The New Finite-Range Droplet-Model Mass Table FRDM(2012)

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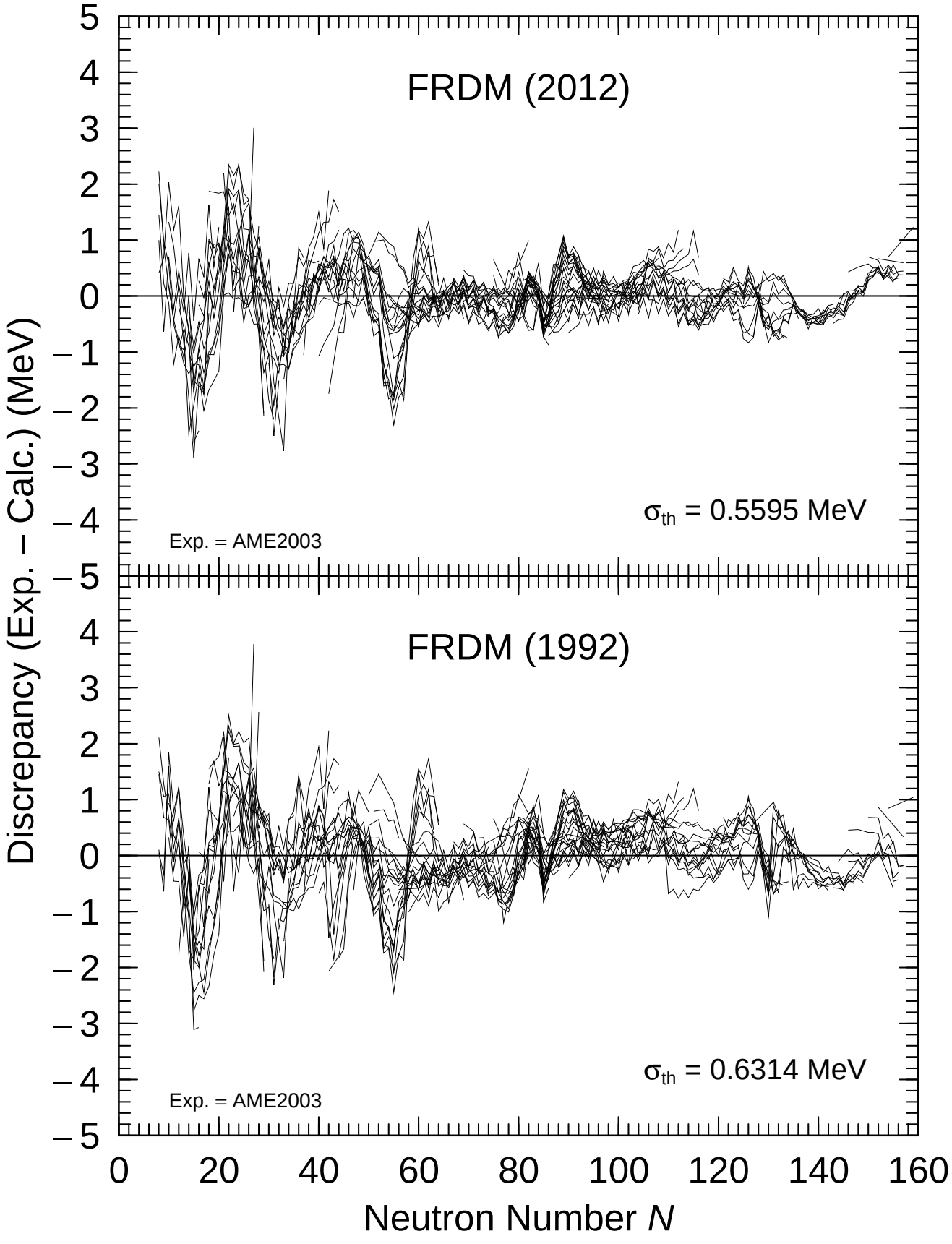
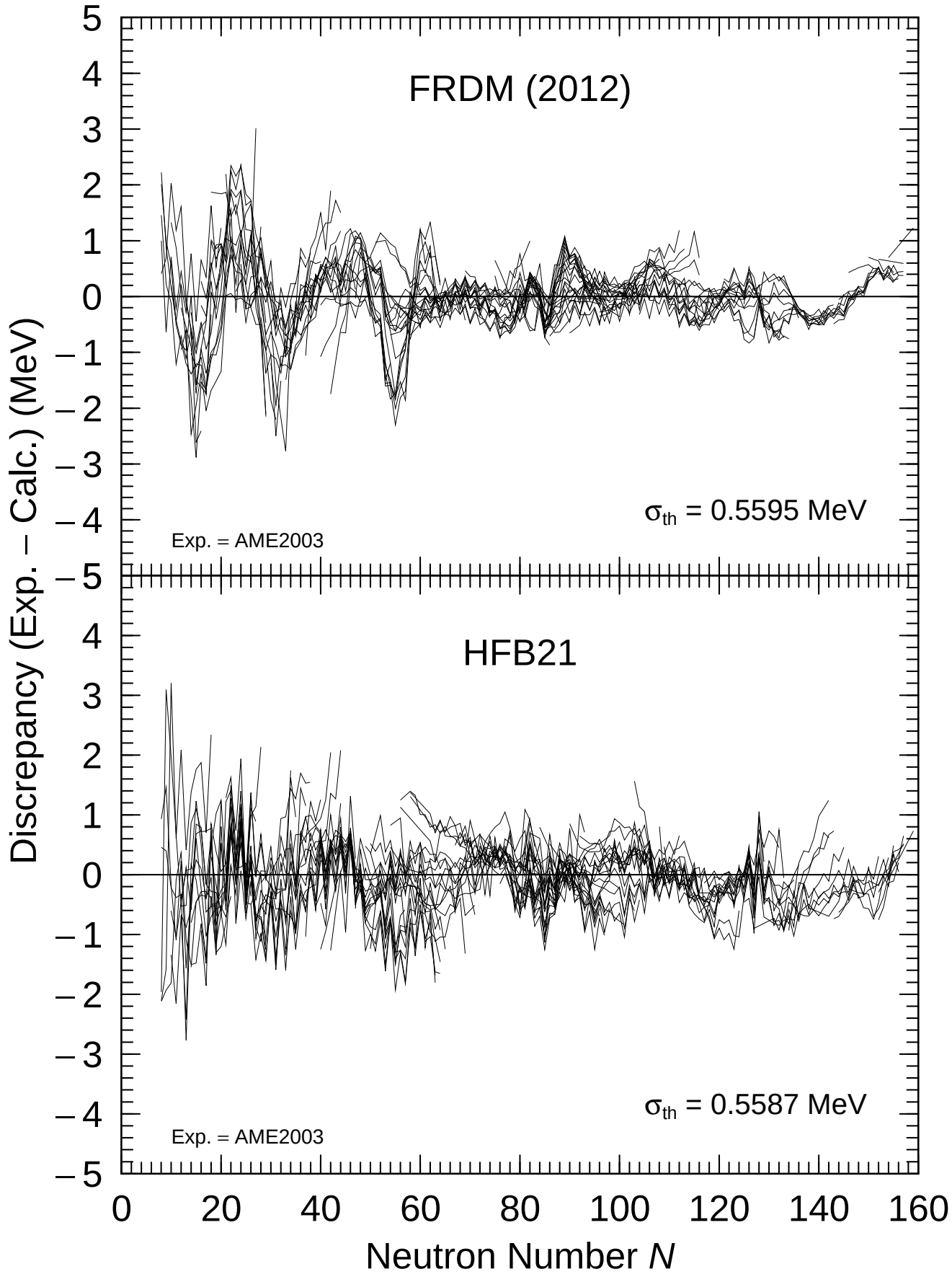
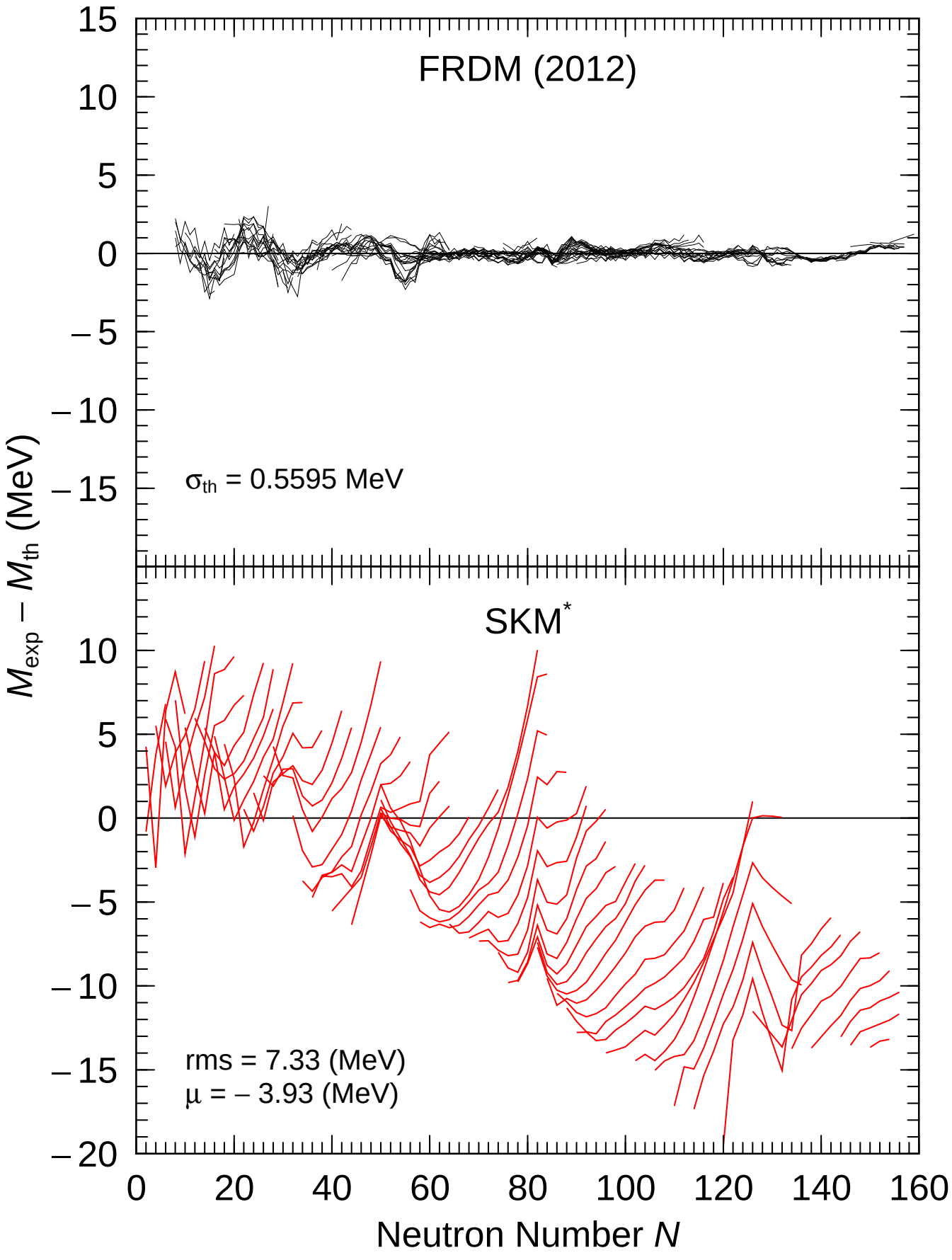
Collaborators on this and other projects:
W. D. Myers, J. Randrup(LBL), H. Sagawa (Aizu), S. Yoshida (Hosei), T. Ichikawa(YITP), A. J. Sierk(LANL), A. Iwamoto (JAEA), S. Aberg (Lund), R. Bengtsson (Lund), S. Gupta (IIT, Ropar), and many experimental groups (e. g. K.-L. Kratz (Mainz), H. Schatz (MSU), A. Andreyev (York) . . .).

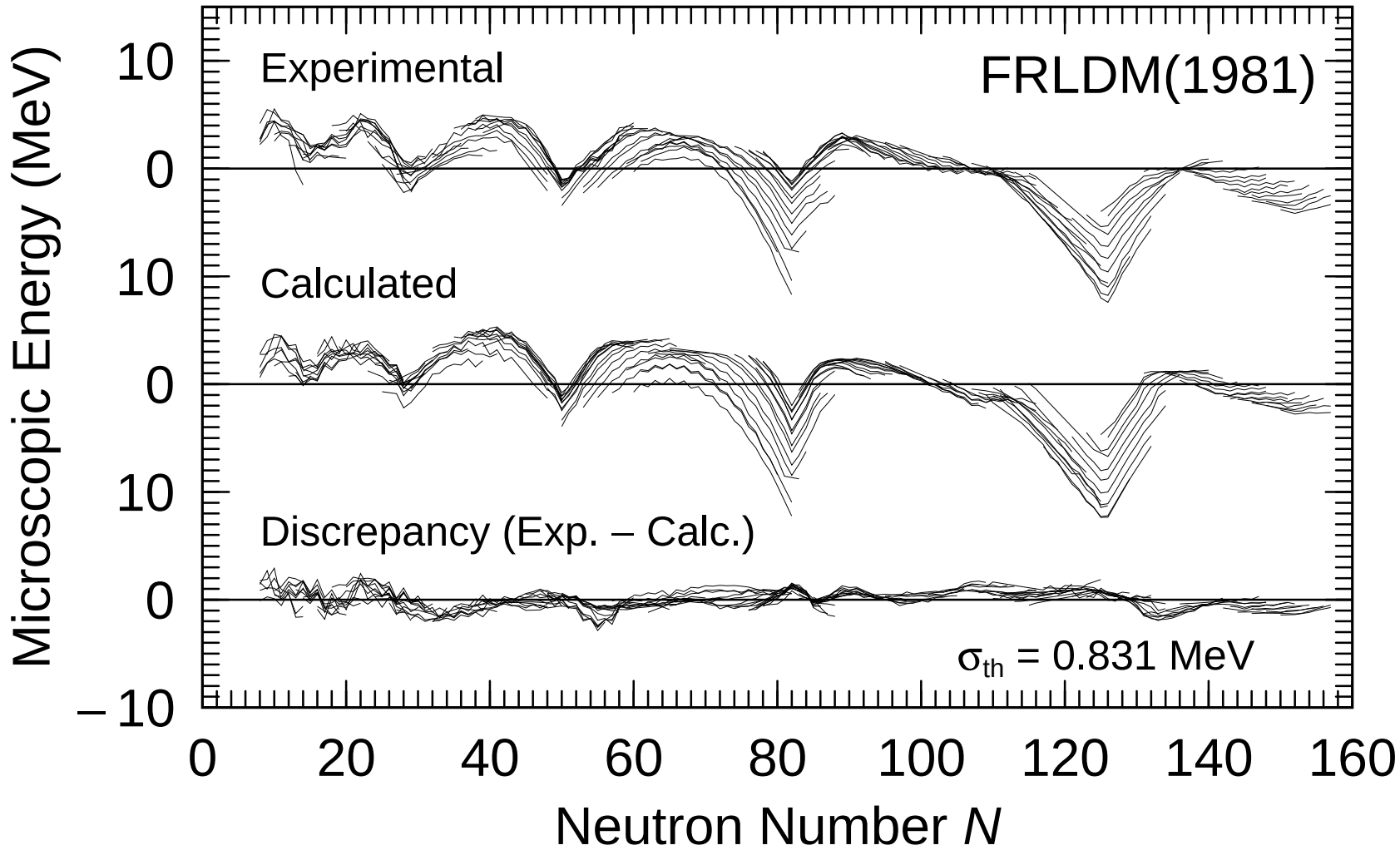
More details about masses, other projects (beta-decay,fission), associated ASCII data files, interactive access to data (type in Z, A and get specific data, contour maps) and figures are at

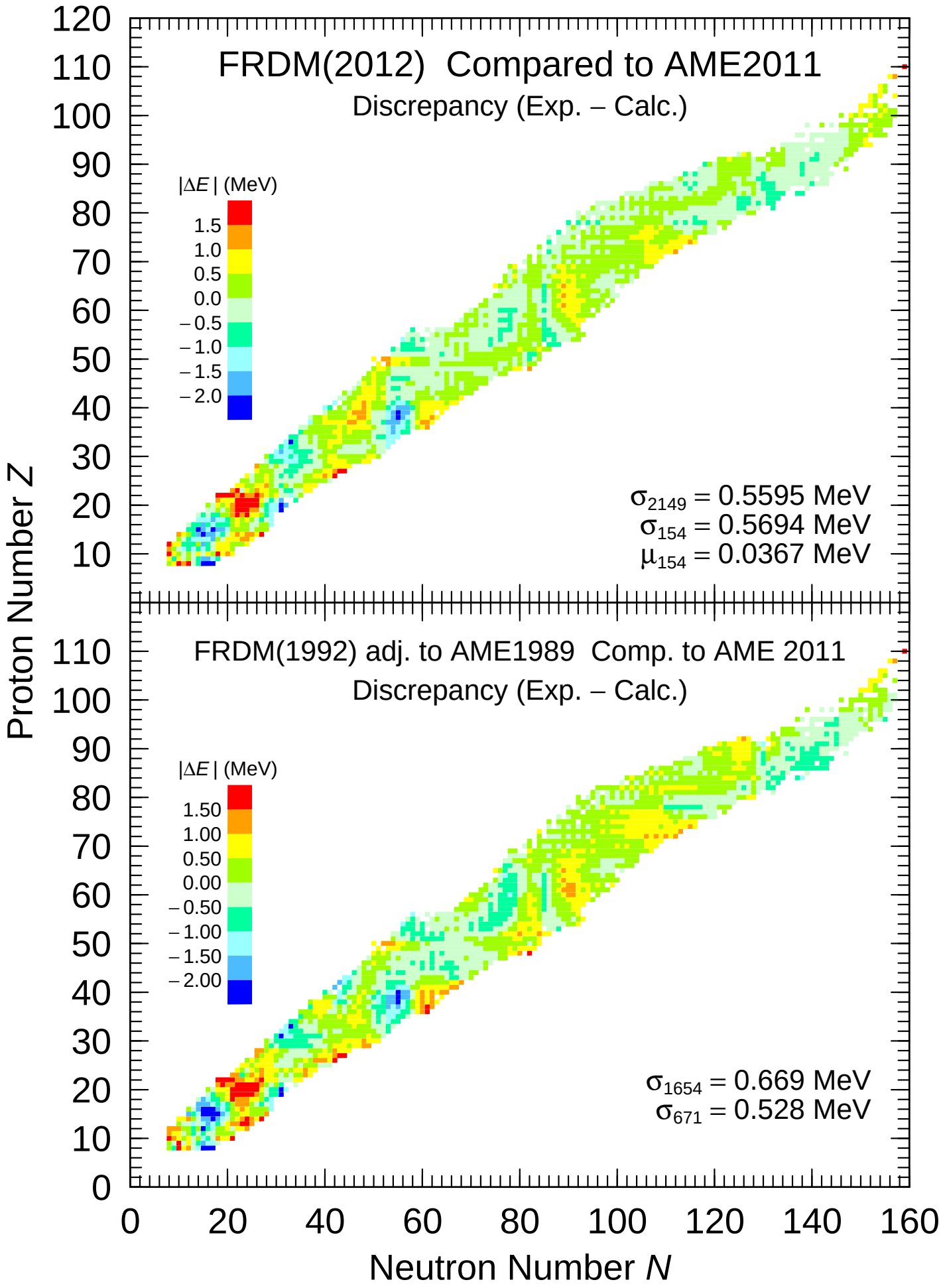
http://t2.lanl.gov/nis/molleretal/


FRDM (2012)
σth = 0.5595 MeV
Exp. = AME2003
FRDM (1992)
σth = 0.6314 MeV
Exp. = AME2003
Discrepancy (Exp. – Calc.) (MeV)
Neutron Number N
5
4
3
2
1
0
–1
–2
–3
–4
–5
0
20
40
60
80
100
120
140
160


FRDM (2012)
σth = 0.5595 MeV
Exp. = AME2003
HFB21
σth = 0.5587 MeV
Exp. = AME2003
Discrepancy (Exp. – Calc.) (MeV)
Neutron Number N
5
4
3
2
1
0
–1
–2
–3
–4
–5
0
20
40
60
80
100
120
140
160


FRDM (2012)
σth = 0.5595 MeV
SKM*
rms = 7.33 (MeV)
μ = – 3.93 (MeV)
Mexp – Mth (MeV)
Neutron Number N
15
10
5
0
–5
–10
–15
–20
0
20
40
60
80
100
120
140
160


Experimental
FRLDM(1981)
Calculated
Discrepancy (Exp. – Calc.)
σth = 0.831 MeV
Microscopic Energy (MeV)
10
0
10
0
10
0
– 10
0
20
40
60
80
100
120
140
160
Neutron Number N


FRDM(2012) Compared to AME2011
Discrepancy (Exp. – Calc.)
|ΔE| (MeV)
1.5
1.0
0.5
0.0
– 0.5
– 1.0
– 1.5
– 2.0
σ2149 = 0.5595 MeV
σ154 = 0.5694 MeV
μ154 = 0.0367 MeV
FRDM(1992) adj. to AME1989 Comp. to AME 2011
Discrepancy (Exp. – Calc.)
|ΔE| (MeV)
1.50
1.00
0.50
0.00
– 0.50
– 1.00
– 1.50
– 2.00
σ1654 = 0.669 MeV
σ671 = 0.528 MeV
Proton Number Z
Neutron Number N
0
10
20
30
40
50
60
70
80
90
100
110
120
140
160

Successive FRDM enhancements

Optimization (2006)

Better search for optimum FRDM parameters.

Accuracy improvement: 0.01 MeV

New mass data base (AME2003) (2006)

Better agreement than with AME1989.

Accuracy improvement: 0.04 MeV

Full 4D energy minimization (2006–2008)

Full 4D minimization($\epsilon_2, \epsilon_3, \epsilon_4, \epsilon_6$) step=0.01.

Accuracy improvement: 0.02 MeV

Axial asymmetry (2002–2006)

Also yields correct SHE gs assignments.

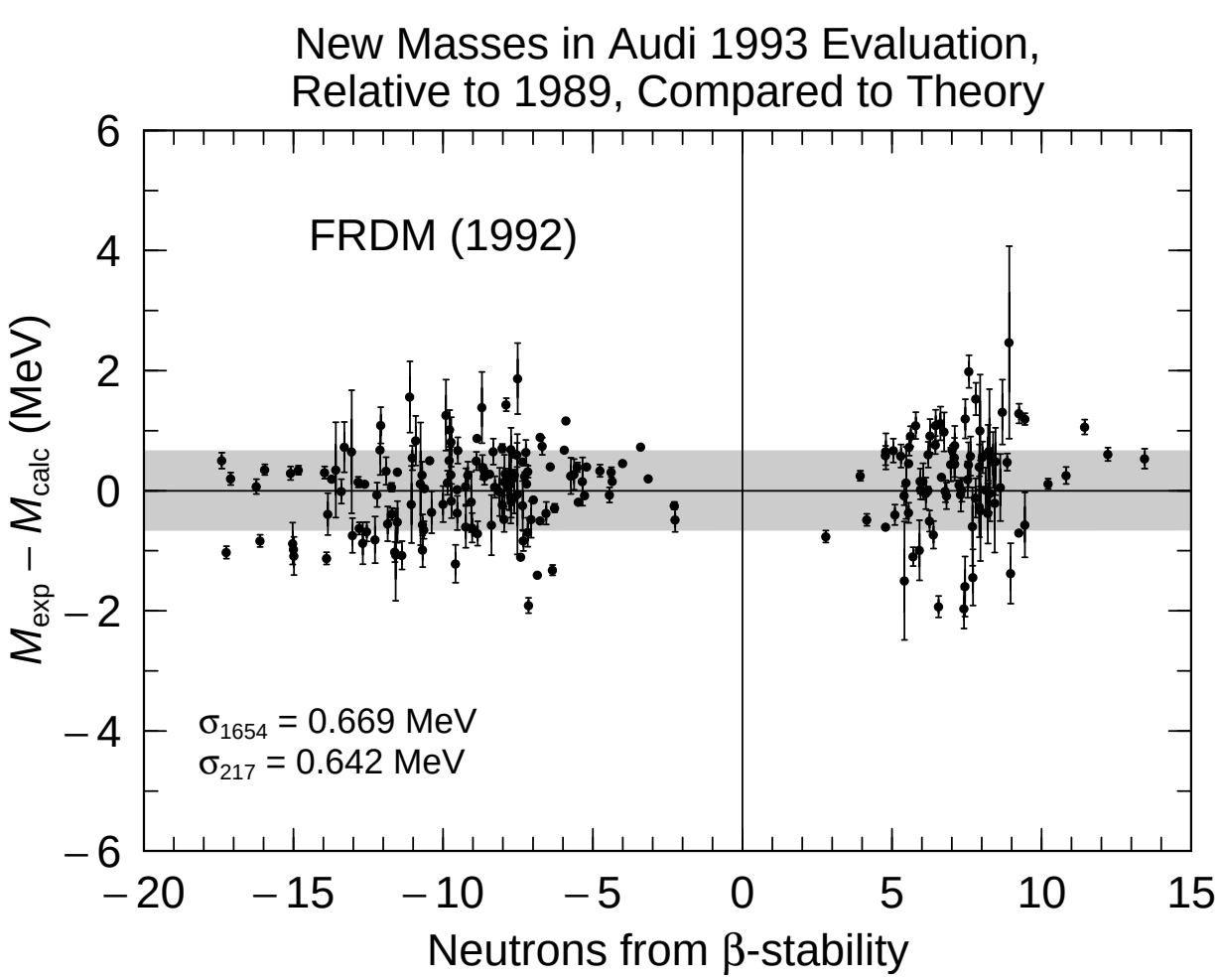
Accuracy improvement: 0.01 MeV

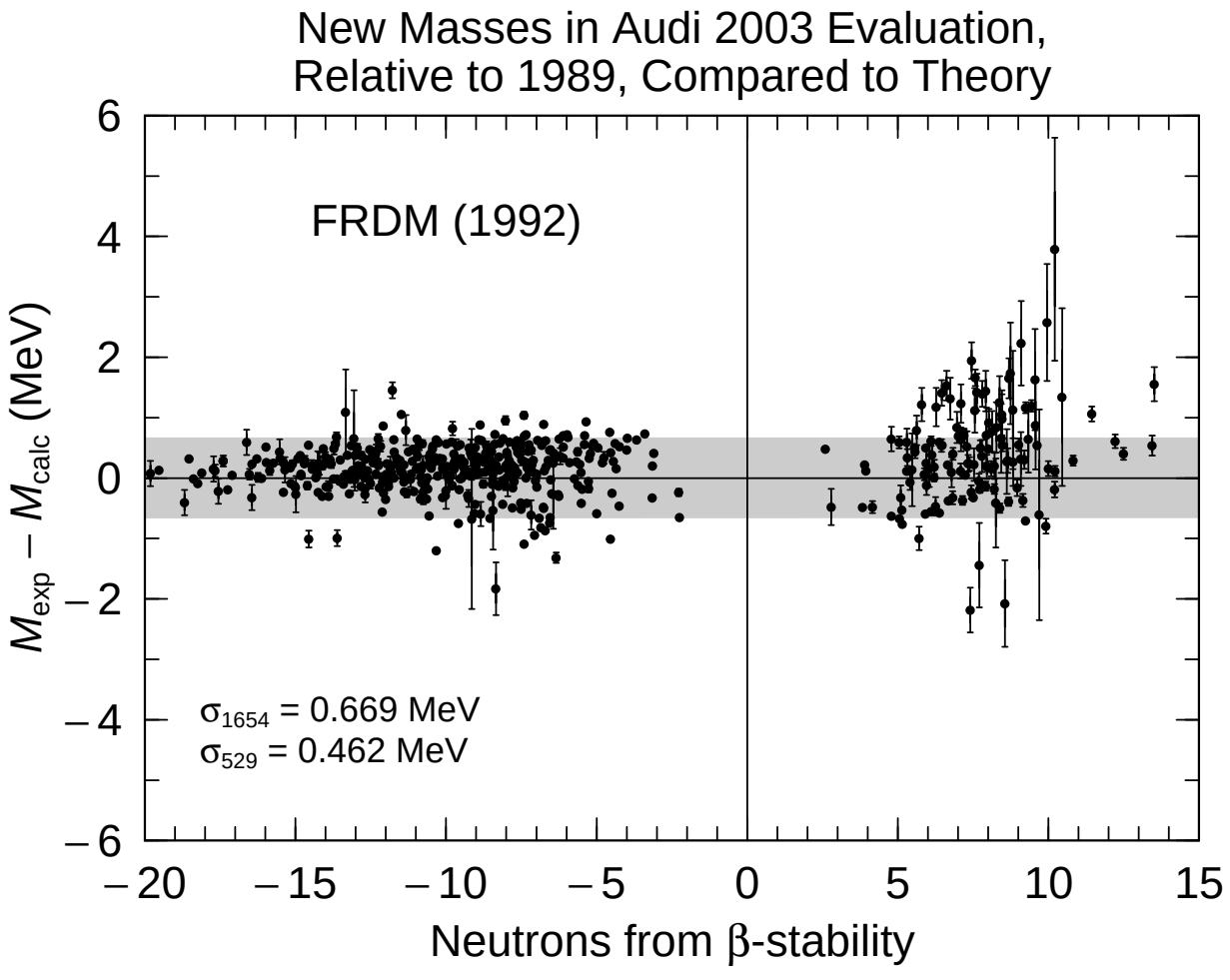
L **variation** (2009–2011)

Accuracy improvement: 0.02 MeV

Improved gs correlation energies (2012)

Accuracy improvement: 0.01 MeV


New Masses in Audi 1993 Evaluation,
Relative to 1989, Compared to Theory
FRDM (1992)
σ1654 = 0.669 MeV
σ217 = 0.642 MeV
Mexp – Mcalc (MeV)
Neutrons from β-stability
6
4
2
0
– 2
– 4
– 6
– 20
– 15
– 10
– 5
0
5
10
15


New Masses in Audi 2003 Evaluation,
Relative to 1989, Compared to Theory
FRDM (1992)
σ1654 = 0.669 MeV
σ529 = 0.462 MeV
Mexp – Mcalc (MeV)
Neutrons from β-stability
6
4
2
0
– 2
– 4
– 6
– 20
– 15
– 10
– 5
0
5
10
15

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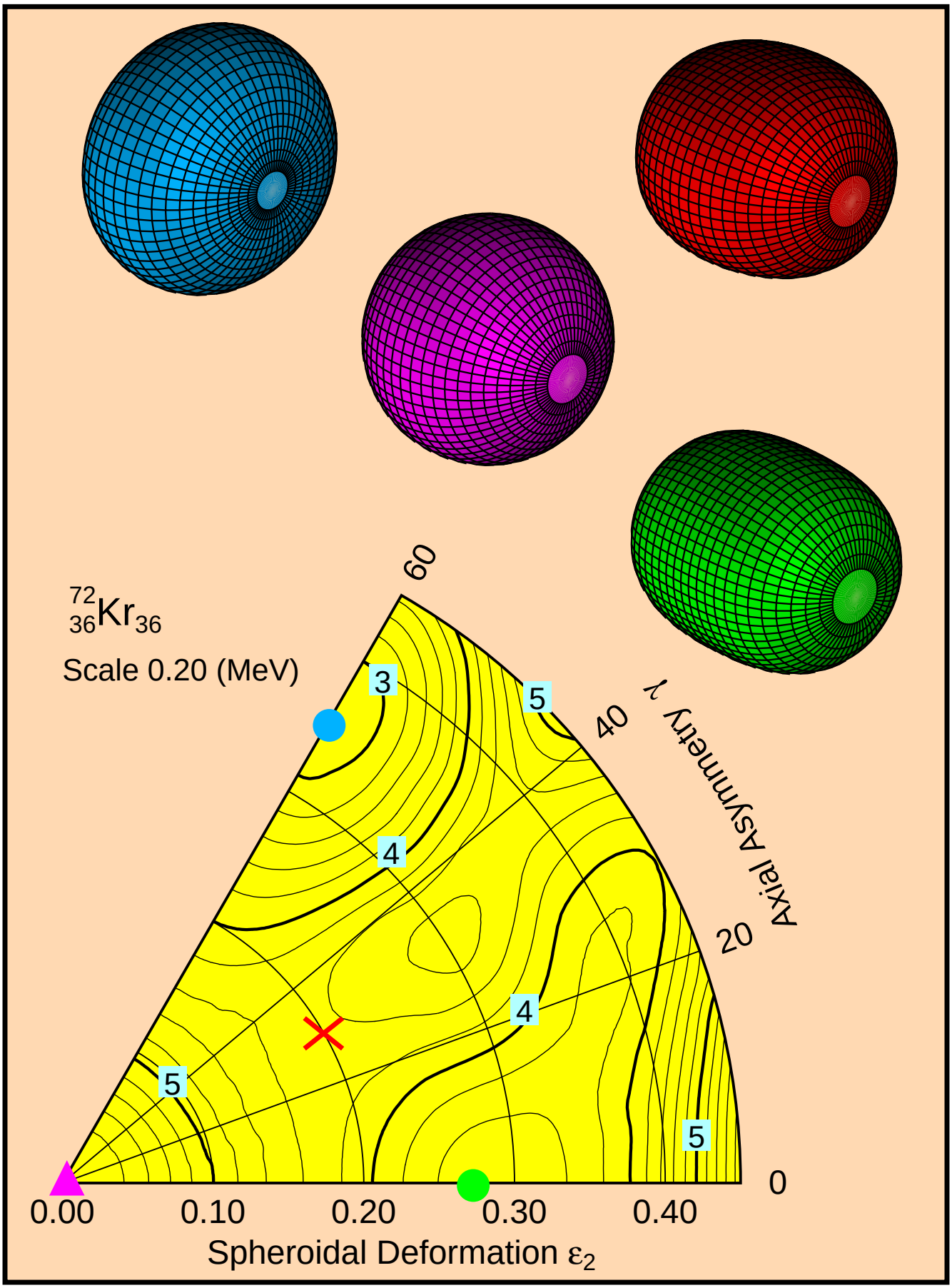
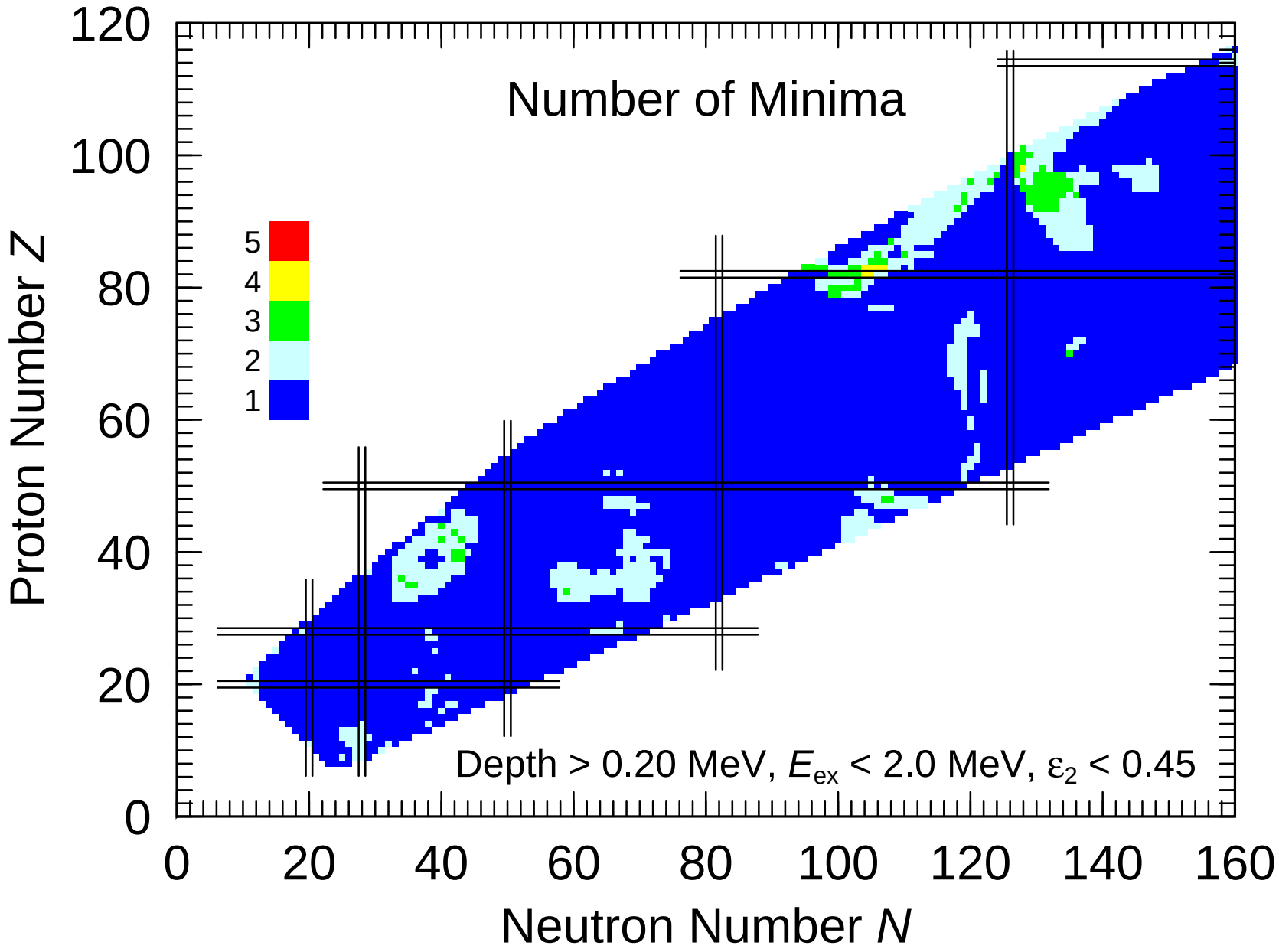
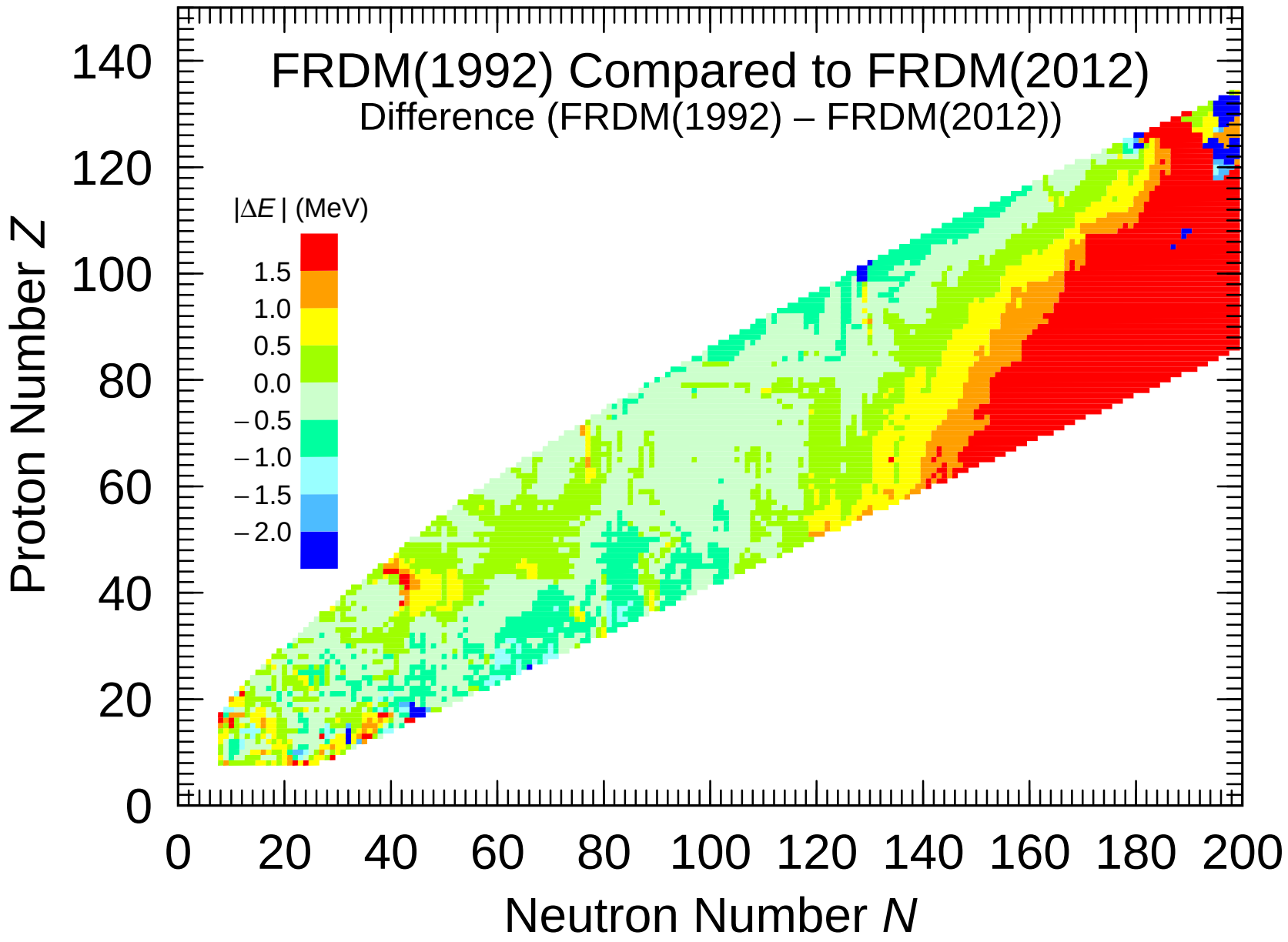
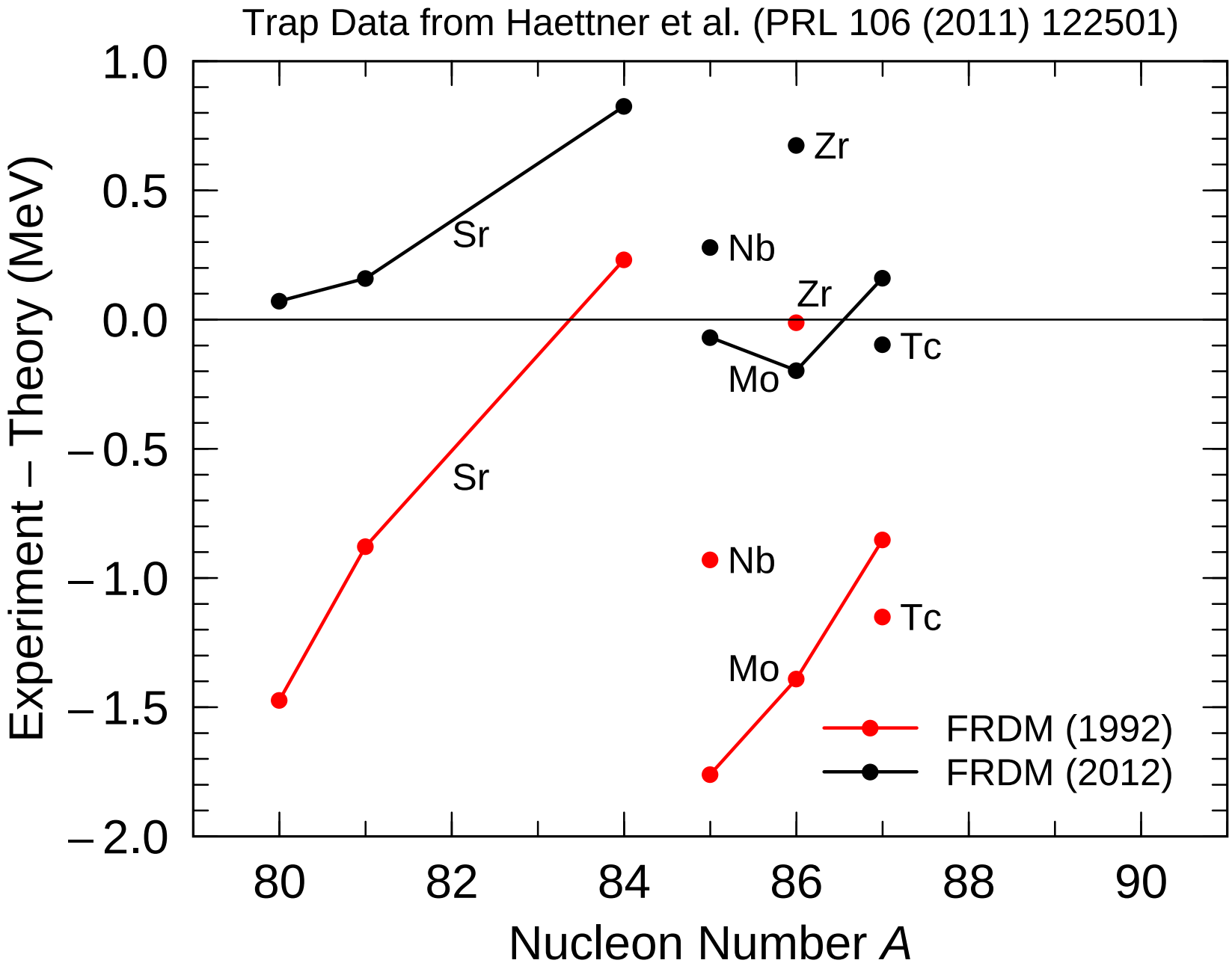

$^{72}_{36}Kr_{36}$
Scale 0.20 (MeV)
60
40
20
0
Axial Asymmetry γ
3
5
4
4
5
5
0.00
0.10
0.20
0.30
0.40
Spheroidal Deformation ε2


Number of Minima
5
4
3
2
1
Depth > 0.20 MeV, E_{ex} < 2.0 MeV, ε_2 < 0.45
Proton Number Z
Neutron Number N
0
20
40
60
80
100
120
140
160


FRDM(1992) Compared to FRDM(2012)
Difference (FRDM(1992) – FRDM(2012))
|ΔE | (MeV)
1.5
1.0
0.5
0.0
– 0.5
– 1.0
– 1.5
– 2.0
Proton Number Z
0
20
40
60
80
100
120
140
Neutron Number N
0
20
40
60
80
100
120
140
160
180
200

Z	A	AME2003 (MeV)	Trap (MeV)	FRDM(1992) (MeV)	Dev.-1992 (MeV)	FRDM(2012) (MeV)	Dev.-2012 (MeV)
38	80	-70.308	-70.313	-68.840	-1.473	-70.385	0.072
38	81	-71.528	-71.528	-70.650	-0.878	-71.688	0.160
38	84	-80.644	-80.648	-80.880	0.232	-81.474	0.826
40	86	-77.800	-77.971	-77.960	-0.011	-78.646	0.675
41	85	-67.150	-66.279	-65.350	-0.929	-66.559	0.280
42	85	-59.100#	-57.510	-55.750	-1.760	-57.441	-0.069
42	86	-64.560	-64.110	-62.720	-1.390	-63.913	-0.197
42	87	-67.690	-66.882	-66.030	-0.852	-67.043	0.161
43	87	-59.120#	-57.690	-56.540	-1.150	-57.786	0.096


Trap Data from Haettner et al. (PRL 106 (2011) 122501)
1.0
0.5
0.0
– 0.5
– 1.0
– 1.5
– 2.0
Experiment – Theory (MeV)
80
82
84
86
88
90
Nucleon Number A
Sr
Zr
Nb
Zr
Tc
Mo
Sr
Nb
Tc
Mo
FRDM (1992)
FRDM (2012)

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Full 4D minimization($\epsilon_2, \epsilon_3, \epsilon_4, \epsilon_6$) step=0.01.

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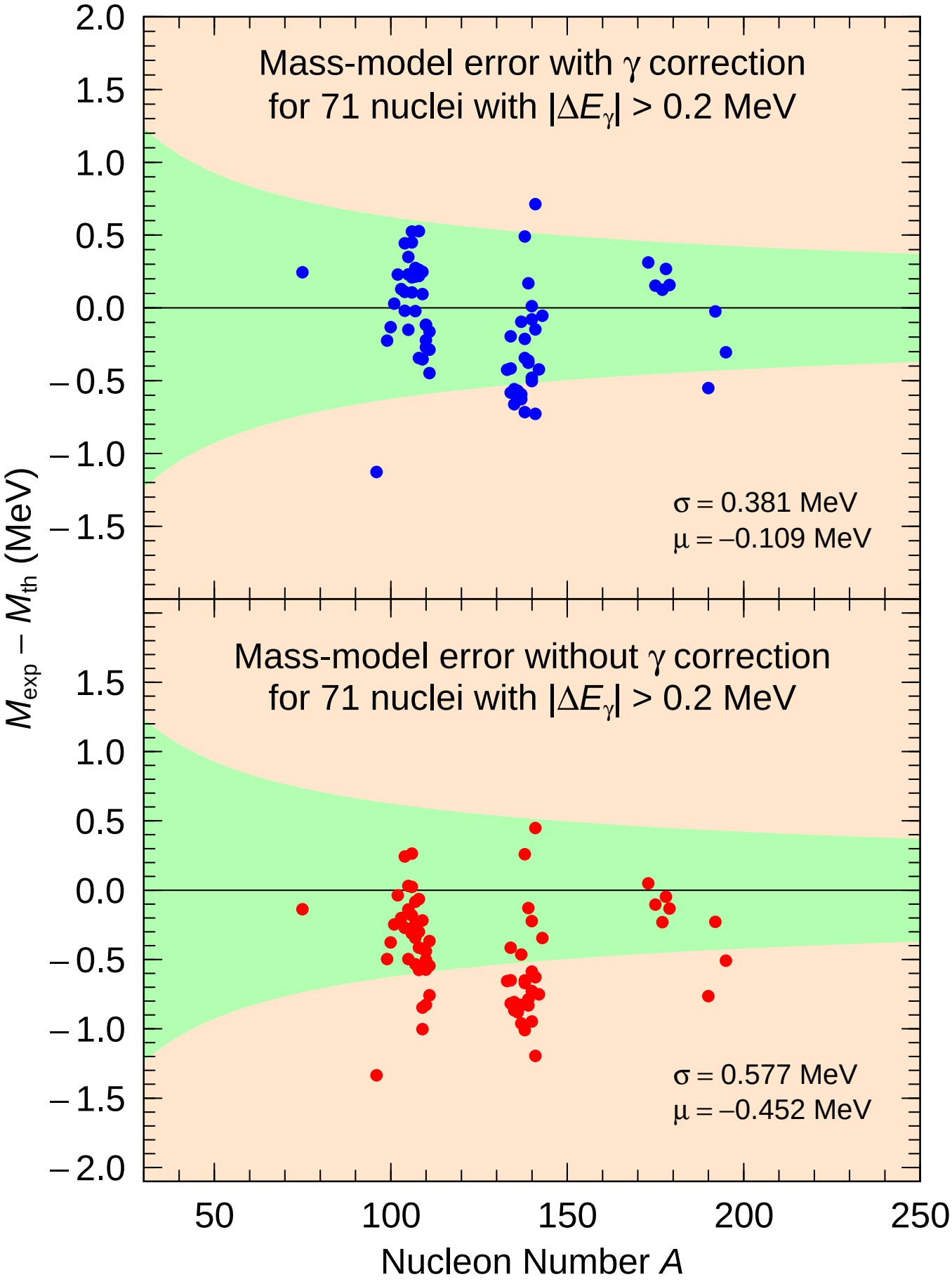
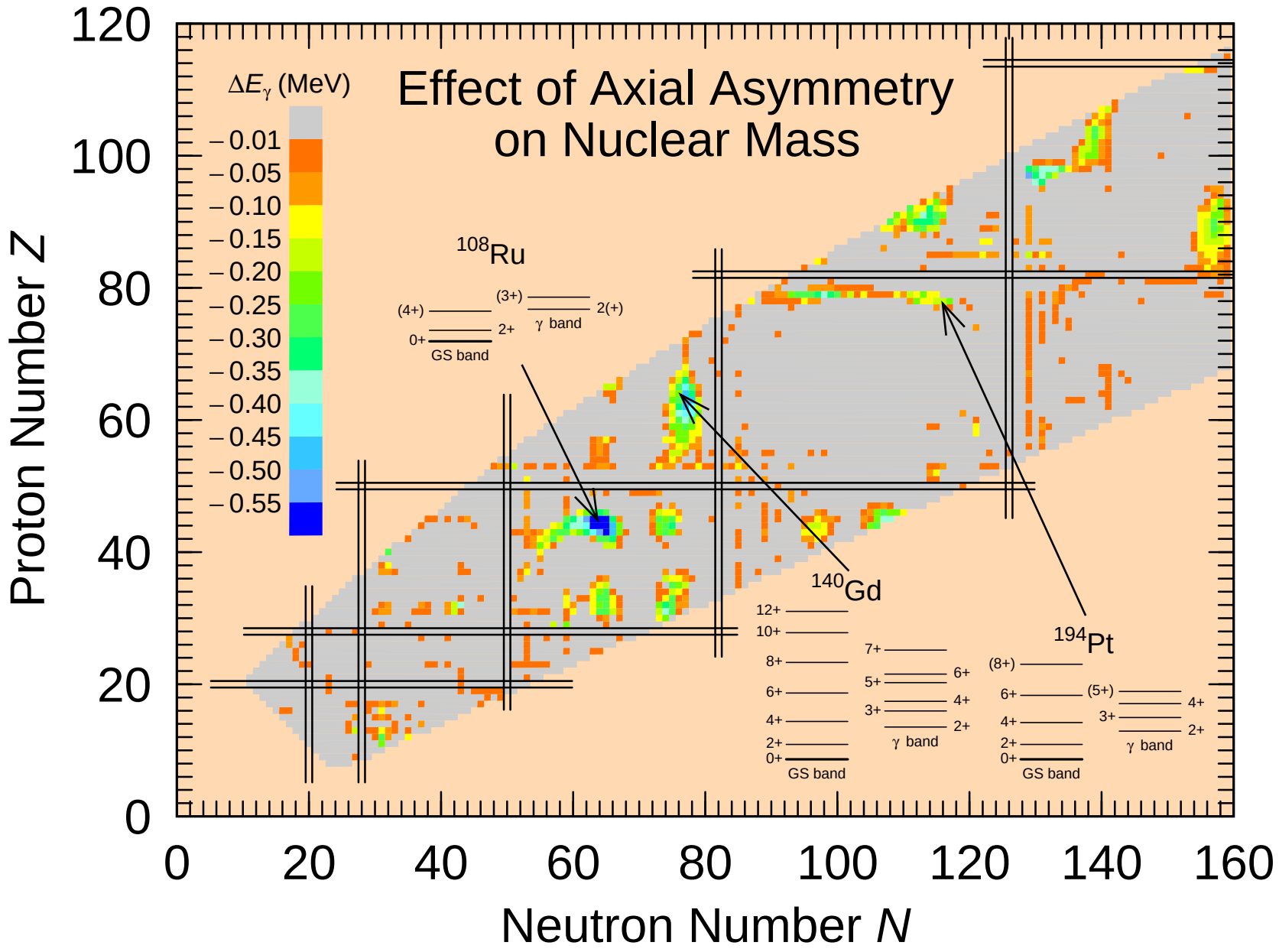
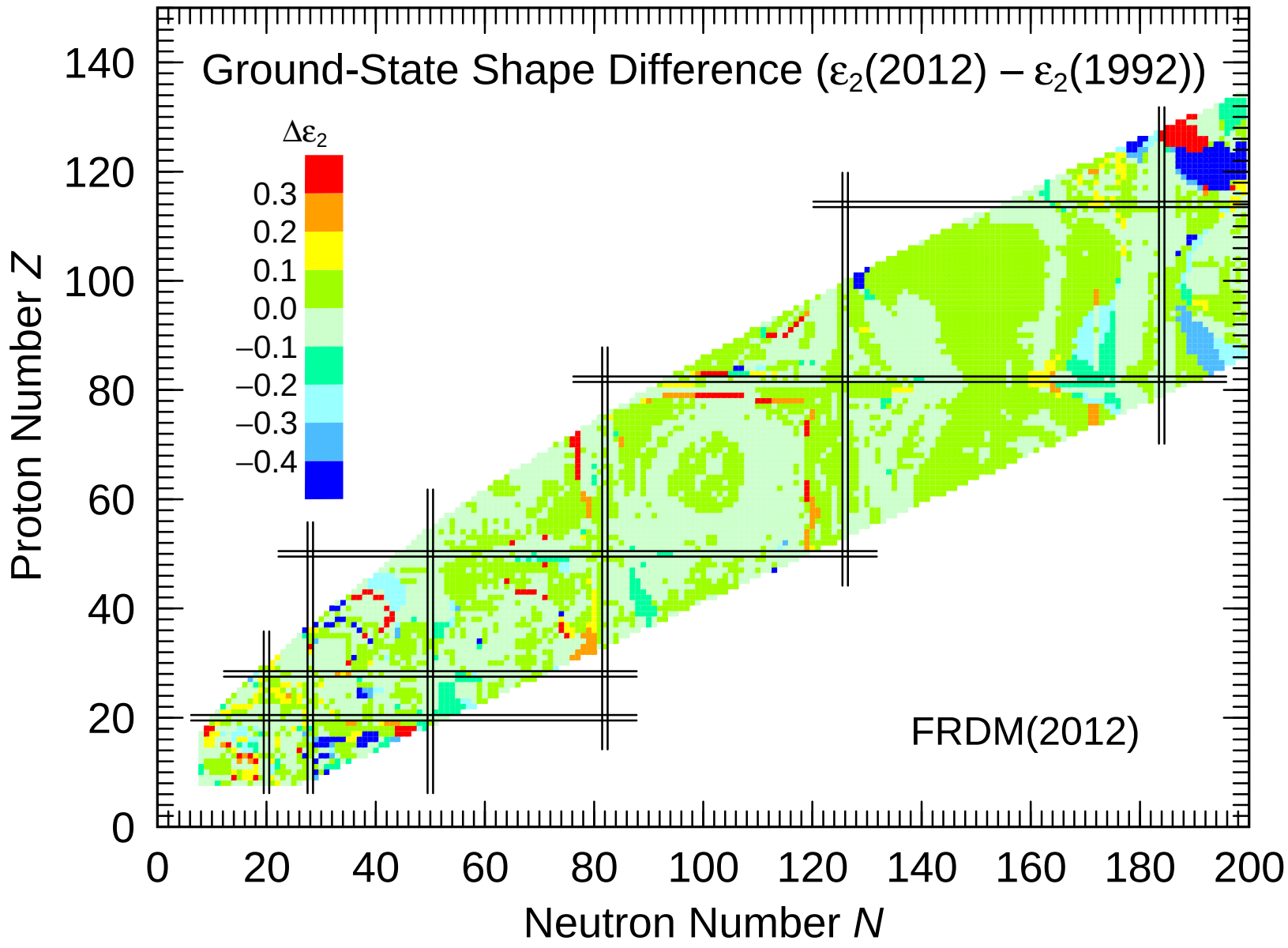
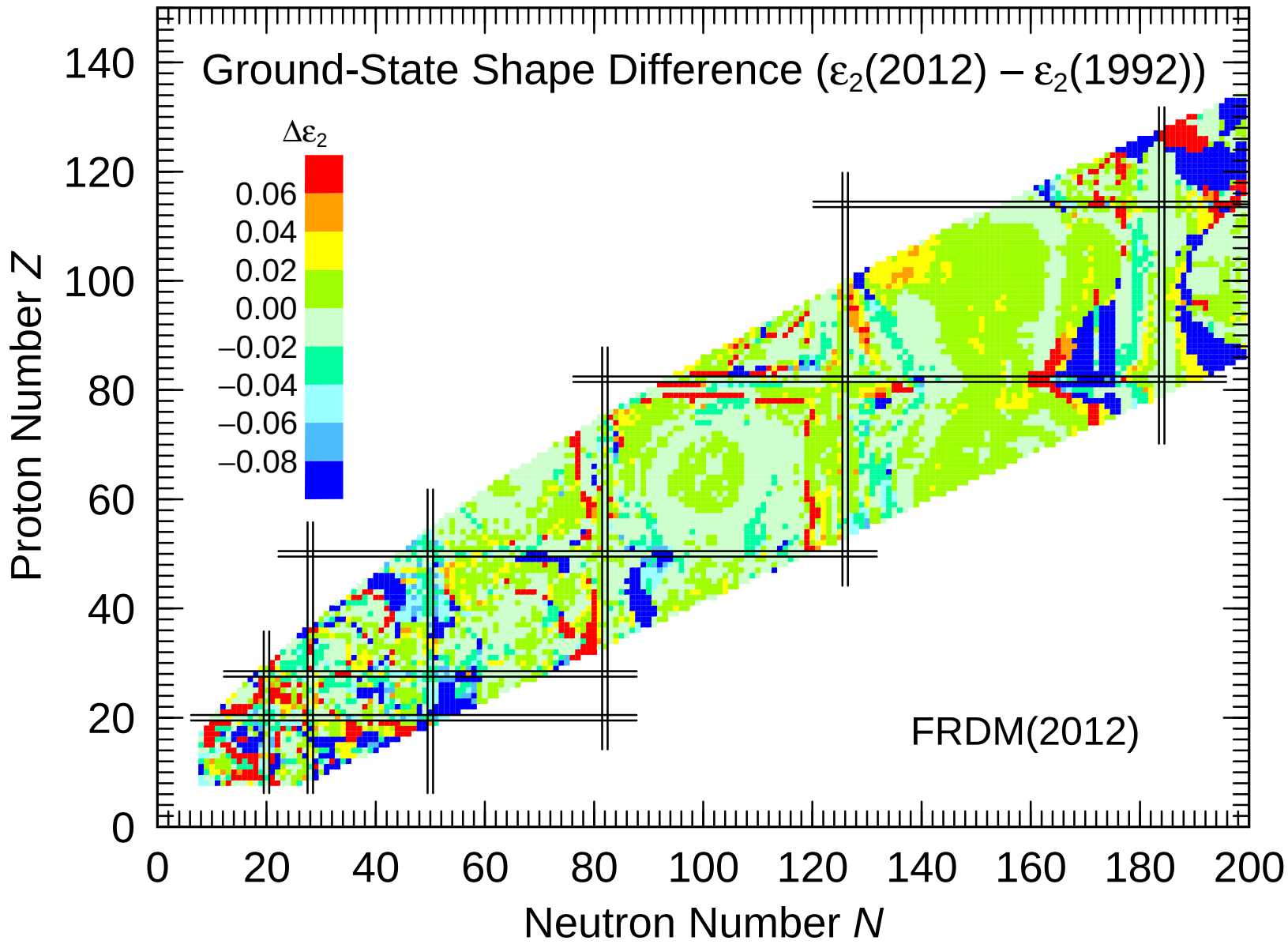
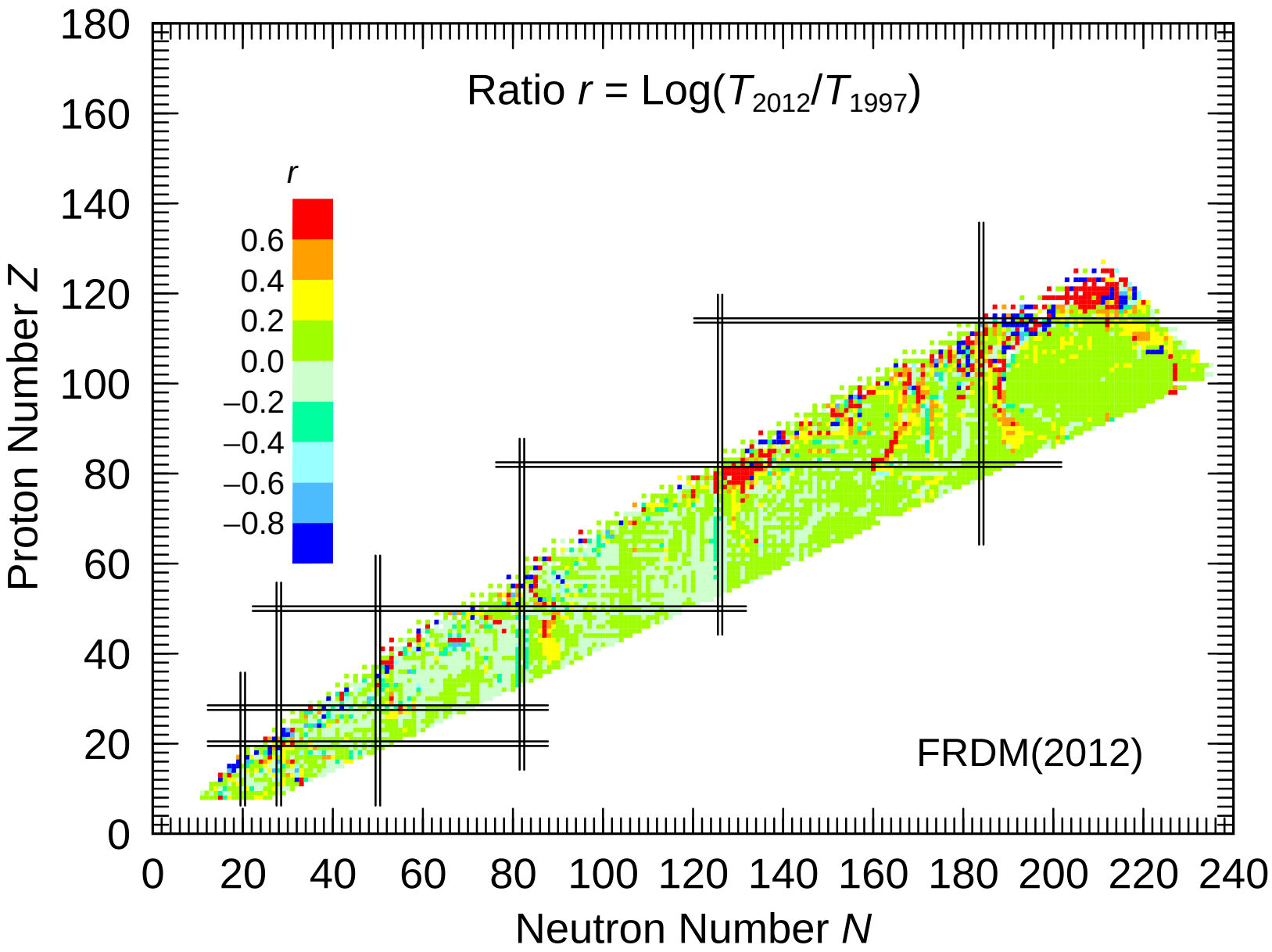

Mass-model error with γ correction
for 71 nuclei with |ΔE_γ| > 0.2 MeV
σ = 0.381 MeV
μ = –0.109 MeV
Mass-model error without γ correction
for 71 nuclei with |ΔE_γ| > 0.2 MeV
σ = 0.577 MeV
μ = –0.452 MeV
M_exp – M_th (MeV)
Nucleon Number A
2.0
1.5
1.0
0.5
0.0
–0.5
–1.0
–1.5
–2.0
50
100
150
200
250


Effect of Axial Asymmetry on Nuclear Mass
ΔE_γ (MeV)
0.01
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
^{108}Ru
(4+)
(3+)
2(+)
2+
0+
γ band
GS band
^{140}Gd
12+
10+
8+
6+
4+
2+
0+
7+
6+
5+
4+
3+
2+
γ band
GS band
^{194}Pt
(8+)
6+
4+
2+
0+
(5+)
4+
3+
2+
γ band
GS band
Proton Number Z
0
20
40
60
80
100
120
Neutron Number N
0
20
40
60
80
100
120
140
160


Ground-State Shape Difference (ε2(2012) – ε2(1992))
Δε2
0.3
0.2
0.1
0.0
–0.1
–0.2
–0.3
–0.4
FRDM(2012)
Proton Number Z
Neutron Number N
0
20
40
60
80
100
120
140
160
180
200

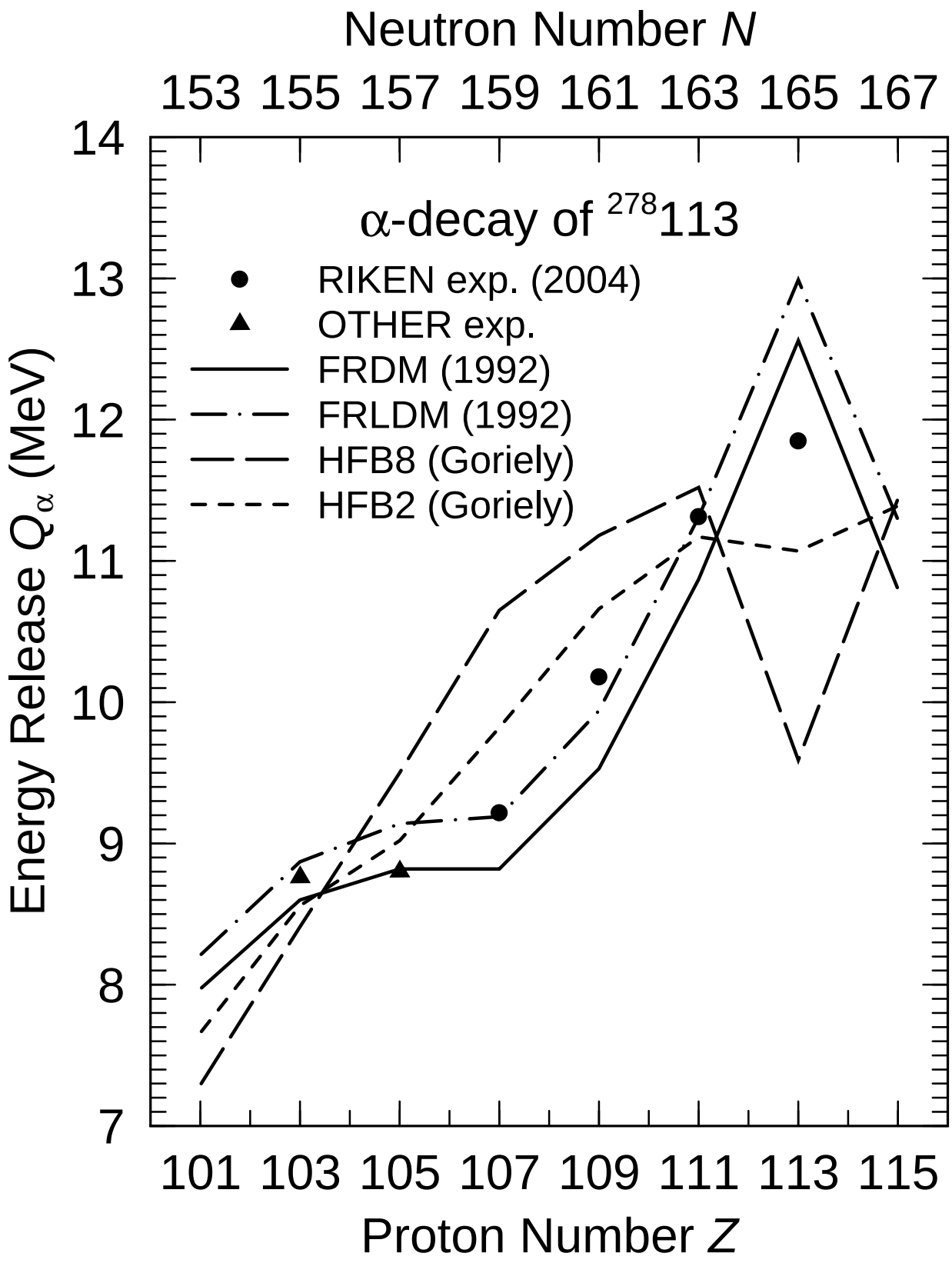

Ground-State Shape Difference (ε2(2012) – ε2(1992))
Δε2
0.06
0.04
0.02
0.00
–0.02
–0.04
–0.06
–0.08
FRDM(2012)
Proton Number Z
Neutron Number N
0
20
40
60
80
100
120
140
160
180
200

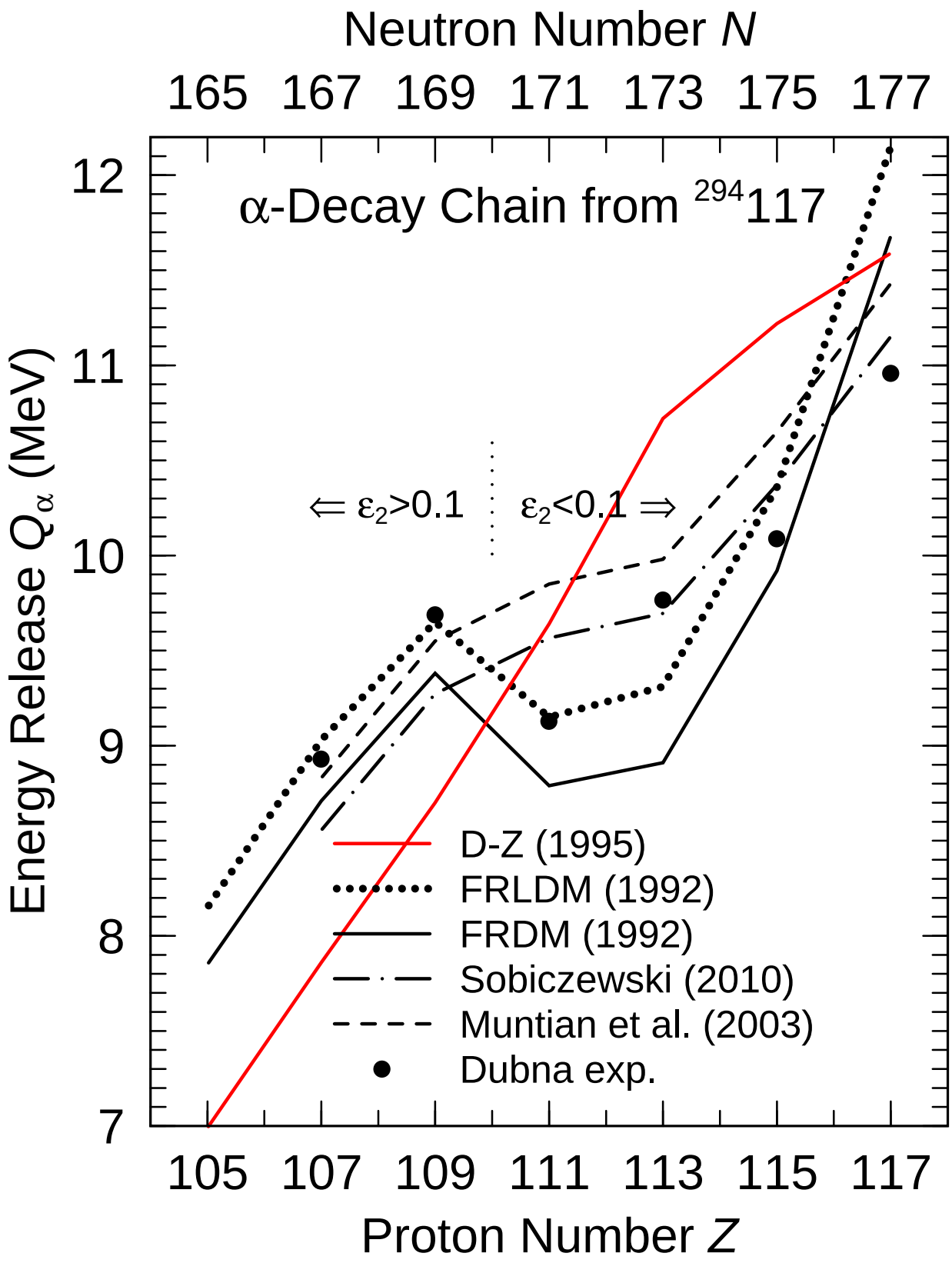

Ratio r = Log(T_{2012}/T_{1997})
r
0.6
0.4
0.2
0.0
–0.2
–0.4
–0.6
–0.8
FRDM(2012)
Proton Number Z
Neutron Number N
0
20
40
60
80
100
120
140
160
180
200
220
240

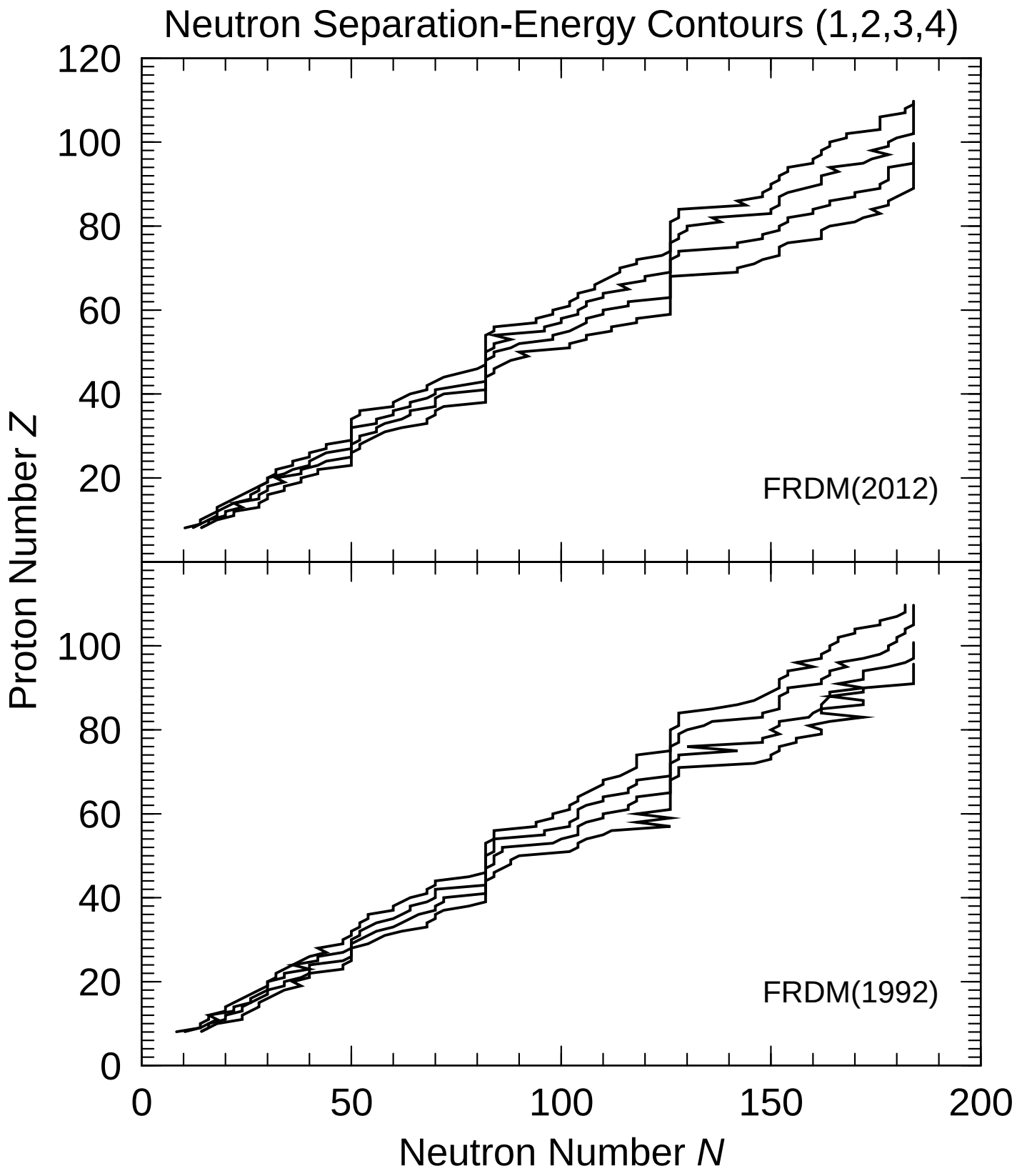
Model	A/C	a_1 (MeV)	a_2 (MeV)	J (MeV)	Q (MeV)	L	μ_{th} (MeV)	$\sigma_{\mathrm{th};\mu=0}$ (MeV)
(92)	1/1	16.247	22.92	32.73	29.21	0	0.0156	0.6688
(92)	1/3						0.1755	0.4617
(92)	1/2						0.0607	0.6314
(92)-a	1/1	16.245	23.02	32.22	30.73	0	0.0000	0.6614
(92)-a	1/3						0.0174	0.4208
(92)-a	1/2						0.0114	0.6180
(92)-b	1/1	16.286	23.37	32.34	30.51	0	0.0000	0.6591
(92)-b	1/3						0.0031	0.4174
(92)-b	1/2						0.0076	0.6157
(06)-a	2/2	16.274	23.27	32.19	30.64	0	0.0000	0.6140
(07)-b	2/2	16.231	22.96	32.11	30.83	0	0.0000	0.5964
(11)-b	2/2	16.231	22.95	32.10	30.78	0	0.0001	0.5863
(11)-c	1/1	16.251	23.10	32.31	30.49	0	−0.0003	0.6300
(11)-c	1/3						0.0144	0.3874
(11)-d	1/1	16.142	22.39	32.98	27.58	85.95	0.0000	0.6092
(11)-d	1/3						0.0545	0.3929
(11)-d	1/2						0.0228	0.5719
(11)-a	2/2	16.147	22.44	32.51	28.54	70.84	−0.0004	0.5700
(11)-a	2/3						-0.0516	0.5618
(12)	**2/2**	**16.195**	**22.763**	**32.30**	**28.72**	**53.50**	**−0.0004**	**0.5595**
(12)	**2/3**						**−0.0367**	**0.5684**

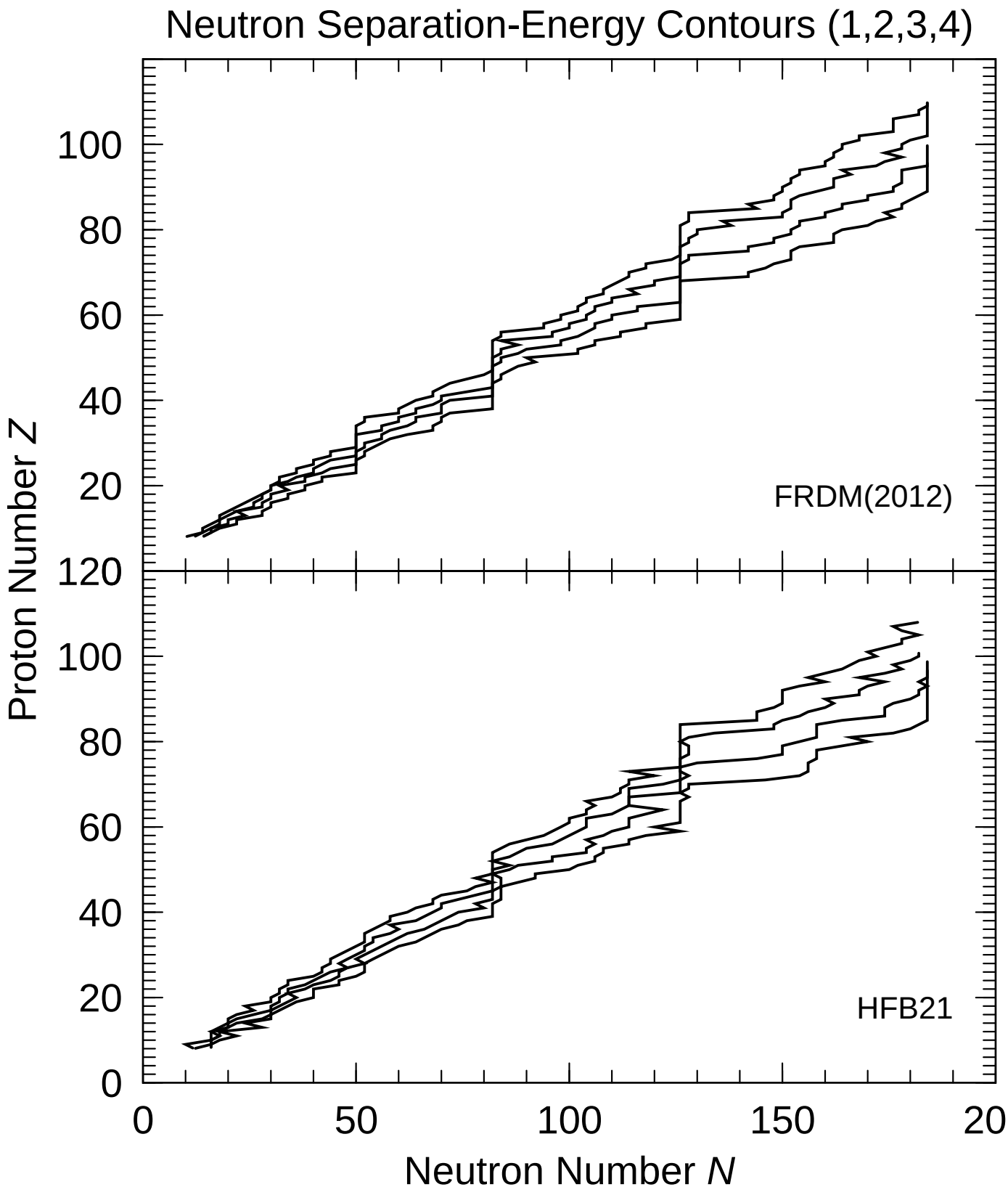
Q_α Deviations beyond $N = 126$

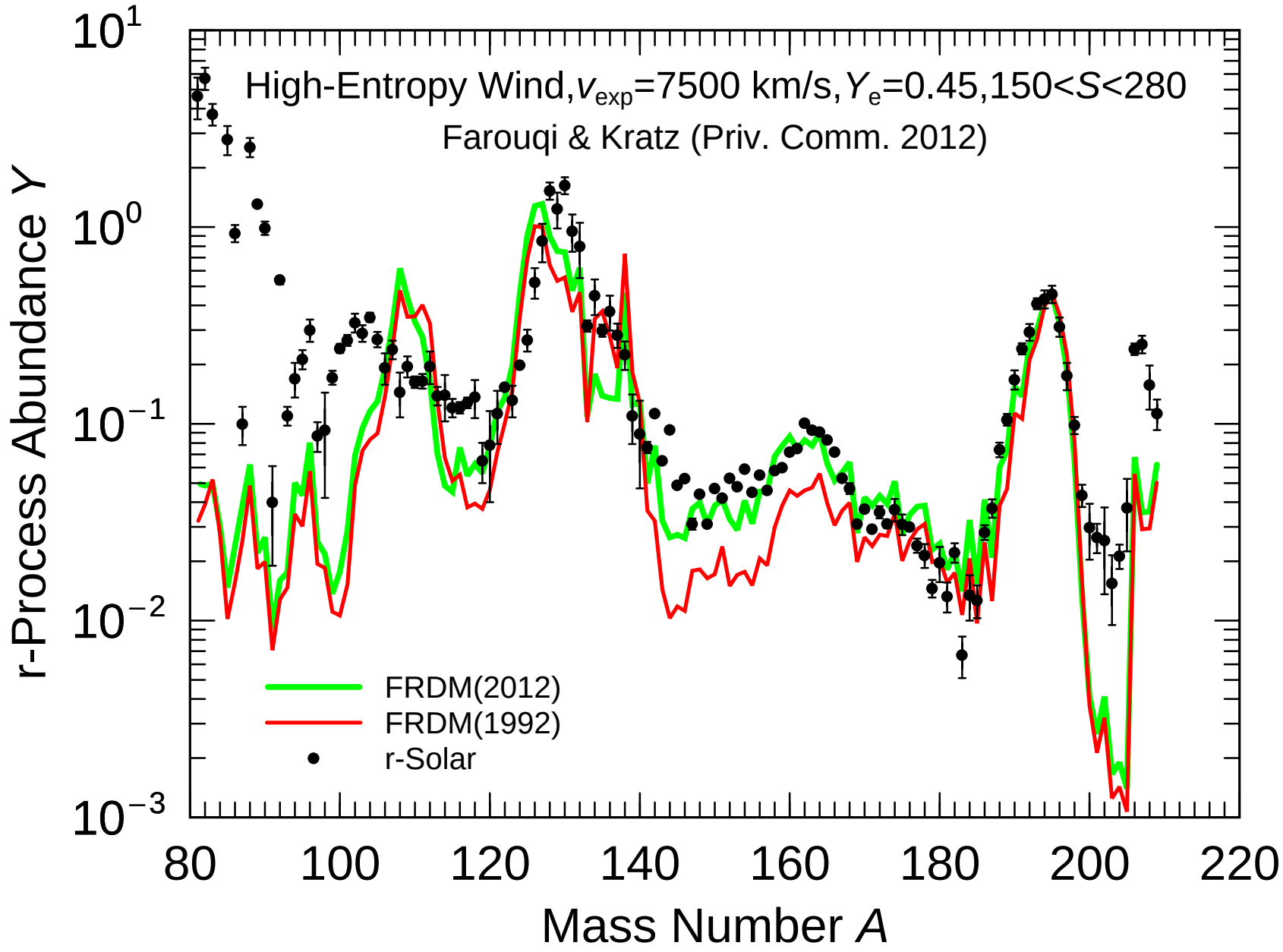
Region	Model	Nuclei	RMS (MeV)
$Z > 82$	SkM*	46	2.6
$Z > 82$	Sly4	46	2.6
$Z > 82$	HFB21	145	0.409
$Z > 82$	FRDM(2012)	145	0.326
$Z > 88$	SkM*	36	1.7
$Z > 88$	Sly4	36	2.2
$Z > 88$	HFB21	101	0.367
$Z > 88$	FRDM(2012)	101	0.274

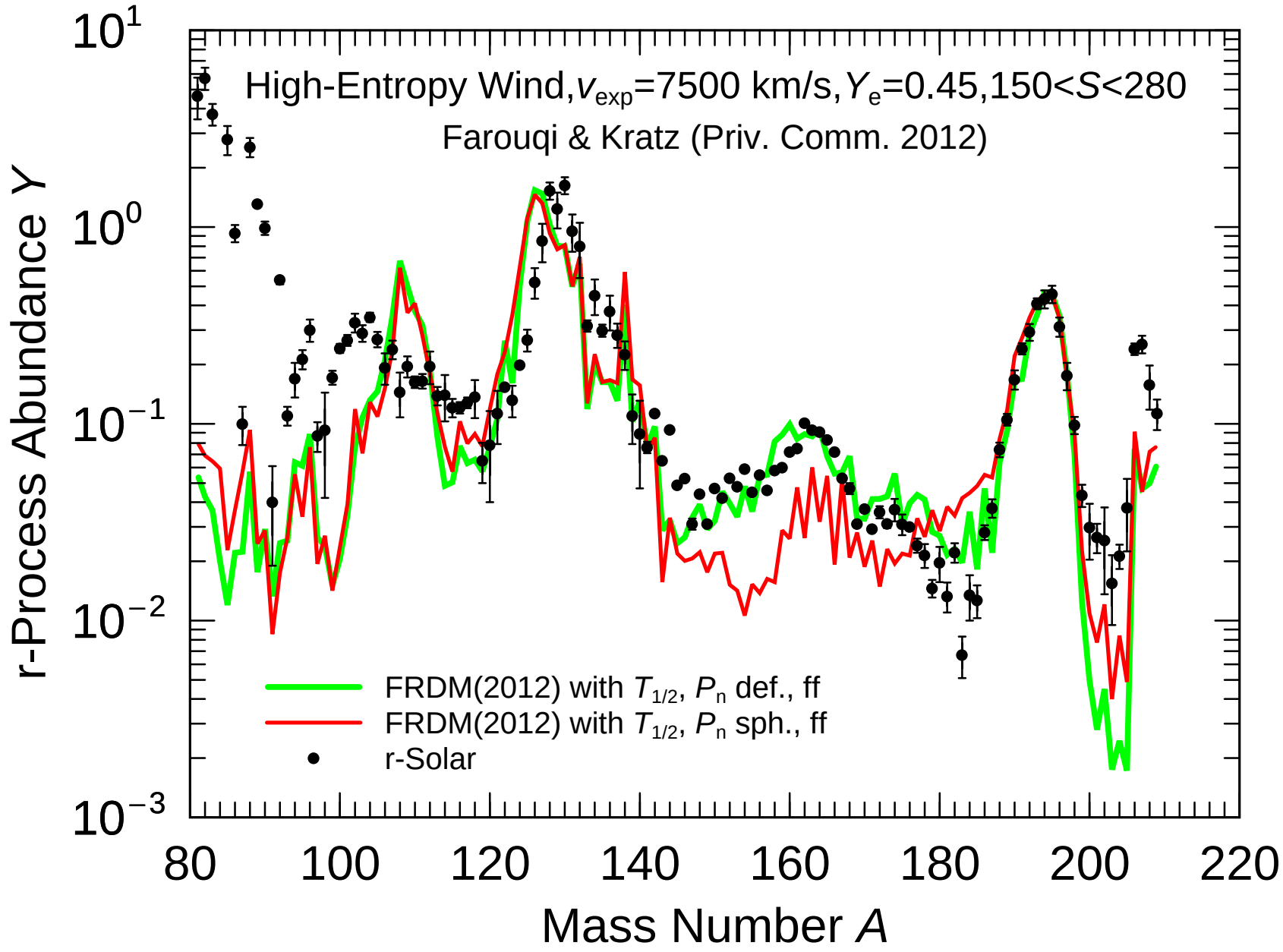

Neutron Number N
153 155 157 159 161 163 165 167
14
13
12
11
10
9
8
7
α-decay of 278113
RIKEN exp. (2004)
OTHER exp.
FRDM (1992)
FRLDM (1992)
HFB8 (Goriely)
HFB2 (Goriely)
Energy Release Qα (MeV)
101 103 105 107 109 111 113 115
Proton Number Z


Neutron Number N
165 167 169 171 173 175 177
α-Decay Chain from 294117
⇐ ε2>0.1
ε2<0.1 ⇒
Energy Release Qα (MeV)
12
11
10
9
8
7
D-Z (1995)
FRLDM (1992)
FRDM (1992)
Sobiczewski (2010)
Muntian et al. (2003)
Dubna exp.
105 107 109 111 113 115 117
Proton Number Z


Neutron Separation-Energy Contours (1,2,3,4)
Proton Number Z
Neutron Number N
FRDM(2012)
FRDM(1992)
0
20
40
60
80
100
120
0
50
100
150
200


Neutron Separation-Energy Contours (1,2,3,4)
Proton Number Z
Neutron Number N
FRDM(2012)
HFB21
0
20
40
60
80
100
120
0
50
100
150
200


High-Entropy Wind, v_{exp}=7500 km/s, Y_e=0.45, 150<S<280
Farouqi & Kratz (Priv. Comm. 2012)
FRDM(2012)
FRDM(1992)
r-Solar
r-Process Abundance Y
Mass Number A
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
80
100
120
140
160
180
200
220


High-Entropy Wind, v_{exp}=7500 km/s, Y_e=0.45, 150<S<280
Farouqi & Kratz (Priv. Comm. 2012)
FRDM(2012) with $T_{1/2}$, P_n def., ff
FRDM(2012) with $T_{1/2}$, P_n sph., ff
r-Solar
r-Process Abundance Y
Mass Number A
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
80
100
120
140
160
180
200
220

C O N C L U S I O N S

- Five distinct improvements in execution of calculations plus better experimental mass data decreased the FRDM(1992) model error by 16.4 %.

- Less staggering in error between neighbors $\Longrightarrow$ much less staggering in $S_{1\mathsf{n}}$, etc.

- Substantial improvement in specific regions, in particular in regions of shape coexistence and where nuclei are axially asymmetric.

- Calculation is performed for 9318 nuclei compared to 8979 nuclei in FRDM(1992).

Next steps?:
Tabulate results and calculate additional quantities, for example
β-decay half-lives and P_{n} values
Ground-state spins
Test impact on astrophysics applications

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- Less staggering in error between neighbors $\Longrightarrow$ much less staggering in $S_{1\mathsf{n}}$, etc.

- Substantial improvement in specific regions, in particular in regions of shape coexistence and where nuclei are axially asymmetric.

- Calculation is performed for 9318 nuclei compared to 8979 nuclei in FRDM(1992).

Next steps?:
Tabulate results and calculate additional quantities, for example
β-decay half-lives and P_{n} values
Ground-state spins
Test impact on astrophysics applications