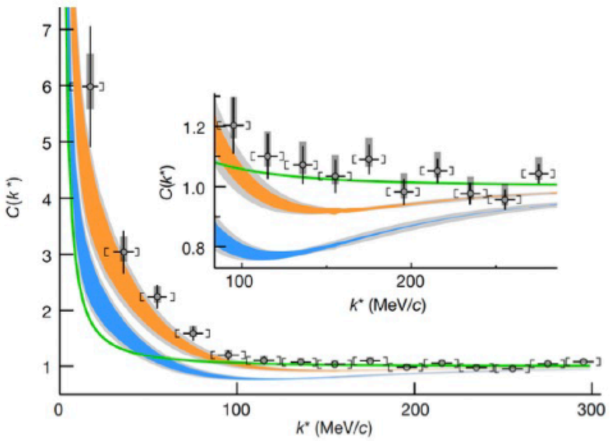
Feed-down correlations taken into account

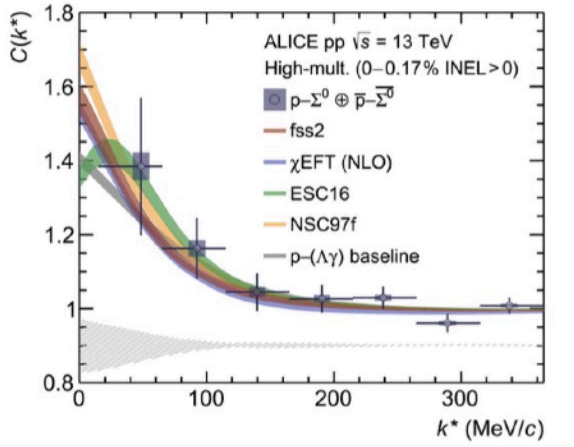
NN, NY, YY interactions at ALICE

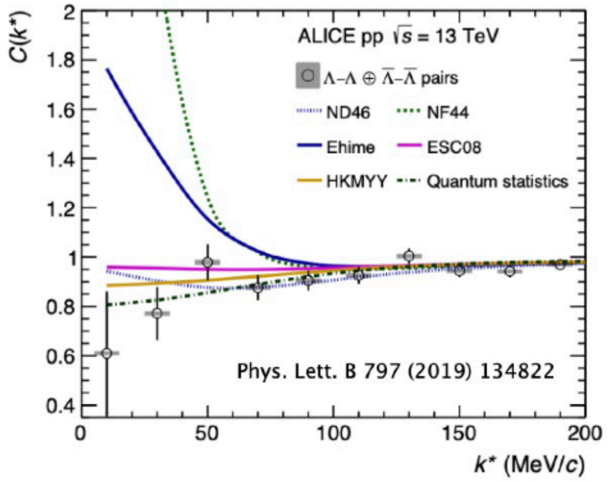
|S| = 0	|S| = 1	|S| = 2	|S| = 3	|S| > 3
NN	NΛ, NΣ	ΛΛ, NΞ	ΛΞ, NΩ	ΞΞ, ΛΩ, ΣΩ, ΞΩ, ΩΩ



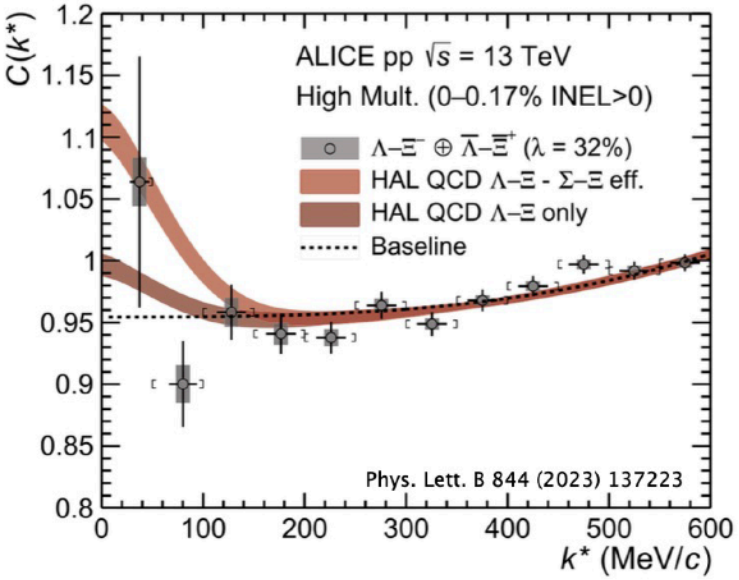








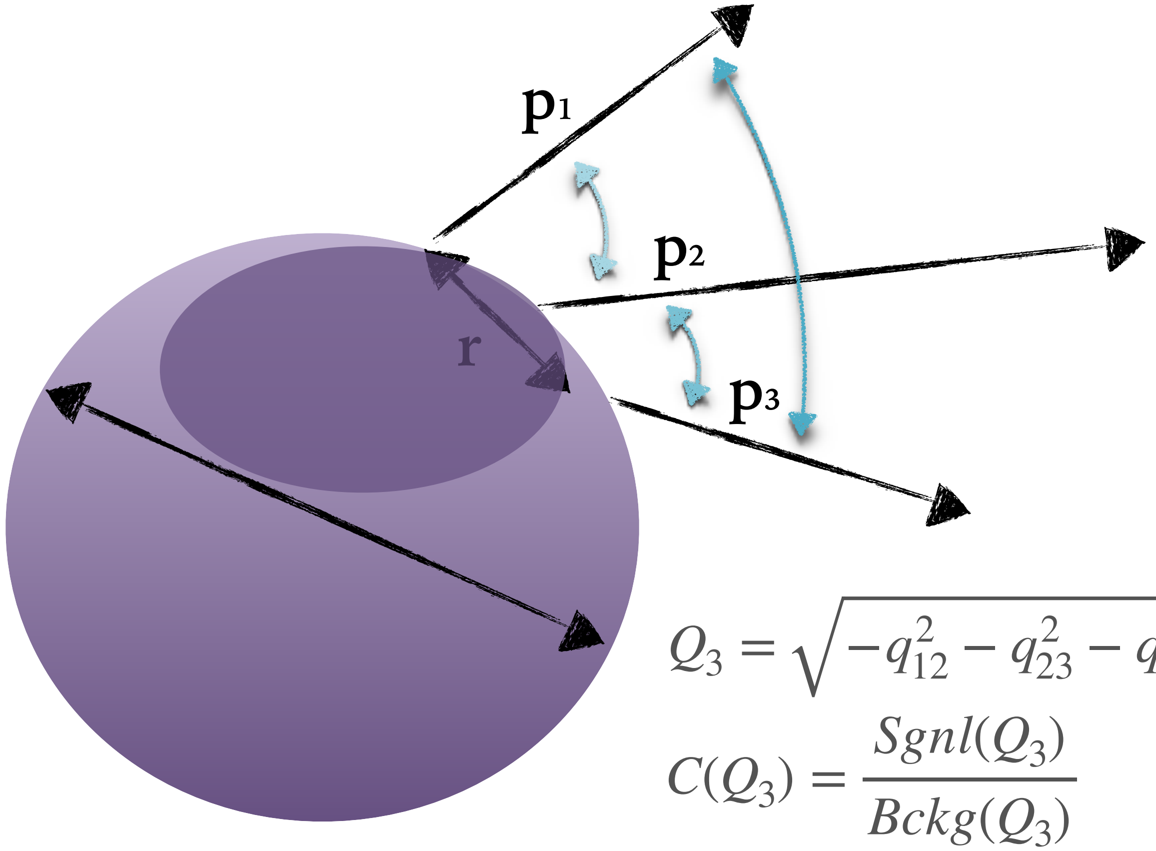
Phys. Lett. B 797 (2019) 134822



Phys. Lett. B 844 (2023) 137223

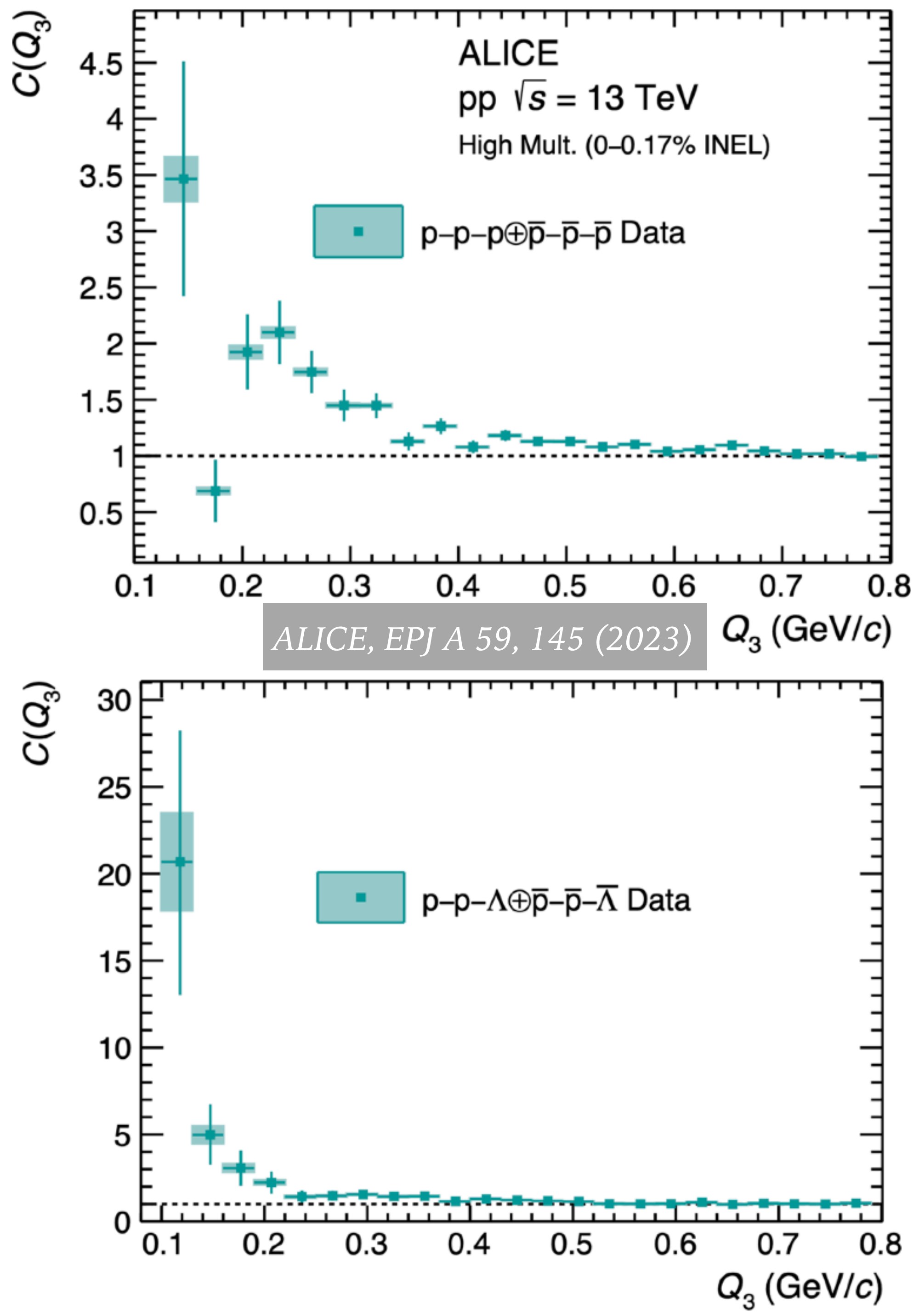
Adding more strangeness content to constrain EoS valid at all measured combinations

3-body interactions at ALICE



$$Q_3 = \sqrt{-q_{12}^2 - q_{23}^2 - q_{13}^2}$$

$$C(Q_3) = \frac{Sgnl(Q_3)}{Bckg(Q_3)}$$



ALICE, EPJ A 59, 145 (2023)

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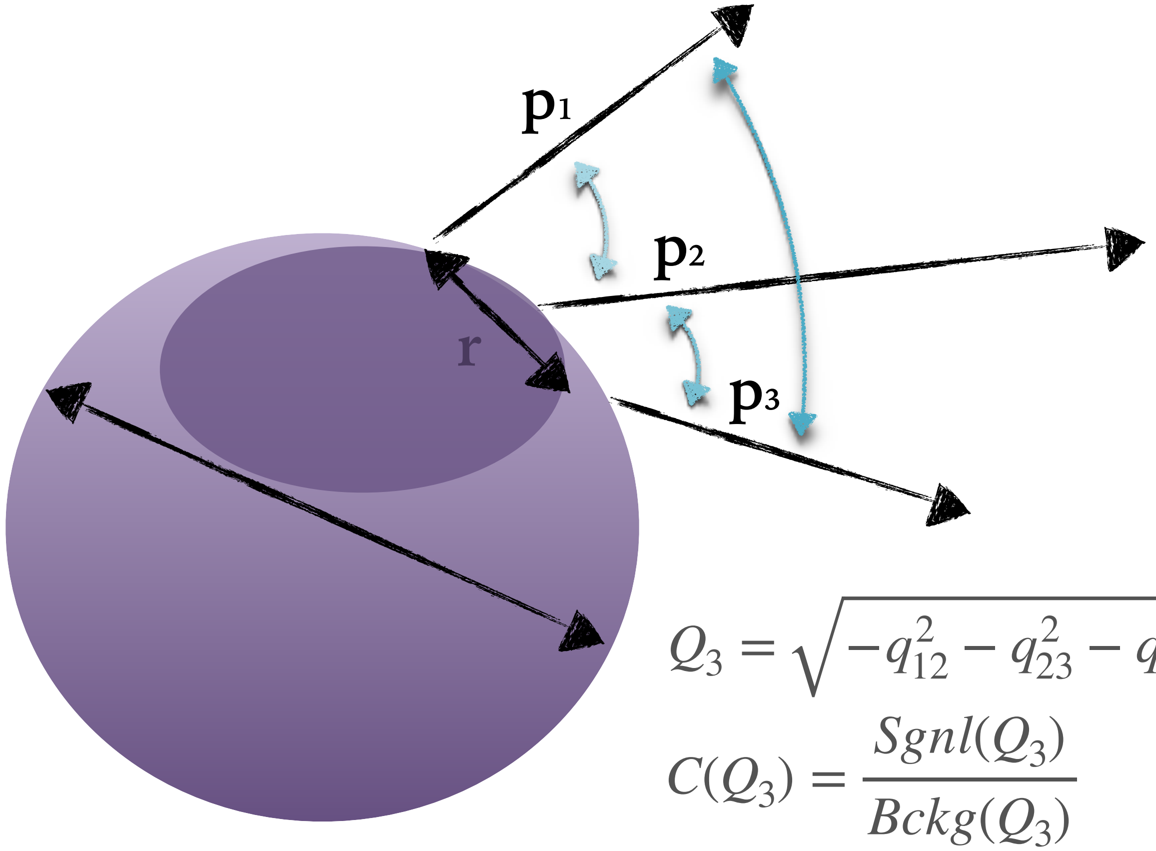

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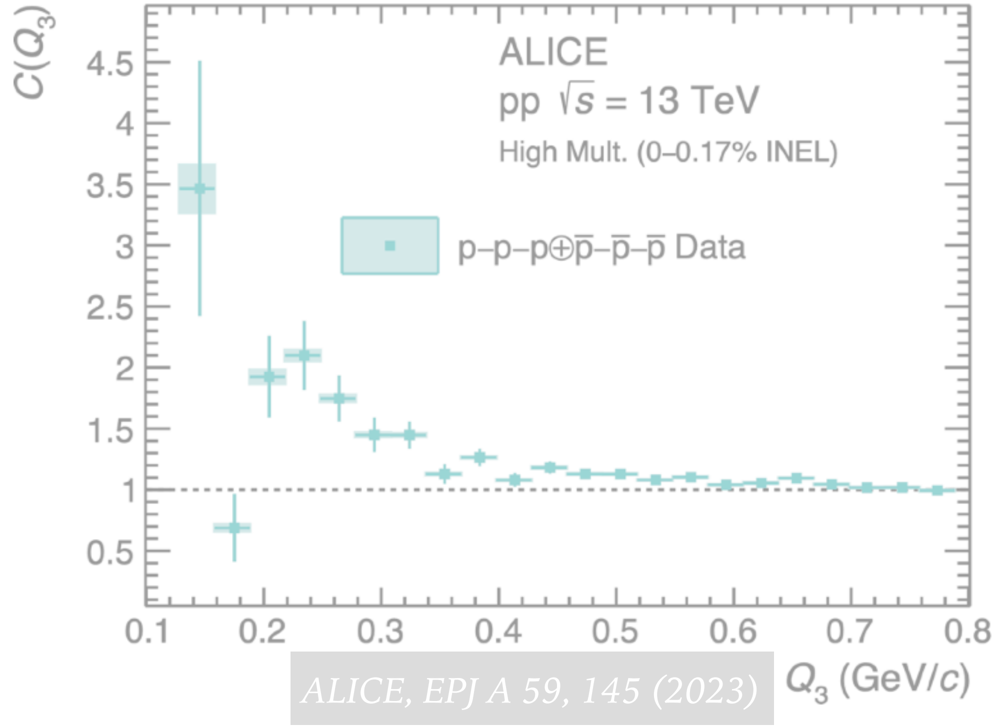
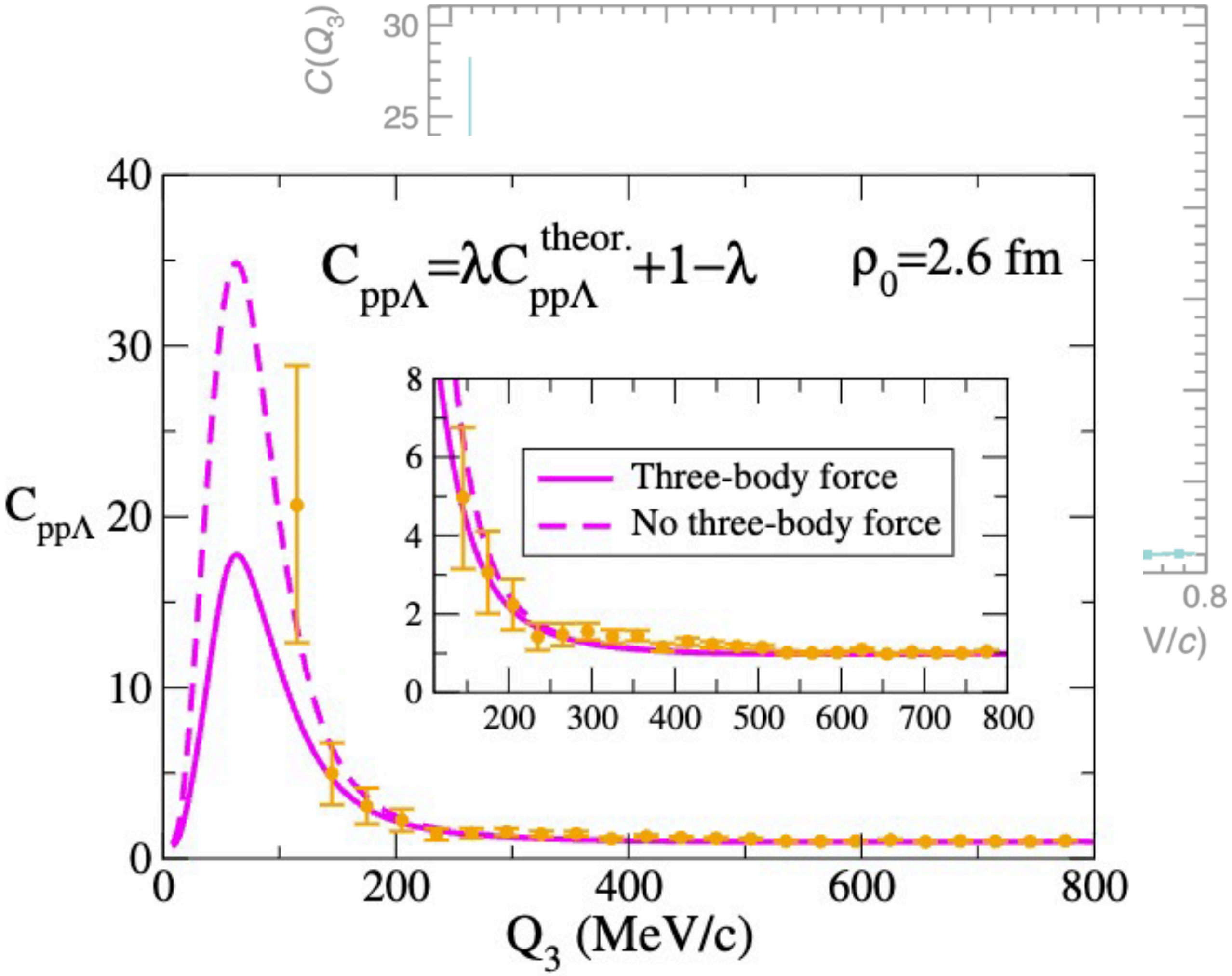


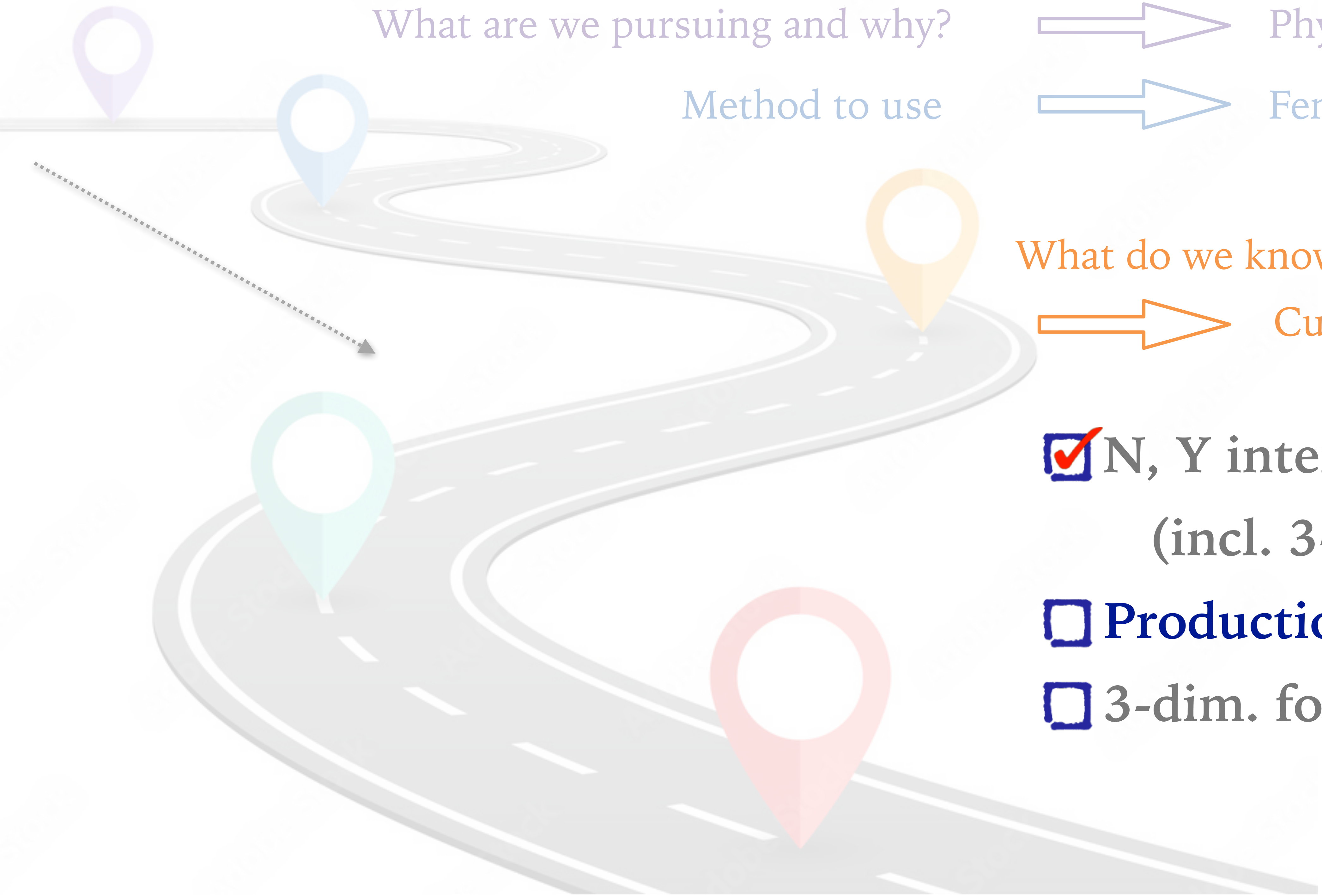
ALICE, EPJ A 59, 145 (2023)



Three-particle emission source implemented as three single-particle emitters constrained to data

- Feed-down corrections included
- Gauss NLO19 (600): 40% effect of three-body interactions
- Most interesting region $Q_3 < 100$ MeV/c not yet accessed by data

Road map



What are we pursuing and why? → Physics motivation

Method to use → Femtoscopy

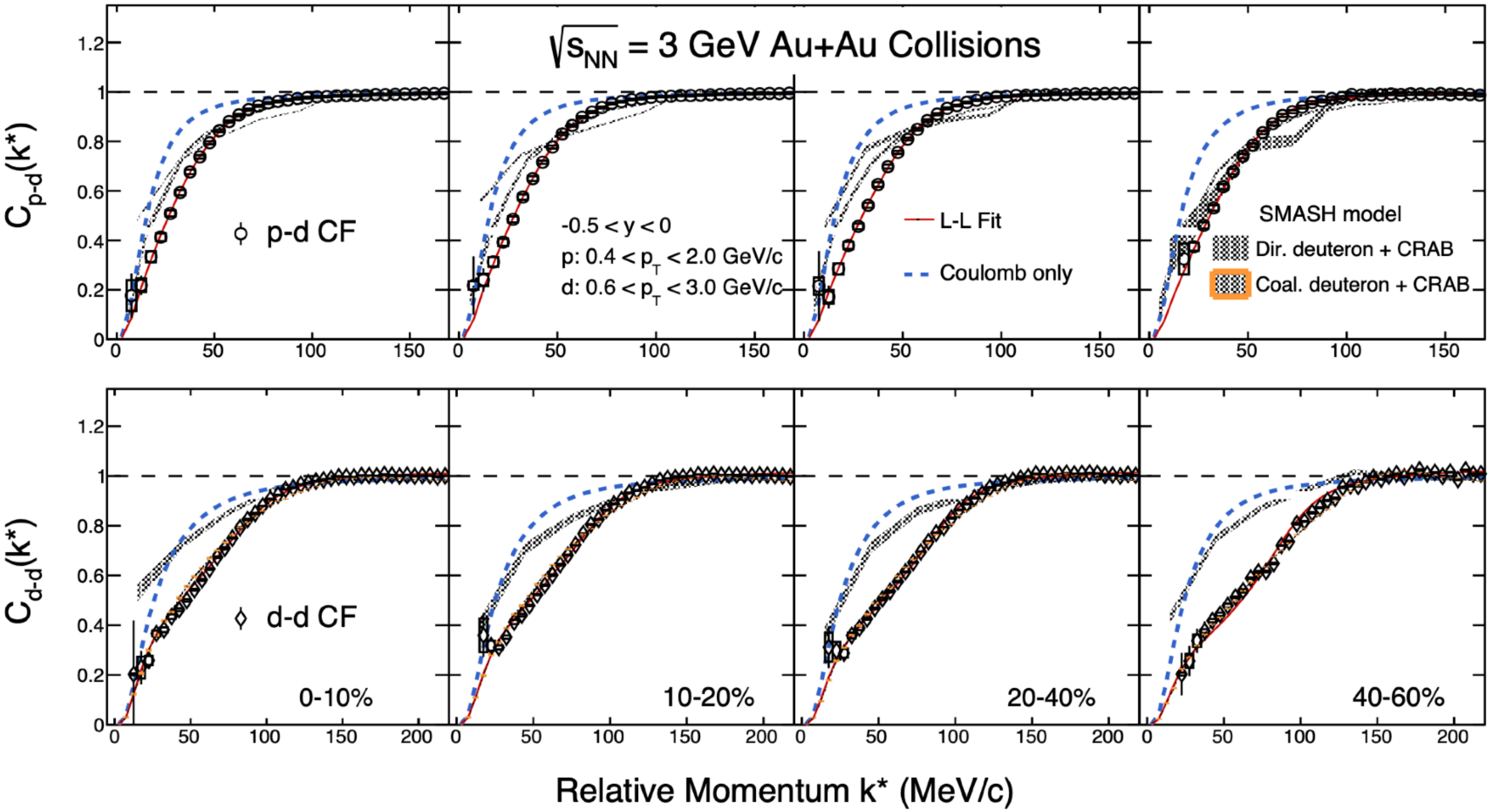
What do we know now?

→ Current status

- ☑ N, Y interactions (incl. 3-body approach)
- ☐ **Production mechanism**
- ☐ 3-dim. form

Hanna Zbroszczyk, QCD at FAIR Workshop, November 11-14, 2024, GSI, Darmstadt, Germany

Light nuclei production at STAR



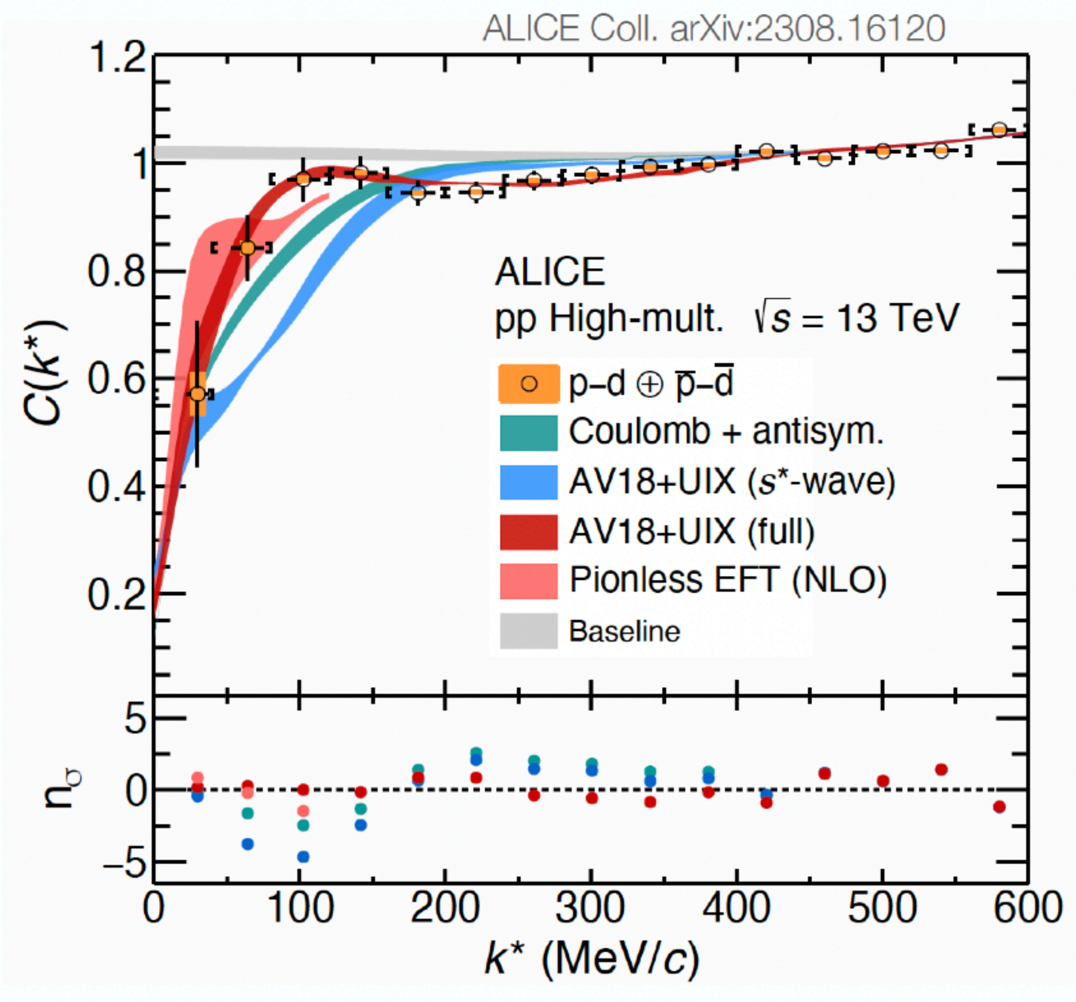
STAR: arXiv:2410.03436v1
SMASH: J. Weil et al. Phys.Rev.C 94 (2016) 5, 054905
Coalescence: W.Zhao et al. Phys. Rev. C.98 (2018) 5,054905 R. Lednicky, et al, Sov.J.Nucl.Phys. 35 (1982) 770

First measurement of **proton-deuteron** and **deuteron-deuteron** correlation functions from STAR

Proton-deuteron and **deuteron-deuteron** correlations qualitatively described by Lednicky-Lyuboshitz model

Deuteron-deuteron correlations described better by the model including **coalescence**. Light nuclei are likely to be formed via coalescence

Light nuclei seen as many-body system at ALICE



ALICE:
arXiv:2308.16120
[1] B. R. B. Wiringa et al. Phys. Rev. C 51, 38
[2] B. S. Pudliner et al. Phys. Rev. Lett. 74, 4396

Coulomb only: disagree!

Argonne v18(2N) + Urbana IX (**genuine three-body force**) potentials[1,2]

s-wave only: more repulsion

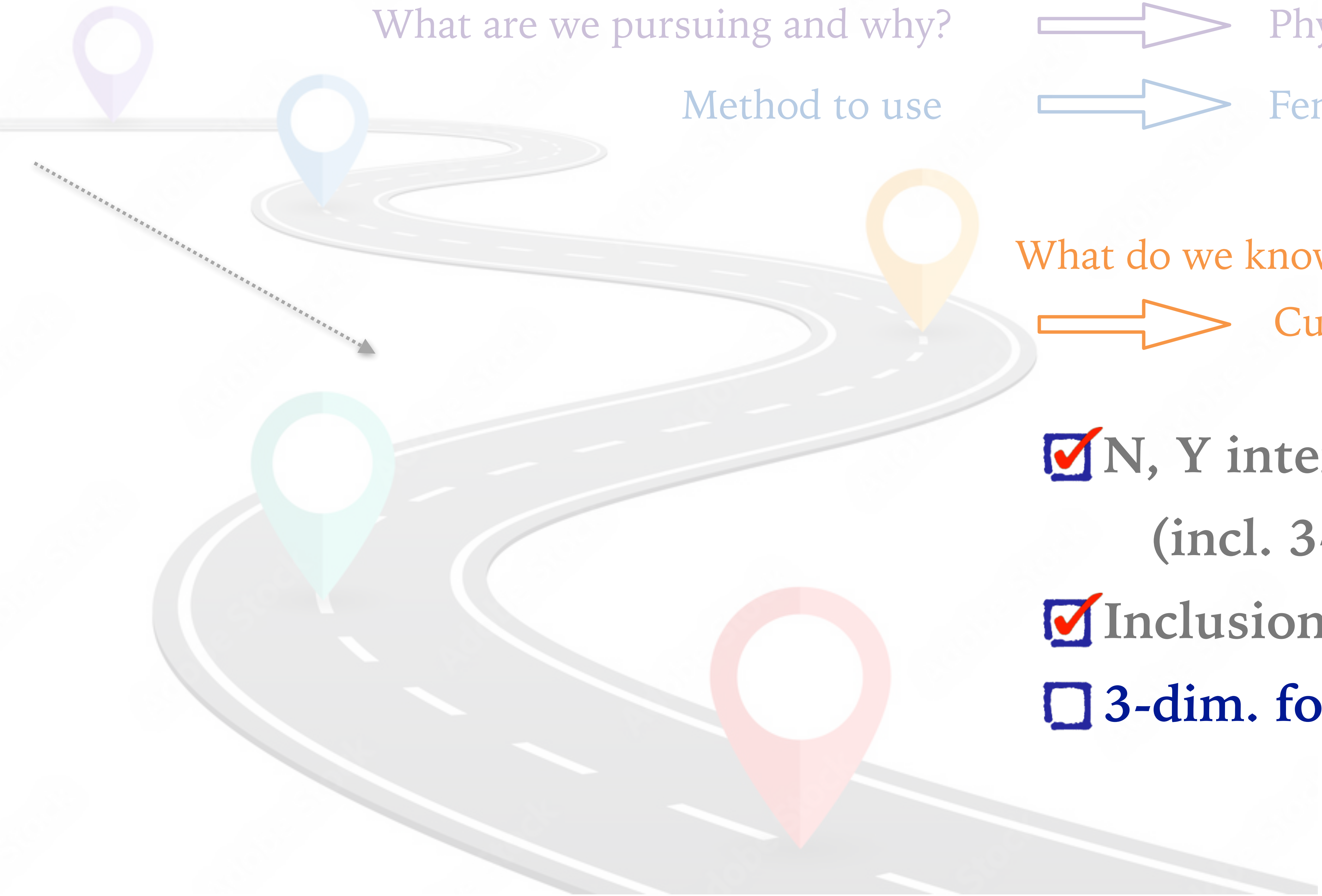
all partial waves up to d-waves: excellent description ($n_\sigma \sim 1$ for k^* up to 400 MeV/c)

Pionless EFT NLO (s+p+d waves):

Agree with data within $n_\sigma \sim 2.5$ for $k^* < 120$ MeV/c

Dynamics of the p–(pn) triplet and higher partial waves at short distances!

Road map



What are we pursuing and why? ⇨ Physics motivation

Method to use ⇨ Femtoscopy

What do we know now?

⇨ Current status

- ☑ **N, Y interactions (incl. 3-body approach)**
- ☑ **Inclusion of K**
- ☐ **3-dim. form**

Bertsch-Pratt parametrization, 3 and 1 dimensions

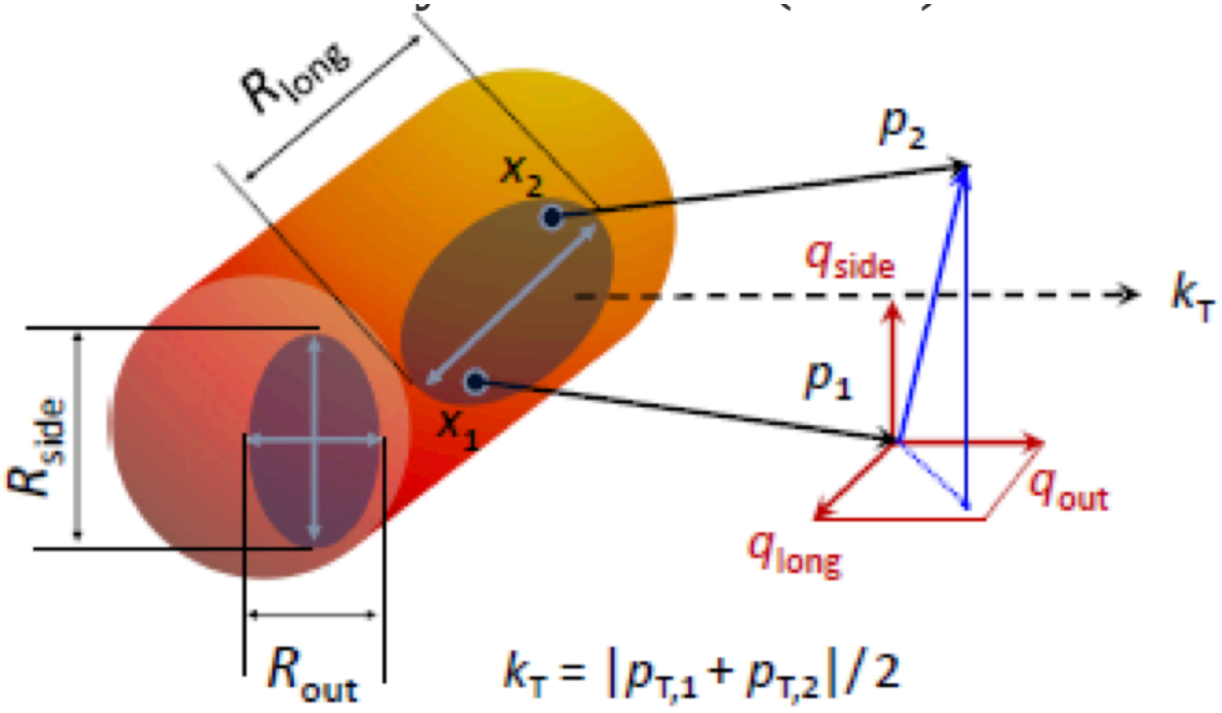
→ R_{side} spatial source evolution in the transverse direction
→ R_{out} related to spatial and time components
→ R_{out}/R_{side} signature of phase transition
→ $R_{out}^2 - R_{side}^2 = \Delta\tau^2 \beta_t^2$; $\Delta\tau$ – emission time
→ R_{long} temperature of kinetic freeze-out and source lifetime

long → beam direction

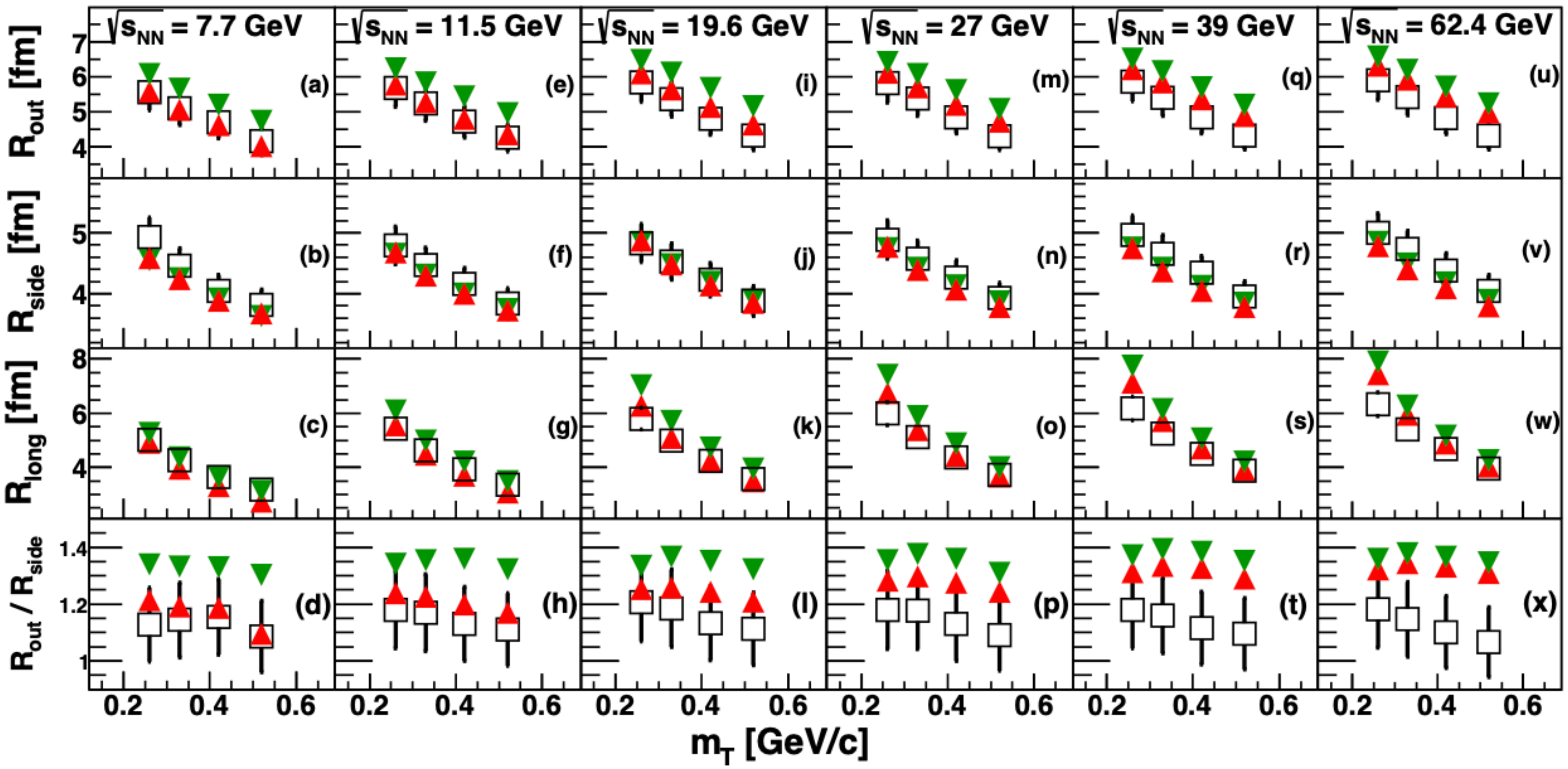
out → pair transverse momentum direction

side → perpendicular to *long* and *side*

3D case is considered if statistics is enough and two-particle correlations are easy to describe (Quantum Statistics and Coulomb FSI).
It is challenging for systems interacting strongly.
1D case is considered then (assuming spherical source).



How to measure phase transition?



☐ STAR Data

▲ 1st order Phase Tr.

▲ Cross-over Tr.

vHLLE (3+1)-D viscous hydrodynamics: Iu. Karpenko, P. Huovinen, H. Petersen, M. Bleicher; Phys.Rev. C 91, 064901 (2015), arXiv:1502.01978, 1509.3751

HadronGas + Bag Model → 1st order PT ; P.F. Kolb, et al, PR C 62, 054909 (2000)

Chiral EoS → crossover PT (XPT); J. Steinheimer, et al, J. Phys. G 38, 035001 (2011)

Pre-thermal → Hydrodynamical phase → Hydronic cascades

UrQMD | vHLEE | UrQMD

vHLEE+UrQMD model verify **sensitivity of HBT measurements to the first-order phase transition**

Phys. Rev. C 96 (2017) no.2, 024911

Road map

What are we pursuing and why? ⇒ Physics motivation

Method to use ⇒ Femtoscopy

What do we know now? ⇒ Current status

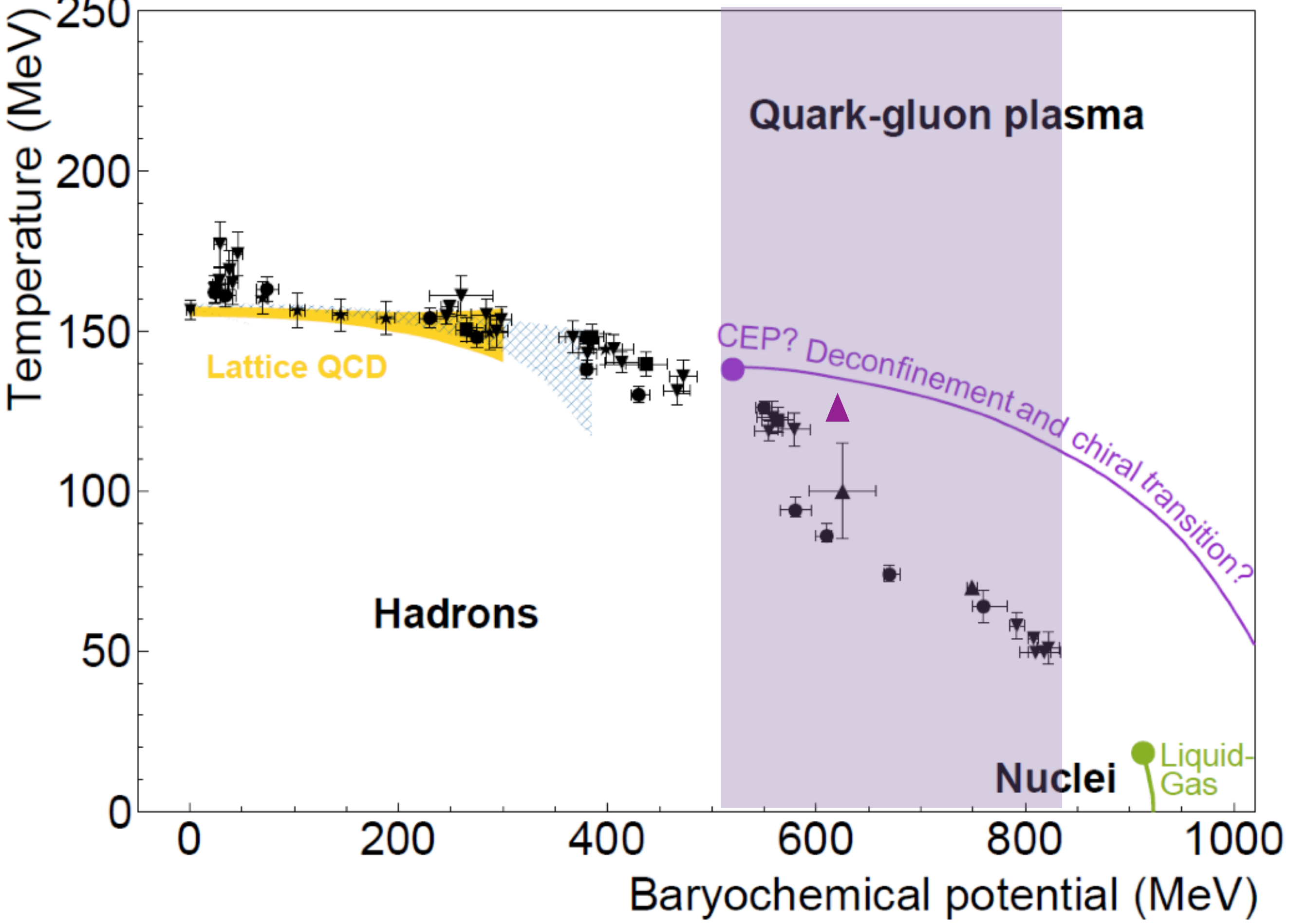
Who else is involved? ⇒ Experimental facilities

Perspectives for future ⇒ **Plans**

Current coverage of the QCD phase diagram

CBM / HADES experimental exploration of the region $\mu_B \sim 520 - 830\ MeV$

HADES; Nature Physics 15 (2019) 10, 1040-1045



Bazavovet al.[HotQCD], PLB 795 (2019) 15-21
Dinget al., [HotQCD], PRL 123 (2019) 6, 062002
Borsanyiet al., PRL125(2020)5,052001
Isserstedt et al. PRD 100 (2019) 074011
Gao, Pawlowski, PLB 820 (2021) 136584

Fu et al., PRD 101 (2020), 054032
Gunkel, Fischer, PRD 104 (2021) 5, 054022

	$\sqrt{s_{NN}}$ (GeV)	μ_B (MeV)
HADES@SIS18	**2-2.5**	**830-760**
CBM@SIS100	**2.3-5.3**	**785-520**
NA61/SHINE@SPS	5.1-17.3	530-220
STAR-COLL@RHIC	7.7-200	400-22
STAR-FXT@RHIC	3-13.7	700-265

A. Andronic, P. Braun- Munzinger, K. Redlich and
B. J . Stachel, Nature 561, no. 7723, 321 (2018)

High μ_B facilities

CBM / HADES@ SIS100 (>2028)

MPD, MB@N@NICA

NA61/SHINE@SPS

HADES@SIS18

NA60@SPS(>2030)

STAR@RHIC

J-PARC-HI

CEE@HIAF (>2027)

Road map

What are we pursuing and why? → Physics motivation

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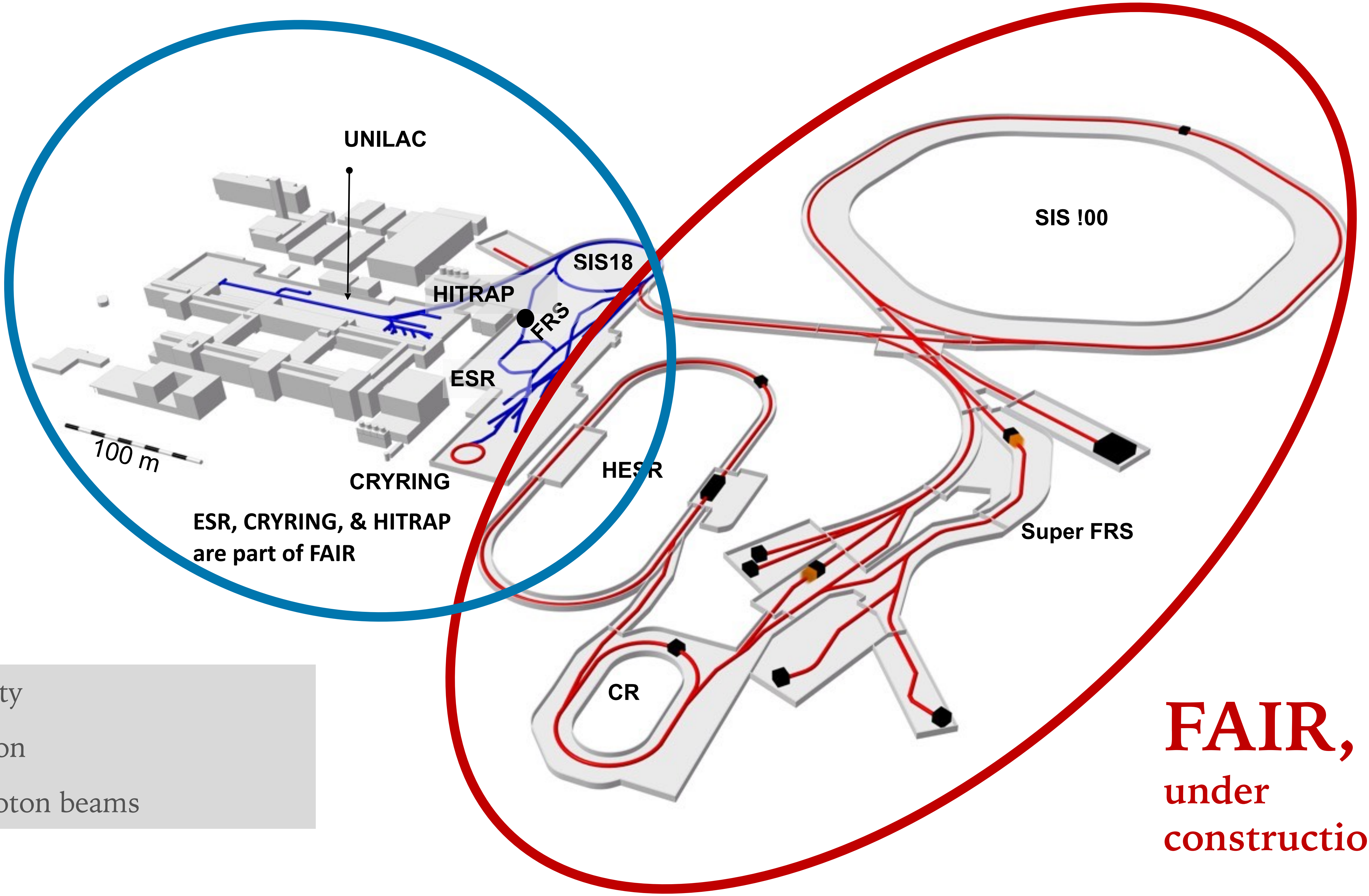
→ **Plans**

GSI GmbH – Helmholtzzentrum für Schwerionenforschung
FAIR GmbH – Facility for Antiproton and Ion Research

GSI, existing (upgraded to integrate with FAIR)



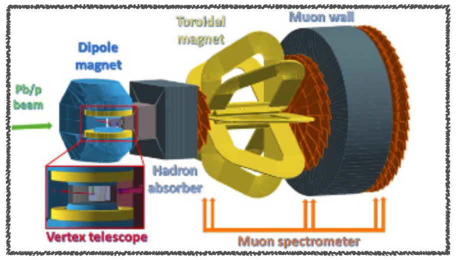
ESR, CRYRING, & HITRAP are part of FAIR

FAIR, under construction

- Intensity
- Precision
- Antiproton beams

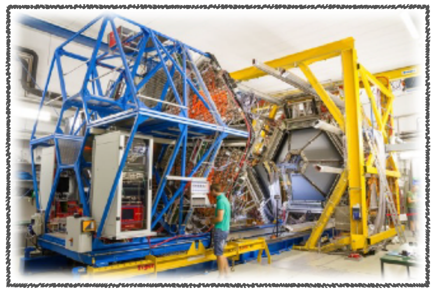
High μ_B facilities

NA60@SPS(>2030)



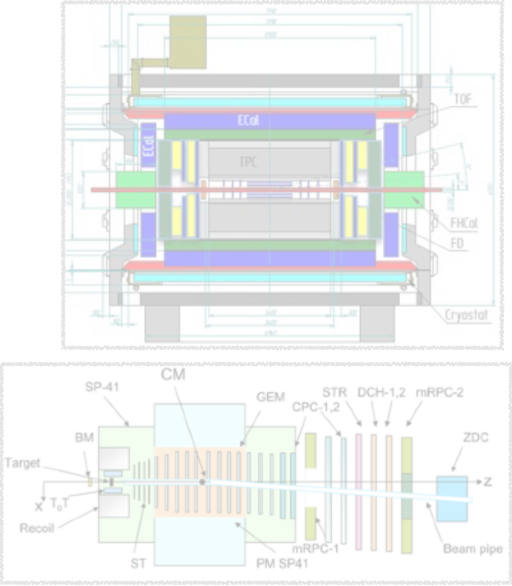
NA61/SHINE@SPS

HADES@SIS18



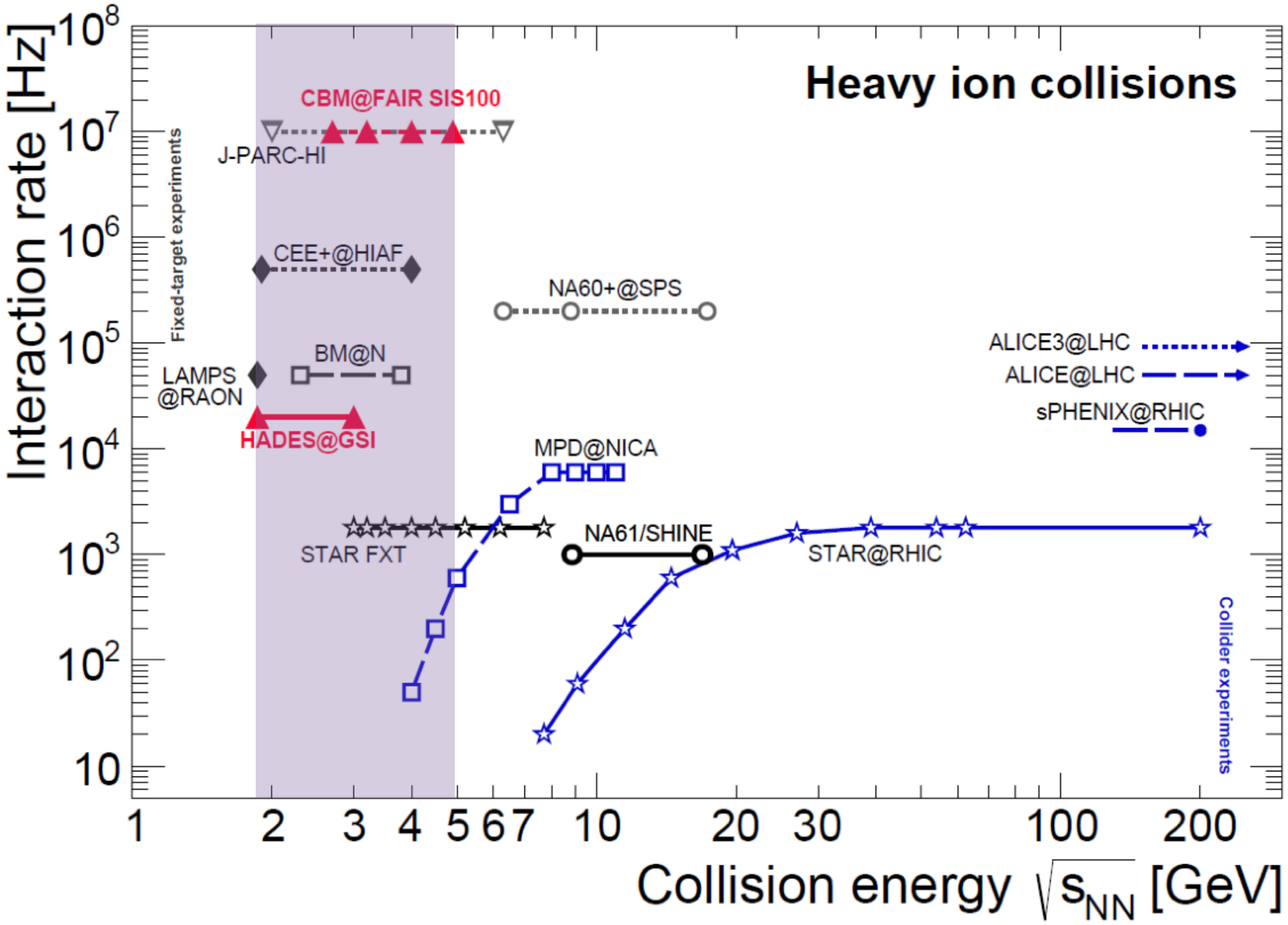
CBM / HADES@ SIS100 (>2028)

MPD, MB@N@NICA



STAR@RHIC

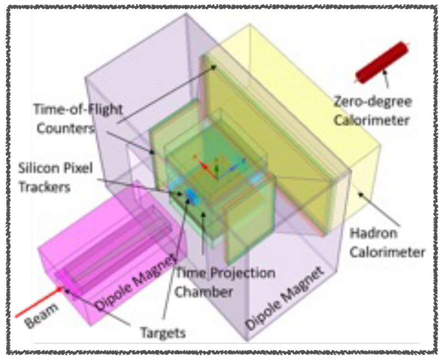
CBM / HADES: operations at $\sqrt{s_{NN}} \sim 2-5\ GeV$



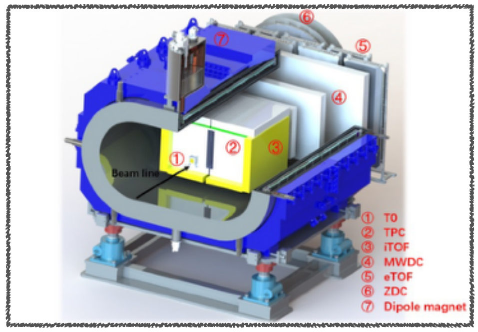
T. Galatyuk, NPA 982 (2019), update 2024 https://github.com/tgalatyuk/interaction_rate_facilities

CBM, EPJA 53 3 (2017) 60

J-PARC-HI



CEE@HIAF (>2027)



Wrap up

What are we pursuing and why?

To answer fundamental questions about the structure of the QCD phase diagram at high μ_B and to explore neutron stars

Method to use

.. that sensitive to the interactions in the final states and significant for determination of the EoS

What do we know now?

Already operating at high μ_B experiments are complete and exploration of new physics needs new facilities

Who else is involved?

Many world-wide existing and planned facilities complement each other programs

Perspectives for future

CBM plans to start these exploration in 2028 to answer fundamental questions in the first year of running

Thank you!

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The Future is Bright

Be the light

Extra slides

Strange hadronic matter in the inner core

The inner core of the neutron star is totally unknown. One of the most probable scenarios is that hyperons (baryons with strange quarks) appear at a density larger than (2–3) ρ_0

Λ hyperons, being free from Pauli exclusion principle by neutrons, are allowed to stay at the bottom of the attractive nuclear potential made by neutrons. When the kinetic energy of a neutron on the Fermi surface of the degenerate neutron matter exceeds the Λ-n mass difference of 176 MeV, it converts into a Λ hyperon via weak interaction.

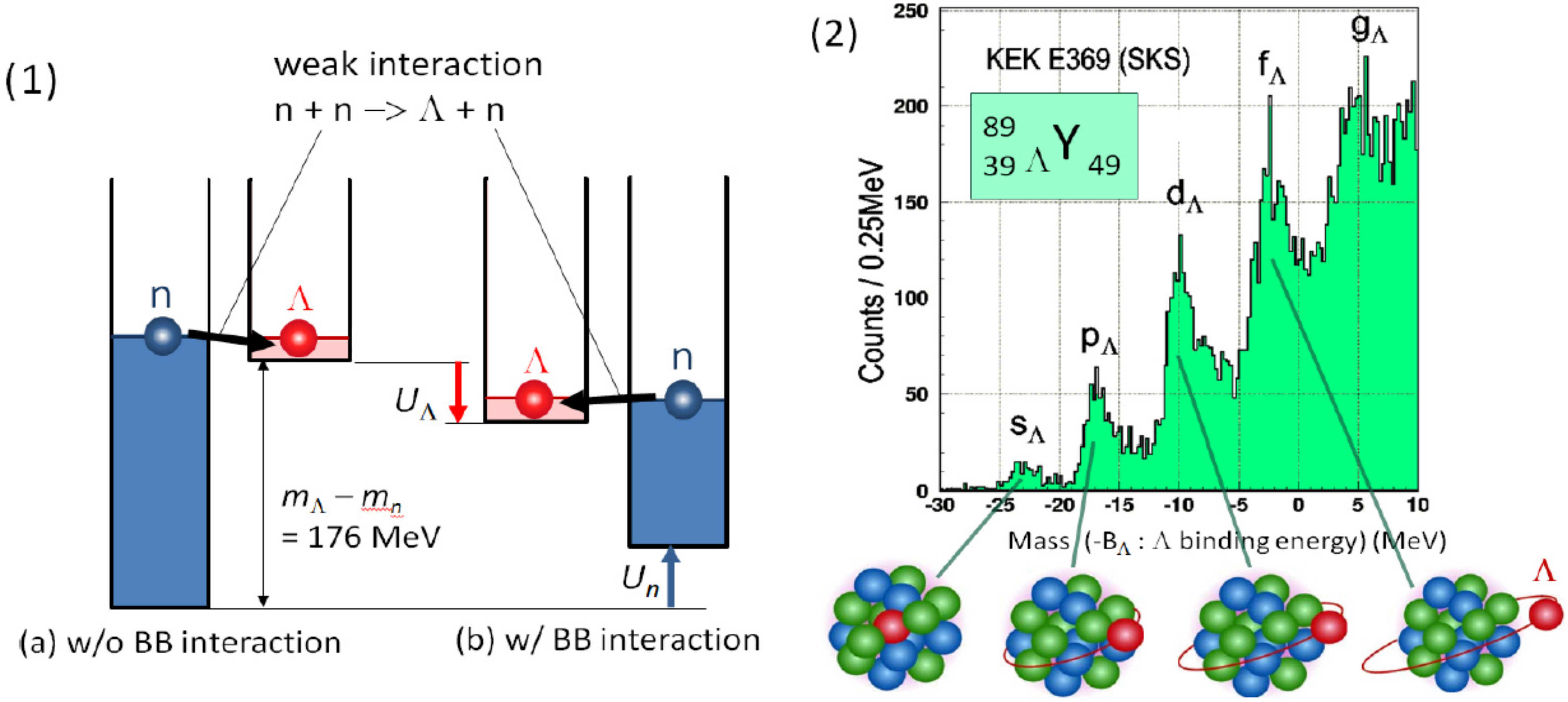


Fig. 3. (1) Energies of neutrons and Λ hyperons in high density neutron matter confined in the potential made by gravity. See text for details. (2) Excitation spectrum of a Λ hypernucleus $^{89}_{\Lambda}$Y via the (π^+,K^+) reaction on ^{89}Y target [6].

Lednicky-Lyuboshitz model

The correlation function can be calculated analytically by averaging **Ψ** over the total spin S and the distribution of the relative distances **S(r*)**

Ref : Lednicky, Richard & Lyuboshits, V.L.. (1982). Sov. J. Nucl. Phys. (Engl. Transl.); (United States). 35:5.

$$C(k*) = \int S(r*)\,|\Psi(r*,k*)|^2 d^3r$$

The normalized pair separation distribution (source function) **S(r*)** is assumed to be Gaussian,

$$S(r^*) = (2\sqrt{\pi}r_0)^{-3} e^{-\frac{r^{*2}}{4r_0^2}},$$

$$\Psi^S(r*,k*) = e^{-ik^*r^*} + f^S(k*)\frac{e^{ik^*r^*}}{r*} \qquad f^S(k*) = (\frac{1}{f_0^S} + \frac{1}{2}d_0^S k*^2 - ik*)^{-1}$$

Strong

$$|\Psi^C(r*,k*)| = \sqrt{A_C}e^{-ik^*r^*}F(-i\eta,1,i\zeta) \qquad A_C(\eta) = \frac{2\pi}{k*a_c}(exp(\pm\frac{2\pi}{k*a_c}) - 1)^{-1}$$

Coulomb

F- confluent hypergeometric function

f_0 and d_0 - parameters of strong interaction.

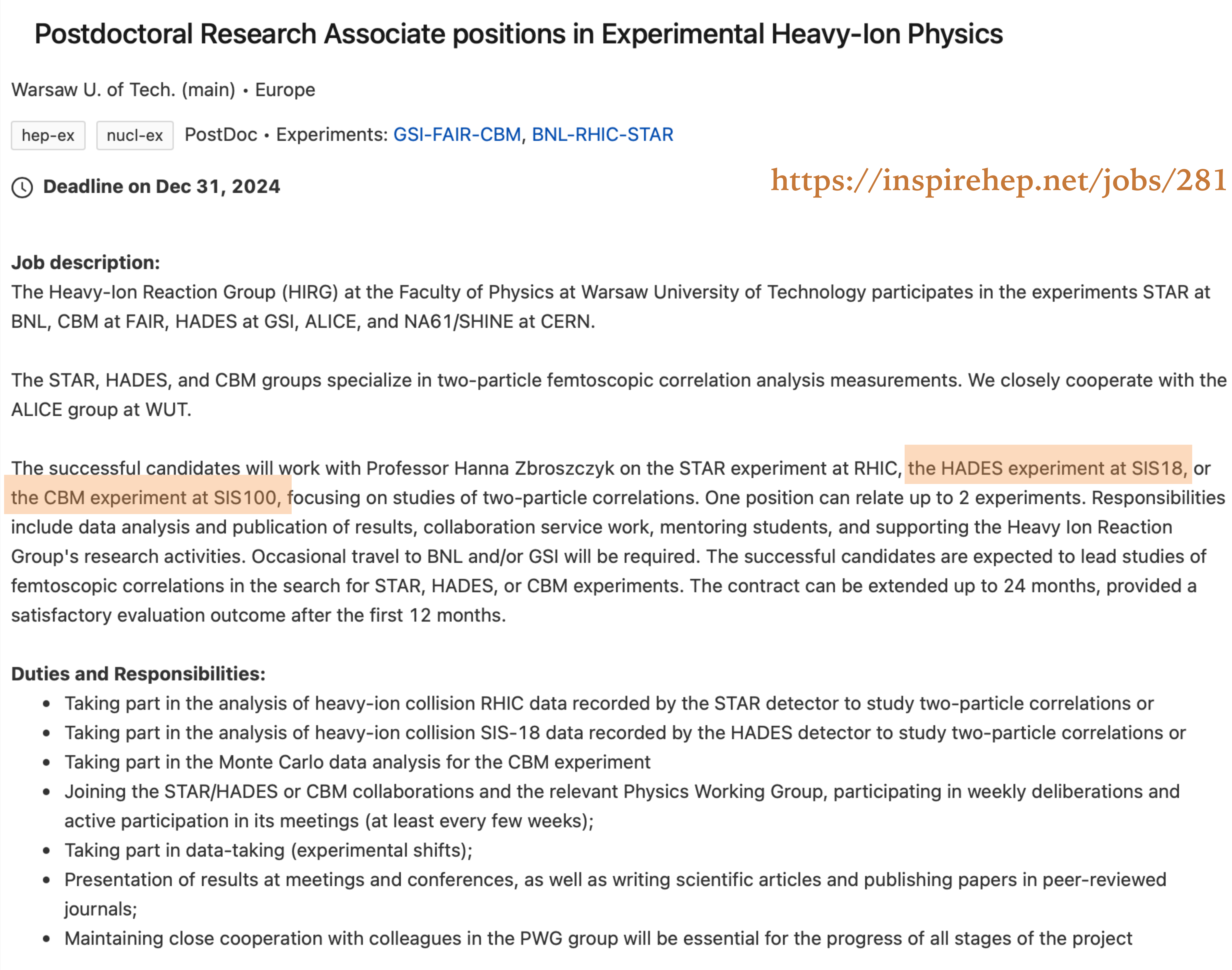
Theoretical correlation function (k*) depends on: R, f_0 and d_0.

f_0 - the scattering length, determines low-energy scattering.

The elastic cross section, σ_e, (at low energies) determined by the scattering length, $\lim_{k\to 0}\sigma_e = 4\pi f_0^2$

d_0 - the effective range, corresponds to the range of the potential (simplified scenario - the square well potential.

For identical systems one has to include QS (Fermi-Dirac / Bose-Einstein) as well.



Postdoctoral Research Associate positions in Experimental Heavy-Ion Physics

Warsaw U. of Tech. (main) • Europe

PostDoc • Experiments: GSI-FAIR-CBM, BNL-RHIC-STAR

Deadline on Dec 31, 2024

https://inspirehep.net/jobs/2811921

Job description:

The Heavy-Ion Reaction Group (HIRG) at the Faculty of Physics at Warsaw University of Technology participates in the experiments STAR at BNL, CBM at FAIR, HADES at GSI, ALICE, and NA61/SHINE at CERN.

The STAR, HADES, and CBM groups specialize in two-particle femtoscopic correlation analysis measurements. We closely cooperate with the ALICE group at WUT.

The successful candidates will work with Professor Hanna Zbroszczyk on the STAR experiment at RHIC, the HADES experiment at SIS18, or the CBM experiment at SIS100, focusing on studies of two-particle correlations. One position can relate up to 2 experiments. Responsibilities include data analysis and publication of results, collaboration service work, mentoring students, and supporting the Heavy Ion Reaction Group's research activities. Occasional travel to BNL and/or GSI will be required. The successful candidates are expected to lead studies of femtoscopic correlations in the search for STAR, HADES, or CBM experiments. The contract can be extended up to 24 months, provided a satisfactory evaluation outcome after the first 12 months.

Duties and Responsibilities:

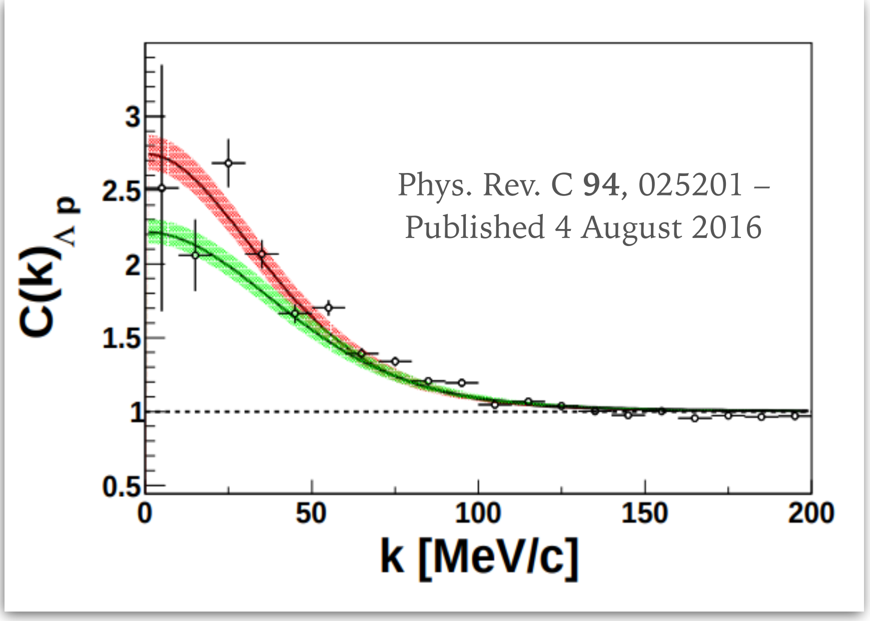
- Taking part in the analysis of heavy-ion collision RHIC data recorded by the STAR detector to study two-particle correlations or
- Taking part in the analysis of heavy-ion collision SIS-18 data recorded by the HADES detector to study two-particle correlations or
- Taking part in the Monte Carlo data analysis for the CBM experiment
- Joining the STAR/HADES or CBM collaborations and the relevant Physics Working Group, participating in weekly deliberations and active participation in its meetings (at least every few weeks);
- Taking part in data-taking (experimental shifts);
- Presentation of results at meetings and conferences, as well as writing scientific articles and publishing papers in peer-reviewed journals;
- Maintaining close cooperation with colleagues in the PWG group will be essential for the progress of all stages of the project

Lednicky-Lyuboshitz model

Model		$f_0^{S=0}$ (fm)	$f_0^{S=1}$ (fm)	$d_0^{S=0}$ (fm)	$d_0^{S=1}$ (fm)	n_σ
ND [77]		1.77	2.06	3.78	3.18	1.1
NF [78]		2.18	1.93	3.19	3.358	1.1
NSC89 [79]		2.73	1.48	2.87	3.04	0.9
NSC97 [80]	a	0.71	2.18	5.86	2.76	1.0
	b	0.9	2.13	4.92	2.84	1.0
	c	1.2	2.08	4.11	2.92	1.0
	d	1.71	1.95	3.46	3.08	1.0
	e	2.1	1.86	3.19	3.19	1.1
	f	2.51	1.75	3.03	3.32	1.0
ESC08 [81]		2.7	1.65	2.97	3.63	0.9
χEFT	LO [25]	1.91	1.23	1.4	2.13	1.8
	NLO [26]	2.91	1.54	2.78	2.72	1.5
Jülich	A [82]	1.56	1.59	1.43	3.16	1.0
	J04 [83]	2.56	1.66	2.75	2.93	1.4
	J04c [83]	2.66	1.57	2.67	3.08	1.1

S. Acharya *et al.* Phys. Rev. C 99, 024001 – Published 13 Feb 2019
https://doi.org/10.1103/PhysRevC.99.024001

parameter scan boundaries : f_0 [0.01, 5.0], d_{0s} [0.01, 2.0] and d_{0t} [0.01, 5.0]

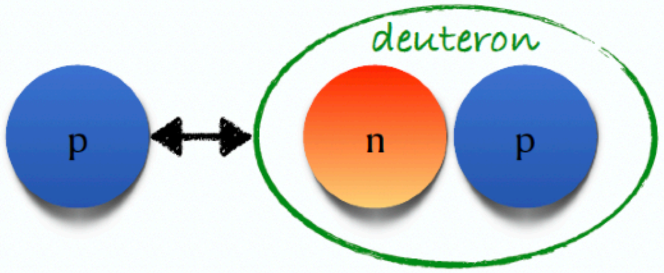


Phys. Rev. C **94**, 025201 – Published 4 August 2016

Parameters	p-Nb (LO)	p-Nb (NLO)
f_{0s}	1.91 fm	2.91 fm
d_{0s}	1.40 fm	2.78 fm
f_{0t}	1.23 fm	1.54fm
d_{0t}	2.13 fm	2.72fm
r_0	1.71 ± 0.10	1.62 ± 0.02

Light nuclei correlation: p-d, d-d correlations

Proton-Deuteron Pair: (SI, Coul)



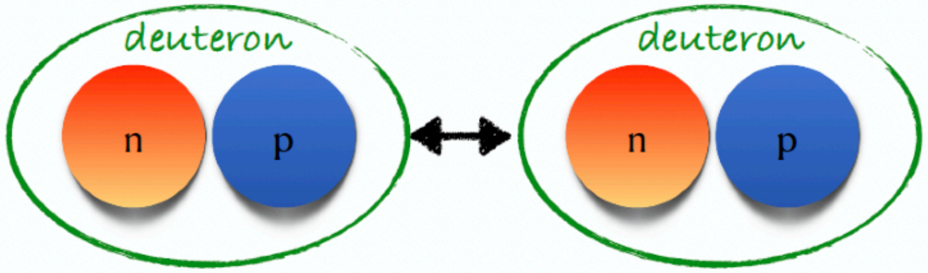
Spin = 1/2 Spin = 1

^{3}He **bound** →

$$C_{pd} = \boxed{\frac{1}{3} C_{doublet,S=\frac{1}{2}}} + \frac{2}{3} C_{quartet,S=\frac{3}{2}}$$

Doublet spin state $^{2}S_{1/2}$		Quartet spin state $^{4}S_{3/2}$		Ref
Scattering Length	**Effective Range**	**Scattering Length**	**Effective Range**	
1.30 +/- 0.2 fm	-	11.40 +/- 1.5 fm	2.05 +/ 0.25 fm	Oers, Brockmann et al, Nucl.Phys.A 561-583
2.73 +/- 0.1 fm	2.27 +/- 0.12 fm	11.88 +/- 0.25 fm	2.63 +- 0.02 fm	J. Arvieux, Nucl.Phys.A 221 253-268 (1973)
4.0 fm	-	11.1 fm	-	E. Huttel et al, Nucl.Phys.A 406 443-455
0.024 fm	-	13.7 fm	-	A. Kievsky et al, PLB 406 292-296 (1997)
-0.13 +/- 0.04 fm	-	14.70 +/- 2.30 fm	-	T. C. Black et al, PLB 471 103-107 (1999)

Deuteron-Deuteron Pair: (SI, Coul, QS)



Spin = 1 Spin = 1

^{4}He **bound** →

$$C_{dd} = \boxed{\frac{1}{9} C_{singlet,S=0}} + \frac{3}{9} C_{triplet,S=1} + \frac{5}{9} C_{quintet,S=2}$$

Triplet: Do not contribute to SI

⇒ Triplet spin (S=1) : irrelevant for s-wave

⇒ Modify the component used in L-L model

$$C_{dd} = \frac{1}{6} C_{singlet,S=0} + \frac{5}{6} C_{quintet,S=2}$$