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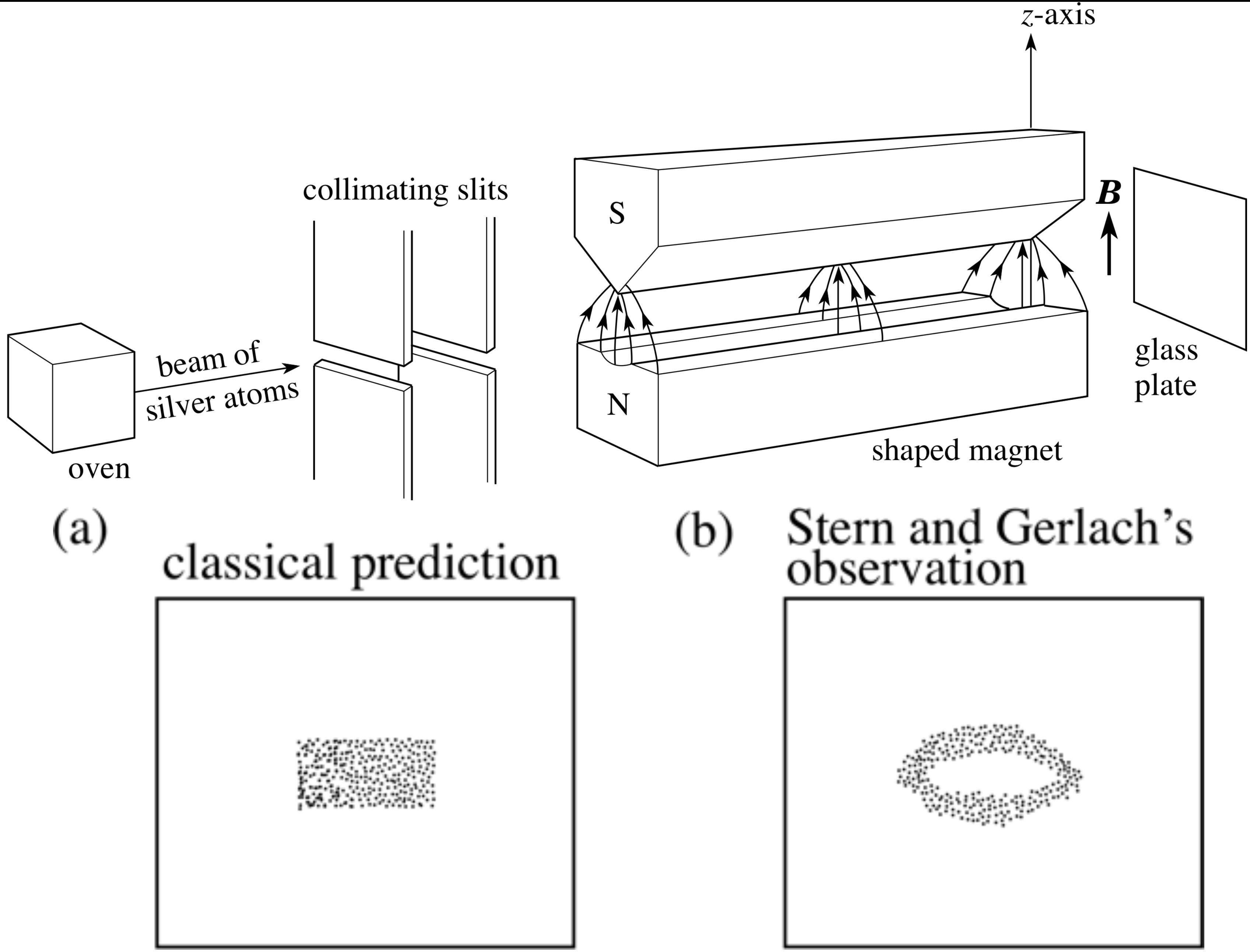
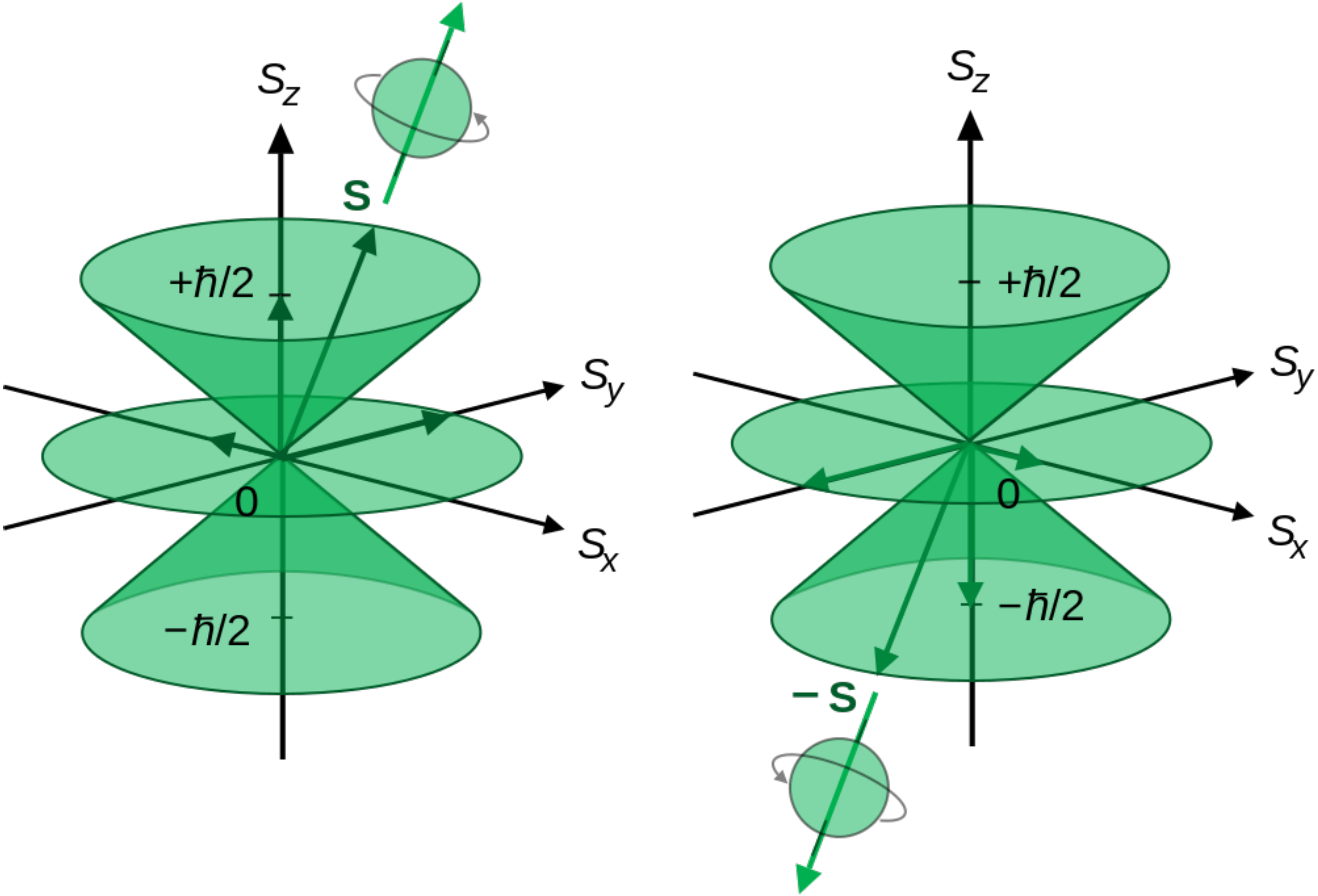
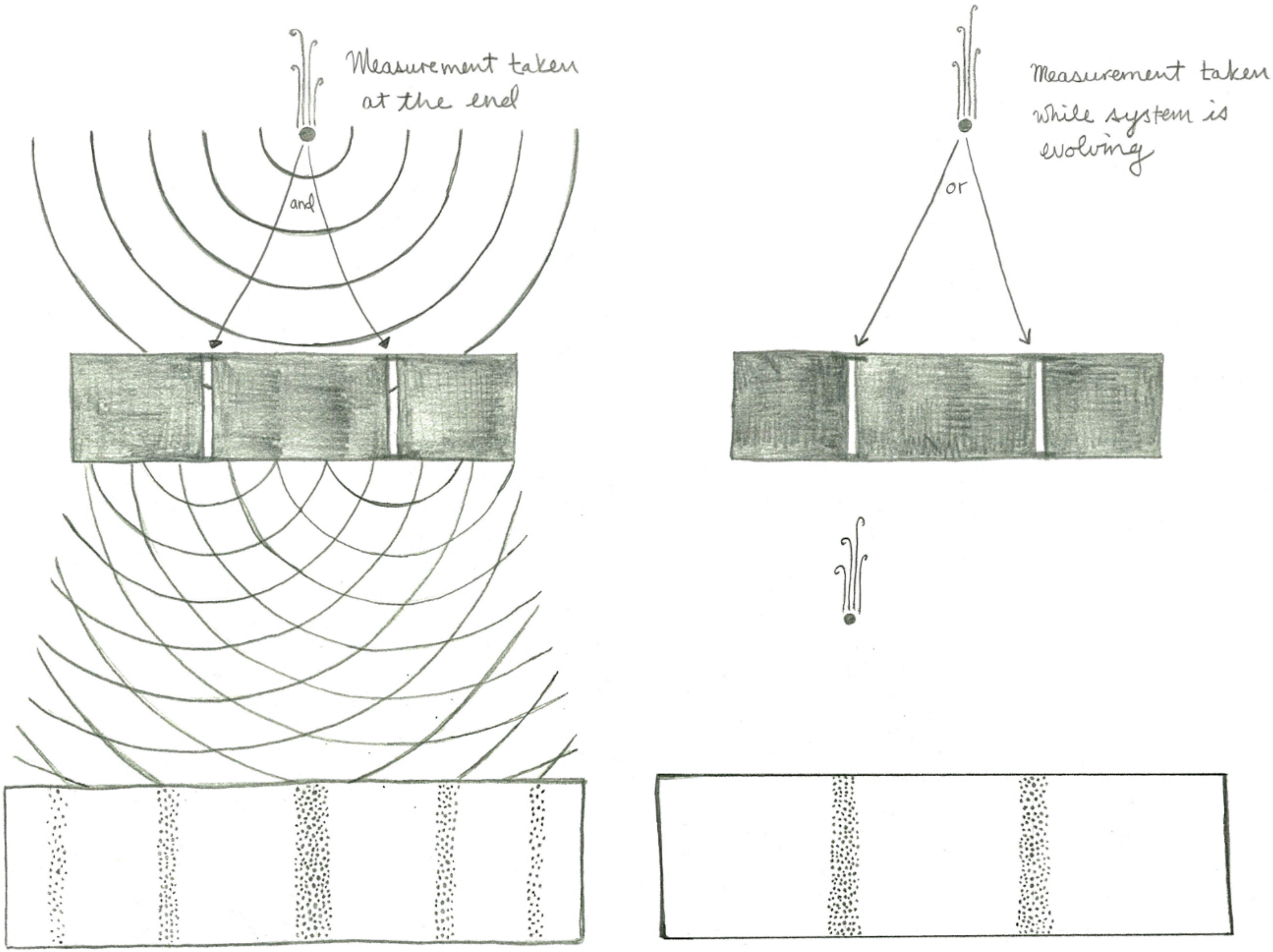
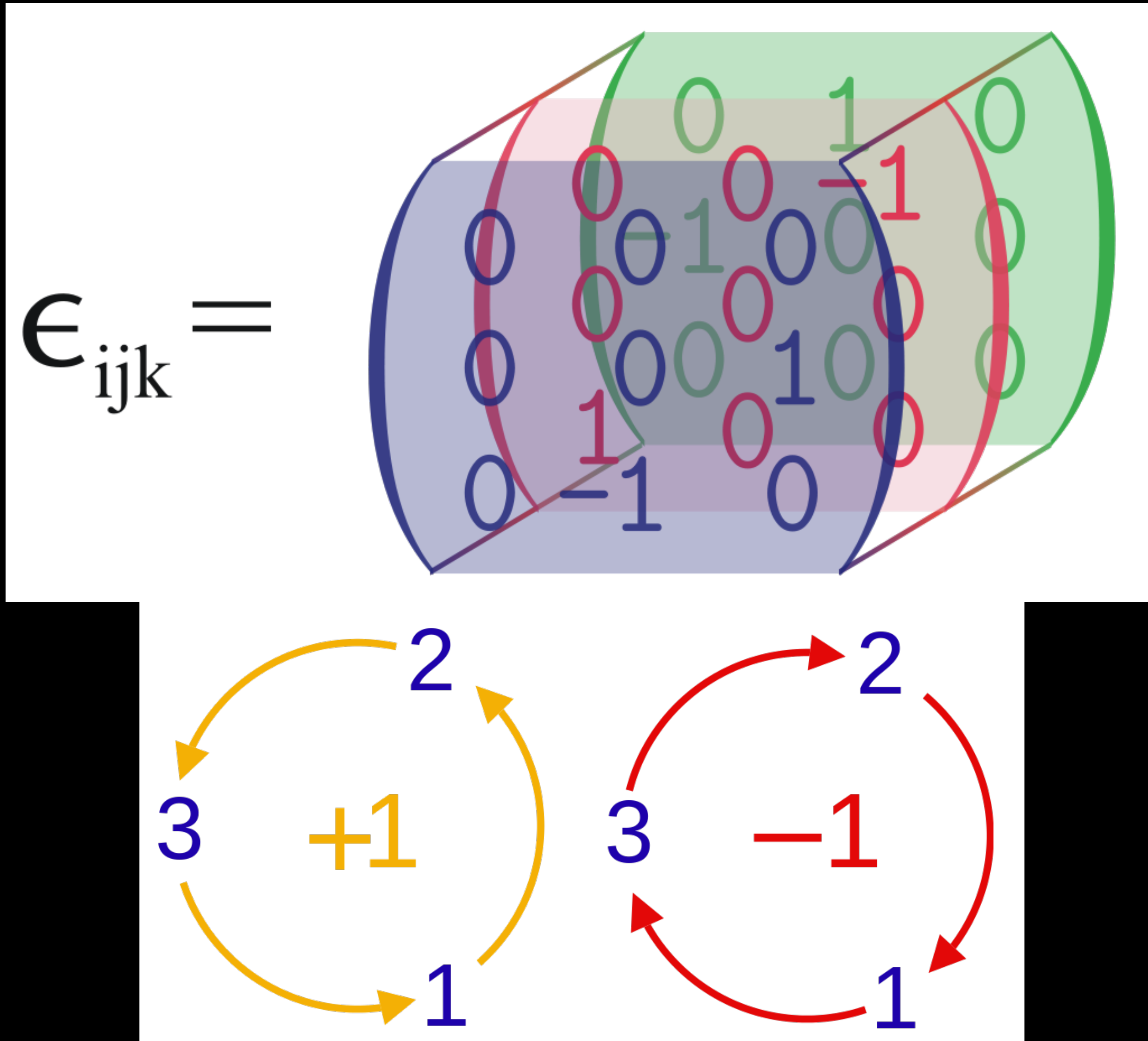

z-axis
collimating slits
S
B
beam of
silver atoms
glass
plate
N
oven
shaped magnet
(a)
(b)
classical prediction
Stern and Gerlach's
observation

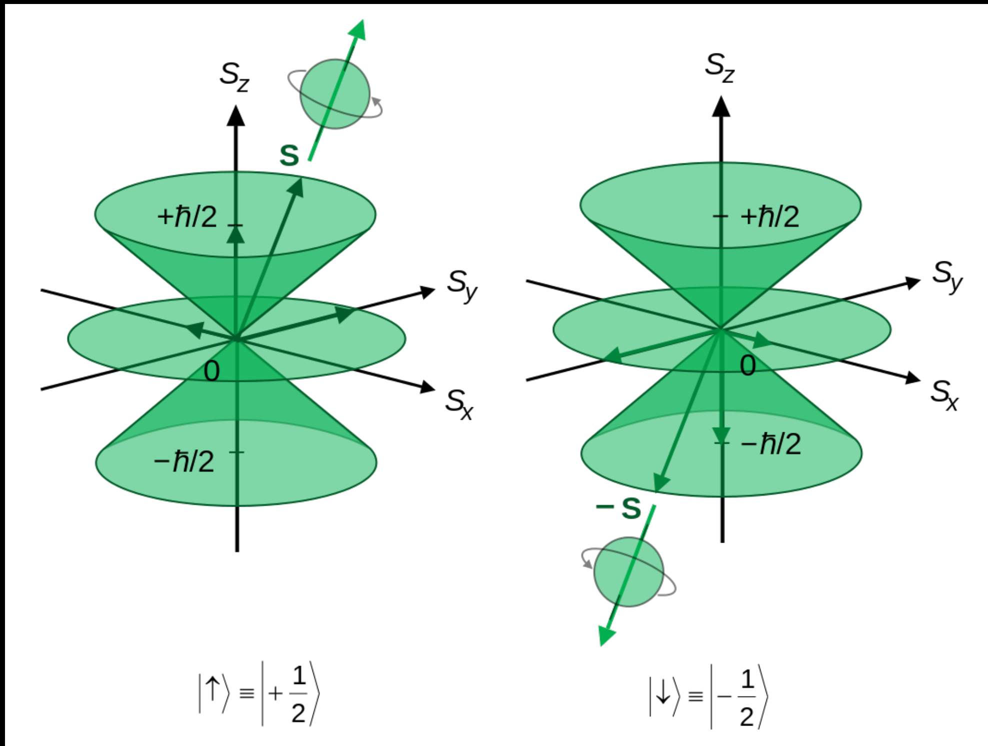


$$\left|\uparrow\right\rangle \equiv \left|+\frac{1}{2}\right\rangle \qquad \left|\downarrow\right\rangle \equiv \left|-\frac{1}{2}\right\rangle$$

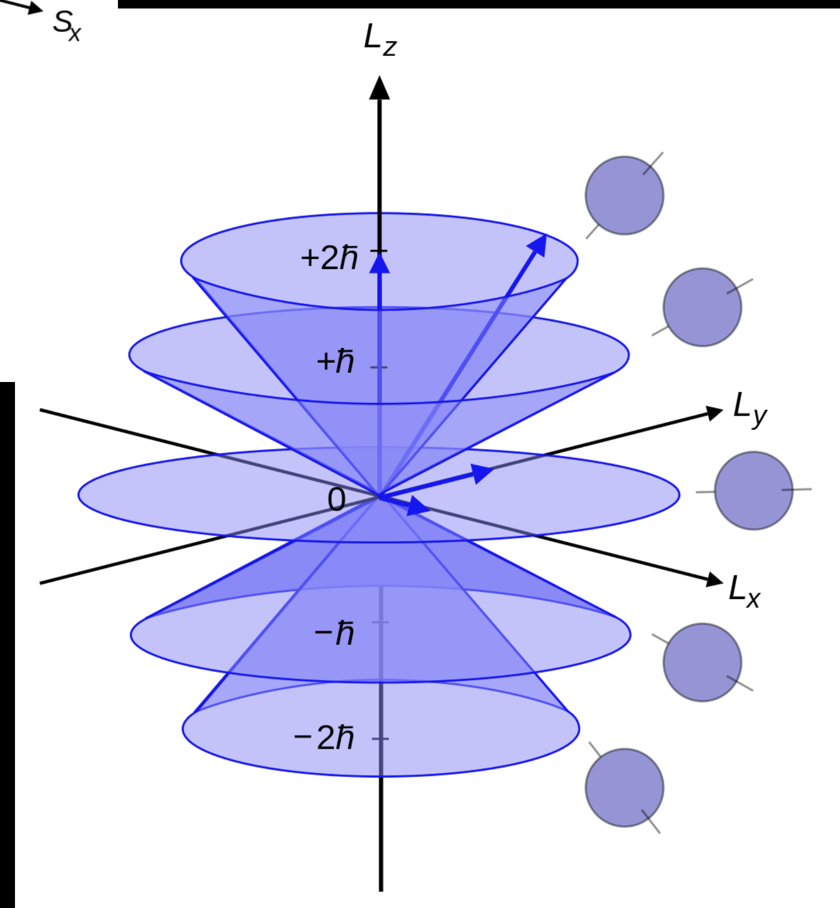

Measurement taken at the end
and
Measurement taken while system is evolving
or



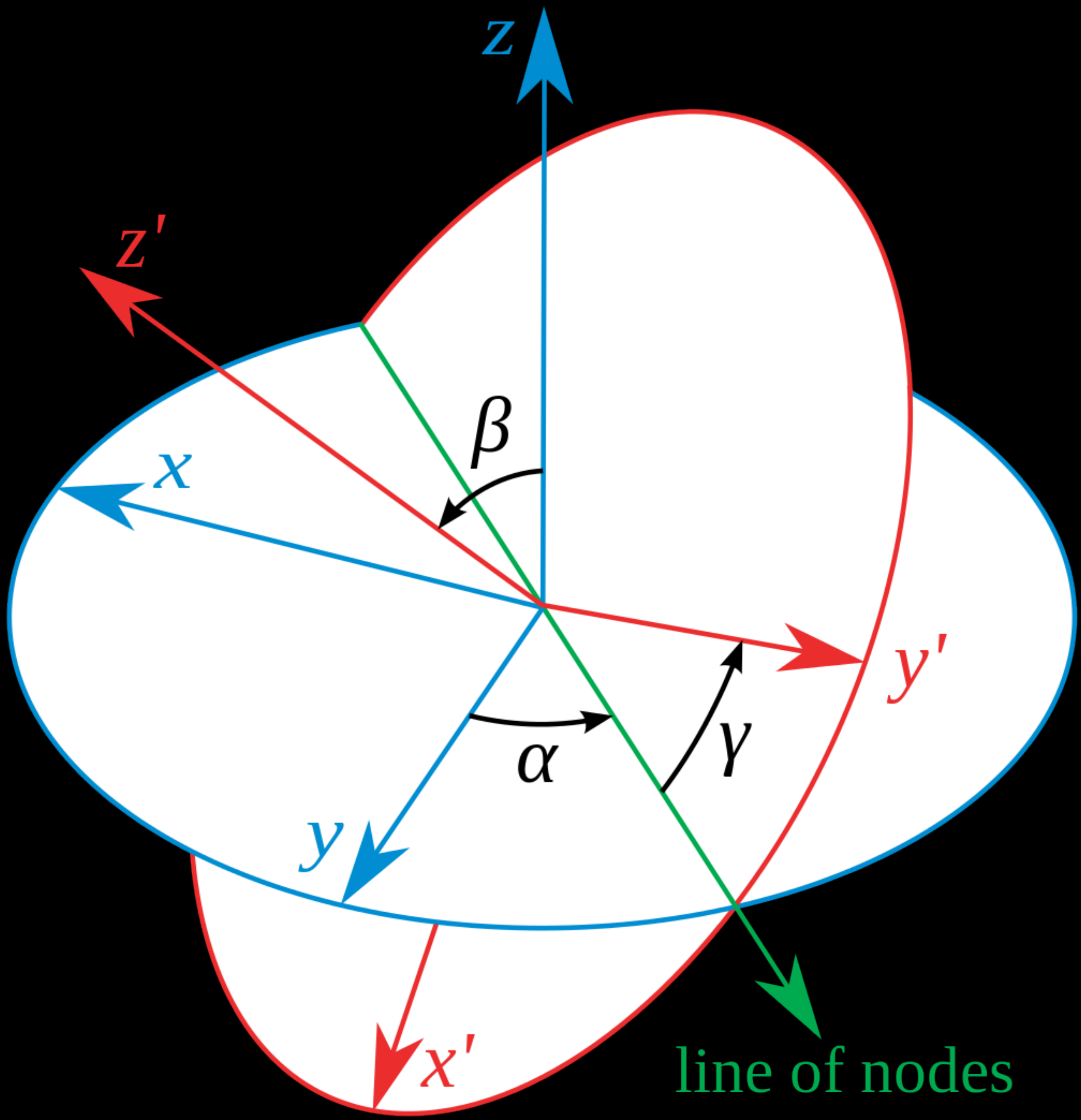
ϵijk =
2
3
+1
1
2
3
−1
1



Spin 2



Spin 1/2



z
z'
β
x
y'
α
γ
y
x'
line of nodes

Wigner d-function

$$d^j_{m'm}(\beta) = [(j+m')!(j-m')!(j+m)!(j-m)!]^{1/2} \sum_s \left[\frac{(-1)^{m'-m+s}}{(j+m-s)!s!(m'-m+s)!(j-m'-s)!} \cdot \left(\cos\frac{\beta}{2}\right)^{2j+m-m'-2s} \left(\sin\frac{\beta}{2}\right)^{m'-m+2s} \right].$$

The sum over s is over such values that the factorials are nonnegative.

The d-matrix elements are related to Jacobi polynomials $P_k^{(a,b)}(\cos\beta)$ with nonnegative a and b. Let

$$k = \min(j+m,\, j-m,\, j+m',\, j-m').$$

$$\text{If} \quad k = \begin{cases} j+m: & a = m'-m; \quad \lambda = m'-m \\ j-m: & a = m-m'; \quad \lambda = 0 \\ j+m': & a = m-m'; \quad \lambda = 0 \\ j-m': & a = m'-m; \quad \lambda = m'-m \end{cases}$$

Then, with $b = 2j - 2k - a$, the relation is

$$d^j_{m'm}(\beta) = (-1)^\lambda \binom{2j-k}{k+a}^{1/2} \binom{k+b}{b}^{-1/2} \left(\sin\frac{\beta}{2}\right)^a \left(\cos\frac{\beta}{2}\right)^b P_k^{(a,b)}(\cos\beta),$$

where $a, b \geq 0$.