Quark-diquark correlations in baryons

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QCD at FAIR Workshop 2024
GSI, Nov 11, 2024

The role of diquarks

- **Diquark correlations in baryons:**

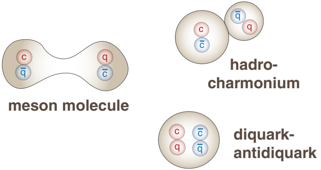
  "Missing resonances": nonrelativistic quark model predicts too many states, pointlike diquarks reduce them by freezing one excitation mode.
  New states in PDG call this into question

  Anselmino et al., RMP 65 (1993), Santopinto, PRC 72 (2005),
  Nikonov, Anisovich, Klempt, Sarantsev, Thoma, PLB 662 (2008)

  Dominance of **qq forces inside baryons**:
  non-pointlike diquark correlations, diquarks exchange roles
  GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016), Barabanov et al., PPNP 116 (2021)

  Diquark mass differences can be studied on lattice
  Francis, de Forcrand, Lewis, Maltman, JHEP 05 (2022)

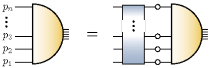
- **Diquark correlations in multiquark states:**
  Tetraquarks as compact diquark-antidiquark states?
  Jaffe, PRD 15 (1977), Esposito, Pilloni, Polosa, Phys. Rept 668 (2017),
  Guo, Hanhart, Meißner et al., RMP 90 (2018), . . .



- Phases at high baryon densities,
  QCD-like theories (SU(2)), ...

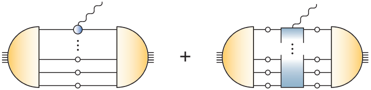
Functional methods

- Hadronic **bound-state equations** (Bethe-Salpeter & Faddeev eqs)

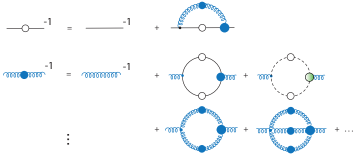


"QFT analogue of Schrödinger eq."
→ hadron masses & "wave functions"
→ **spectroscopy calculations**

- **Structure calculations:** form factors, PDFs, GPDs, TMDs, two-photon processes, ...



- Ingredients: **QCD's n-point functions,** computed from DSEs (quantum eqs. of motion) or FRG (functional renormalization group)

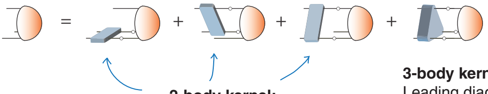


→ Dynamical mass generation, gluon mass gap, confinement, QCD phase diagram, ...

Baryons

Three-quark BSE (Faddeev equation) for baryons:

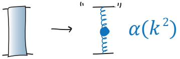
GE, Alkofer, Nicmorus, Krassnigg, PRL 104 (2010)

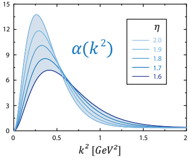


2-body kernel:
fixed in meson sector

3-body kernel:
Leading diagram (3-gluon vertex) vanishes by color trace,
higher-order diagrams small (?)
2-quark correlations dominant?

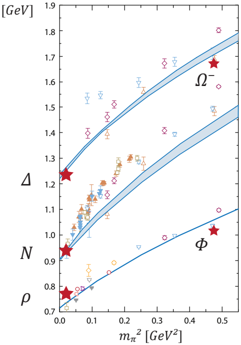
Rainbow-ladder





Scale set by f_π,
shape parameter → bands

Maris, Tandy, PRC 60 (1999)



- Analogous results for many **form factors**

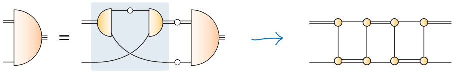
  Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. 91 (2016)

- Relativistically, nucleon also has **p waves!**

see also:
Qin, Roberts, Schmidt, PRD 97 (2018)

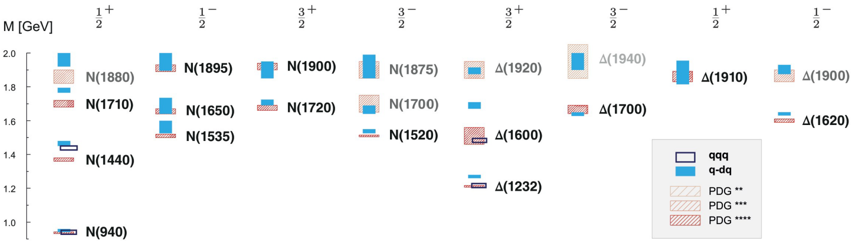
Diquark correlations

- **Quark-diquark** (two-body) equation
  Oettel et al., PRC 58 (1998), GE et al., Ann. Phys. 323 (2008), Cloet et al., FBS 46 (2009), Segovia et al., PRL 115 (2015)

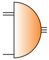


Diquark clustering in baryons?
Barabanov et al., Prog. Part. Nucl. Phys. 116 (2021)

- **Three-quark** and **quark-diquark** results very similar
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



Diquarks 101



Wave function: Dynamics ⊗ $\underbrace{\text{Flavor}}_{\mathcal{A},\,\mathcal{S}}$ ⊗ $\underbrace{\text{Color}}_{\mathcal{A}}$

attractive

$$\mathbf{3} \otimes \mathbf{3} = \overline{\mathbf{3}} \oplus \mathbf{6}$$

Flavor:

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1}_A \oplus \mathbf{3}_S$$
$$\mathbf{3} \otimes \mathbf{3} = \overline{\mathbf{3}}_A \oplus \mathbf{6}_S$$
$$\mathbf{4} \otimes \mathbf{4} = \mathbf{6}_A \oplus \mathbf{10}_S$$

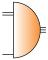
"good"	"bad"
$[ud]$	$\{uu\}$, $\{ud\}$, $\{dd\}$
$[us]$, $[ds]$	$\{us\}$, $\{ds\}$, $\{ss\}$
$[uc]$, $[dc]$, $[sc]$	$\{uc\}$, $\{dc\}$, $\{sc\}$, $\{cc\}$

I = 0
I = 1
I = 1/2

Quark content	q-dq	Isospin	contributes to
sss	s {ss}	0	Ω
ccc	c {cc}	0	Ω_c
uud	u [ud]	1/2	p
	u {ud}	1/2, 3/2	p, Δ^+
	d {uu}	1/2, 3/2	p, Δ^+
nns	n [ns]	0, 1	Λ, Σ
n = u, d	n {ns}	0, 1	Λ, Σ
	s [nn]	0	Λ
	s {nn}	1	Σ

- Nucleon has "good" and "bad" diquarks, Delta only "bad"
- Λ: n [ns], n {ns}, s [nn]
  Σ: n [ns], n {ns}, s {nn}

Diquarks 101



Wave function: $\underbrace{\text{Dynamics}}_{\mathcal{A},\,\mathcal{S}} \otimes \underbrace{\text{Flavor}}_{\mathcal{A},\,\mathcal{S}} \otimes \underbrace{\text{Color}}_{\mathcal{A}} \stackrel{!}{=} \mathcal{A}$

- $\gamma_5 C$, ... **scalar** $\mathcal{A}$: $(\gamma_5 C)^T = -\gamma_5 C$ ⇔ **"good"**
- $\gamma^\mu C$, ... **axialvector** $\mathcal{S}$: $(\gamma^\mu C)^T = \gamma^\mu C$ ⇔ **"bad"**
- C, ... **pseudoscalar** $\mathcal{A}$
- $\gamma^\mu \gamma_5 C$, ... **vector** $\mathcal{A}$

(pseudoscalar and vector: **"ugly"**)

- ...

Total wave function:

$$\Gamma^{(\mu)}(q,P) = \sum_i \underbrace{f(q^2, q\cdot P, P^2=-m^2)}_{\text{Dressing functions calculated from BSE}} \, \underbrace{\tau_i^{(\mu)}(q,P)}_{\text{Lorentz/Dirac tensors}} \otimes \text{Flavor} \otimes \text{Color}$$

Diquark correlations

Mesons and diquarks are closely related:

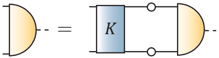
attractive

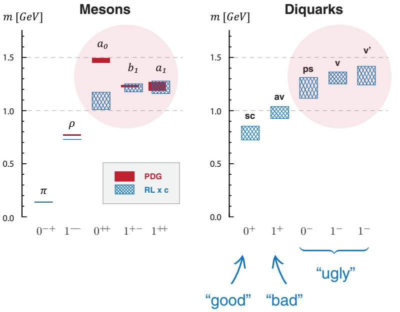
$$\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{1} \otimes \mathbf{8}$$

attractive

$$\mathbf{3} \otimes \mathbf{3} = \overline{\mathbf{3}} \oplus \mathbf{6}$$

In BSE this comes out naturally: Maris, FBS 32 (2002)





Lowest-lying diquarks are dominant for ground-state octet & decuplet baryons

pseudoscalar mesons	⇔	**scalar diquarks** (~0.8 GeV)
vector mesons	⇔	**axialvector diquarks** (~1 GeV)

Higher-lying diquarks are subleading, but contribute to excited states & remaining channels

scalar mesons	⇔	**pseudoscalar diquarks** (~1.2 GeV)
axialvector mesons	⇔	**vector diquarks** (~1.3 GeV)

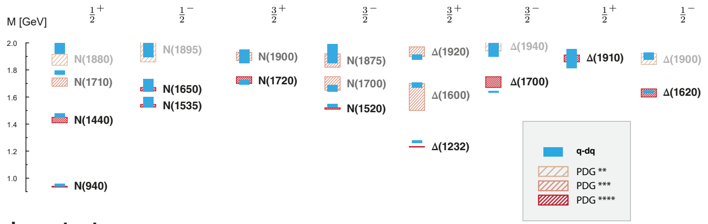
In RL, these are too strongly bound; simulate beyond-RL effects by (one) strength parameter c

Roberts, Chang, Cloet, Roberts, FBS 51 (2011)
GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

Diquark correlations

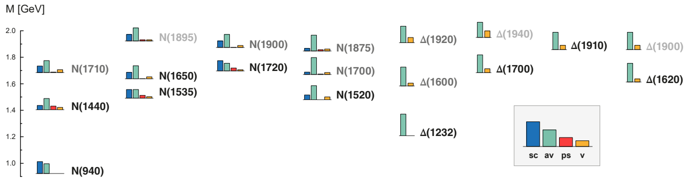
Light baryon spectrum

GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



Diquark content:

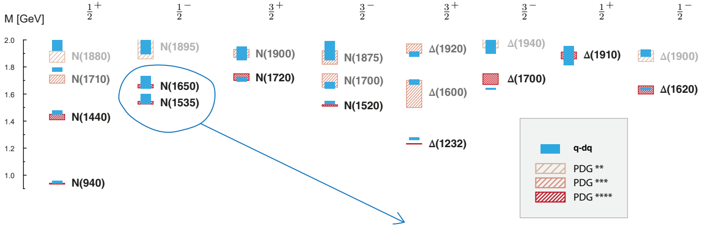
Barabanov et al., PPNP 116 (2021)



Diquark correlations

Light baryon spectrum

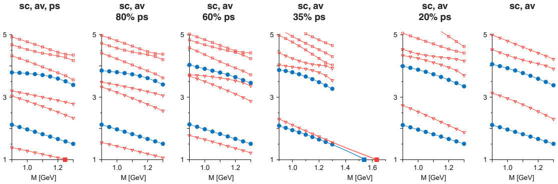
GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



RL, all diquarks:
"N(1535)" too low

"Beyond RL":
N(1535), N(1650)

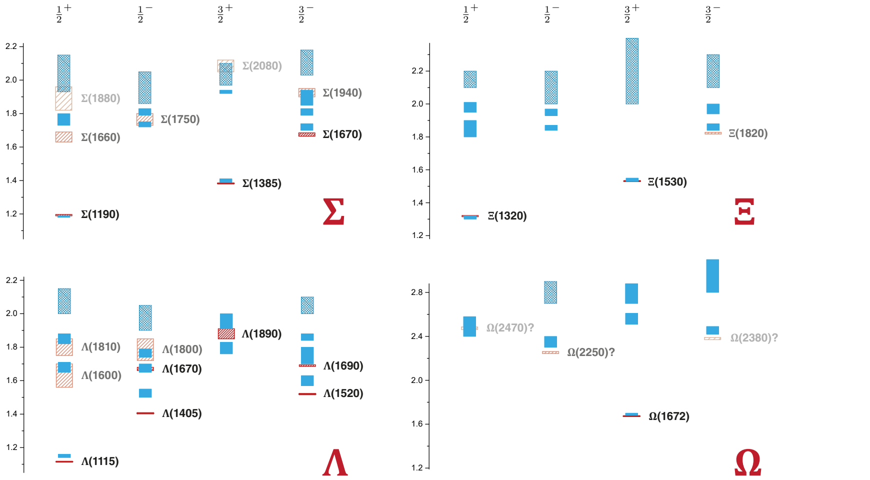
RL, sc+av only:
"N(1650)" too high



- **Level ordering** determined by diquark dynamics
- Diquarks are not pointlike, also here **rich spectrum!**

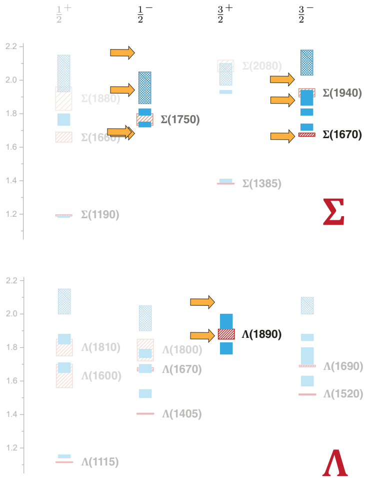
Barabanov et al., PPNP 116 (2021)

Strange baryons



GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

Strange baryons



New states from Bonn-Gatchina

Sarantsev, Matveev, Nikonov, Anisovich, Thoma, EPJA 55 (2019)

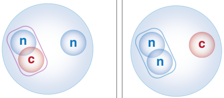
GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

Heavy baryons

Quark content	q-dq	Isospin	contributes to
nnc	n [nc]	0, 1	Λ_c , Σ_c
↑	n {nc}	0, 1	Λ_c , Σ_c
n = u, d	c [nn]	0	Λ_c
	c {nn}	1	Σ_c

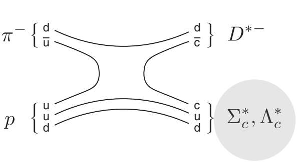
$\Rightarrow$ Λ_c: n [nc], n {nc}, c [nn]

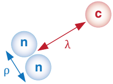
Σ_c: n [nc], n {nc}, c {nn}



Sometimes these are assumed as dominant components, e.g. **J-PARC** charm baryon spectroscopy program (high-p)

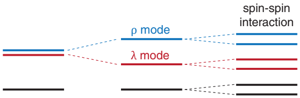
Kim, Hosaka, Kim, Noumi, Shirotori, PTEP 2014 (2014), 10, 103D01,
Shim, Hosaka, Kim, PTEP 2020 (2020) 5, 053D01





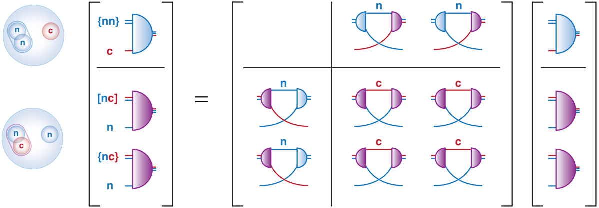
Decay of ρ mode → $\Sigma_c \pi$
Decay of λ mode → pD

Assumptions on spectrum & production rates



Heavy baryons

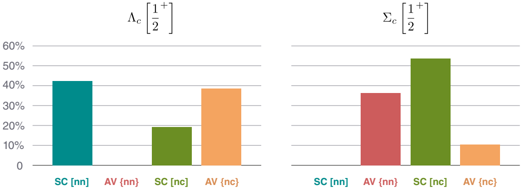
Quark-diquark BSE would not work under this assumption, e.g., Σ_c:

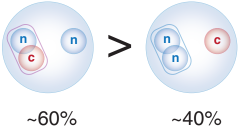


n[nc], n{nc} necessary, otherwise no equation. Presumably these are also dominant: cannot switch off n[nc], n{nc}, but c{nn}

Analogous for Λ_c and hyperons

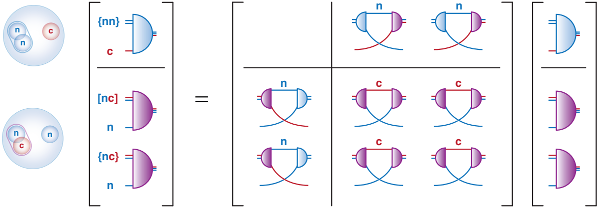
Results: **wave function contributions** Torcato, Arriaga, GE, Peña, FBS 64 (2023)





Heavy baryons

Quark-diquark BSE would not work under this assumption, e.g., Σ_c:

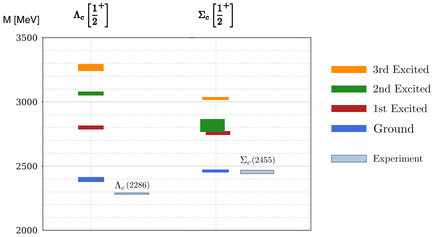


n[nc], n{nc} necessary, otherwise no equation. Presumably these are also dominant: cannot switch off n[nc], n{nc}, but c{nn}

Analogous for Λ_c and hyperons

Results: **spectrum**

Torcato, Arriaga, GE, Peña, FBS 64 (2023)

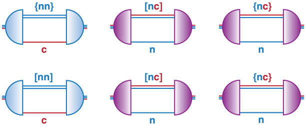


see also:
Yin, Chen, Krein, Roberts, Segovia, PRD 100 (2019)

How to test this?

Σ_c:

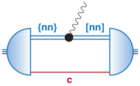
Λ_c:

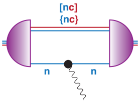


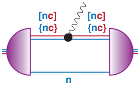
Form factors:

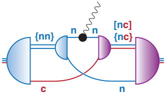
Couple currents (photons, pions, ...) to all possible places: quarks, diquarks, exchange diagrams, seagulls

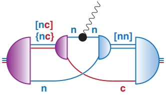
e.g. $\Sigma_c \to \Lambda_c$ transition form factors (analogous to $\Sigma \to \Lambda$)

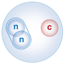






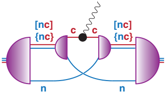




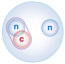


Leading (?) contribution: direct coupling to light quarks

Coupling to charm quark inside diquark

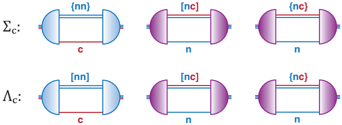


Quark exchange diagrams (+ seagulls, not shown)



If this were the only configuration, we would never probe charm quarks

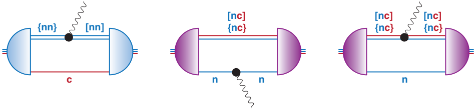
How to test this?



Form factors:

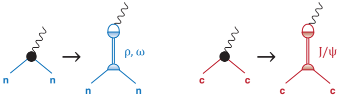
Couple currents (photons, pions, ...) to all possible places: quarks, diquarks, exchange diagrams, seagulls

e.g. $\Sigma_c \to \Lambda_c$ transition form factors (analogous to $\Sigma \to \Lambda$)

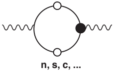


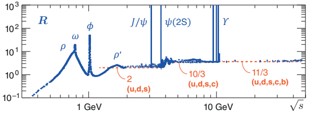
+ exchange + seagull diagrams

Timelike form factors:
Quark-photon vertices contain vector-meson poles:



Analogous to hadronic vacuum polarization (R ratio):

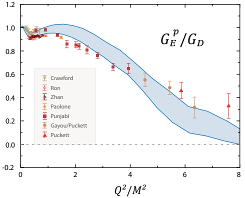




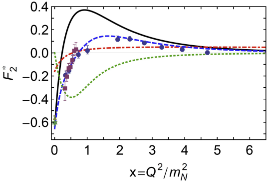
Form factors

Many **form factor calculations** in qqq or q(qq) approaches available:

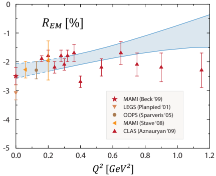
Nucleon electromagnetic FFs
GE, PRD 84 (2011)



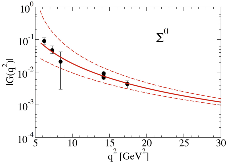
Roper em. transition FFs
Segovia et al., PRL 115 (2015)



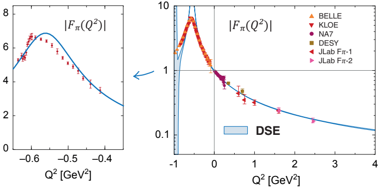
Δ-baryon em. transition FFs
GE, Nicmorus, PRD 85 (2012)



Timelike em. strangeness FFs
Ramalho, Peña, PRD 101 (2020)



Timelike pion FF dynamically develops ρ pole with $\pi\pi$ decay channel
Miramontes, Sanchis-Alepuz, Alkofer, PRD 103 (2021)



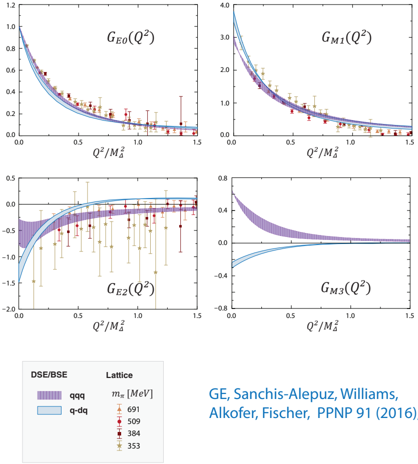
GE, Fischer, Williams, PRD 101 (2020),

Thank you!

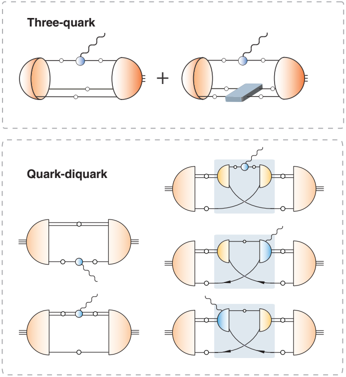
Backup slides

Form factors

Like for spectrum, similar results from qqq and q(qq) approaches,
e.g. Δ **electromagnetic form factors**:



GE, Sanchis-Alepuz, Williams,
Alkofer, Fischer, PPNP 91 (2016),

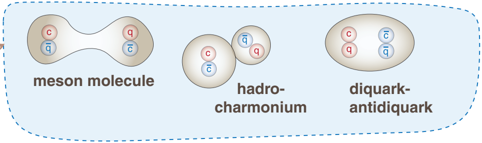


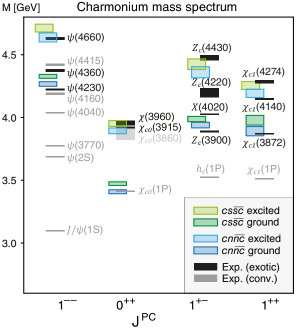
Four-quark states

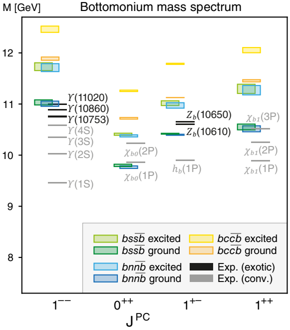
Hidden charm, hidden bottom

Hoffer, GE, Fischer, PRD 109 (2024)

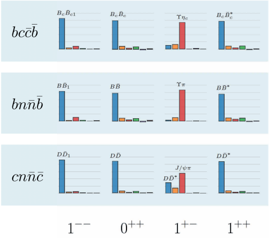
$cq\bar{q}\bar{c}$







Wave-function components:

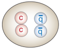


Four-quark states

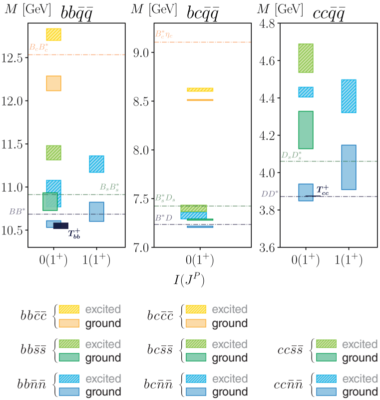
Open-flavor states

Hoffer, GE, Fischer, in preparation

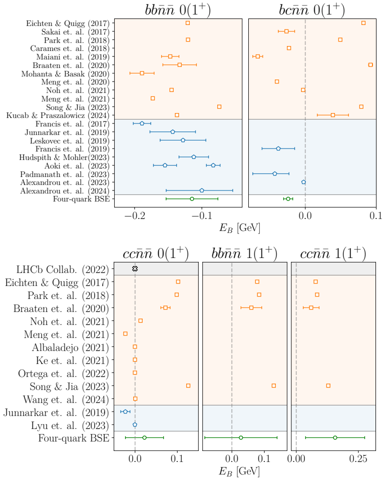
meson molecule



diquark-antidiquark



Binding energies (ground states):

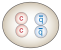


Four-quark states

Open-flavor states

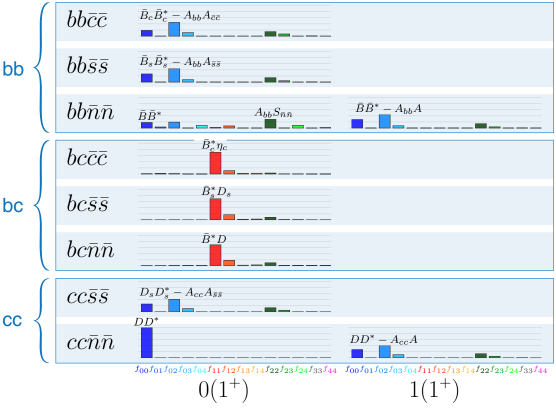
Hoffer, GE, Fischer, in preparation

meson molecule



diquark-antidiquark

Wave-function components:



$I(J^P)$	Physical components					
	$\mathbf{1}\otimes\mathbf{1}$		$\bar{\mathbf{3}}\otimes\mathbf{3}$	$\mathbf{8}\otimes\mathbf{8}$		$\bar{\mathbf{6}}\otimes\mathbf{6}$
	f_0	f_1	f_2	f_3	f_4	f_5
$0(1^+)$ $bb\bar{n}\bar{n}$	BB^*	B^*B^*	$A_{bb}S$	BB^*	B^*B^*	$S_{bb}A$
$bc\bar{n}\bar{n}$	BD^*	B^*D	$A_{bc}S$	BD^*	B^*D	$S_{bc}A$
$cc\bar{n}\bar{n}$	DD^*	D^*D^*	$A_{cc}S$	DD^*	D^*D^*	$S_{cc}A$
$bb\bar{s}\bar{s}$	$B_sB_s^*$	–	$A_{bb}A_{ss}$	$B_sB_s^*$	–	–
$bc\bar{s}\bar{s}$	$B_sD_s^*$	$B_s^*D_s$	$S_{bc}A_{ss}$	$B_sD_s^*$	$B_s^*D_s$	$A_{bc}S_{ss}$
$cc\bar{s}\bar{s}$	$D_sD_s^*$	–	$A_{cc}A_{ss}$	$D_sD_s^*$	–	–
$1(1^+)$ $bb\bar{q}\bar{q}$	BB^*	–	$A_{bb}A$	BB^*	–	–
$cc\bar{q}\bar{q}$	DD^*	–	$A_{cc}A$	DD^*	–	–

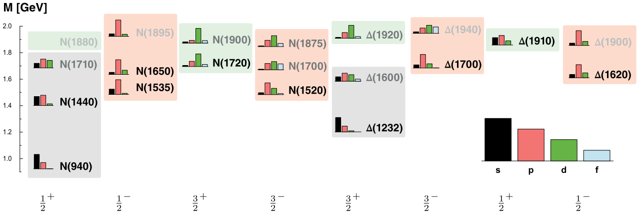
→ T_{bb} is complicated mixture of molecule and diquark-antidiquark

Can **quantify** meson-meson and diquark-antidiquark contributions

→ T_{cc} sits on top of threshold, pure molecule

Relativistic effects

Orbital angular momentum: clear traces of nonrelativistic quark model, but strong relativistic effects (in some cases even dominant)



Quark model:

L = 0

L = 1

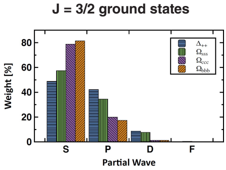
L = 2

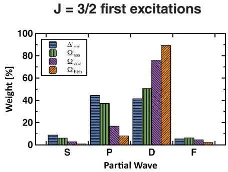
GE, FBS 58 (2017)

Relativistic contributions

even up to bottom baryons!

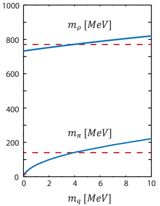
Qin, Roberts, Schmidt, PRD 97 (2018)



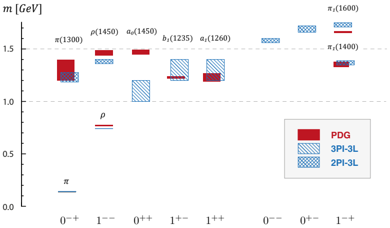


Mesons

- Pion is **Goldstone boson:** $m_\pi^2 \sim m_q$



- **Light meson spectrum** beyond rainbow-ladder

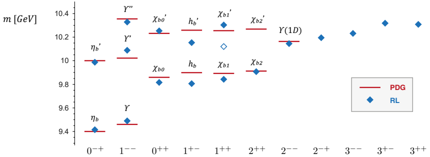


Williams, Fischer, Heupel, PRD 93 (2016)

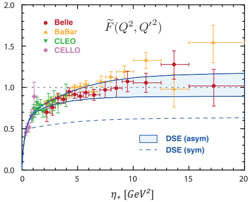
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- **Bottomonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)



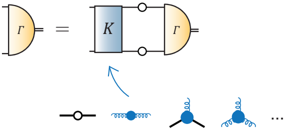
- **Pion transition form factor**



GE, Fischer, Weil, Williams, PLB 774 (2017)

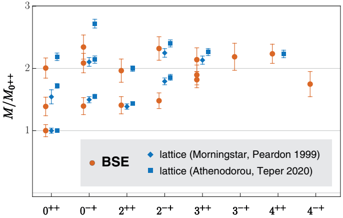
Towards ab-initio

- **Goal:** go towards ab-initio calculations by calculating **higher n-point functions**

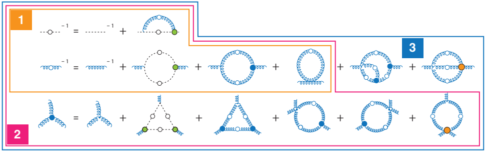


. . . ,
Williams, Fischer, Heupel, PRD 93 (2016),
Cyrol et al., PRD 97 (2018),
Oliveira, Silva, Skullerud, Sternbeck, PRD 99 (2019),
Aguilar et al., EPJ C 80 (2020),
Huber, PRD 101 (2020),
Qin, Roberts, Chin. Phys. Lett. 38 (2021),
GE, Pawlowski, Silva, PRD 104 (2021),
. . .

- **Glueball spectrum** agrees with lattice QCD
Huber, Fischer, Sanchis-Alepuz, EPJ C 80 (2020), EPJ C 81 (2021)



- Coupled **Yang-Mills DSEs**
Huber, PRD 101 (2020),
GE, Pawlowski, Silva, PRD 104 (2021)



truncation error:

1 60% 2 10% 3 4%