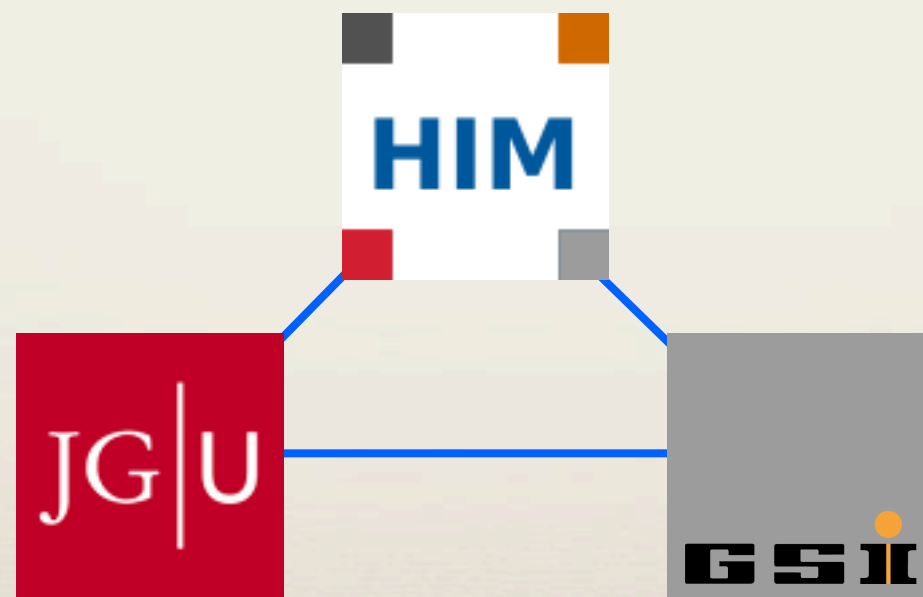


# Application of the QCD factorization in SL and TL regions

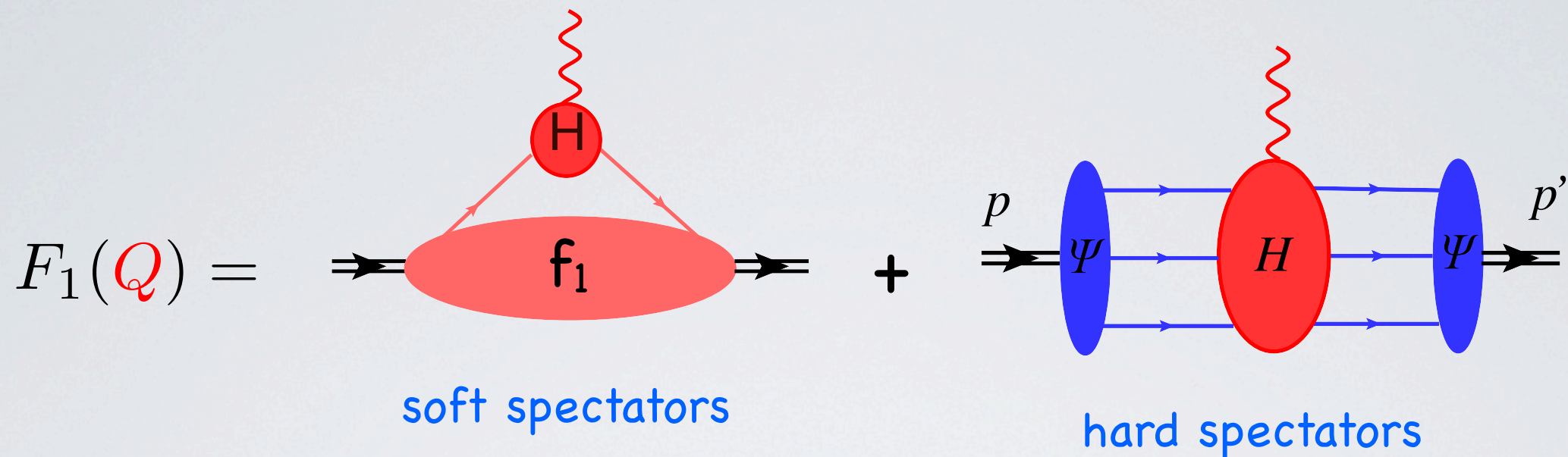
**Nikolay Kivel**

Helmholtz Institute Mainz, Germany





# QCD factorization at large $Q^2$



- the same power at large  $Q^2$

$$F_1^{(s)}(Q^2) \sim f^{(s)}(\ln Q^2/\Lambda^2)/Q^4$$

Duncan, Mueller 1980

Milshtein, Fadin 1981/82

- Large at moderate values of  $Q^2$

Isgur, Smith 1984

Nesterenko, Radyushkin 1989

Braun et al, 2002, 2006

- SCET factorization scheme at large  $Q^2$

NK, Vanderhaeghen PRD, 2010

Brodsky, Lepage 1979

Chernyak, Zhitnitsky 1977

Efremov, Radyushkin 1980

- model independent QCD prediction

$$F_1^{(h)}(Q^2) \sim f(\ln Q^2/\Lambda^2)/Q^4$$

- Non-perturbative  $\Psi$  is UNIVERSAL, well defined objects  $\langle p | O_i | 0 \rangle$

QCD Sum Rules

Estimates: Lattice calculations

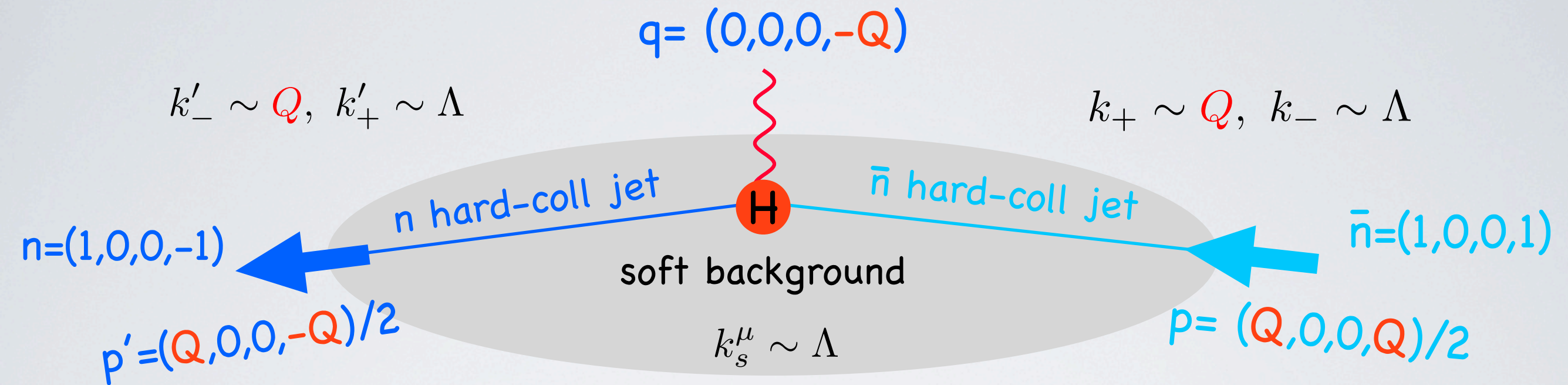
Low energy models: QSM, ...



# Soft spectator contribution

hard-collinear scale:  $Q\Lambda \lesssim m_N^2$

hard-collinear = collinear + soft



quark "jets"  $\chi_{\bar{n}} = \text{P exp} \left\{ ig \int_{-\infty}^0 ds n \cdot A_{hc}(sn) \right\} \frac{1}{4} \not{n} \not{\bar{n}} \psi_{hc}(0)$

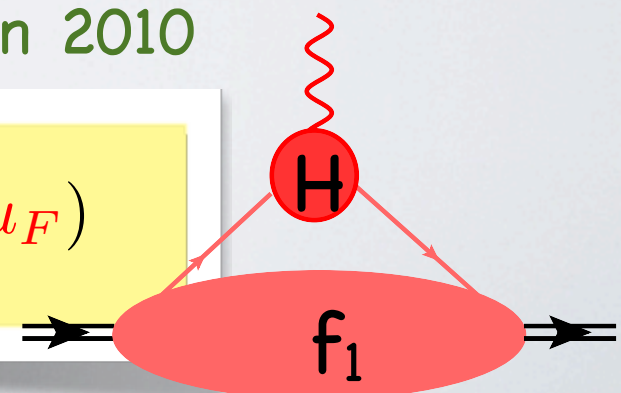
Soft-Collinear Effective Theory Form Factor

NK, Vanderhaeghen 2010

$$\langle p' | \bar{\chi}_n \gamma_{\perp \mu} \chi_{\bar{n}} + \bar{\chi}_{\bar{n}} \gamma_{\perp \mu} \chi_n | p \rangle_{SCET} = \bar{N}(p') \frac{\not{n} \not{\bar{n}}}{4} \gamma_{\perp \mu} N(p) f_1(\Lambda Q, \mu_F)$$

quark

antiquark



# Soft spectator contribution

moderate values of  $Q^2$ :  $Q\Lambda \sim m_N^2$  hard-collinear scale is not large

$$\left. \begin{array}{l} Q^2 = 9 - 25 \text{GeV}^2 \\ \Lambda \simeq 0.3 \text{GeV} \end{array} \right| \Rightarrow Q\Lambda \simeq 0.9 - 1.5 \text{GeV}^2$$

NK, Vanderhaeghen PRD,2010

$$Q^2 \gg Q\Lambda \sim m_N^2$$

$$F_1(Q) = \text{Diagram 1} + \text{Diagram 2}$$

Diagram 1: A triangle diagram with a grey oval labeled  $f_1$ . A blue line enters from the left, a red wavy line exits from the top, and a cyan line exits to the right. The top vertex is a red dot.

Diagram 2: A diagram with two blue ovals labeled  $\Psi$  and a red oval labeled  $H$ . A blue line enters from the left with momentum  $p$ , passes through the first  $\Psi$  oval, then the  $H$  oval, and finally the second  $\Psi$  oval, exiting to the right with momentum  $p'$ . A red wavy line exits from the top of the  $H$  oval.

$$F_2(Q) = \text{Diagram 3} + \frac{4m_N^2}{Q^2} f_1 + \text{Diagram 4}$$

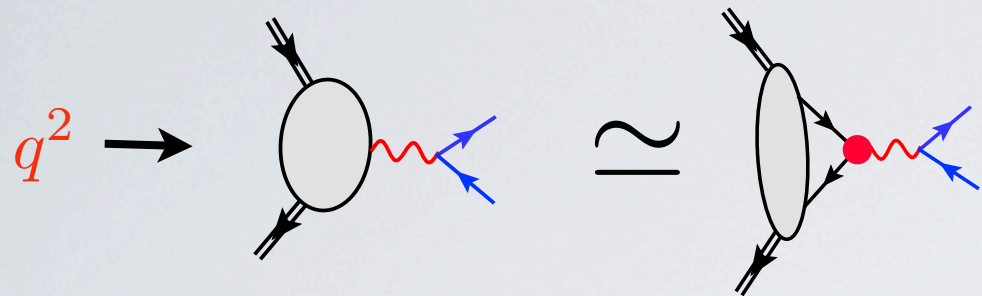
Diagram 3: A triangle diagram with a grey oval labeled  $f_2$ . A blue line enters from the left, a red wavy line exits from the top, and a cyan line exits to the right. A blue wavy line connects the two bottom vertices.

Diagram 4: A diagram with two blue ovals labeled  $\Psi$  and a red oval labeled  $H$ . A blue line enters from the left with momentum  $p$ , passes through the first  $\Psi$  oval, then the  $H$  oval, and finally the second  $\Psi$  oval, exiting to the right with momentum  $p'$ . A red wavy line exits from the top of the  $H$  oval.



# Soft spectator mechanism in SL and TL regions

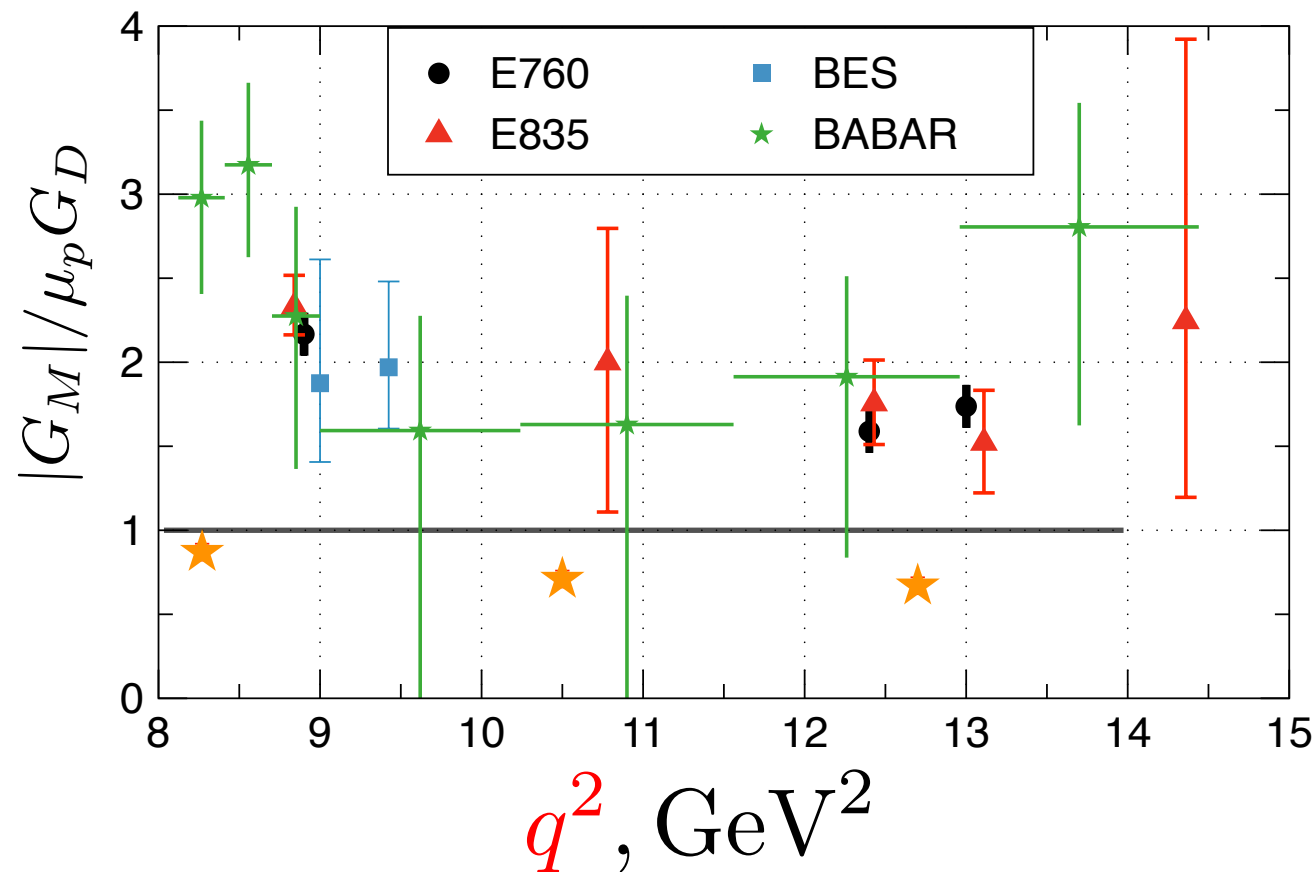
$q^2 \gg \Lambda$     $q\Lambda \lesssim m_N^2$    (timelike scattering)



$$G_M(Q^2 \rightarrow \infty) = |G_M(q^2 \rightarrow \infty)|$$

analytic function in  $q^2$

(Phragmen-Lindelöf theorem)



Hard spectator mechanism  
can not explain TL enhancement

Current experiment:      used assumptions

TL    $|G_M|/\mu_p G_D \sim 2$     $|G_E| \approx |G_M|$

SL    $G_M/\mu_p G_D \sim 0.9 - 0.7$

SL data

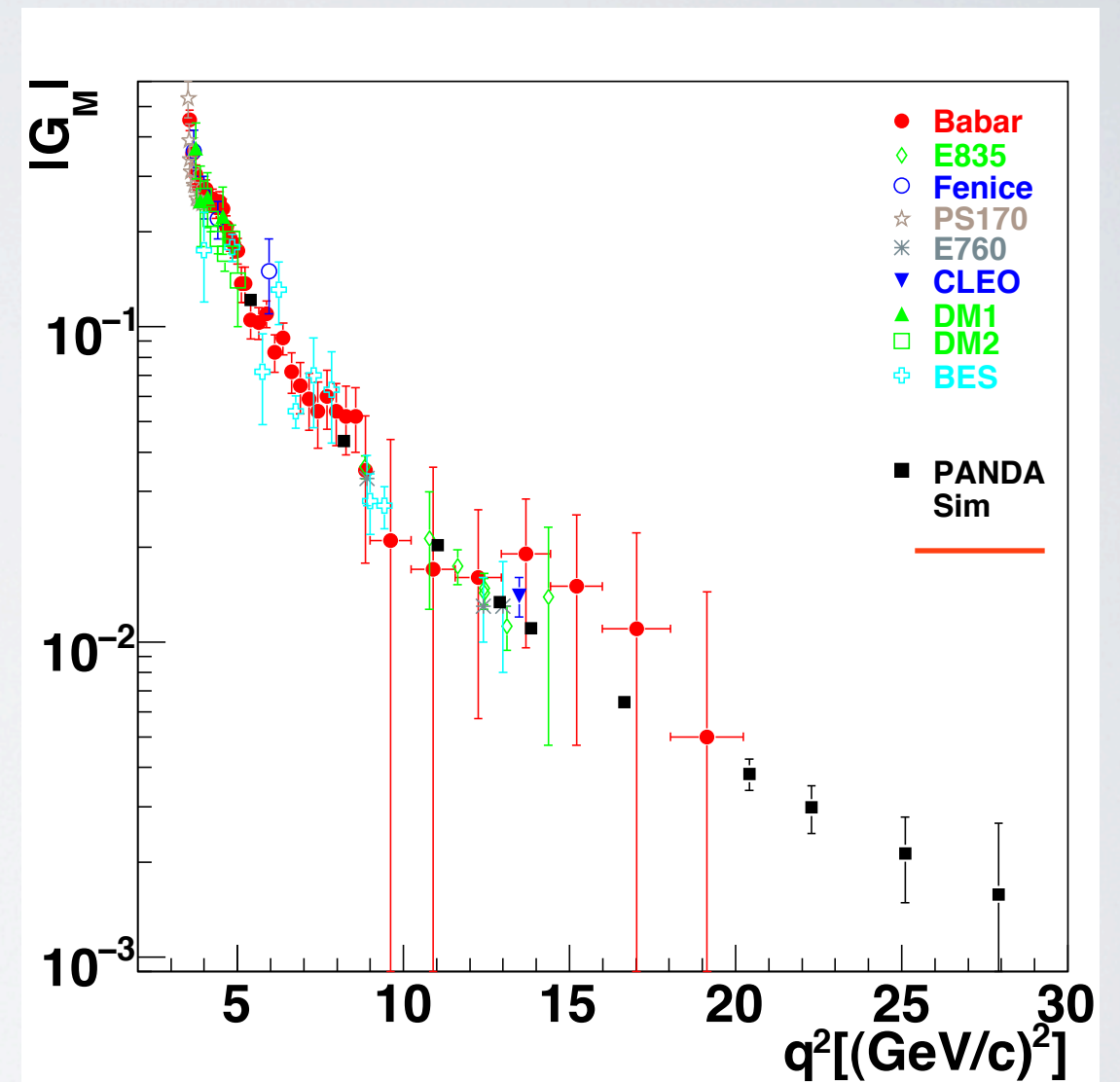
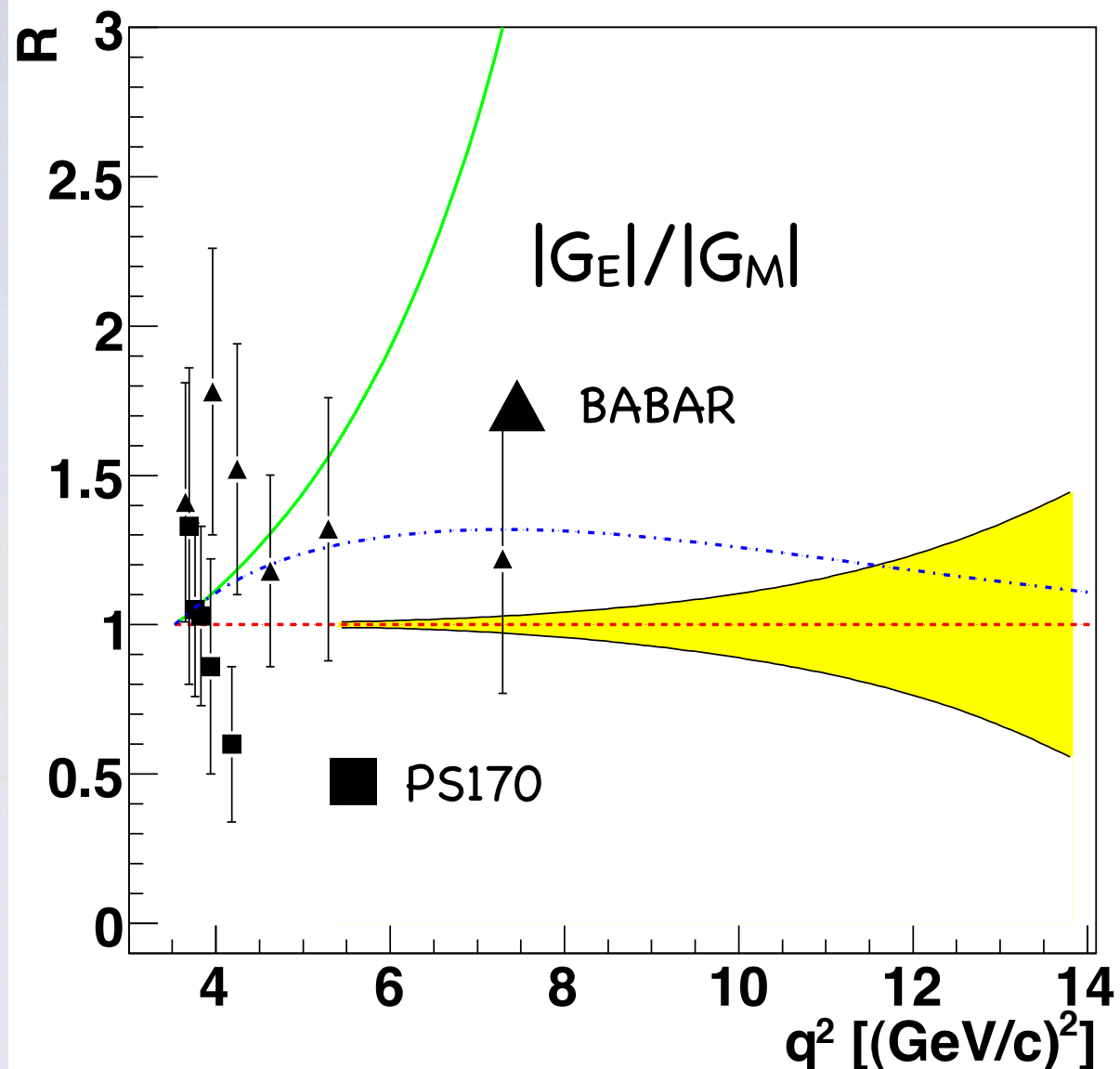
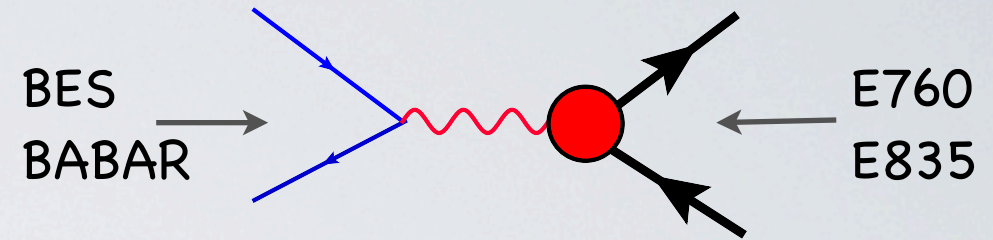
★ SLAC, Sill et al, 1993    $G_E \approx G_M/\mu_p$

Soft spectator contribution:

Sudakov logs provide  $\approx 40\%$   
enhancement at large TL  $q^2$

# Proton FFs in the time-like region $q^2 > 0$

$$p + \bar{p} \rightarrow e^- + e^+$$



Sudol et al, 2009

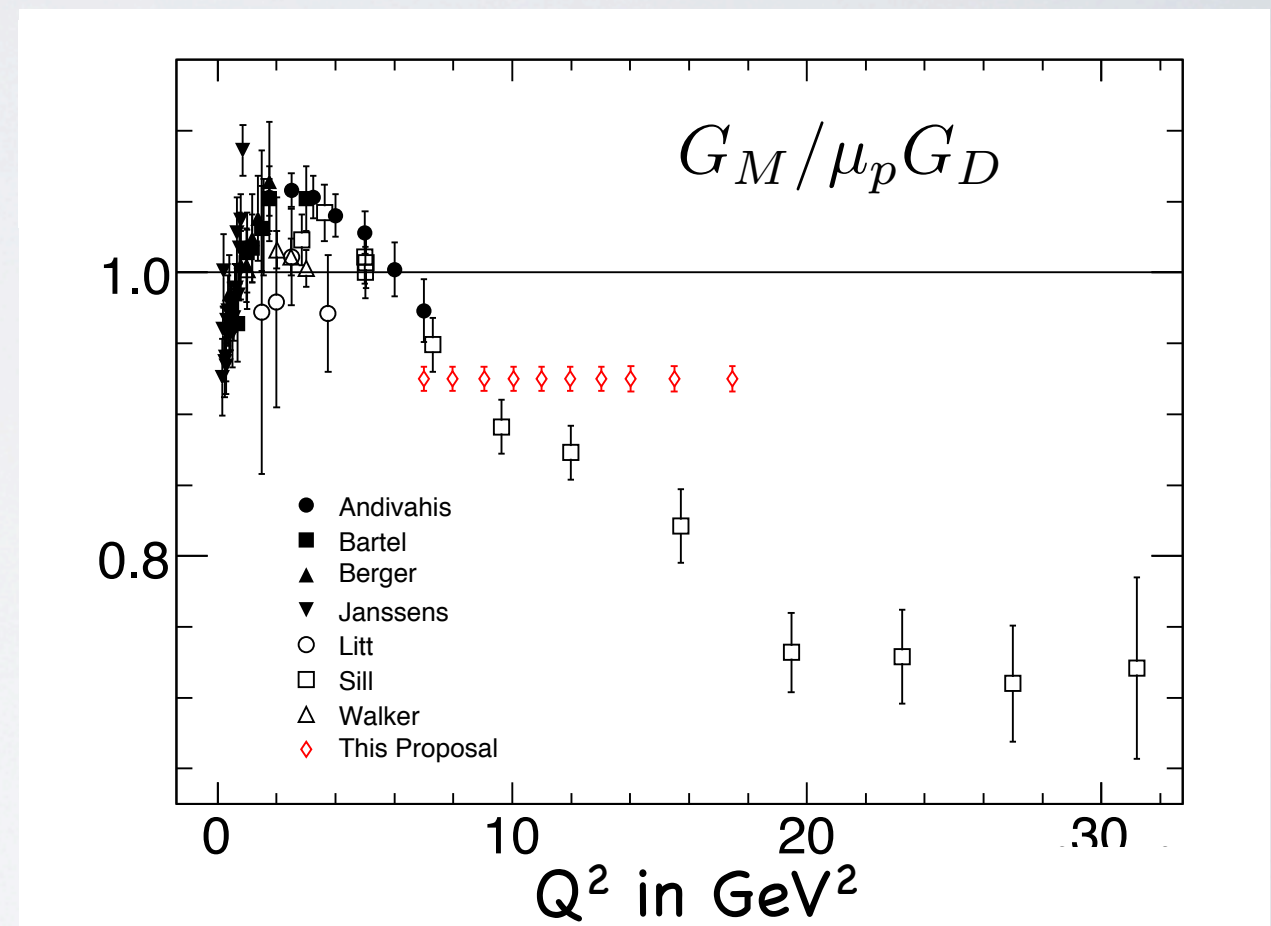
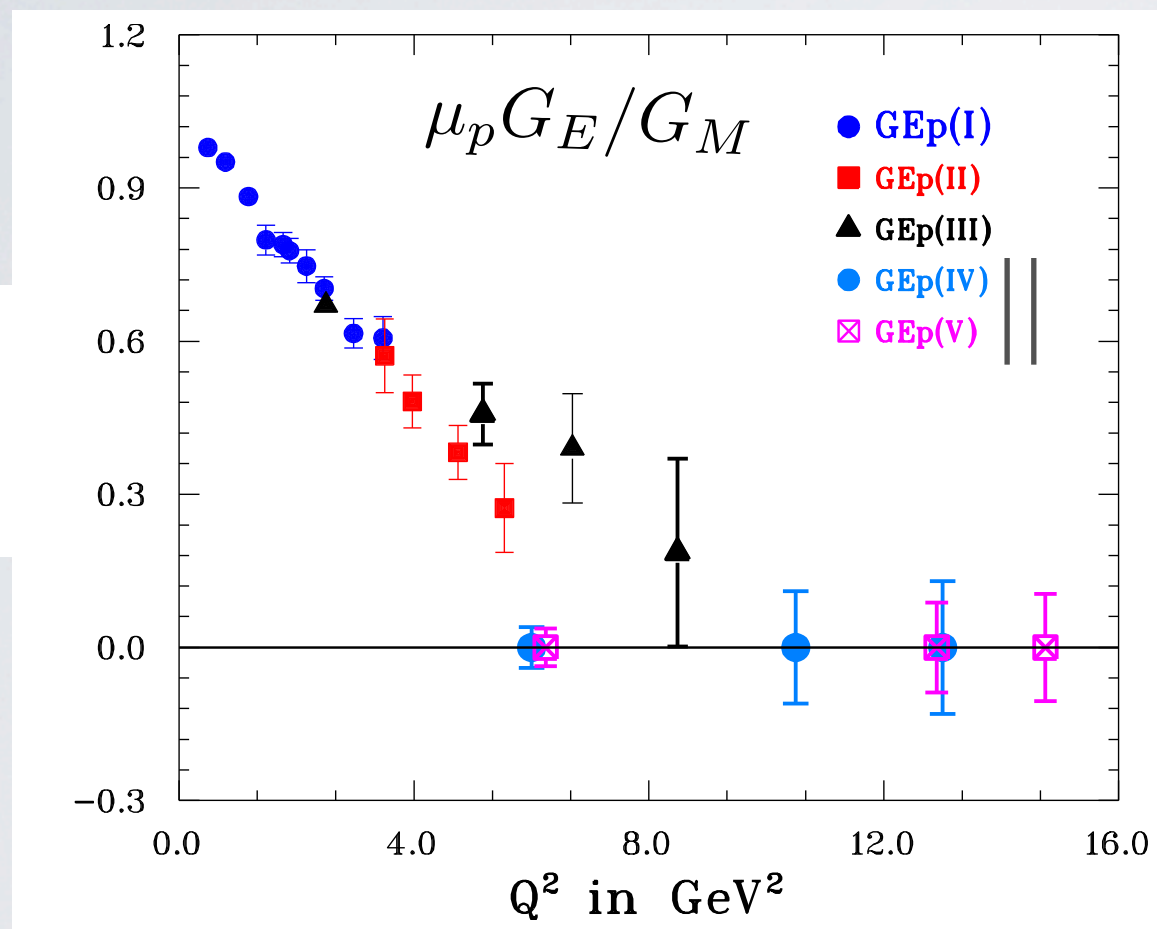




# Future experiments in JLab, SL:

$$G_E/G_M: \quad \frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}} \frac{G_E}{G_M}$$

$$G_M: \quad \sigma_R = G_M^2 + \frac{\varepsilon}{\tau} G_E^2$$



- ★ JLab Hall A E1207-109  $G_E/G_M$   
recoil pol  $Q^2=6-14.8 \text{ GeV}^2$
- ★ JLab Hall C E1209-001  $G_E/G_M$   
recoil pol  $Q^2=6-13 \text{ GeV}^2$

- ★ JLab Hall A E1207-108  
 $\sigma_R$  unpol  $Q^2=7-17.5 \text{ GeV}^2$ ,  
total err. < 2%

# The ratio $G_E/G_M$ in SL and TL regions

➡ Assume that the soft spectator contribution dominates in  $G_E$  and  $G_M$

$$G_M(Q) \approx \text{diagram with } f_1 = f_1(Q) \quad G_E(Q) \approx -\frac{1}{4} \text{diagram with } f_2 = -\frac{1}{4} f_2(Q)$$

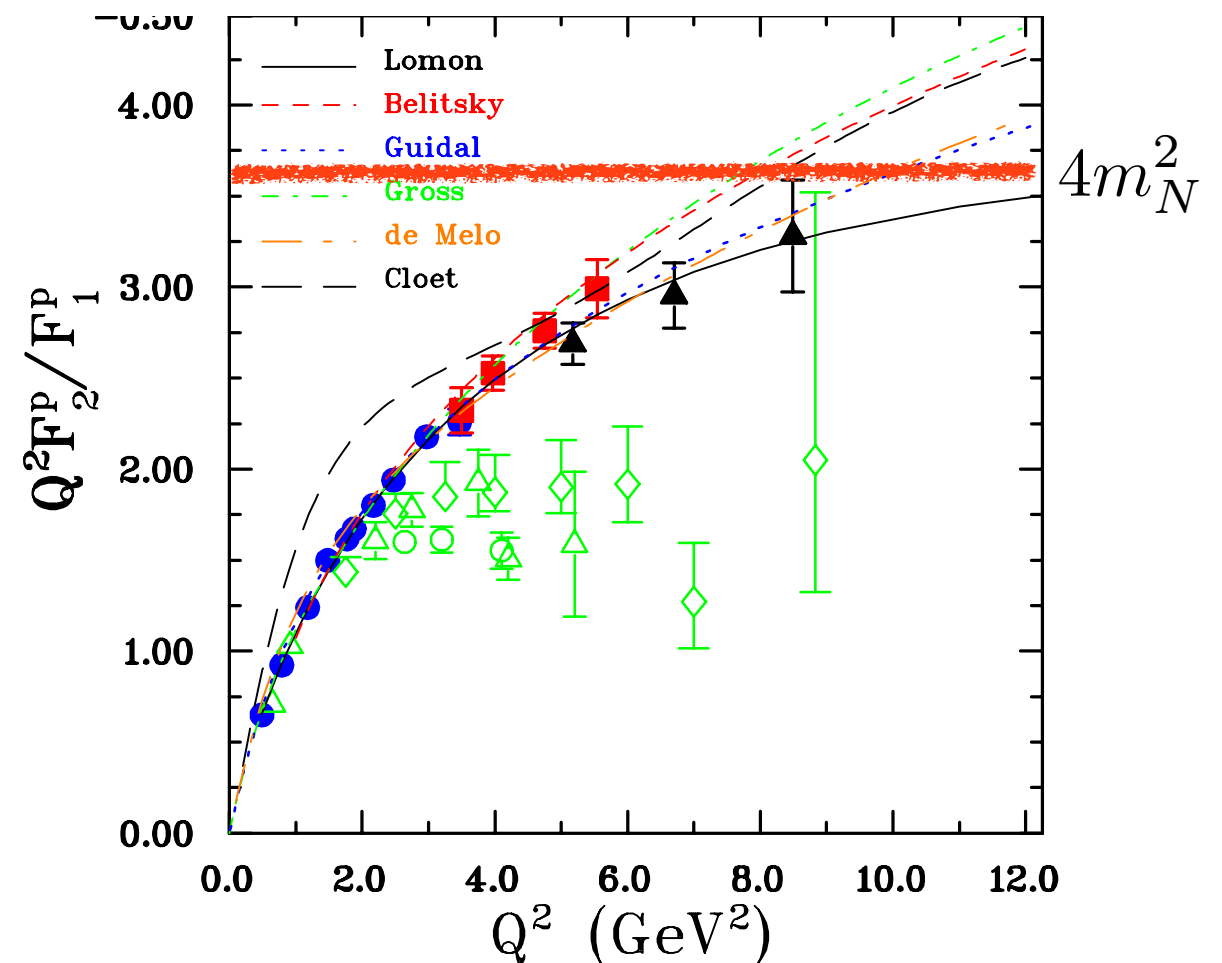
SL

$$\frac{G_E}{G_M} = -\frac{1}{4} \frac{f_2(Q)}{f_1(Q)} + \mathcal{O}(1/Q^2) \approx 0$$

assuming  $f_2(Q) \ll f_1(Q)$

$$\Rightarrow \frac{Q^2 F_2(Q^2)}{F_1(Q^2)} \approx \frac{\frac{1}{4} m^2 f_2 + 4m^2 f_1}{f_1} \simeq 4m_N^2$$

This is not pQCD asymptotic behavior!





# The ratio $G_E/G_M$ in SL and TL regions

➡ Assume that the soft spectator contribution dominates in  $G_E$  and  $G_M$

$$G_M(Q) \approx \text{diagram with } f_1 = f_1(Q) \qquad G_E(Q) \approx -\frac{1}{4} \text{diagram with } f_2 = -\frac{1}{4} f_2(Q)$$

$$\text{TL} \quad \frac{|G_E|}{|G_M|} = \frac{1}{4} \frac{|f_2(Q)|}{|f_1(Q)|} + \mathcal{O}(1/Q^2) \ll 1$$

we can also expect  $|f_2(Q)| \ll |f_1(Q)|$

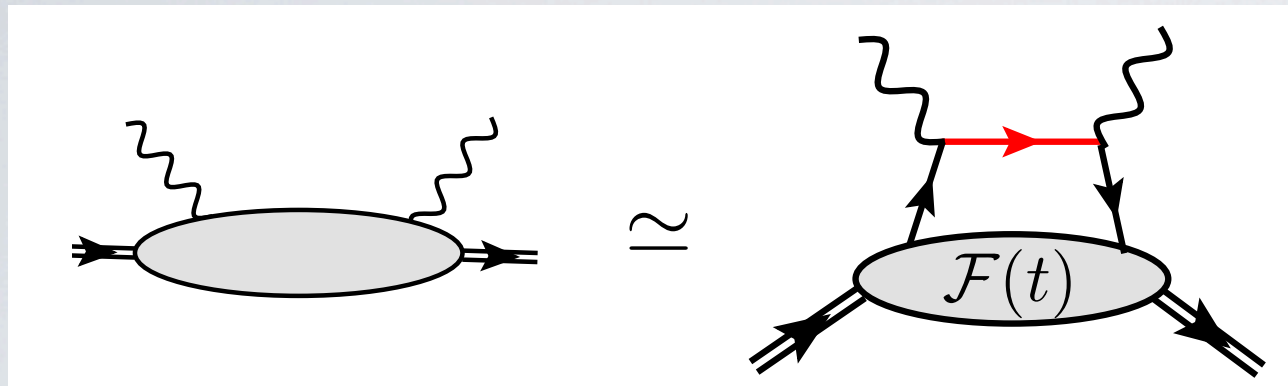
- Sudakov factors cancel in the ratio therefore  
TL modification can be weaker comparing FFs
- If  $G_E$  is small then the Rosenbluth separation is more sensitive to the TPE corrections



# Wide Angle Compton Scattering in SL region

$$-t \sim -u \sim s \sim Q^2 \gg \Lambda^2 \quad Q\Lambda \lesssim m_N^2$$

Dominance of the soft  
spectator scattering



GPD (handbag)-model

Radyshkin, 1998

Kroll et al, 2005

SCET approach

Kivel, Vanderhaeghen, 2012

$$G_M \quad f_1 = e_u(f_1^u - f_1^{\bar{u}}) + e_d(f_1^d - f_1^{\bar{d}})$$

$$WACS \quad \mathcal{F} = e_u^2(f_1^u + f_1^{\bar{u}}) + e_d^2(f_1^d + f_1^{\bar{d}})$$

SCET  
power counting

$$\frac{f_1^{\bar{q}}}{f_1^q} \sim \mathcal{O}((Q\Lambda)^{-2})$$

quarks dominate at large  
hard collinear scale



# Wide Angle Compton Scattering in SL region

Kivel, Vanderhaeghen, 2012

ratio  $\mathcal{F}(t) \simeq \frac{T_2(s, t)}{H_2(s, t)} \simeq \frac{T_4(s, t)}{H_4(s, t)} \simeq \frac{T_6(s, t)}{H_6(s, t)}$       amplitude  
hard coeff. f.      approximately  
s-independent at LO

$$\frac{d\sigma}{dt} = \frac{d\sigma^{\gamma q \rightarrow \gamma q}(s, t)}{dt} |\mathcal{F}(t)|^2$$

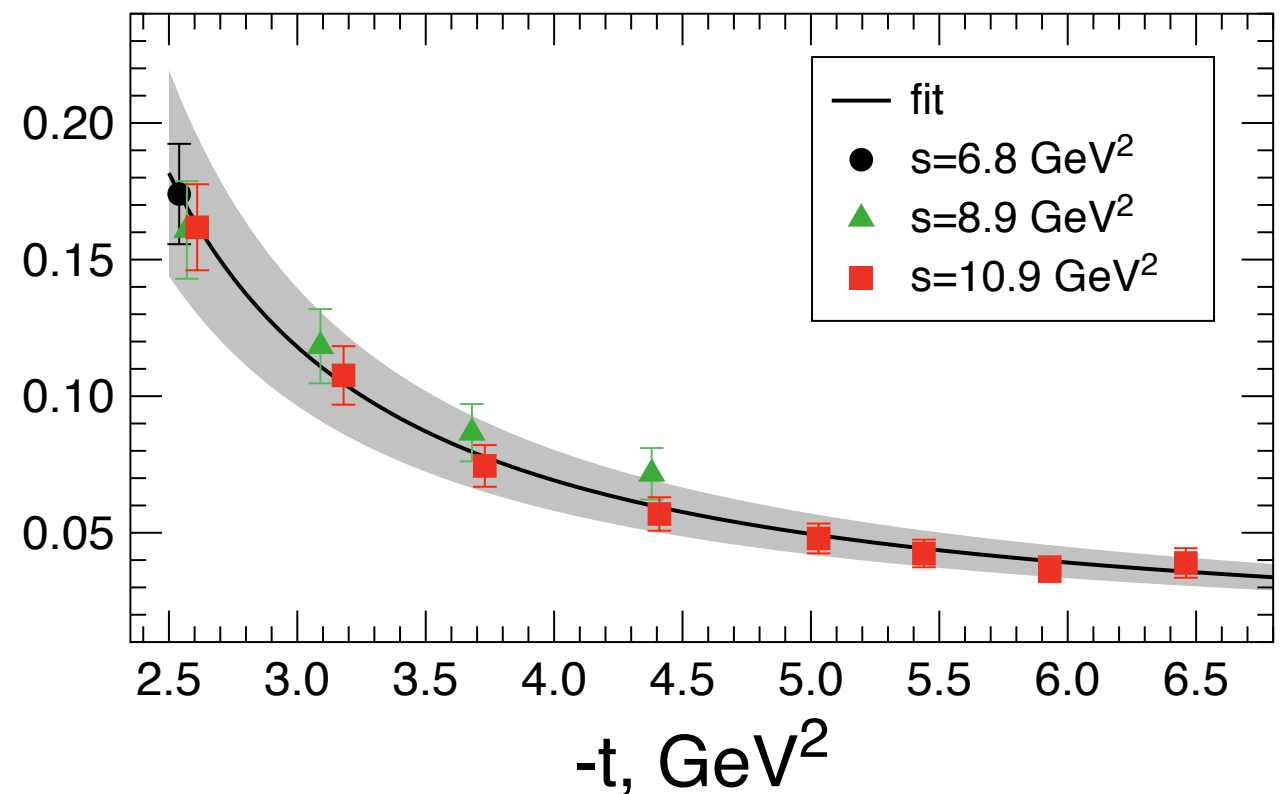
fit  $\mathcal{F}(t) = \frac{c}{\ln^2[-t/\Lambda^2]}$

$$c = 0.10 \pm 0.01$$

$$\Lambda = 1.08 \pm 0.03$$

$$\chi^2/d.o.f. = 0.35$$

used data: JLab, Hall A, 2007



# Dominance of the soft spectator contribution in WACS

NK, Vanderhaeghen, 2012

circular photon polarizations (R,L)

recoiled proton: longitudinal pol.

$$K_{LL} = \frac{\sigma_{||}^R - \sigma_{||}^L}{\sigma_{||}^R + \sigma_{||}^L}$$

SCET results:  $K_{LL} \simeq K_{LL}^{\text{KN}}$

SCET FFs cancel and asymmetry is not sensitive to proton structure

recoiled proton: transverse pol.

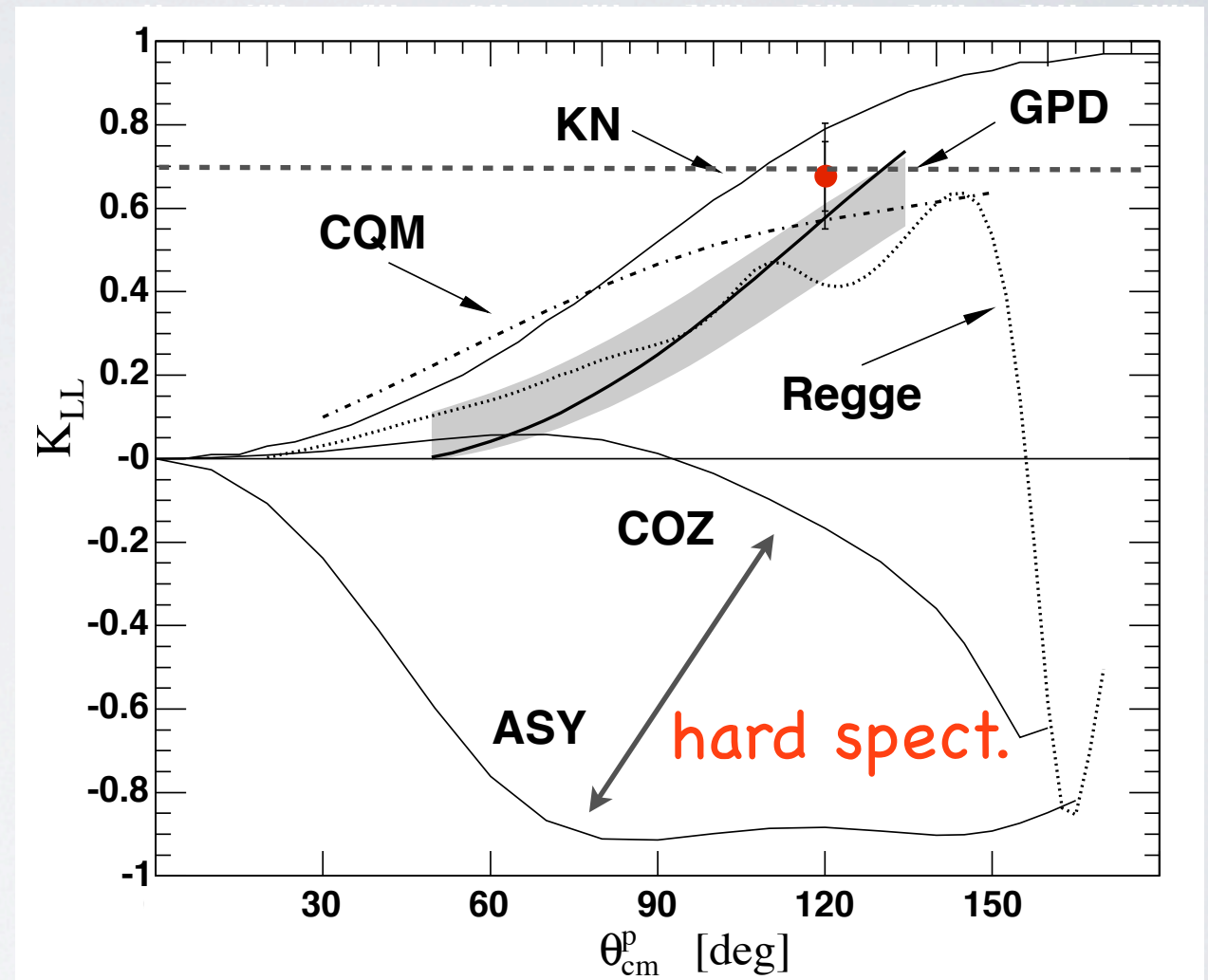
$$K_{LS} = \frac{\sigma_{\perp}^R - \sigma_{\perp}^L}{\sigma_{\perp}^R + \sigma_{\perp}^L}$$

SCET results:  $K_{LS} \simeq \mathcal{O}(m^2/Q)$

defined by power subleading terms (sensitive to helicity flip amplitudes)

$u = -0.84 \text{ GeV}^2$

JLAB, 2004  $s = 6.9 \text{ GeV}^2$   $t = -4 \text{ GeV}^2$



$$K_{LL} = 0.68 \pm 0.08 \quad (0.70)$$

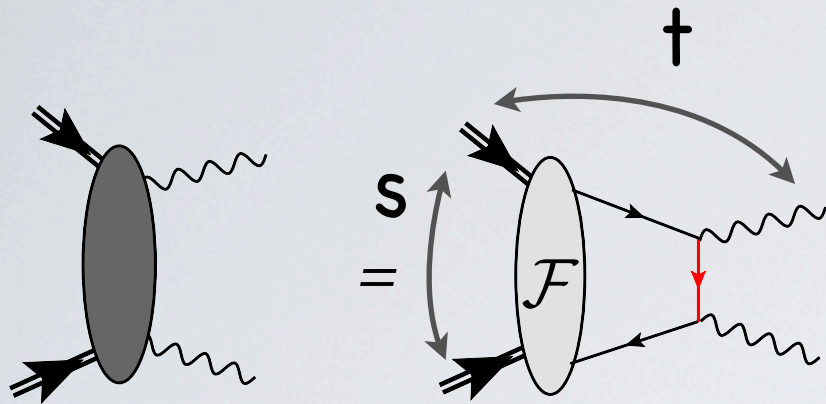
$$K_{LS} = 0.114 \pm 0.08 \quad (0.066)$$



# Wide Angle Compton Scattering in TL region

Handbag model: Diehl, Kroll, Vogt 2002

SCET: Kivel, Vanderhaeghen (in progress)

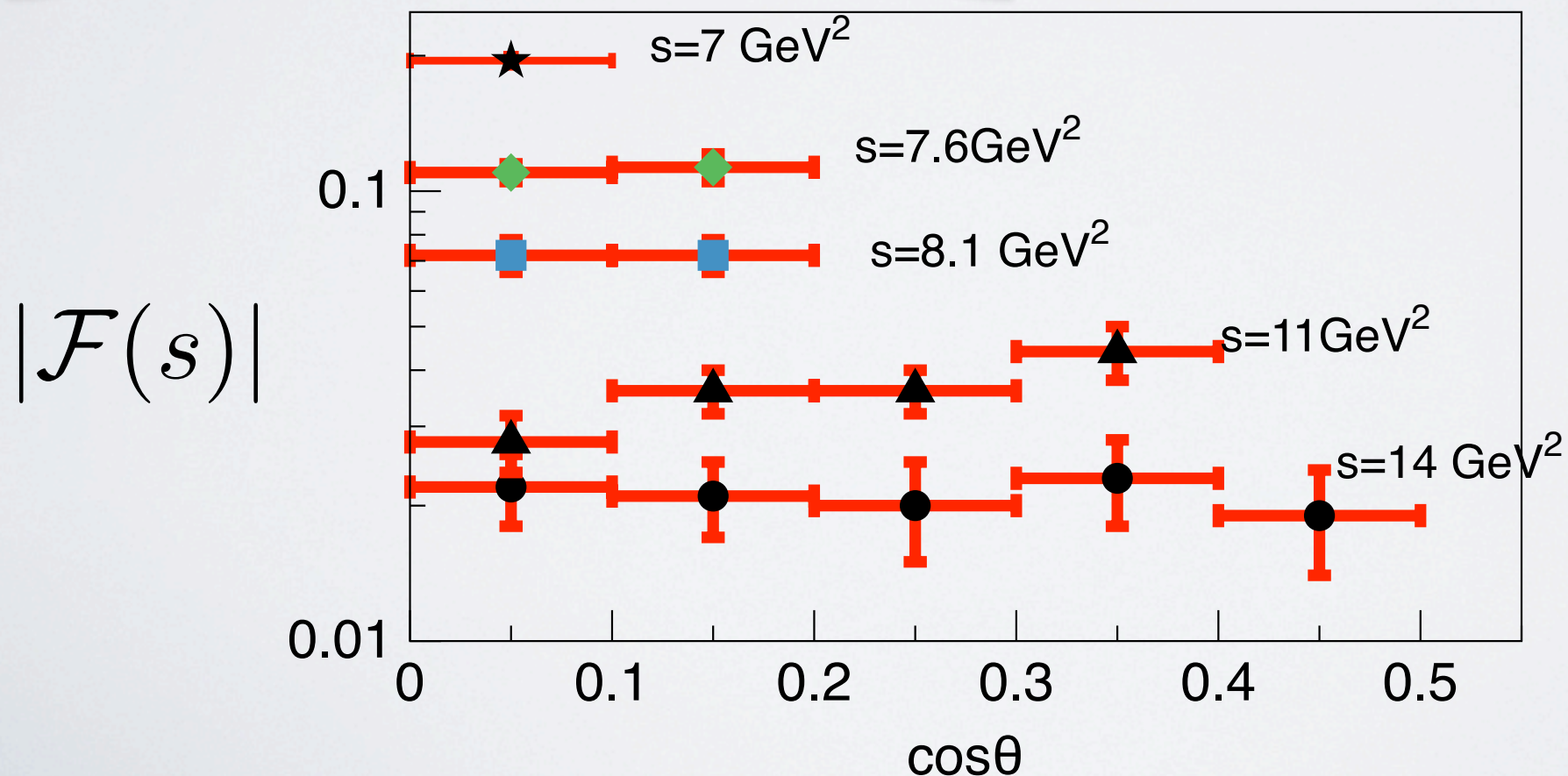


$$\mathcal{F}(s) \simeq \frac{T_2(s, \cos \theta)}{H_2(s, \cos \theta)} \simeq \frac{T_4(s, \cos \theta)}{H_4(s, \cos \theta)} \simeq \frac{T_6(s, \cos \theta)}{H_6(s, \cos \theta)}$$

$\cos \theta$  -independent!

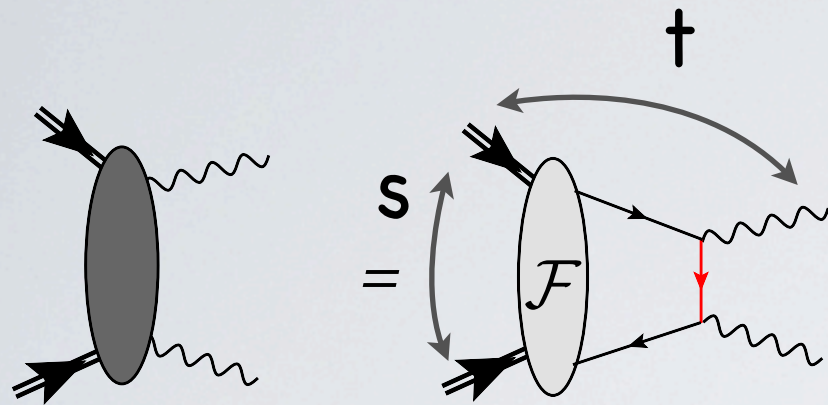
$$\frac{d\sigma}{d\cos\theta} = \frac{d\sigma^{q\bar{q} \rightarrow \gamma\gamma}(s, \cos\theta)}{d\cos\theta} |\mathcal{F}(s)|^2$$

used data: BELLE, 2005  $\gamma\gamma \rightarrow p\bar{p}$



$|t|, |u| > 2.5 \text{ GeV}^2$

# $B\bar{B}$ or Two Photon Production



Kivel, Vanderhaeghen (in progress)

$$\mathcal{F}(s) \simeq \frac{T_2(s, \cos \theta)}{H_2(s, \cos \theta)} \simeq \frac{T_4(s, \cos \theta)}{H_4(s, \cos \theta)} \simeq \frac{T_6(s, \cos \theta)}{H_6(s, \cos \theta)}$$

$\cos \theta$

$$\frac{d\sigma}{d\cos\theta} = \frac{d\sigma^{q\bar{q} \rightarrow \gamma\gamma}(s, \cos\theta)}{d\cos\theta} |\mathcal{F}(s)|^2$$

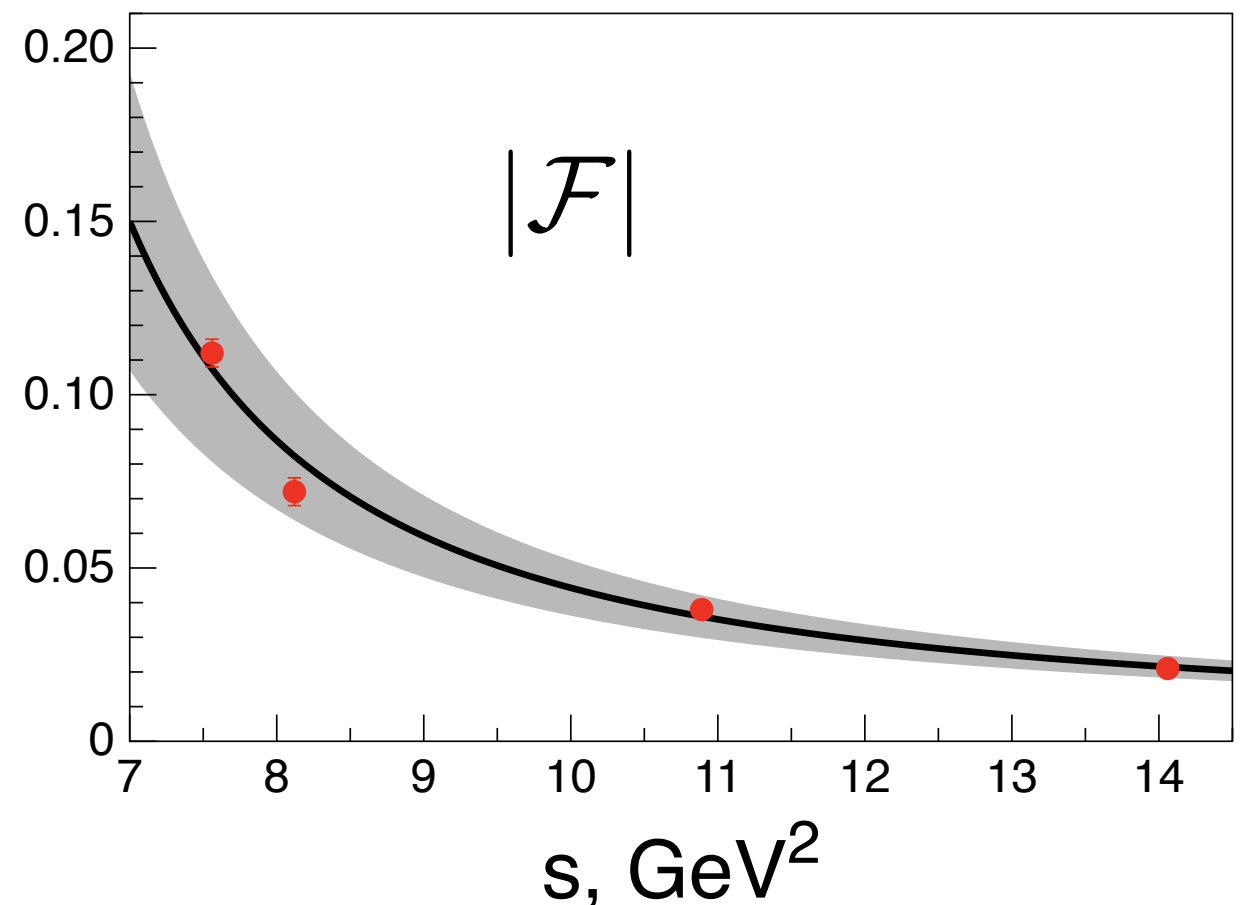
$$\text{fit} \quad |\mathcal{F}(s)| = \frac{c}{\ln^2[s/\Lambda^2]}$$

$$c = 0.027 \pm 0.003$$

$$\Lambda = 2.13 \pm 0.04$$

$$\chi^2/d.o.f. = 4.5$$

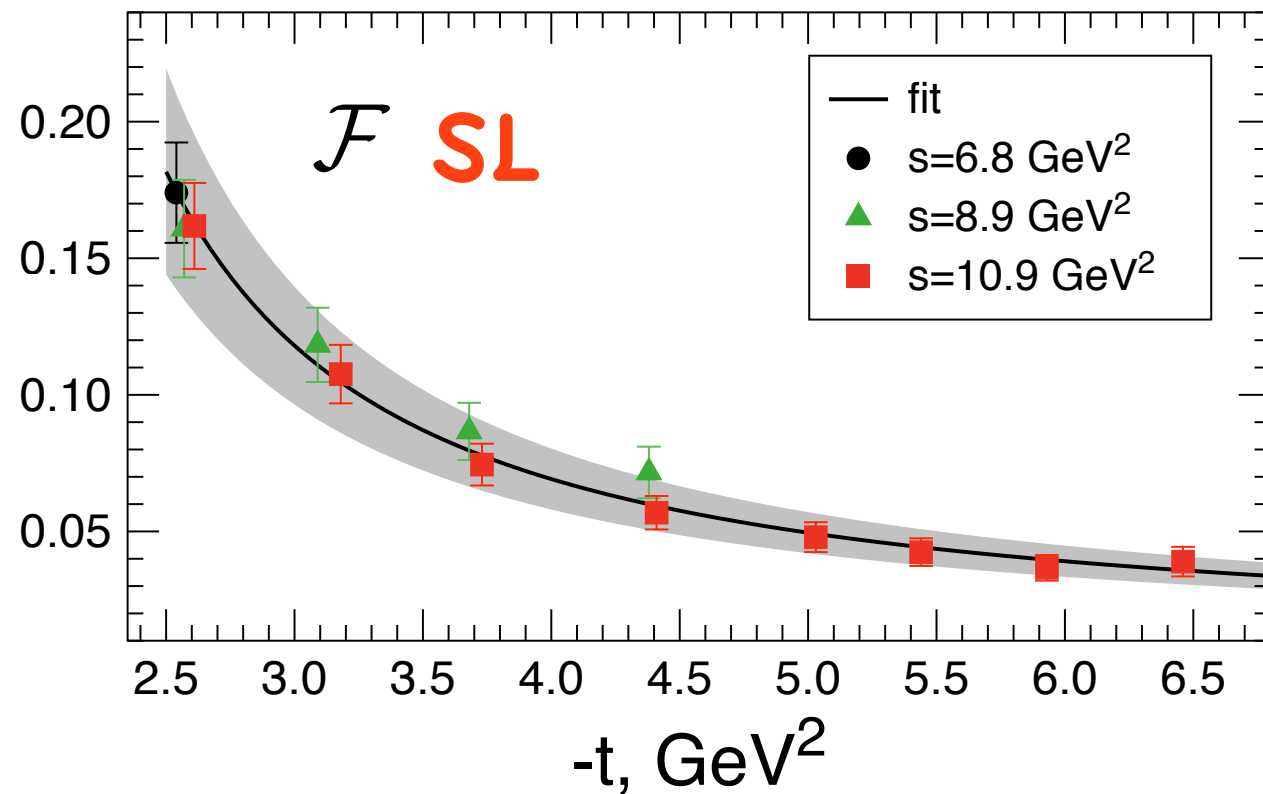
used data: BELLE, 2005  $\gamma\gamma \rightarrow p\bar{p}$



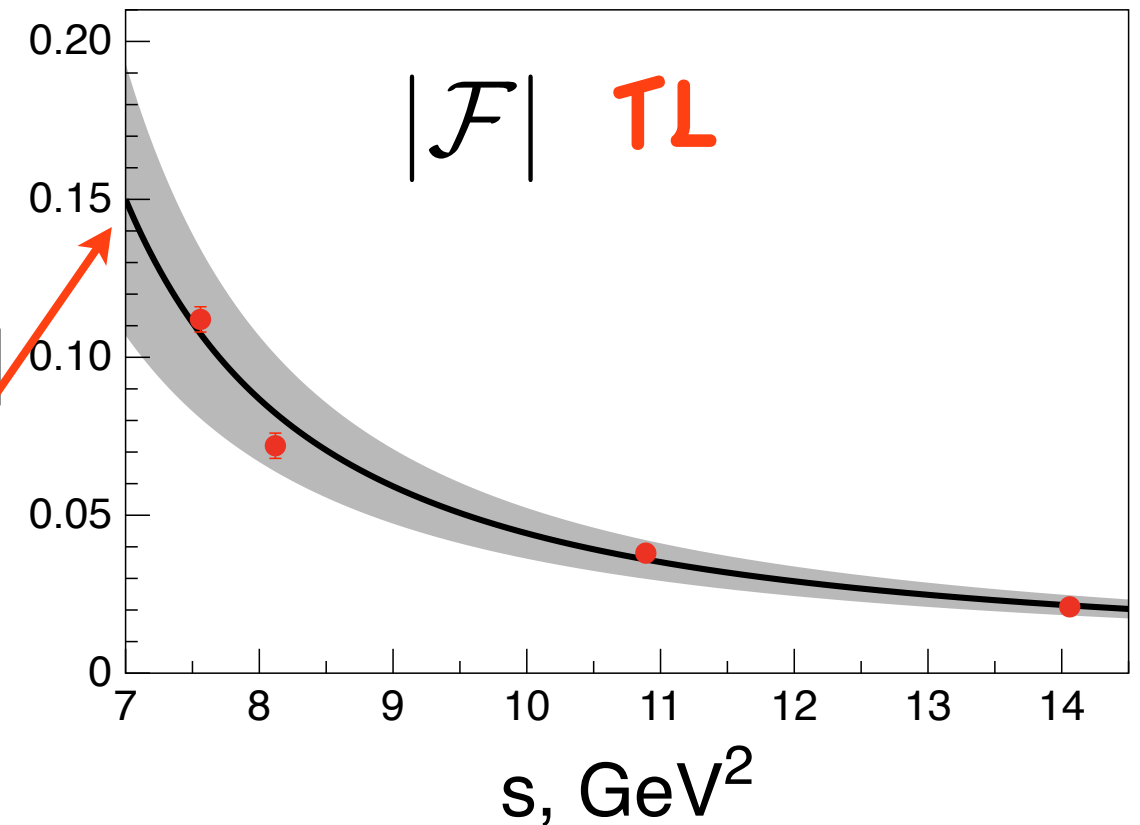


# SCET FFs in SL and TL regions

used data: JLab, Hall A, 2007



used data: BELLE, 2005  $\gamma\gamma \rightarrow p\bar{p}$

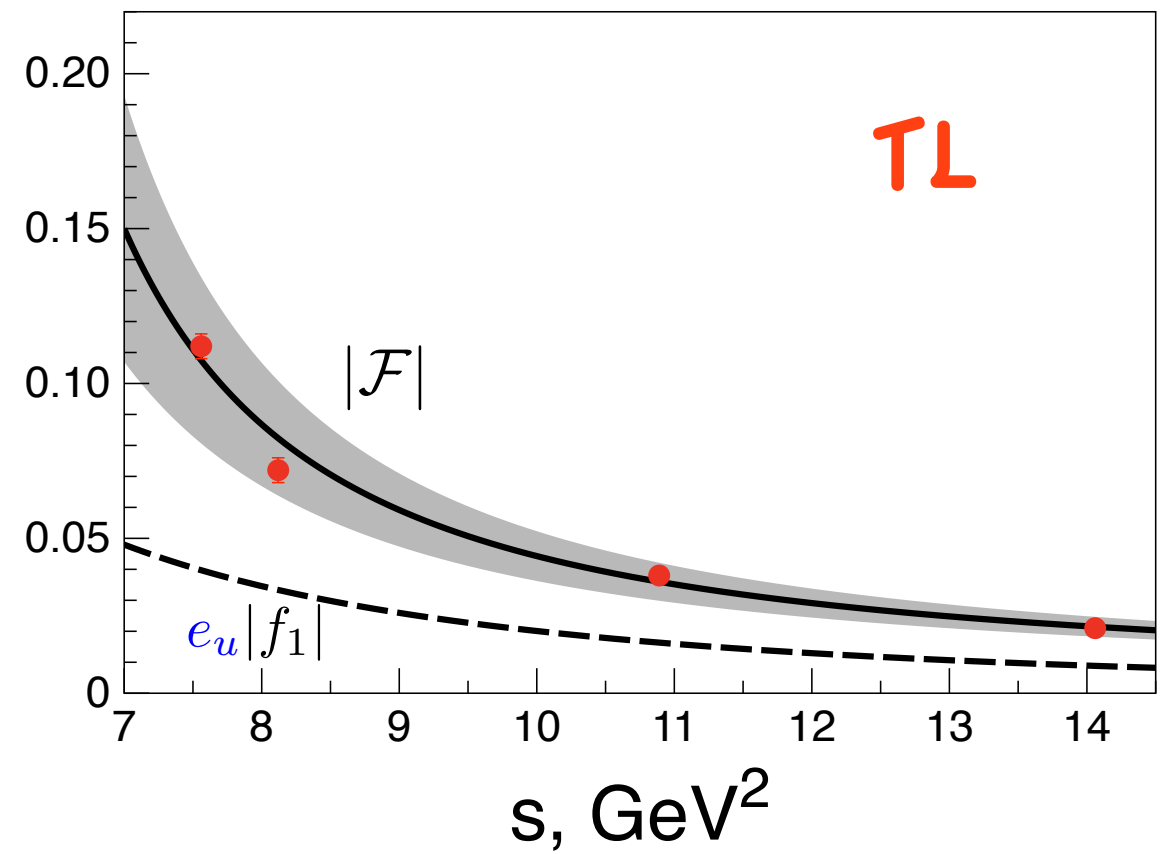
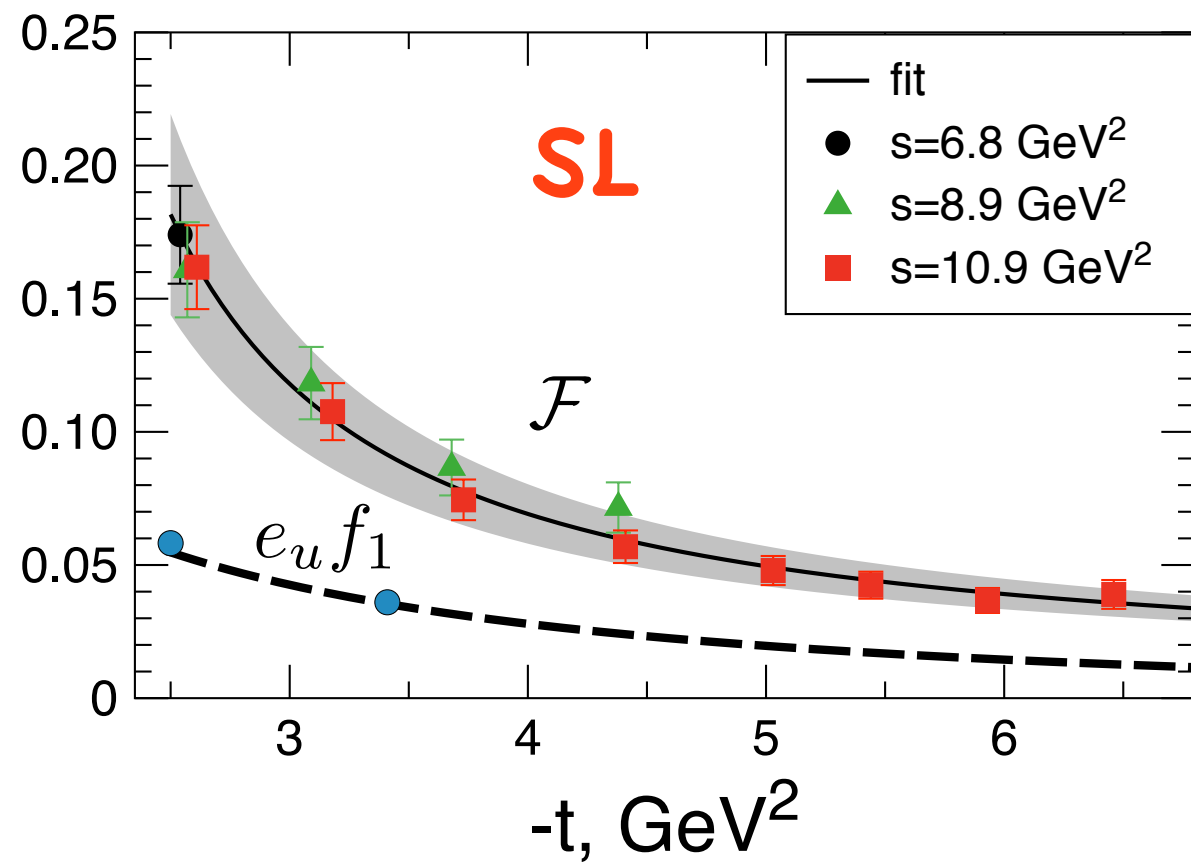


The large soft spectator contribution predicts

$$\frac{\text{TL}}{\text{SL}} \frac{|\mathcal{F}(s)|}{\mathcal{F}(s)} > 1$$

enhancement in TL region as in FF case

# FFs in SL and TL regions



$$\mathcal{F} = e_u^2(f_1^u + f_1^{\bar{u}}) + e_d^2(f_1^d + f_1^{\bar{d}})$$

$G_M$

$$|\mathcal{F}| = |e_u^2(f_1^u + f_1^{\bar{u}}) + e_d^2(f_1^d + f_1^{\bar{d}})|$$

$$e_u f_1 = e_u^2(f_1^u - f_1^{\bar{u}}) + e_u e_d(f_1^d - f_1^{\bar{d}})$$

WACS

$$e_u |f_1| = |e_u^2(f_1^u - f_1^{\bar{u}}) + e_u e_d(f_1^d - f_1^{\bar{d}})|$$

$$e_u f_1 = e_u^2(f_1^u - f_1^{\bar{u}}) + e_u e_d(f_1^d - f_1^{\bar{d}})$$

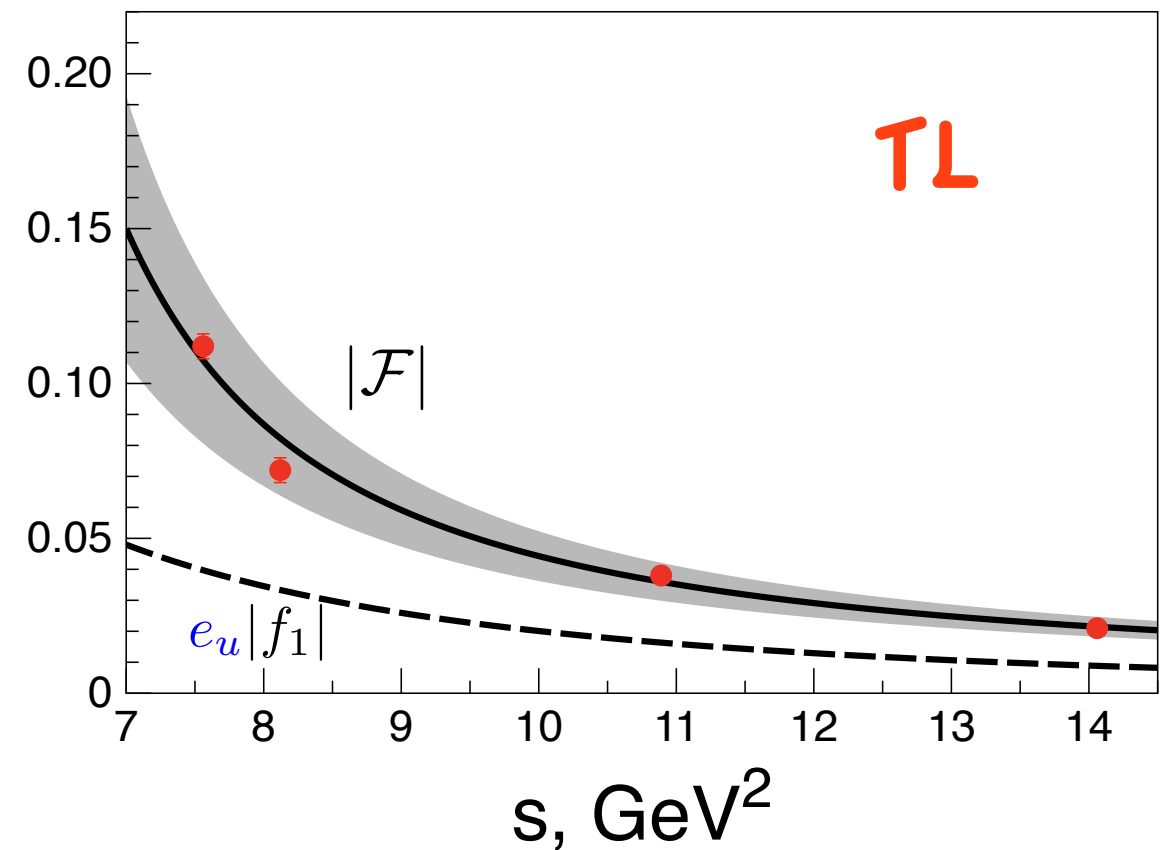
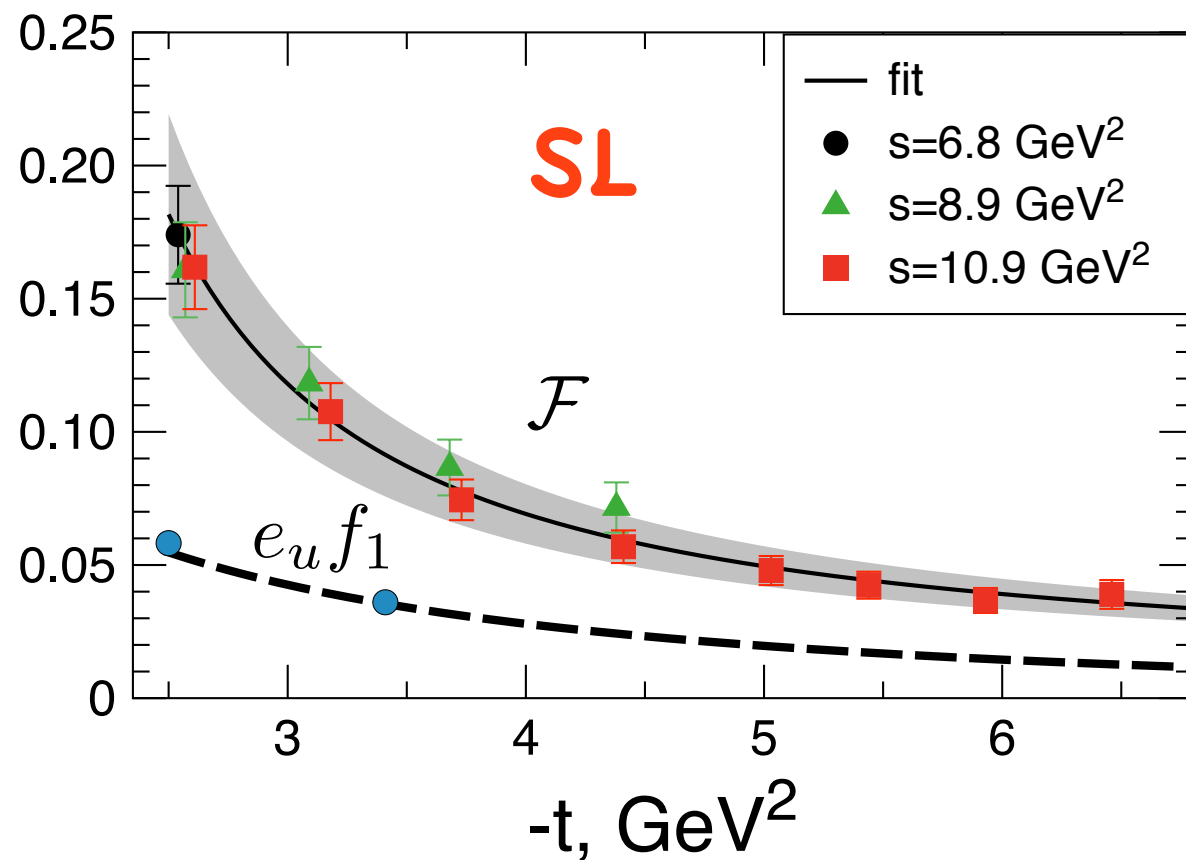
$$|f_1| \simeq |G_M|$$

$$f_1 \simeq G_M \frac{\tau + R}{1 + \tau} \quad \tau = Q^2/4m_N^2$$

$$R = G_E/G_M$$



# FFs in SL and TL regions



$$\mathcal{F} = e_u^2(f_1^u + f_1^{\bar{u}}) + e_d^2(f_1^d + f_1^{\bar{d}})$$

$G_M$

$$|\mathcal{F}| = |e_u^2(f_1^u + f_1^{\bar{u}}) + e_d^2(f_1^d + f_1^{\bar{d}})|$$

$$e_u f_1 = e_u^2(f_1^u - f_1^{\bar{u}}) + e_u e_d(f_1^d - f_1^{\bar{d}})$$

$WACS$

$$e_u |f_1| = |e_u^2(f_1^u - f_1^{\bar{u}}) + e_u e_d(f_1^d - f_1^{\bar{d}})|$$

$$e_u f_1 = e_u^2(f_1^u - f_1^{\bar{u}}) + e_u e_d(f_1^d - f_1^{\bar{d}})$$

Is it really antiquarks? (from DIS we know that # of antiquarks at large  $x$  is very small !!!)

# SCET ffs in TL regions: alternative approach

NK, Vanderhaeghen (in progress)

Assume that antiquark contribution is very small  $f_1^{\bar{q}} \ll f_1^q$

$$|\mathcal{F}| \approx |e_u^2 f_1^u + e_d^2 f_1^d| \quad |f_1| \approx |e_u f_1^u + e_d f_1^d|$$

- use  $|G_{\text{eff}}|$  from the FF data  $|G_{\text{eff}}|(s) \simeq \frac{C}{s^2 \ln^2[s/\Lambda^2]}$  with  $C=66.8\text{GeV}^2$   
 $\Lambda=300\text{MeV}$

- consider as a free parameters

$\Delta\phi$  relative phase between  $F_1$  and  $F_2$  ( $\cos \Delta\phi < 0$ )

$r = |f_1^d|/|f_1^u|$  ratio of the abs. values of the quark ffs

$\delta$  relative phase between  $f_1^d$  and  $f_1^u$

- kinematical power corrections have been added



# SCET ffs in TL regions: alternative approach

NK, Vanderhaeghen (in progress)

Assume that antiquark contributions are very small  $f_1^{\bar{q}} \ll f_1^q$

$$|\mathcal{F}| \approx |e_u^2 f_1^u + e_d^2 f_1^d| \quad |f_1| \approx |e_u f_1^u + e_d f_1^d|$$

$$|f_1| \simeq |G_M|$$

Our model

$$|\mathcal{F}|^2 \approx \frac{e_u^2 |f_1|^2}{1 - \cos^2 \Delta\phi \frac{4\tau}{(1+\tau)^2}} \frac{1+r/2+r^2/16}{1-r+r^2/4}$$

$$r = |f_1^d|/|f_1^u|$$

$$\tau = \frac{s}{4m_N^2}$$

assuming (NB!)

$$|G_M| = |G_E| \Leftrightarrow |F_2| \simeq \frac{-2 \cos \Delta\phi}{1 + \tau} |F_1|$$

$$\frac{4\tau}{(1 + \tau)^2} = \frac{16m_N^2}{s} + \mathcal{O}(1/\tau)$$

large

assuming

$$\Re f_1^u f_1^{*d} \approx |f_1^u| |f_1^d|$$

we need

$$\cos \Delta\phi \simeq -0.95$$

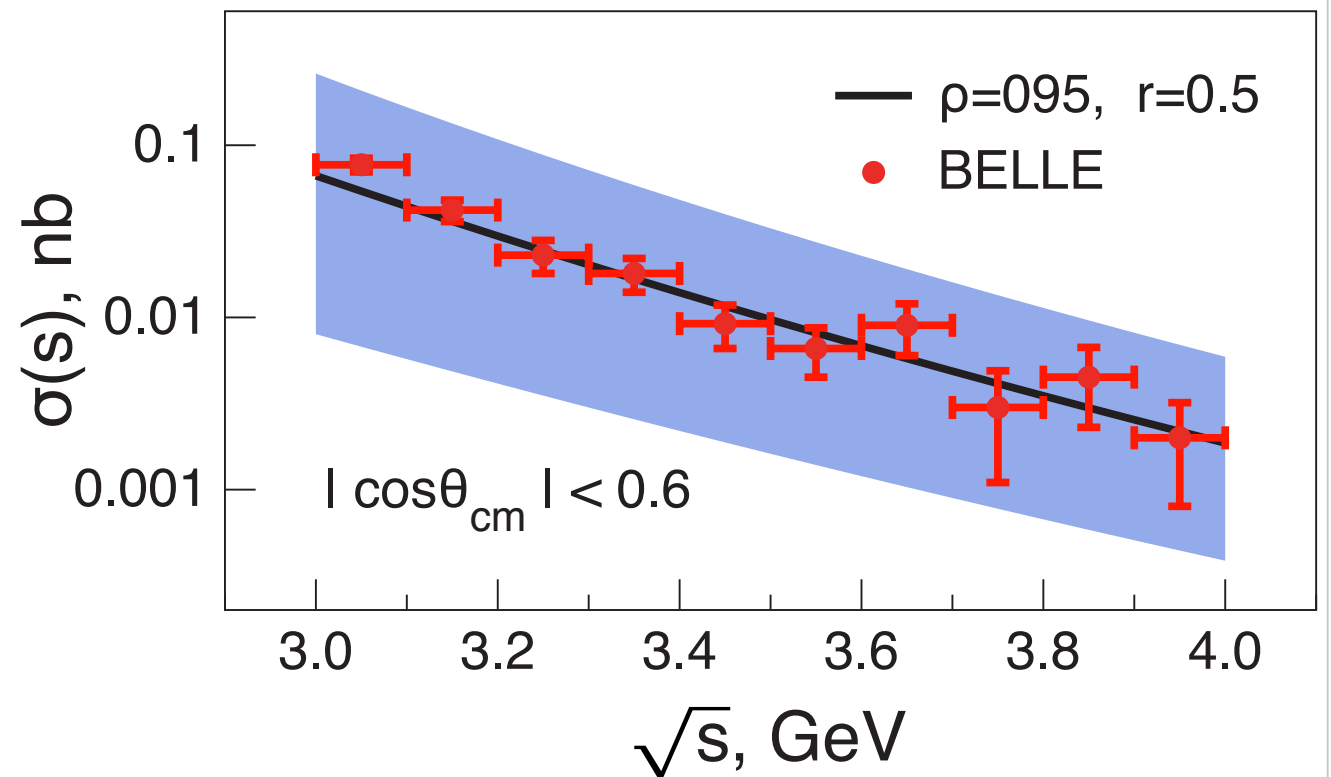
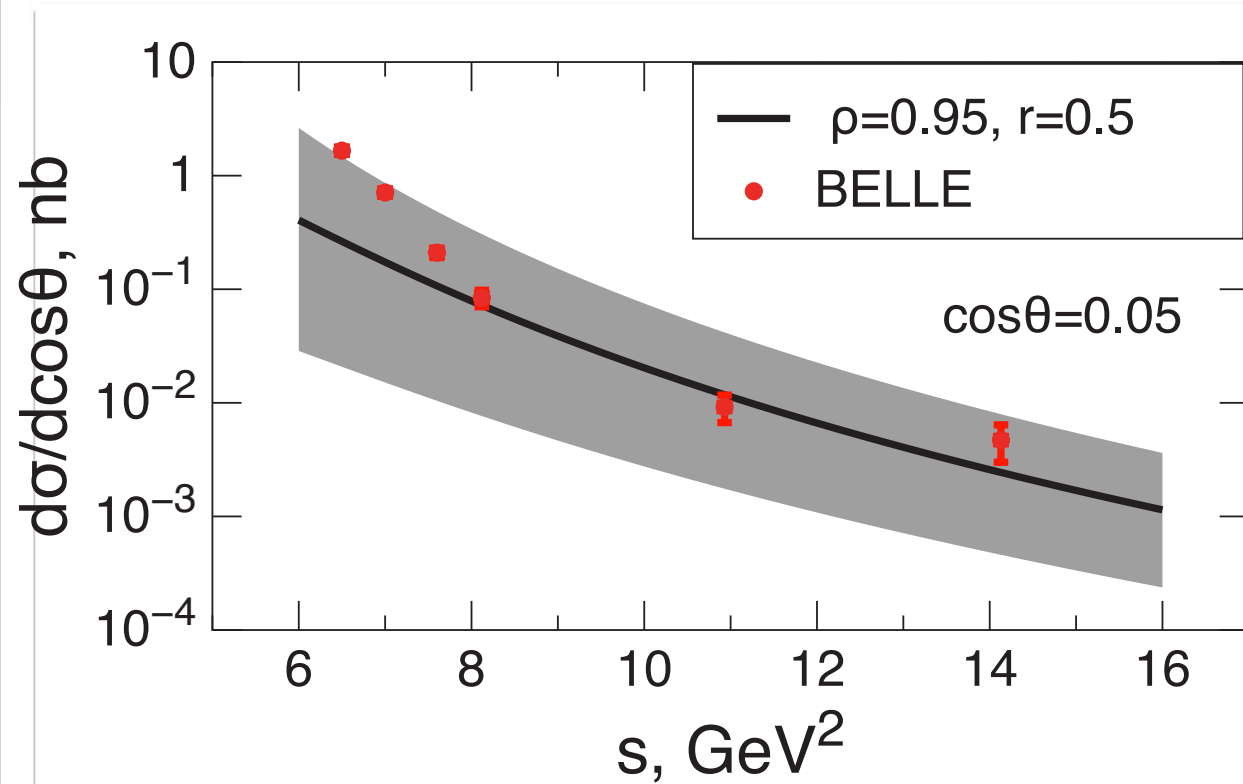
$$r \simeq 0.5 - 0.6$$

for  $s=11-14 \text{ GeV}^2$

# Description $B\bar{B}$ production

$\gamma\gamma \rightarrow p\bar{p}$  data Belle collab., 2005

NK, Vanderhaeghen (in progress)



$$|G_{eff}|(s) \simeq \frac{C}{s^2 \ln^2[s/\Lambda^2]} \quad \text{with } C=66.8\text{GeV}^2 \quad \Lambda=300\text{MeV}$$

shaded area

$$0 < r = |f_1^d|/|f_1^u| < 1$$

$$0 < \rho = -\cos \Delta\phi < 1$$

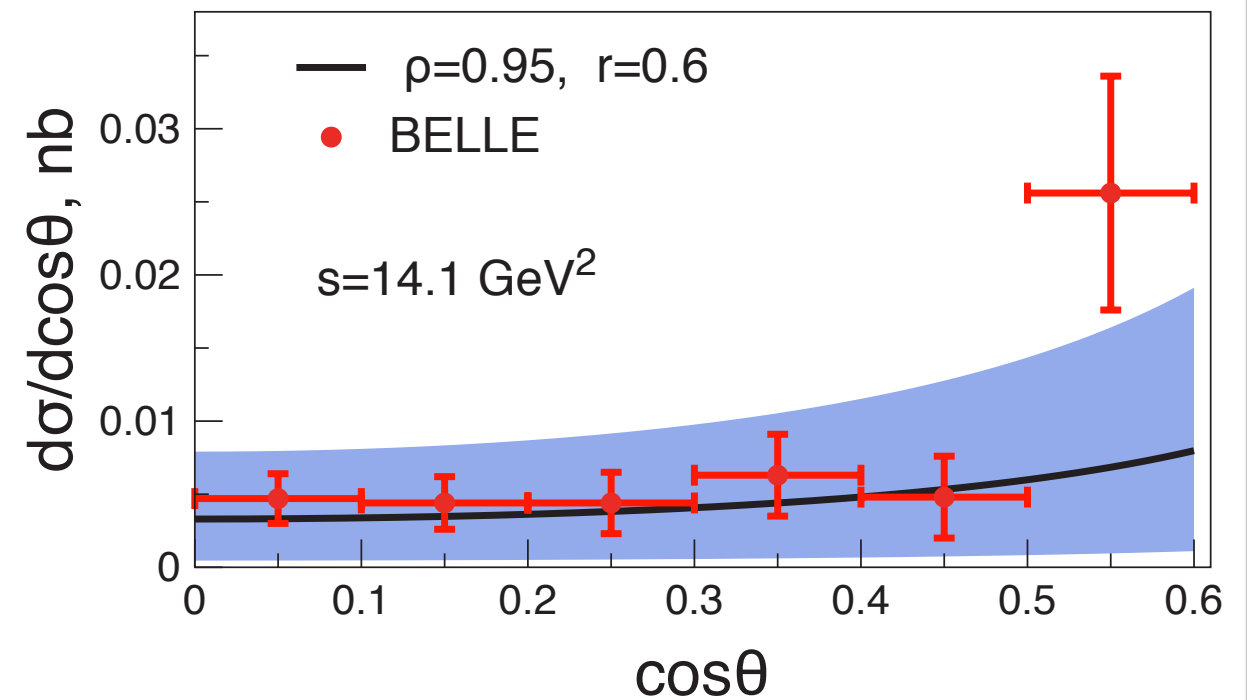
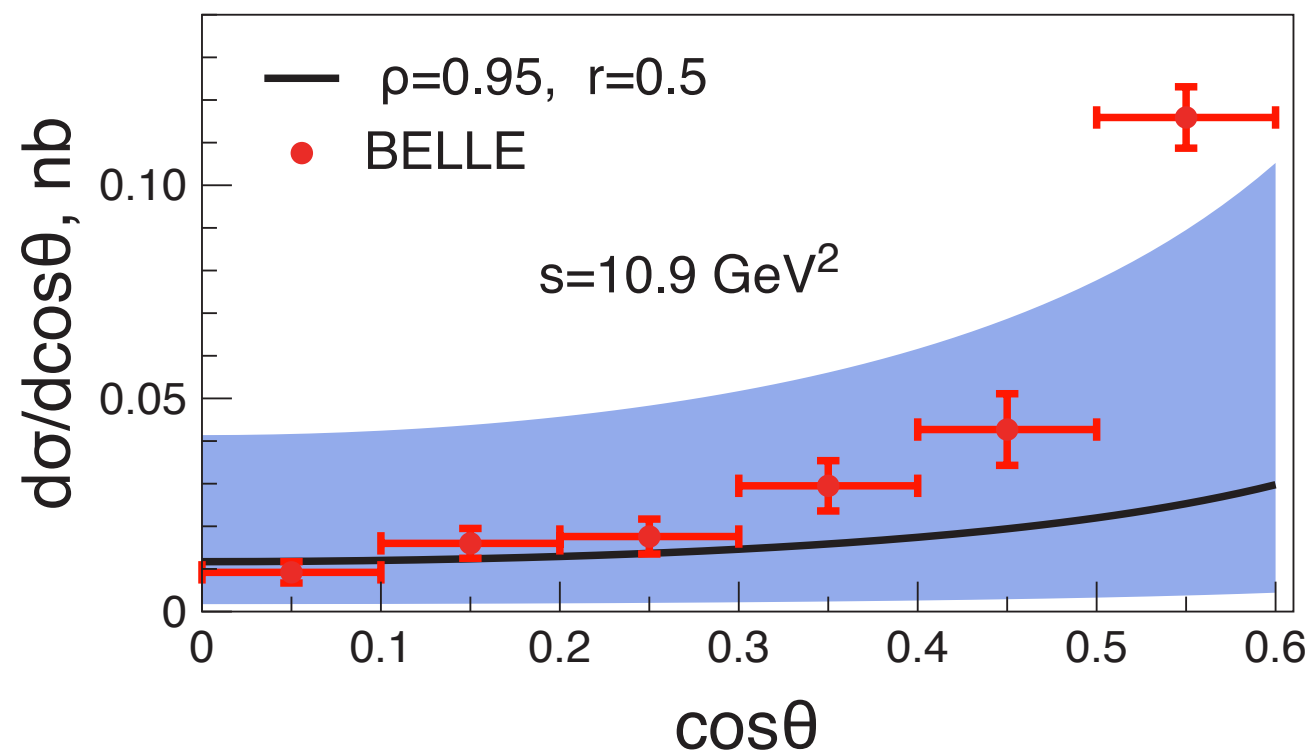
relative phase between  $F_1$  and  $F_2$



# Description $B\bar{B}$ production

$\gamma\gamma \rightarrow p\bar{p}$  data Belle collab., 2005

NK, Vanderhaeghen (in progress)



$$|G_{eff}|(s) \simeq \frac{C}{s^2 \ln^2[s/\Lambda^2]} \quad \text{with } C=66.8\text{GeV}^2 \quad \Lambda=300\text{MeV}$$

shaded area

$$0 < r = |f_1^d|/|f_1^u| < 1$$

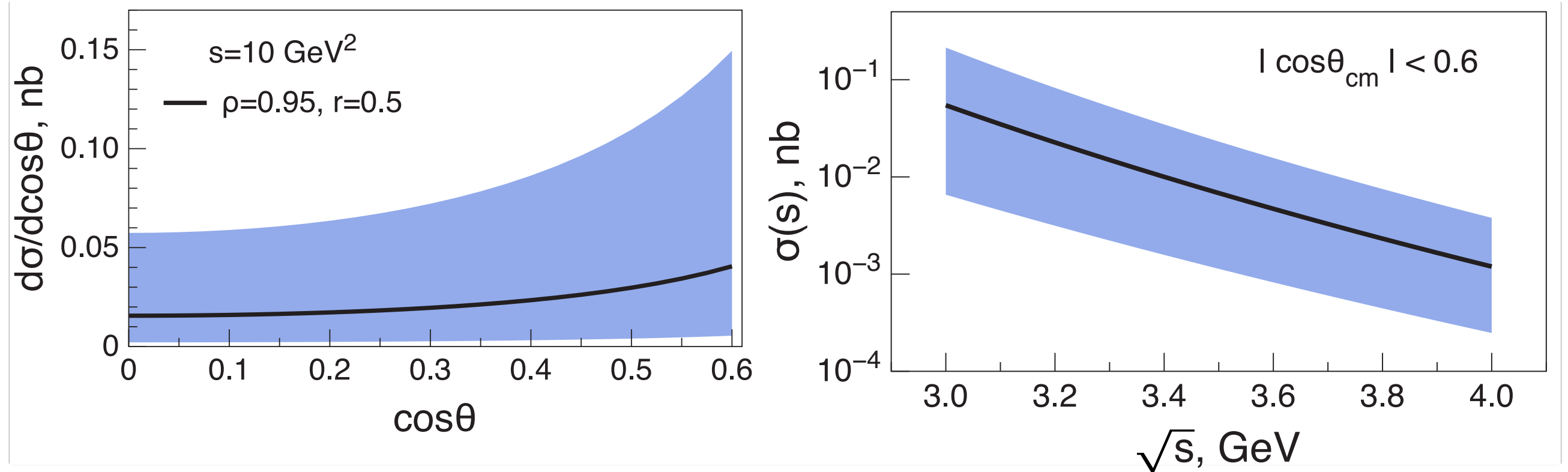
$$0 < \rho = -\cos \Delta\phi < 1 \quad \text{relative phase between } F_1 \text{ and } F_2$$

# Description $B\bar{B}$ production

PANDA  $p\bar{p} \rightarrow \gamma\gamma$

predictions

NK, Vanderhaeghen (in progress)



$$|G_{eff}|(s) \simeq \frac{C}{s^2 \ln^2[s/\Lambda^2]} \quad \text{with } C=66.8\text{GeV}^2 \quad \Lambda=300\text{MeV}$$

shaded area  $0 < r = |f_1^d|/|f_1^u| < 1$

$0 < \rho = -\cos \Delta\phi < 1$  relative phase between  $F_1$  and  $F_2$



# Conclusions

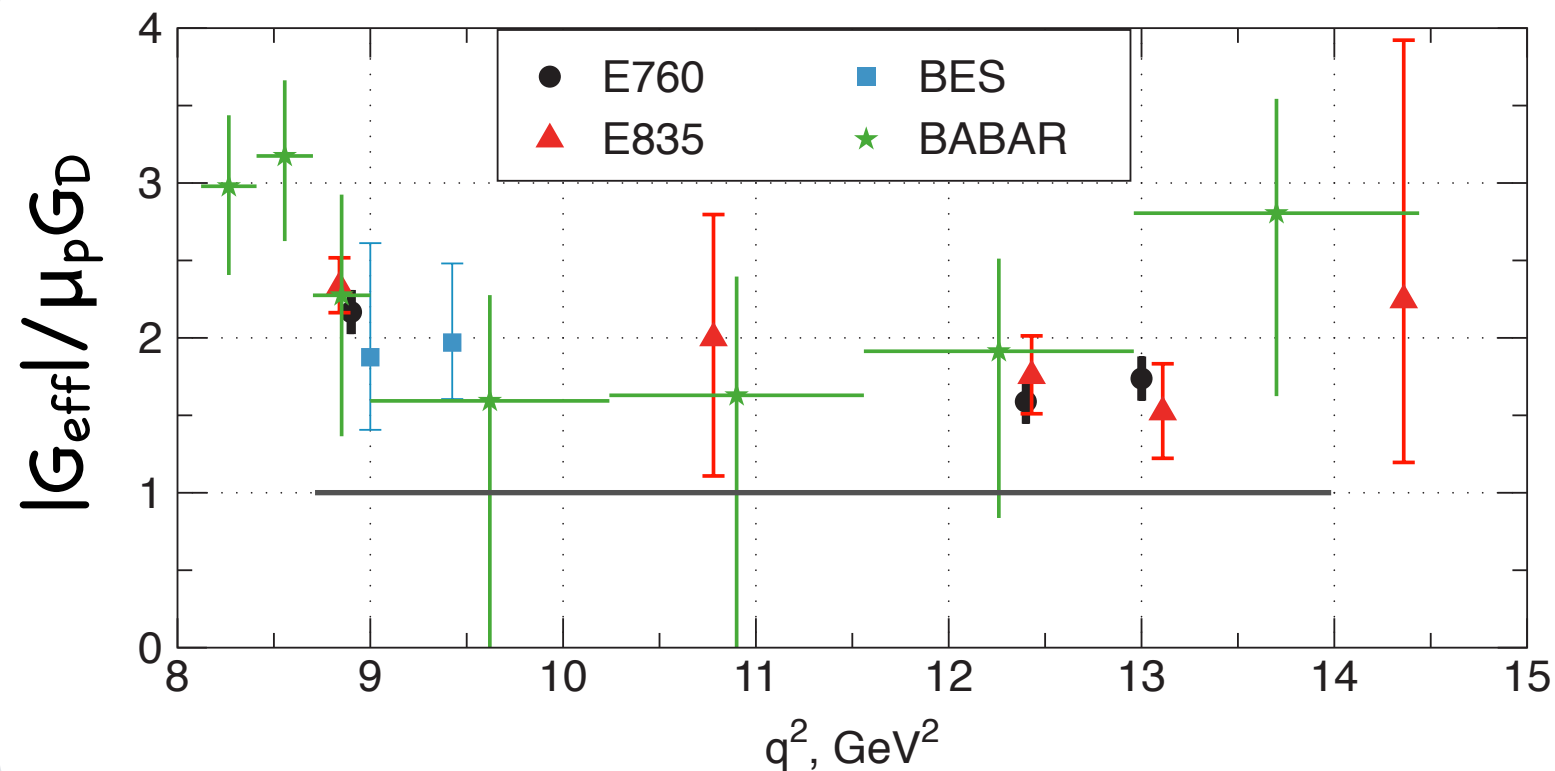
- QCD factorization for e.m. FFs includes hard and soft spectator contributions.
- There are indications that the soft spectator contribution is large or even dominant for moderate values of  $Q^2$  :  $Q\Lambda \sim m_N^2$   
FF enhancement in the TL region
- The full  $Q^2$  dependence of the soft spectator contribution can not be computed from pQCD but can be described by universal form factors within SCET factorization framework
- QCD factorization allows us to study the soft spectator mechanism in other reactions: WACS in SL and  $2\gamma$ (or  $B\bar{B}$ )-production in TL regions
- Soft overlap in WACS and e.m. FF are not completely independent and the new data (SL and TL) will help to understand much better corresponding long-distance dynamics
- SCET factorization provides predictions which can be checked experimentally in SL (JLab) and TL (PANDA) regions







# FFs ratio: timelike vs. spacelike region



enhancement  
in TL region

$$\frac{|G_{eff}|}{\mu_p G_D} \simeq 2$$

$$\frac{|F_1|_{\text{TL}}}{|F_1|_{\text{SL}}} \simeq \frac{|e^{-S}U_1|_{\text{TL}}}{|e^{-S}U_1|_{\text{SL}}} \frac{|f_1(q)|_{\text{TL}}}{|f_1(Q)|_{\text{SL}}}$$

Sudakov logs provide  
enhancement at large time-like  $q^2$

