An update on the Proton Radius Puzzle

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Outline

1. Introduction - Meaning
2. The Mainz measurements
3. Challenges - Radius Puzzle, TPE
4. Summary
5. Outlook - ISR and deuteron radius

Proton / Neutron

mass

$m_p = 938.272046(21)\,\mathrm{MeV/c^2}$

Discovered by E. Rutherford (1919)

$m_n = 939.565379(21)\,\mathrm{MeV/c^2}$

Discovered by J. Chadwick (1939)

size

moments of electric charge and magnetization distribution derived from **form factor** measurements

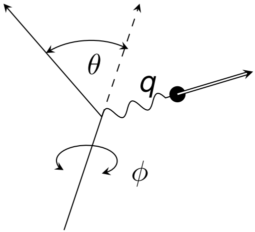
shape

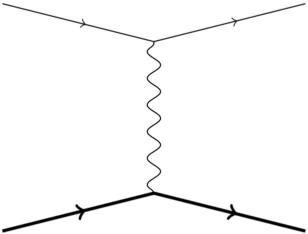
departure from spherical symmetry determined in $N \longrightarrow \Delta$ measurements

stiffness

electric and magnetic polarizabilities extracted from Virtual Compton scattering (VCS)

Cross section and form factors for elastic e-p scattering





The cross section:

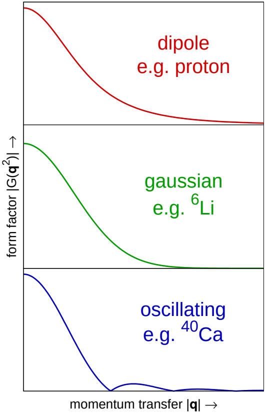
$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = \frac{1}{\varepsilon\left(1+\tau\right)}\left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$

with:

$$\tau = \frac{Q^2}{4m_p^2}, \quad \varepsilon = \left(1 + 2\left(1+\tau\right)\tan^2\frac{\theta_e}{2}\right)^{-1}$$

Classical picture

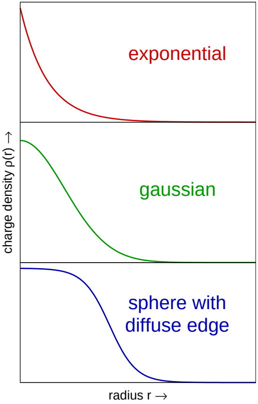
form factor: $G(q^2) = \frac{1}{e} \int_0^\infty \rho(r) \frac{\sin qr}{qr} 4\pi r^2 \, dr$



Fourier

⇔

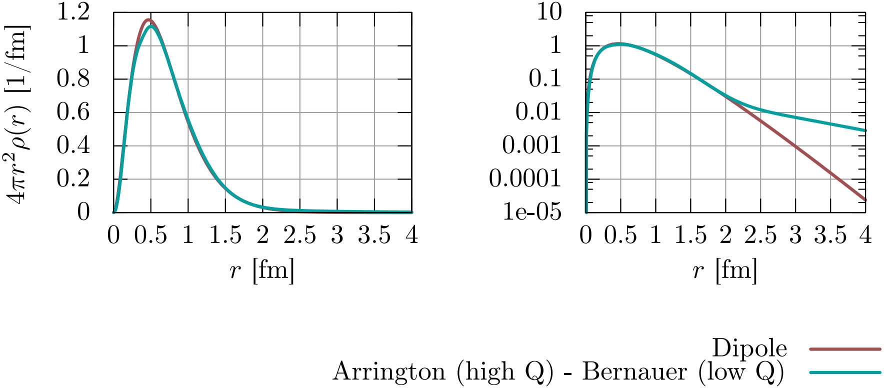
Transform



charge distribution: $\rho(r) = \frac{e}{(2\pi)^3} \int_0^\infty G(q^2) \frac{\sin qr}{qr} 4\pi q^2 \, dq$

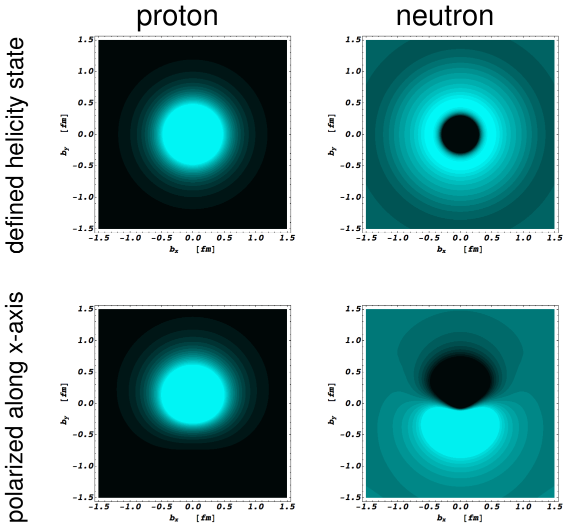
Charge density of the proton

Relativity: Transformation possible in Breit frame
(vanishing energy transfer to the nucleon).
No straight forward interpretation in rest frame!



M.O.D., J.C. Bernauer, Th. Walcher, Phys. Lett. **B696**, 343 (2011)

Infinite Momentum Frame: 3D distribution gets "squashed".



From: M. Vanderhaeghen, Th. Walcher, arXiv:1008.4225v1

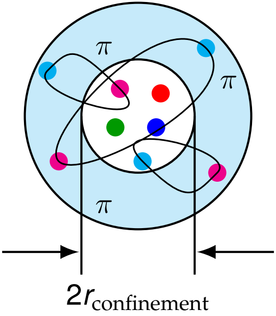
Low-Q form factors

What more can we learn from the form factors?

Low-Q form factors

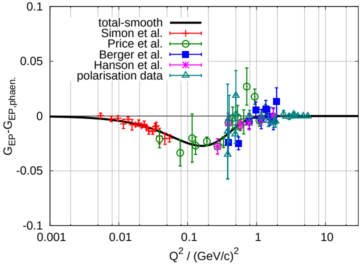
What more can we learn from the form factors?

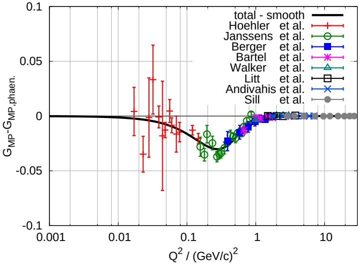
- Low-Q $\Longleftrightarrow$ Long range structure
- How big is the proton?
- Is there evidence for a pion cloud?



$$\left\langle r_E^2 \right\rangle = -6\hbar^2 \left. \frac{dG_E}{dQ^2} \right|_{Q^2=0} \qquad \left\langle r_M^2 \right\rangle = -6\hbar^2 \left. \frac{d\left(G_M/\mu_p\right)}{dQ^2} \right|_{Q^2=0}$$

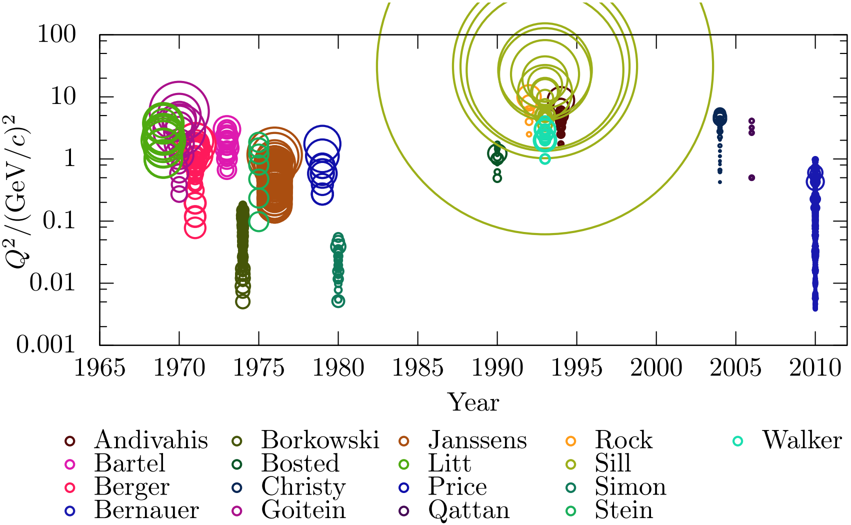
Motivation: Structure



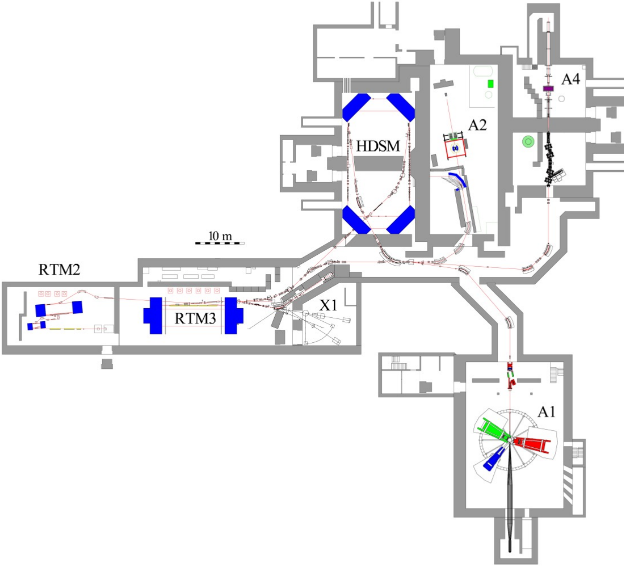


(see J. Friedrich and Th. Walcher, Eur. Phys. J. A **17** (2003) 607)

Timeline of proton cross section data



The Mainz Microtron MAMI



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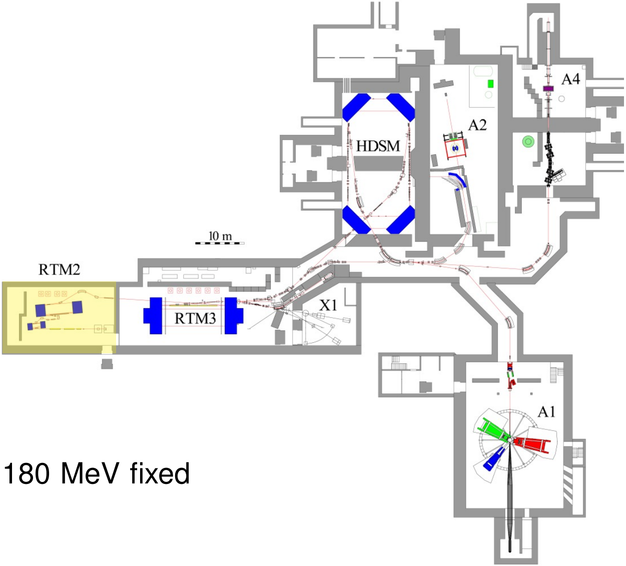

- MAMI-A: 180 MeV fixed

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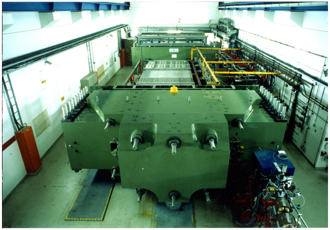

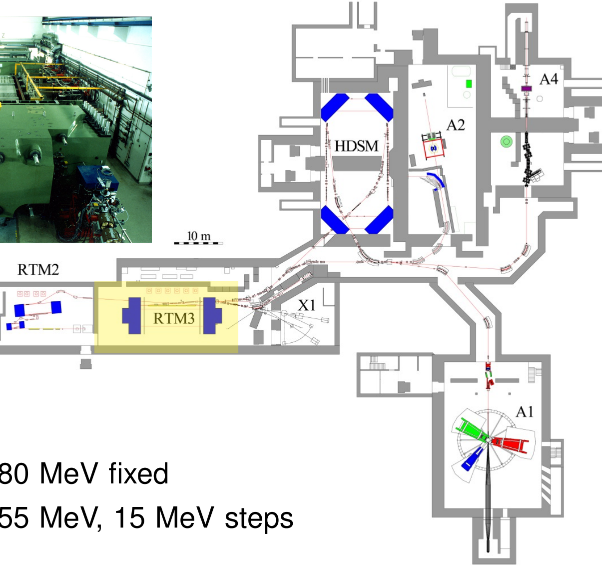

- MAMI-A: 180 MeV fixed
- MAMI-B: 855 MeV, 15 MeV steps

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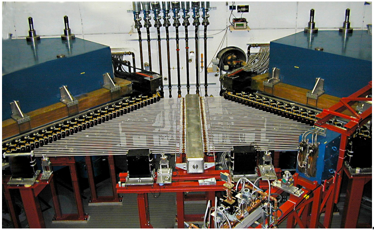

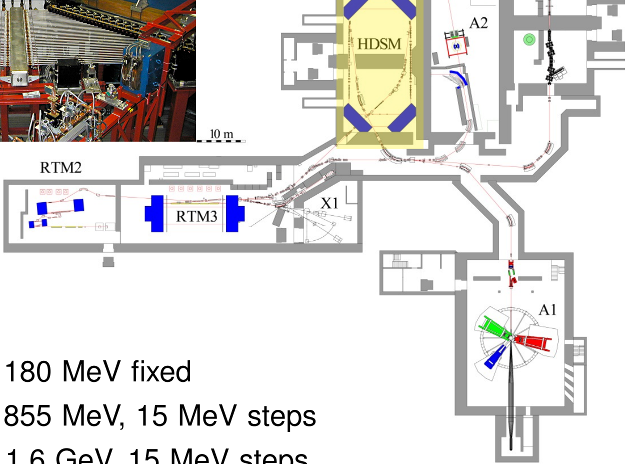

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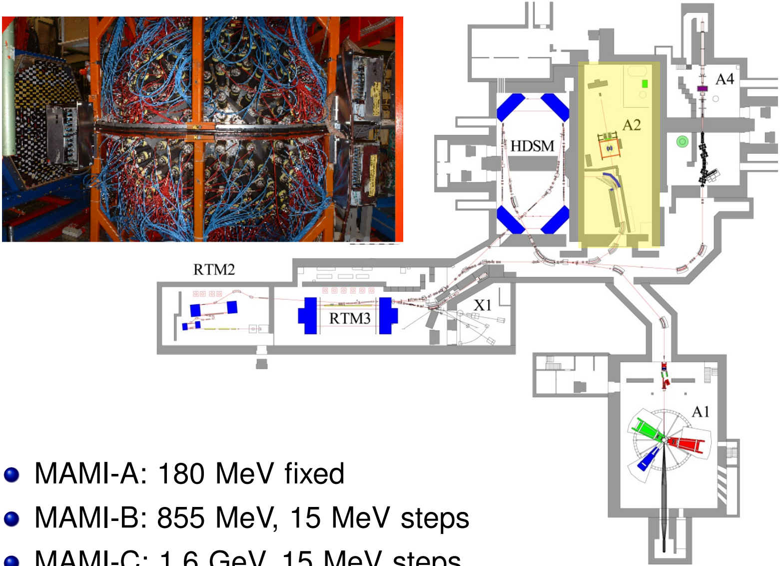

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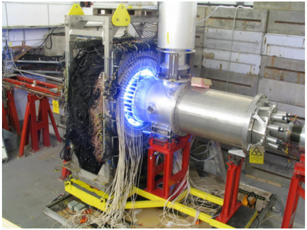

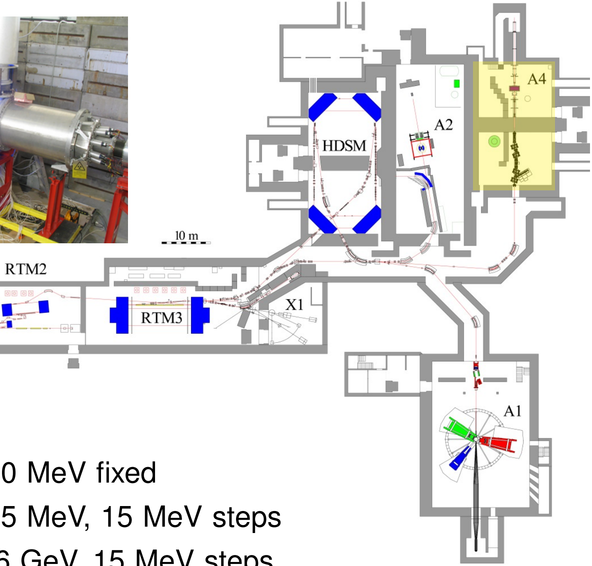

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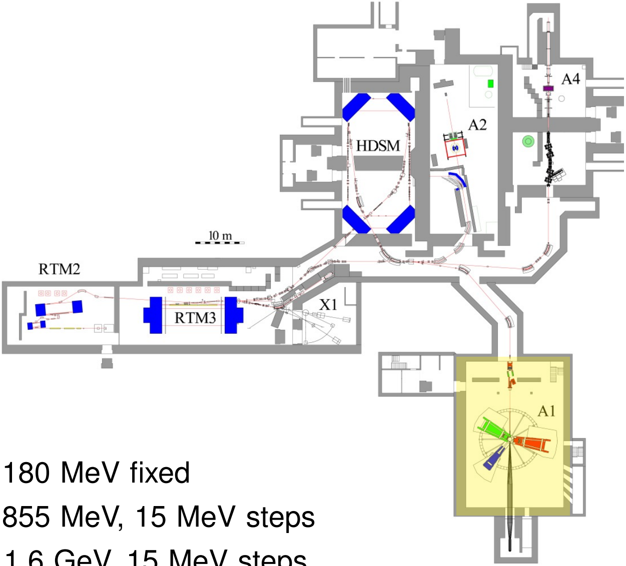

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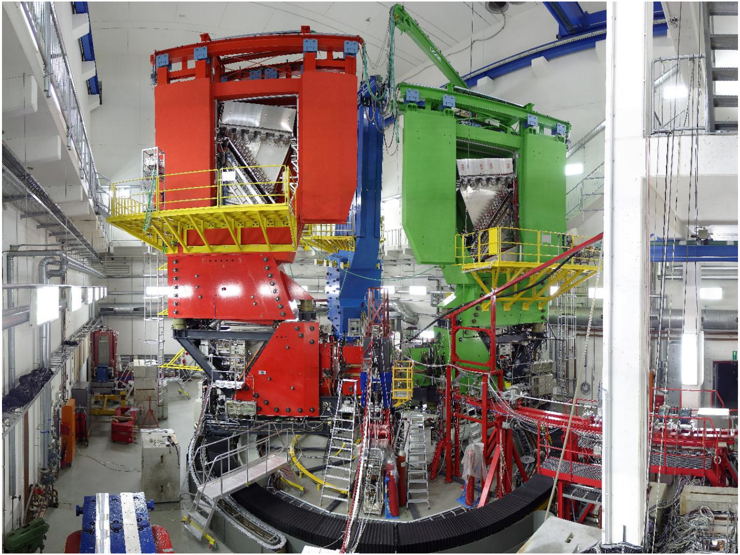
The Mainz high-precision p(e,e')p measurement: Three spectrometer facility of the A1 collaboration



Design goal: High precision

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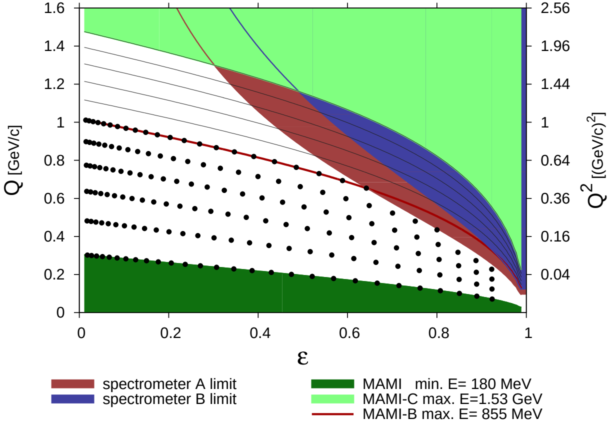
  - Overlapping acceptance
  - Where possible: Measure at the same scattering angle with two spectrometers

Measured settings and future (high Q^2) expansion

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{1}{\varepsilon(1+\tau)} \left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$



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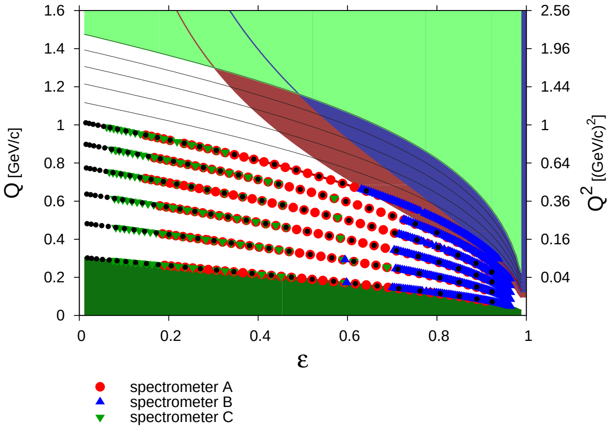
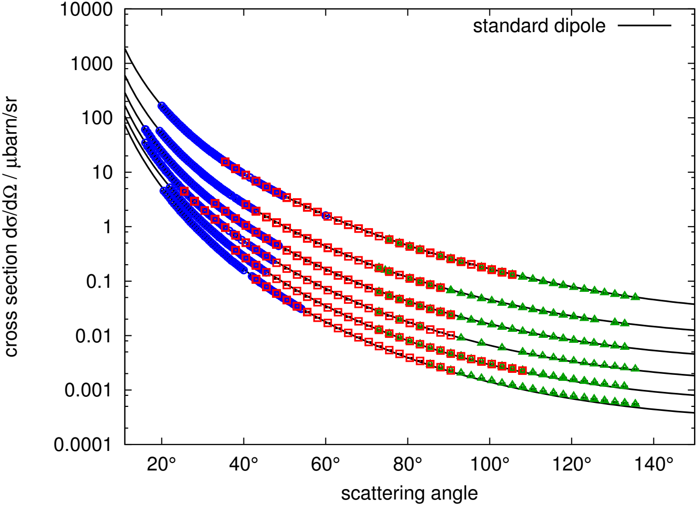
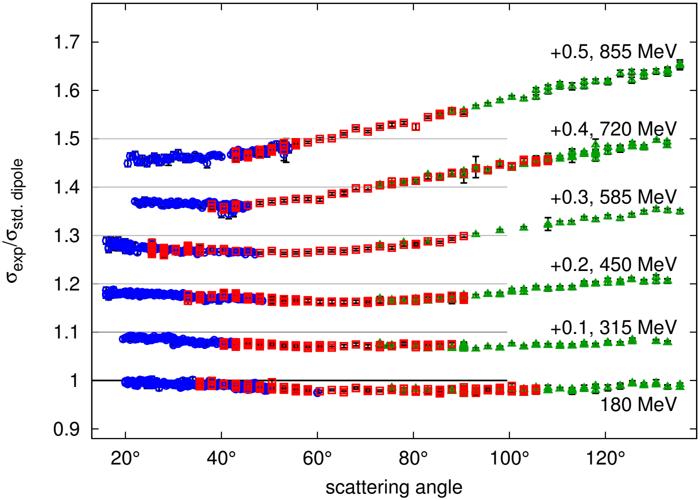


- $\sim$90 GByte on disc
- $\sim$1400 Settings
- $> 10^9$ events.

Cross sections

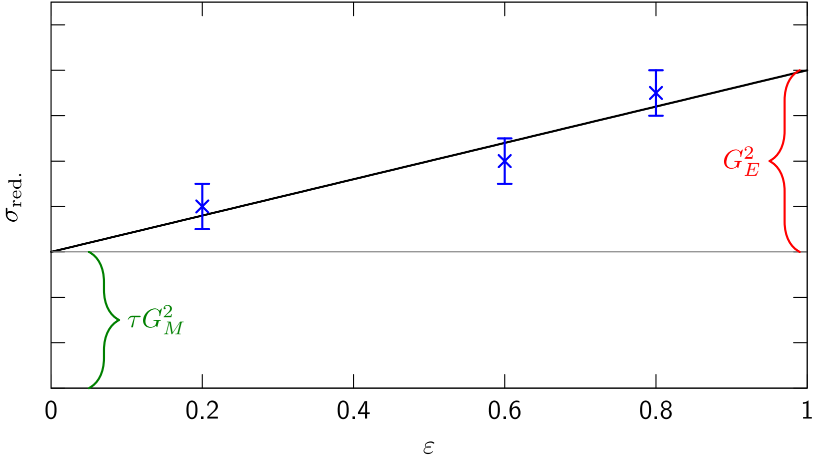


Cross sections / standard dipole



Rosenbluth method

$$\sigma_{red} = \varepsilon (1+\tau) \frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = \left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$



Rosenbluth with a twist!

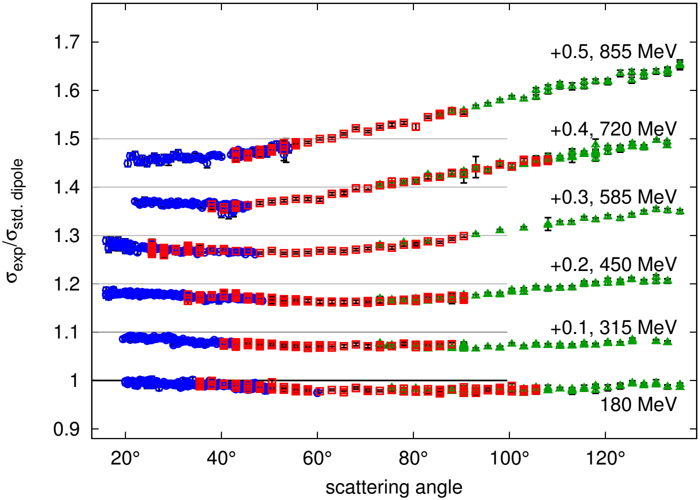
Instead of independent separation at discrete Q^2:

Super-Rosenbluth separation

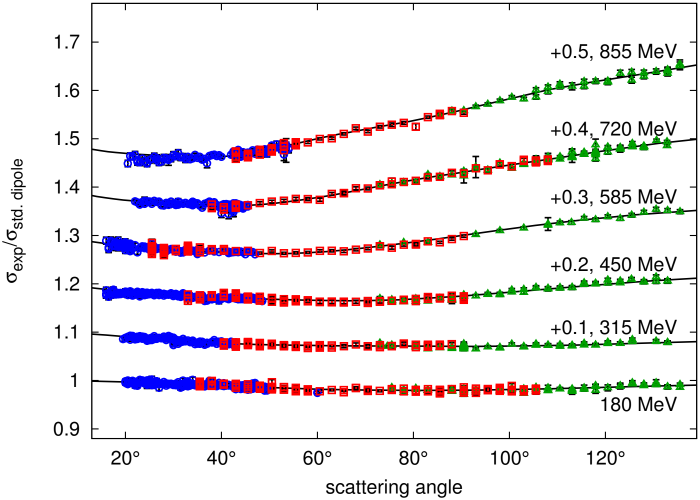
Take (many, flexible) models for form factors, plug them into cross section formula. Fit to all cross section data in one go!

- Feasible due to fast computers.
- All data at all Q^2 and ε values contribute to the fit, i.e. full kinematical region used, no projection (to specific Q^2) needed.
- Global normalization fixed to static limits, $G_E(0) = 1$, $G_M(0) = \mu_p$.

Cross sections / standard dipole

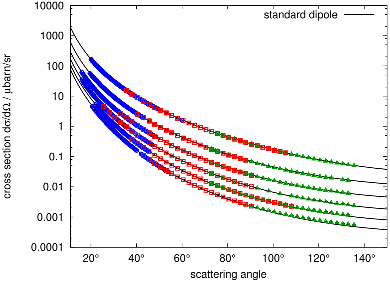


Cross sections + spline fit



Advantages of the Super-Rosenbluth Separation

Rosenbluth formular gives additional constraints

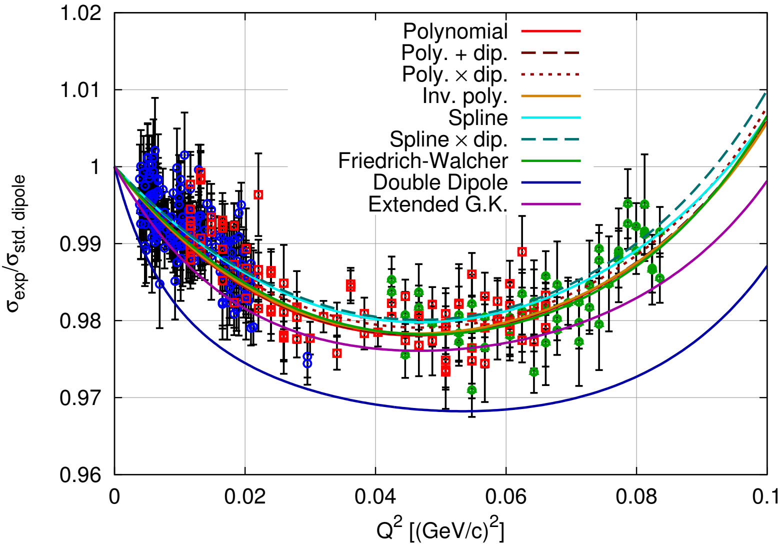


31 normalization parameters allow a statistical analysis.

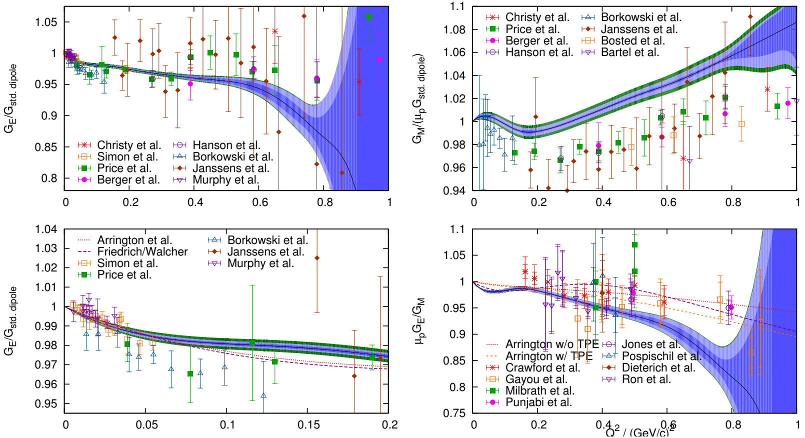
The luminosity is a major source of the systematic error

$\longrightarrow$ **Here, it becomes a statistical error**

Result: Cross section fits (180 MeV)



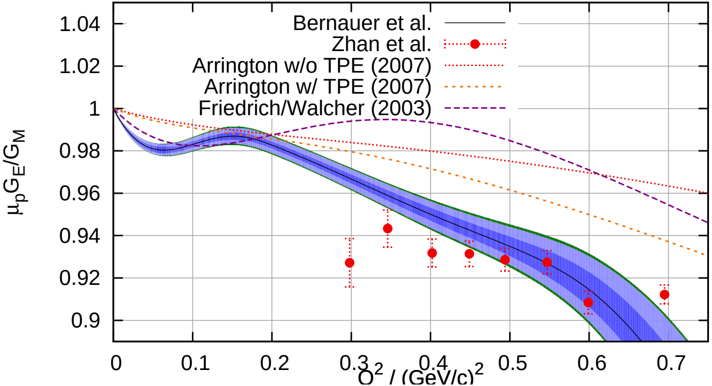
Form factor results



Jan C. Bernauer *et al.*, "High-precision determination of the electric and magnetic form factors of the proton",

PRL 105, 242001 (2010), arXiv:1007.5076

Form factor results: G_E/G_M ratio



Jan C. Bernauer *et al.*, PRL 105, 242001 (2010), arXiv:1007.5076

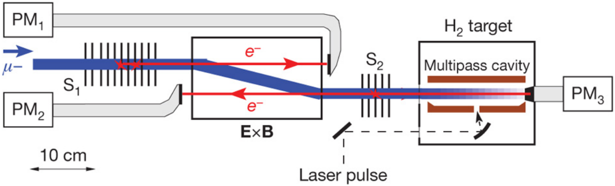
X. Zhan *et al.*, arXiv:1102.0318

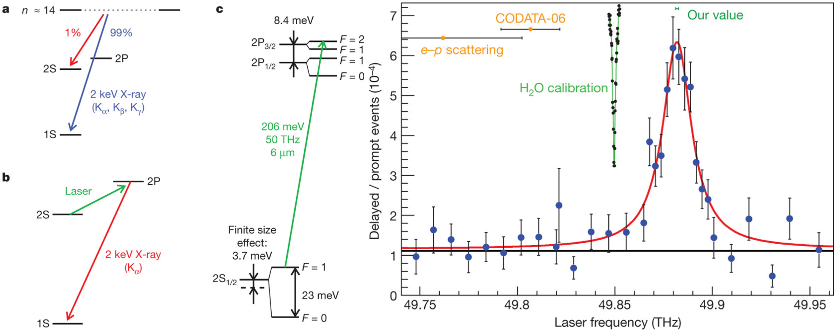
J. Arrington *et al.*, Phys. Rev. C76 (2007) 035205, arXiv:0707.1861

Challenges

The radius puzzle – Lamb shift in μH

Nature **466**, 213-216 (8 July 2010)





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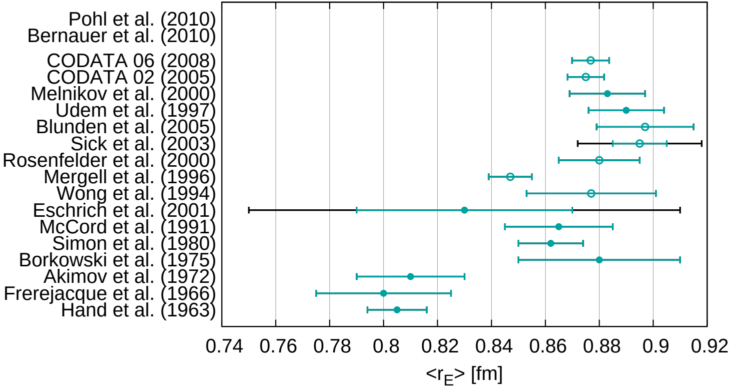

Filled dots: Results from new measurements.
Hollow dots: Reanalysis of existing data.

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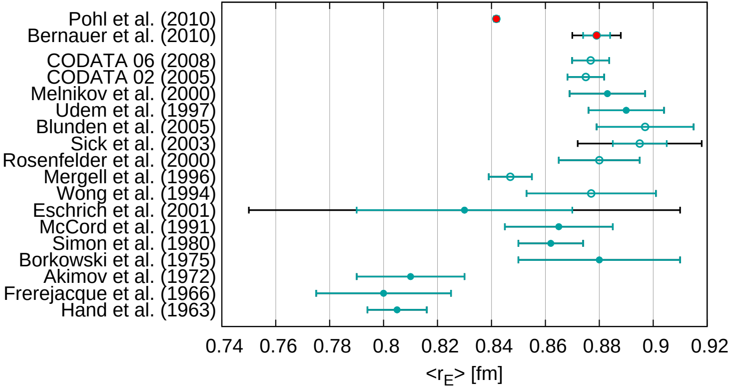

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Discussion of the Lamb shift / electron scattering discrepancy

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Discrepancy is between
muonic **and** *electronic* **measurements**

What could be wrong?

The muonic/electronic puzzle of the charge radius

What could be wrong? or Is it “new” physics?

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- Models don’t extrapolate right to $Q^2 \to 0$?
  But, a plathora of models tried. All give same result.
- Coulomb corrections, resp. two photon exchange (TPE) is incomplete?
  But, effect on charge radius $< r_E >$ is negligible at $Q^2 \lesssim 1\,(\mathrm{GeV/c})^2$ for all TPE calculations.

Possible explanations of the discrepancy

- **Exotic particles**
  e.g. V. Barger *et al.*, arXiv:1011.3519 and references.
- **Contributions to the Lamb shift in μp**
  C.E. Carlson and M. Vanderhaeghen, arXiv:1101.5965
  U.D. Jentschura, Annals Phys. **326**, 500-515 (2011)
  E. Borie, arXiv:1103.1772
- **Higher moments of the charge distribution and Zemach radii**
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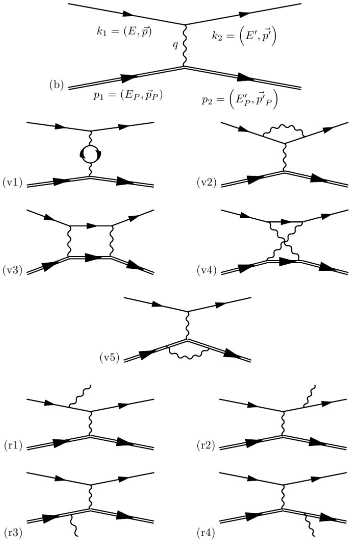
still an unsolved problem

Feynman graphs of leading and next to leading order for elastic scattering

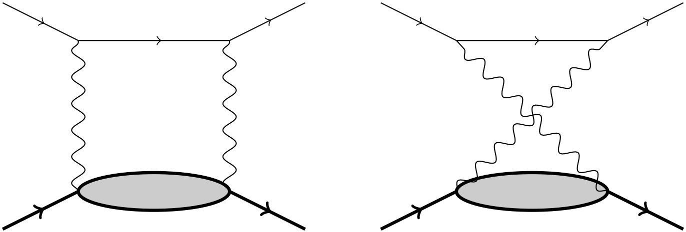


All graphs are taken into account:

- vacuum polarization (v1):
  e, (μ, τ)
  Maximon/Tjon (2000) and Vanderhaeghen et al. (2000)
- electron vertex correction
- Coulomb distortion
  (two photon exchange)
- real photon emission

Two-Photon-Exchange

Are we at the limit of the first Born approximation?
$\Longrightarrow$Higher orders are important!



Two-Photon-Exchange

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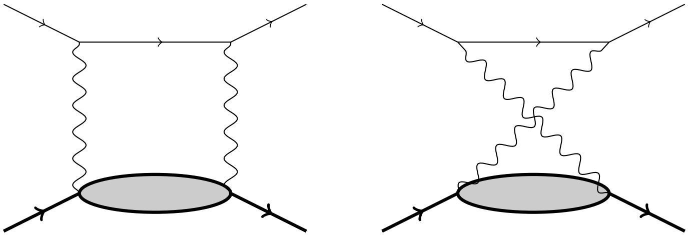

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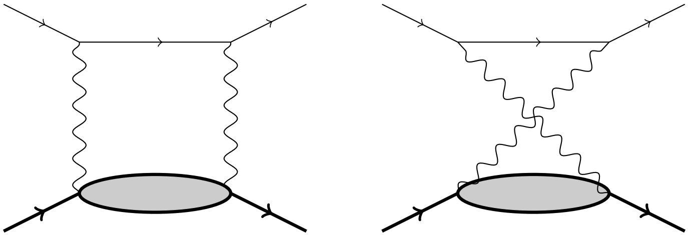

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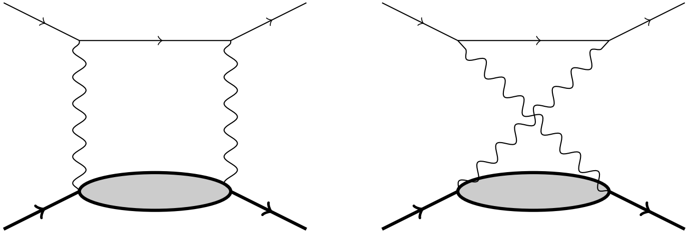


- Hard to calculate because of intermediate states. Different calculations using GPDs, Dispersion-relations,... + phenomenological extractions
- Seems to only account for half of the discrepancy
- Spawned lively theoretical and experimental program (JLab, Novosibirsk, OLYMPUS).

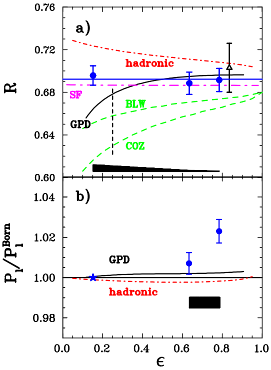
Comments on Coulomb distortion and TPE

- Coulomb distortion:
  Exchange of one hard and multiple soft photons
  Feshbach (1948), Mo and Tsai (1969).

- Two-photon exchange (TPE) with and w/o excited intermediate states:
  Exchange of two hard photons

  Still not reliable and highly debated

  Figure shows a recent experimental result from JLab.



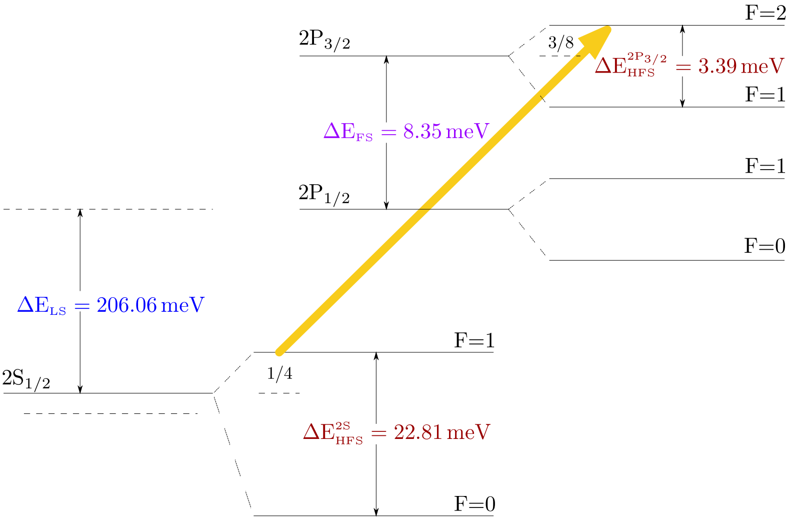
Meziane, M. et al.: *Search for effects beyond the Born approximation in polarization transfer observables in $\vec{e}p$ elastic scattering*, PRL 106, 132501 (2011), arXiv:1012.0339

Proton structure from muonic hydrogen spectroscopy:

Claim that muonic hydrogen spectroscopy and electron scattering are “compatible” and “in agreement”

Randolf Pohl, private communication, October 8, 2012.

2S – 2P splitting in muonic hydrogen



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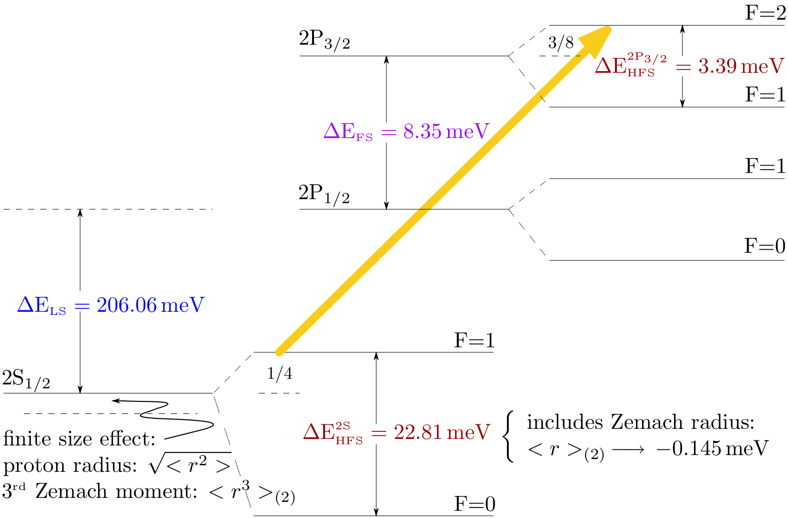


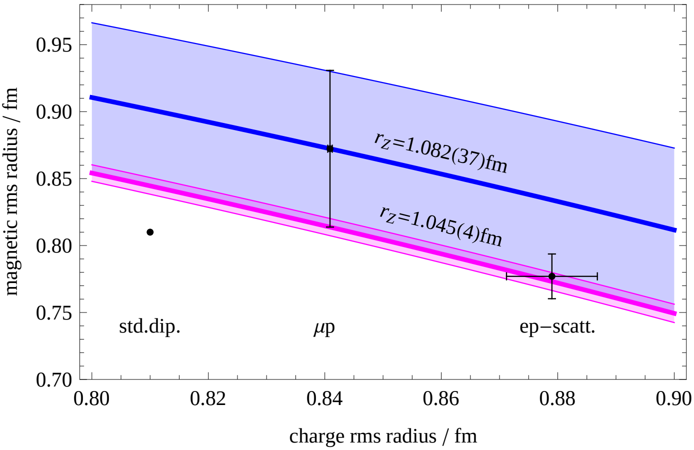
Proton structure from muonic hydrogen

Aldo Antognini et al. report on Zemach and magnetic radii

Assuming dipole form of proton (em-) radii:

$$\begin{aligned}
G_{\mathrm{e,m}} &= \left(1+\left(\frac{r_{\mathrm{e,m}}q}{2\sqrt{3}\hbar c}\right)^2\right)^{-2} \\
r_{\mathrm{Z}} &= \frac{4}{\pi}\int_0^{\infty}\frac{dq}{q^2}\left(G_{\mathrm{e}}(q)G_{\mathrm{m}}(q)-1\right) \\
&= \frac{3r_{\mathrm{e}}^4+9r_{\mathrm{e}}^3r_{\mathrm{m}}+11r_{\mathrm{e}}^2r_{\mathrm{m}}^2+9r_{\mathrm{e}}r_{\mathrm{m}}^3+3r_{\mathrm{m}}^4}{2\sqrt{3}(r_{\mathrm{e}}+r_{\mathrm{m}})^3} \\
r_{\mathrm{m}} &= f\left(r_{\mathrm{Z}}, r_{\mathrm{e}}\right)
\end{aligned}$$

Proton structure from muonic hydrogen



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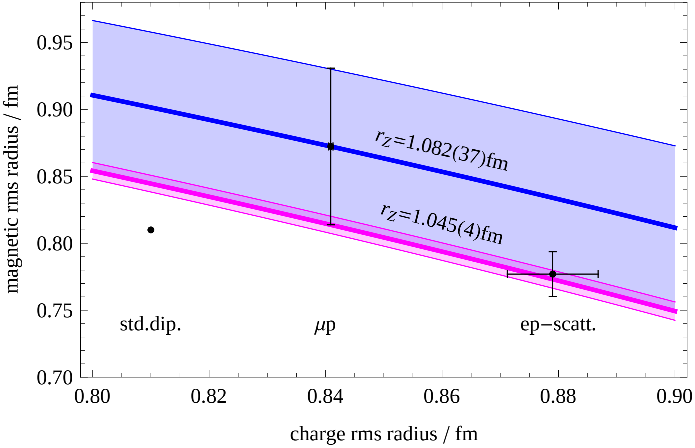
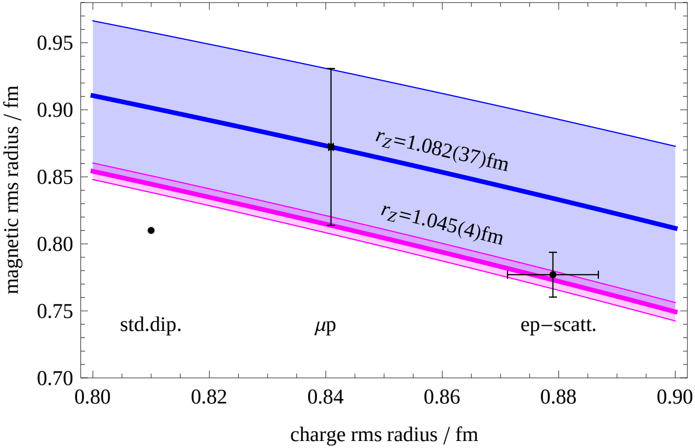


- Combining $r_Z = 1.082(37)\,\mathrm{fm}$ and $r_e = 0.84089(39)\,\mathrm{fm}$ Antognini et al. get $\mathbf{r_m = 0.87(6)\,fm}$

Proton structure from muonic hydrogen



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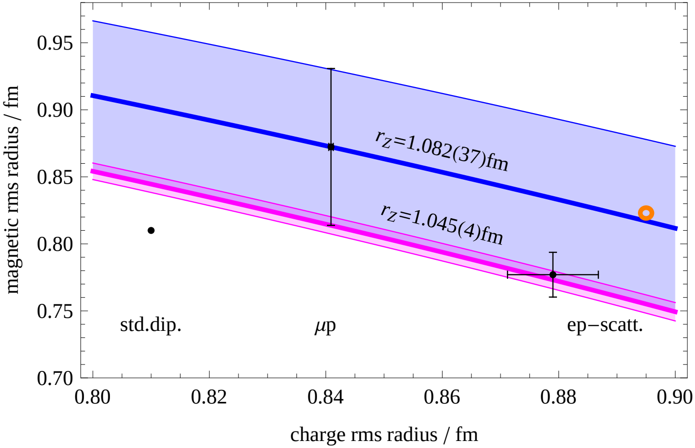

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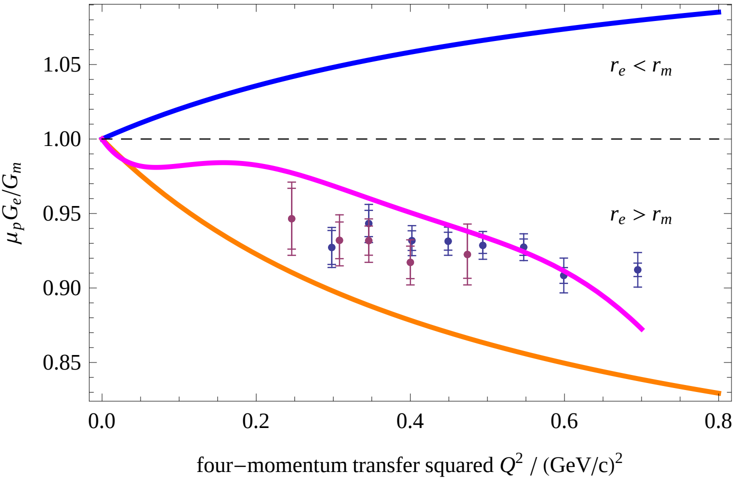
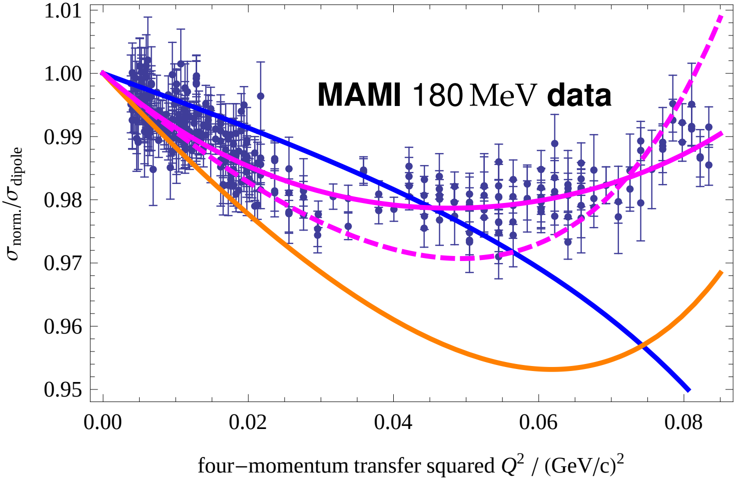


$r_e = 0.84\,\mathrm{fm},\ r_m = 0.87\,\mathrm{fm}$ Bernauer fit

$r_e = 0.90\,\mathrm{fm},\ r_m = 0.82\,\mathrm{fm}$

Proton structure from muonic hydrogen



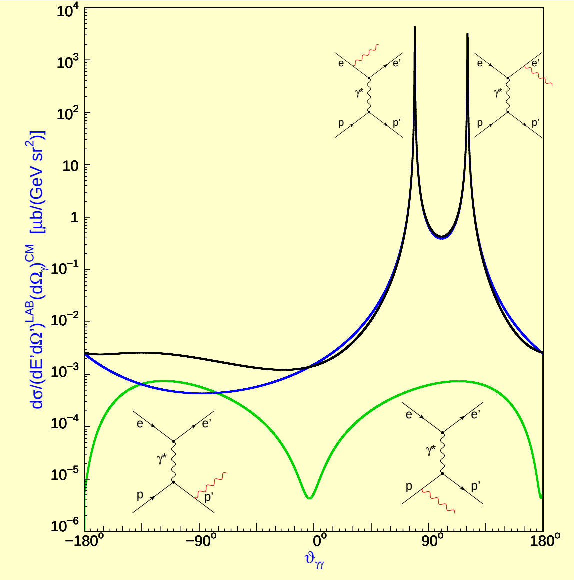
$r_e = 0.84\,\text{fm},\ r_m = 0.87\,\text{fm}$ Bernauer fit (solid) and

$r_e = 0.90\,\text{fm},\ r_m = 0.82\,\text{fm}$ $r_e = 0.88\,\text{fm},\ r_m = 0.78\,\text{fm}$

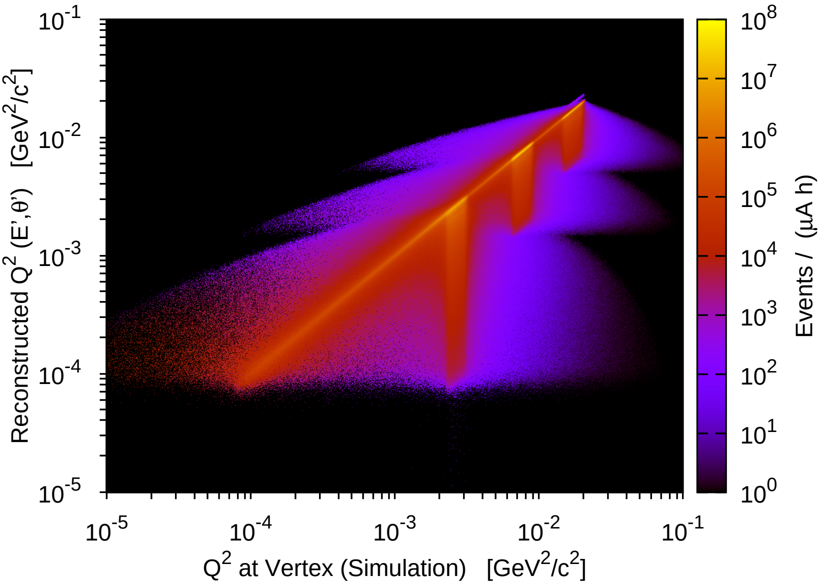
Summary

1. Form factors - Meaning
   - Pictures of the charge distribution
2. The Mainz measurements
   - High-precision p(e,e')p cross sections
3. Challenges
   - Radius puzzle: electronic vs. muonic determination
   - Two photon exchange
4. Outlook: ISR and Deuteron form factor

Outlook: Initial state radiation

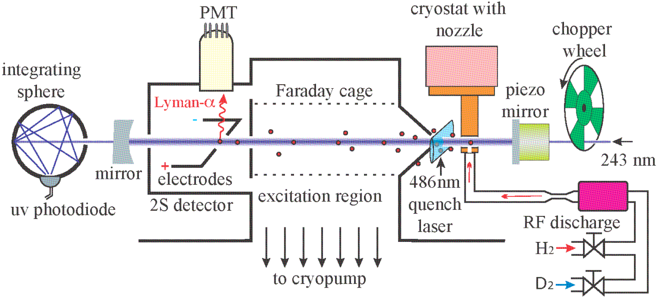


Outlook: Initial state radiation



Precision Measurement of the Hydrogen-Deuterium 1S–2S Isotope Shift

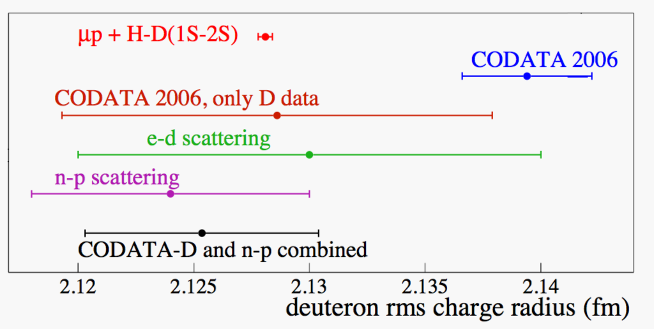
Parthey, Christian G. and Matveev, Arthur and Alnis, Janis and Pohl, Randolf and Udem, Thomas and Jentschura, Ulrich D. and Kolachevsky, Nikolai and Hänsch, Theodor W.: *Precision Measurement of the Hydrogen-Deuterium 1S–2S Isotope Shift*, Phys. Rev. Lett. **104**, 233001 (2010).



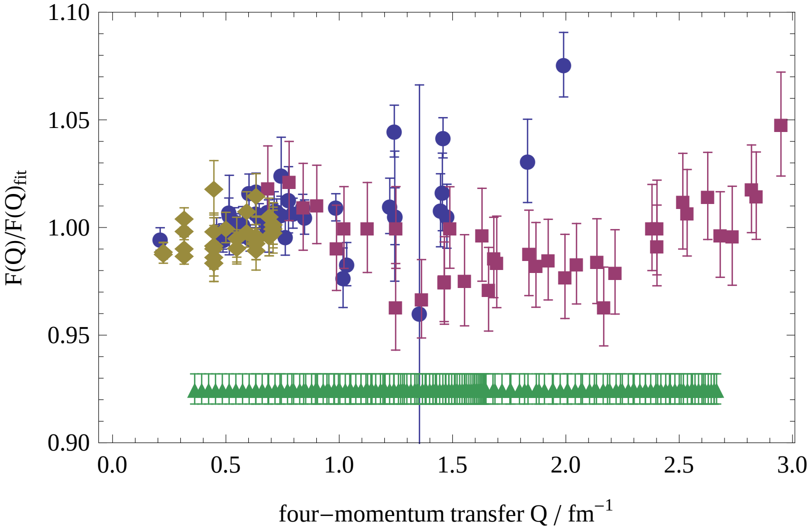
Beam apparatus for two-photon spectroscopy on the hydrogen / deuterium atomic beam.

Deuteron radius from the H-D isotope shift and muonic hydrogen

$$\left.\begin{aligned} r_d^2 - r_p^2 &= 3.82007(65)\,\mathrm{fm}^2 \\ r_p &= 0.84184(67)\,\mathrm{fm} \end{aligned}\right\} \Rightarrow r_d = 2.12808(27)\,\mathrm{fm}$$



World low Q^2 data and predicted errors



♦ Berard (1973), • Simon (1981), ■ Platchkov (1990), ▲ MAMI

New Experiments

- MAMI
  - 'A' rated proposal: Measurement of the elastic $A(Q^2)$ form factor of the deuteron at very low momentum transfer
  - Initial state radiation
- JLab
  - Very low Q^2 experiment, near 0° (A. Gasparian)
  - Form factor ratio at very low Q^2
- PSI
  - MUSE: elastic μp scattering
  - Lamb shift measurements on muonic helium

Backup

Speculation about the discrepancy

- Reminder: The muon g-2 experiment has a $2-3\sigma$ discrepancy. Hadronic corrections may provide an explanation.
- The main contribution to the **Lamb shift** in ...

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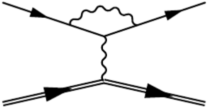
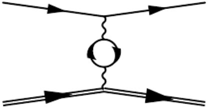




	'electronic' hydrogen	muonic hydrogen
vertex and self-energy	$1011.41\,\mathrm{MHz}$	
vacuum polarization	$-27.13\,\mathrm{MHz}$	$-205.028\,\mathrm{meV}$
anom. magn. moment	$67.82\,\mathrm{MHz}$	
+ higher order		
theoretical value	$1057.864(14)\,\mathrm{MHz}$	$-206.057\,\mathrm{meV}$
experimental value	$1057.862(20)\,\mathrm{MHz}$	$\Delta:0.341\,\mathrm{meV}$

Nuclear Finite Size Effects In Light Muonic Atoms

J. L. Friar, Annals Phys. **122** (1979) 151.

Finite-size shift in the energy of the nth s-state through order $(Z\alpha)^6$:

$$\begin{aligned}
\Delta E_n &= \frac{2\pi}{3} |\phi_n(0)|^2 Z\alpha \times \\
&\quad \left(\left\langle r^2 \right\rangle - \frac{Z\alpha\mu}{2} \left\langle r^3 \right\rangle_{(2)} + (Z\alpha)^2 F_{\mathrm{REL}} + (Z\alpha\mu)^2 F_{\mathrm{NR}} \right) \\
&= 5200 \, \frac{\mu\mathrm{eV}}{\mathrm{fm}^2} \left\langle r^2 \right\rangle - 9.1 \, \frac{\mu\mathrm{eV}}{\mathrm{fm}^3} \left\langle r^3 \right\rangle_{(2)} \\
&\quad + 0.28 \, \frac{\mu\mathrm{eV}}{\mathrm{fm}^2} F_{\mathrm{REL}} + 0.064 \, \frac{\mu\mathrm{eV}}{\mathrm{fm}^4} F_{\mathrm{NR}} \\
&\simeq 5200 \, \frac{\mu\mathrm{eV}}{\mathrm{fm}^2} \left\langle r^2 \right\rangle - 9.1 \, \frac{\mu\mathrm{eV}}{\mathrm{fm}^3} \left\langle r^3 \right\rangle_{(2)} \\
&\simeq 5200 \, \frac{\mu\mathrm{eV}}{\mathrm{fm}^2} r_p^2 - 35 \, \frac{\mu\mathrm{eV}}{\mathrm{fm}^3} r_p^3
\end{aligned}$$

with $\dfrac{\left\langle r^3 \right\rangle_{(2)}}{\left\langle r^2 \right\rangle^{3/2}} = \dfrac{35\sqrt{3}}{16}$ (valid for Dipole FF, only)

How small are F_{REL} and F_{NR}?

Dipole form factor

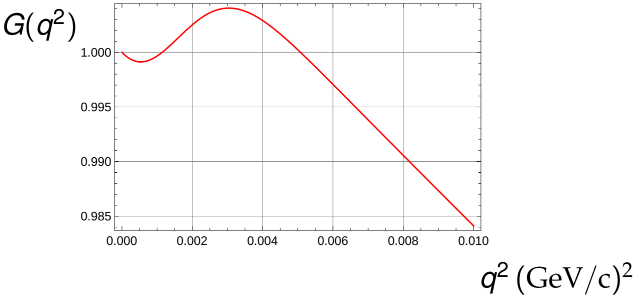
$$
\begin{aligned}
0.28\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^2} \times F_{\text{REL}} &= 0.28\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^2} \times 4.7\,\mathrm{fm}^2 = 1.3\,\mu\mathrm{eV} \\
0.064\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^4} \times F_{\text{NR}} &= 0.064\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^4} \times 0.54\,\mathrm{fm}^4 = 0.035\,\mu\mathrm{eV}
\end{aligned}
$$

Now we construct a FF that **"resolves"** the discrepancy:

$$G(q^2) = (1-x)\left(1+\frac{q^2}{m^2}\right)^{-2} + x\,\exp\left(-\frac{q^4}{2\sigma^2}\right)$$

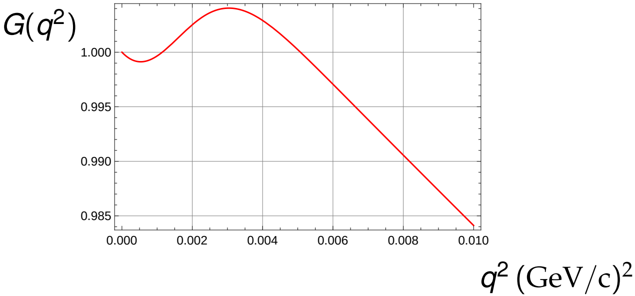
$$\text{with} \quad x = -1.7\%,\ m = 0.776,\ \sigma = 0.04^2$$

"Thorn" form factor



Nuclear Finite Size Effects In Light Muonic Atoms

"Thorn" form factor



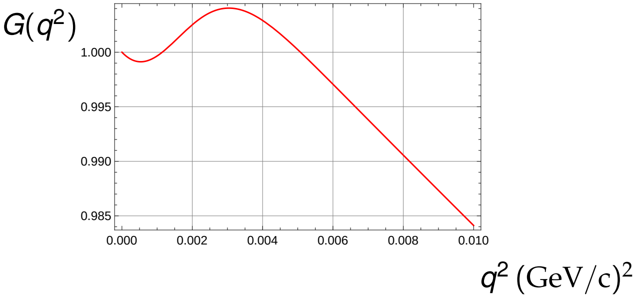
$$0.28\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^2} \times F_{\mathrm{REL}} = 0.28\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^2} \times -58\,\mathrm{fm}^2 = 16\,\mu\mathrm{eV}$$

$$0.064\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^4} \times F_{\mathrm{NR}} = 0.064\,\frac{\mu\mathrm{eV}}{\mathrm{fm}^4} \times 7100\,\mathrm{fm}^4 = 450\,\mu\mathrm{eV}$$

The "Dipole" approximation completely breaks down

Nuclear Finite Size Effects In Light Muonic Atoms

"Thorn" form factor



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The "Dipole" approximation completely breaks down

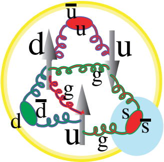
Similar conclusion: J. D. Carroll, A. W. Thomas, J. Rafelski and G. A. Miller, "Proton form-factor dependence of the finite-size correction to the Lamb shift in muonic hydrogen," arXiv:1108.2541

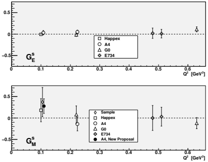
Implications

Parity violating electron scattering

The strangeness content of the nucleon

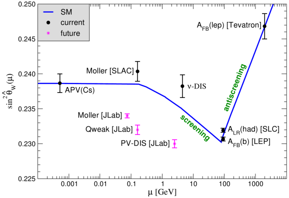
Strange form factors G_E^s, G_M^s,





Precision measurements of the weak mixing angle

The weak charge of the proton $Q^p_{weak} = 1 - 4\sin^2\theta_W$



Impact on PV Asymmetries

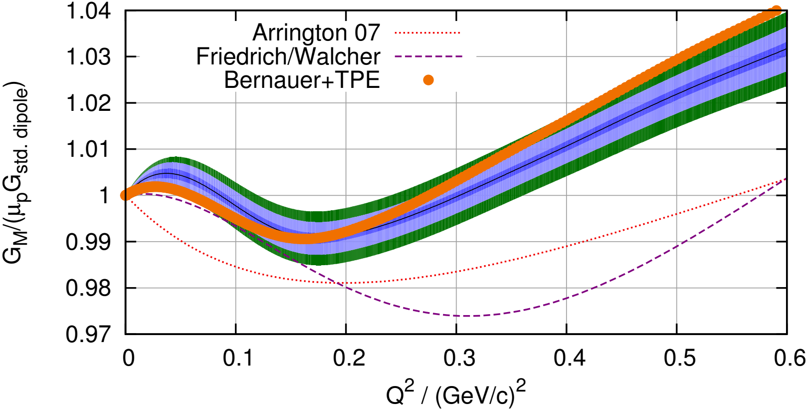
PV asymmetry

$$A_{LR}(\vec{e}p) = A_V + A_s + A_A$$

in elastic scattering cross section of right- and left-handed electrons.

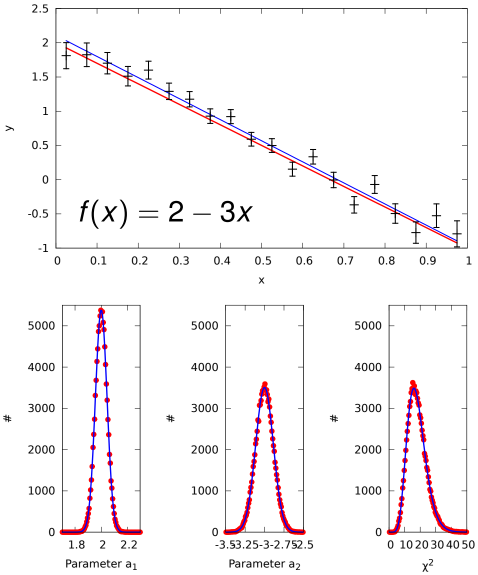
$$\begin{aligned}
A_V &= -a\rho'_{eq}\{(1 - 4\hat{\kappa}'_{eq}\hat{s}^2_Z) - \frac{\epsilon G^p_E G^n_E + \tau G^p_M G^n_M}{\epsilon(G^p_E)^2 + \tau(G^p_M)^2}\}, \\
A_s &= a\rho'_{eq}\frac{\epsilon G^p_E G^s_E + \tau G^p_M G^s_M}{\epsilon(G^p_E)^2 + \tau(G^p_M)^2}, \\
A_A &= a\frac{(1 - 4\hat{s}^2_Z)\sqrt{1-\epsilon^2}\sqrt{\tau(1+\tau)}G^p_M \tilde{G}^p_A}{\epsilon(G^p_E)^2 + \tau(G^p_M)^2}.
\end{aligned}$$

Impact on PV Asymmetries - magn. Form Factor



ChiSquare and the True Curve

χ^2 and the true curve



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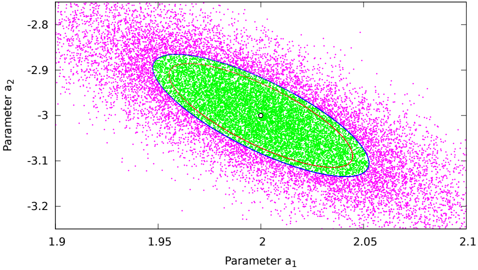

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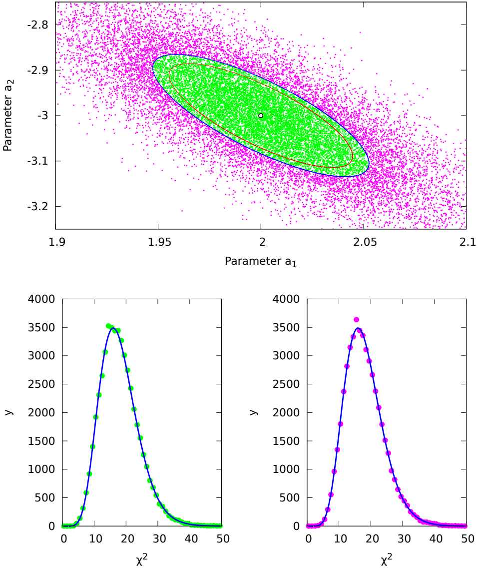
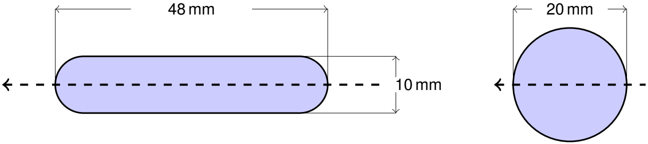


Background

Background



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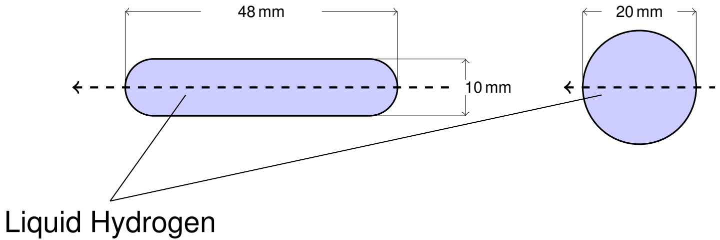

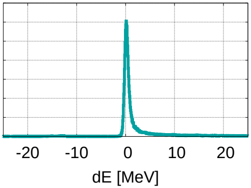

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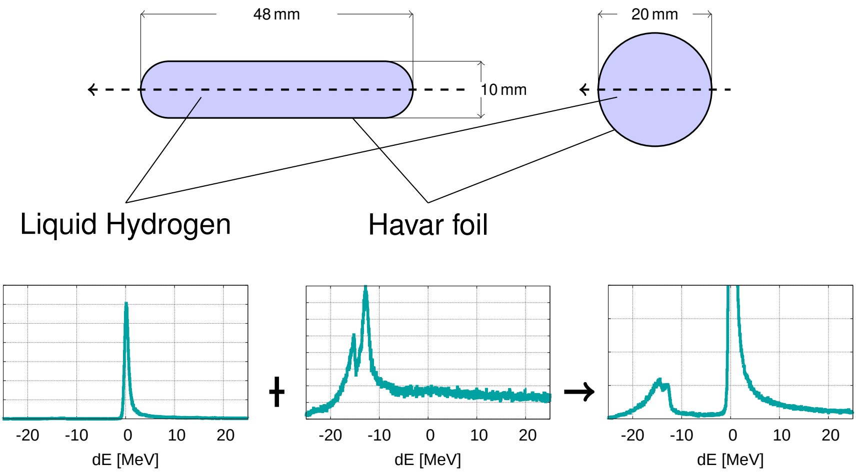
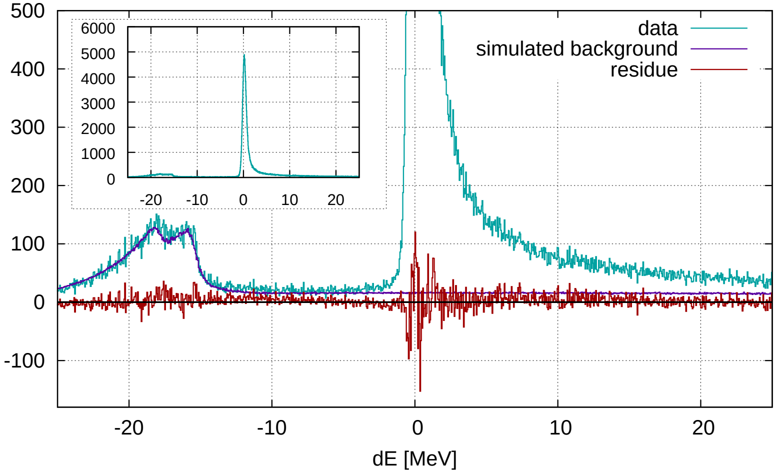


Data⟺Simulation matching

Simulation:

- Model for energy loss and small angle scattering
- Input: momentum-, angular-, vertex resolution



Models

Models: Dipols

Dipole (different b for G_E and G_M):

$$G_D\left(Q^2, b\right) = \frac{1}{\left(1 + \frac{Q^2}{b}\right)^2}$$

Double Dipole (as in Friedrich/Walcher phenomenological fit [Eur. Phys. J. A **17** (2003) 607]):

$$G_{DD}\left(Q^2, a, b_1, b_2\right) = aG_D\left(Q^2, b_1\right) + (1 - a)\, G_D\left(Q^2, b_2\right)$$

Models: Polynomial

Polynomial

$$G_P\left(Q^2, a_1, \ldots, a_n\right) = 1 + \sum_{i=1}^{n} a_i Q^{2 \cdot i}$$

Polynomial + standard Dipole

$$G_{PAD}\left(Q^2, a_1, \ldots, a_n\right) = G_D\left(Q^2, 0.71\right) + \sum_{i=1}^{n} a_i Q^{2 \cdot i}$$

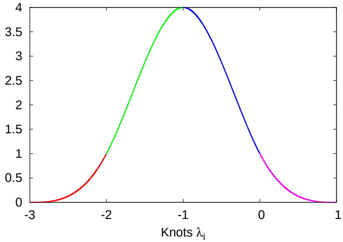
Polynomial × standard Dipole

$$G_{PMD}\left(Q^2, a_1, \ldots, a_n\right) = G_D\left(Q^2, 0.71\right) \cdot \left(1 + \sum_{i=1}^{n} a_i Q^{2 \cdot i}\right)$$

Models: Splines

Uniform cubic splines

$$spline\left(Q^2, a_1, \ldots, a_n\right)$$



Spline:

$$G_{Spline}\left(Q^2, a_1, \ldots, a_n\right) = 1 + Q^2 \cdot spline\left(Q^2\right)$$

Spline $\times$ standard Dipole

$$G_{SMD}\left(Q^2, a_1, \ldots, a_n\right) = G_D\left(Q^2, 0.71\right) \cdot \left(1 + Q^2 \cdot spline\left(Q^2\right)\right)$$

Models: Misc

Also:

- Friedrich / Walcher phenomenological ansatz
- extended Gari-Krümpelmann (VMD), Lomon et al.
- Arrington type:

$$\frac{P^N}{P^{N+2}}$$