# Femtoscopy, production, and exotic spectroscopy

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QCD@FAIR 2024
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# Outline

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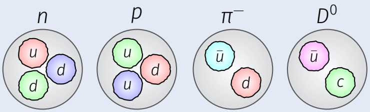
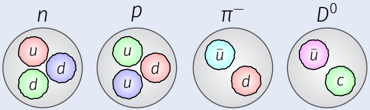



- Conventional hadrons:
  - Mesons: $q\bar{q}'$: $\pi^+ = u\bar{d}$, $D^0 = c\bar{u}$, ...
  - Baryons: $q_1q_2q_3$: $p = uud$, $n = udd$, ...

- Constituent quark models have succesfully described most of (but not all!) the hadrons discovered so far.
- Only possibilities? No, the only requirement is to be color singlets. There can be tetraquarks ($q_1q_2q_3q_4$), pentaquarks ($\bar{q}_1q_2q_3q_4$), hybrids ($\bar{q}_1q_2g$), glueballs ($gg$), hadronic molecules ($MM'$, $MB$, $BB'$),...

# Spectroscopy (conventional and exotic). The recent LHCb $T_{cc}^{+}$ "tetraquark"

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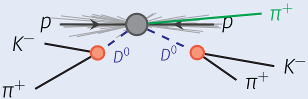
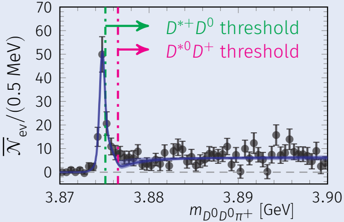
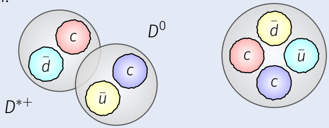
[Nature Phys., **18**, 751('22); Nature Com., **13**, 3351('22)]



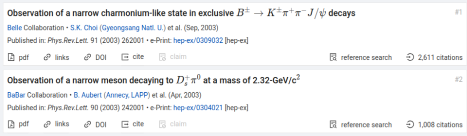

- In 2021, the LHCb collaboration discovers $T_{cc}^{+}$, with quark content $cc\bar{u}\bar{d}$, and very close to $D^{*+}D^0$ threshold
- Even if something is explicitly exotic, you still have to understand how are quarks distributed inside the hadron.




- [MA, PL,**B829**,137052('21); MA, Nieves, EPJ,**C82**,724('22)]
- Our analysis favours the molecular picture.
- Also, relevant to understand QCD confinement, and color combinations.

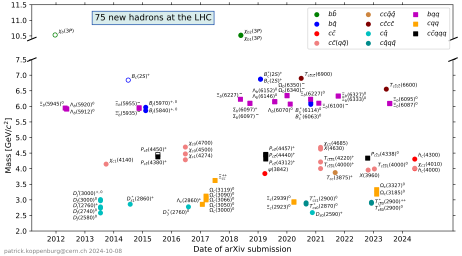
# Spectroscopy: precision and exotics

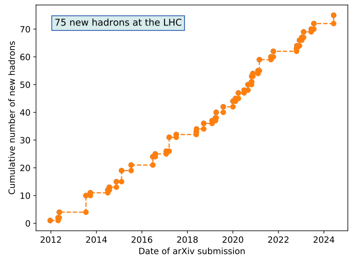
- The case of $T_{cc}^{+}(3875)$ is only a recent example...
- 2nd November Revolution started 2003 with the discovery of:
  - $X(3872)$ (hidden charm)
    Belle, Phys. Rev. Lett., **91**, 262001 (2003)
  - $D_{s0}^{*}(2317)^{+}$ (charm and strange)
    BaBar, Phys. Rev. Lett., **90**, 242001 (2003)


**Observation of a narrow charmonium-like state in exclusive $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$ decays** #1
Belle Collaboration • S.K. Choi (Gyeongsang Natl. U.) et al. (Sep, 2003)
Published in: *Phys.Rev.Lett.* 91 (2003) 262001 • e-Print: hep-ex/0309032 [hep-ex]
pdf links DOI cite claim reference search 2,611 citations

**Observation of a narrow meson decaying to $D_s^{+}\pi^{0}$ at a mass of 2.32-GeV/c$^2$** #2
BaBar Collaboration • B. Aubert (Annecy, LAPP) et al. (Apr, 2003)
Published in: *Phys.Rev.Lett.* 90 (2003) 242001 • e-Print: hep-ex/0304021 [hep-ex]
pdf links DOI cite claim reference search 1,008 citations

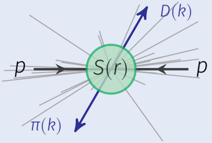
- We have entered the era of precision and exotic spectroscopy
- The LHC (mainly LHCb) is largely contributing on the experimental side, but also many other experiments: BES, Belle, GlueX...




- Theoreticians have a lot of work to do...

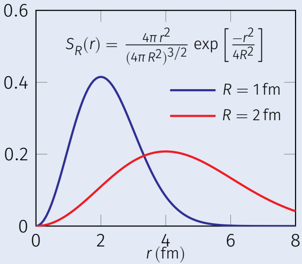
# Outline



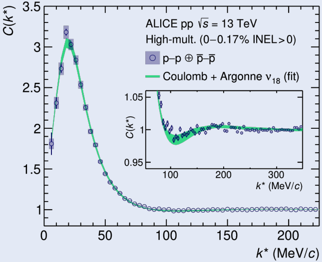

$$C_{\rm exp}(k) = \xi(k)\,\frac{N_{\rm same}(k)}{N_{\rm mixed}(k)}$$

$$C_{\rm th}(k) = \sum_{\ell=0}(2\ell+1)\int dr\, S_R(r)\,|\psi_\ell(r,k)|^2$$

$$\psi_\ell^{[\rm asy,nr]}(k,r) = j_\ell(kr) + f_\ell(k)\,\frac{e^{i(kr-\pi\ell/2)}}{r}$$

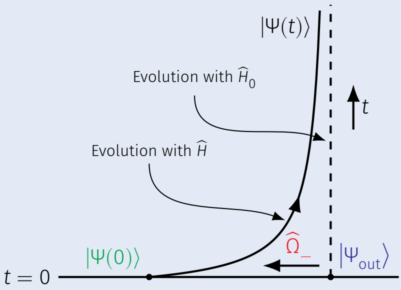



[ALICE PL,B805,135419('20)]




- Known the source $S(r)$, explore interactions (encoded in the wave function)
- A new method to explore hadron interactions
- Lot of attraction. In HADRON 2023:
  - M. Janik [Mon. 11:00]
  - V. Mantovani [Mon. 14:30]
  - D. Mihaylov [Mon. 17:40]
  - M. Albaladejo [Tue. 15:10]
  - L. Graczykowski [Wed. 14:00]
  - W. Rzesa [Wed. 14:24]
  - L. Serksnyte [Wed. 15:12]
  - E. Oset [Thu. 14:30]
  - M. Lesch [Thu. 15:12]
  - R. Lea [Thu. 15:42]

[Fabbietti, Mantovani, Vázquez-Doce, ARNPS,71,377('21)]
Check those talks for more references!




- w.f. in terms of initial or asymptotic states:

$$|\Psi(t)\rangle = e^{-i\widehat{H}t}\,|\Psi(t=0)\rangle\,,$$
$$|\Psi(t\to+\infty)\rangle = \lim_{t\to+\infty} e^{-i\widehat{H}_0 t}\,|\Psi_{\text{out}}\rangle\,,$$

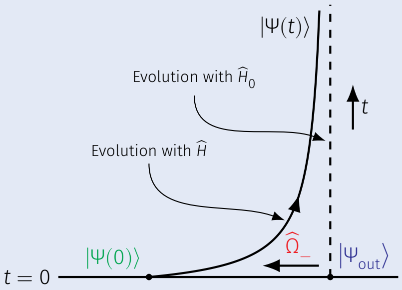
- Master formula:

$$|\Psi(t=0)\rangle = \widehat{\Omega}_-|\Psi_{\text{out}}\rangle = |\Psi_{\text{out}}\rangle + \frac{1}{E-\widehat{H}_0-i\epsilon}\,\widehat{T}^{\text{QM}}(E-i\epsilon)\,|\Psi_{\text{out}}\rangle\,,$$

- The $\widehat{T}^{\text{QM}}(W)$ satisfies LSE:

$$\widehat{T}^{\text{QM}}(W) = \widehat{V}^{\text{QM}} + \widehat{V}^{\text{QM}}\frac{1}{W-\widehat{H}_0}\widehat{T}^{\text{QM}}(W)\,,$$

# Wave function and interactions: «*as we all know...*»

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Final formula for the (c.c. of the) wave function:

$$\left\langle \vec{r}\,\middle|\Psi(t=0);\vec{k}\right\rangle^* = \psi^*(\vec{r};\vec{k}\,) = e^{-i\vec{k}\cdot\vec{r}} + \int d^3\vec{p}\,e^{-i\vec{p}\cdot\vec{r}}\,\langle\vec{p}\,|\frac{1}{E-\widehat{H}_0-i\epsilon}\hat{T}^{\text{QM}}(E-i\epsilon)|\vec{k}\,\rangle^*$$
$$= e^{-i\vec{k}\cdot\vec{r}} + \int d^3\vec{p}\,\frac{e^{-i\vec{p}\cdot\vec{r}}}{E-\frac{\vec{p}^{\,2}}{2\mu}+i\epsilon}\,\langle\vec{k}\,|\hat{T}^{\text{QM}}(E+i\epsilon)|\vec{p}\,\rangle\,.$$

# Koonin-Pratt formula for $C(k)$

- Now, let us consider just $S$-wave interaction:

$$\langle \vec{k}\,|\hat{T}^{\rm QM}(E)|\vec{p}\,\rangle = T^{\rm QM}_{\ell=0}(E;k\leftarrow p)$$

- Partial wave proj. of the previous expression for $\psi(\vec{r},\vec{k}\,) = \sum_\ell i^\ell(2\ell+1)\psi_\ell(r;k)P_\ell(\hat{r}\cdot\hat{k})$:

$$\psi_\ell(r;k) = j_\ell(kr)\ \text{for}\ \ell\neq 0$$
$$\psi^*_{\ell=0}(r;k) = j_0(kr) + 4\pi\int p^2 {\rm d}p\,\frac{j_0(pr)}{E-\frac{p^2}{2\mu}+i\epsilon}\,T^{\rm QM}_{\ell=0}(E;k\leftarrow p)$$

- Inserting this into the expression for $C(k)$, we obtain Koonin-Pratt formula:

$$C(k) = \sum_\ell (2\ell+1)\int {\rm d}r\,S_R(r)\,|\psi_\ell(r,k)|^2 = 1+\int {\rm d}r\,S_R(r)\left[|\psi_{\ell=0}(r;k)|^2 - j_0(kr)^2\right]$$
$$= 1+\int {\rm d}r\,S_R(r)\left[\left|j_0(kr)+4\pi\int p^2{\rm d}p\,\frac{j_0(pr)}{E-\frac{p^2}{2\mu}+i\epsilon}\,T^{\rm QM}_{\ell=0}(E;k\leftarrow p)\right|^2 - j_0(kr)^2\right]$$

1. From here, under some assumptions, one derives the LL formula (more, later).
2. Note that the w.f. language has been replaced with amplitudes language.

# Outline

# Definining a "production" correlation function

MA, Feijoo, Nieves, Oset, Vidaña, 2410.08880

- Original formula for CF $C(k)$:

$$C(k) = 1 + \int \mathrm{d}r\, S_R(r) \left[ \left| j_0(kr) + 4\pi \int p^2 \mathrm{d}p\, \frac{j_0(pr)}{E - \frac{p^2}{2\mu} + i\epsilon}\, T^{\mathrm{QM}}_{\ell=0}(E; k \leftarrow p) \right|^2 - j_0(kr)^2 \right]$$

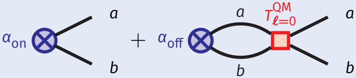
- Change integration of order:

$$C(k) = 1 + 2\mathrm{Re} \left( 4\pi \int p^2 \mathrm{d}p\, \frac{T^{\mathrm{QM}}_{\ell=0}(k \leftarrow p; E)}{E - \frac{p^2}{2\mu} + i\epsilon} F_R(k,p) \right) + (4\pi)^2 \iint p^2 \mathrm{d}p\, p'^2 \mathrm{d}p'\, \frac{T^{\mathrm{QM}}_{\ell=0}(k \leftarrow p; E)[T^{\mathrm{QM}}_{\ell=0}(k \leftarrow p'; E)]^*}{\left(E - \frac{p^2}{2\mu} + i\epsilon\right)\left(E - \frac{p'^2}{2\mu} - i\epsilon\right)} F_R(p,p')$$

with a "form factor" $F_R(q,q') = \int \mathrm{d}r\, S_R(r)\, j_0(qr)\, j_0(q'r) = \frac{e^{-(q^2+q'^2)R^2} \sinh(2qq'R^2)}{2qq'R^2}$

- Define $C^{\mathrm{prod}}(k)$ as:

$$C^{\mathrm{prod}}(k) = \left| 1 + 4\pi \int p^2 \mathrm{d}p\, \frac{T^{\mathrm{QM}}_{\ell=0}(E; k \leftarrow p)}{E - \frac{p^2}{2\mu} + i\epsilon} \tilde{F}_R(k,p) \right|^2 \qquad \text{with: } \tilde{F}_R(k,q) = F_R(k,q)/F_R(k,k)$$

- Think of this as an *ansatz*: 

, where $\tilde{F}_R \to \frac{\alpha_{\mathrm{off}}}{\alpha_{\mathrm{on}}}$

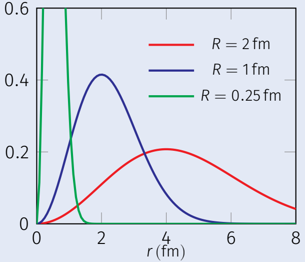
# Point-like production limit ($R \to 0$)

MA, Feijoo, Nieves, Oset, Vidaña, 2410.08880

- $C(k)$ and $C^{\text{prod}}(k)$ can be written in terms of the w.f. as:

$$C(k) = \int dr\, S_R(r)\, \left|\psi_{\ell=0}(r,k)\right|^2$$

$$C^{\text{prod}}(k) = \int dr\, S_R(r)\, j_0(kr)\, \psi_{\ell=0}(r,k)^*$$

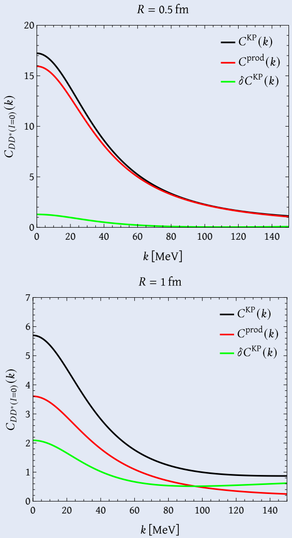



- In the limit $R \to 0$:
  - The source tends to a Dirac delta, $S_R(r) \to \delta(r)$:
  - The functions $F_R(q,q') = 1 + \mathcal{O}(R^2) = \tilde{F}_R(q,q')$
- In the point-like limit, both $C(k)$ and $C^{\text{prod}}(k)$ are equal:

$$\lim_{R\to 0} C(k) = \lim_{R\to 0} C^{\text{prod}}(k) = \left|\psi_{\ell=0}(r=0;k)\right|^2 = \left|1 + 4\pi \int p^2 dp\, \frac{T^{\text{QM}}_{\ell=0}(E; k \leftarrow p)}{E - \frac{p^2}{2\mu} + i\epsilon}\right|^2$$

- This is the usual formula we would use for exclusive processes
- For inclusive, high-multiplicity events, coherence is lost (in a sense, partially).
- This shows the connection between femtoscopy (ALICE, STAR, ...) and "usual" production experiments (LHCb, BES, ...) that measure invariant mass spectra.
- Two generalizations [MA *et al.*, 2410.08880]:
  - Relativistic phase space / loop / normalization
  - Coupled channels

MA, Feijoo, Nieves, Oset, Vidaña, 2410.08880

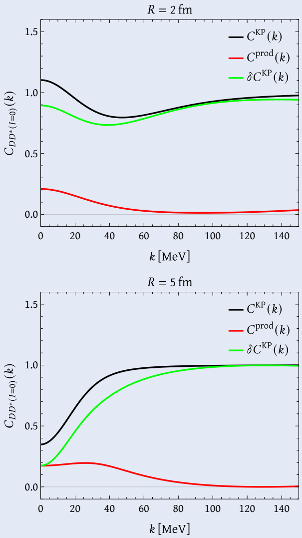



- Simple model for $T$-matrix: isospin limit, single channel, $T^{-1}(E) = V^{-1} - G_\Lambda(E)$, $V$ constant.
- The model produces a reasonable $T_{cc}(3875)^+$, bound by 860 keV in the isospin limit.
- Scattering length well reproduced.
- For small $R$, both functions give similar results. This reflects the result proven earlier that:

$$\lim_{R\to 0} C(k) = \lim_{R\to 0} C^{\mathrm{prod}}(k) = |\psi(\vec{r}=\vec{0};\vec{k}\,)|^2$$

$$= \left|1 + \int \frac{p^2\,\mathrm{d}p}{2\pi^2}\,\frac{\omega_D(p)+\omega_{D^*}(p)}{2\omega_D(p)\omega_{D^*}(p)}\,\frac{T(k\leftarrow p;E)}{s-(\omega_D(p)+\omega_{D^*}(p))^2+i\epsilon}\right|^2$$

MA, Feijoo, Nieves, Oset, Vidaña, 2410.08880

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- For larger $R$ (away from the point-like production limit), the discrepancies are larger.‘

# Outline

- $S$-wave partial w.f. ($k$ on-shell momentum, $E = k^2/(2\mu)$)

$$\psi^*_{\ell=0}(r;k) = j_0(kr) + 4\pi \int p^2 \mathrm{d}p \, \frac{j_0(pr)}{E - \frac{p^2}{2\mu} + i\epsilon} \, T^{\mathrm{QM}}_{\ell=0}(E;k \leftarrow p)$$

- LL approach: consider $T^{\mathrm{QM}}_{\ell=0}(E;p) = T^{\mathrm{QM}}_{\ell=0}(E)$, no off-shell dependence

$$\psi^*_{\ell=0}(r;k) = j_0(kr) + \underbrace{\left( 4\pi \int p^2 \mathrm{d}p \, \frac{j_0(pr)}{E - \frac{p^2}{2\mu} + i\epsilon} \right)}_{-4\pi^2\mu \frac{\exp(ikr)}{r}} \underbrace{T^{\mathrm{QM}}_{\ell=0}(E)}_{-f_0(k)/(4\pi^2\mu)}$$

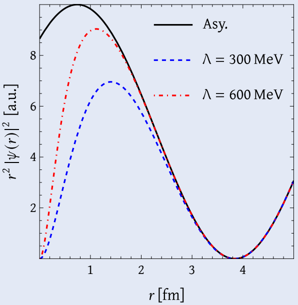
$$= j_0(kr) + f_0(k)\frac{e^{ikr}}{r} \, , \quad \text{which is the usual QM formula for asymptotic partial w.f.}$$

- Inserting this into $C(k)$, one gets a useful, closed formula in terms of $f_0(k)$ and $R$:

$$C_{\mathrm{LL}}(k) = 1 + \frac{2\mathrm{Re}\, f_0(k)}{\sqrt{\pi}R} F_1(2kR) + \frac{|f_0(k)|^2}{2R^2} e^{-(2kR)^2} \, , \quad \text{with } F_1(x) = \int_0^x \mathrm{d}t \, \frac{e^{t^2-x^2}}{x} \overset{x\to 0}{\simeq} 1 - \frac{2x^2}{3} + \cdots$$

The LL formula is equivalent to considering the asymptotic ($r \to \infty$) wave function to be valid for every distance $r$

MA, Feijoo, Nieves, Oset, Vidaña, 2410.08880




- $C_{\rm LL}(k)$ diverges for $R \to 0$:
  - Loop function does not converge for $r \to 0$
  - So does the w.f.
- Natural suggestion: regularize the loop function / the w.f.:

$$\psi^*_{\ell=0}(r;k) = j_0(kr) + 4\pi \int p^2 \mathrm{d}p \, \frac{j_0(pr)}{E - \frac{p^2}{2\mu} + i\epsilon} \, T^{\rm QM}_{\ell=0}(E;p)$$

$$= j_0(kr) + 4\pi \int p^2 \mathrm{d}p \, \frac{j_0(pr)}{E - \frac{p^2}{2\mu} + i\epsilon} \, T^{\rm QM}_{\ell=0}(E) \, \frac{\Lambda^2 + k^2}{\Lambda^2 + p^2}$$

$$= j_0(kr) + f_0(k) \, \frac{e^{ikr} - e^{-\Lambda r}}{r}$$

- This gives an expression for the CF that we denote $C^{\rm LL\Lambda}(k)$:

$$C_{\rm LL\Lambda}(k) = 1 + \frac{2{\rm Re} f_0(k)}{\sqrt{\pi}R} \left[ F_1(x) + \frac{\sqrt{\pi}\,{\rm Im}\,F_4(x^+_\Lambda)}{x} \right] + \frac{|f_0(k)|^2}{2R^2} \left[ e^{-x^2} + F_4(2x_\Lambda) - 2{\rm Re}\,F_4(x^+_\Lambda) \right]$$

- We can also insert this regularized w.f. into the expression for $C^{\rm prod}(k) \to C^{\rm prod}_{\rm LL\Lambda}(k)$

$$C^{\rm prod}_{\rm LL\Lambda}(k) = 1 + \frac{2{\rm Re} f_0(k)}{\sqrt{\pi}R} \left[ F_3(x) + \frac{\sqrt{\pi}\,{\rm Im}\,F_4(x^+_\Lambda)}{F_2(x)} \right] + \frac{|f_0(k)|^2}{2R^2} \left[ \frac{2}{\pi} \left( F_3(x) + \frac{\sqrt{\pi}\,{\rm Im}\,F_4(x^+_\Lambda)}{F_2(x)} \right)^2 - \frac{x^2}{2} \right]$$

- One can relate $\Lambda$ and $r_0$:

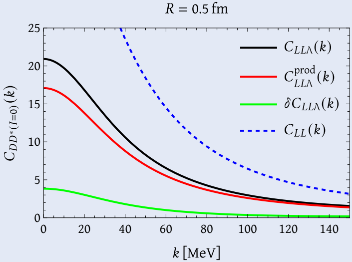
$$r_0 = -\frac{2}{a_0^2} \int \mathrm{d}r \, \left( |\psi(r;k=0)|^2 - \left|\psi_{\rm asy}(r;k=0)\right|^2 \right) = \frac{3}{\Lambda} \left( 1 - \frac{4}{3a_0\Lambda} \right)$$

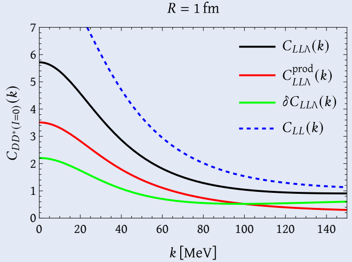
- Take ERE parameterization for the amplitude:

$$f_0(k)^{-1} = -\frac{1}{a_0} + \frac{1}{2} r_0^2 - ik$$

- Use $a_0 = 5.37\,\mathrm{fm}$ and $r_0 = 0.95\,\mathrm{fm}$ [MA, PL,B829, 137052('22)]
(This fixes $E_B = 860\,\mathrm{keV}$ and $\Lambda = 570\,\mathrm{MeV}$)

- For small $R$ we see that $C_{\mathrm{LL}\Lambda}(k)$ and $C^{\mathrm{prod}}_{\mathrm{LL}\Lambda}(k)$ agree. Also, they reproduce the features of $C_{\mathrm{LL}}(k)$ and $C^{\mathrm{prod}}_{\mathrm{LL}}(k)$ seen before.



# Comparison of $C_{\rm LL}(k)$, $C_{\rm LL\Lambda}(k)$, and $C_{\rm LL\Lambda}^{\rm prod}(k)$

MA, Feijoo, Nieves, Oset, Vidaña, 2410.08880

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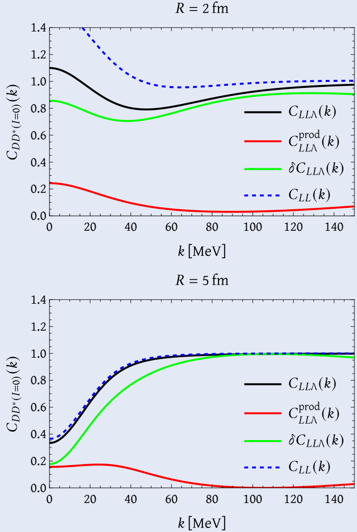
(This fixes $E_B = 860\,\text{keV}$ and $\Lambda = 570\,\text{MeV}$)

- For small $R$ we see that $C_{\rm LL\Lambda}(k)$ and $C_{\rm LL\Lambda}^{\rm prod}(k)$ agree. Also, they reproduce the features of $C_{\rm LL}(k)$ and $C_{\rm LL}^{\rm prod}(k)$ seen before.

- For $R = 2$ and $5\,\text{fm}$, $C_{\rm LL\Lambda}^{\rm prod}(k)$ at $k \simeq 150\,\text{MeV}$ is far from the asymptotic value $C(k) \to 1$.



# Outline

MA, Phys. Lett., B829, 137052 (2021)

- Coupled $T$-matrix for the $D^{*+}D^0$, $D^{*0}D^+$ channels:

$$T^{-1}(E) = V^{-1}(E) - \mathcal{G}(E)$$

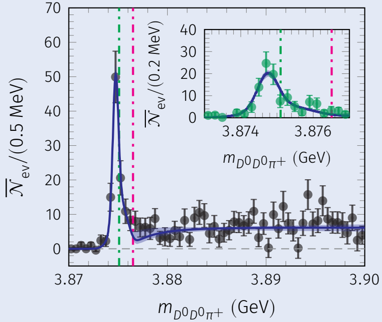
- $V(E)$: interaction kernels written in terms of $C_{I=0,1}$ (constants):

$$V(E) = \frac{1}{2}\begin{pmatrix} C_0 + C_1 & C_1 - C_0 \\ C_1 - C_0 & C_0 + C_1 \end{pmatrix}$$

- Pole position (wrt $D^{*+}D^0$ threshold):

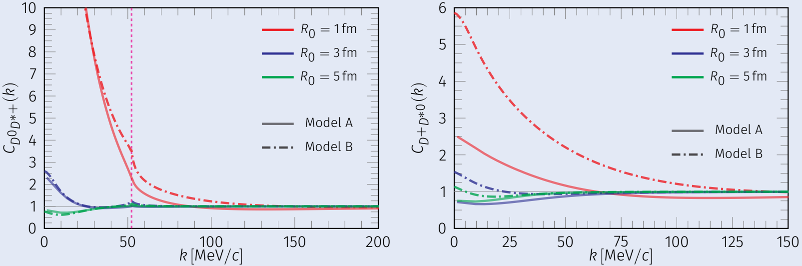
| $\Lambda$ (GeV) | $\delta M_{T_{cc}^+}$ (keV) | $\Gamma_{T_{cc}^+}$ (keV) |
|---|---|---|
| 1.0 | −357(29) | 77(1) |
| 0.5 | −356(29) | 78(1) |
| [2109.01056] | −360(40) | 48(2) |
| [2109.01038] | −273(61) | 410(165) |

- Good agreement with LHCb determination



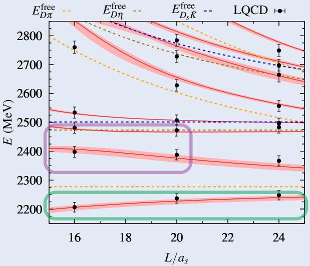

- Many other interesting works: [Feijoo *et al.*, 2108.02730], [Ling *et al.*, 2108.00947], [Du *et al.*, 2110.13765], [Abolnikov *et al.*, 2407.04649], ...

- $D^0D^{*+}$ channel: $C(k)$ positive and very large ($\simeq 30$) for $R = 1\,\text{fm}$ at the threshold.
- $D^+D^{*0}$ channel: Also positive and larger than 1 (but substantially smaller than for $D^0D^{*+}$)
- These predictions could be compared with future measurements by ALICE



# $D\pi$, $D\eta$, $D_s\overline{K}$ and $D^*(2300)$: Comparison with LQCD

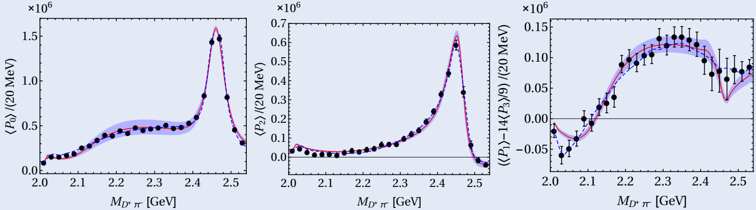
MA, P. Fernández-Soler, F.-K. Guo, J. Nieves, Phys. Lett., **B767**, 465 (2017)
M.-L. Du, MA, P. Fernández-Soler, F.-K. Guo, C. Hanhart, U.-G. Meißner, J. Nieves, D.-L. Yao, Phys. Rev., **D98**,094018 (2018)




- $E_n(L)$ LQCD simulation. [G. Moir *et al.*, JHEP 1610, 011 (2016)]
- Red Bands: [MA *et al.*, Phys. Lett. B **767**, 465 (2017)]
- No fit is performed (LECs previously determined)
- Level below threshold, associated with a bound state.
- Second level has large shifts w. r. t. non-interacting levels.

| | $M$ (MeV) | $\Gamma/2$ (MeV) |
|---|---|---|
| Low pole | $2105^{+6}_{-8}$ | $102^{+10}_{-12}$ |
| High pole | $2451^{+36}_{-26}$ | $134^{+7}_{-8}$ |

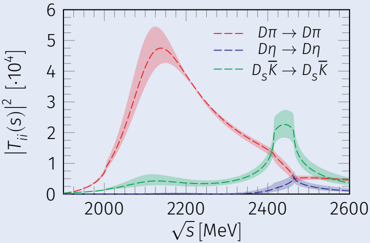
- $B^- \to D^+\pi^-\pi^-$ [LHCb Collab., PR,D94,072001('16)]

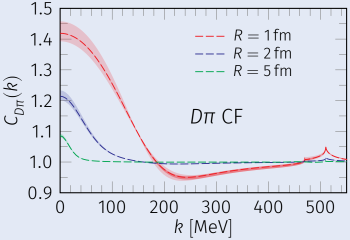



The $D_0^*(2300)$ structure actually consists of two different states (with complicated interferences with thresolds)

Previously reported in:
Kolomeitsev, Lutz, Phys. Lett. B **582**, 39 (2004)
Guo *et al.*, Phys. Lett. B **641**, 278 (2006)
Guo *et al.*, Eur. Phys. J. A **40**, 171 (2009)

- Lower pole, peak at 2135 MeV would correspond to $k_{D\pi} = 215$ MeV. However, we find a minimum at $C_{D\pi}(k)$.



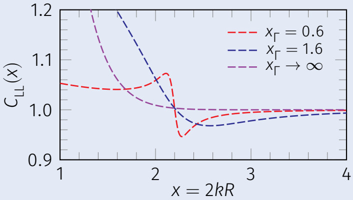




- Take the LL approximation to $C(k)$, and $f(k)$ in terms of $\delta(k)$:

$$C_{LL}(k) = 1 + \frac{2\sin^2\delta(k)}{x^2}\left(e^{-x^2} + \frac{2xF_1(x)}{\sqrt{\pi}}\cot\delta(k)\right)$$

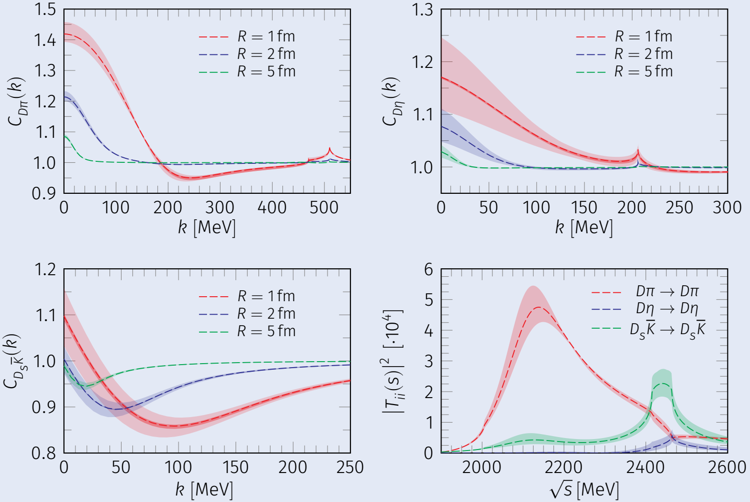
  - For a simple BW: $C_{LL}(k_R) = 1$, $\quad C'_{LL}(k_R) < 0$

Conclusion: the minimum at $k_{D\pi} = 215$ MeV is a clear signature of the lowest pole.



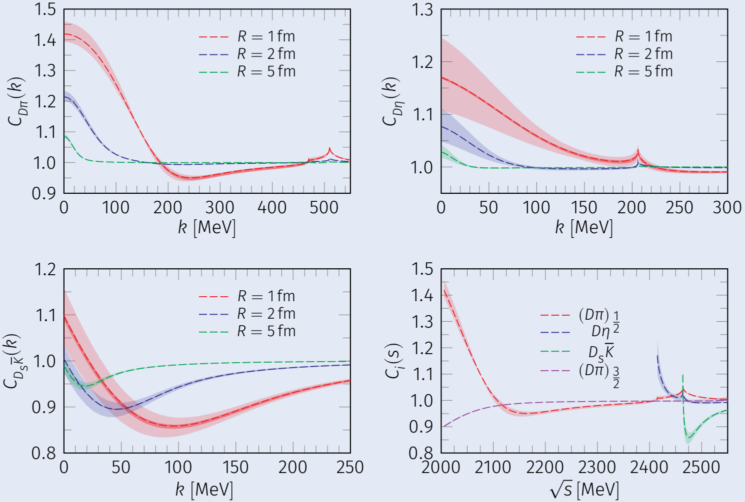
# Open charm femtoscopy: results $S = 0$, $I = 1/2$ $[D\pi, D\eta, D_s\bar{K}]$

MA, Nieves, Ruiz–Arriola, PRD,108,014020('23)




- Two different minima at $\sqrt{s} \simeq 2135$ MeV ($D\pi$ CF) and 2475 MeV ($D_s\bar{K}$ CF), produced by the two different $D_0^*$ states, can be observed
- Their observation would constitute a strong additional support of the two-state pattern.

MA, Nieves, Ruiz–Arriola, PRD,108,014020('23)

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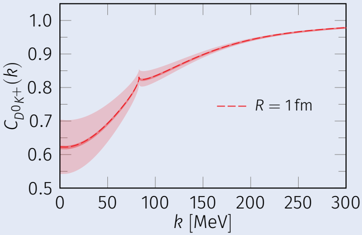
MA, Nieves, Ruiz–Arriola, PRD,108,014020('23)

- $D_{s0}^*(2317)$ is a (mostly) $DK$ $I = 0$ bound state, with $B = 45\,\text{MeV}$, $p_B = 190i\,\text{MeV}$
- Why use physical channels?
  - $E^{\text{th}}_{D^+K^0} - E^{\text{th}}_{D^0K^+} \simeq 9\,\text{MeV}$ $[k_{D^0K^+} \simeq 83\,\text{MeV}]$
  - Also, $C_{D^0K^+} = C_{D^+K^0} = \dfrac{C_0^{DK} + C_1^{DK}}{2}$
- Clear depletion at threshold, related to the presence of $D_{s0}^*(2317)$
- Similar trends in recent works, only small differences in the values:
  - [Liu, Lu, Geng, PR,**D107**,074019('23)]
  - [Ikeno, Toledo, Oset, 2305.16431]



# Open charm femtoscopy: results $S=1$, $I=0,1$ [$D_s^+\pi^0$, $D^0K^+$, $D^+K^0$, $D_s^+\eta$]

MA, Nieves, Ruiz–Arriola, PRD,108,014020('23)

- $D_{s0}^*(2317)$ is a (mostly) $DK$ $I=0$ bound state, with $B=45\,\text{MeV}$, $p_B=190i\,\text{MeV}$
- Why use physical channels?
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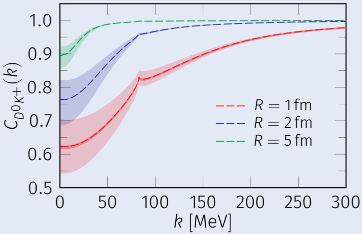

# Summary and conclusions

- Femtoscopy is becoming a popular and useful tool to study hadron interactions
- We have checked that, in the point-like limit ($R \to 0$) the CF reduces to a usual invariant mass spectrum
- We have defined a would-be CF, $C^{\text{prod}}(k)$, that one would expect from a QM/QFT inspiration
- We have proposed modifications to the LL approach that correct its ill behaviour for $R \to 0$
- We have shown that the application of femtoscopy to exotic states ($T_{cc}(3875)^+$ and $D_0^*(2300)$) can help pinning down its existence and nature

# Thank you for your attention