

# The Laura++ Dalitz-plot Fitter

Thomas Latham



# History

- Package originally developed within BaBar by members of the Warwick group
- The LAURA (Fortran) package was written in 2002 by Paul Harrison
- Converted into C++ by John Back in 2003
- Developed subsequently by John Back and TL, adding many extra features
- TL now main developer (with several contributors)

# History (cont.)

- Used in many Dalitz-plot (and other) analyses published by BaBar collaboration
  - Mainly charmless 3-body B decays
- Already being used by several analyses in LHCb ( $B \rightarrow K_S hh$ ,  $B \rightarrow Dhh$ , etc.)
- First public version released on HepForge on Monday!

<http://laura.hepforge.org>

# Main features

- Can form isobar models for the decay of spin-0 parent into 3 spin-0 children
- Many resonance line-shape options available
- Different forms for isobar coefficients implemented
- Extended maximum likelihood fitter
- Toy MC generation from PDFs
- Can include additional discriminating variables
- Many different PDF shapes for non-DP variables, including forms that allow for correlation with the DP position

# Main features (cont.)

- Can fit for direct CP violation
- Also possible to model time-dependent CP violation (code currently being updated from being rather B-factory specific to more general – not yet available in public release)
- Can add as many different background categories as needed
- Possible to model variation of reconstruction efficiency over the DP, as well as migration of mis-reconstructed events
- Automated calculation and storage of per-event likelihoods and sPlot weights from fit results

# Technical introduction

- A set of C++ classes that are built into a (shared) library
- Package depends only on ROOT
- Several example programs that can be built into executables to generate and fit toy data
- Built with handmade Makefiles at present, will use autotools/cmake soon (should give greater portability)
- Code comes with doxygen documentation

# Amplitude models

- As we've seen already this week, we want to describe each contributing amplitude in the decay and the interference between them
- Most common technique known as isobar model
- Total amplitude is sum of contributing amplitudes, each with a complex coefficient:

$$A(x, y) = \sum_{j=1}^N c_j F_j(x, y)$$

Complex coefficient

Decay dynamics  
(function of DP position)

# Amplitude models

- You set the  $F_j$  forms by choosing the resonance lineshapes as well as their mass, width and spin

```
106 // Create the isobar model
107 LauIsobarDynamics* sigModel = new LauIsobarDynamics(daughters, effModel);
108 sigModel->addResonance("rho0(770)", 1, "RelBW"); // resPairAmpInt = 1 => resonance mass is m23.
109 sigModel->addResonance("rho0(1450)", 1, "RelBW");
110 sigModel->addResonance("f_0(980)", 1, "Flatte");
111 sigModel->addResonance("f_2(1270)", 1, "RelBW");
112 sigModel->addResonance("NonReson", 0);
```



# Amplitude models

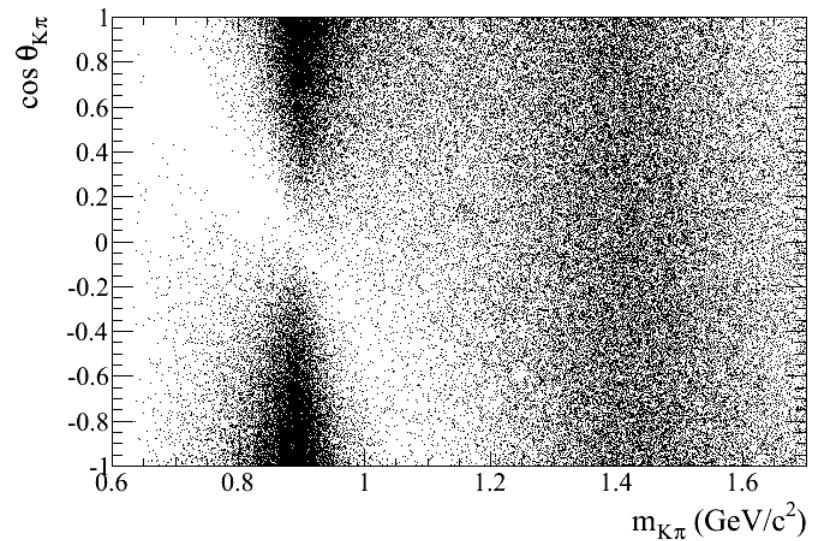
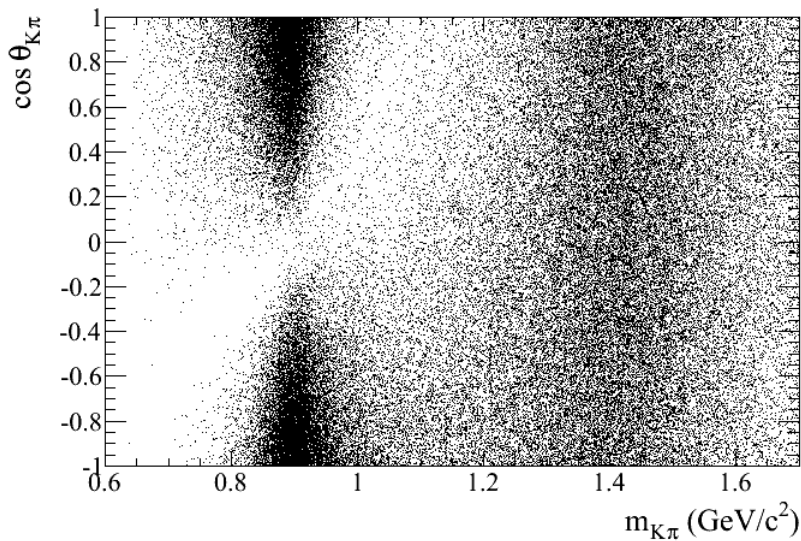
- The  $c_j$  can be set in the generation of toy MC and in the fits are the free parameters that you wish to determine
- Need to be cast into a pair real numbers to become fit parameters, e.g.:
  - $c_j = x_j + i y_j$
  - $c_j = a_j e^{i\theta_j}$

```
121 // Create the fit model
122 LauSimpleFitModel* fitModel = new LauSimpleFitModel(sigModel);
123
124 // Create the complex coefficients for the isobar model
125 std::vector<LauAbsCoeffSet*> coeffset;
126 coeffset.push_back( new LauMagPhaseCoeffSet("rho0(770)", 1.00, 0.00, kTRUE, kTRUE) );
127 coeffset.push_back( new LauMagPhaseCoeffSet("rho0(1450)", 0.37, 1.99, kFALSE, kFALSE) );
128 coeffset.push_back( new LauMagPhaseCoeffSet("f_0(980)", 0.27, -1.59, kFALSE, kFALSE) );
129 coeffset.push_back( new LauMagPhaseCoeffSet("f_2(1270)", 0.53, 1.39, kFALSE, kFALSE) );
130 coeffset.push_back( new LauMagPhaseCoeffSet("NonReson", 0.54, -0.84, kFALSE, kFALSE) );
131 for (std::vector<LauAbsCoeffSet*>::iterator iter=coeffset.begin(); iter!=coeffset.end(); ++iter) {
132     fitModel->setAmpCoeffSet(*iter);
133 }
```

# Examples

$K^*(892)$

$K_0^*(1430)$



- Both resonances modelled as relativistic BW in this toy example
- Different relative phases lead to different interference patterns

# Integration Trick

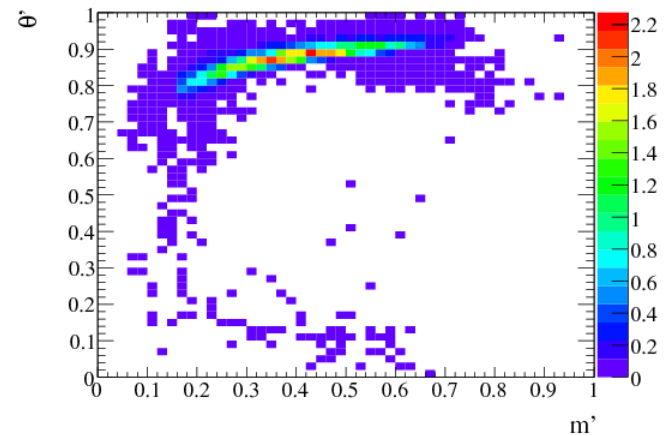
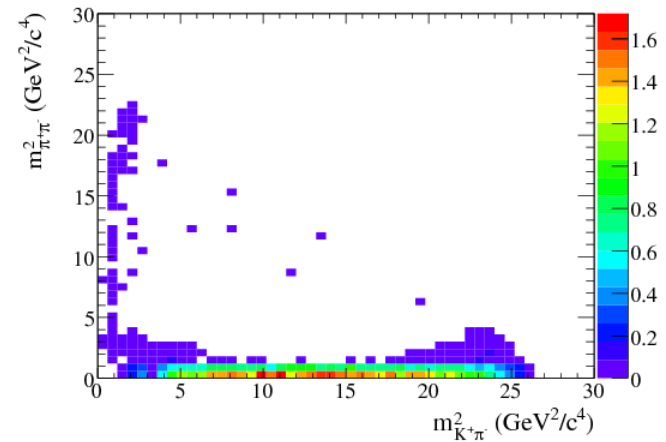
- When calculating the normalisation of the amplitude, find that the  $c_j$ 's factorise:

$$\int |A(x, y)|^2 dx dy = |c_1|^2 \int |F_1(x, y)|^2 dx dy + c_1 c_2^* \int F_1(x, y) F_2^*(x, y) dx dy + \dots$$

- Can calculate all the integrals once and cache
  - Assumes resonance parameters are fixed
- Gives massive speed benefits!

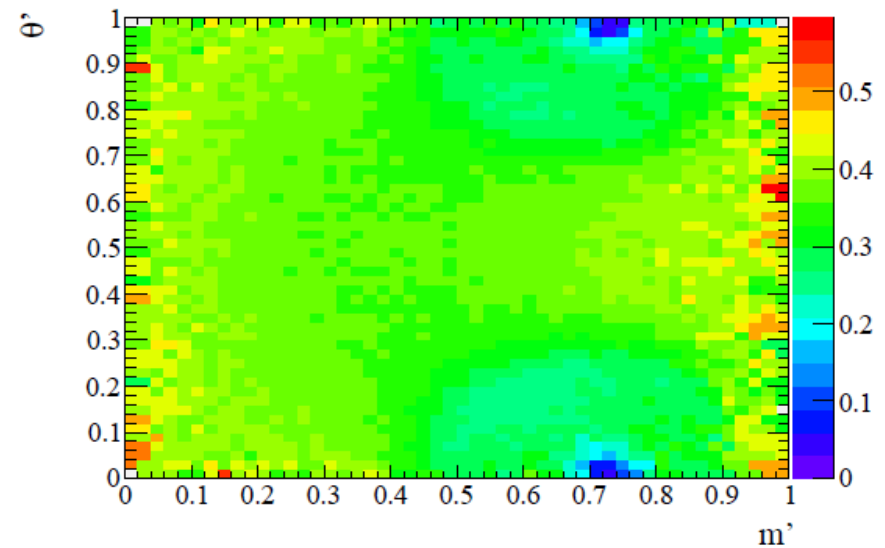
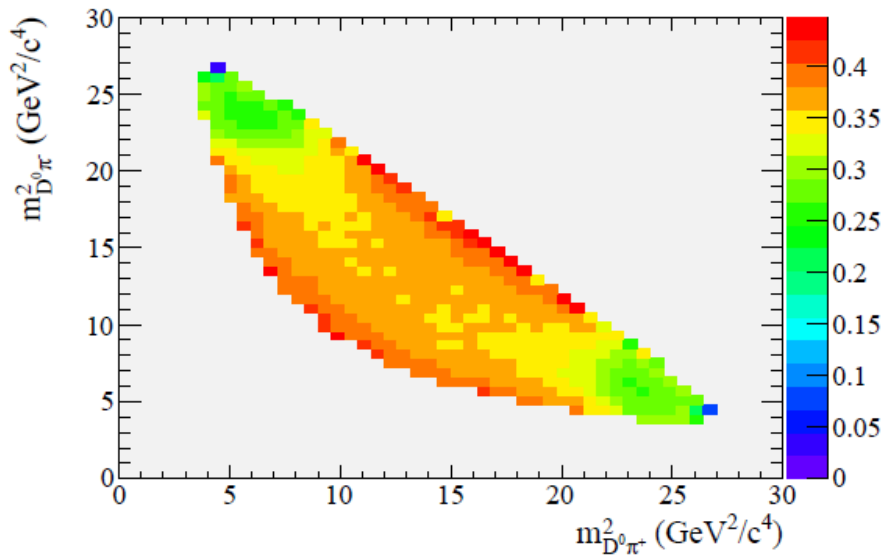
# Backgrounds

- Background model included as 2D histogram
- Either in conventional DP co-ordinates or in the “square DP”
- Can optionally use linear interpolation to smooth shape

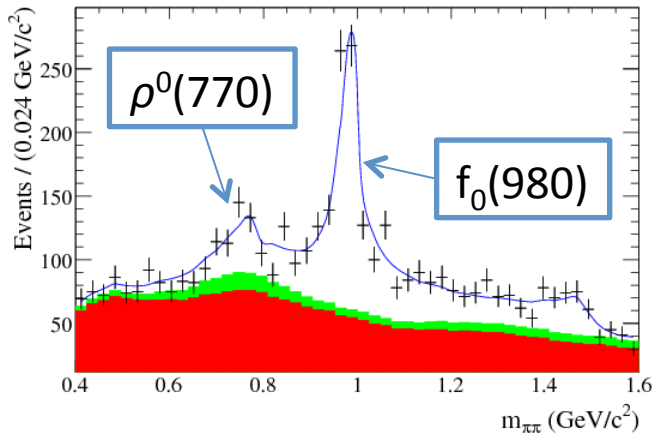


# Efficiency variation

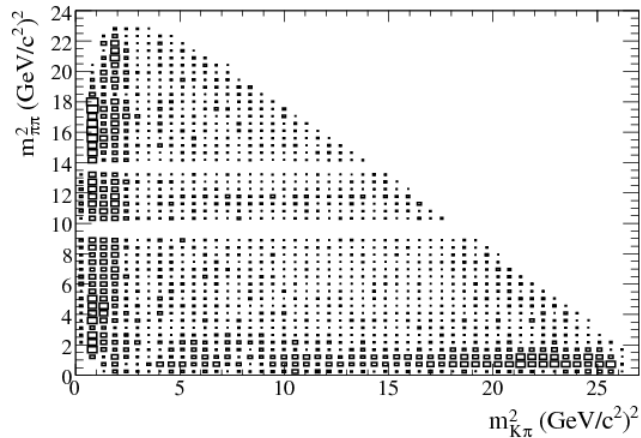
- Implemented in similar way to backgrounds
- 2D histogram with optional linear interpolation
- Will soon be able to use 2D splines for smoothing



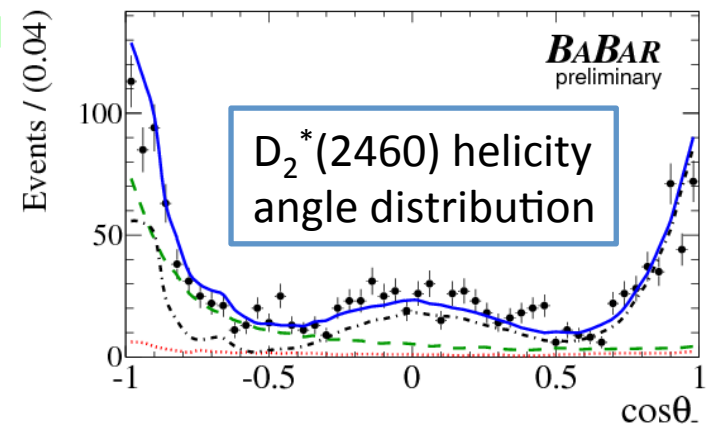
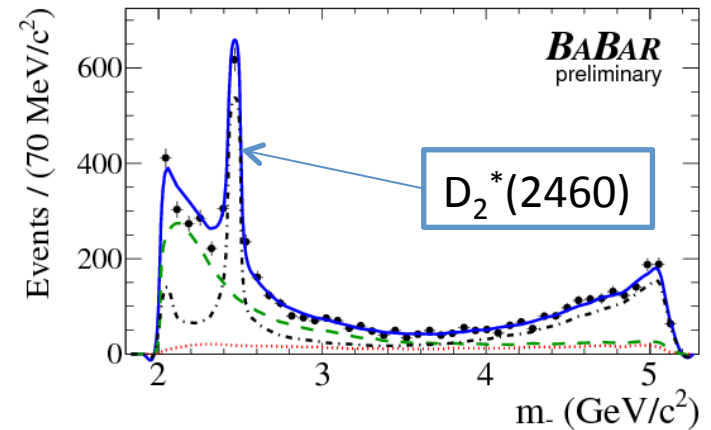
# Some example fits



**Total Fit Result**  
**Continuum background**  
**BB background**



**B+ -> K+ pi+ pi-**



**B0 -> D0 pi+ pi-**



# Summary

- Laura++ is a mature package, used for many published analyses
  - Well tested, lots of features, e.g. K-matrix, sPlot, etc.
- Specialised fitter for DP analysis – very fast!!
- Under active development – new features being added
- Current major items on the TODO list:
  - Model for time-dependent Dalitz-plot analysis for time-dependent CP asymmetry measurements
  - Allow floating of resonance parameters, e.g. masses and widths
  - Extend to deal with initial-state and final-state particles with non-zero spin, e.g.  $\Lambda_b$  decays

# Conclusion

- Please take a look:  
<http://laura.hepforge.org>
- There is documentation on the website to help you get started, plus several examples
- A quick-start tutorial is also on the indico agenda (along with the slides for this talk)
- A full user guide is in preparation
- A paper is also in preparation, which will document the conventions and formulae used



# Thanks!

The screenshot shows a web browser window with the title "Laura++ - Hepforge". The address bar contains "laura.hepforge.org/acknowledgements.html". The browser's navigation bar includes links for Apple, iCloud, Facebook, Twitter, Wikipedia, Yahoo!, News, and Popular. A status bar at the bottom of the browser indicates "Laura++ is hosted by Hepforge, IPPP Durham".

On the left side of the page, there is a navigation menu with the following items:

- Home
- Documentation
- Subversion
- Bug Tracker
- Licence
- Acknowledgements
- Contact

## Acknowledgements

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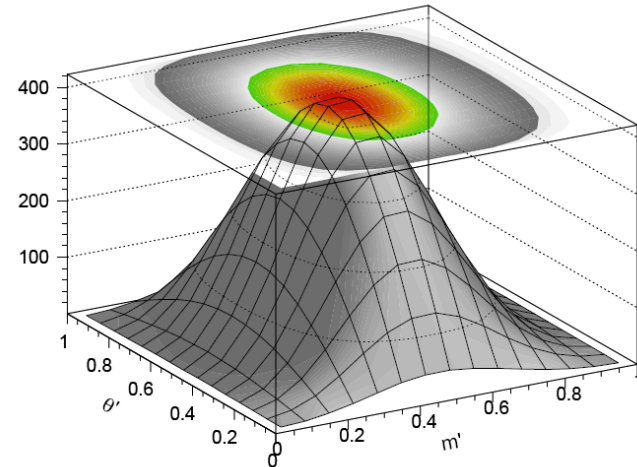
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# BACKUP

# The square Dalitz plot

$$m' \equiv \frac{1}{\pi} \arccos \left( 2 \frac{m_{K^+\pi^+} - m_{K^+\pi^+}^{\min}}{m_{K^+\pi^+}^{\max} - m_{K^+\pi^+}^{\min}} - 1 \right),$$
$$\theta' \equiv \frac{1}{\pi} \theta_{K^+\pi^+},$$



- Transformation of coordinates
- “Zooms” into the areas around the boundary of the conventional Dalitz plot
- Increases resolution in those areas of interest
- Also avoids effect from kinematic boundary not being aligned with bin edges

# List of DP papers

- B. Aubert et al. (BABAR Collaboration), Dalitz-plot analysis of the decays  $B^+ \rightarrow K^+ \pi^+ \pi^-$ ,
  - Phys. Rev. D 72, 072003 (2005) [Erratum-ibid. D 74, 099903 (2006)], arXiv:hep-ex/0507004
- B. Aubert et al. (BABAR Collaboration), An amplitude analysis of the decay  $B^+ \rightarrow \pi^+ \pi^+ \pi^-$ ,
  - Phys. Rev. D 72, 052002 (2005), arXiv:hep-ex/0507025
- B. Aubert et al. (BABAR Collaboration), Evidence for Direct CP Violation from Dalitz-plot analysis of  $B^+ \rightarrow K^+ \pi^+ \pi^-$ ,
  - Phys. Rev. D 78, 012004 (2008), arXiv:0803.4451 [hep-ex]
- T. Latham and T. Gershon, A Method to Measure  $\cos(2\beta)$  Using Time-Dependent Dalitz Plot Analysis of  $B^0 \rightarrow D_{CP} \pi^+ \pi^-$ ,
  - J. Phys. G 36, 025006 (2009), arXiv:0809.0872 [hep-ph]
- B. Aubert et al. (BABAR Collaboration), Dalitz plot analysis of  $B^+ \rightarrow \pi^+ \pi^+ \pi^-$  decays,
  - Phys. Rev. D 79, 072006 (2009), arXiv:0902.2051 [hep-ex]
- B. Aubert et al. (BABAR Collaboration), Time-dependent amplitude analysis of  $B^0 \rightarrow K_S \pi^+ \pi^-$ ,
  - Phys. Rev. D 80, 112001 (2009), arXiv:0905.3615 [hep-ex]
  - (Used as a cross-check, not for the main results.)
- P. del Amo Sanchez et al. (BABAR Collaboration), Dalitz-plot Analysis of  $B^0 \rightarrow D^0 \pi^+ \pi^-$ ,
  - Presented at ICHEP 2010, arXiv:1007.4464 [hep-ex]

# List of other papers

- B. Aubert et al. (BABAR Collaboration), Observation of the Decay  $B^+ \rightarrow K^+ K^- \pi^+$ ,
  - Phys. Rev. Lett. 99, 221801 (2007), arXiv:0708.0376 [hep-ex]
- B. Aubert et al. (BABAR Collaboration), Search for the highly suppressed decays  $B^- \rightarrow K^+ \pi^- \pi^-$  and  $B^- \rightarrow K^- K^- \pi^+$ ,
  - Phys. Rev. D 78, 091102(R) (2008), arXiv:0808.0900 [hepex]
- B. Aubert et al. (BABAR Collaboration), Search for the decay  $B^+ \rightarrow K_S K_S \pi^+$ ,
  - Phys. Rev. D 79, 051101(R) (2009), arXiv:0811.1979 [hep-ex]
- P. del Amo Sanchez et al. (BABAR Collaboration), Observation of the Rare Decay  $B^0 \rightarrow K_S K^+ \pi^-$ ,
  - Phys. Rev. D 82, 031101(R) (2010), arXiv:1003.0640 [hep-ex]
- J. P. Lees et al. (BABAR Collaboration), Observation of the rare decay  $B^+ \rightarrow K^+ \pi^0 \pi^0$  and measurement of the quasi-two body contributions  $B^+ \rightarrow K^*(892)^+ \pi^0$ ,  $B^+ \rightarrow f_0(980) K^+$  and  $B^+ \rightarrow \chi_{c0} K^+$ ,
  - Phys. Rev. D 84, 092007 (2011), arXiv:1109.0143 [hep-ex]