Introduction

Lectures at the "School on Concepts of Modern Amplitude Analysis Techniques"

Klaus Peters
GSI Darmstadt and GU Frankfurt
Flecken-Zechlin, September 2013

Books

Dynamics

J.M. Blatt & V.F. Weisskopf – *Theoretical Nuclear Physics*

J.R. Taylor – *Scattering Theory*

M.L. Goldberger & K.M. Watson – *Collision Theory*

J. Gillespie – *Final State Interactions*

H. Burkhardt – *Dispersion Relation Dynamics*

Kinematics and more

M. Nikolic – *Kinematics and Multiparticle Systems*

E. Byckling & K. Kajantie – *Particle Kinematics*

Spin

M.E. Rose – *Elementary Theory of Angular Momentum*

Overview

D.V. Bugg (Edtitor) – Nato School on *Hadron Spectroscopy and the Confinement Problem*

Reviews and Articles

General

R.S. Longacre – *Techniques in Meson Spectroscopy,* BNL 49445

K. Peters – *A Primer on Partial Wave Analysis,* Int.J.Mod.Phys. A21 (2006) 5618-5624

Spin

S.U. Chung – *Spin Formalisms,* CERN Yellow Report 71-8

V. Filippini et al. – *Covariant Spin Tensors in Meson Spectroscopy,* PRD 51(1995) 2247

Dynamics

S.U. Chung et al. – *Partial wave analysis in K matrix formalism,* Annalen Phys. 4 (1995) 404-430

F.v. Hippel, C. Quigg – *Centrifugal-Barrier Effect in Resonance Partial Decay Widths, Shapes, and Production Amplitudes* , PRD 5 (1972) 624

I.J.R Aitchison – *K-matrix Formalism For Overlapping Resonances*, Nucl.Phys. A189 (1972) 417-423

Introduction

- Mission
- Concepts
- Procedures
- Use Cases

What is the mission ?

Particle physics at small distances is quite well understood

One Boson Exchange, Heavy Quark Limits

This is not true at large distances

Hadronization, Light mesons

are barely understood compared to their abundance

Understanding interaction/dynamics of light hadrons will

improve our knowledge about non-perturbative QCD

parameterizations will give provide
toolkit to analyze heavy quark processes

thus an important tool also for precise standard model tests

We need

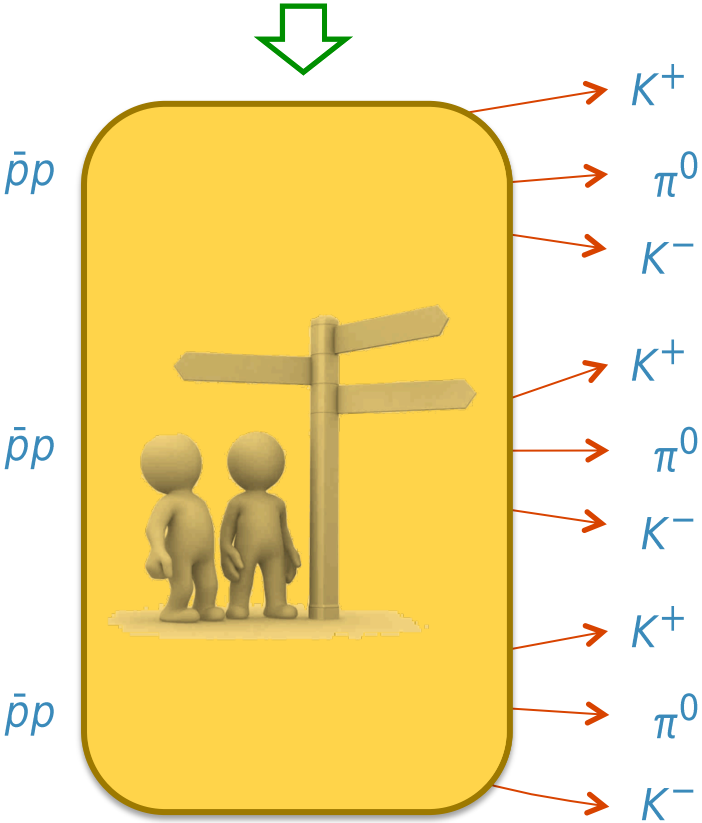
Appropriate parameterizations for the multi-particle phase space

A translation from the parameterizations to effective degrees of freedom for a deeper understanding of QCD

The Need for Partial Wave Analysis

Example: Consider the reaction $\bar{p}p \to K^+K^-\pi^0$

What *really* happened...



... etc.

What you may see is always the same ...

PWA = technique to find out what happens in between

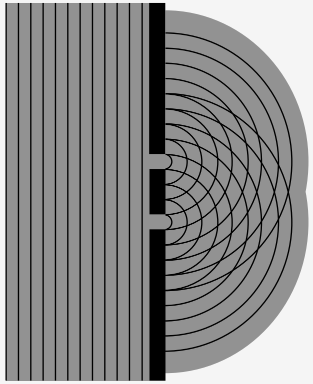
Solve the interference problem

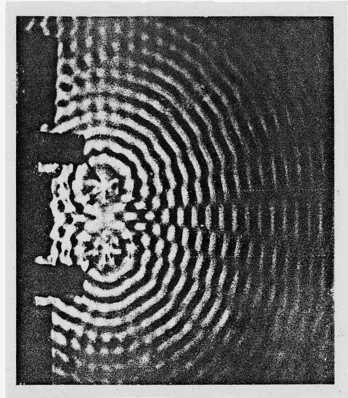
PWA

The phase space diagram in hadron physics shows a pattern due to interference and spin effects

This is the unbiased measurement

What has to be determined ?





Analogy Optics ⇔ PWA

lamps ⇔ # level

slits ⇔ # resonances

positions of slits ⇔ masses

sizes of slits ⇔ widths

→ only if spins are properly assigned

bias due to hypothetical spin-parity assumption

Optics

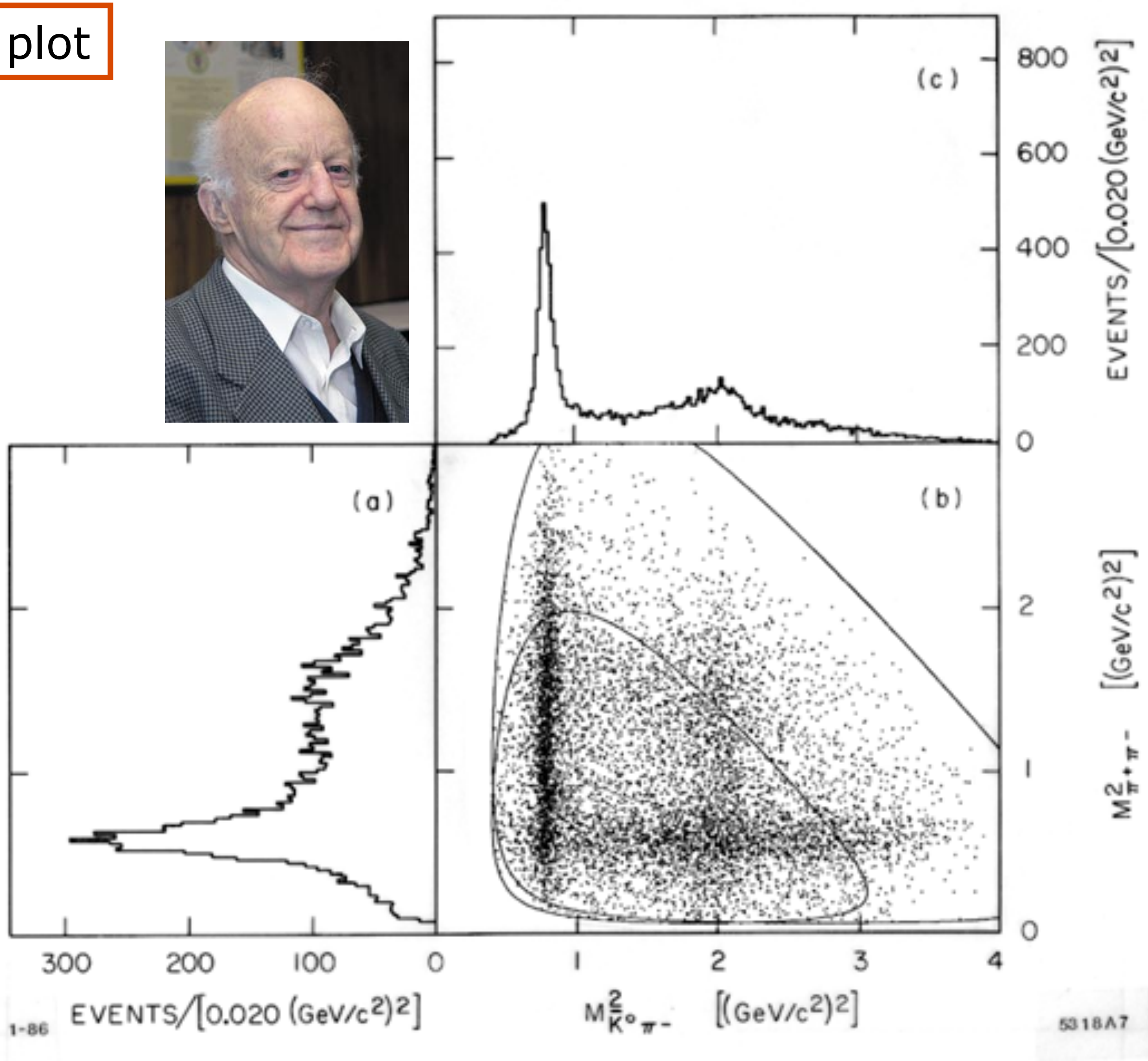
$$I(x) = \left| A_1(x) + A_2(x) e^{i\varphi} \right|^2$$

Dalitz plot

$$I(m) = \left| A_1(m) + A_2(m) e^{i\varphi} \right|^2$$

n-Particle Phase space, *n*=3

Dalitz plot



Intermediate State Mixing

Many states may contribute to a final state

- not only ones with well defined (already measured) properties
- not only expected ones

Many mixing parameters are poorly known

- *K*-phases
- SU(3) phases

In addition

- also *D/S* mixing (b_1, a_1 decays)

Isoscalar Mixing:

strong Int.: I^G und J^{PC} identical

η-η' or f_2- f_2' and/or Glueballs

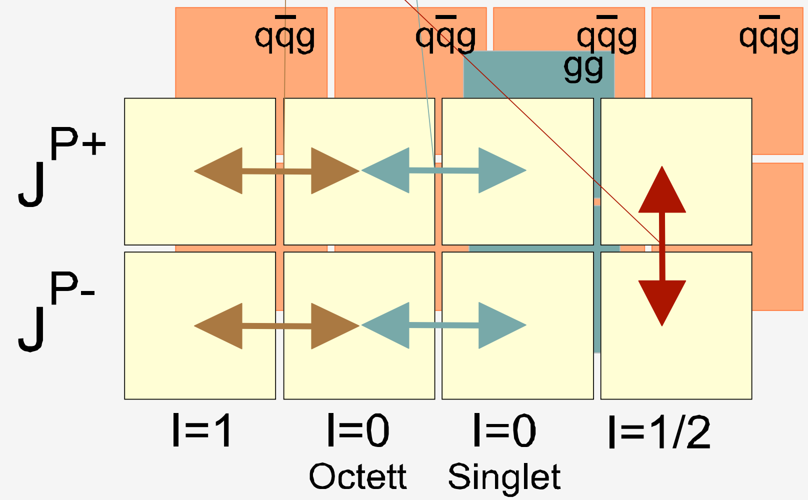
I=0/I=1-Mixing:

elm. Int.: $\Delta I=1$: ρ-ω

Kaonmixing:

strong Int.: C undef., I^G and J^P identical

K_{1A}-K_{1B}



Goal

For whatever you need the parameterization of the *n*-Particle phase space

- It contains the static properties of the unstable (resonant) particles within the decay chain like
  - mass
  - width
  - spin and parities
- as well as properties of the initial state
- and some constraints from the experimental setup/measurement

The main problem is, you don‘t need just a good description, you need the right one

- Many solutions may look alike, but only one is right

But…

the mission is way more general,

…there are many more questions, which can only be answered with a correct phase space description

whenever states mix and need to be unambiguously disentangled

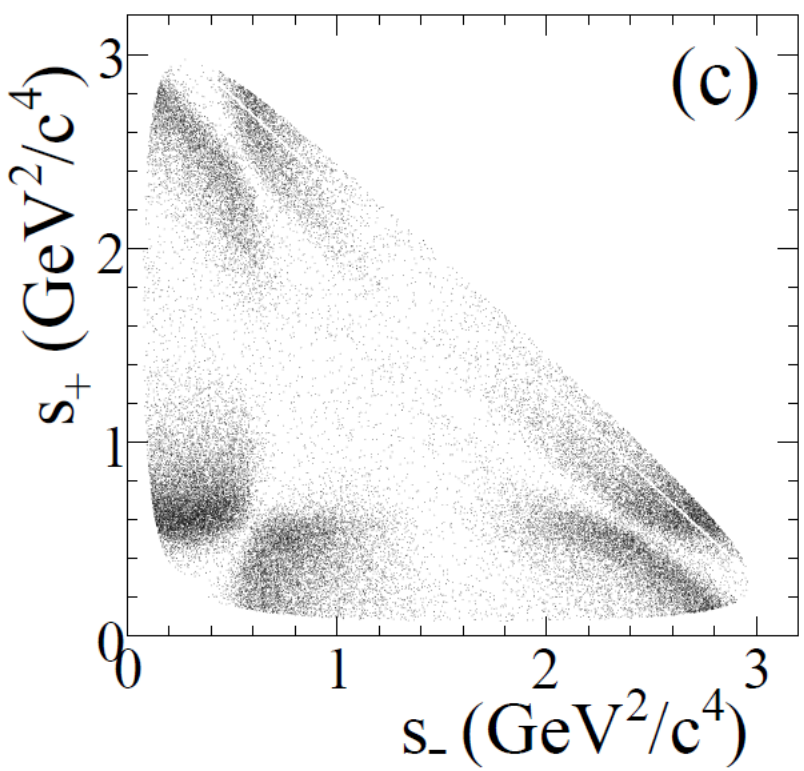
the focus then moves away
from masses and line shapes to yields and phases

example: $D^0\bar{D}^0$-Mixing and CPV

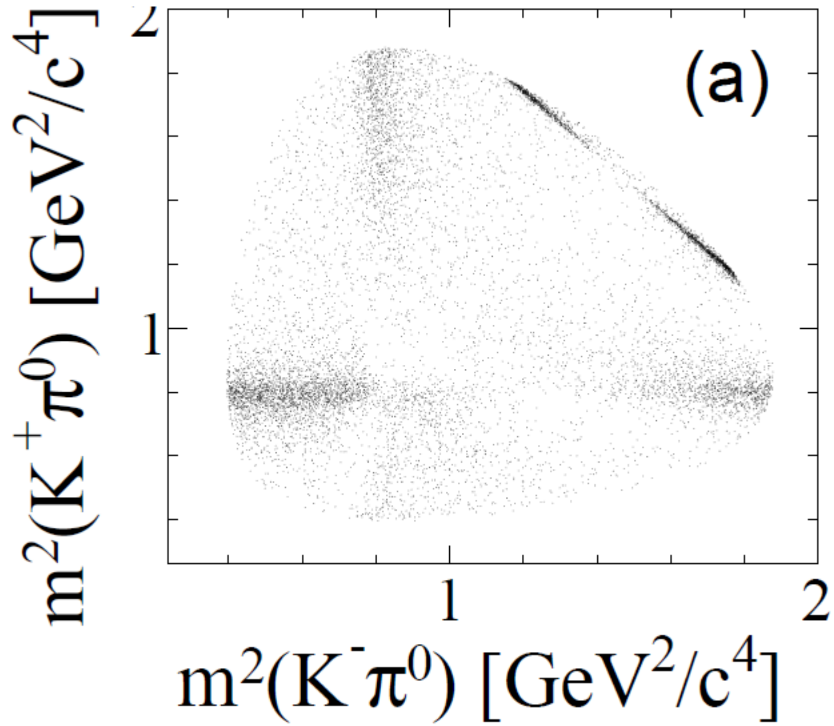
Data from BaBar

$D^0 \rightarrow h^+h^-\pi^0$, $h=K,\pi$

$D^0 \rightarrow \pi^-\pi^+\pi^0$



$D^0 \rightarrow K^-K^+\pi^0$

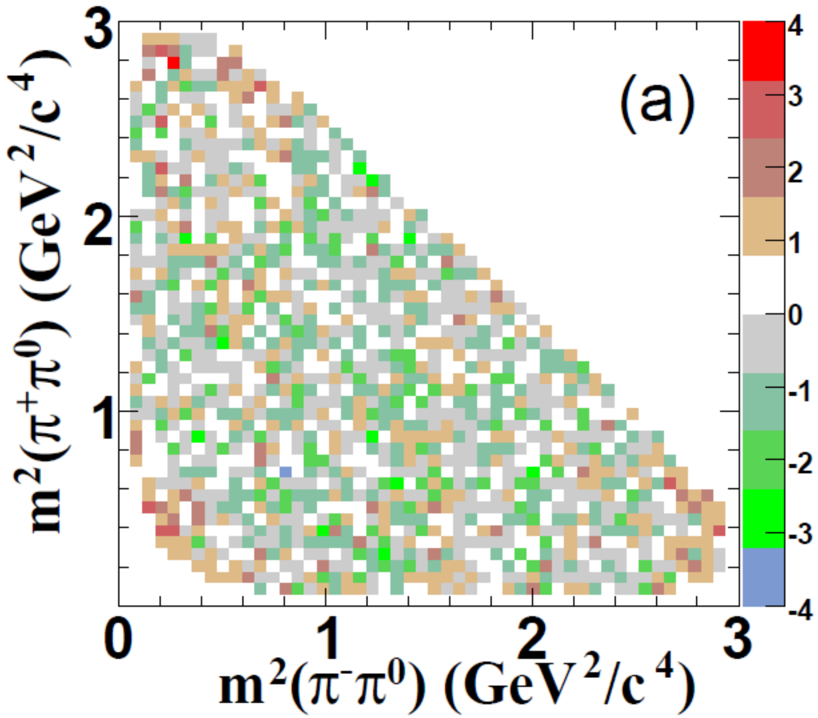


search for asymmetry in production cross section
or in branching fractions

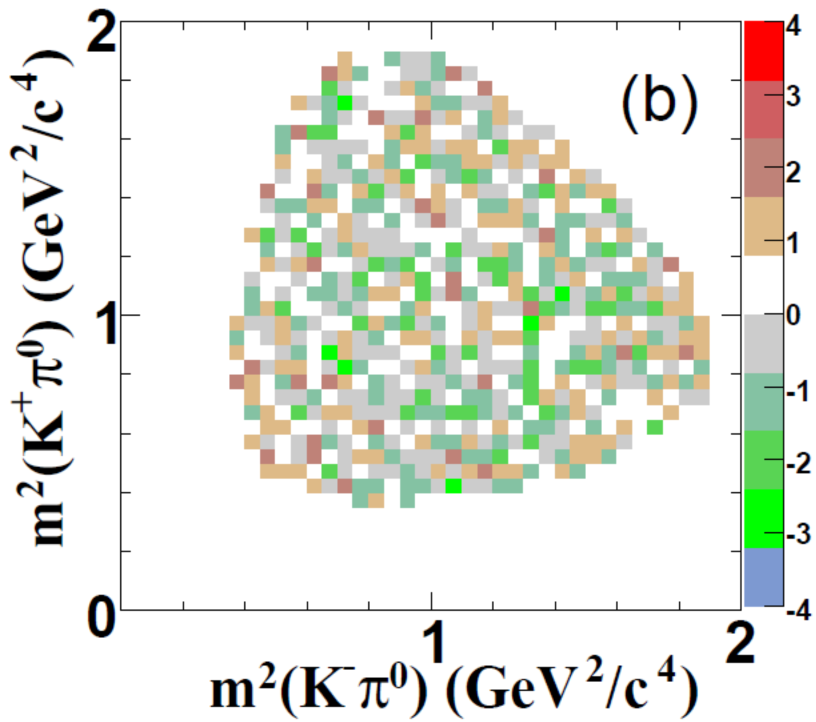
example: $D^0\bar{D}^0$-Mixing and CPV

Data from BaBar

$D^0 \to \pi^-\pi^+\pi^0$



$D^0 \to K^-K^+\pi^0$



χ^2-distribution shows: no observed CP-violation

not enough statistics to verify SM prediction

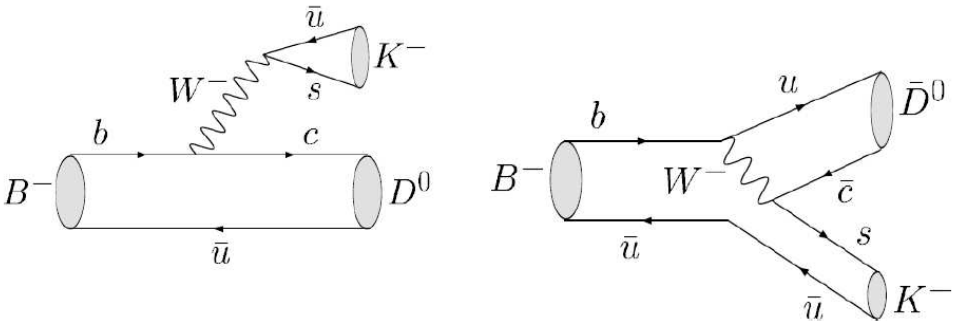
example: CKM Angle γ in $B^- \to D^0K^-$ (+c.c.)

Direct *CP* violation in interference between $b \to c\bar{c}s$, $u\bar{u}s$

$B^- \to \tilde{D}^{(*)0}K^{(*)-}$

Interference, if

$D^0 \to f \leftarrow \bar{D}^0$



$$\frac{\mathcal{A}(B^- \to \bar{D}^0K^-)}{\mathcal{A}(B^- \to D^0K^-)} = r_B e^{i(\delta_B - \gamma)}, \quad \frac{\mathcal{A}(B^+ \to D^0K^+)}{\mathcal{A}(B^+ \to \bar{D}^0K^+)} = r_B e^{i(\delta_B + \gamma)}$$

r_B Ratio of magnitudes of amplitudes, small

δ_B *CP* invariant strong phase

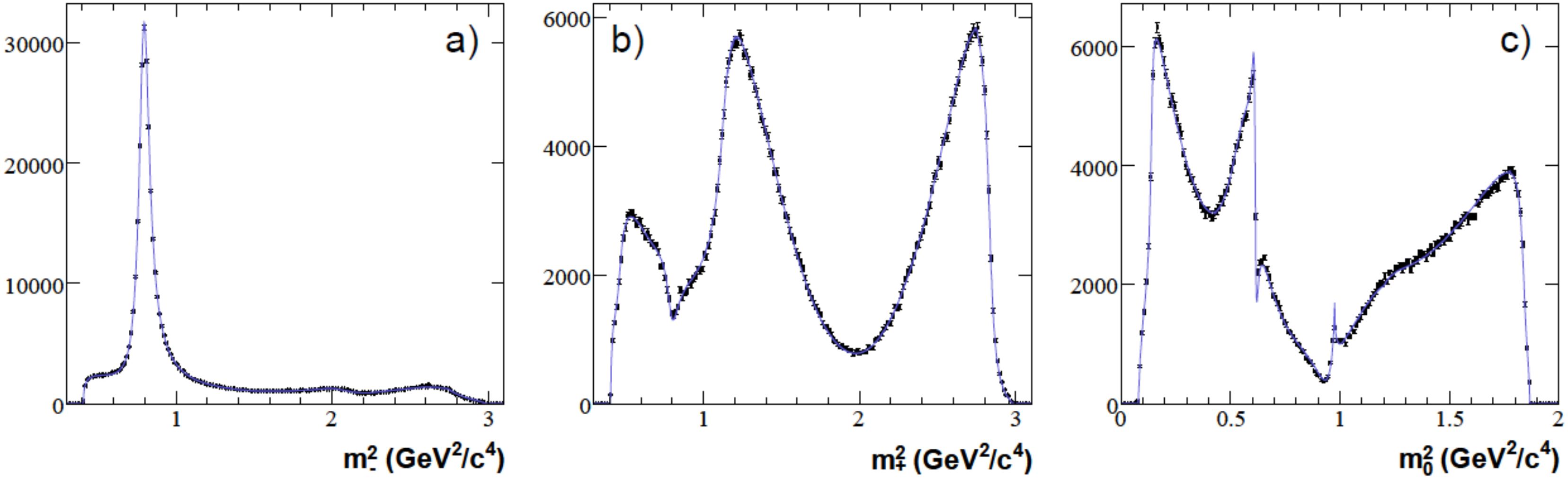
Most sensitive channel to date: $\tilde{D}^0 \to K_S^0\pi^+\pi^-$:

GGSZ, Phys. Rev. D **68**, 054018 (2003), BP, Eur. Phys. Jour. **47**, 347 (2006)

Requires a detailed understanding of the *D⁰* decay as input

High Quality is needed

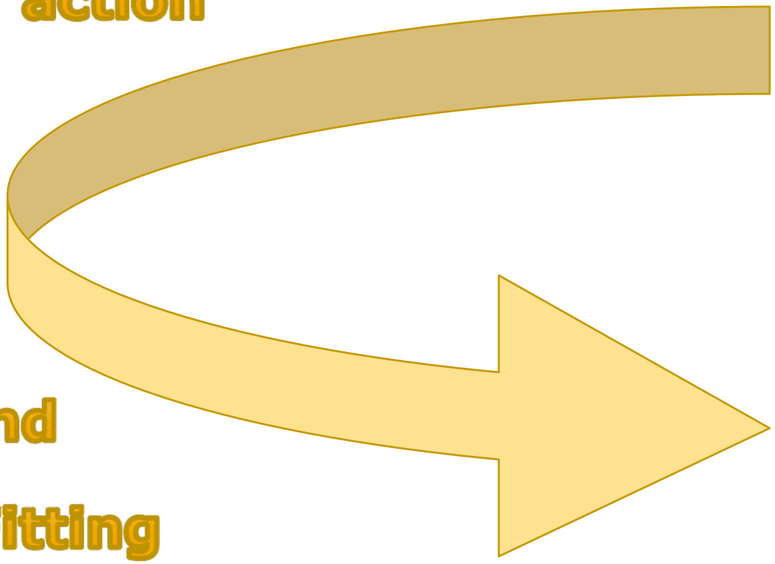
and achievable…



this week basically about how to model the input for such fits

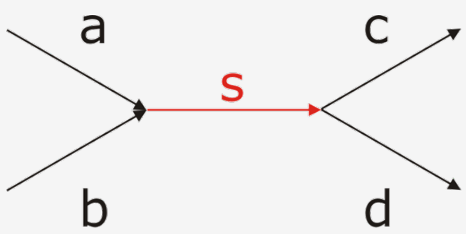
to reveal all the physics of a multi-particle reaction

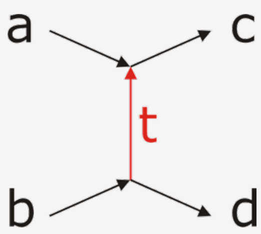
Important aspects…

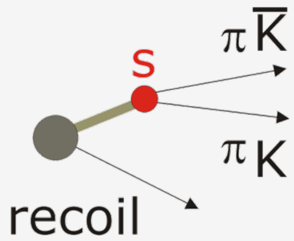


- General considerations
- Course of action
- Phase space
- Observables
- Hypotheses
- Background
- Fitting
- Mathematical problems
- Quality Assurance

Experimental Techniques







Scattering Experiments

πN - *N** measurement

πN - meson spectroscopy

E818, E852 @ AGS, GAMS
Compass, VES

pp meson threshold production

WASA @ Celsius, COSY

pp or *π*p in the central region

WA76, WA91, WA102

γN – photo production

Cebaf, Mami, Elsa, Graal

"At-rest" Experiments

N̄N @ rest at LEAR

Asterix, Obelix, Crystal Barrel
PANDA

J/ψ decays

MarkIII,DM2,BES,CLEO-c

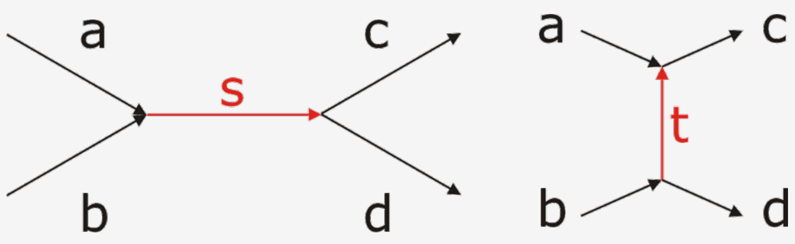
$\phi(1020)$ decays

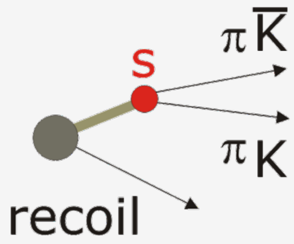
Kloe @ Dafne, VEPP

D and D_s decays

FNAL, Babar, Belle, Belle-II

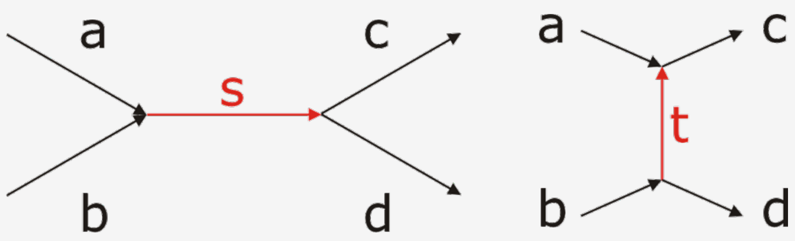
Experimental Techniques

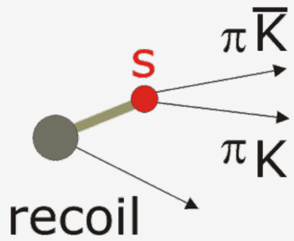




Scattering Experiments	"At-rest" Experiments
partial waves decomposition → via moment analysis	ad-hoc introduction of waves
systematic studies to limit #waves	ad-hoc introduction of dynamic amplitudes ("resonances")
dynamics appear as amplitude variations	systematic studies to limit #waves and #resonances
resonance parameters from fits to amplitudes	resonance parameters appear as fit parameters

Experimental Techniques





Scattering Experiments

exchange model needed

ad−hoc intermediate resonances

→ parameters fixed for wave decomposition

"At-rest" Experiments

independent of production model

→ intermediate resonances treated identically to final state resonances

crossing bands may provide high resolution interferometer

General considerations (I)

Which processes take place?

Interactions?

Basic processes – scattering vs. decay – which scattering

(Physics of) Initial State – recoils – inclusive/exclusive

Physics background

Leading effects

Scales?

Dynamics – range parameters

Approximations – low energy or threshold expansions

do scales differ for different sub-processes?

factorization of dynamics, like in open-charm decays

General considerations (II)

What are conserved properties?

kinematics

energy/momentum conservation

kinematicaly fitted data?

quantum numbers

quark/isospin conservation/symmetries

good and bad quantum numbers (isospin, parity, CP)

impact on spin formalisms

misc

interferences of Feynman graphs

phase space

full set of observables?

integrate over part of the phase-space

Example: Isospin Dependence

$\bar{p}p$ initial states differ in isospin

$^1S_0$ $\quad I^G(J^{PC}) = 1^-(0^{-+})$

$^3S_1$ $\quad I^G(J^{PC}) = 0^+(1^{--})$

Calculate isospin Clebsch-Gordan

$$\rho^0\pi^0 \;\rightarrow\; (1010|00) = -\sqrt{\frac{1}{3}}$$

$$\rho^0\pi^0 \;\rightarrow\; (1010|10) = 0$$

$$\rho^\pm\pi^\mp \;\rightarrow\; (1(\pm 1)\ 1(\mp 1)|00) = \sqrt{\frac{1}{3}}$$

$$\rho^\pm\pi^\mp \;\rightarrow\; (1(\pm 1)\ 1(\mp 1)|10) = \pm\sqrt{\frac{1}{2}}$$

$^1S_0$ destructive interferences

$^3S_1$ $\rho^0\pi^0$ forbidden

General considerations (III)

What are the relevant parameters?

Order of magnitude

relevant for coding?

leading terms?

Parameter too small e.g. different parameterization

Parameter too small, e.g. drop terms

Relations

are the parameters related to each other? (D/S, phases, …)

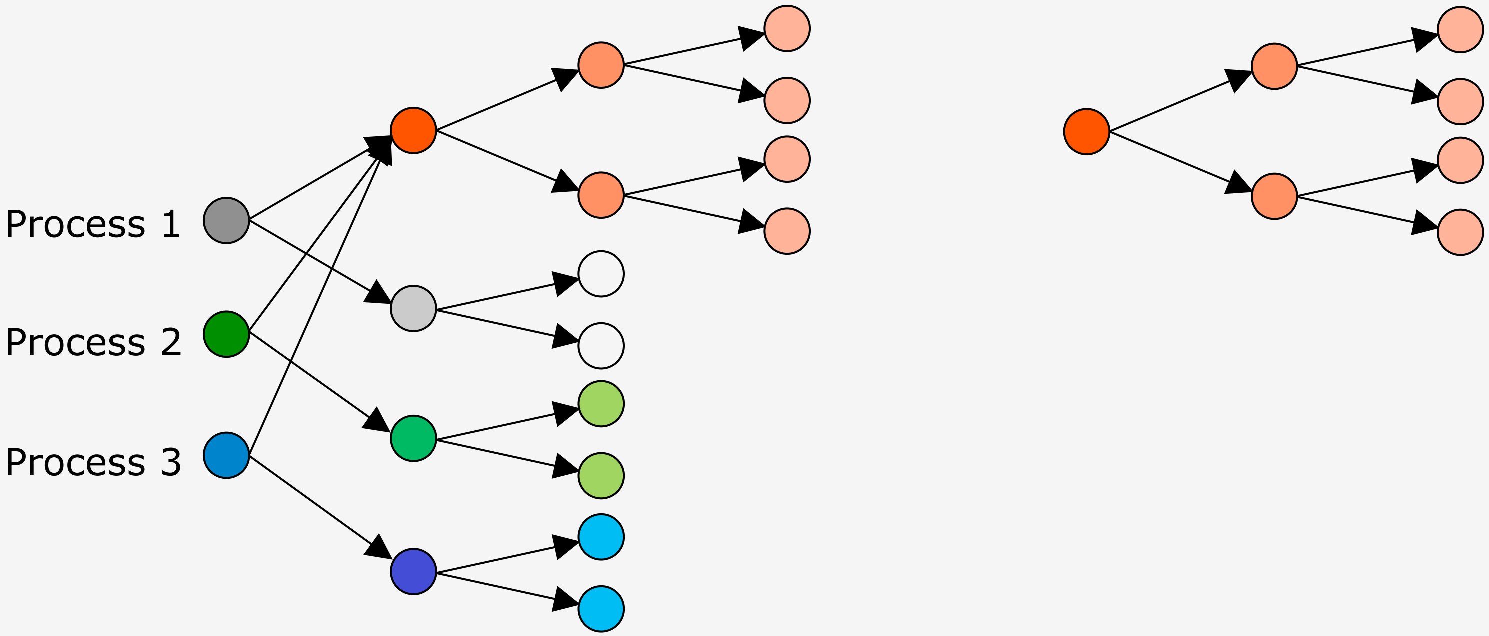
which one is the master and which the slave?

Normalization/Constraints

e.g. couplings normalized to 1

Can the process be factorized or simplified?

Whole tree needed? or is a leave sufficient



Rules

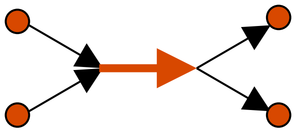
which rules/conditions can be used to formulate the model

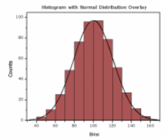
which rules/conditions have to be applied during the fit

e.g. what is fixed by definitions

Course of action

1. Data Analysis
2. Modeling
3. Fitting
4. Quality Assurance
5. Review and Publication





Course of action (I)

Data analysis

Data

extract relevant data set(s) with appropriate statistics, high purity and high efficiency

MC

signal MC, may be mixed due to experimental conditions

Background

extract from data and/or generate via Monte Carlo data sets from potential background channels

Representation

represent the data in *n*-tuples of relevant (transformed?) observables for the fit and the visualization

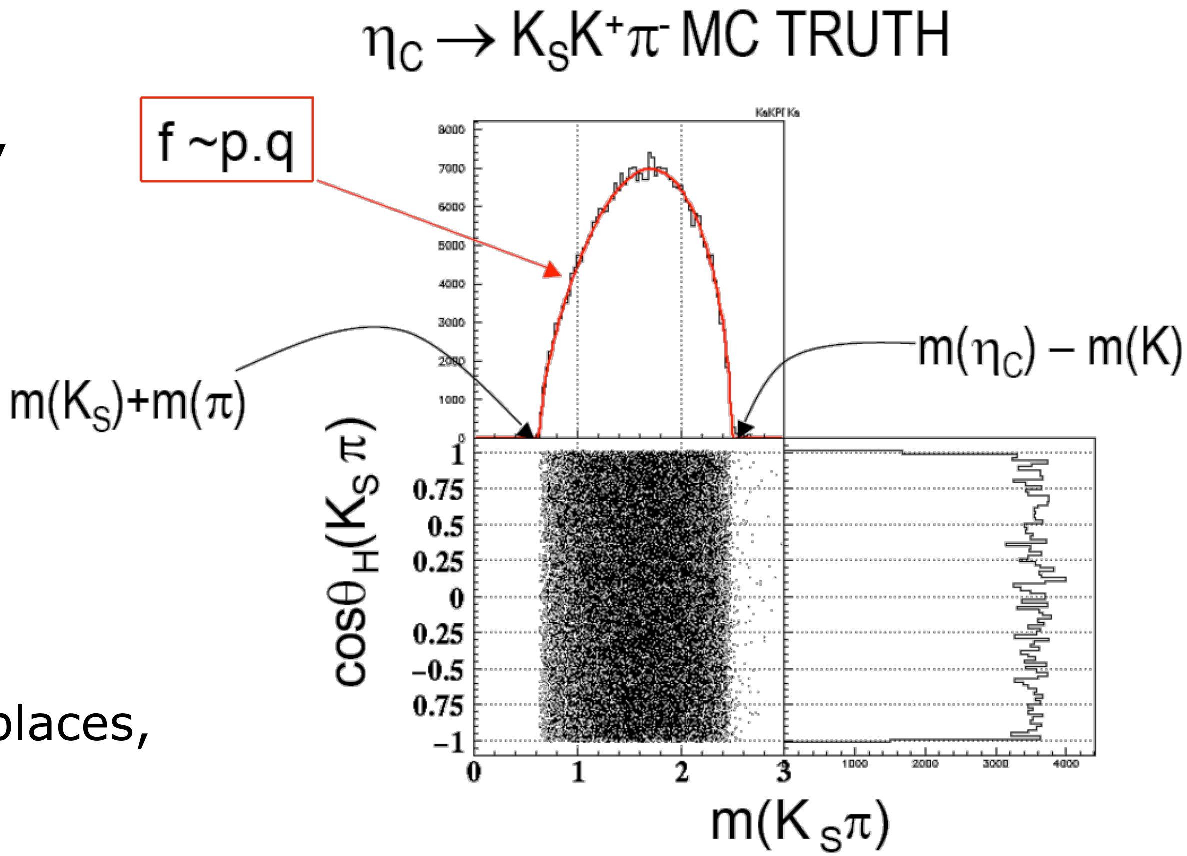
Obervables

Observables should be aligned with the problem/process

- is polarization relevant?
- is dynamics present in all particle pairs?
- are there isolated structures or regions with strong correlations?

Typical observables are

- *m^2 (s)*
- invariant mass square,
- Mandelstam *s*
- *T*
- kinetic energy
- $\cos\theta$
- decay angle of
- resonances
- $\cos\psi$
- angle between decay places,
- ….. a.m.o.m.



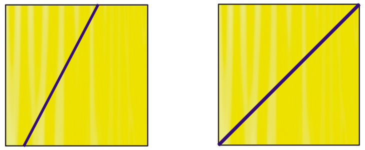
Observables, cont’d

are there symmetries in the phase space?

unique assignment of phase space coordinates is important to avoid double counting

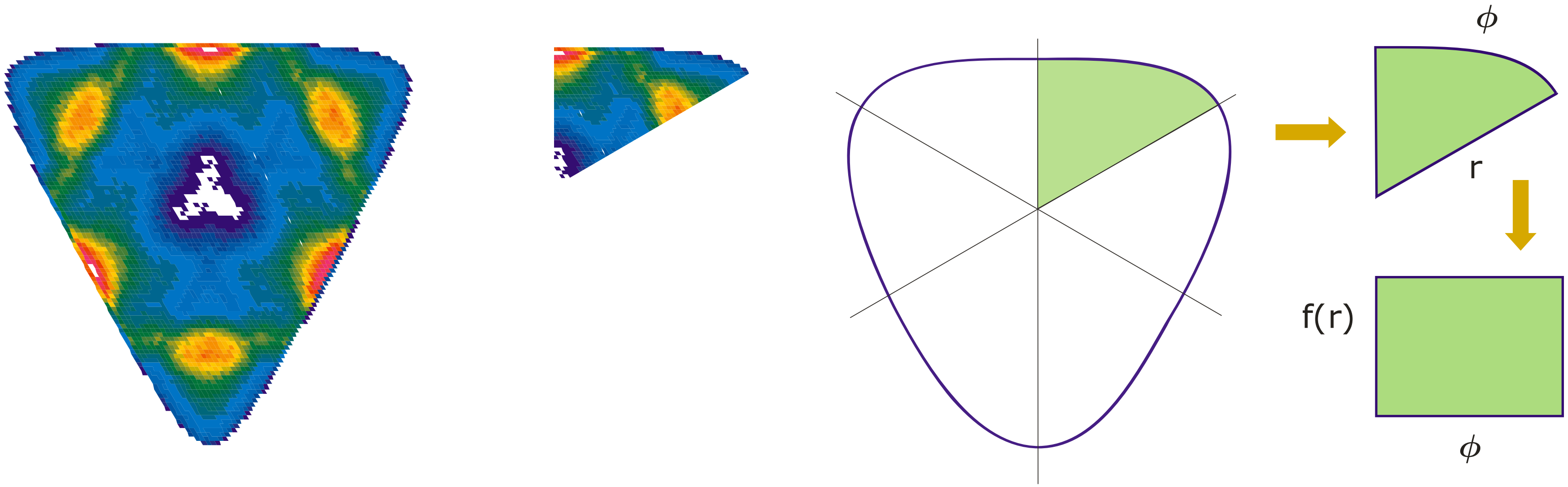
transformation necessary?

Most Dalitz plots are symmetric:



Problem: sharing of events

Possible solution: transform DP



Course of action (II)

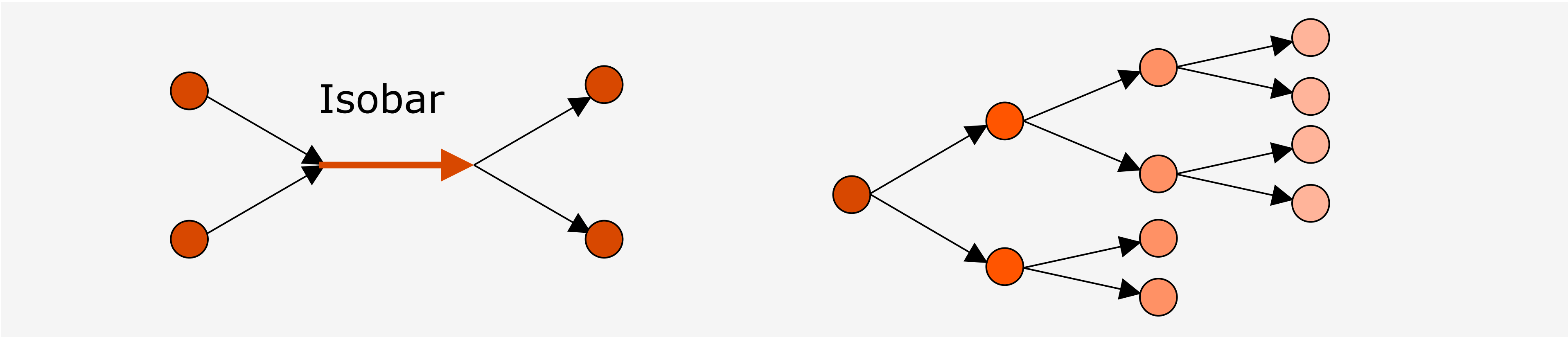
Modeling

Data

Visual inspection of the data !!

Physics

create list of hypotheses (incl. production, spins, dynamics and if so, background)



Mathematics

optimize the mathematical form

may improve speed and may reduce numerical instabilities

Phase space

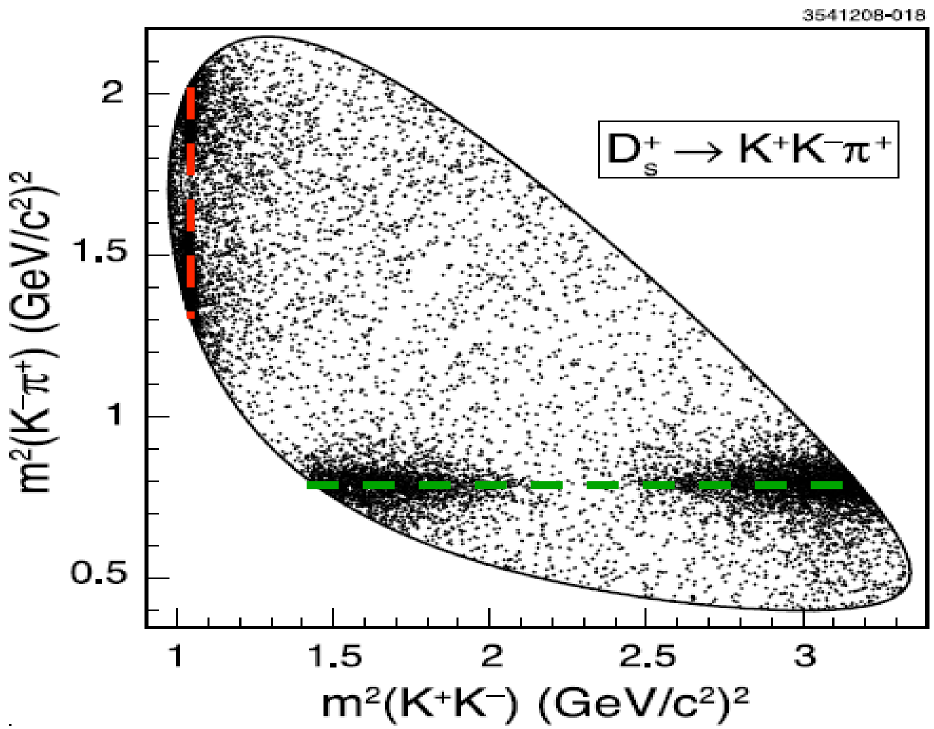
visual inspection of the phase space distribution

are the structures?

structures from signal or background?

are there strong interferences, threshold effects, potential resonances?

$\phi \;\; K^+K^-$



$K^*(892) \;\; K^-\pi^+$

Phase space (cont'd)

do you expect phase space distortions?

- for example from varying efficiencies
  - example: $\varepsilon(p) \neq$ const.

how strong is the event displacement?

- due to resolution
  - example: m^2 has Gaussian smeared
  - may end up in a different bin
- due to wrong particle assignments
  - example: 15 combinations of 6γ may form $3\pi^0$
  - a wrong assignment is still reconstructed but with different coordinates
- has it impact on the model and/or the method?

Kinematical Reflections

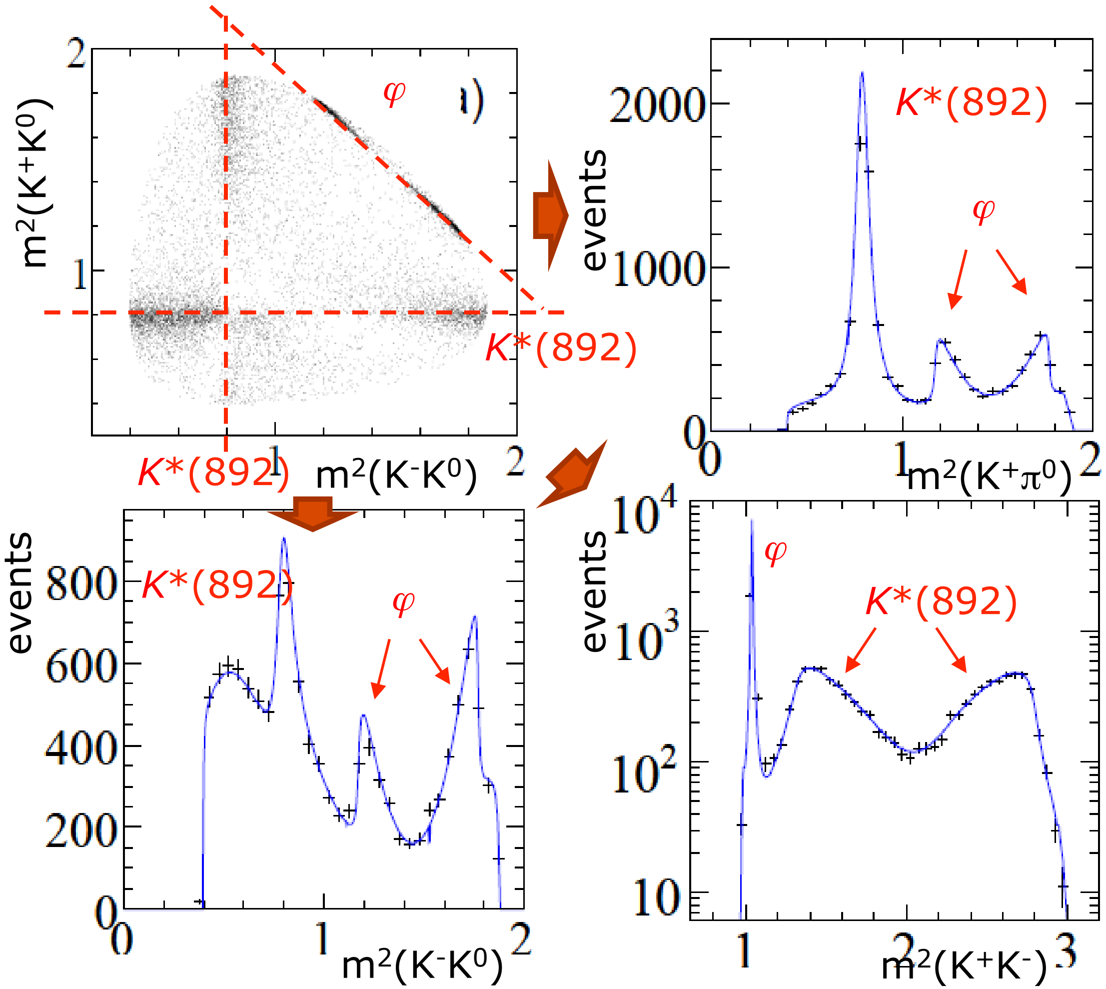
Kinematic situation can produce mass peaks
not being true resonances → called Reflections

Example:

Dalitz plot of

$$D_s^+ \rightarrow K^+ K^- \pi^0$$

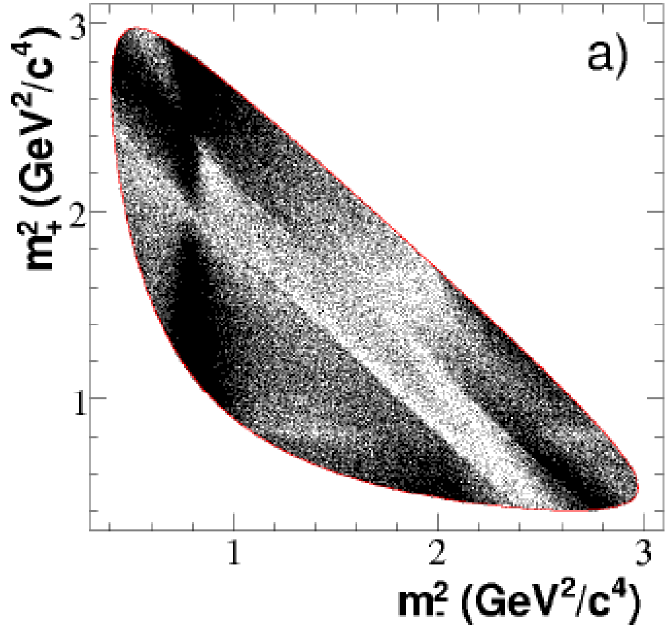
in this case „fakes“
are simple to spot...

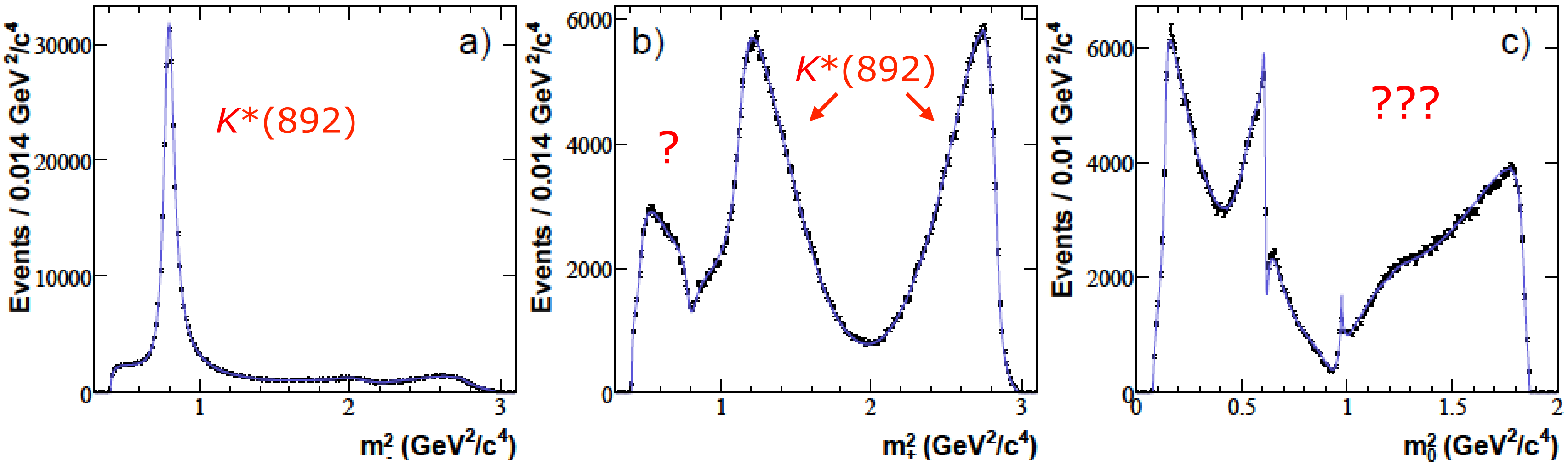


Kinematical Reflections, cont'd

... but it can be much less obvious!

Example: $D_s^+ \rightarrow K_S^0 \pi^+ \pi^-$





Hypotheses

Select basic model

- usually isobar model, is not appropriate in all cases
- rescattering, t-channel and Deck effects may lead to artifacts

Select formalism to handle the spin

- select basis (helicity reflectivity, canonical….)
- or tensors (Zemach, covariant or Lorentz-invariant)
- depends on the process and the goals

Select set of dynamical functions

- which resonances and thresholds are known
- which do you guess from inspection
- how much freedom is needed,
  how well do I know the processes involved
- analysis of angular moments might be helpful as a start

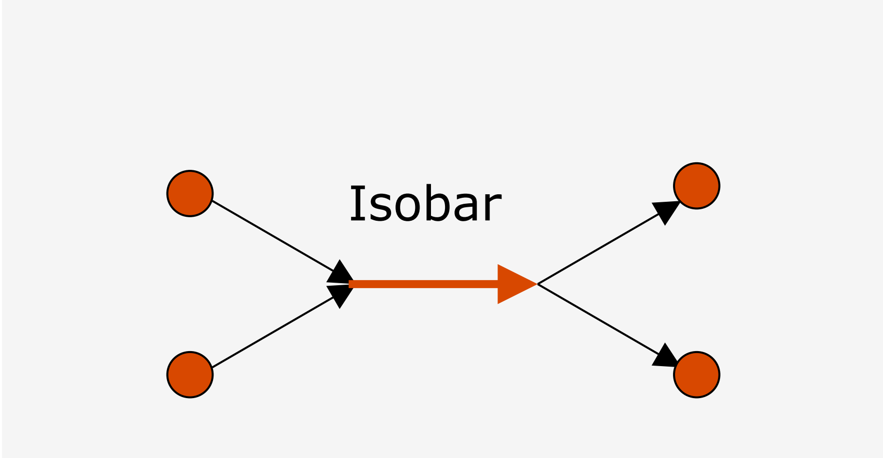
Selection of parameters and optimization

First results may indicate that the assumptions are wrong and one has to start over

Isobar Model

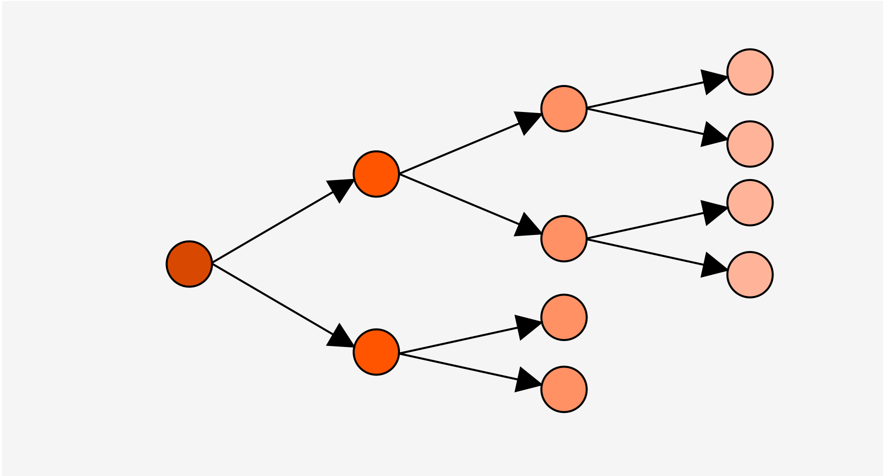
Generalization

- the overall process is dominated by two-body processes
- construct any many-body system
  tree of subsequent two-body decays
- the two-body systems behave identical in each reaction
- different initial states may interfere



We need

- need two-body “spin”-algebra
  various formalisms
- need two-body scattering formalism
  final state interaction
  e.g. Breit-Wigner (pars pro toto)



Course of action (III)

Fitting

fit model(s) to the data

likelihood definition, what is to be minimized

(max. Likelihood ($-\log\mathcal{L}$), Chi^2,…)

needs a strategy to find the best solution

systematic studies for a variety of hypotheses

vary initial stats, resonances, parameterizations

need a strategy for each fit

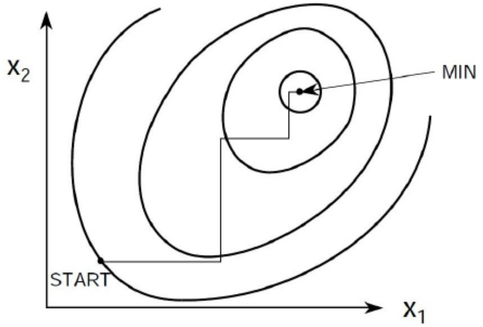
optimizer (gradient/random/genetic)

sequence (different optimizers, fixation and release of parameters)

criteria for convergence and termination

Remains one important question: where to start?

Minimization

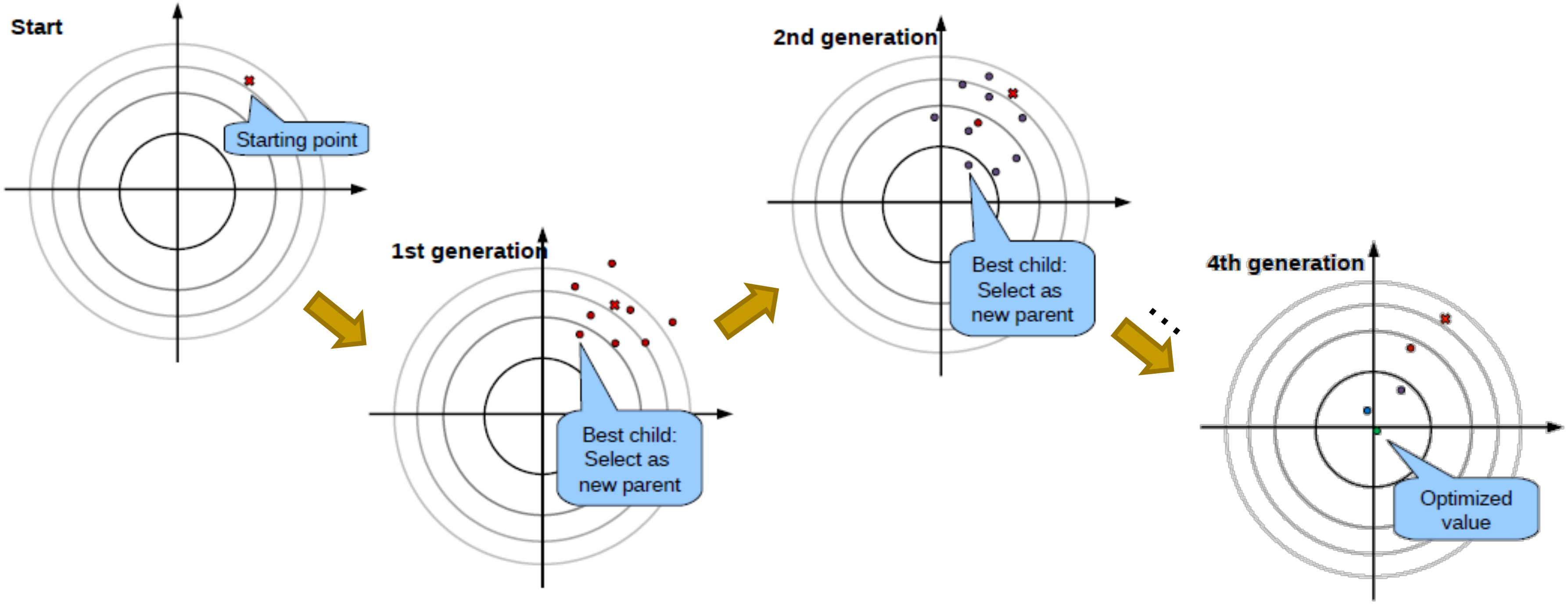


MINUIT2 = classical gradient descent

may be stuck in local minima

Alternative: Evolutionary Strategy GenEvA

→ new solutions created from previous ones (offspring)



Course of action (IV)

Quality Assurance

Documentation

excellent documentation! is the key

what was done? formulae!

(intermediate) results!

Validation

validation of the result (for example with toy MC)

Significance

scrutinize the significance of new findings

check various methods to investigate the goodness-of-fit

Errors

determination of statistical and systematic errors

Course of action (V)

Review and Publication

Scrutinize everything – and be prepared to redo certain tasks

Wikipedia
The Free Encyclopedia

In Greek mythology Sisyphus (/ˈsɪsɪfəs/;[1] Greek: Σίσυφος, Sísyphos) was a king of Ephyra (now known as Corinth) punished for chronic deceitfulness by being compelled to roll an immense boulder up a hill, only to watch it roll back down, and to repeat this action forever.