

# Precision theory for charge radii of light nuclei

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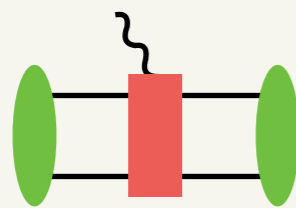
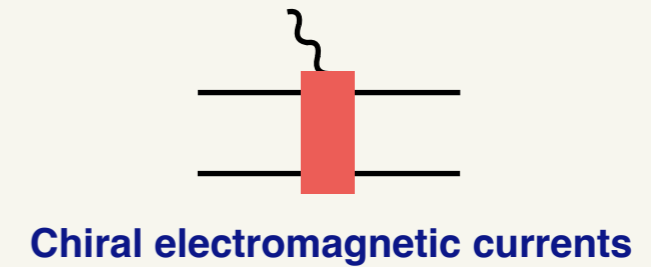
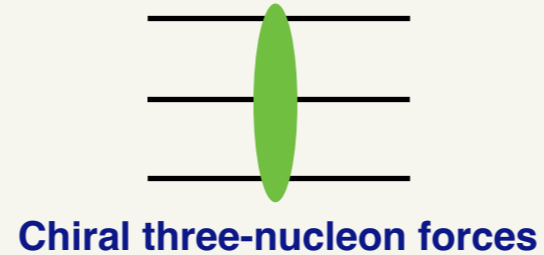
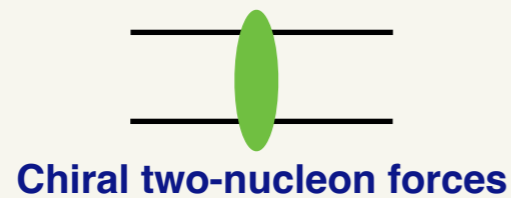
in collaboration with

V. Baru, E. Epelbaum, C. Körber, H. Krebs, D. Möller, A. Nogga, and P. Reinert

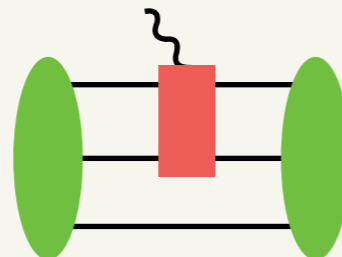
PRL 124 082501 (2020)  
Phys.Rev.C 103 024313 (2021)

# Nuclear structure using chiral effective field theory

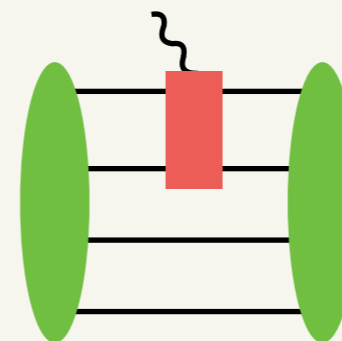
Low-energy **chiral effective field theory** of the standard model



$^2\text{H}$

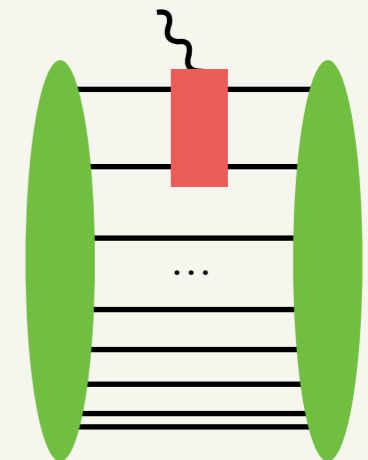


$^3\text{He}$   $^3\text{H}$



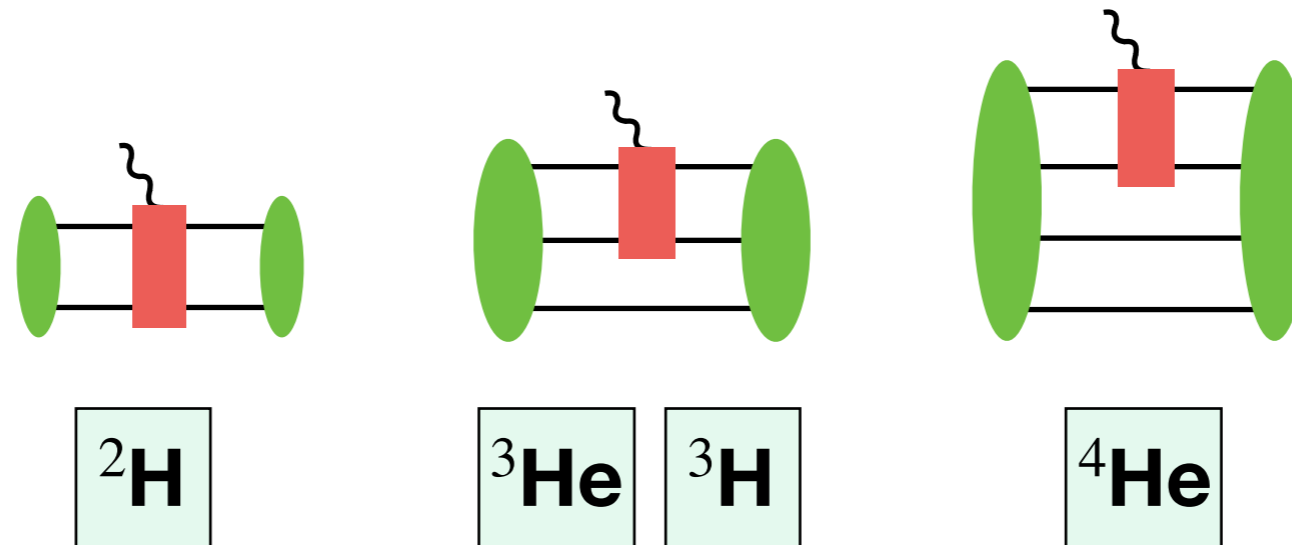
$^4\text{He}$

Charge form factors and radii of super-light nuclei



structure of  
medium-mass nuclei

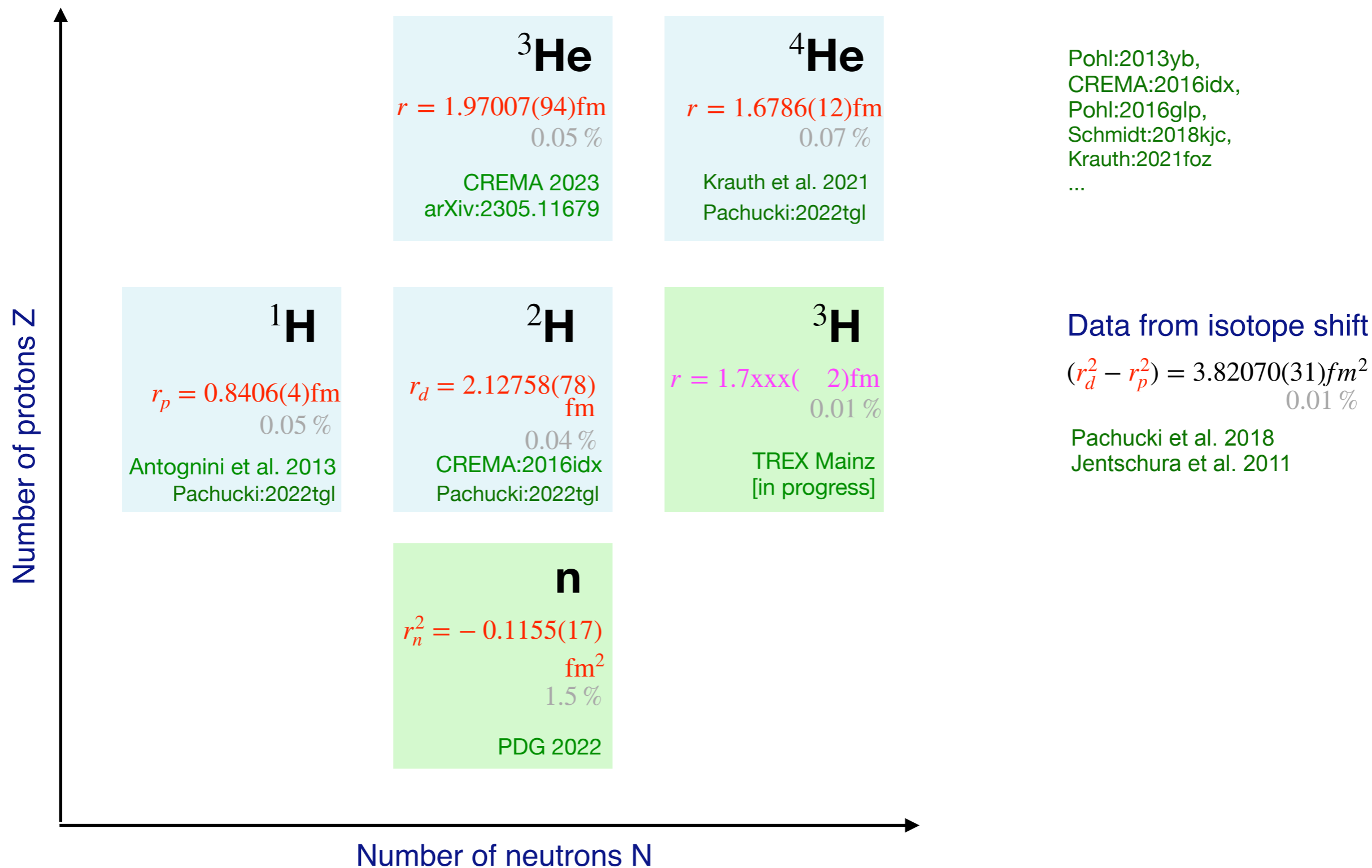
# Precise calculations of charge radii of super-light nuclei



## Motivation:

- Precision **tests of nuclear chiral effective field theory (Chiral EFT)**
- Help to resolve long-standing issue with **underpredicted radii of medium-mass and heavy nuclei**
- A new way to **extract the neutron and the proton charge radii** from few-nucleon data
- Search for **Beyond-Standard-Model** physics (lepton universality breaking)

# Charge radii of $A \leq 4$ nuclei — experimental data



# Basic features of chiral effective field theory

## Chiral effective field theory

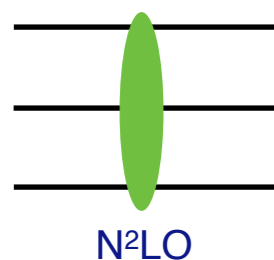
- Low-energy **effective field theory of QCD**
- Degrees of freedom are **pions and nucleons**
- Most general Lagrangian consistent with **symmetries and symmetry-breaking pattern of QCD**
- Expansion parameter  $Q = \frac{\max(m_\pi, p)}{\Lambda_\chi}$ 
  - $m_\pi$  = pion mass
  - $p$  = typical momentum scale
  - $\Lambda_\chi$  = chiral symmetry breaking scale
- **Systematically improvable**
- Observables are calculated order by order
  - LO = Leading Order
  - NLO = Next-to-leading order
  - N2LO = Next-to-next-to-leading order
  - ...
- Higher orders contain more free parameters, which have to be fitted to data

# Chiral effective field theory - **precise, accurate and consistent**



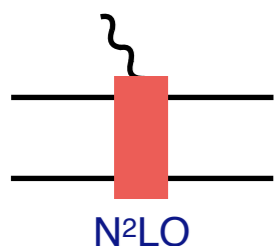
## New **high-precision chiral NN forces** ( $N^4LO^+$ ) Reinert et al. PRL 126, 092501 (2021)

- Nearly perfect description of pp and pn scattering data up to pion production threshold



## **Chiral 3N forces** (general $N^2LO$ ; selected terms at $N^4LO$ ) Epelbaum:2019kcf

- LECs  $cD$  and  $cE$  ( $N^2LO$ ) are fitted to **RIKEN** Nd DCS data and  $^3He$  binding energy
- **Consistent** regularisation of  $N^3LO$  is also in progress
- Charge radii of  $^3N$  and  $^4He$  are not sensitive to 3N forces beyond  $N^2LO$  if binding energies of  $A=3, 4$  are reproduced

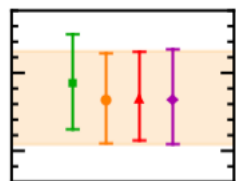


**Isoscalar:  $N^4LO$**

## **2N Chiral electromagnetic currents** (general $N^2LO$ ; isoscalar $N^4LO$ )

- $N^2LO$  (**isoscalar  $N^4LO$** ) is derived and regularised consistently with the chiral NN forces
- Consistent regularisation of  $N^3LO$  (isovector) is in progress

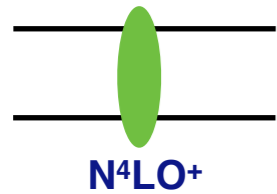
Kolling:2009iq  
Kolling:2012cs  
Krebs:2019aka  
Krebs:2020pii (Review)



## **Reliable methods to quantify truncation uncertainty** of the EFT expansion

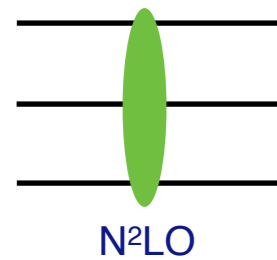
Epelbaum et al. EPJA 51 (2015); Furnstahl et al. PRC 92, 024005 (2015); Melendez et al. PRC 96, 024003 (2017),  
Wesolowski et al. J. Phys. G 46, 045102 (2019); Melendez et al. PRC 100, 044001 (2019), ...

# Chiral effective field theory - **precise, accurate and consistent**



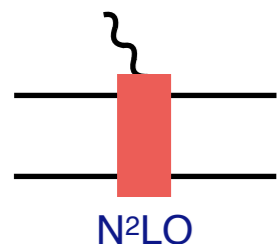
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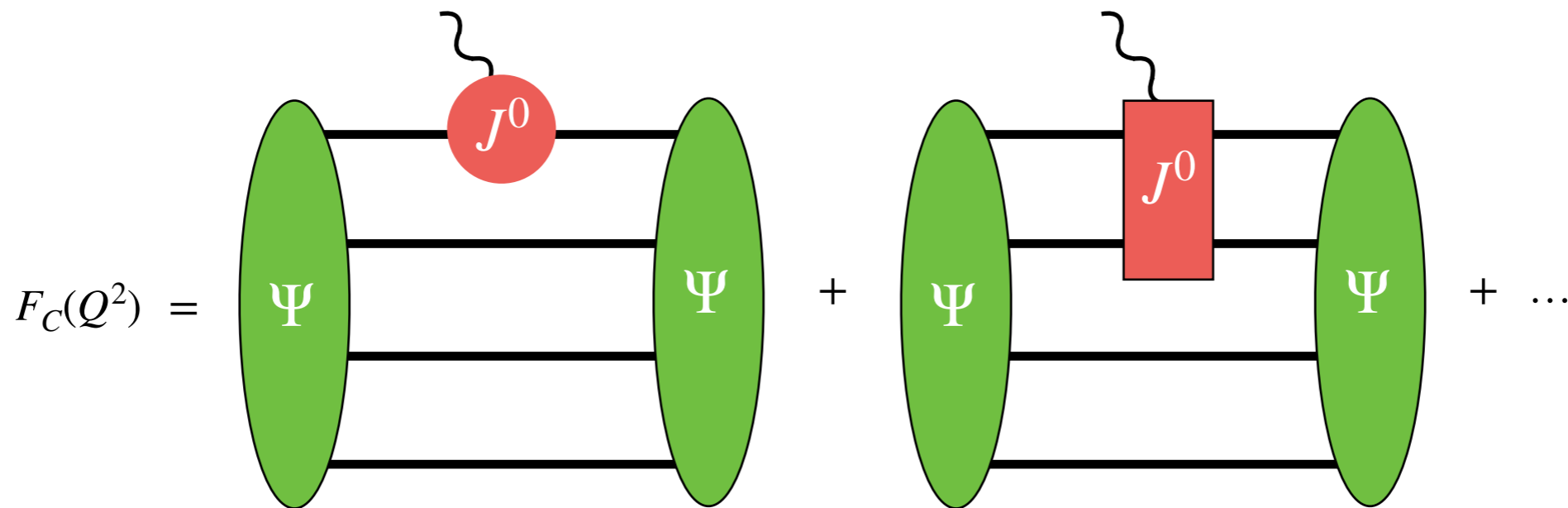
Kolling:2009iq  
Kolling:2012cs  
Krebs:2019aka  
Krebs:2020pii (Review)

## **Goals of this study:**

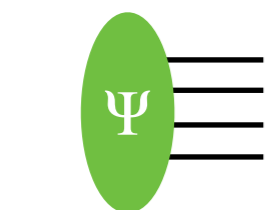
- consistent  $\chi EFT$  calculation of **isoscalar structure radii** of  $A = 2, 3, 4$  nuclei
- aim at  $N^4LO$  level of accuracy even in the incomplete calculation
- careful estimation of uncertainties (truncation, statistical, numerical and other)

# Chiral EFT calculation of the nuclear charge radius

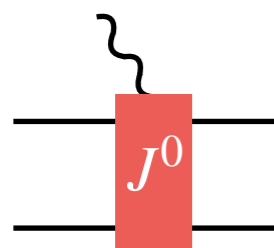
Charge radius  $r_C$  is related to the charge form factor  $F_C(Q^2)$   $r_C^2 = (-6) \frac{\partial}{\partial Q^2} F_C(Q^2) \Big|_{Q=0}$



The matrix element is a convolution of nuclear wave function and charge density operator



Nuclear wave function based on high-precision chiral EFT interactions



Charge density operator - consistent with chiral nuclear forces



# Structure radius

Nuclear **charge radius** can be decomposed into **structure**, **proton** and **neutron** radii

$$r_C^2 = r_{str}^2 + \left( r_p^2 + \frac{3}{4m_p^2} \right) + \frac{N}{Z} r_n^2$$

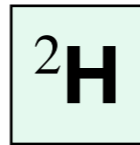
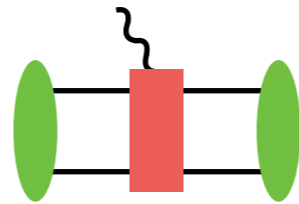
## Nuclear structure radius

- depends on distribution of matter (proton and neutrons) inside the nuclei
- depends on many-body electromagnetic currents
- **can be accurately calculated using chiral nuclear forces and EM currents**

Structure radius = **charge radius** if protons and neutrons have point-like charge distributions

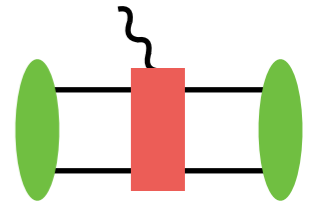
# Deuteron structure radius

& extraction of the neutron charge radius

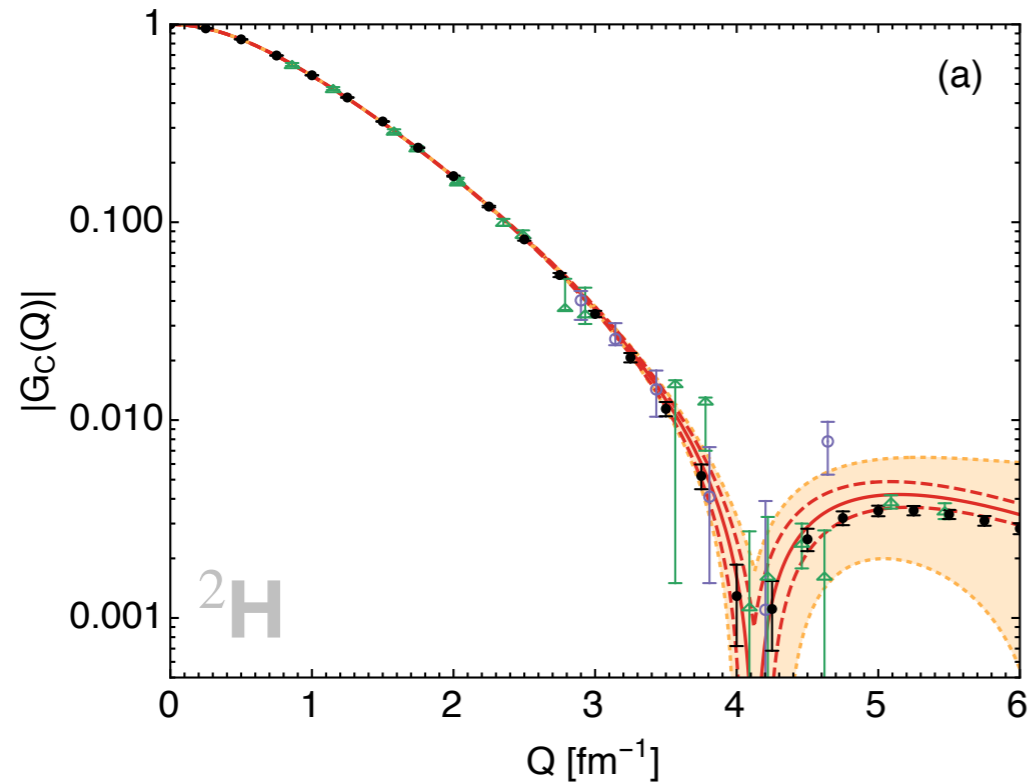


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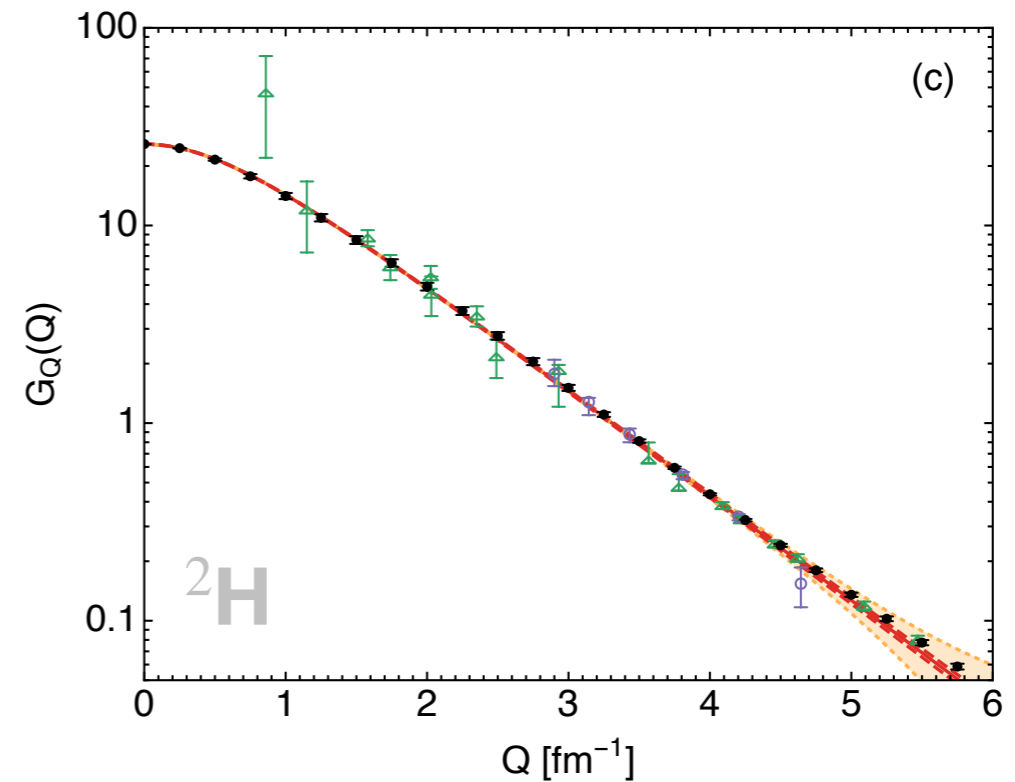
# Deuteron charge and quadrupole form factors



Deuteron charge form factor



Deuteron quadrupole form factor



our result + N<sup>4</sup>LO truncation uncertainty

statistical uncertainty

Experimental data

Parameterisation by I.Sick (not used in the fit)

Extraction of deuteron structure radius and quadrupole moment

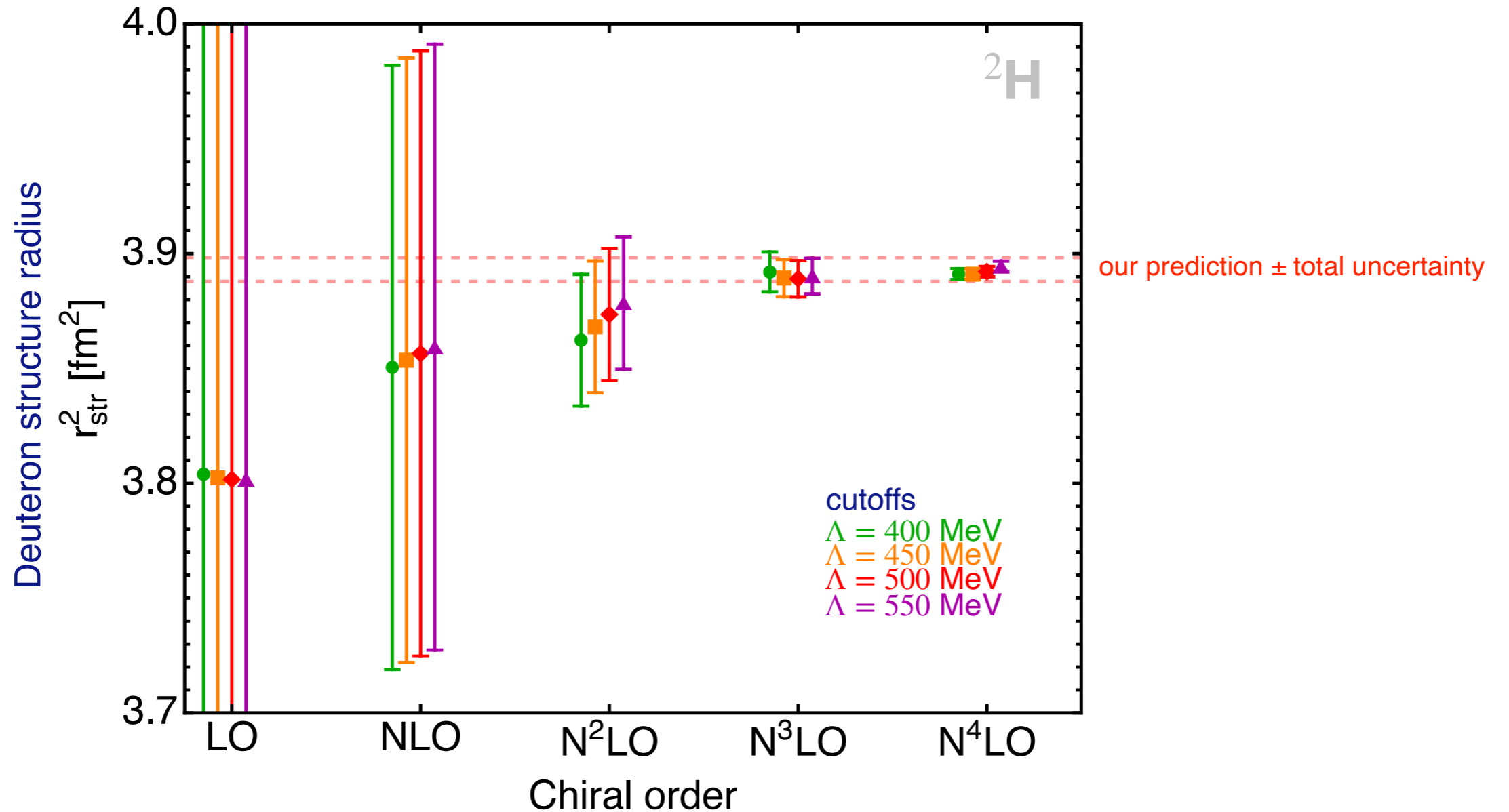
$$r_{str} = 1.9729^{+0.0015}_{-0.0012} fm$$

$$Q_d = 0.2854^{+0.0038}_{-0.0017} fm^2$$

AF, Möller, Baru, Epelbaum, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

# Truncation uncertainty of $^2\text{H}$ structure radius

Using Bayesian model to estimate truncation uncertainty at each order [Epelbaum et al. EPJA 56, 92 \(2020\)](#)

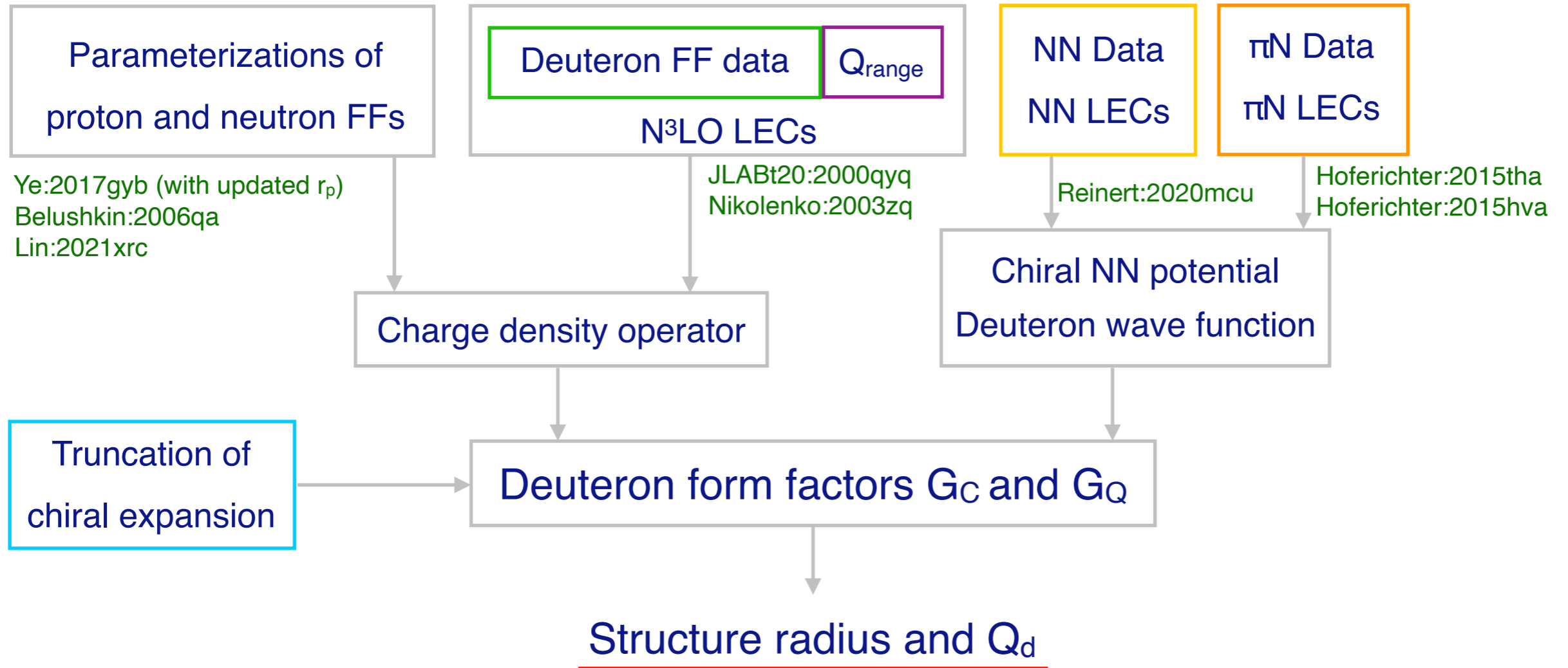


Chiral EFT expansion converges very well

Cutoff dependence is smaller than the truncation uncertainty

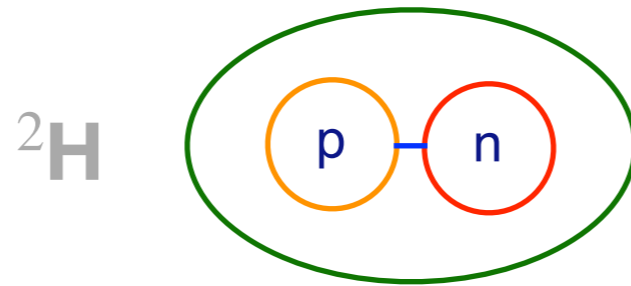
# Uncertainty analysis of deuteron structure radius

We propagate uncertainties from multiple sources



|                                       | <u>Central</u> | Truncation   | $\rho_{\text{Cont}}^{\text{reg}}$ | $\pi$ N LECs RSA | 2N LECs and $f_i^2$ | $Q$ range          | Total              |
|---------------------------------------|----------------|--------------|-----------------------------------|------------------|---------------------|--------------------|--------------------|
| $r_{\text{str}}^2$ (fm <sup>2</sup> ) | 3.8925         | $\pm 0.0030$ | $\pm 0.0024$                      | $\pm 0.0003$     | $\pm 0.0025$        | +0.0035<br>-0.0005 | +0.0058<br>-0.0046 |
| $Q_d$ (fm <sup>2</sup> )              | 0.2854         | $\pm 0.0005$ | $\pm 0.0007$                      | $\pm 0.0003$     | $\pm 0.0016$        | +0.0035<br>-0.0005 | +0.0038<br>-0.0017 |

# Neutron charge radius from high-accuracy $\chi$ EFT calculation of deuteron structure radius



$$r_d^2 = r_{str}^2({}^2\text{H}) + \left( r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

Deuteron-proton radius difference

$$(r_d^2 - r_p^2) = 3.82070(31) fm^2$$

Atomic spectroscopy

Hydrogen-deuterium 1S-2S isotope shift

+ QED corrections

Pachucki et al., PRA 97, 062511 (2018)

Jentschura et al. PRA 83 (2011)

Our accurate  $\chi$ EFT calculation of the deuteron structure radius

New method of determination of the neutron charge radius

$$r_n^2 = (r_d^2 - r_p^2) - \frac{3}{4m_p^2} - r_{str}^2({}^2\text{H})$$

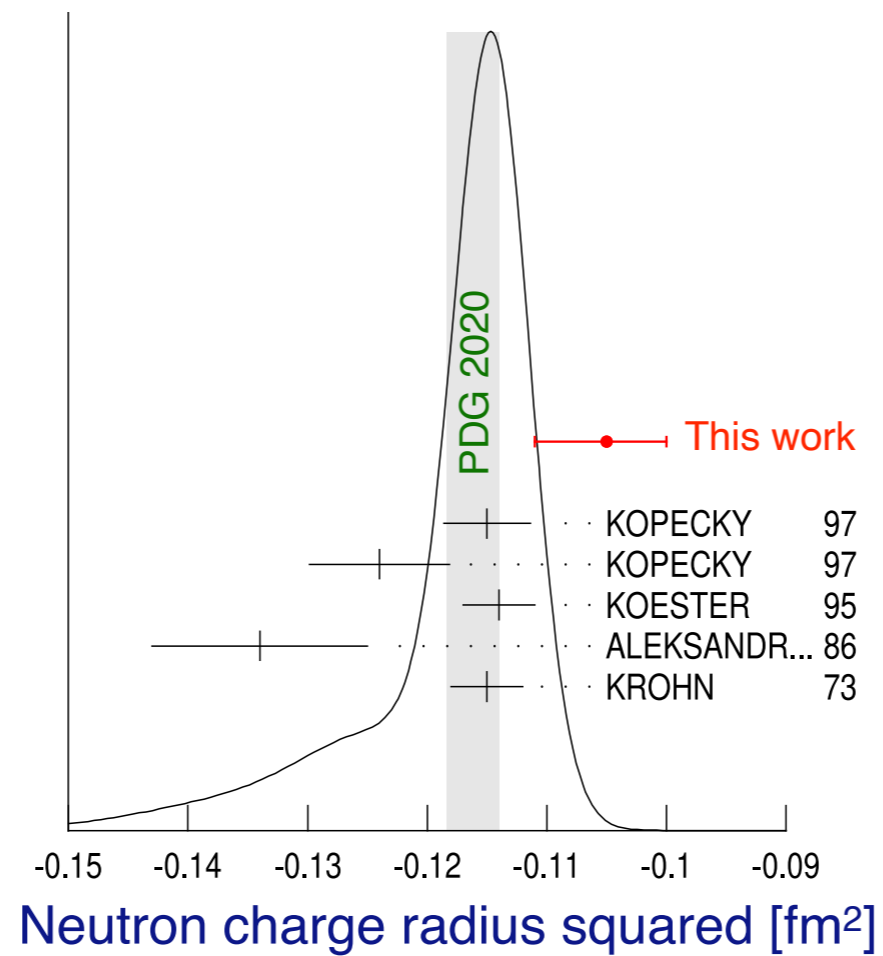
$$r_n^2 = -0.105^{+0.005}_{-0.006} fm^2$$

AF, Möller, Baru, Epelbaum, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

# Our extraction of the neutron charge radius

$$r_n^2 = -0.105^{+0.005}_{-0.006} \text{fm}^2$$

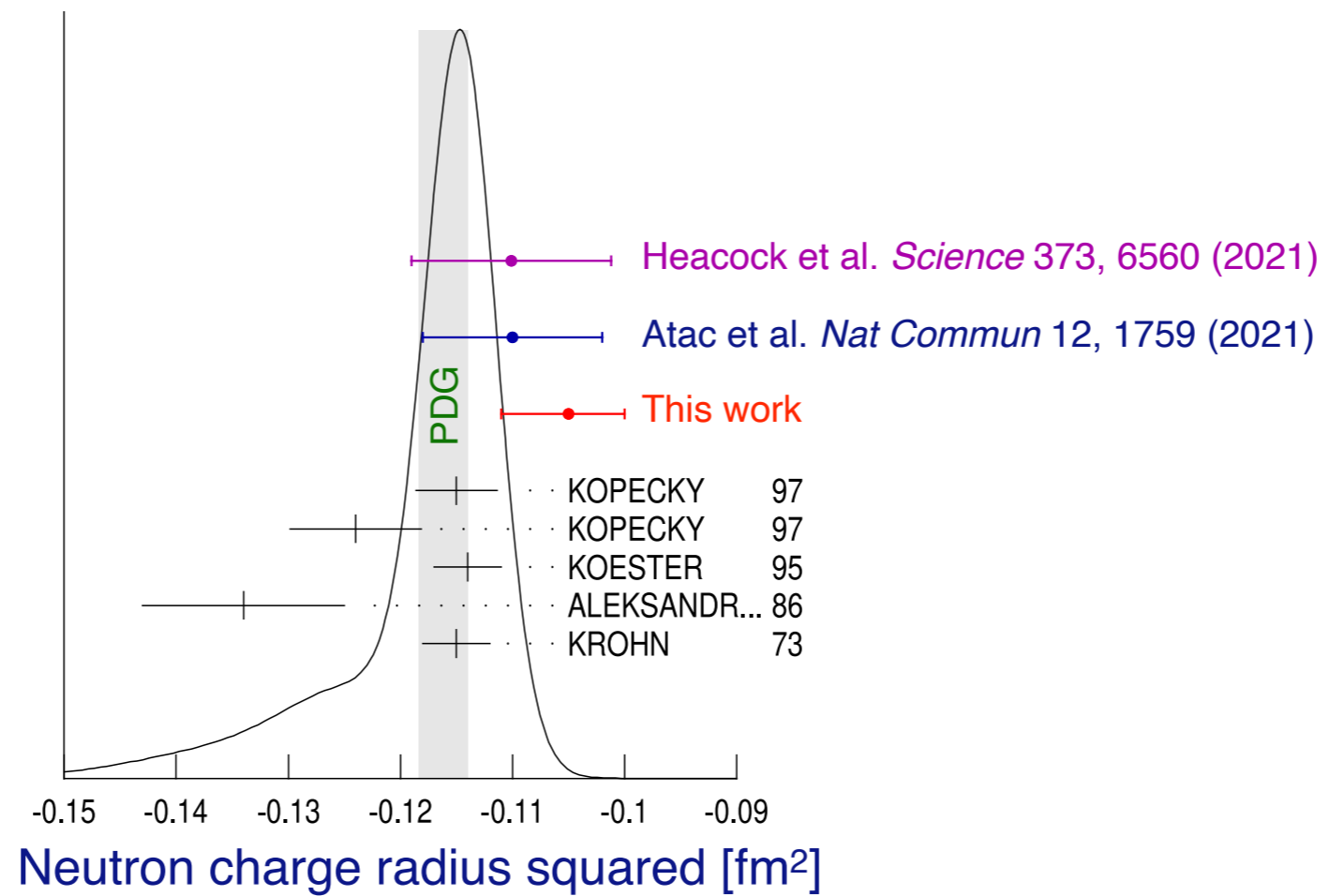
~2σ deviation from the PDG (2022) weighted average  $r_n^2 = -0.1155(17)\text{fm}^2$



# Our extraction of the neutron charge radius

$$r_n^2 = -0.105^{+0.005}_{-0.006} \text{fm}^2$$

~2 $\sigma$  deviation from the PDG (2022) weighted average  $r_n^2 = -0.1155(17)\text{fm}^2$





# Neutron charge radius in PDG 2022

R.L. Workman et al. (Particle Data Group), Prog.Theor.Exp.Phys. 2022, 083C01 (2022) and 2023 update

## $n$ MEAN-SQUARE CHARGE RADIUS

| <i>VALUE</i> (fm <sup>2</sup> )   | <i>DOCUMENT ID</i>      | <i>COMMENT</i>                   |
|---|-------------------------|----------------------------------|
| <b>−0.1155±0.0017 OUR AVERAGE</b>   |                         |                                  |
| −0.115 ±0.002 ±0.003  | KOPECKY 97              | $ne$ scattering (Pb)             |
| −0.124 ±0.003 ±0.005  | KOPECKY 97              | $ne$ scattering (Bi)             |
| −0.114 ±0.003   | KOESTER 95              | $ne$ scattering (Pb, Bi)         |
| −0.115 ±0.003   | <sup>1</sup> KROHN 73   | $ne$ scattering (Ne, Ar, Kr, Xe) |
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● |                         |                                  |
| −0.1101±0.0089  | <sup>2</sup> HEACOCK 21 | $n$ interferometry               |
| −0.106 $\begin{matrix} +0.007 \\ -0.005 \end{matrix}$                         | <sup>3</sup> FILIN 20   | chiral EFT analysis              |
| −0.117 $\begin{matrix} +0.007 \\ -0.011 \end{matrix}$                         | BELUSHKIN 07            | Dispersion analysis              |
| −0.113 ±0.003 ±0.004  | KOPECKY 95              | $ne$ scattering (Pb)             |
| −0.134 ±0.009   | ALEKSANDR...86          | $ne$ scattering (Bi)             |
| −0.114 ±0.003   | KOESTER 86              | $ne$ scattering (Pb, Bi)         |
| −0.118 ±0.002   | KOESTER 76              | $ne$ scattering (Pb)             |
| −0.120 ±0.002   | KOESTER 76              | $ne$ scattering (Bi)             |
| −0.116 ±0.003   | KROHN 66                | $ne$ scattering (Ne, Ar, Kr, Xe) |

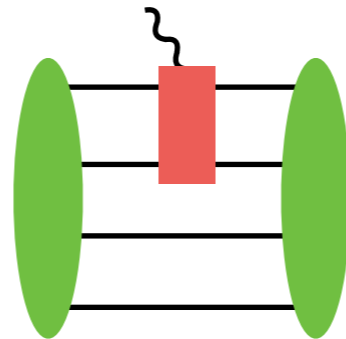
<sup>1</sup> KROHN 73 measured  $-0.112 \pm 0.003$  fm<sup>2</sup>. This value is as corrected by KOESTER 76.

<sup>2</sup> HEACOCK 21 extract the value from Pendelloesung interferometry to measure the neutron structure factors of silicon. This value is strongly anti-correlated with the mean-square thermal atomic displacement.

<sup>3</sup> FILIN 20 extract the value based on their chiral-EFT calculation of the deuteron structure radius and use as input the atomic data for the difference of the deuteron and proton charge radii.

# $^4\text{He}$ charge radius

Precision test of the chiral EFT for  $^4\text{He}$



$^4\text{He}$

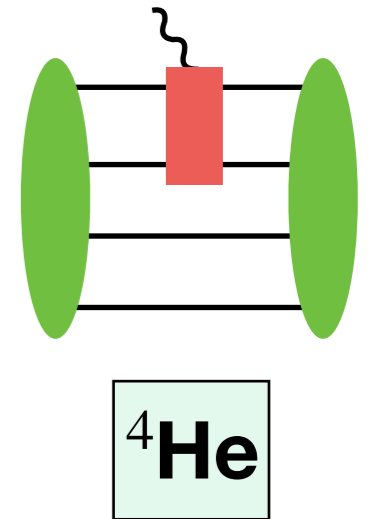
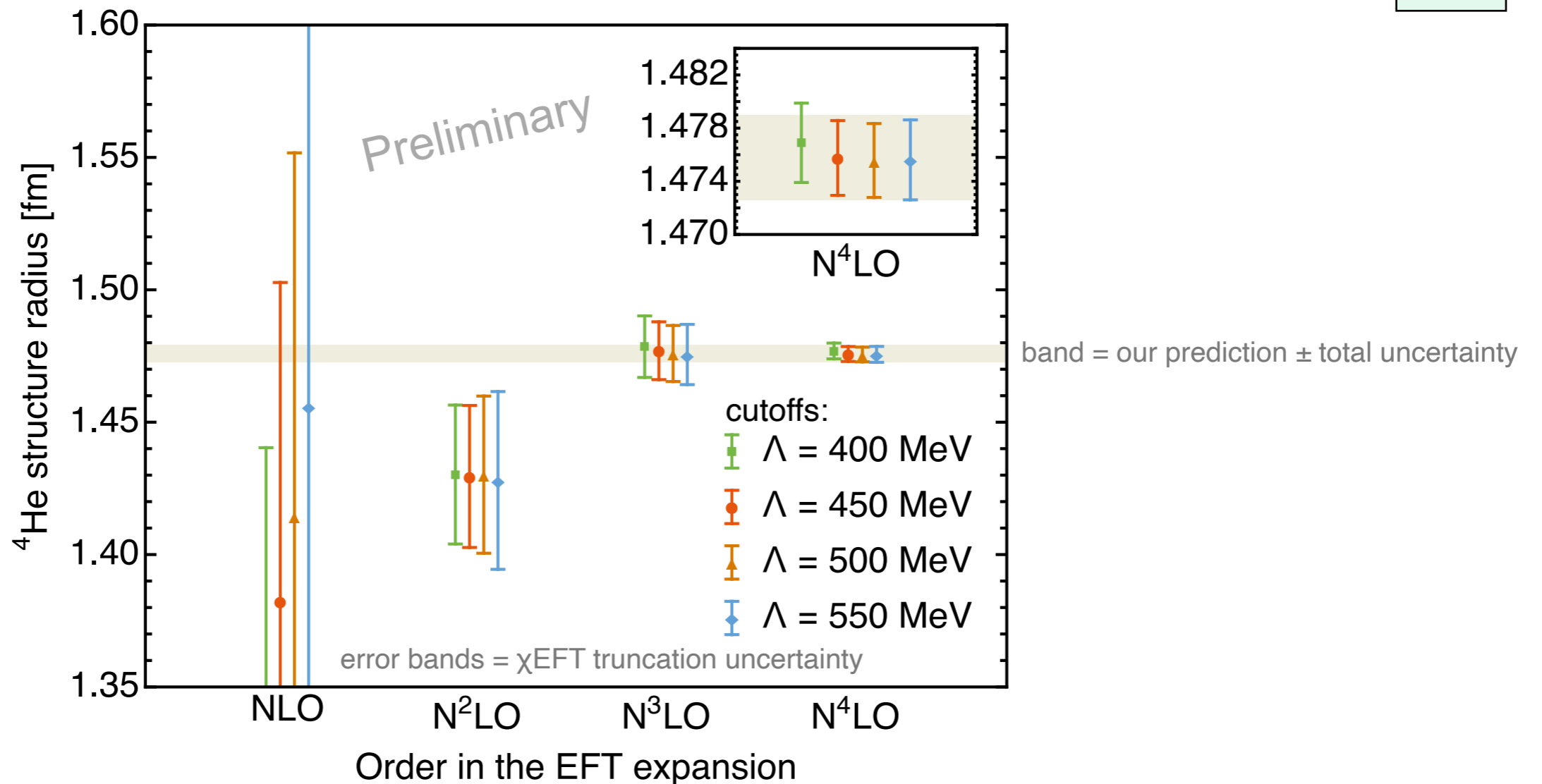
Preliminary results

# Prediction of $^4\text{He}$ structure radius

Our preliminary prediction for  $^4\text{He}$  structure radius:

$$r_{str}(^4\text{He}) = 1.4758 \pm 0.0028_{\text{trunc}} \pm 0.0011_{\text{stat}} \pm 0.0010_{\text{nucIFF}} \text{ fm (Preliminary)}$$

Estimation of truncation error:

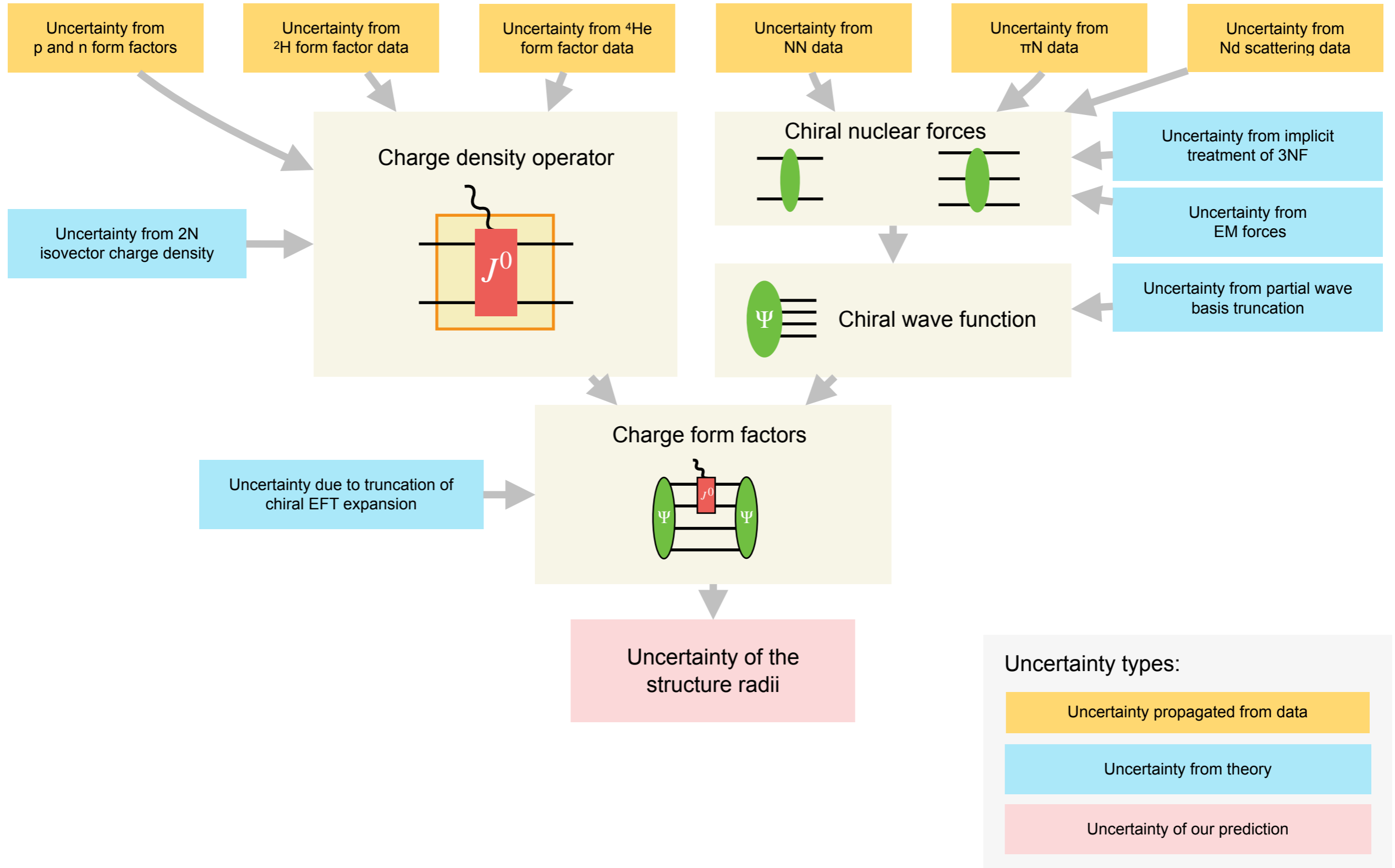


Cutoff dependence is smaller than the truncation uncertainty

Chiral EFT expansion converges well

# Extensive uncertainty analysis

## Propagation of uncertainties from data and theory



# Prediction for ${}^4\text{He}$ charge radius

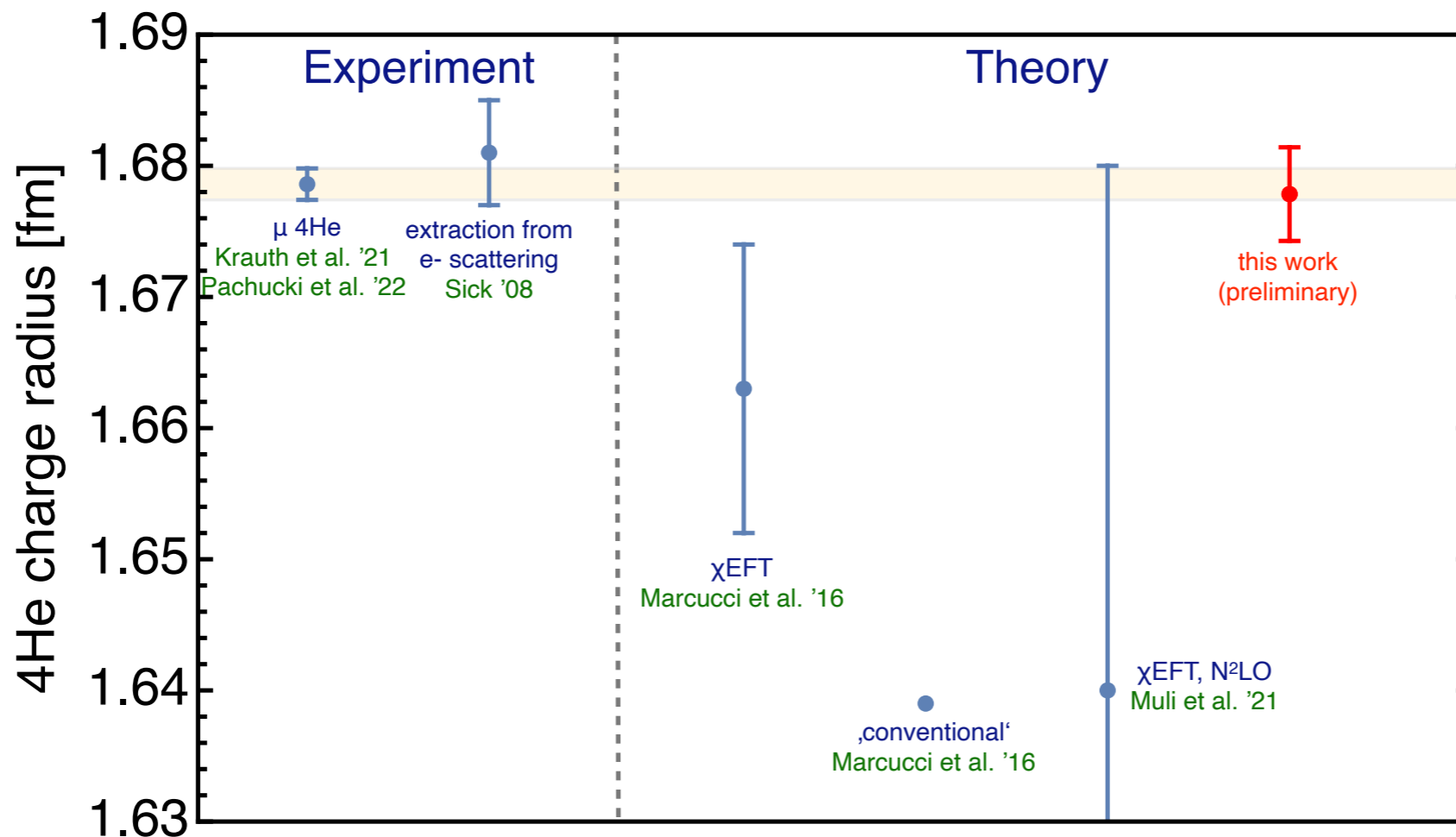
$$r_{str}({}^4\text{He}) = 1.4758 \pm 0.0028_{\text{trunc}} \pm 0.0011_{\text{stat}} \pm 0.0010_{\text{nucIFF}} \text{ fm} \text{ (Preliminary)}$$

Our prediction for  ${}^4\text{He}$  **charge** radius

$$r_C({}^4\text{He}) = (1.6775 \pm 0.0035) \text{ fm}$$

$$r_C({}^4\text{He}) = r_{str}^2({}^4\text{He}) + \left( r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

preliminary, using CODATA 2018  $r_p$  and own determination of  $r_n$



Our prediction for  ${}^4\text{He}$  charge radius is fully consistent with the muonic-atom spectroscopy

# Indications of BSM physics?

All data used to constrain chiral EFT LECs are from strong interaction / electron-based experiments:

$\pi$  N Roy-Steiner analysis [Hoferichter:2015tha](#), [Hoferichter:2015hva](#)

NN pn and pp scattering data, deuteron BE [Reinert:2020mcu](#)

Deuteron charge and quadrupole FF data [JLABt20:2000qyq](#), [Nikolenko:2003zq](#)

Deuteron-proton radii difference from atomic spectroscopy [Pachucki:2018yxe](#), [Jentschura et al. PRA 83 \(2011\)](#)

Proton charge radius [CODATA2018](#)

$^4\text{He}$  form factor data [Erich:1971rhg](#), [Mccarthy:1977vd](#), [VonGunten:1982yna](#), [Ottermann:1985km](#), [Frosch:1967pz](#),

[Arnold:1978qs](#), [Camsonne:2013df](#)

Binding energies of  $^3\text{He}$  and  $^4\text{He}$

Nd DCS minimum @ 70 MeV [RIKEN data](#)

No muonic data is used in our chiral EFT predictions

Our prediction for  $^4\text{He}$  charge radius is consistent with the muonic experiment

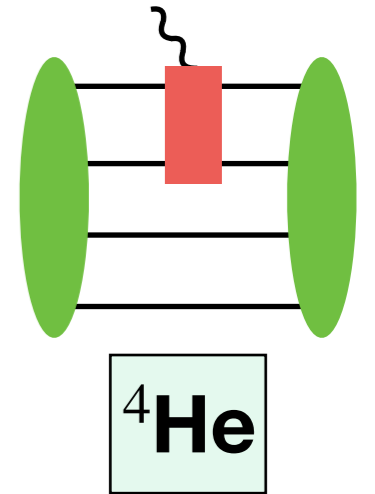
**No indication of lepton universality breaking at this accuracy level**

# Isoscalar nucleon charge radius from data on $^4\text{He}$

Our prediction for  $^4\text{He}$  **structure** radius

Experimental  $^4\text{He}$  charge radius  
Krauth et al., Nature 589 (2021) 7843, 527-531  
+ theory update from Pachucki et al. (2022)

$$r_C(^4\text{He}) = r_{str}^2(^4\text{He}) + \left( r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$



Determination  
of the isoscalar nucleon charge radius  
 $(r_n^2 + r_p^2) = (0.607 \pm 0.010) \text{fm}^2$

preliminary

# Proton charge radius from isoscalar nucleon radius

Our determination of the  
isoscalar nucleon charge radius from  $^4\text{He}$

$$(r_n^2 + r_p^2) = (0.607 \pm 0.010) \text{ fm}^2 \text{ preliminary}$$

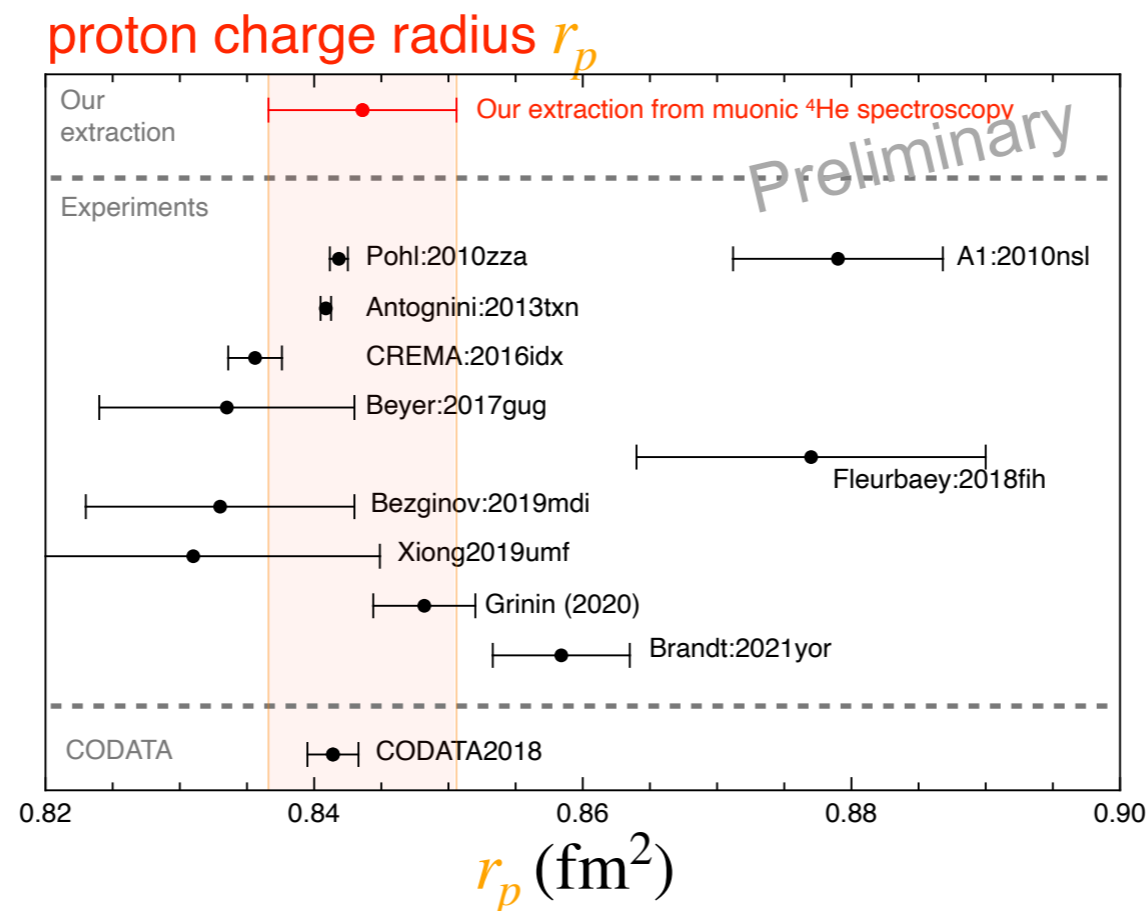
Our determination of the  
neutron charge radius from  $^2\text{H}$

$$r_n^2 = -0.105^{+0.005}_{-0.006} \text{ fm}^2$$

AF, Möller, Baru, Epelbaum, Krebs, Reinert,  
PRL 124 (2020) 082501; PRC 103 (2021) 024313

New determination of the proton charge radius:  $r_p = (0.844 \pm 0.007) \text{ fm}$

preliminary



Our extraction supports the „small“ proton radius



# Prediction for isoscalar 3N charge radius

With all LECs being fixed, we can **predict the isoscalar 3N charge radius:**  $r_C^{isoscalar3N} = \sqrt{\frac{1}{3}(r_C^{3H})^2 + \frac{2}{3}(r_C^{3He})^2}$

$$r_C^{isoscalar3N} = (1.9061 \pm 0.0026) fm$$

preliminary, using CODATA 2018  $r_p$  and own determination of  $r_n$

Our result is 10x more precise than current experimental data:

the  $^3H$  charge radius from  $e^-$  scattering experiments:

$$r_C^{3H} = (1.7550 \pm 0.0860) fm \quad \text{Amroun et al. '94 (world average)} \\ \text{5 \%}$$

the  $^3He$  charge radius from muonic  $^3He$ :

$$r_C^{3He} = (1.9701 \pm 0.0009) fm \quad \text{CREMA 2023 arXiv:2305.11679} \\ \text{0.05 \%}$$

**Exp. 3N isoscalar charge radius:** (using muonic  $^3He$  and old  $^3H$ )

$$r_{C,exp.}^{isoscalar3N} = (1.9010 \pm 0.0260) fm \\ \text{1.4 \%}$$

T-REX experiment in Mainz [Pohl et al.] aims at measuring  $r_C^{3H}$  within  $\pm 0.0002 fm$  (400x more precise)

The isoscalar 3N radius will be then known within  $\pm 0.0009 fm$

⇒ **precision tests of nuclear chiral EFT!**

# Summary

Precise and accurate calculation of  $A = 2, 3, 4$  isoscalar charge radii in chiral effective field theory

Extensive uncertainty analysis

**Nuclear structure calculations with sub-percent accuracy!**

Charge radii of neutron and proton from light nuclei:

- ${}^2\text{H}$   $r_{\text{str}}$  combined with isotope-shift data => **extracted the neutron charge radius** ( $2\sigma$  tension with PDG)
- ${}^4\text{He}$   $r_{\text{str}}$  combined with spectroscopic data => **extracted isoscalar nucleon and proton charge radii** preliminary

${}^4\text{He}$  calculation: preliminary

- calculated  **${}^4\text{He}$  charge radius (0.2% accuracy)** agrees with the new  $\mu^4\text{He}$  measurement
- no indications of lepton universality breaking at this accuracy level

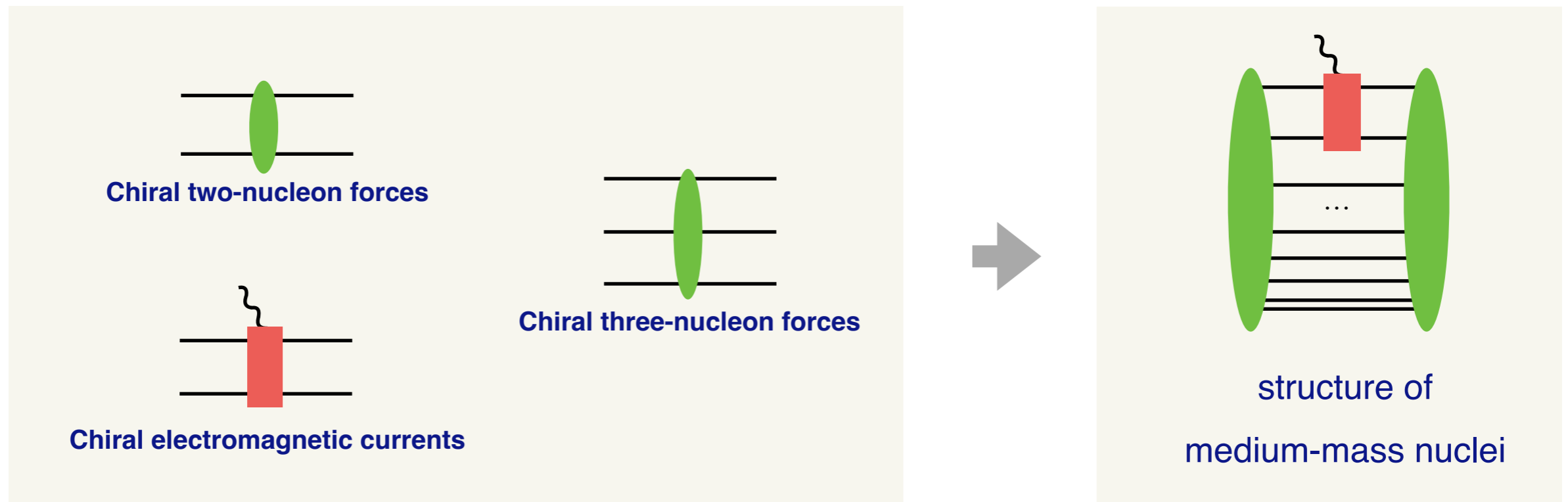
${}^3\text{H}$ - ${}^3\text{He}$ : preliminary

- predicted the **isoscalar 3N charge radius  $r_C$  (0.1% accuracy)**
- our  $r_C$  is in agreement with the current exp. value (which has 10x larger errors)
- the ongoing T-REX ( ${}^3\text{H}$ ) exp. in Mainz will allow for a **precision test of nuclear chiral EFT**

# Outlook

## In progress:

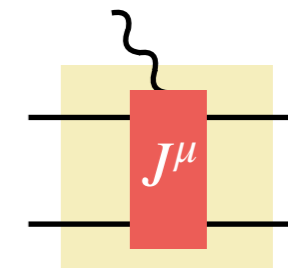
- **Consistent** inclusion of **isovector 2N currents at N<sup>3</sup>LO and N<sup>4</sup>LO**
- **Consistent** inclusion of **N<sup>3</sup>LO and N<sup>4</sup>LO three-nucleon forces**
- Analysis of **magnetic form factors**
- Application to processes with two photons (**polarizabilities, ...**)
- Calculation of isoscalar N<sup>4</sup>LO **charge radii for nuclei with A>4** (work in progress by LENPIC collaboration)



**Spare**

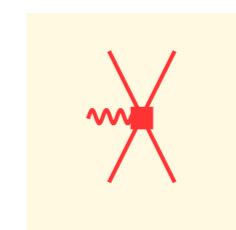
# Nuclear electromagnetic currents

Kolling:2009iq, Kolling:2012cs, Krebs:2019aka  
 Review: H. Krebs, EPJA 56 (2020) 240



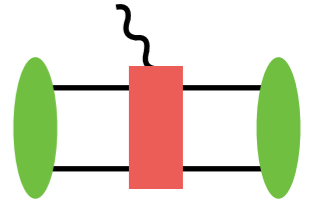
|                   | single-nucleon                                    | two-nucleon   |
|-------------------|---|---|
| LO                |   |   |
| NLO               | <p>current      charge</p>                        |   |
| N <sup>2</sup> LO |   |   |
| N <sup>3</sup> LO | <p>can be parametrised in terms of nucleon FF</p> | <p>depend on <math>d_8, d_9, d_{18}, d_{21}, d_{22}</math>, no 1/m corrections...      parameter-free</p> <p>parameter-free static two-pion exchange</p> <p>depend on <math>C_2, C_4, C_5, C_7 + L_1, L_2</math>; no loop corrections      depend on <math>C_T</math></p> |

Ye:2017gyb  
 Belushkin:2006qa  
 Lin:2021umz



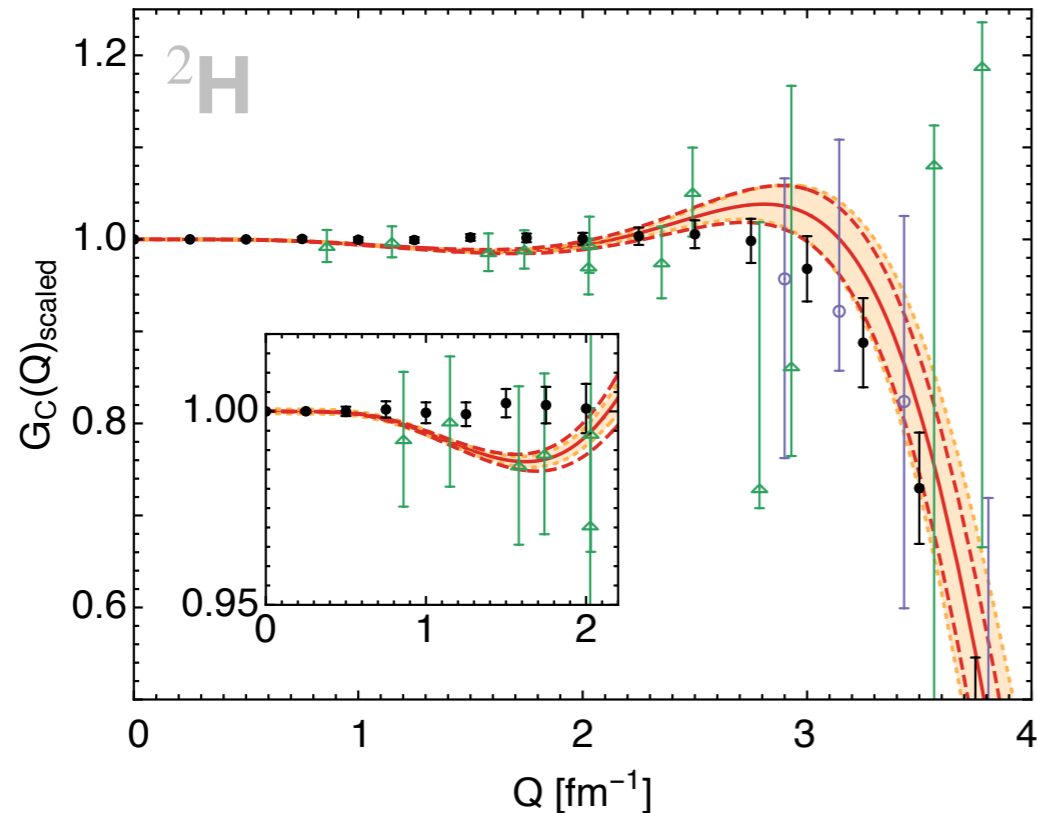
depend on **3 parameters (LECs)**  
 $^3S_1$ - $^3S_1$  - can be fitted to deuteron FF data  
 $^3S_1$ - $^3D_1$  - this one too  
 $^1S_0$ - $^1S_0$  - can be fitted to  $^4\text{He}$  FF data  
 Chen, Rupak, Savage '99;  
 Phillips '07  
 AF et al. '20

# Deuteron charge and quadrupole form factors

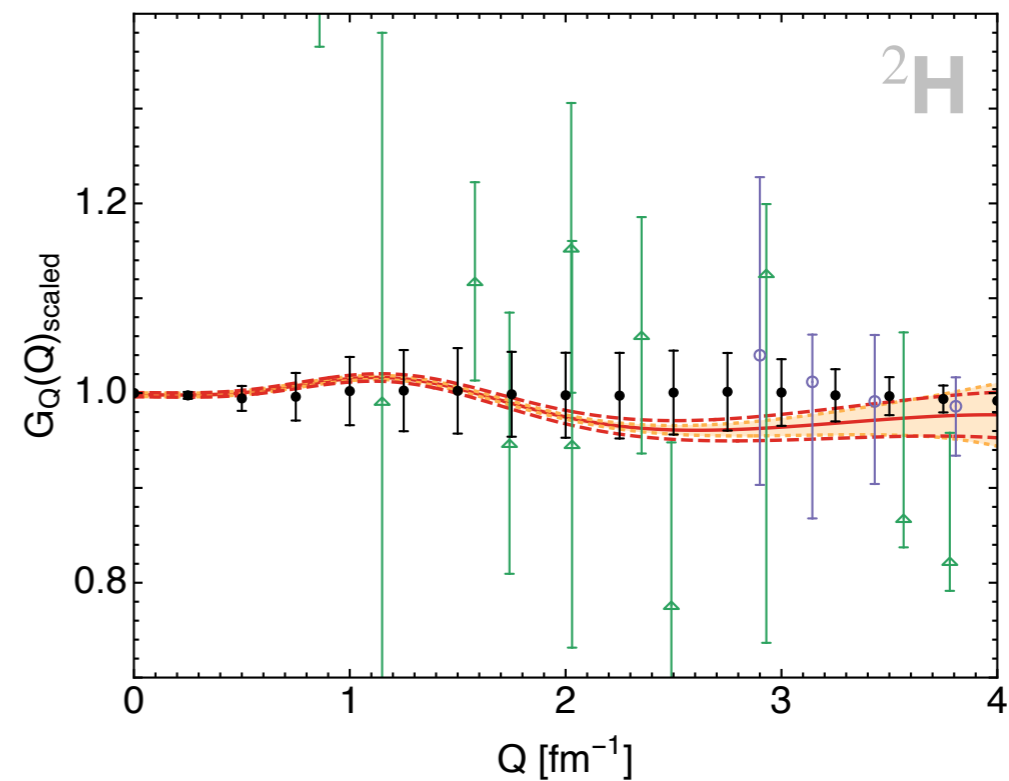


## Deuteron form factors at low Q

Charge FF zoomed at low Q



Quadrupole FF zoomed at low Q



Deuteron charge FF can be used to extract neutron charge FF

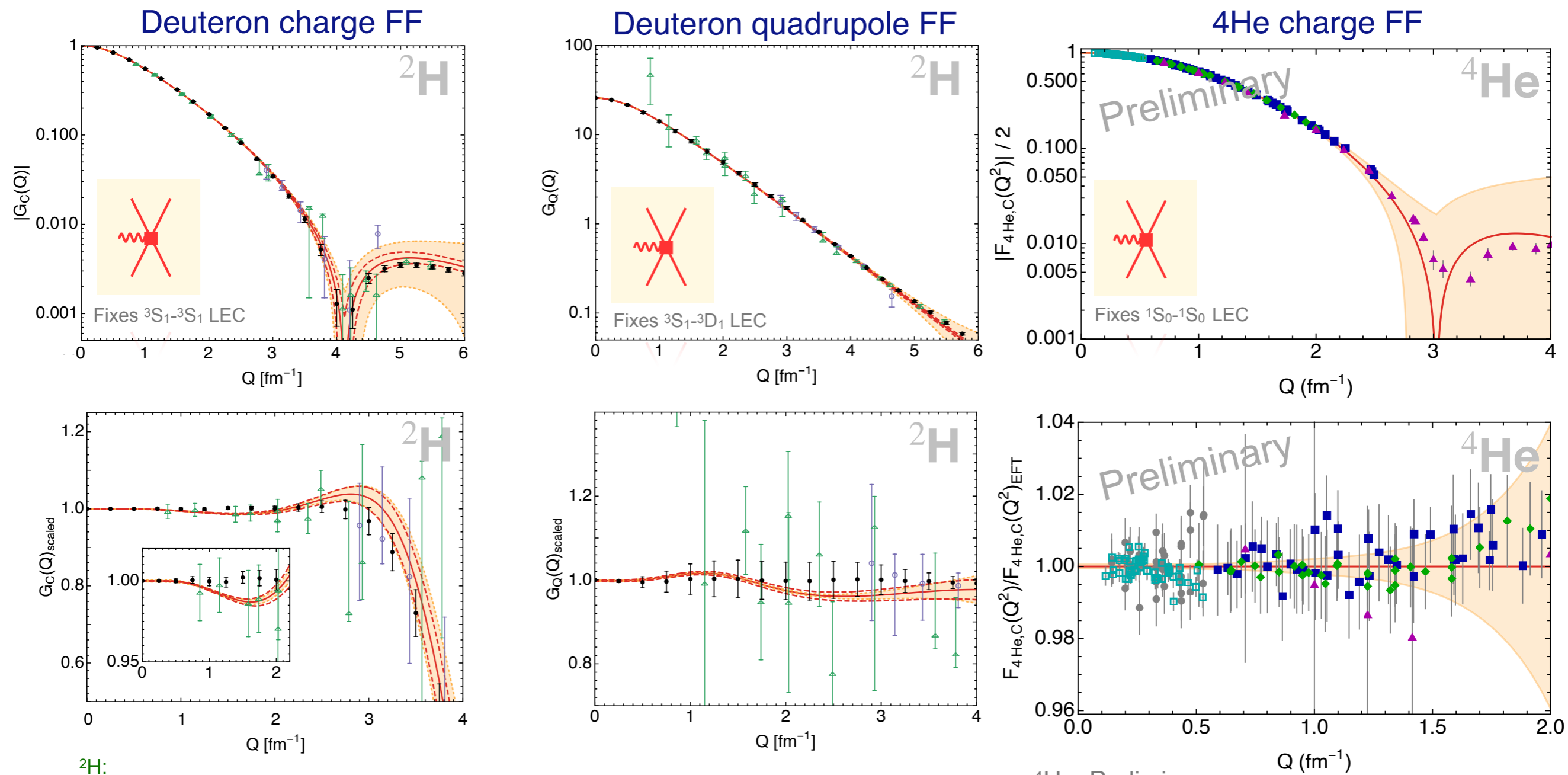
Extraction of deuteron structure radius and quadrupole moment

$$r_{str} = 1.9729^{+0.0015}_{-0.0012} fm$$

$$Q_d = 0.2854^{+0.0038}_{-0.0017} fm^2$$

AF, Möller, Baru, Epelbaum, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

# Low-energy constants from a fit to charge and quadrupole form factors



$^2\text{H}$ :  
 AF, Möller, Baru, Epelbaum, Krebs, Reinert,  
 PRL 124 (2020) 082501; PRC 103 (2021) 024313

— best fit + N<sup>4</sup>LO truncation uncertainty

3 parameters (LECs) in 2N charge density  $J^0$  are fixed from the form factor data of deuteron and  $^4\text{He}$

# Prediction for ${}^4\text{He}$ charge radius

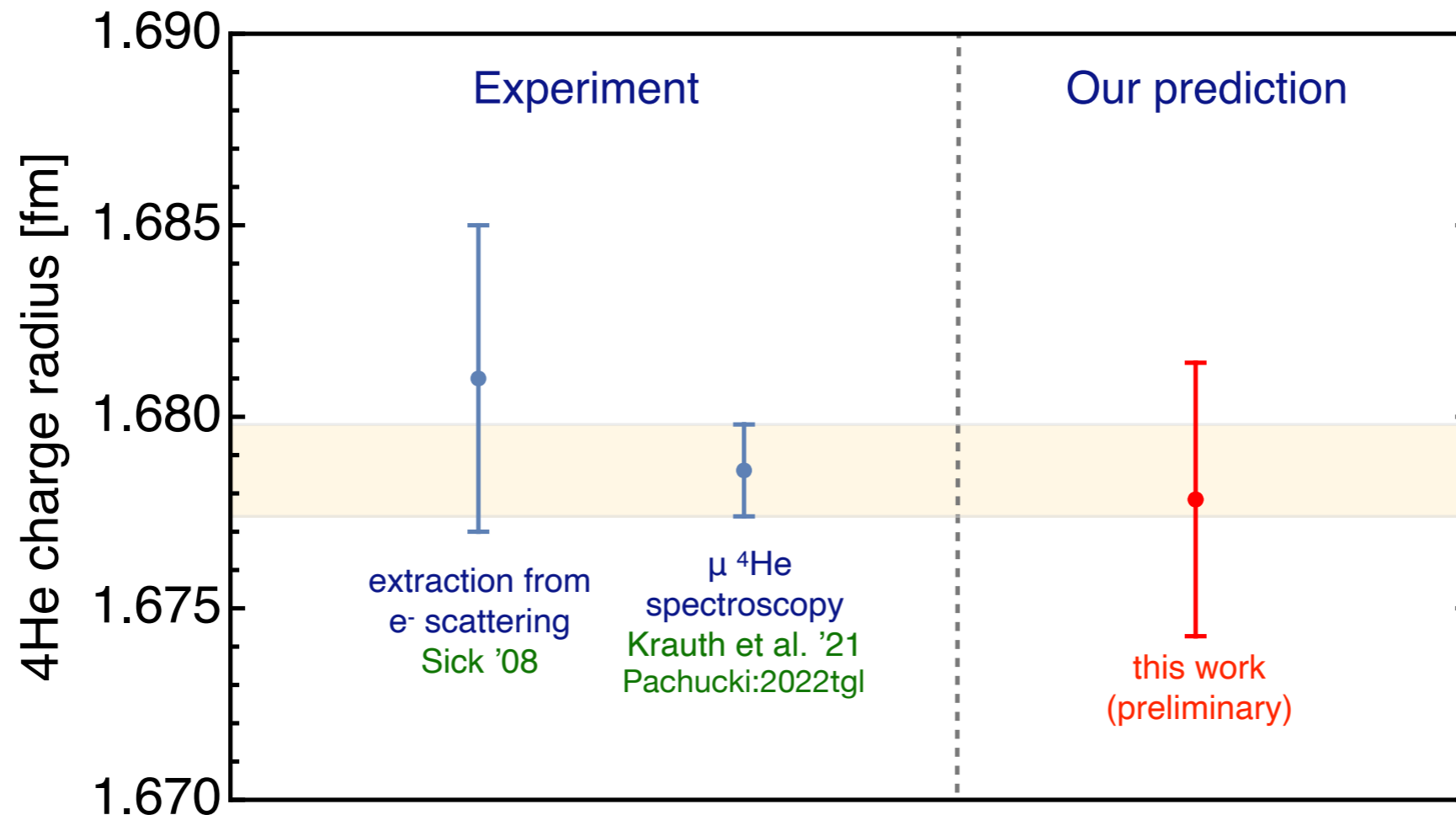
$$r_{str}({}^4\text{He}) = 1.4758 \pm 0.0028_{\text{trunc}} \pm 0.0011_{\text{stat}} \pm 0.0010_{\text{nuclFF}} \text{ fm (Preliminary)}$$

Our prediction for  ${}^4\text{He}$  **charge** radius

$$r_C({}^4\text{He}) = (1.6775 \pm 0.0035) \text{ fm}$$

$$r_C({}^4\text{He}) = r_{str}^2({}^4\text{He}) + \left( r_p^2 + \frac{3}{4m_p^2} \right) + r_n^2$$

preliminary, using CODATA 2018  $r_p$  and own determination of  $r_n$

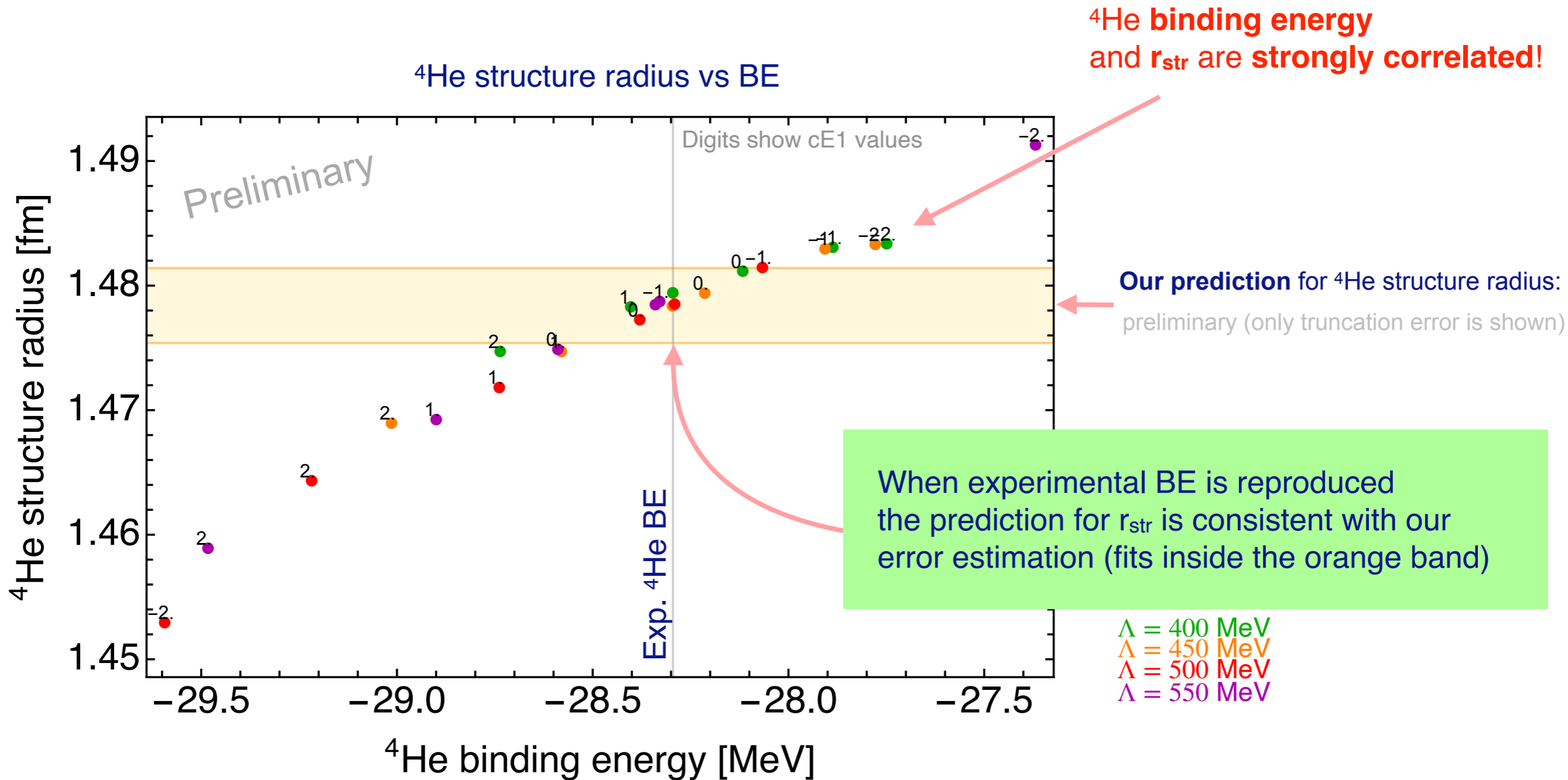


Our prediction for  ${}^4\text{He}$  charge radius is fully consistent with the muonic-atom spectroscopy



# Correlation between $^4\text{He}$ structure radius and binding energy

using variation of  $\mathbf{cE1}$  (N<sup>4</sup>LO contact 3NF LEC)



Many effects (higher-order 3NFs, some relativistic corrections) change BE and radius simultaneously. Once BE is fitted back to physical value the residual effect is usually ver small.

# Estimation of $^3\text{H}$ charge radius

Our preliminary prediction for **isoscalar 3N charge** radius:

$$r_C^{isoscalar3N} = (1.9061 \pm 0.0026) fm$$

preliminary, using CODATA 2018  $r_p$  and own determination of  $r_n$

Isoscalar 3N charge radius definition:

$$(r_C^{isoscalar3N})^2 = \frac{(r_C^{3H})^2 + 2(r_C^{3He})^2}{3}$$

Expression for  $^3\text{H}$  radius:

$$(r_C^{3H})^2 = 3(r_C^{isoscalar3N})^2 - 2(r_C^{3He})^2$$

$^3\text{He}$  charge radius [CREMA 2023 2305.11679]

$$r_C^{3He} = (1.9701 \pm 0.0009) fm$$

Coefficients 2 and 3 amplify both theoretical and experimental uncertainties

Our  $^3\text{H}$  radius estimation:

$$r_C^{(3H)} = (1.7714 \pm 0.0087) fm$$

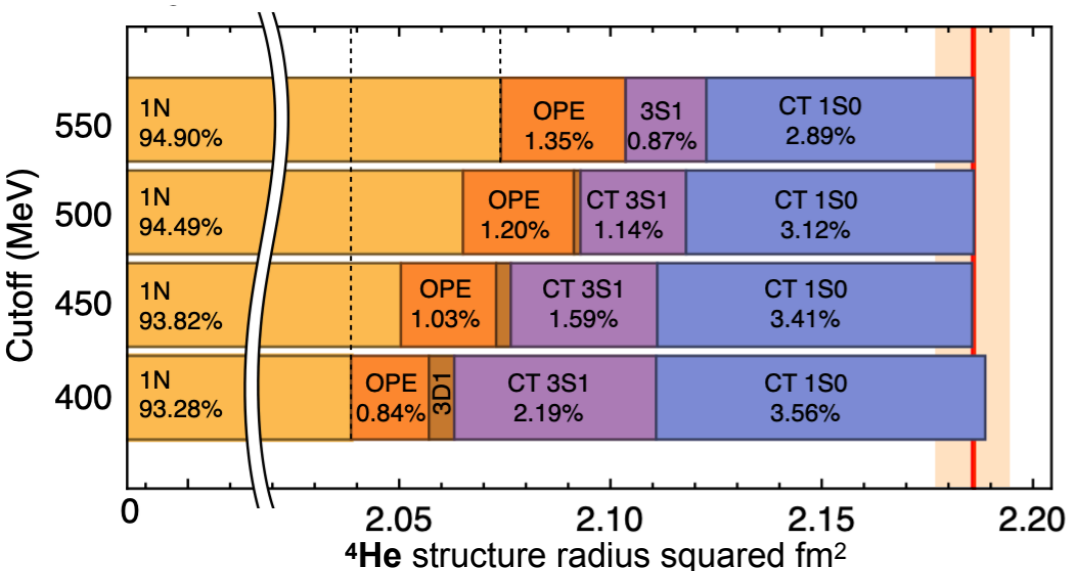
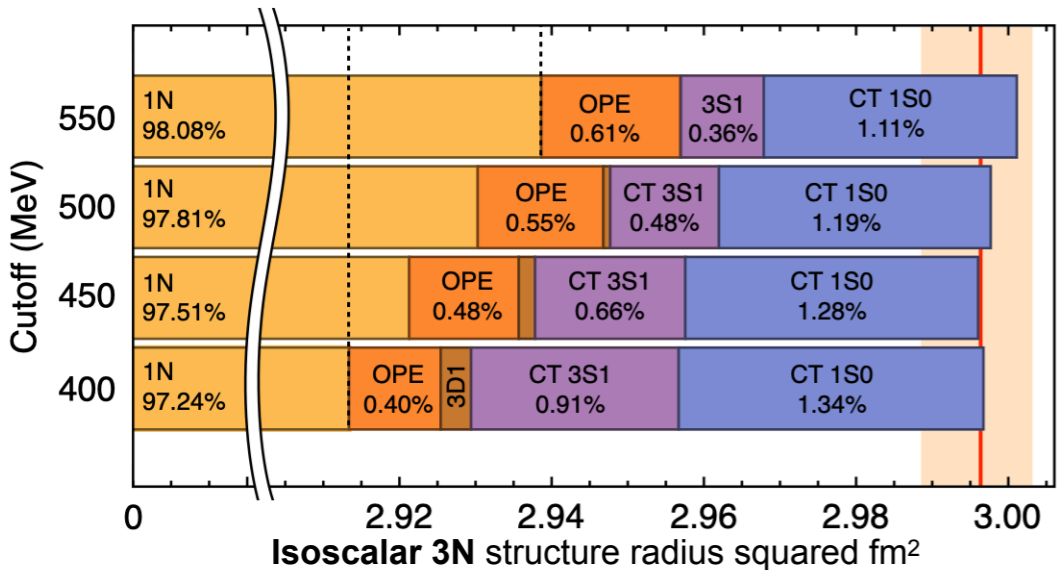
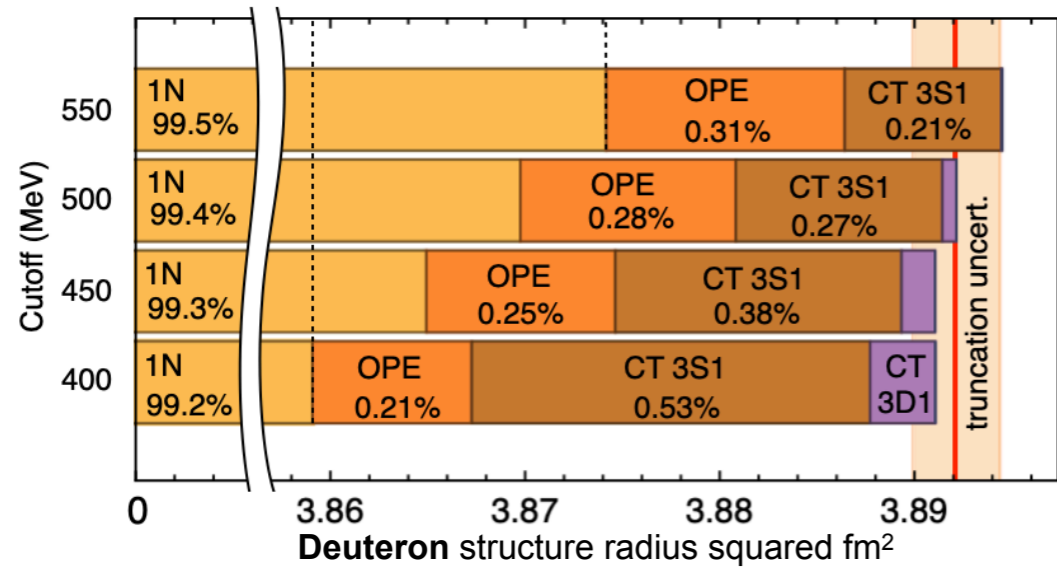
preliminary

This estimation is 10x more precise than  $e^-$  data  $r_C^{3H} = (1.7550 \pm 0.0860) fm$  Amroun et al. '94 (world average)

But it suffers from parametric amplification of uncertainties (both from theory and from  $^3\text{He}$  data)

=> isoscalar 3N charge radius should be used for precision tests

# Importance of 2N charge density



Individual contributions to  $A=2,3,4$  structure radii from

- single-nucleon charge density (1N)
- 2N one-pion exchange density (OPE)
- 2N contact densities (CT 3S1, 3D1, 1S0)

2N charge density contribution to structure radii squared:

deuteron  $\sim 0.7\%$

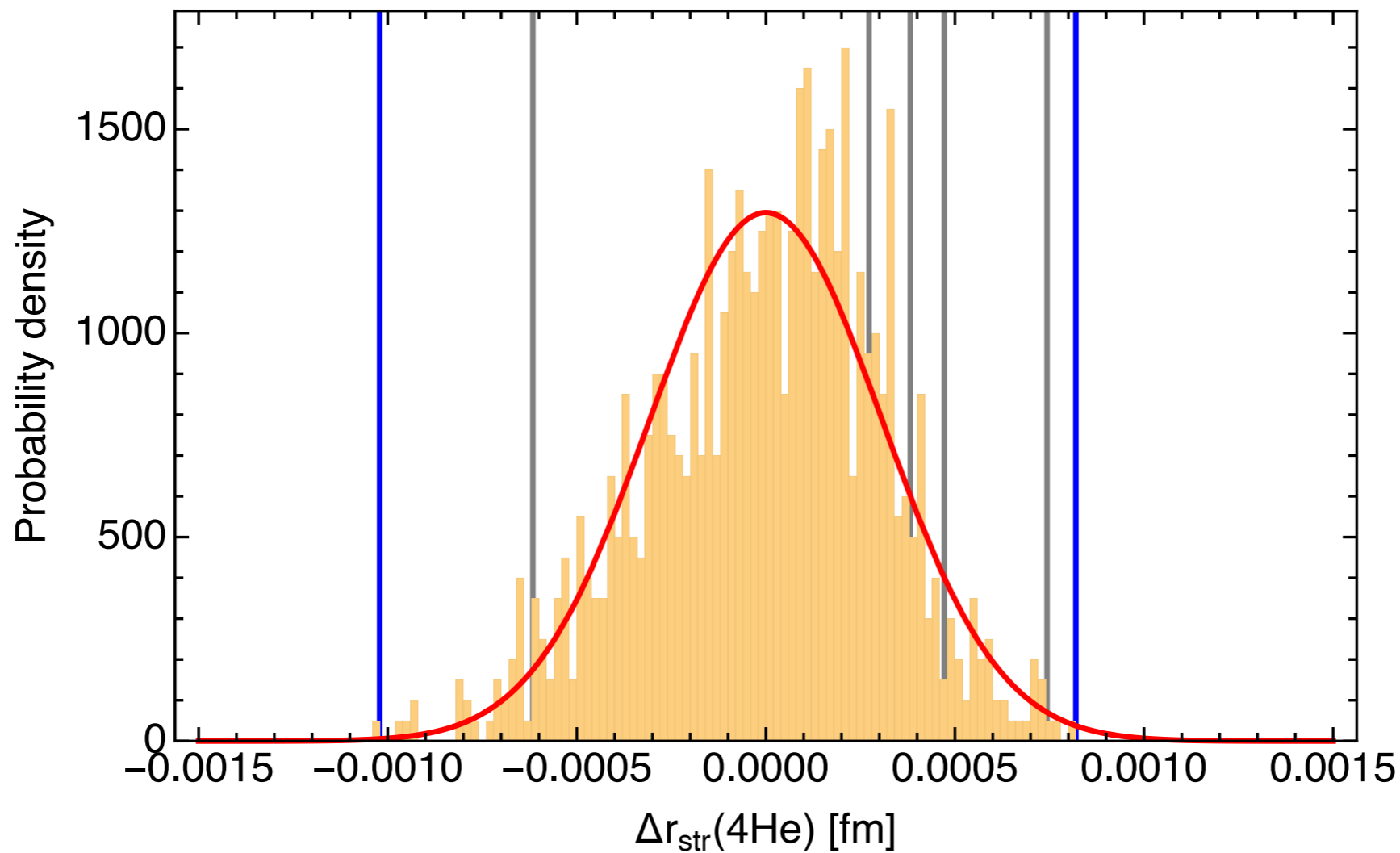
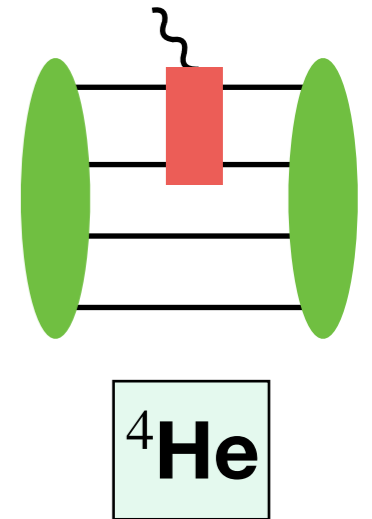
isoscalar 3N  $\sim 2.5\%$

$^4\text{He}$   $\sim 6\%$

For  $A=2,3,4$  importance of 2N charge grows with  $A$

# Sensitivity to nucleon FF parameterisation

Results for  $^4\text{He}$  structure radius fit with  $\sim 1000$  variations of nucleon isoscalar FF  
GES from dispersive analysis of Bonn group [Lin, Hammer and Meißner, PRL 128 2022](#)



Uncertainty from FF parameterisation:

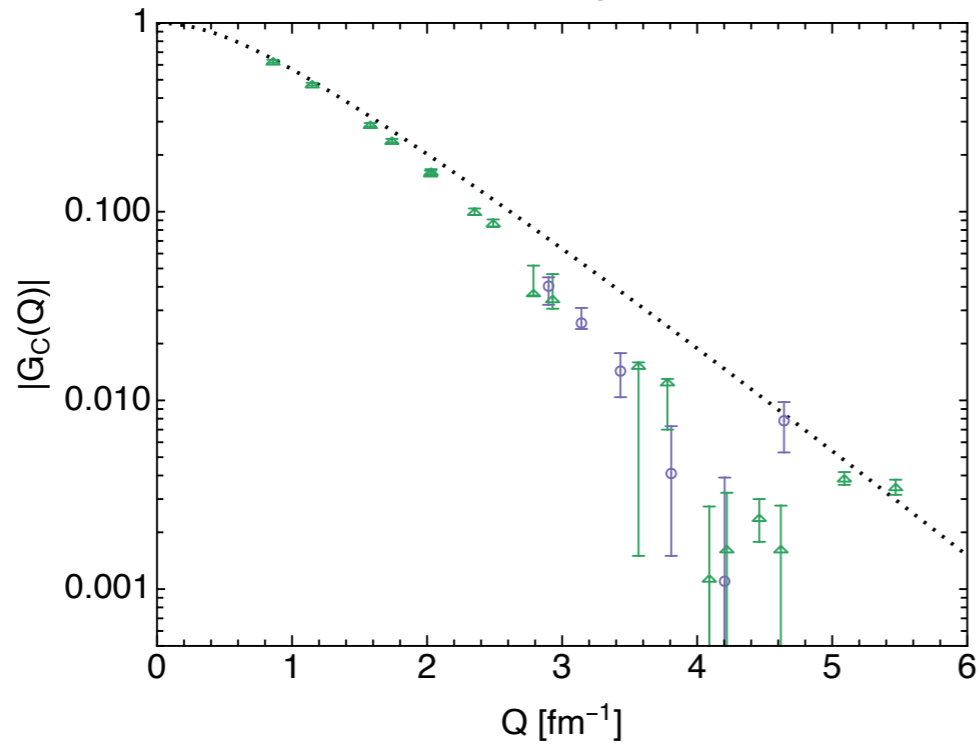
$$r_{str}(^4\text{He}) = 1.4758 \pm 0.0010_{\text{nuc|FF}} \text{ fm} \text{ (Preliminary)}$$

We have also tested other modern nucleon FF parameterisations and they produce results in the same range

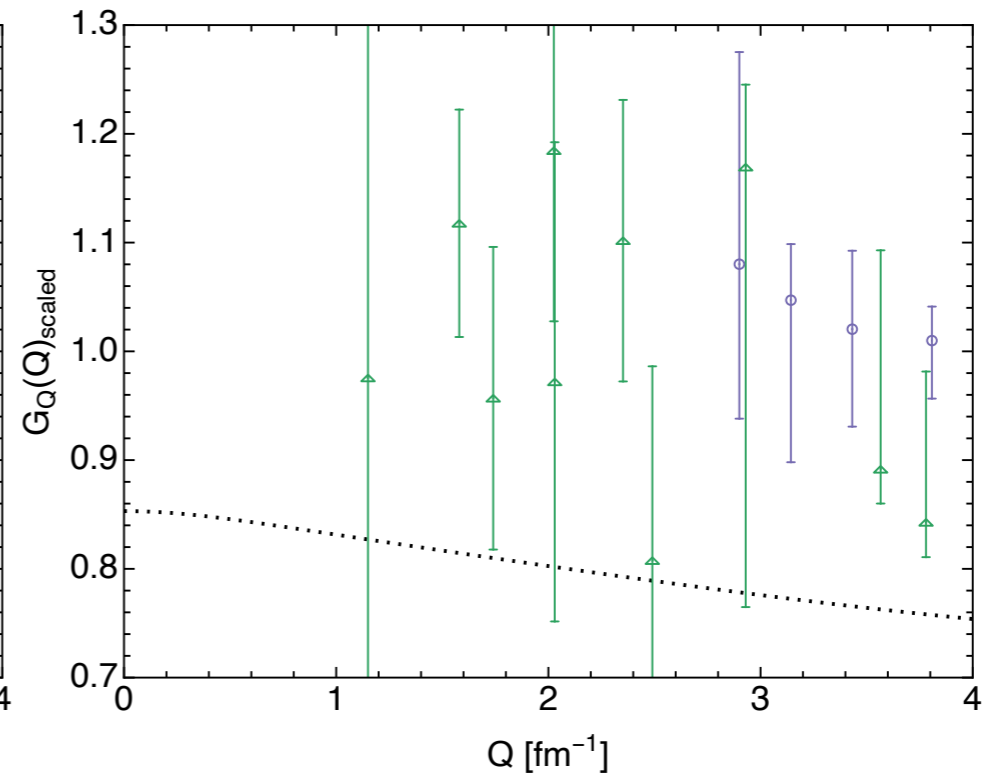
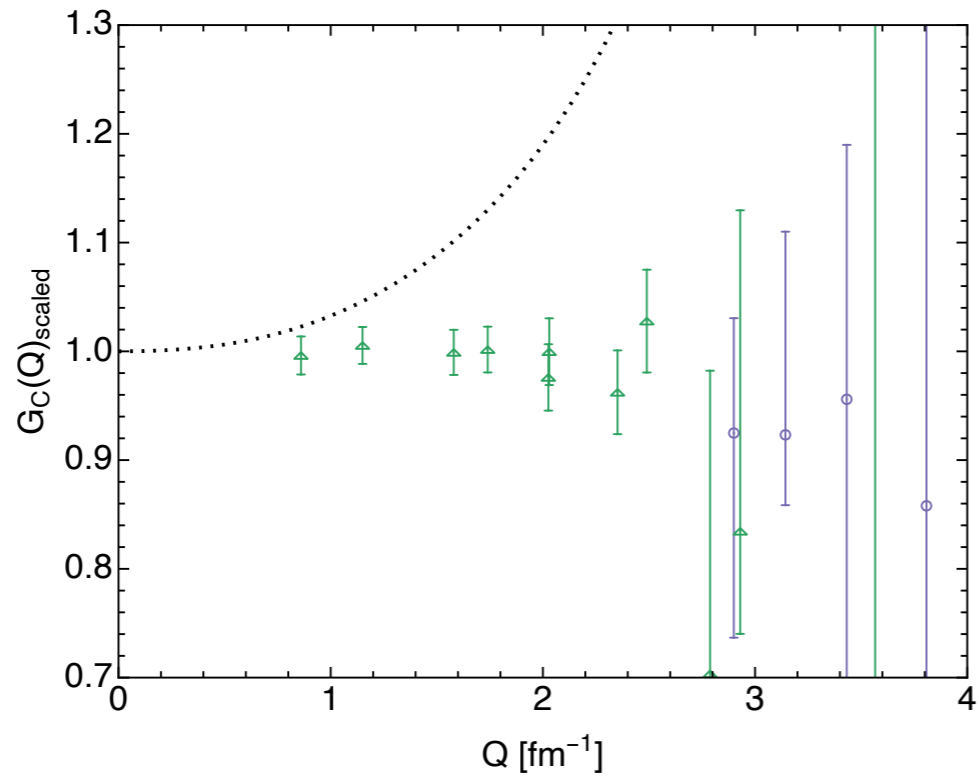
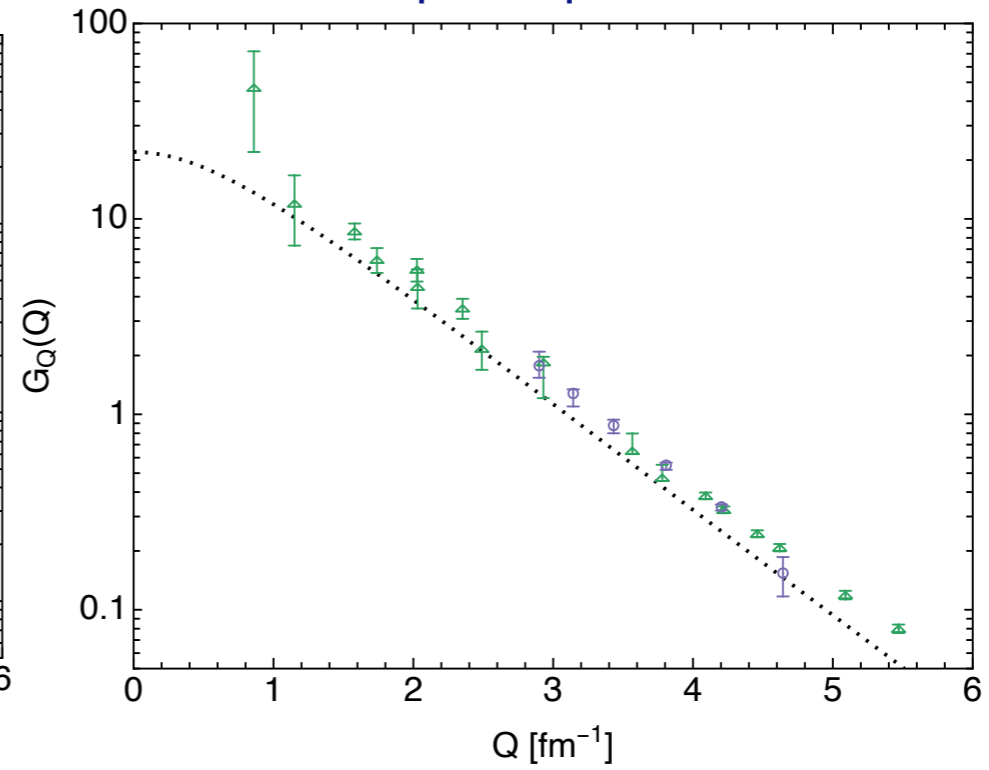
# Deuteron charge and quadrupole form factors

LO

### Deuteron charge form factor



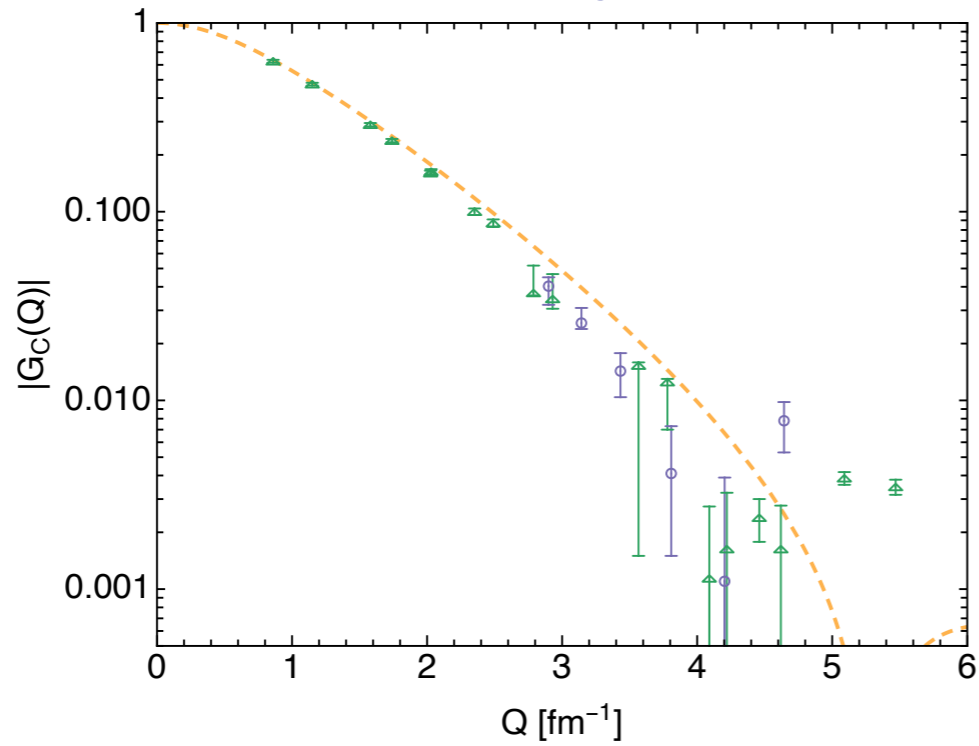
### Deuteron quadrupole form factor



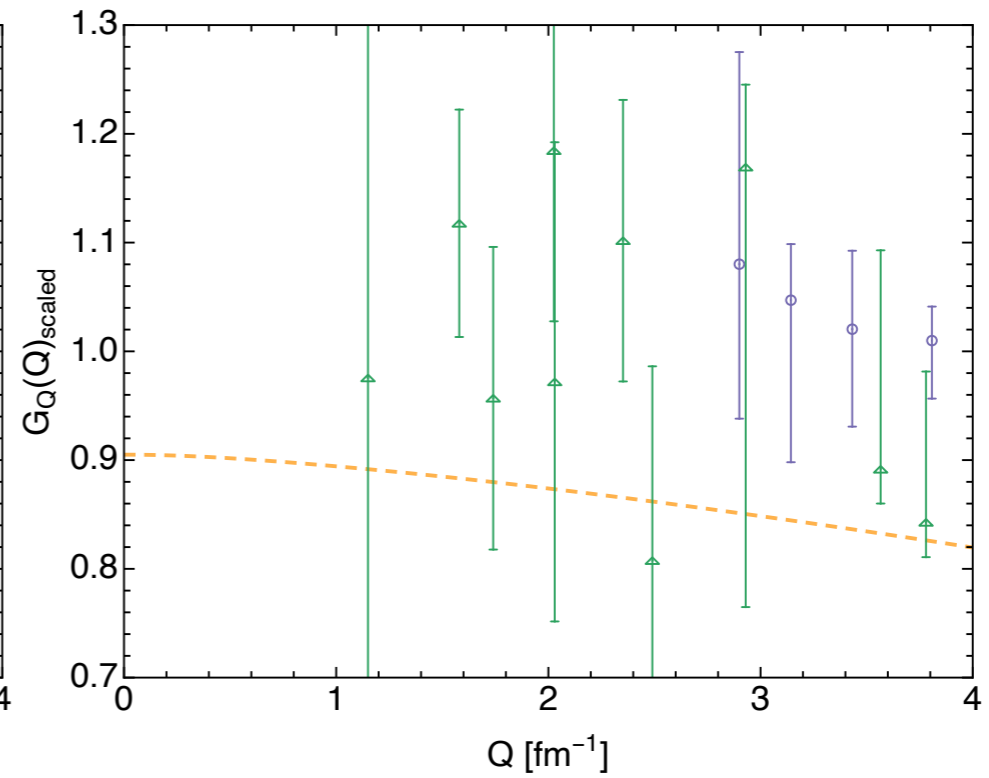
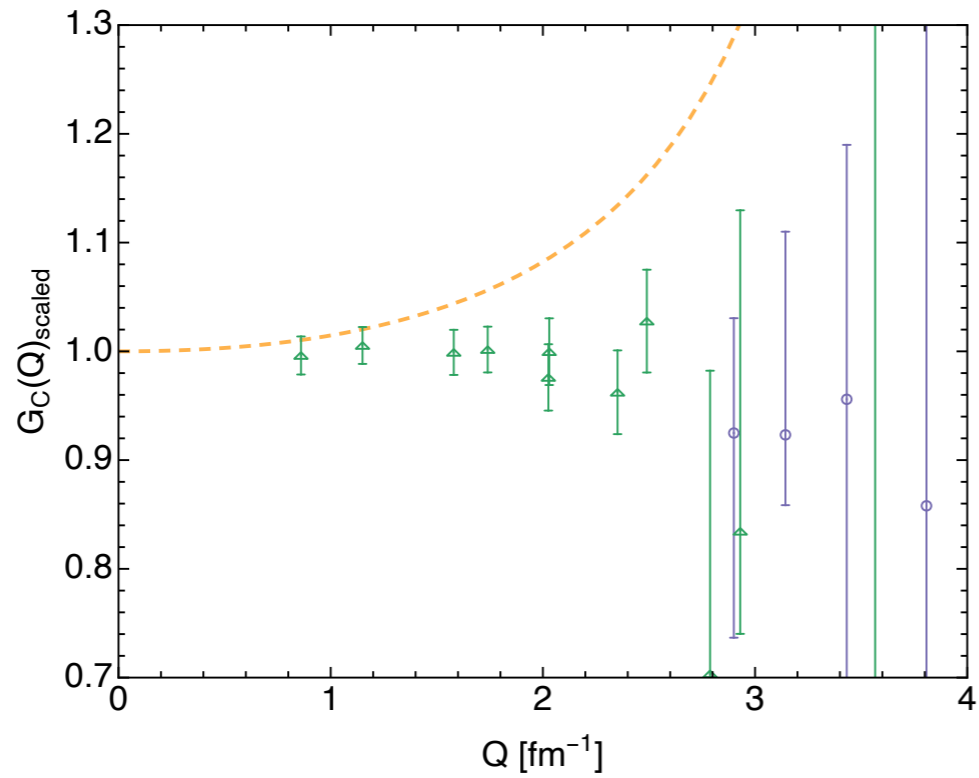
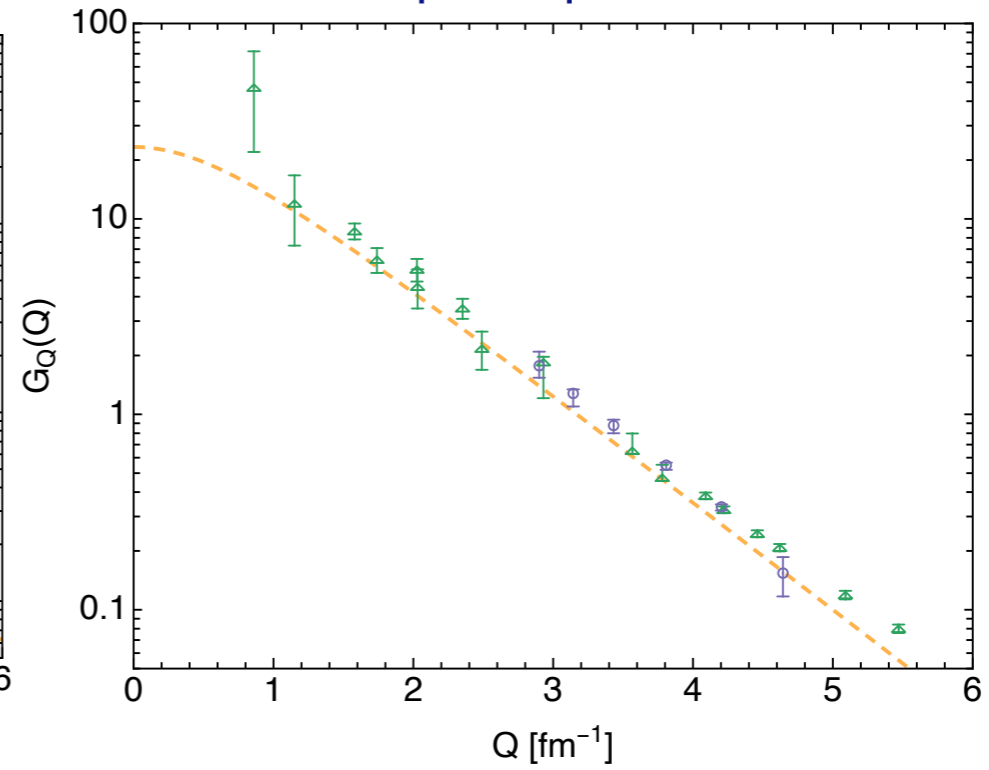
# Deuteron charge and quadrupole form factors

NLO

## Deuteron charge form factor



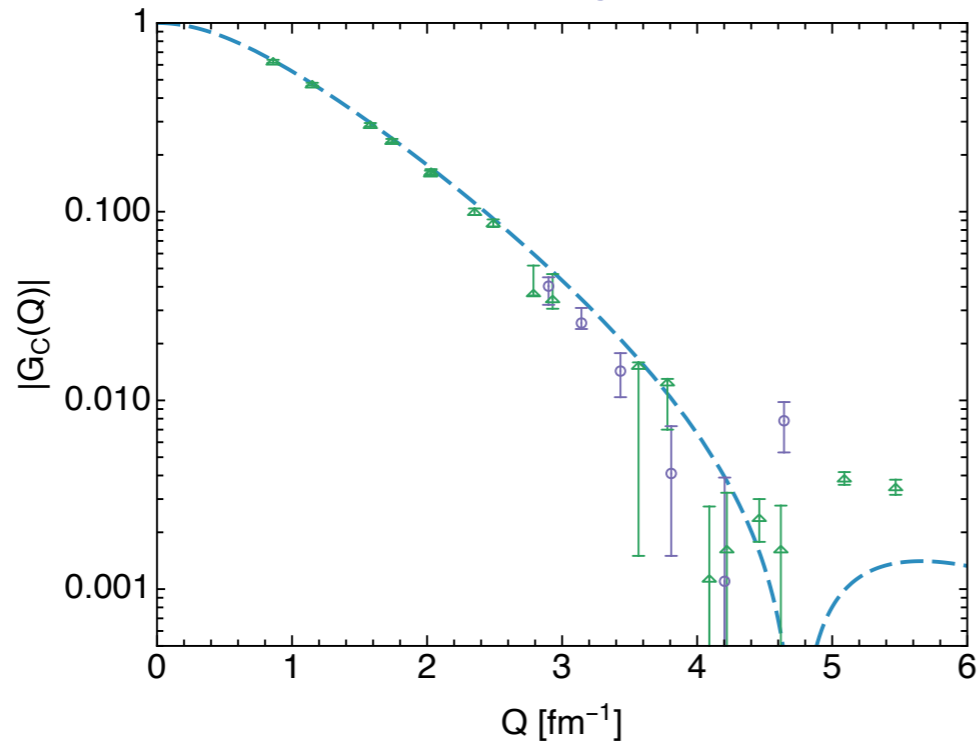
## Deuteron quadrupole form factor



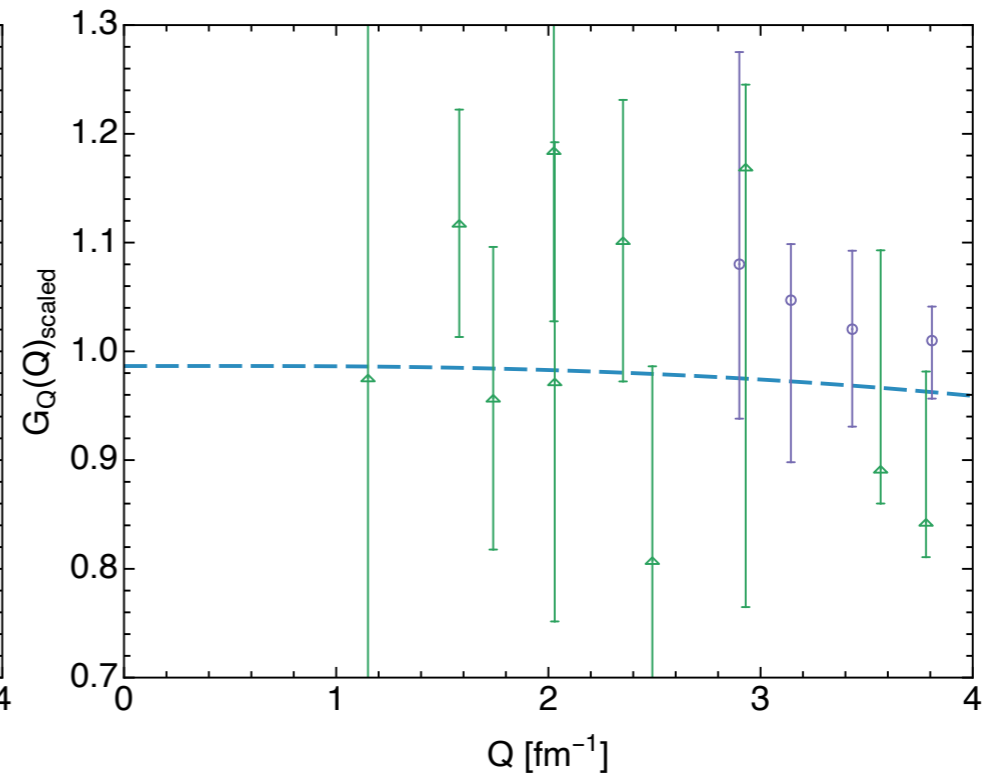
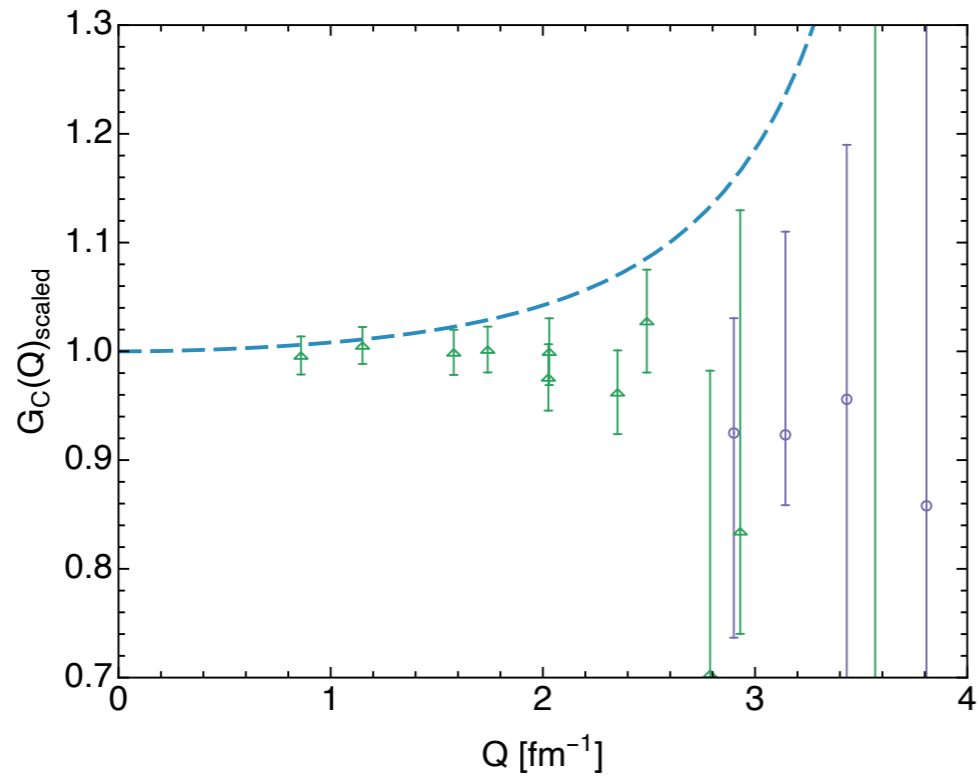
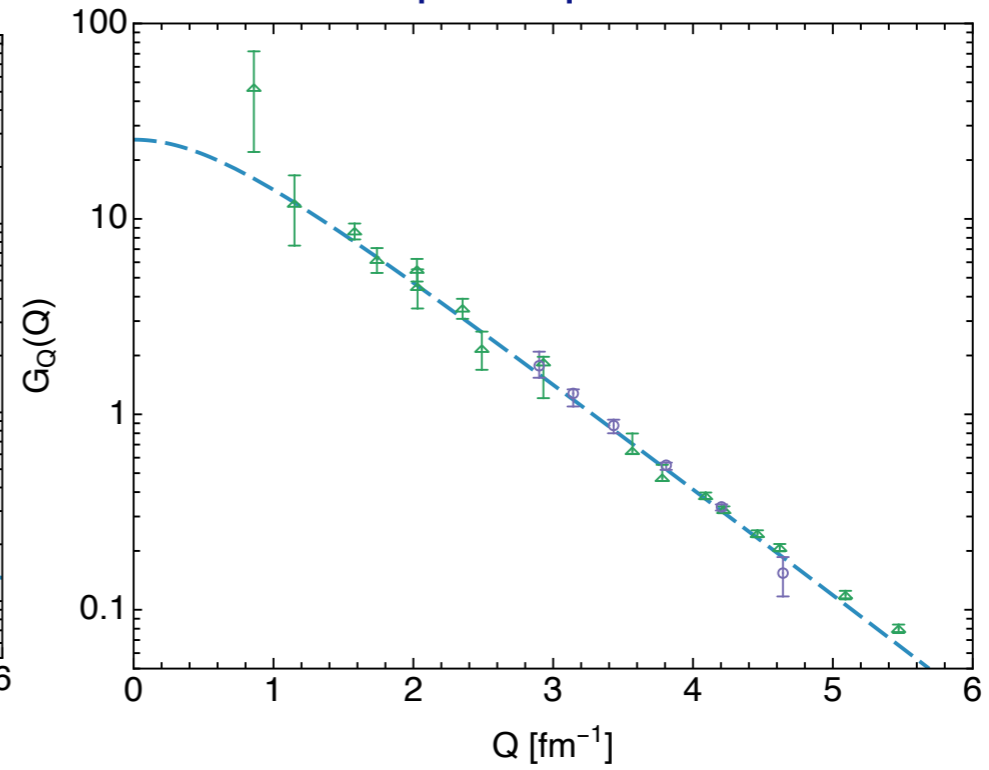
# Deuteron charge and quadrupole form factors

N2LO

## Deuteron charge form factor



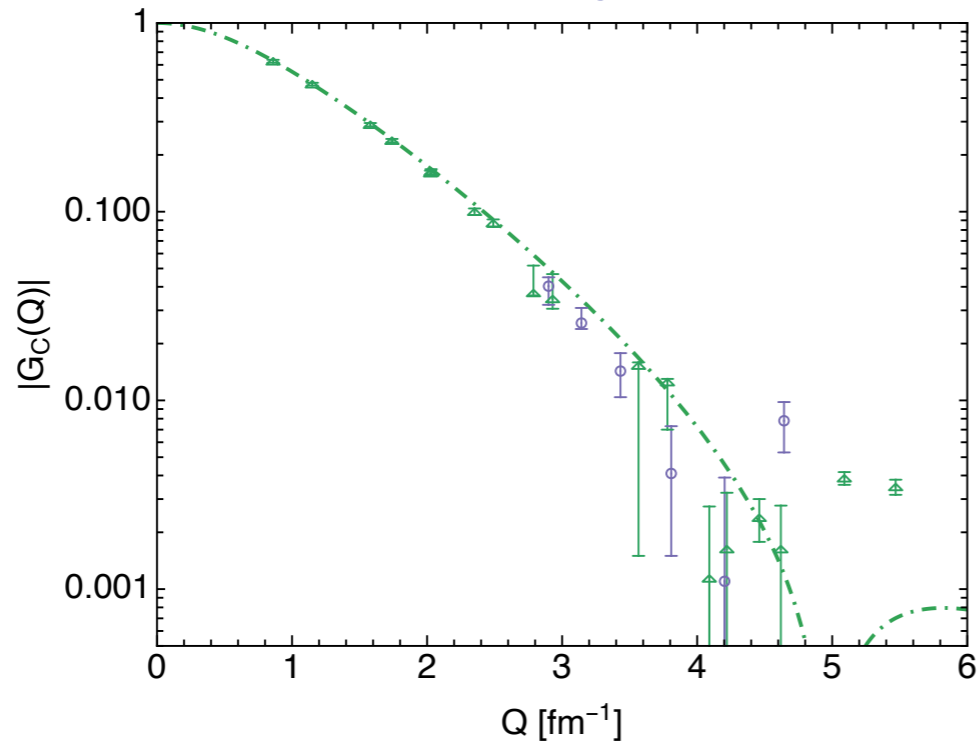
## Deuteron quadrupole form factor



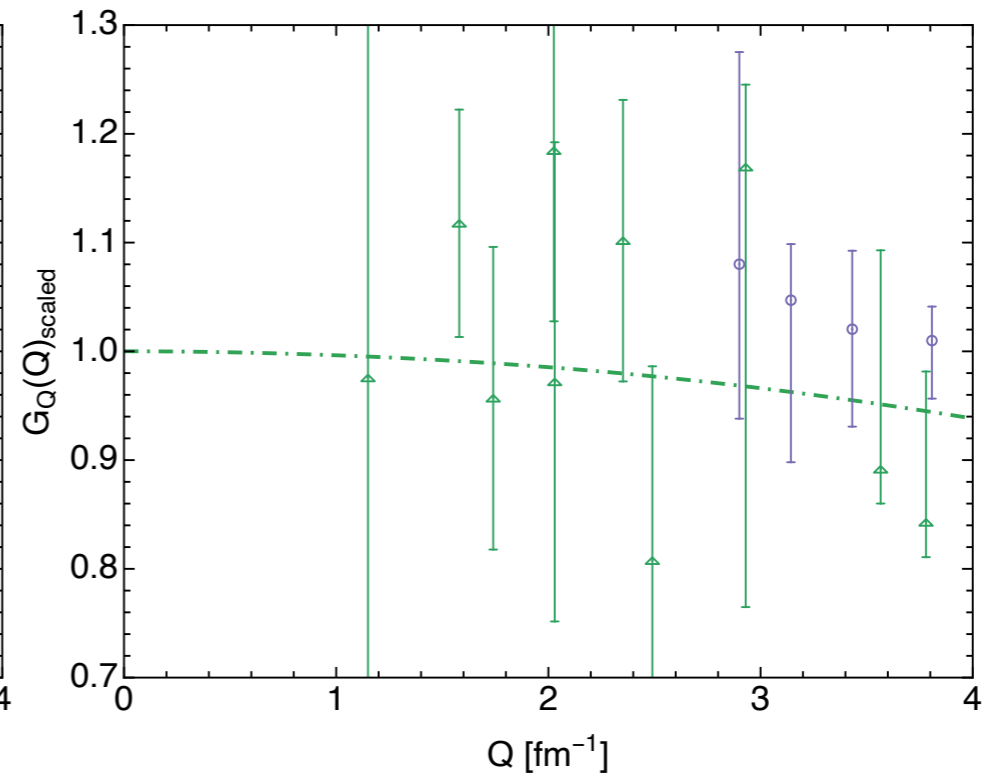
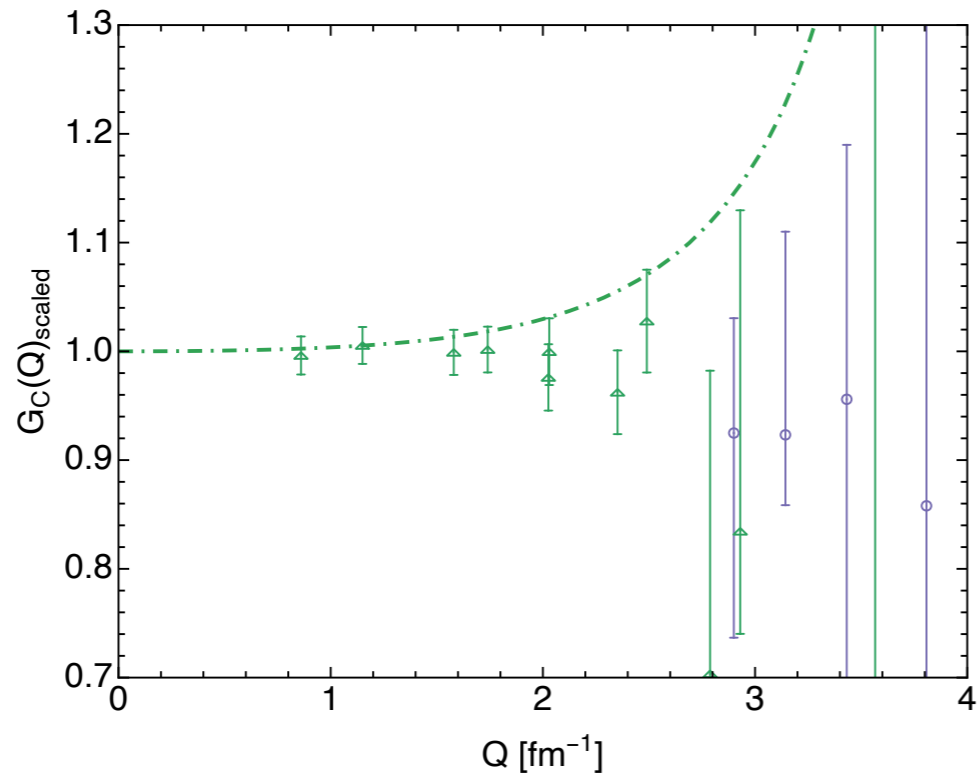
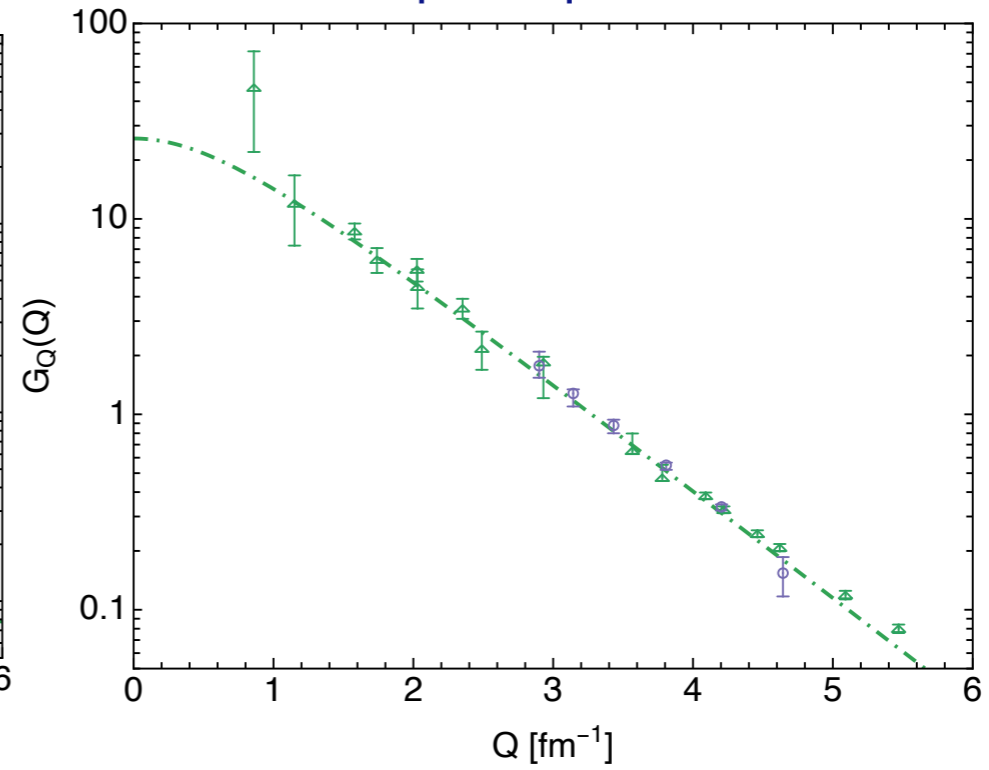
# Deuteron charge and quadrupole form factors

N3LO  
without  
2N charge  
density

## Deuteron charge form factor

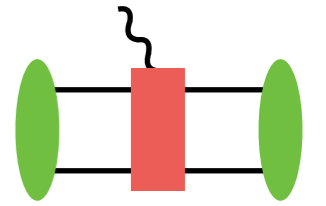


## Deuteron quadrupole form factor

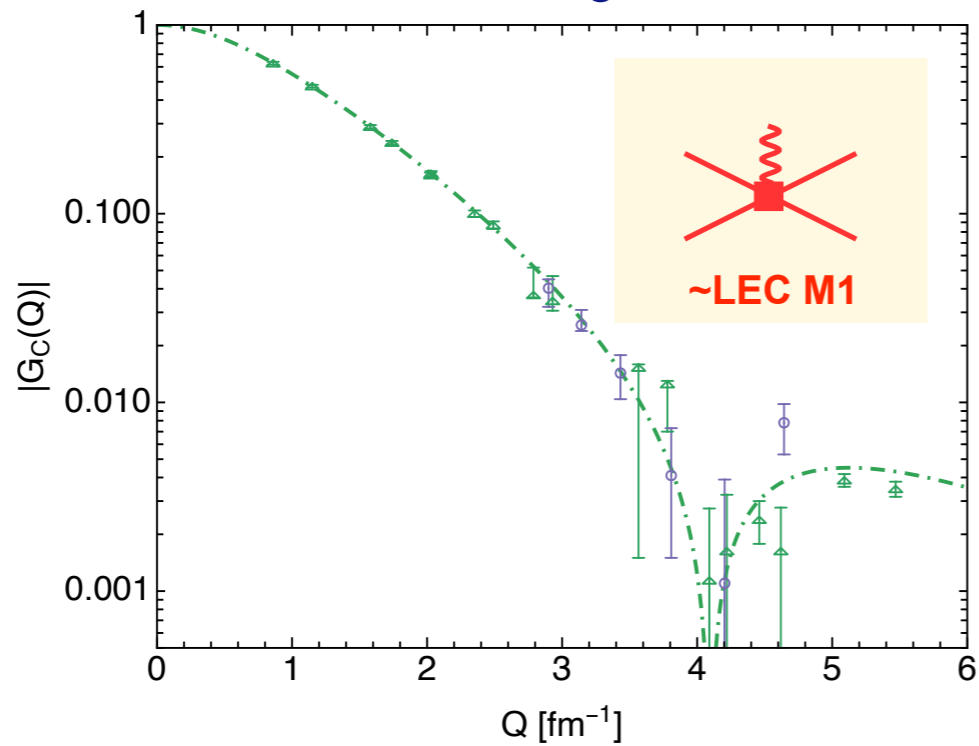




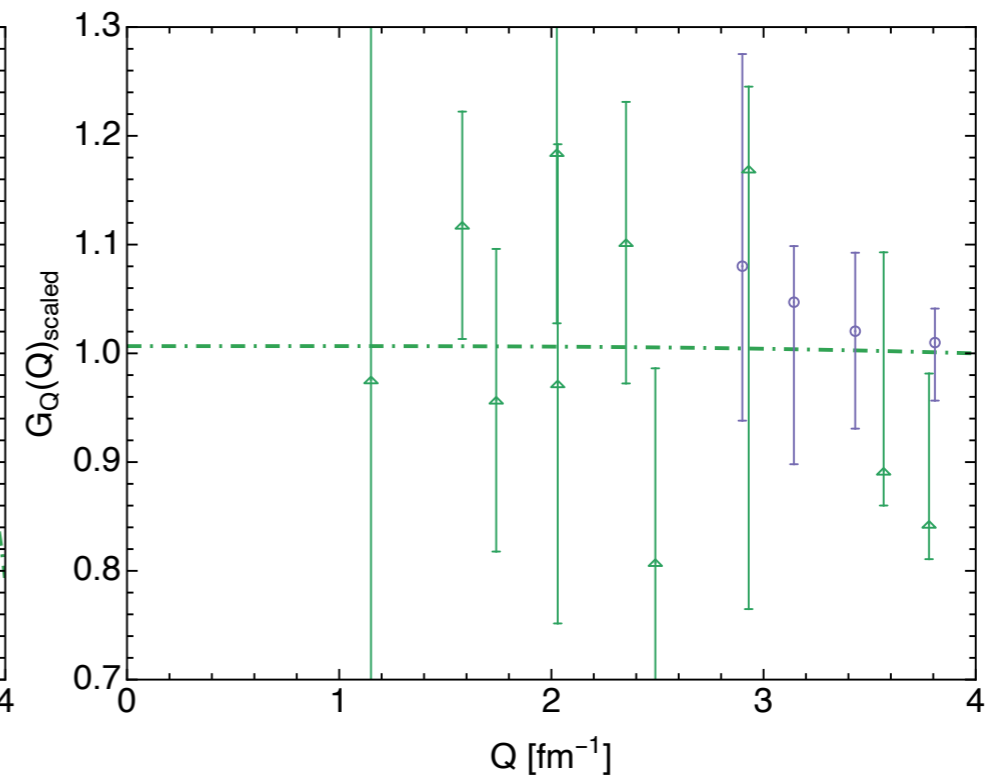
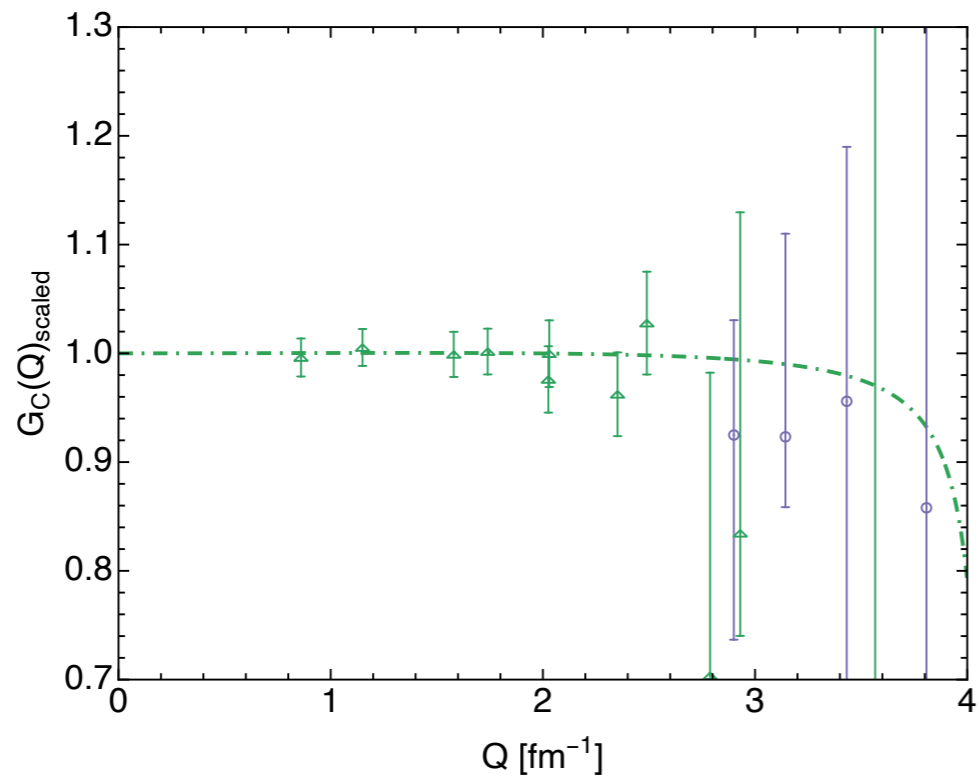
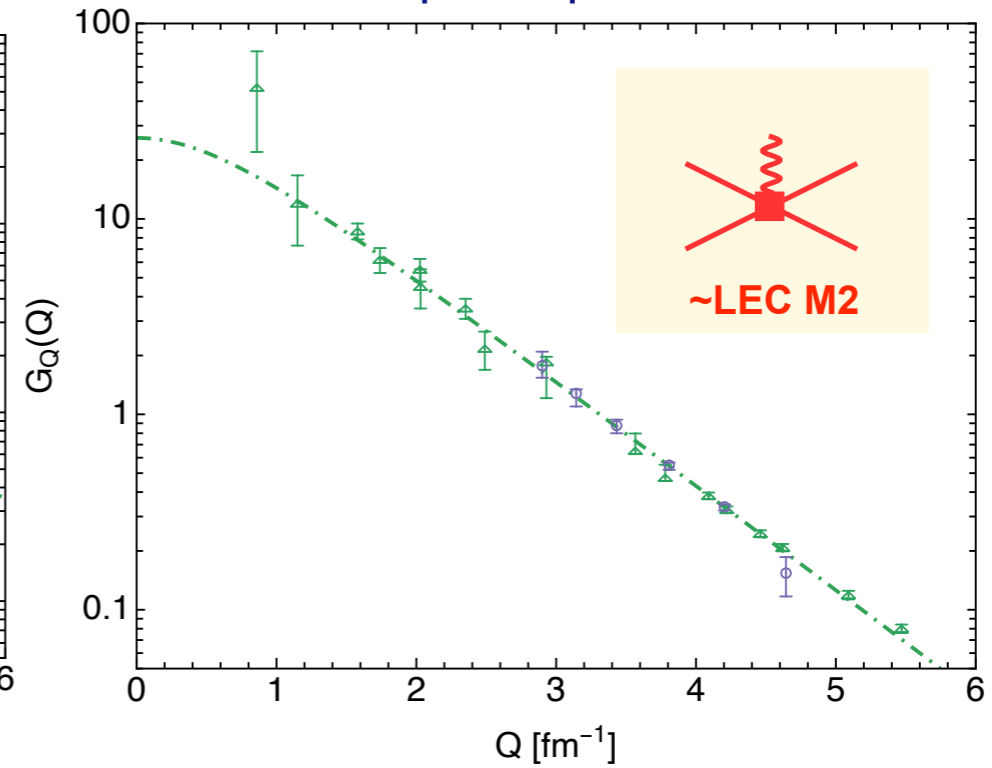
# Deuteron charge and quadrupole form factors



## Deuteron charge form factor

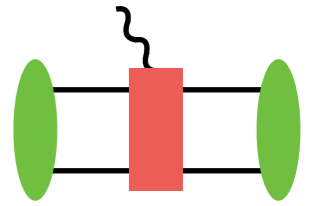


## Deuteron quadrupole form factor

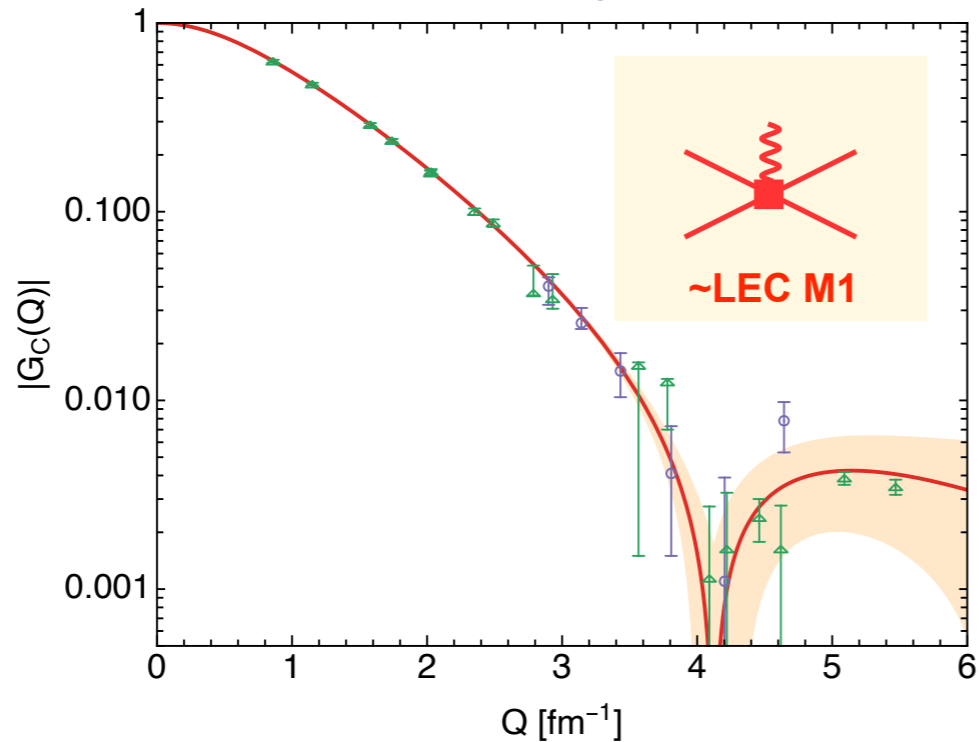


N3LO  
with  
2N charge  
density

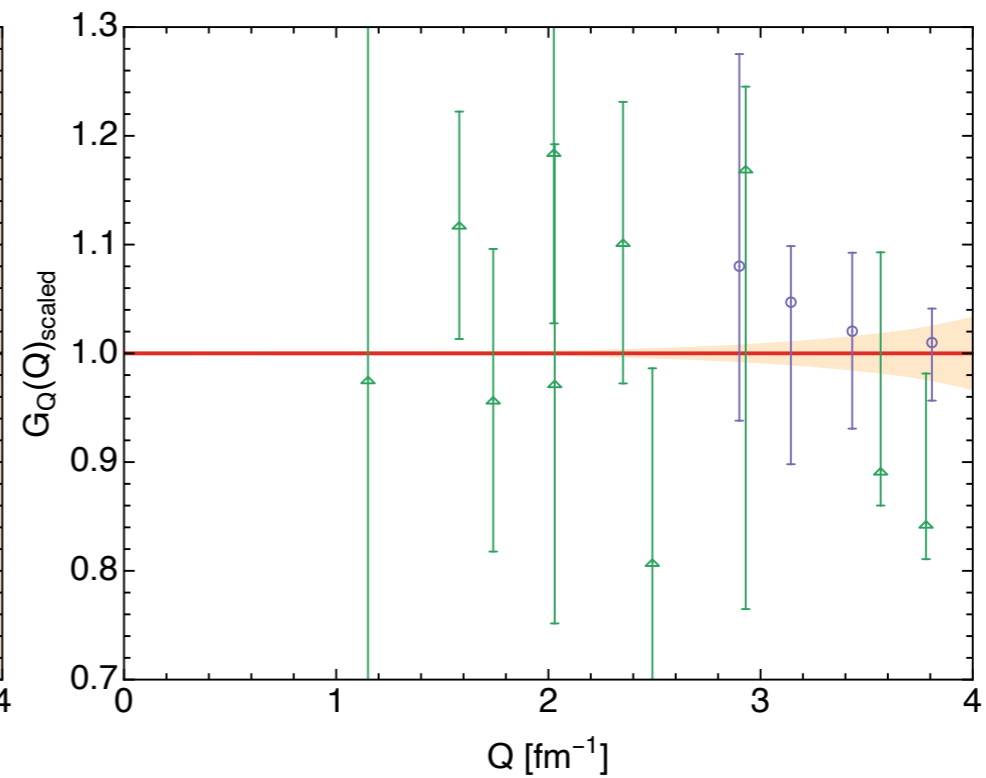
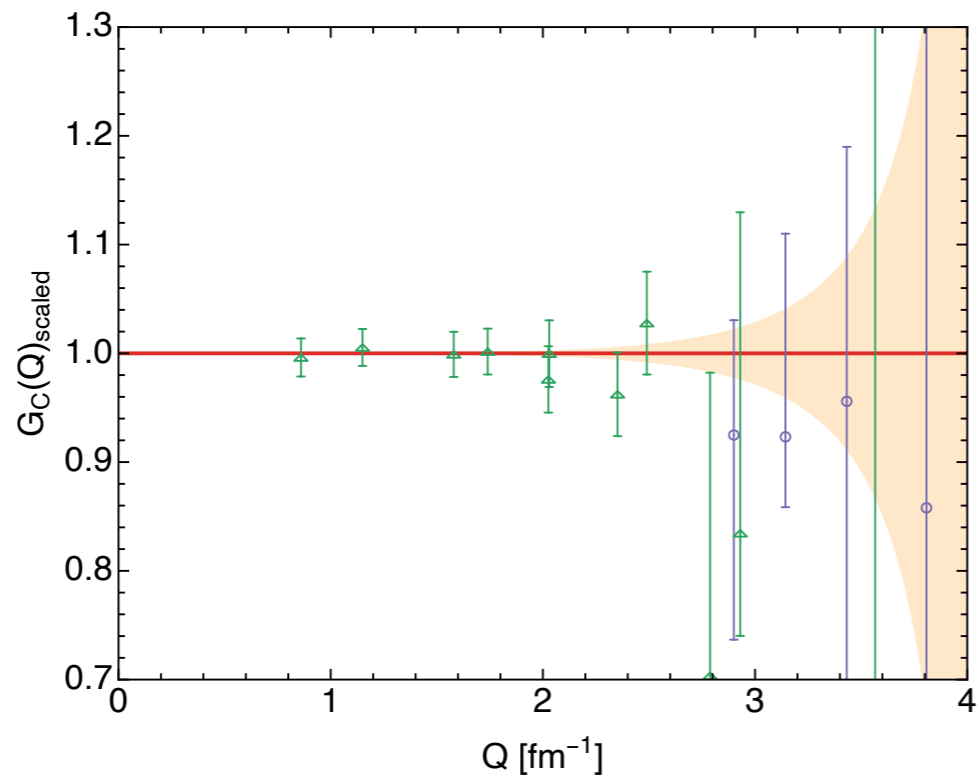
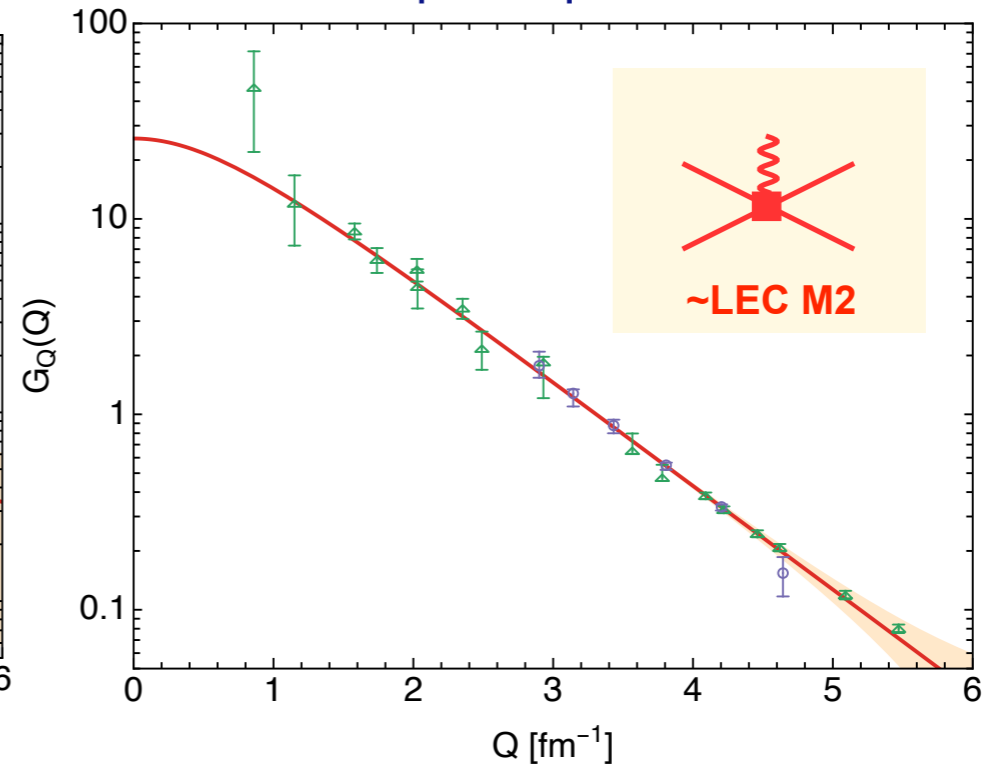
# Deuteron charge and quadrupole form factors



## Deuteron charge form factor



## Deuteron quadrupole form factor



N4LO  
with  
2N charge  
density

LECs M1 and M2 are fitted to GC and GQ data

Good description of low-Q behaviour and also node position