Two-Photon Exchange in the Spacelike and Timelike Regions

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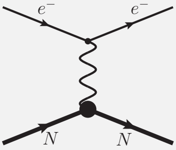
Electromagnetic Form Factors of the Nucleon

Spacelike Region ($q^2 < 0$)

Elastic eN-scattering:



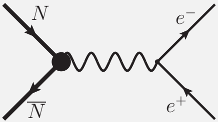
$$\langle N(p')|J^\mu_{em}|N(p)\rangle =$$

$$\bar{u}(p')\left[\boldsymbol{F_1(Q^2)}\gamma^\mu + \boldsymbol{F_2(Q^2)}\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Form factors are real functions of $Q^2 = -q^2$

Timelike Region ($q^2 > 0$)

$p\bar{p}$-Annihilation



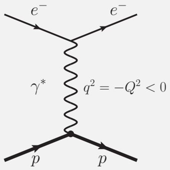
$$\langle 0|J^\mu_{em}|N(p)\,\overline{N}(p')\rangle =$$

$$\bar{v}(p')\left[\boldsymbol{F_1(q^2)}\gamma^\mu - \boldsymbol{F_2(q^2)}\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Form factors are complex functions of q^2

Form Factor Measurements: ep-Scattering

Rosenbluth Separation



Cross section in 1γ-approximation

$$d\sigma = d\sigma_{\text{Mott}} \frac{\tau}{\varepsilon(1+\tau)} \underbrace{\left[G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2) \right]}_{\text{reduced cross section } \sigma_R}$$

with $\tau = \frac{Q^2}{4m_N^2} > 0$ and $\varepsilon = \left(1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right)^{-1}$

Form Factor Measurements: ep-Scattering

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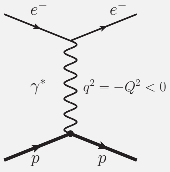

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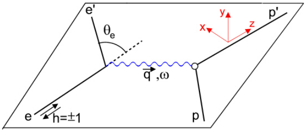
with $\tau = \dfrac{Q^2}{4m_N^2} > 0$ and $\varepsilon = \left(1 + 2(1+\tau)\tan^2\dfrac{\theta}{2}\right)^{-1}$

Polarization Tranfer:

Scatter polarized e^- beam & measure proton polarization:

$$\mu_p \frac{P_t}{P_l} = -\mu_p \sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}} \frac{G_E(Q^2)}{G_M(Q^2)}$$

$$\vec{e} + p \rightarrow e + \vec{p}$$



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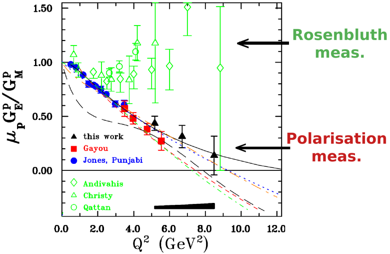
Form Factor Measurements: ep-Scattering

Results:

- The two methods give different results:

  Rosenbluth exp:
  $\frac{\mu_p G_E}{G_M} \approx 1$

  Polarization exp.:
  linear decrease of $\frac{G_E}{G_M}$ **with** Q^2

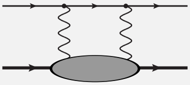


(Puckett et al., PRL (2010))

A possible explanation for the discrepancy:

Two-photon exchange corrections

(Guichon, Vanderhaeghen (2003), Blunden et al. (2003), ...)

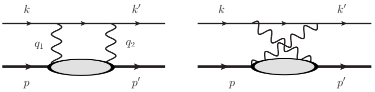


Two-Photon Exchange

- Influence of 2γ-exchange can be described by 3 amplitudes:

  $$Y_M(Q^2,\epsilon),\ \ Y_E(Q^2,\epsilon),\ \ Y_3(Q^2,\epsilon)$$

  $\Rightarrow$ have to be calculated within a model description or extracted from existing data



- 2γ-exchange corrections strongly influence Rosenbluth extraction

$$\frac{\sigma_R}{G_M^2} = 1 + \frac{\varepsilon}{\tau}\frac{G_E^2}{G_M^2} + 2\,Y_M + 2\varepsilon\frac{G_E}{\tau G_M}\,Y_E + 2\varepsilon\left(1+\frac{G_E}{\tau G_M}\right)Y_3 + \mathcal{O}(e^4)$$

- Small effect on results of polarization experiments

$$\frac{P_t}{P_l} \propto \frac{G_E}{G_M} + Y_E - \frac{G_E}{G_M}\,Y_M + \left(1 - \frac{2\varepsilon}{1+\varepsilon}\frac{G_E}{G_M}\right)Y_3 + \mathcal{O}(e^4)$$

Two-Photon Exchange

GEp2γ experiment (JLab/HallC)

P_t/P_l and P_l at Q^2 = 2.5 GeV^2 :

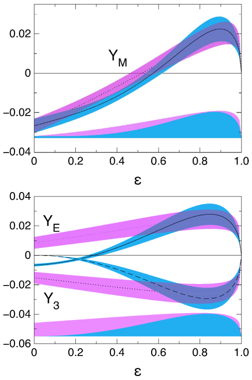
- no effect on P_t/P_l
- clear effect on P_l ($\sim$ 2 %)

Determination of 2γ-amplitudes:

- New data of polarization observables for Q^2 = 2.5 GeV^2
- Rosenbluth measurements of σ_R at Q^2 = 2.64 GeV^2

$\Rightarrow$ Extraction of Y_M, Y_E, Y_3

Empirical extraction of 2γ-amplitudes Q^2=2.5 GeV^2:



(JG, Meziane, Kivel, Vanderhaeghen, 2011)

$\Rightarrow$ 2γ-amplitudes are in **2-3% range**

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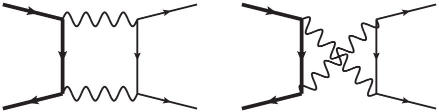
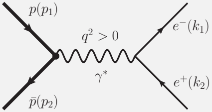
Two-Photon Exchange in the Timelike Region



Timelike Form Factors

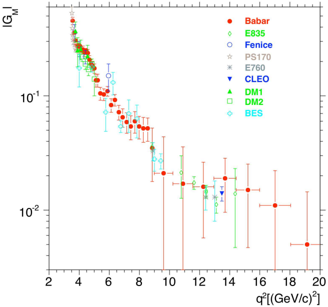
$p\bar{p} \to e^+e^-$: Cross Section



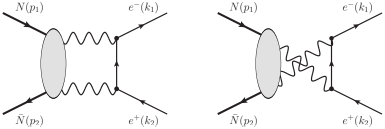
Cross section in 1γ approximation:

$$d\sigma_{1\gamma} \propto \left[\left|G_M\right|^2 (1 + \cos^2\theta) + \frac{1}{\tau} \left|G_E\right|^2 \sin^2\theta \right]$$

- $G_M(q^2), G_E(q^2)$: complex functions of q^2 in the timelike region
- Experiments planned at PANDA@FAIR and BES-III



Two-Photon Exchange in Timelike Processes?



Matrix element parametrized by

3 generalized form factors $\tilde{G}_M$, $\tilde{G}_E$, $\tilde{F}_3$:

$$\mathcal{M} = \frac{e^2}{q^2}\,\bar{u}(k_1)\,\gamma_\mu\, v(k_2)$$

$$\times\, \bar{N}(p_2)\left[\widetilde{G}_M\gamma^\mu - \widetilde{F}_2\frac{P^\mu}{m_N} + \widetilde{F}_3\frac{\not{K}P^\mu}{m_N^2}\right]N(p_1)$$

- complex functions of 2 variables, e.g. q^2 and t

2γ-amplitudes:

$$\widetilde{G}_M(q^2,t) = G_M(q^2) + \delta\widetilde{G}_M(q^2,t)$$

$$\widetilde{F}_2(q^2,t) = F_2(q^2) + \delta\widetilde{F}_2(q^2,t)$$

$$\widetilde{F}_3(q^2,t) = 0 + \delta\widetilde{F}_3(q^2,t)$$

$$\Uparrow \qquad\qquad \Uparrow$$

proton FF **2γ amplitude**

Two-Photon Exchange in Timelike Processes?

Cross section including 2γ-exchange corrections:

$$\begin{aligned} d\sigma = \mathcal{C}(q^2)\Big[& |G_M|^2(1+\cos^2\vartheta) + \frac{1}{\tau}|G_E|^2\sin^2\vartheta \\ & + 2\mathrm{Re}[G_M\delta\widetilde{G_M}](1+\cos^2\vartheta) + 2\frac{1}{\tau}\mathrm{Re}[G_E\delta\widetilde{G_E}]\sin^2\vartheta \\ & + 2\left(\mathrm{Re}[G_M\widetilde{F_3}^*] - \frac{1}{\tau}\mathrm{Re}[G_E\widetilde{F_3}^*]\right)\sqrt{\tau(\tau-1)}\cos\vartheta\sin^2\vartheta\Big] \end{aligned}$$

Two-Photon Exchange in Timelike Processes?

Cross section including 2γ-exchange corrections:

$$
\begin{aligned}
d\sigma = \mathcal{C}(q^2)\Big[& |G_M|^2(1+\cos^2\vartheta) + \frac{1}{\tau}|G_E|^2\sin^2\vartheta \\
& + 2\mathrm{Re}[G_M\delta\widetilde{G_M}](1+\cos^2\vartheta) + 2\frac{1}{\tau}\mathrm{Re}[G_E\delta\widetilde{G_E}]\sin^2\vartheta \\
& + 2\left(\mathrm{Re}[G_M\widetilde{F_3}^*] - \frac{1}{\tau}\mathrm{Re}[G_E\widetilde{F_3}^*]\right)\sqrt{\tau(\tau-1)}\cos\vartheta\sin^2\vartheta\Big]
\end{aligned}
$$

Symmetry Relations:

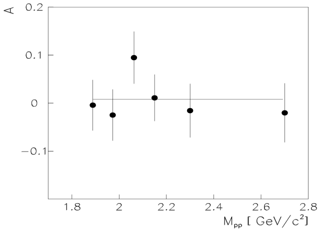
$$\delta\widetilde{G}_{E,M}(q^2,-\cos\theta) = -\delta\widetilde{G}_{E,M}(q^2,\cos\theta)$$

$$\delta\widetilde{F}_3(q^2,-\cos\theta) = \delta\widetilde{F}_3(q^2,\cos\theta)$$

$\Longrightarrow$ **Asymmetry in angular distribution of dσ**

$$A_{\cos\theta} = \frac{d\sigma(\theta) - d\sigma(\pi-\theta)}{d\sigma(\theta) + d\sigma(\pi-\theta)}$$

(Gakh, Tomasi-Gustafsson NPA 2006)



$\Rightarrow \quad A_{\cos\theta} = 0.01 \pm 0.02$

(Tomasi-Gustafsson et al., PLB 2008)

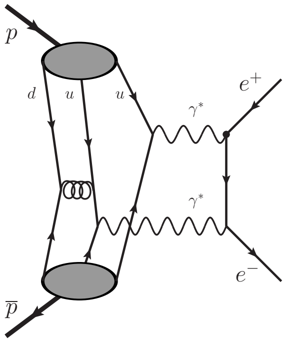
Leading pQCD Analysis of 2γ-Exchange

2γ-exchange at large q^2
$\Rightarrow$ consider factorization approach:

2γ-exchange correction as convolution of:

- Hard amplitude (calculable in pQCD)
- nonperturbative contribution: Nucleon **D**istribution **A**mplitudes φ_N

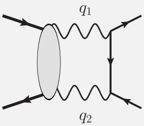
$$\varphi_N(x_i') * T_H(q^2, \varepsilon) * \varphi_N(x_i)$$



Leading pQCD Analysis of 2γ-Exchange

both photons have large virtualities:

$$q_1^2 \sim q_2^2 \sim q^2$$



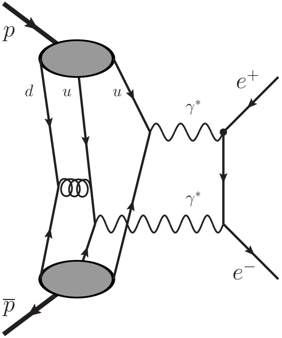
2γ-exchange amplitudes:

helicity conserving amplitudes:

$$\delta \widetilde{G}_M \sim \frac{s}{m^2}\widetilde{F}_3 \sim \frac{\alpha_{em}\alpha_s}{q^4}$$

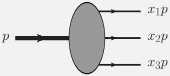
helicity flip amplitude:

$$\delta \widetilde{F}_2 \sim \frac{1}{q^6} \quad \text{(suppressed)}$$



Nucleon Distribution Amplitudes

Distribution Amplitude φ_N:



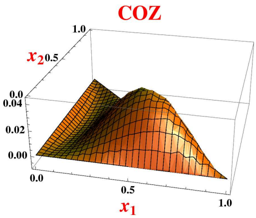
describes how the longitudinal momentum is shared between the constituents

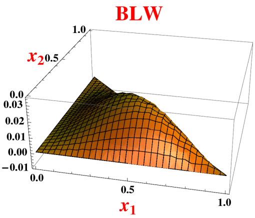
Model for asymptotic behavior of the DAs and the first corrections:

$$\varphi_N(x_i) \simeq 120\, \boldsymbol{f_N}\, x_1 x_2 x_3\, (1 + \boldsymbol{r_-}(x_1 - x_2) + \boldsymbol{r_+}(1 - 3x_3) + \ldots)$$

DAs include 3 parameters: f_N, r_-, r_+

	f_N (10^{-3} GeV^2)	r_-	r_+
COZ[1]	5.0	4.0	1.1
BLW[2]	5.0	1.37	0.35

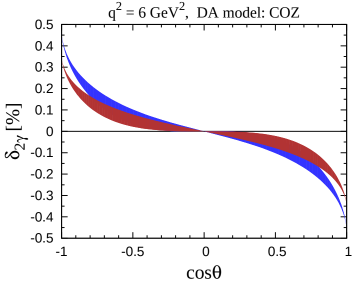


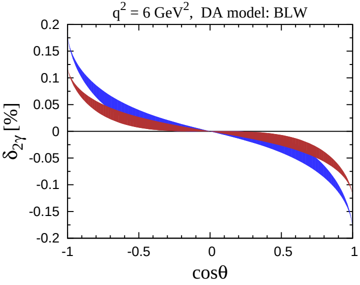


[1] Chernyak et al., Z.Phys C (1989)

[2] Braun et al., PRD (2006)

Results





(JG, Kivel, Vanderhaeghen, PRD(2011))

Two-Photon Contribution $\boldsymbol{\delta_{2\gamma}}$: $d\sigma_{1\gamma+2\gamma} = d\sigma_{1\gamma}\left(1 + \boldsymbol{\delta_{2\gamma}}\right)$

Nucleon form factors:

- QCD inspired fit: (Brodsky et al.(2004))

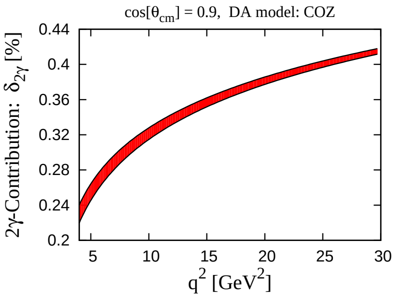
  $|G_M(q^2)| = {}^{C}/_{q^4} \log^2\left(\frac{-q^2}{\Lambda^2}\right)$

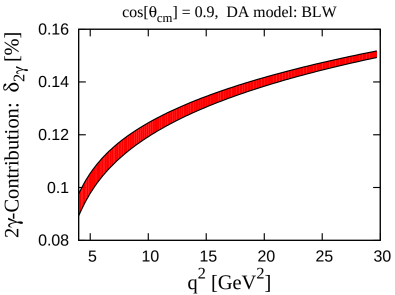
- VMD model

  (Iachello et al. (2004))

- $\delta\widetilde{G}_E = \lambda\,\delta\widetilde{G}_M$, with $-1 < \lambda < 1$ (bands)

Results





(JG, Kivel, Vanderhaeghen, PRD(2011))

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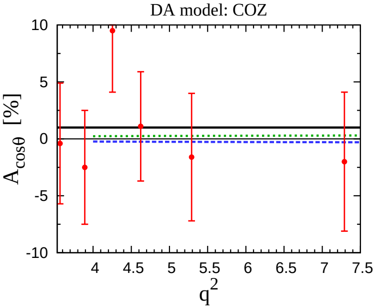
Results



Asymmetry $\quad A_{\cos\theta} = \dfrac{d\sigma(\cos\theta) - d\sigma(-\cos\theta)}{d\sigma(\cos\theta) + d\sigma(-\cos\theta)}$

black curve: $A_{\cos\theta} = 1\%$ (Tomasi-Gustafsson et al., PLB (2008)))

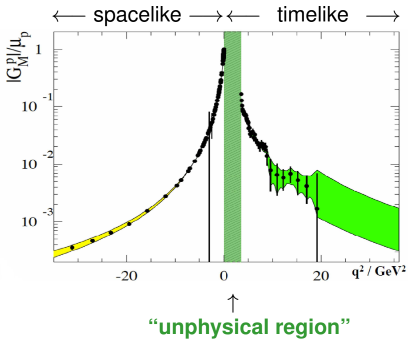
blue curve: $A_{\cos\theta}$ using pQCD approach for $\cos\theta = 0.9$

green curve: $A_{\cos\theta}$ using pQCD approach for $\cos\theta = -0.9$

Nucleon Form Factors in the Unphysical Region

Form Factors in the Unphysical Region

- Unphysical region:
  $0 < q^2 < 4m_N^2$
- Not accessible by process
  $p + \bar{p} \rightarrow e^+ + e^-$
- Idea: Consider process
  $$\boldsymbol{p + \bar{p} \rightarrow \pi^0 + \gamma^*}$$
  $$\boldsymbol{\rightarrow \pi^0 + e^+ + e^-}$$
  (Dubničková et al., ZPhys. (1996), Adamuščín et al., PRC (2007))
- Form factor extraction requires model



Comparison with $p\bar{p} \to \pi^0\gamma$ Data

Regge trajectory exchange model

- kinematic region: $s \gg |t|, |u|$
- consider exchange of dominant **Regge trajectories α**: Nucleon & Δ trajectory
- adding up orbital excitations

Feynman pole $\to$ Regge pole

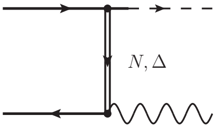
$$\frac{1}{t - m_N^2} \to \frac{s^{\alpha_N - \frac{1}{2}}}{\Gamma[\alpha_N + \frac{1}{2}]} \pi\alpha'_N \frac{e^{-i\pi(\alpha_N + \frac{1}{2})}}{\sin\pi(\alpha_N + \frac{1}{2})}$$

$\boldsymbol{\alpha_N(t)}$: Regge trajectory

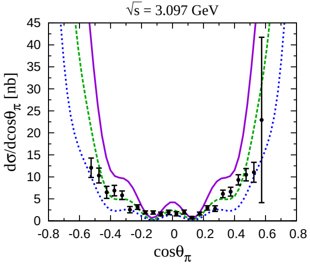
Data $p\bar{p} \to \pi^0\gamma$

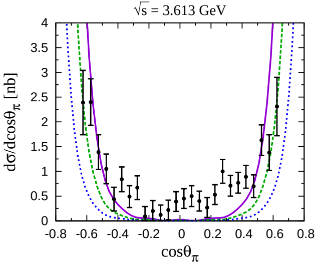
E760 at Fermilab:

$\cos\theta$-dependence of cross section for $8.5 \leq s \leq 14$ GeV^2



Comparison with $p\bar{p} \to \pi^0\gamma$ Data





Regge description:

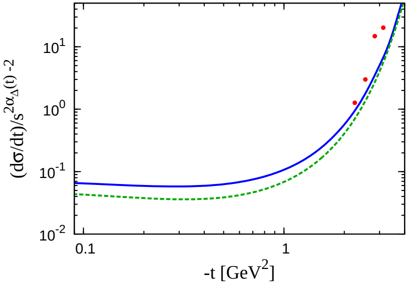
blue curve: N trajectory exchange

purple curve: $(N + \Delta)$ trajectory exchange

green curve: $(N + \Delta)$ trajectory exchange including reduction of Δ pole:

$$\mathcal{M} = \mathcal{M}^N + \mathcal{F} \cdot \mathcal{M}^\Delta, \qquad \mathcal{F} \approx 0.5$$

Comparison with $p\bar{p} \to \pi^0\gamma$ Data



$d\sigma/dt$ divided by s-dependence of leading Regge trajectory:

$\sqrt{s}$ = 3.686 GeV

$\sqrt{s}$ = 10 GeV

Regge description: $\frac{d\sigma}{dt} \propto F(t)s^{2\alpha(t)-2}$

$p\bar{p} \to \pi^0 e^+ e^-$: Model Independent Calculation

Separation of the cross section

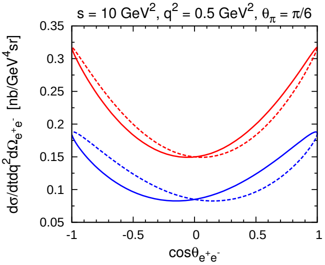
$$\frac{d\sigma}{dt\, dq^2\, d\Omega_{e^+e^-}} = \frac{1}{16\pi^2 s(s-4m_N^2)} \frac{e^2}{(4\pi)^2\, 2\, q^2} \frac{4\pi}{3} \cdot \mathcal{W}(\theta_{e^+e^-}, \Phi_{e^+e^-})$$

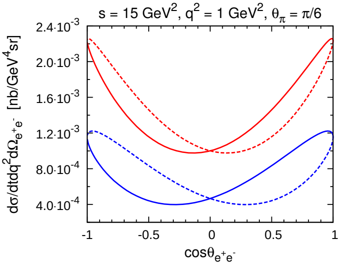
$$\begin{aligned} \mathcal{W}(\theta_{e^+e^-}, \Phi_{e^+e^-}) = \frac{3}{4\pi} \Big[& \sin^2\theta_{e^+e^-}\, \rho_{00} + (1+\cos^2\theta_{e^+e^-})\rho_{11} \\ & + \sqrt{2}\sin 2\theta_{e^+e^-} \cos\Phi_{e^+e^-}\, \mathrm{Re}[\rho_{10}] \\ & + \sin^2\theta_{e^+e^-} \cos 2\Phi_{e^+e^-}\, \mathrm{Re}[\rho_{1-1}] \Big] \end{aligned}$$

Density matrix: $\rho_{\lambda\lambda'} = \left(\mathcal{M}^\mu \varepsilon^*_\mu(q,\lambda)\right) \cdot \left(\mathcal{M}^\mu \varepsilon^*_\mu(q,\lambda')\right)^*$

$p\bar{p} \to \pi^0 e^+ e^-$: Results in the Regge framework

Cross Section: $\frac{d\sigma}{(dtdq^2 d\Omega_{e^+e^-})}$ $\left[\frac{nb}{GeV^4 sr}\right]$





N-trajectory exchange: $\Phi_{e^+e^-} = 0$ (solid) and $\Phi_{e^+e^-} = \pi$ (dashed)

(N+Δ)-trajectory exchange: $\Phi_{e^+e^-} = 0$ (solid) and $\Phi_{e^+e^-} = \pi$ (dashed)

Summary

2γ-exchange in the **spacelike region**:

- Candidate to explain form factor discrepancy
- Combined analysis of data: 2γ-amplitudes $\sim$ 2-3 %

2γ-exchange in the **timelike region**:

- Estimate of 2γ-exchange for the reaction $p\bar{p} \to e^+e^-$ using a **pQCD factorization** approach
- Contribution $\delta_{2\gamma} \lesssim$ **1%**

Proton form factor in the **unphysical region**:

- Accessible in the process $p\bar{p} \to \pi^0 e^+ e^-$
- **Regge description** in good agreement with data of $p\bar{p} \to \pi^0\gamma$