Constraining the $D \to K^+\pi^-\pi^+\pi^-$ Coherence Factor Using D Mixing

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Introduction

- Four Body Phase Space
- D Mixing
  - General Case
  - Wrong Sign $D^0 \to K^+\pi^-\pi^+\pi^-$ Decays
- CKM Phase γ
- Constraining the $D \to K^+\pi^-\pi^+\pi^-$ Coherence Factor
- Model Inspired Binning
- Conclusions

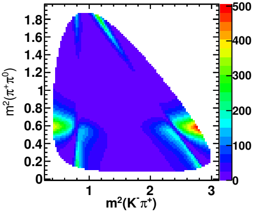
4 Body Phase Space

From 3 Body to 4 Body

$$D^0 \to K^-\pi^+\pi^0$$

(2-dim phase space)

$m^2(K^-, \pi^+)$ vs. $m^2(\pi^+, \pi^0)$



Commonly seen

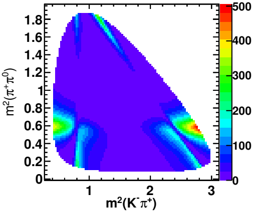
'Dalitz Plot'

From 3 Body to 4 Body

$$D^0 \to K^-\pi^+\pi^0$$

(2-dim phase space)

$m^2(K^-,\pi^+)$ vs. $m^2(\pi^+,\pi^0)$



Commonly seen

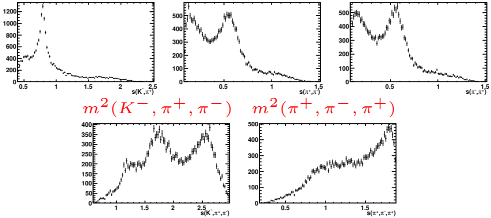
'Dalitz Plot'

$$D^0 \to K^-\pi^+\pi^-\pi^+$$

(5-dim phase space)

$m^2(K^-,\pi^+)$ $m^2(\pi^+,\pi^-)$ $m^2(\pi^-,\pi^+)$

$m^2(K^-,\pi^+,\pi^-)$ $m^2(\pi^+,\pi^-,\pi^+)$



1D projections of a 5D space

Not easy to visualise!

General D-Mixing Formalism

Mixing Formalism

- The mass eigenstates $|D^0_1\rangle$ and $|D^0_2\rangle$ can be defined as a superposition of the flavour eigenstates $|D^0\rangle$ and $|\overline{D}^0\rangle$ (assuming no indirect CPV or CPV in mixing)

$$|D_1\rangle = \frac{1}{\sqrt{2}}\left(|D^0\rangle + |\overline{D}^0\rangle\right)$$
$$|D_2\rangle = \frac{1}{\sqrt{2}}\left(|D^0\rangle - |\overline{D}^0\rangle\right)$$

- A D meson is produced in a flavour eigenstate then evolves as a superposition of its mass eigenstates.
- This gives us mixing!

Mixing Formalism

- The 'amount' of mixing we get is characterised by the dimensionless parameters x and y.

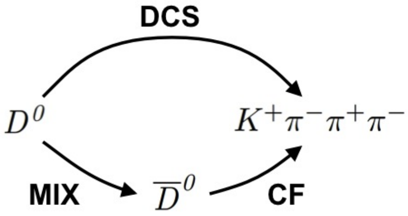
$$x \equiv \frac{m_1 - m_2}{\Gamma} \qquad y \equiv \frac{\Gamma_1 - \Gamma_2}{2\Gamma}.$$

- m_1 and m_2 are the masses of the mass eigenstates.
- Γ_1 and Γ_2 are the widths of the mass eigenstates.
- Γ is the average width.

$$\Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$$

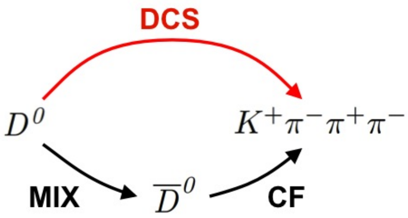
D-Mixing Formalism for $D^0 \to K^+\pi^-\pi^+\pi^-$

Wrong Sign $D^0 \to K^+\pi^-\pi^+\pi^-$ Decays

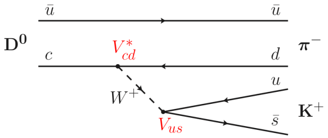


- Now look at the specific case of mixing in Wrong Sign $D^0 \to K^+\pi^-\pi^+\pi^-$ decays.
- There are two routes from the initial to the final state...

Wrong Sign $D^0 \to K^+\pi^-\pi^+\pi^-$ Decays



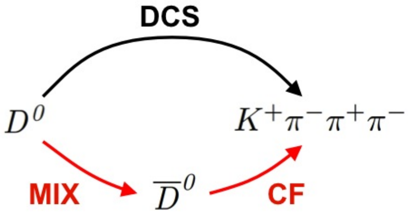
Doubly Cabibbo Suppressed (DCS) Amplitude



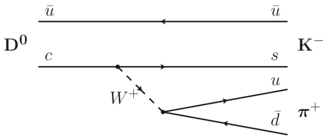
$$\mathrm{R}\left[D^0 \to K^+\pi^-\pi^+\pi^-\right](\mathbf{p},t) = e^{-\Gamma t}\left[|\mathcal{A}_{\mathrm{DCS}}(\mathbf{p})|^2 + |\mathcal{A}_{\mathrm{DCS}}(\mathbf{p})||\mathcal{A}_{\mathrm{CF}}(\mathbf{p})|y'\Gamma t + |\mathcal{A}_{\mathrm{CF}}(\mathbf{p})|^2 \frac{x^2+y^2}{4}(\Gamma t)^2\right]$$

$\mathcal{A}_{\mathrm{DCS}}(\mathbf{p})$ - Doubly Cabibbo Suppressed amplitude, varies as a function of phase space

Wrong Sign $D^0 \to K^+\pi^-\pi^+\pi^-$ Decays



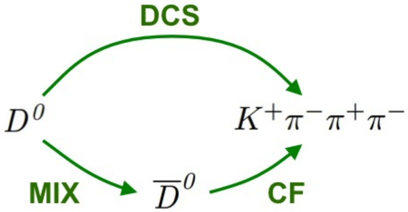
Cabibbo Favoured (CF) Amplitude

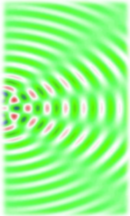


$$\mathrm{R}\left[D^0 \to K^+\pi^-\pi^+\pi^-\right](\mathbf{p}, t) = e^{-\Gamma t}\left[|\mathcal{A}_{\mathrm{DCS}}(\mathbf{p})|^2 + |\mathcal{A}_{\mathrm{DCS}}(\mathbf{p})||\mathcal{A}_{\mathrm{CF}}(\mathbf{p})|y'\Gamma t + |\mathcal{A}_{\mathrm{CF}}(\mathbf{p})|^2 \frac{x^2+y^2}{4}(\Gamma t)^2\right]$$

$\mathcal{A}_{\mathrm{CF}}(\mathbf{p})$ - Cabibbo Favoured amplitude, varies as a function of phase space

Wrong Sign $D^0 \to K^+\pi^-\pi^{+-}\pi^-$ Decays





$$\mathrm{R}\left[D^0 \to K^+\pi^-\pi^+\pi^-\right](\mathbf{p},t) = e^{-\Gamma t}\left[|\mathcal{A}_{\mathrm{DCS}}(\mathbf{p})|^2 + |\mathcal{A}_{\mathrm{DCS}}(\mathbf{p})||\mathcal{A}_{\mathrm{CF}}(\mathbf{p})|y'\Gamma t + |\mathcal{A}_{\mathrm{CF}}(\mathbf{p})|^2 \frac{x^2+y^2}{4}(\Gamma t)^2\right]$$

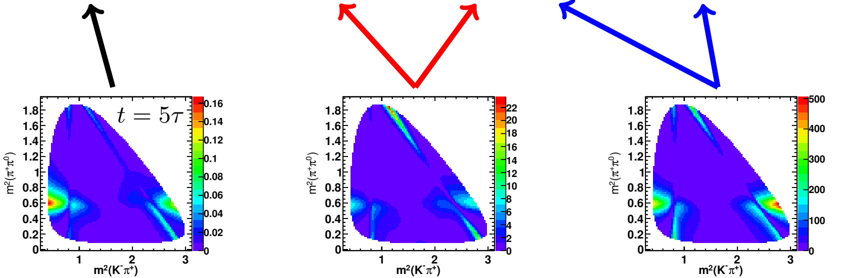
$$y' = y\cos\delta_D^{K3\pi}(\mathbf{p}) - x\sin\delta_D^{K3\pi}(\mathbf{p})$$

$\delta_D^{K3\pi}(\mathbf{p})$ - Strong phase difference between CF and DCS amplitudes

Wrong Sign $D^0 \to K^+\pi^-\pi^0$ Decays

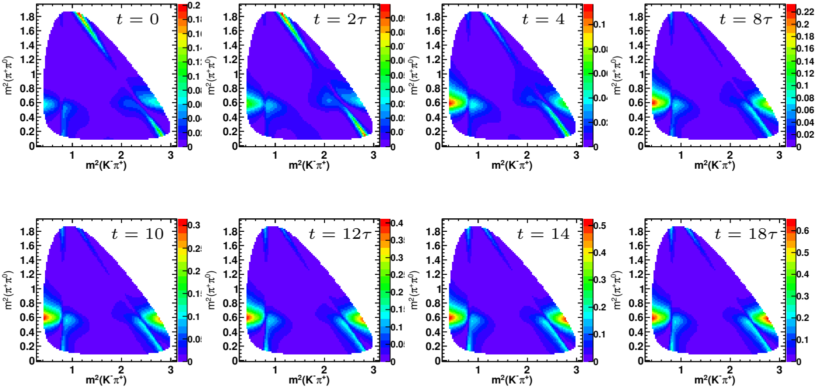
- Due to mixing there is time dependent amplitude structure.
- To visualise this, it's easier to look at the three body $D^0 \to K^+\pi^-\pi^0$.

$$\mathrm{R}\left[D^0 \to K^+\pi^-\pi^0\right](\mathbf{p}, t) = e^{-\Gamma t}\left[|\mathcal{A}_{\mathrm{DCS}}(\mathbf{p})|^2 + |\mathcal{A}_{\mathrm{DCS}}(\mathbf{p})||\mathcal{A}_{\mathrm{CF}}(\mathbf{p})|y'\Gamma t + |\mathcal{A}_{\mathrm{CF}}(\mathbf{p})|^2 \frac{x^2+y^2}{4}(\Gamma t)^2\right]$$



NOTE: Toy MC with a made up amplitude structure, and parameters fiddled to exaggerate the effect of mixing.

Time Dependent Amplitude Analysis of $D^0 \to K^+\pi^-\pi^0$



- All of this toy data was generated with MINT - A fitter written by Jonas Rademacker that specialises in 4 body amplitude analyses.

Time Dependent Amplitude Analysis of $D^0 \rightarrow K^+\pi^-\pi^+\pi^-$

- What would be more in-keeping with this school, and very interesting, is a time dependent amplitude analysis.
- Realistically this would be difficult...
  - Very clean samples are desirable for an amplitude analysis.
  - Would have to work in 5+1 dimensions - MINT cannot yet do this
- At the moment such an analysis is out of reach - but all is not lost...
  - Go Model Independent!

Model Independent Analysis of $D^0 \to K^+\pi^-\pi^+\pi^-$

- To go model independent we integrate over phase space...

$$\mathrm{R}\left[D^0 \to K^+\pi^-\pi^+\pi^-\right](t) = \int \mathrm{R}\left[D^0 \to K^+\pi^-\pi^+\pi^-\right](\mathbf{p}, t)\mathrm{d}\mathbf{p}$$
$$= e^{-\Gamma t}\left[\mathcal{A}^2_{\mathrm{DCS}} + \mathcal{A}_{\mathrm{DCS}}\mathcal{A}_{\mathrm{CF}} R_D^{K3\pi} y'\Gamma t + \mathcal{A}^2_{\mathrm{CF}}\frac{x^2+y^2}{4}(\Gamma t)^2\right]$$

where now $\quad y' = y\cos\delta_D^{K3\pi} - x\sin\delta_D^{K3\pi}$

- We now have some new quantities in the rate...

$\mathcal{A}^2_{\mathrm{CF}} = \int |\mathcal{A}_{\mathrm{CF}}(\mathbf{p})|^2 \mathrm{d}\mathbf{p}$

$\mathcal{A}^2_{\mathrm{DCS}} = \int |\mathcal{A}_{\mathrm{DCS}}(\mathbf{p})|^2 \mathrm{d}\mathbf{p}$

$\delta_D^{K3\pi}$ - *Average* Strong Phase Difference

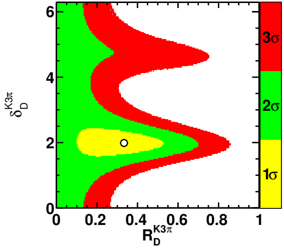
$R_D^{K3\pi}$ - Coherence Factor $\in [0, 1]$

Coherence Factor & Average Strong Phase Difference

$$\frac{\int \mathcal{A}(\mathbf{p})_{\mathrm{DCS}}\mathcal{A}^*(\mathbf{p})_{\mathrm{CF}}\ \mathrm{d}\mathbf{p}}{\mathcal{A}_{\mathrm{DCS}}\mathcal{A}_{\mathrm{CF}}} \equiv R_D^f e^{-i\delta_D^f}$$

$$\mathcal{A}^2_{\mathrm{DCS}} = \int |\mathcal{A}(\mathbf{p})_{\mathrm{DCS}}|^2 \mathrm{d}\mathbf{p}$$

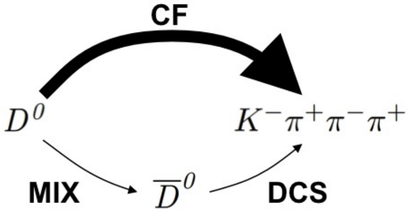
$$\mathcal{A}^2_{\mathrm{CF}} = \int |\mathcal{A}(\mathbf{p})_{\mathrm{CF}}|^2 \mathrm{d}\mathbf{p}$$



- The coherence factor $R_D^{K3\pi}$ gives a measure of how much the interference is diluted from integrating over phase space.
- δ_D^f is the average strong phase difference between amplitudes.
- Current constraints on $R_D^{K3\pi} - \delta_D^{K3\pi}$ from CLEO-c shown in the figure. [1]

[1] Phys.Rev. D80, 031105, 2009

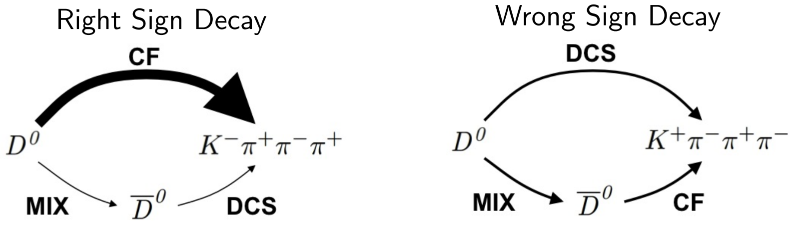
Right Sign $D^0 \to K^-\pi^+\pi^-\pi^+$ Decays



$$\mathrm{R}\left[D^0 \to K^-\pi^+\pi^-\pi^+\right](t) = \mathcal{A}^2_{\mathrm{CF}} e^{-\Gamma t}$$

- Use Right Sign decays as a normalisation channel
- These are completely dominated by the Cabibbo Favoured Amplitude (no Mixing).

WS to RS ratio



$$r(t) = \frac{\mathrm{R}\left[D^0 \to K^+\pi^-\pi^+\pi^-\right](t)}{\mathrm{R}\left[D^0 \to K^-\pi^+\pi^-\pi^+\right](t)} = r_D^2 + r_D R_D^{K3\pi} y' \Gamma t + \frac{x^2+y^2}{4} (\Gamma t)^2$$

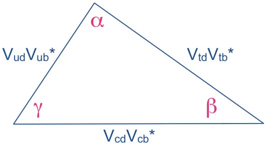
- By taking the ratio of WS to RS decays it is possible to cancel many detection and selection efficiencies
- r_D is the ratio $\mathcal{A}_{\mathrm{DCS}}/\mathcal{A}_{\mathrm{CF}}$

CKM phase γ

CKM complex phase γ

- CP violation enters the standard model through complex phases in the CKM matrix.
- By requiring the CKM matrix to be unitary, one can define several 'unitary triangles'
- The most interesting of these triangles is:

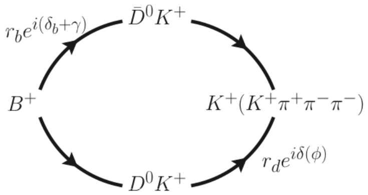
$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$



- γ is the least constrained angle of the triangle.

CKM complex phase γ

- γ is only accessible when an amplitude involving a $b \to u$ transition, interferes with other amplitudes.
- One method is using $B^+ \to DK^+$ decays where here D represents a D^0 or a $\bar{D}^0$.
- If the D meson decays to a final state that is accessible from both a D^0 and a $\bar{D}^0$, we have the required interference to extract gamma i.e. $D \to K^+\pi^-\pi^+\pi^-$.



From $R_D^{K3\pi}$ and $\delta_D^{K3\pi}$ to gamma

- These are the rates for a model independent (phase space integrated) measurement of γ from $B^+ \to DK^+$ where $D \to K^-\pi^+\pi^-\pi^+$

$$\mathrm{R}\big(B^- \to DK^-,\ D \to K^+\pi^-\pi^+\pi^-\big) \propto r_B^2 + r_D^2 + 2 r_B r_D R_D^{K3\pi} \cos(\delta_D^{K3\pi} + \delta_B - \gamma)$$

$$\mathrm{R}\big(B^+ \to DK^+,\ D \to K^-\pi^+\pi^-\pi^+\big) \propto r_B^2 + r_D^2 + 2 r_B r_D R_D^{K3\pi} \cos(\delta_D^{K3\pi} + \delta_B + \gamma)$$

- The highlighted parameters also appear in our model independent mixing rates.
- Maybe we can learn something about these through mixing? Then apply these to a γ measurement.

Constraining the $D \to K^+\pi^-\pi^+\pi^-$ Coherence Factor Using D Mixing

WS to RS ratio

- As a reminder, this is the theoretical rate for the WS to RS ratio:

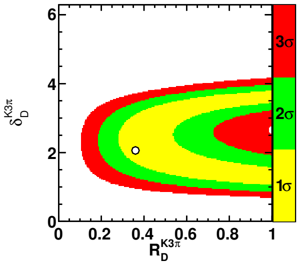
$$r(t) = \frac{\mathrm{R}\left[D^0 \to K^+\pi^-\pi^+\pi^-\right](t)}{\mathrm{R}[D^0 \to K^-\pi^+\pi^-\pi^+](t)} = r_D^2 + r_D R_D^{K3\pi} y' \Gamma t + \frac{x^2+y^2}{4} (\Gamma t)^2$$

$$y' = y \cos \delta_D^{K3\pi} - x \sin \delta_D^{K3\pi}$$

- Usually one would associate this with a mixing analysis i.e. constraining x and y.
- We are turning this around, using previous measurements of x and y to constrain $R_D^{K3\pi}$ and $\delta_D^{K3\pi}$.

Toy Simulation Studies

- Generate 8 million Right Sign + Wrong Sign toy events using the CLEO-c central values for $R_D^{K3\pi}$ and $\delta_D^{K3\pi}$.
- This is the approximate statistics expected from 2011+2012 data taking at LHCb

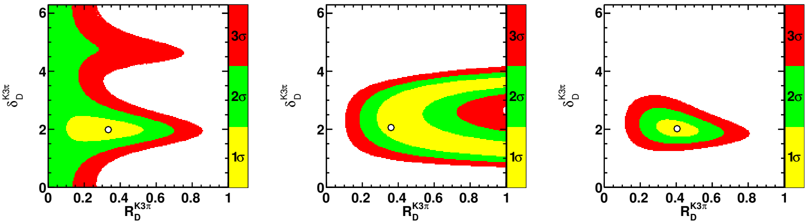


- Use the toy data to extract constraints on $R_D^{K3\pi}$ and $\delta_D^{K3\pi}$. [2]

[2] arXiv:1309.0134

Toy Simulation Studies

CLEO-C DATA **TOY SIMULATION** **COMBINATION**



- The real power of this analysis can be seen when combining with CLEO-c. [2]
  - Considerable improvement in $R_D^{K3\pi}$ and $\delta_D^{K3\pi}$ constraints.
- Analysis with LHCb data in progress.

[2] arXiv:1309.0134

Model Inspired Binning

Binned Coherence Factor

- So far we have considered R_D^f and δ_D^f obtained by integrating over the entire kinematically allowed region.
- Can also measure the same quantities for subsets of this region...

$$\frac{\int_\Omega \mathcal{A}(\mathbf{p})_{\mathrm{DCS}}\mathcal{A}^*(\mathbf{p})_{\mathrm{CF}}\ \mathrm{d}\mathbf{p}}{\mathcal{A}_{\mathrm{DCS}}\mathcal{A}_{\mathrm{CF}}} \equiv R_D^{f\in\Omega} e^{-i\delta_D^{f\in\Omega}}$$

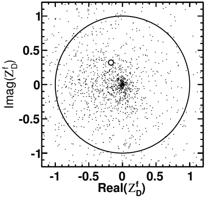
- Measure γ though an interference effect.
  - Want the dilution of the interference to be as small as possible.
  - Therefore want $R_D^{f\in\Omega}$ close to 1.0
- Can we devise a binning strategy that makes $R_D^{f\in\Omega}$ as large as possible?

Model Inspired Binning

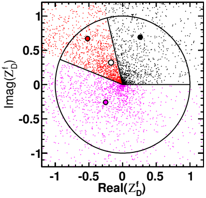
- A DCS and CF model would allow us to bin in areas of similar strong phase difference.
- Small dots show the integrand evaluated at random points in phase space - the coherence factor is the average of these.

$$I(\mathbf{p}) = \frac{\mathcal{A}(\mathbf{p})_{\mathrm{DCS}}\mathcal{A}^*(\mathbf{p})_{\mathrm{CF}}}{\mathcal{A}_{\mathrm{DCS}}\mathcal{A}_{\mathrm{CF}}} \qquad \int_\Omega I(\mathbf{p})\ \mathrm{d}\mathbf{p} \equiv R_D^{f\in\Omega} e^{-i\delta_D^{f\in\Omega}} \equiv Z_D^{f\in\Omega}$$

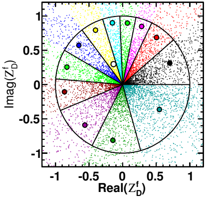
1 BIN



3 BINS

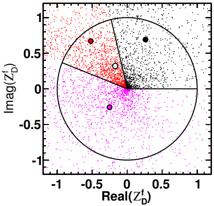


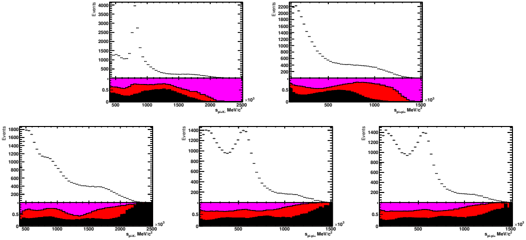
12 BINS



Binned Coherence Factor

- For interest, we can look how these bins are distributed in the projections...





- IMPORTANT: Such a binning requires a CF and a DCS amplitude model - this toy study assumes perfect models.
- This is called 'model inspired' binning

Conclusions

- LHCb is expected to make a significant improvement to the constraints on the $K\pi\pi\pi$ Coherence Factor and Strong Phase Difference.
- These quantities let us sweep the rich amplitude structure under the carpet...
  - Negatives: Loss in sensitivity due to less information.
  - Positives: Much simpler method, and no model systematics.
- In the future sensitivity to γ could be increased by using a **model inspired** binning.