The muon $g-2$ and the role of hadron physics

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14 March 2024

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WISSENSCHAFT

11. APRIL 2021 NR. 14 SEITE 57

Braut sich da was zusammen? Im Sommer 2013 wurde der Myonen-Speicherring (auf dem Lastwagen) am Fermilab nahe Chicago angeliefert. Jetzt wurden erste Ergebnisse verkündet. Foto Fermilab/Reidar Hahn

Die Macht der Myonen

Ein Teilchen schickt sich an, eine beispiellos erfolgreiche Theorie zu sprengen. Viele Physiker freuen sich wie Bolle. Andere warnen, dazu sei es noch zu früh.

Von Ulf von Rauchhaupt

Neue Zürcher Zeitung

Vor zwei Wochen bekam das Standardmodell der Teilchenphysik nasse Füsse. Jetzt steht ihm das Wasser bis zum Hals

Christian Speicher
07.04.2021, 17.00 Uhr

Frankfurter Allgemeine

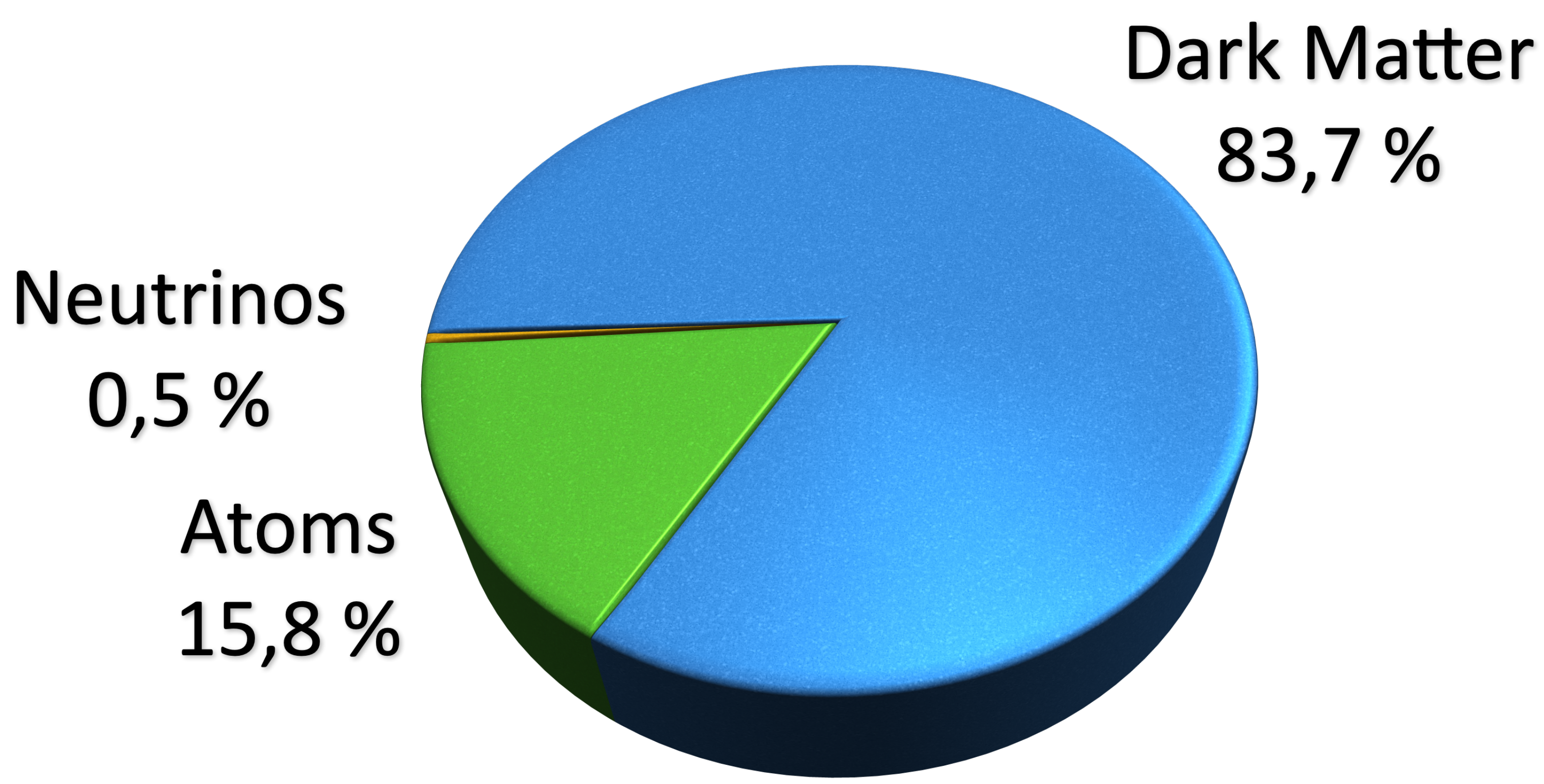
HERAUSGEGEBEN VON GERALD BRAUNBERGER, JÜRGEN KAUBE, CARSTEN KNOP, BERTHOLD KOHLER

RÄTSELHAFTE MYONEN

Abschied vom Standardmodell?

VON MANFRED LINDINGER - AKTUALISIERT AM 07.04.2021 - 18:55

Motivation: The Dark Matter Puzzle

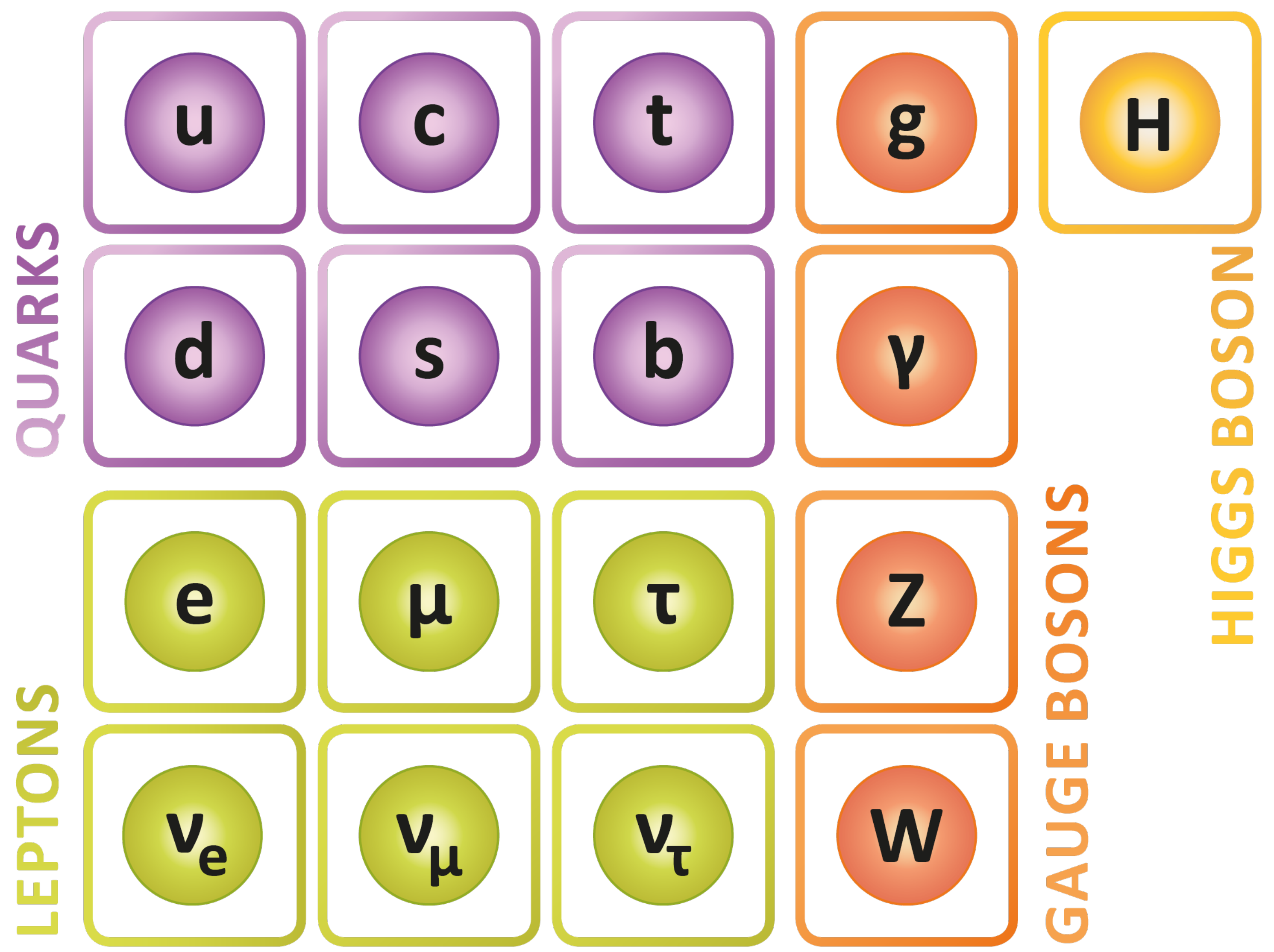


Astrophysical observations:

- Dark Matter dominates matter density of universe
- No clues on nature of Dark Matter from laboratory experiments

Standard Model of Particle Physics:

- Quantitative framework for description of known constituents of visible matter
- No Dark Matter candidate among SM particles
- No explanation for baryon asymmetry in universe



Source: Tanja Labs for PRISMA+

Standard Model does not provide a complete description of Nature

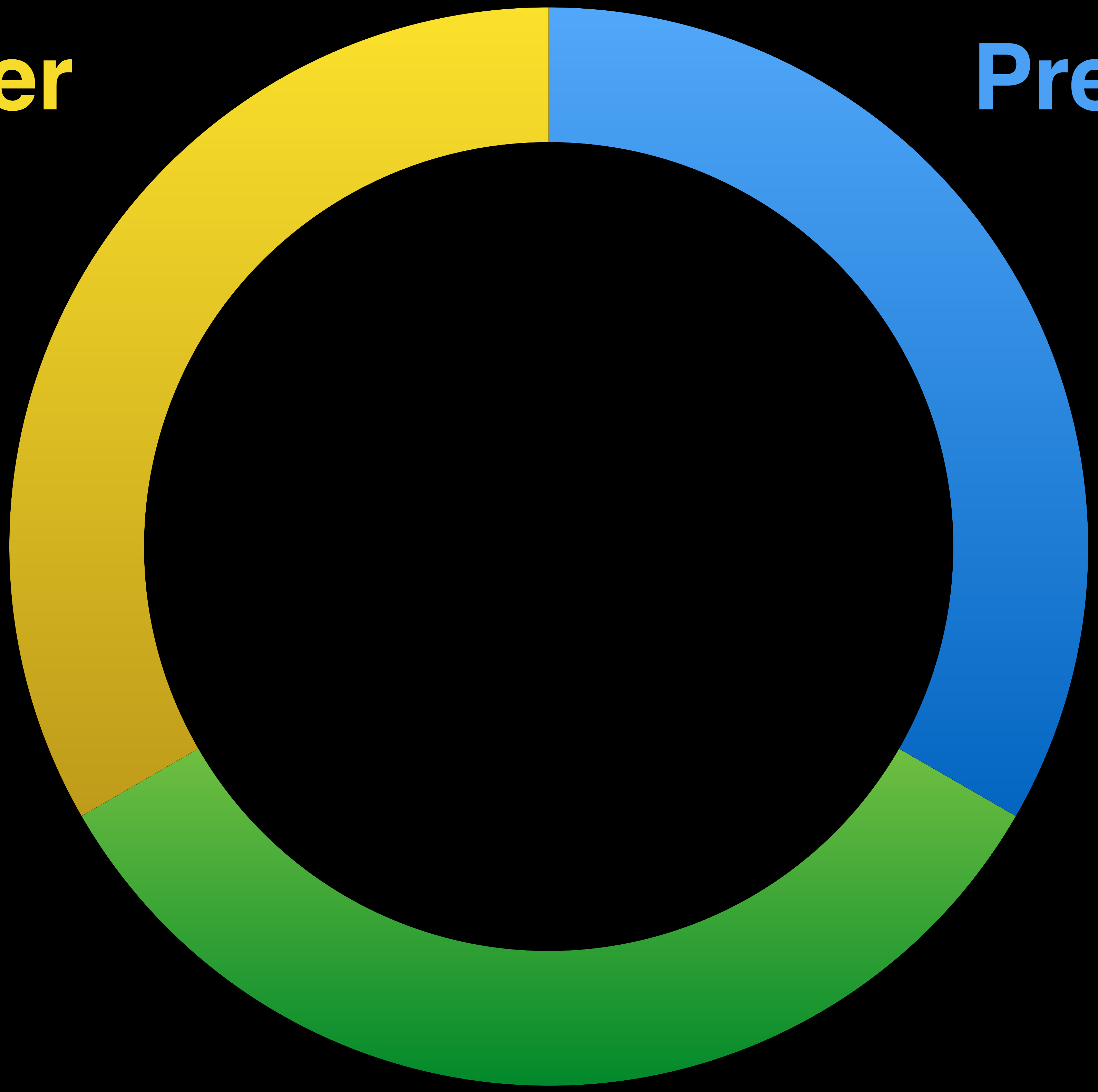
The Quest for New Physics

Energy Frontier

New particles and interactions at colliders

Precision Frontier

Comparison of precision observables to Standard Model prediction



Intensity Frontier

Enhancement of rare phenomena

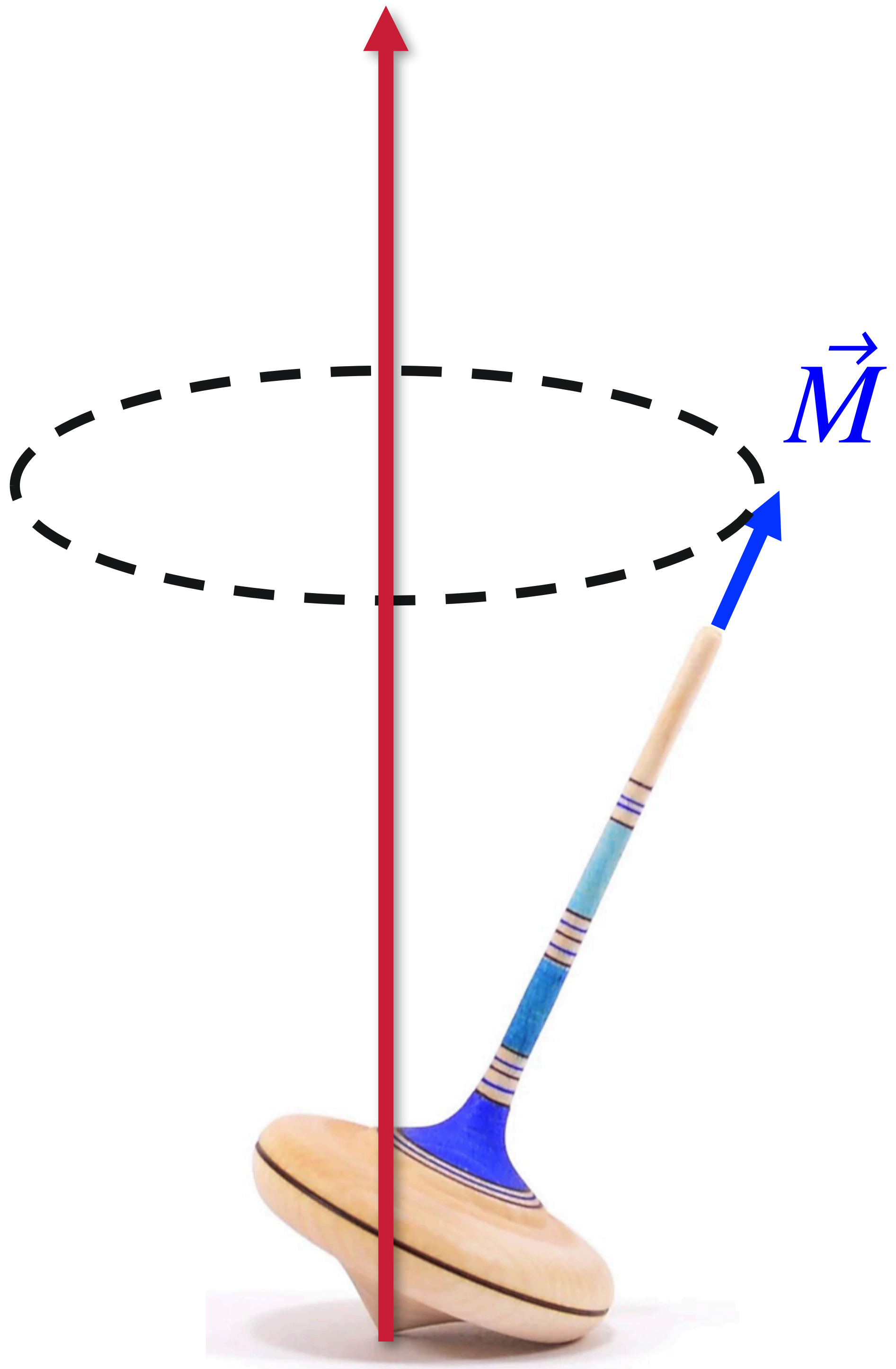
The anomalous magnetic moment of the muon

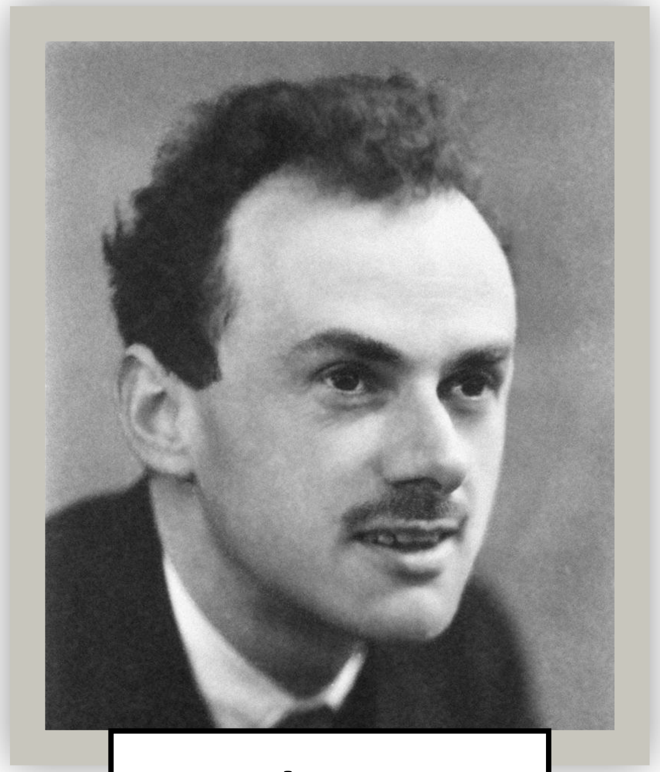
Magnetic moment of particle with mass m and charge e :

$$\vec{M} = g\,\frac{e\hbar}{2m}\,\vec{S}$$

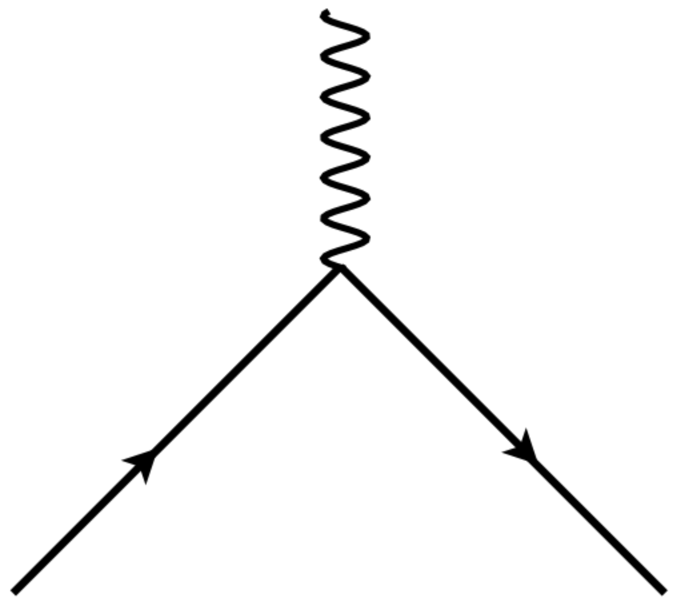
g-factor

Magnetic field $\vec{B}$




Paul Dirac

$$g = 2$$



Quantum corrections modify Dirac's prediction $g = 2$

$g = 2(1 + a), \quad a :$ anomalous magnetic moment

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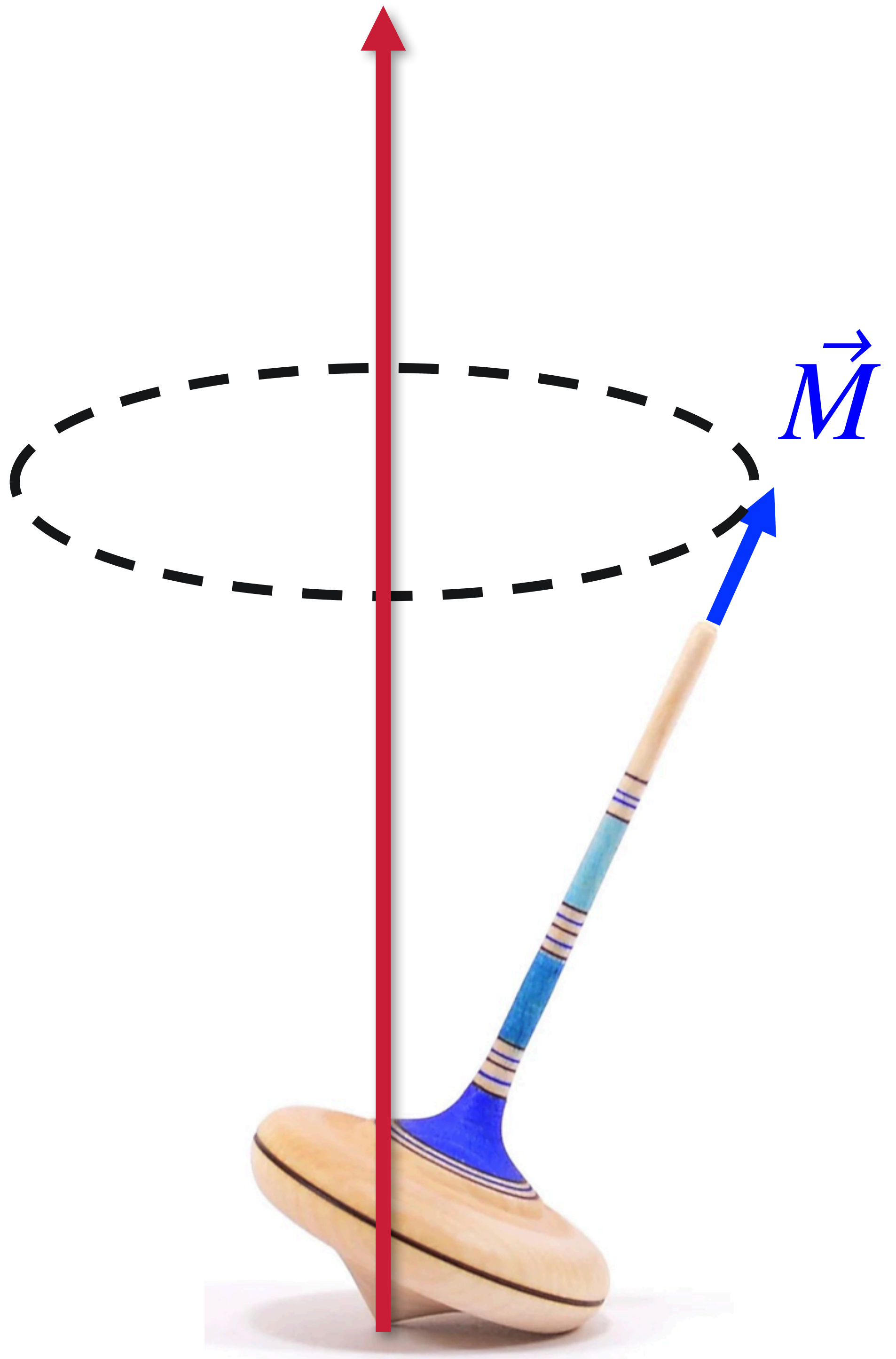
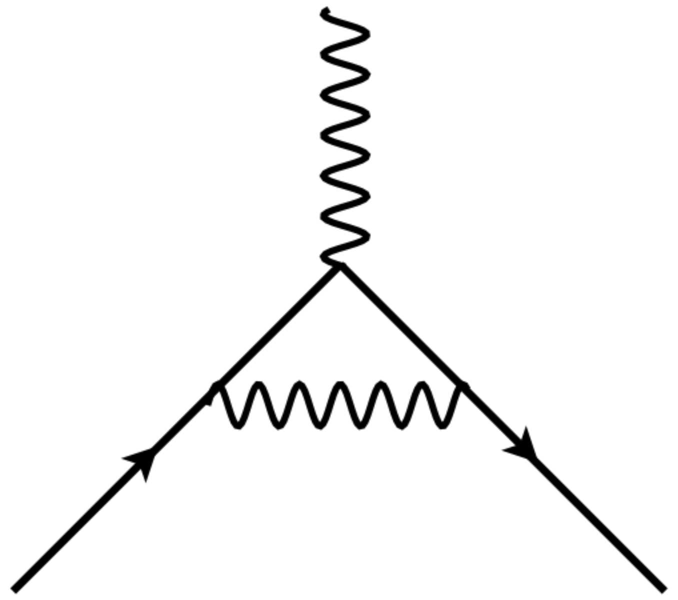


Julian Schwinger

$$g = 2\left(1 + \frac{\alpha}{2\pi}\right)$$



Quantum corrections modify Dirac's prediction $g = 2$

$g = 2(1+a), \quad a :$ anomalous magnetic moment

Electromagnetic, weak and strong interactions contribute to a

The anomalous magnetic moment of the muon

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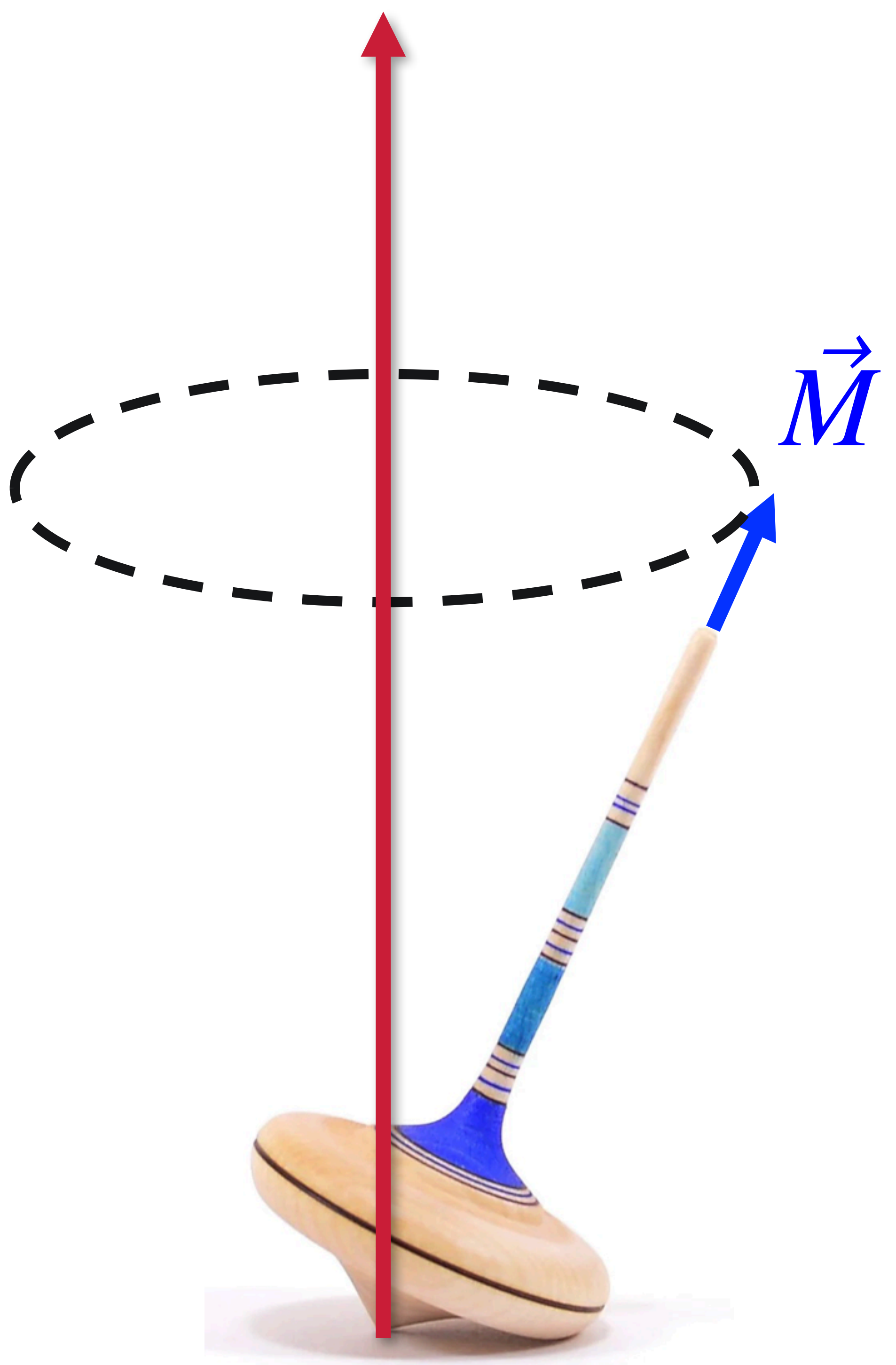
g-factor

Magnetic field $\vec{B}$



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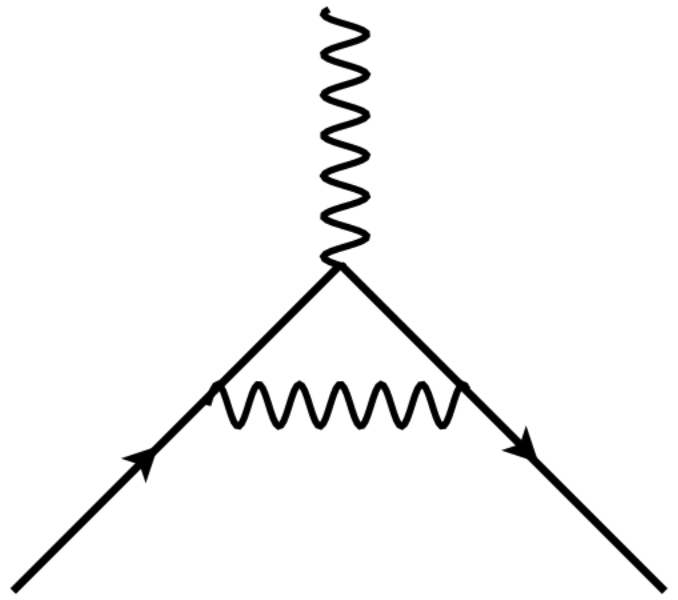

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$$g = 2(1 + a), \quad a: \quad \text{anomalous magnetic moment}$$

Beyond leading order: distinct values of a_e, a_μ and a_τ

Muon $g-2$ Theory Initiative

- Founded in 2017
- Agree on common SM prediction
- Focus on hadronic contributions
- Prospects for increased precision

Mainz 2018

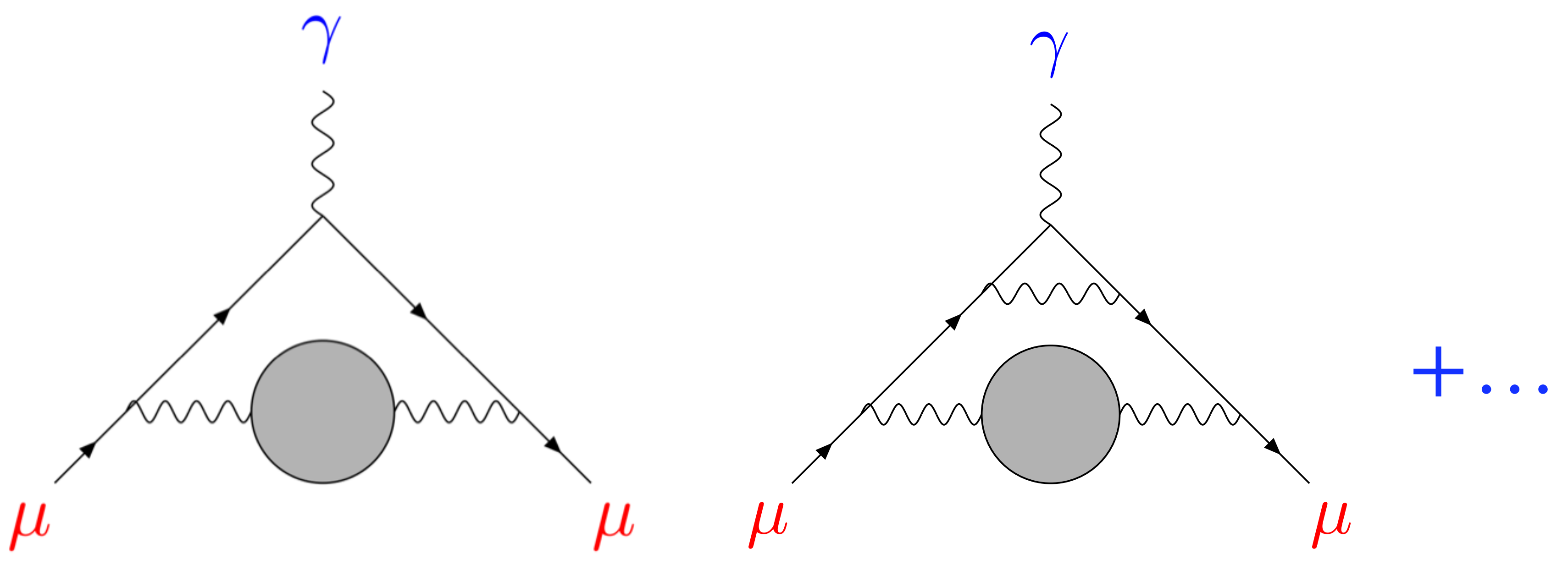
Zoom 2020

Bern 2023

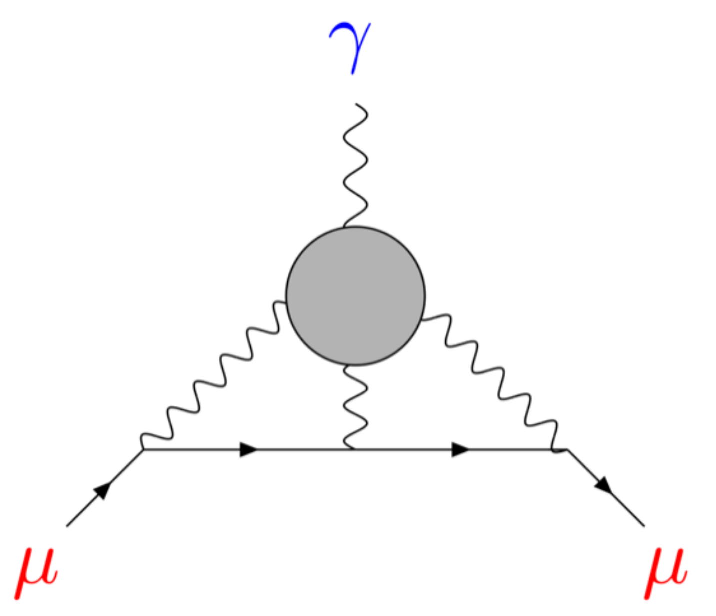
Standard Model prediction for muon $g-2$ [2020 White Paper]

QED:	$116\,584\,718.9\,(1) \times 10^{-11}$	0.001 ppm	
Weak:	$153.6(1.0) \times 10^{-11}$	0.01 ppm	
Hadronic vacuum polarisation:	$6845(40) \times 10^{-11}$	0.34 ppm	[0.6%]
Hadronic light-by-light scattering:	$92(18) \times 10^{-11}$	0.15 ppm	[20%]

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm hvp} + a_\mu^{\rm hlbl} = 116\,591\,810(43) \times 10^{-11} \qquad 0.37 \text{ ppm}$$



Hadronic vacuum polarisation (HVP)



Hadronic light-by-light scattering (HLbL)

[Aoyama et al., Phys. Rep. 887 (2020) 1, arXiv:2006.04822]

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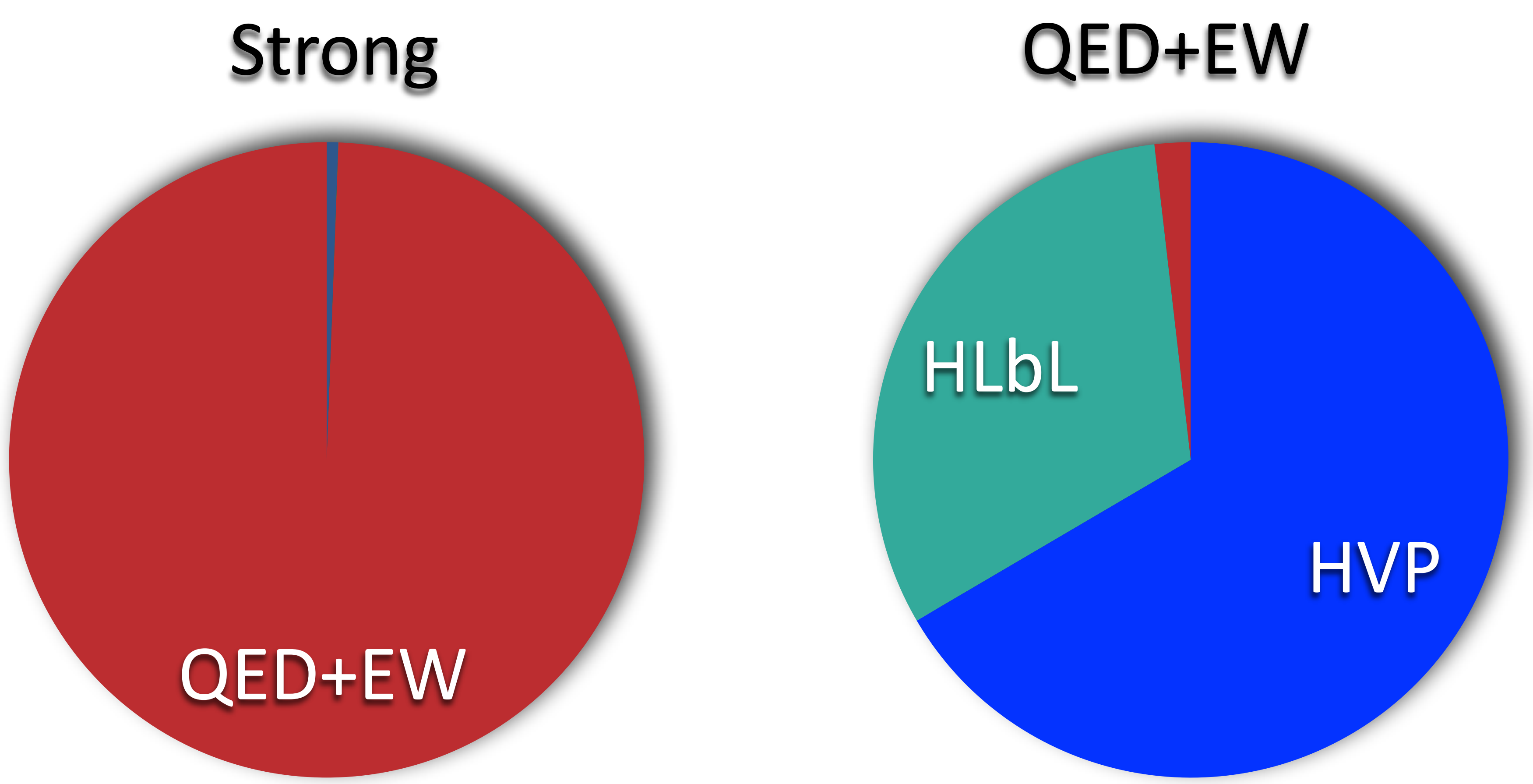
- QED and electroweak contributions account for 99.994% of the SM prediction for a_μ
- Error is dominated by strong interaction effects



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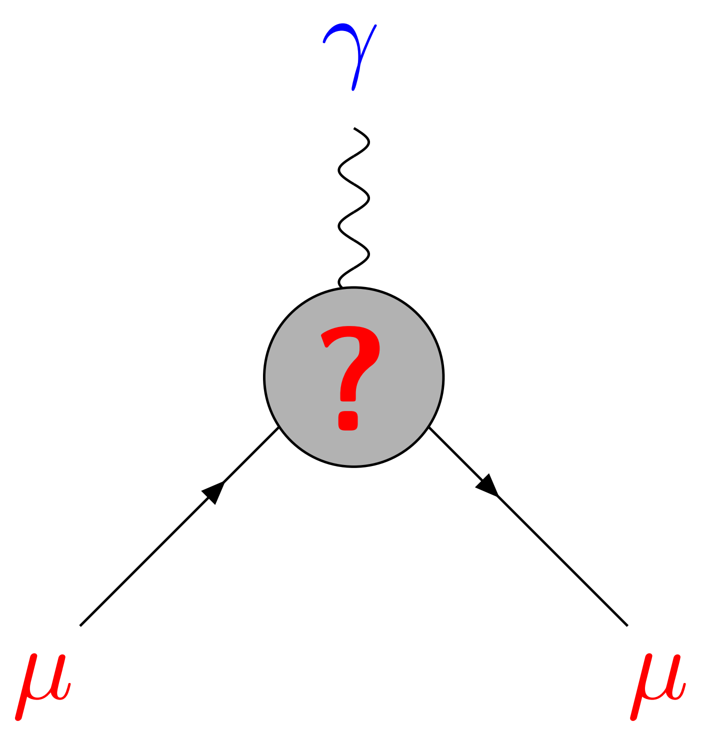
Standard Model vs. experiment: $a_\mu^{\rm exp} \stackrel{?}{=} a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm hvp} + a_\mu^{\rm hlbl} + a_\mu^{\rm BSM}$

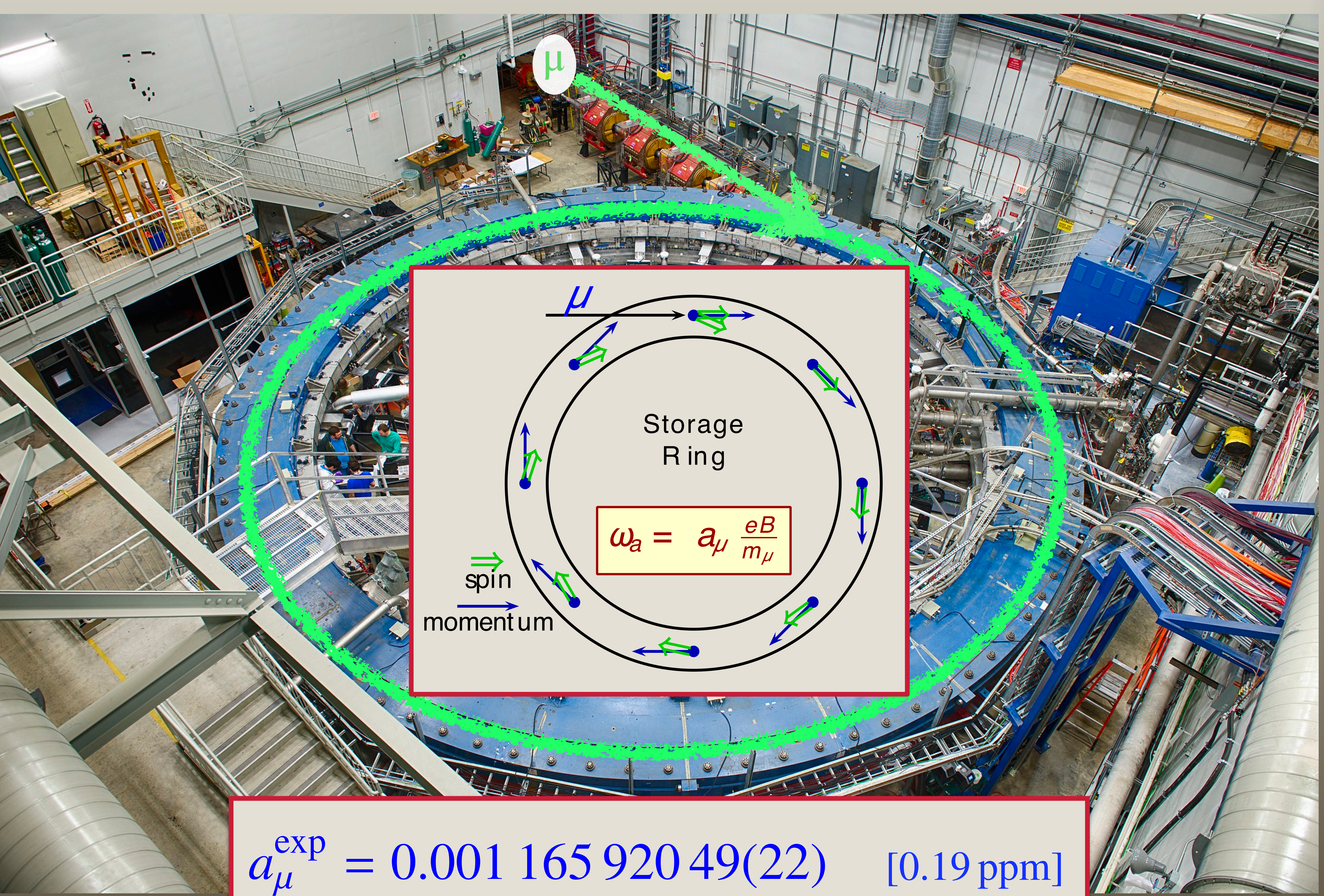


Why the muon?

$$a_\ell^{\rm BSM} \propto m_\ell^2 / M_{\rm BSM}^2 \qquad \ell = e, \mu, \tau$$

$\rightarrow$ sensitivity of a_μ enhanced by $(m_\mu/m_e)^2 \approx 4.3 \times 10^4$ relative to a_e

Fermilab experiment E989



$$a_\mu^{\text{exp}} = 0.001\,165\,920\,49(22) \qquad [0.19\ \text{ppm}]$$

[Aguillard et al., Phys Rev Lett 131 (2023) 16, 161802]

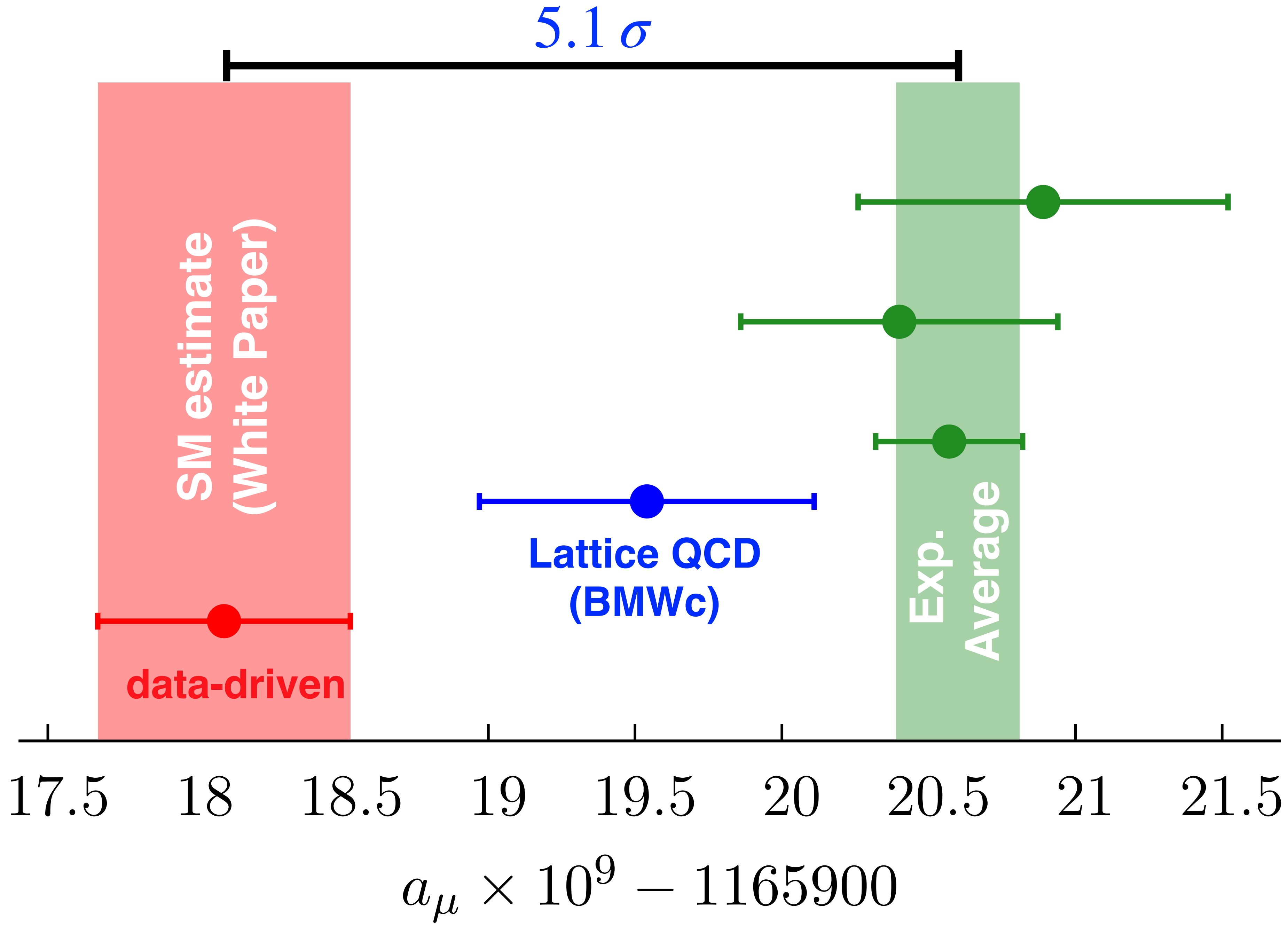
New Physics on the horizon?

Confronting the SM prediction with the E989 measurement

$$a_\mu^{\rm exp} = 116\,592\,049(22)\times 10^{-11} \quad [0.19\,\text{ppm}]$$

$$a_\mu^{\rm SM} = 116\,591\,810(43)\times 10^{-11} \quad [0.37\,\text{ppm}]$$

$$\Rightarrow \quad a_\mu^{\rm exp} - a_\mu^{\rm SM} = (249 \pm 48)\cdot 10^{-11} \quad [5.1\sigma]$$



Standard Model prediction:

- White paper estimate based on "data-driven" evaluation of (leading-order) HVP contribution: dispersion integrals and hadronic cross sections
- Lattice QCD result for HVP with comparable precision *[Borsányi et al., Nature 593 (2021) 7857]*

$$a_\mu^{\rm exp} - a_\mu^{\rm SM}\Big|_{\rm BMWc}^{\rm hvp,\,LO} = (105 \pm 61)\cdot 10^{-11} \quad [1.7\,\sigma]$$

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Confronting the SM prediction with the E989 measurement

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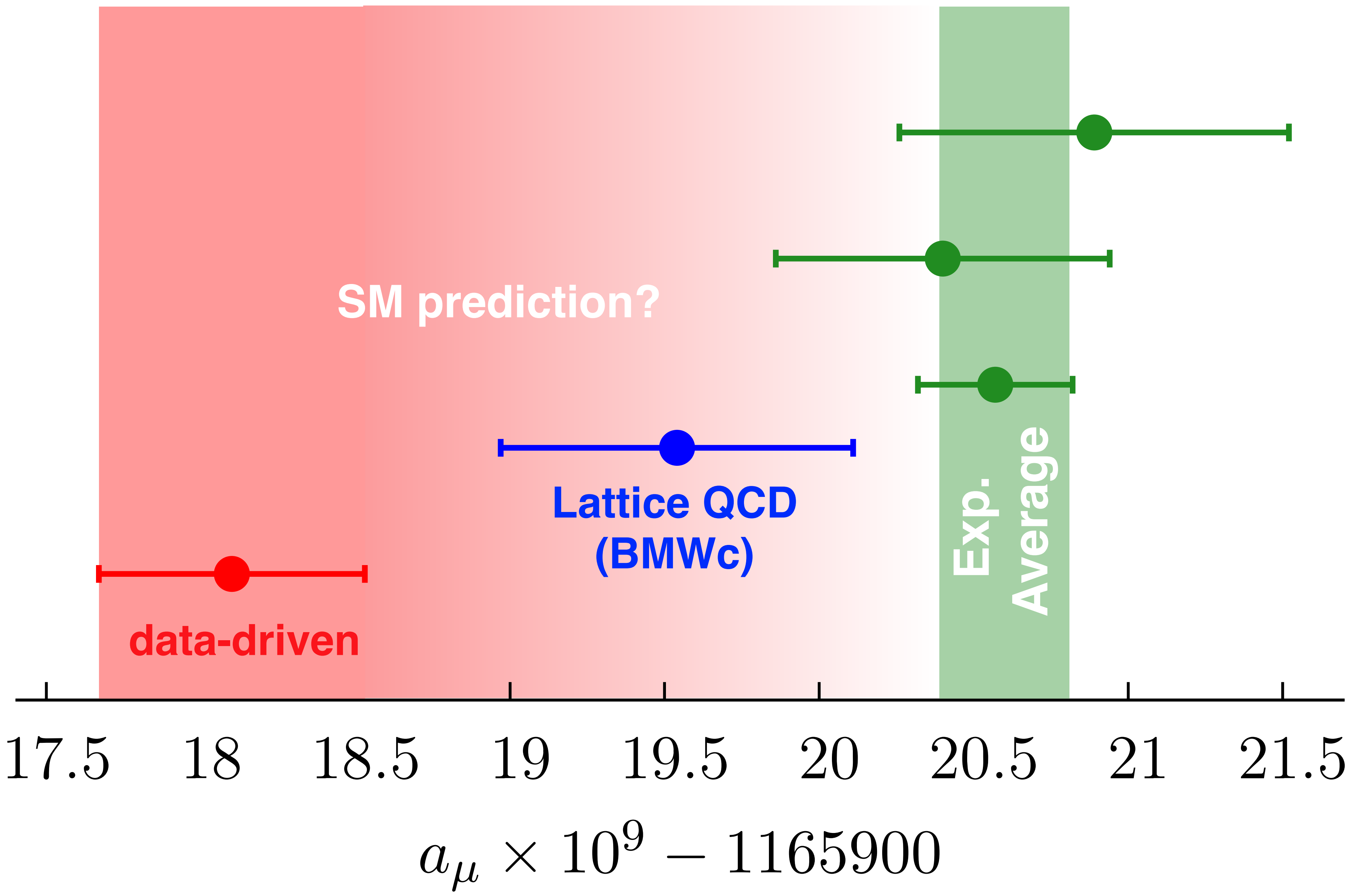

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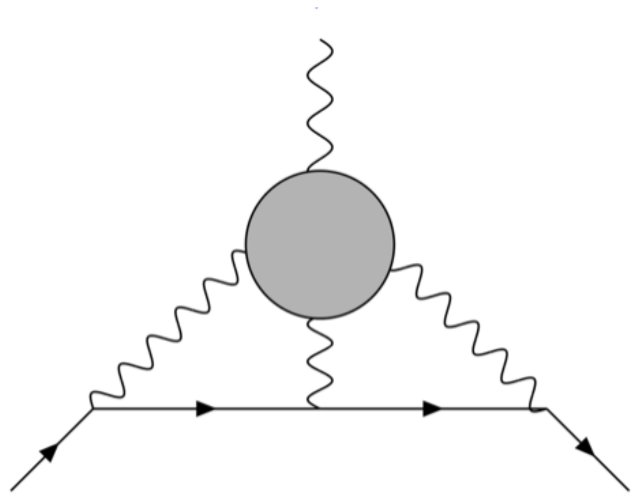
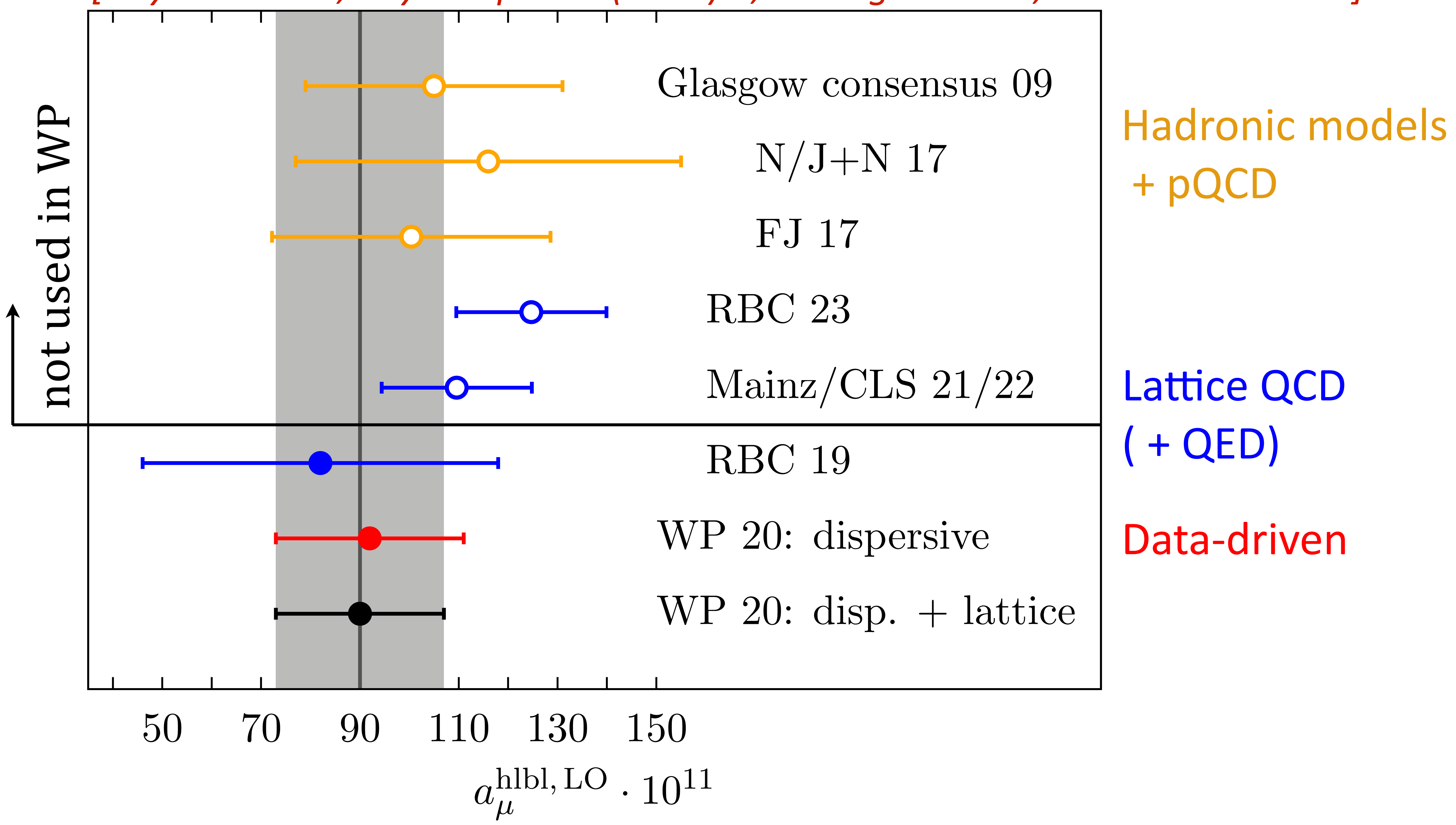
$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}}\Big|_{\text{BMWc}}^{\text{hvp, LO}} = (105 \pm 61) \cdot 10^{-11} \quad [1.7\,\sigma]$$

Requires independent confirmation

Hadronic light-by-light scattering



[Aoyama et al., Phys. Rep. 887 (2020) 1; Colangelo et al., arXiv:2203.15810]



Hadronic models, data-driven method and Lattice QCD produce compatible results

White paper recommended value:

$$a_\mu^{\rm hlbl} = (92 \pm 18)\cdot 10^{-11}$$

Recent lattice calculations:

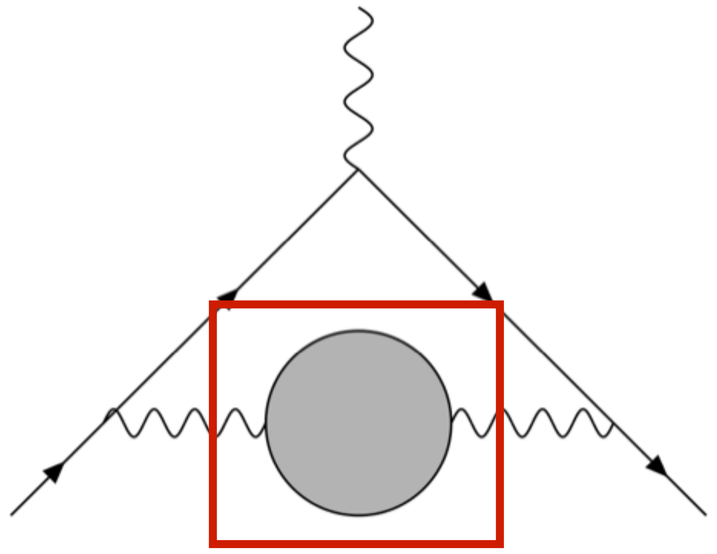
$$a_\mu^{\rm hlbl,\,LO} = \begin{cases} (109.6 \pm 14.7)\cdot 10^{-11} & \text{Mainz/CLS} \\ (124.7 \pm 15.2)\cdot 10^{-11} & \text{RBC} \end{cases}$$

[Chao et al., EPJC 81 (2021) 651; EPJC 82 (2022) 664; Blum et al., arXiv:2304.04423]

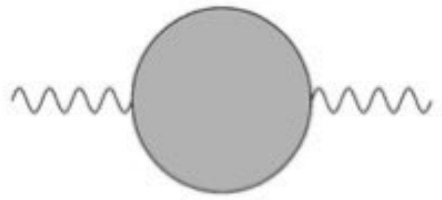
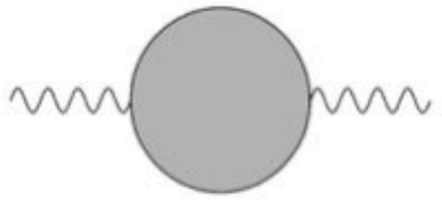
$a_\mu^{\rm hlbl}$: **Uncontroversial** — contributes $0.15\,\rm ppm$ to the total SM uncertainty of $0.37\,\rm ppm$

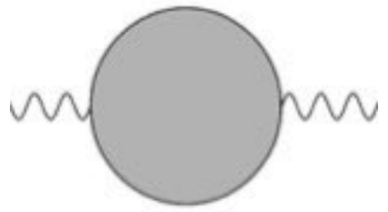
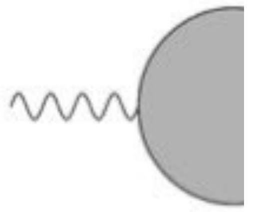
→ Focus on refinements and further reduction of uncertainty

Hadronic vacuum polarisation from dispersion theory



Analyticity, unitarity & optical theorem imply:

 $= \int \frac{ds}{\pi(s-q^2)}\,\mathrm{Im}$ 

$2\,\mathrm{Im}$  $= \sum_{\mathrm{had}} \int d\Phi \,\Big|$  $\Big|^2$

$\propto \sigma(e^+e^- \to \mathrm{hadrons})$

$$a_\mu^{\mathrm{hvp,\,LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R_{\mathrm{had}}(s)\hat{K}(s)}{s^2}, \qquad R_{\mathrm{had}}(s) = \frac{3s}{4\pi\,(\alpha(s))^2}\,\sigma(e^+e^- \to \mathrm{hadrons})$$

"R-ratio"

Hadronic effects cannot be treated in perturbation theory

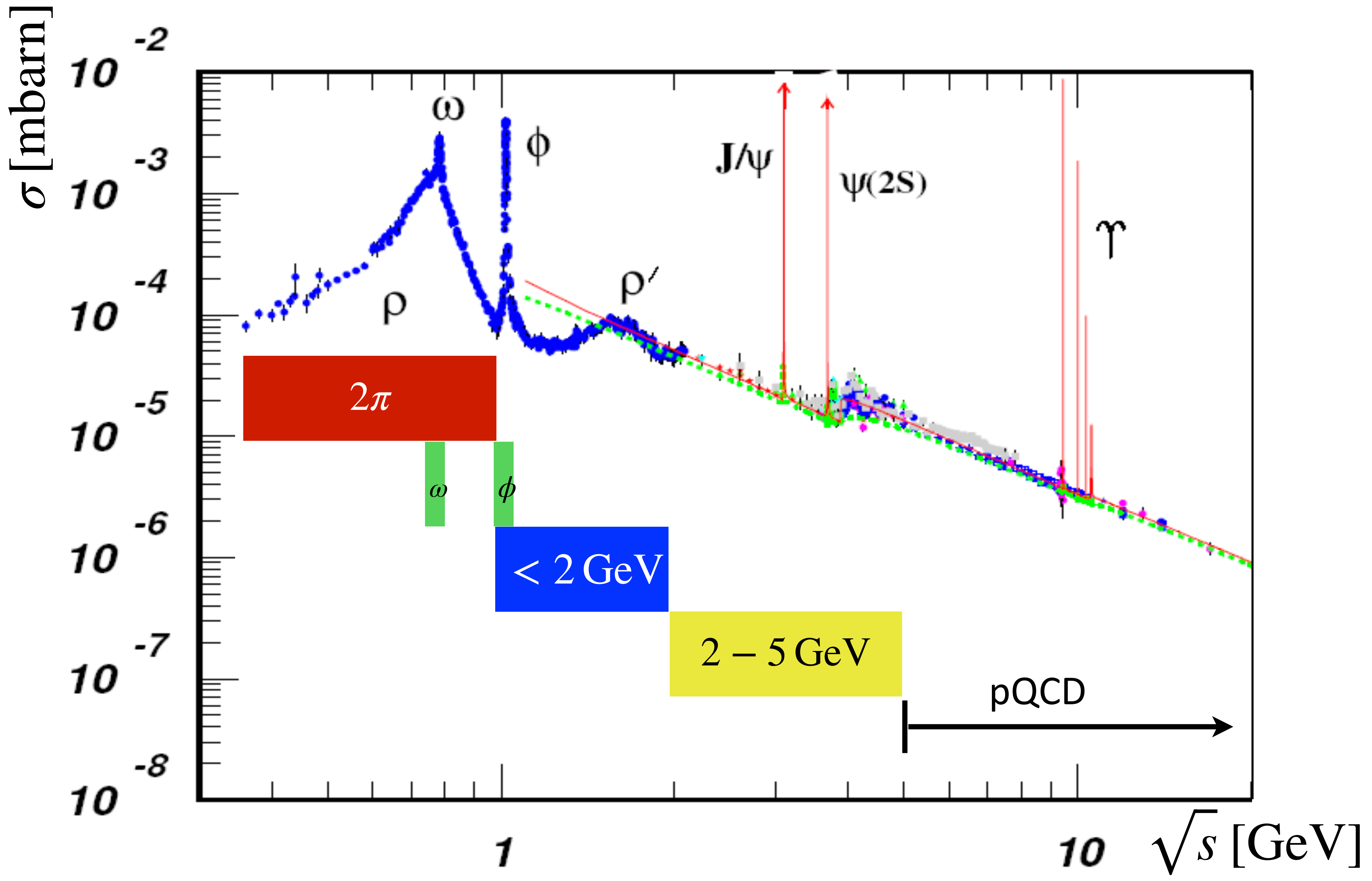
- Use experimental data for $R_{\mathrm{had}}(s)$ in the low-energy regime ("data-driven approach")
- Standard Model prediction is subject to experimental uncertainties

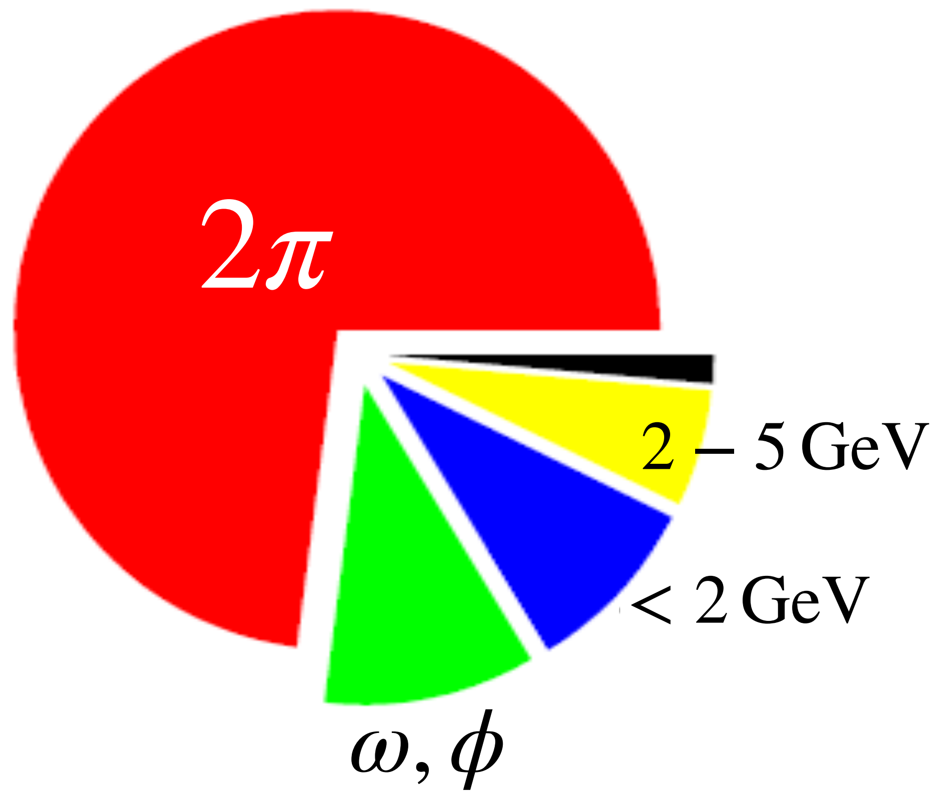
Hadronic vacuum polarisation: Data-driven approach

Decade-long effort to measure e^+e^- cross sections

$$a_\mu^{\rm hvp,\,LO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R_{\rm had}(s)\hat{K}(s)}{s^2}$$

- $\sqrt{s} \lesssim 2\,\mathrm{GeV}$: sum of exclusive channels
- $\sqrt{s} > 2\,\mathrm{GeV}$: inclusive channels, narrow resonances, perturbative QCD





Two-pion channel contributes $\approx 70\,\%$

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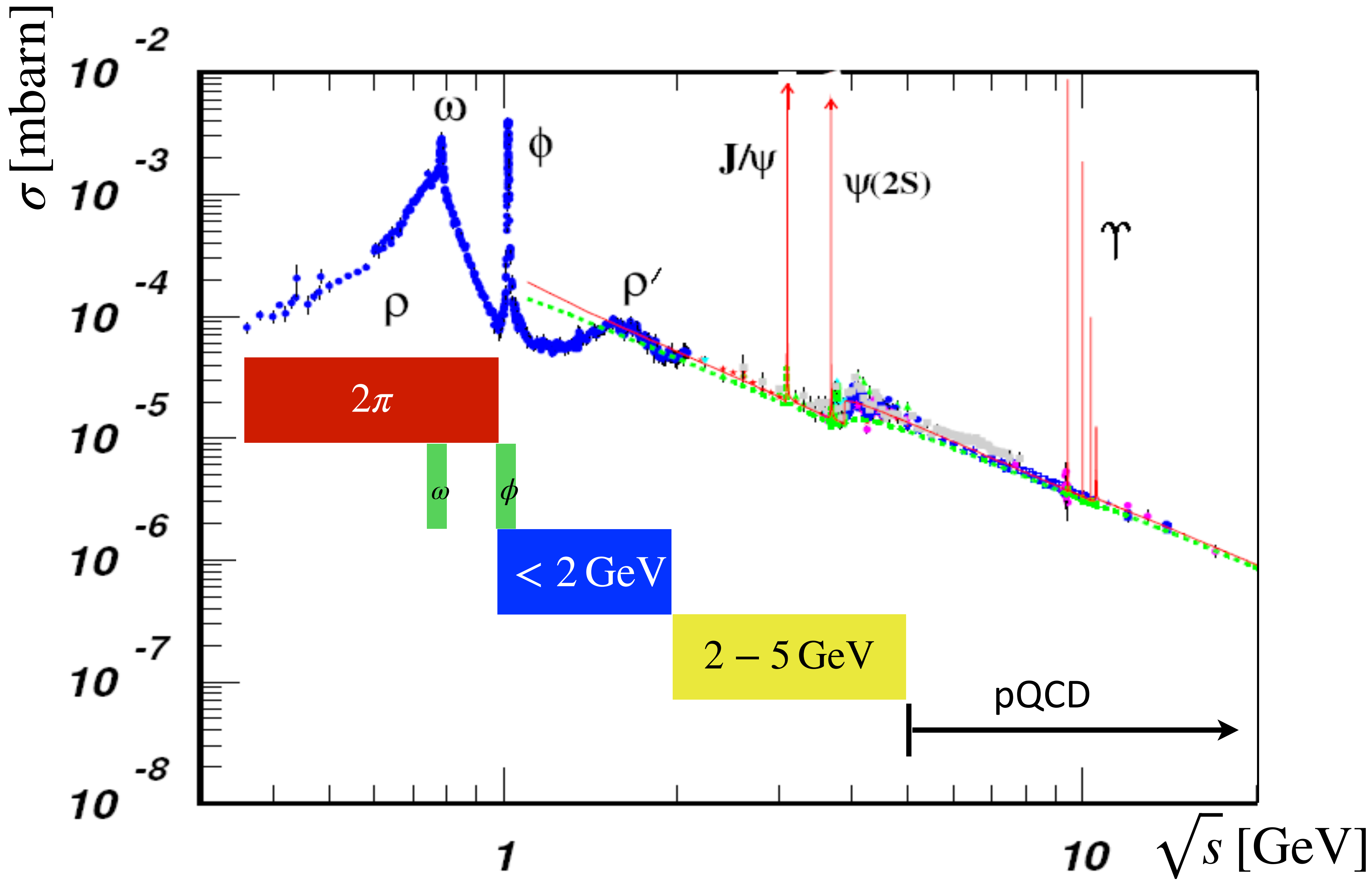

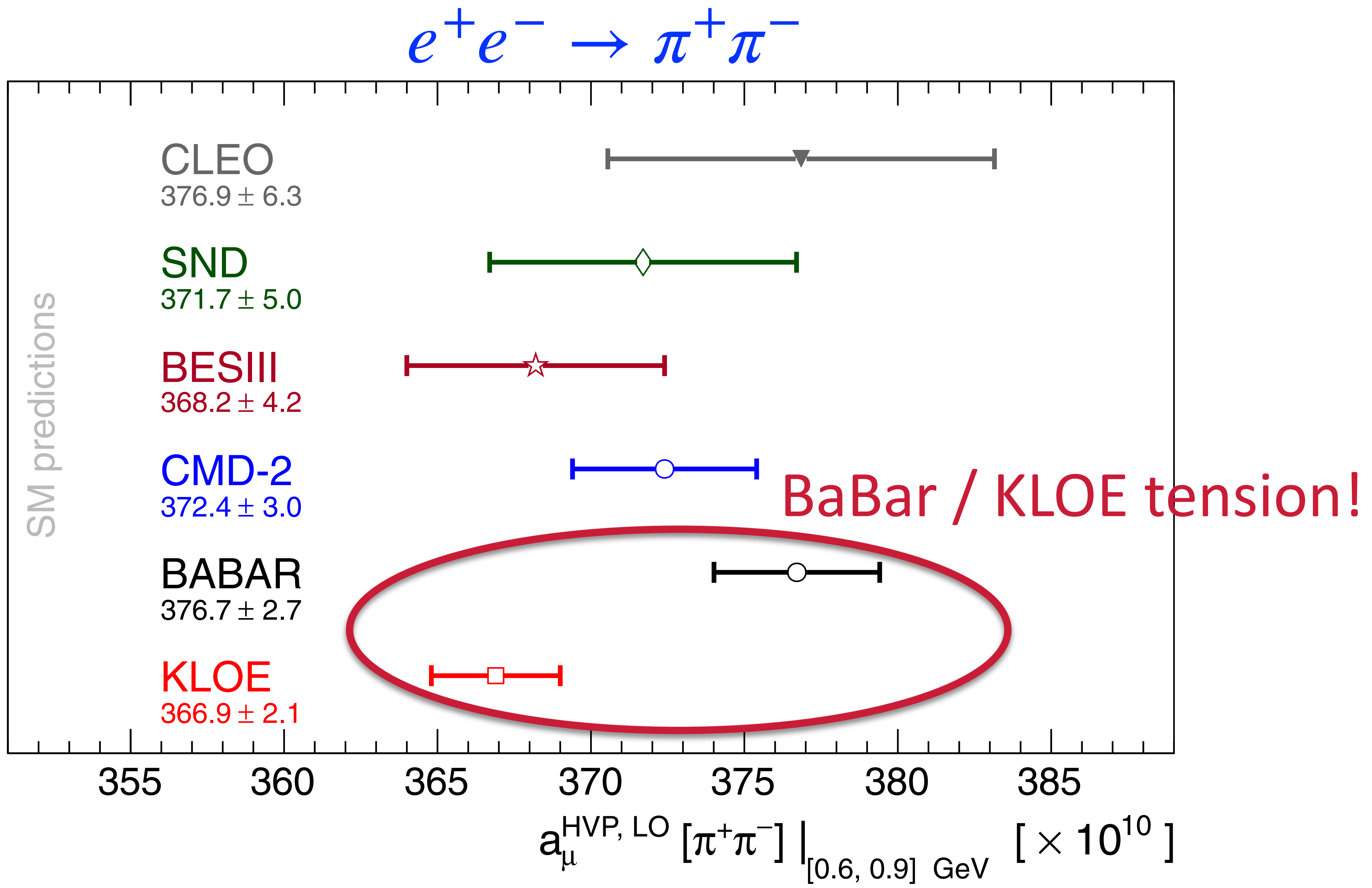

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- White Paper recommended value (2020):

$$a_\mu^{\mathrm{hvp,LO}} = 693.1(2.8)_{\mathrm{exp}}(2.8)_{\mathrm{syst}}(0.7)_{\mathrm{DV+QCD}} \times 10^{-10}$$

$$= 693.1(4.0) \times 10^{-10} \qquad [0.6\%]$$

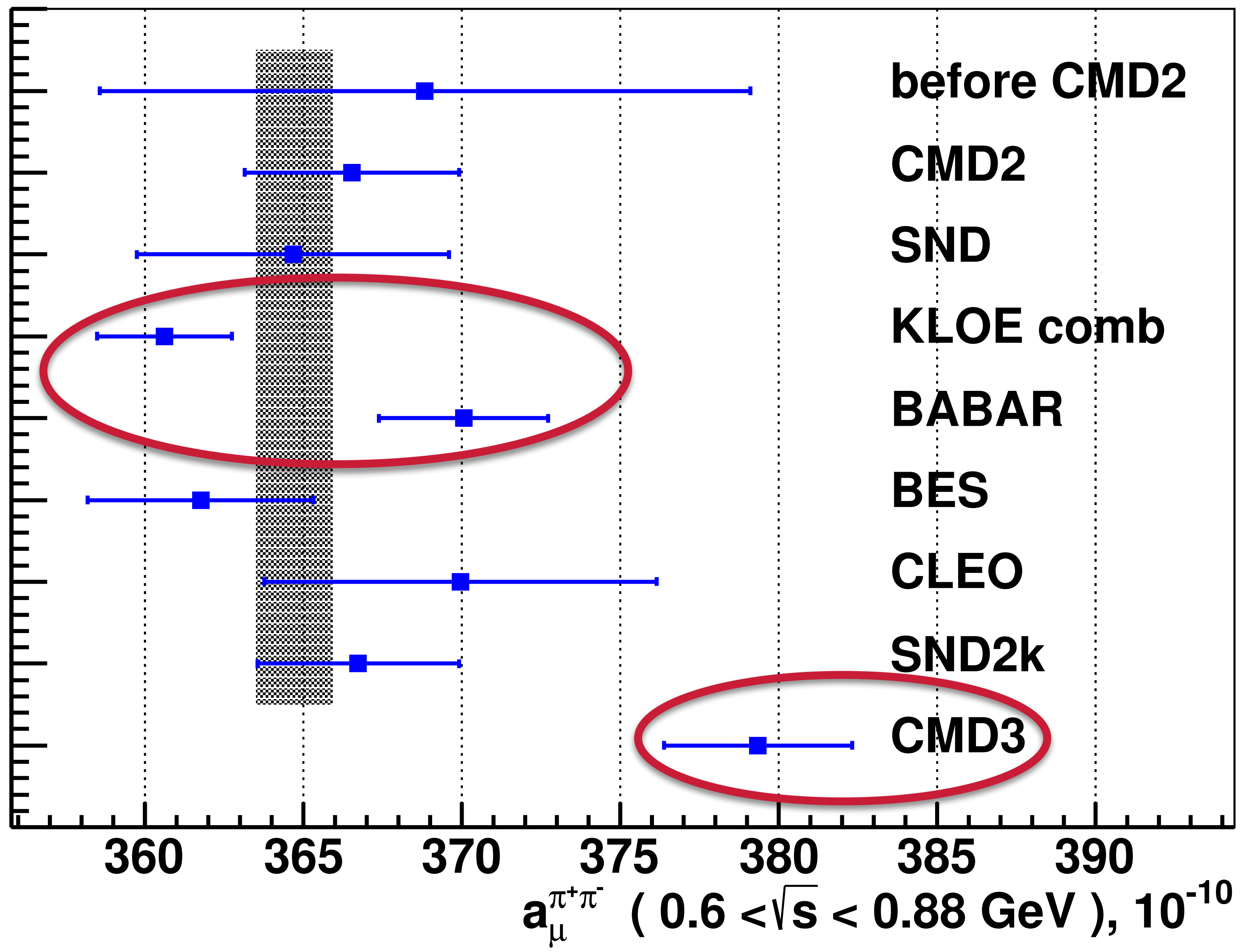
(accounts for tensions in the data and differences between analyses)

- Recent results in the $\pi^+\pi^-$ channel by CMD-3:

$\rightarrow$ further tension among e^+e^- data

$$a_\mu^{\mathrm{hvp,LO}} = 707.6(3.4)_{\mathrm{exp}}(0.7)_{\mathrm{DV+QCD}} \times 10^{-10}$$

(my own estimate)



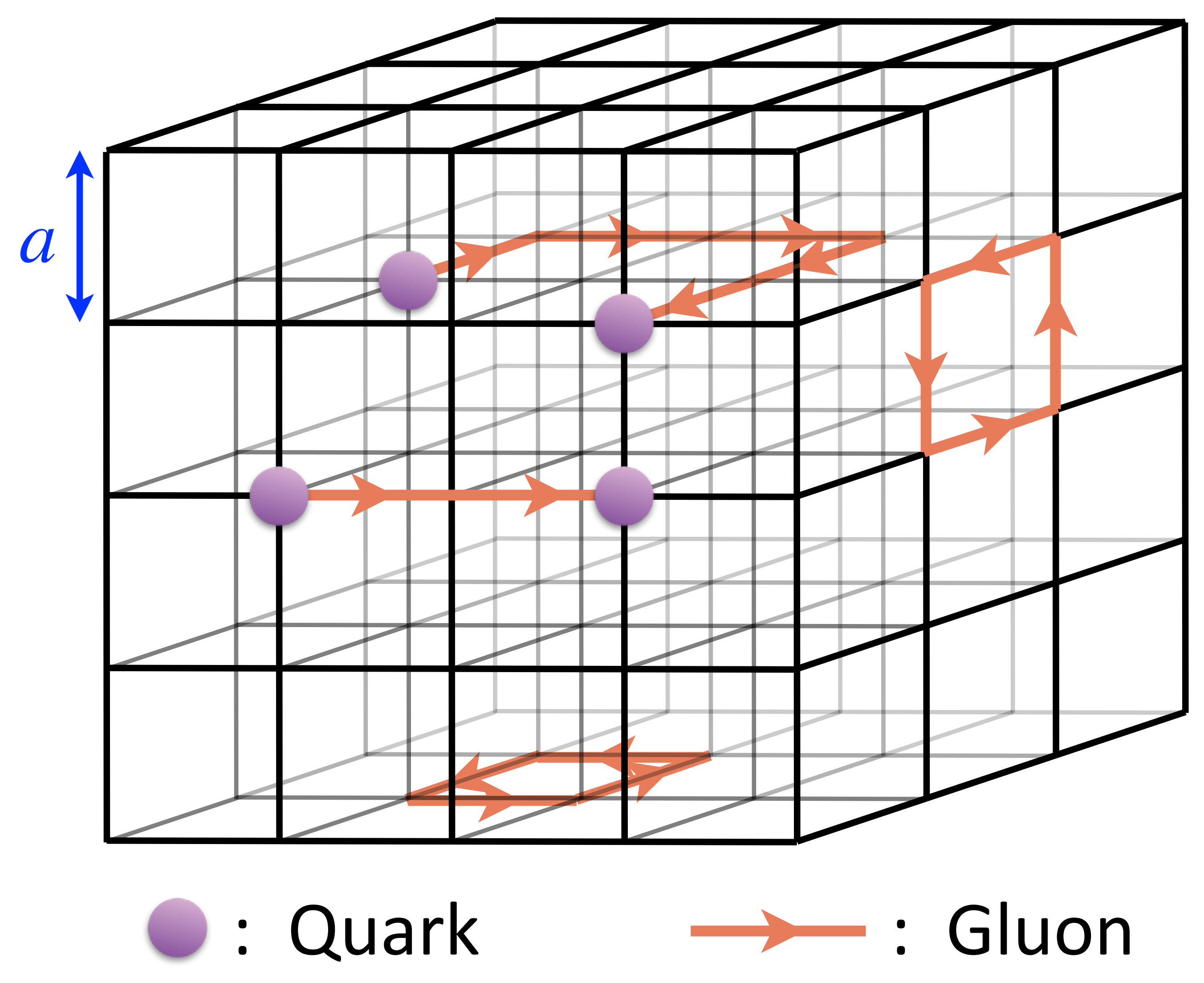
[Ignatov et al. (CMD-3 Collab.), arXiv:2302.08834]

Lattice QCD

Non-perturbative treatment of strong interaction via regularised Euclidean path integrals

Lattice spacing: $a, \quad x_\mu = n_\mu a, \quad a^{-1} = \Lambda_{\mathrm{UV}}$

Expectation value: $\langle\Omega\rangle = \frac{1}{Z}\int \prod_{x,\mu} dU_\mu(x)\, \Omega\, \mathrm{e}^{-S_{\mathrm{G}}^{\mathrm{eff}}[U]}$



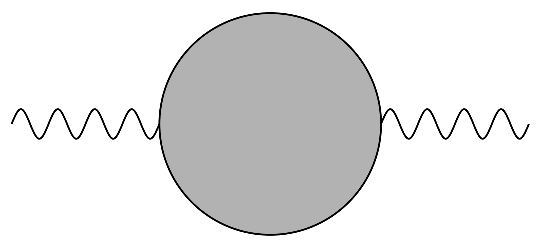
Procedure:

- Choose discretisation of QCD action
- Evaluate $\langle\Omega\rangle$ via Monte Carlo Integration:
  generate ensembles of gauge configurations via a Markov chain
- Ensemble average: $\langle\Omega\rangle \simeq \overline{\Omega}$ Statistical error: $\sqrt{\overline{\Omega^2} - \overline{\Omega}^2} \propto 1/N_{\mathrm{cfg}}^{1/2}$
- Extrapolate observables to the continuum limit: $a \to 0$ and tune quark masses to physical values

Hadronic vacuum polarisation from Lattice QCD

Lattice QCD does NOT determine the R-ratio from first principles

Time-momentum representation (TMR): *[Bernecker & Meyer EPJA 47 (2011) 148]*



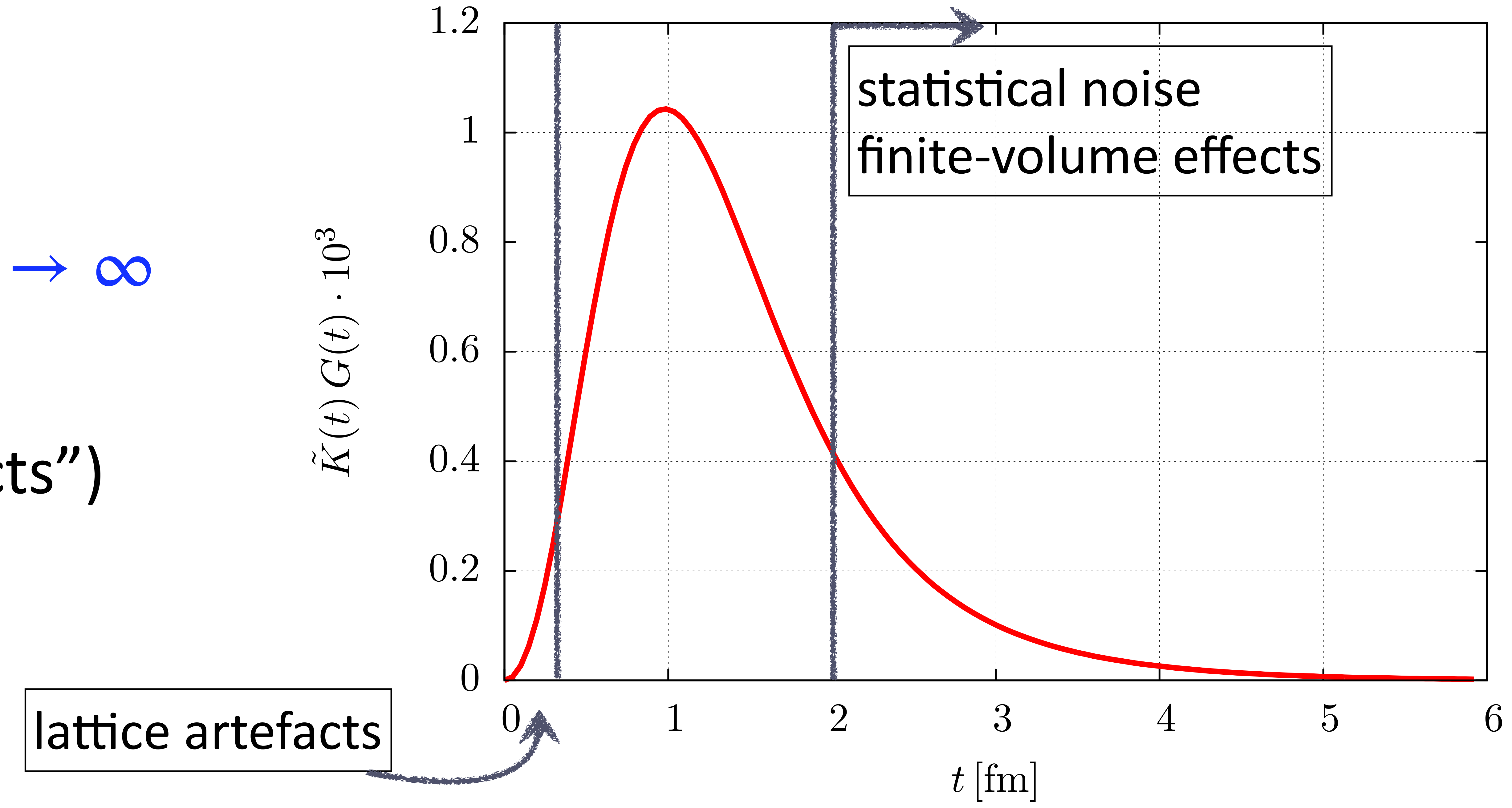
$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{K}(t)\, G(t), \quad G(t) = -a^3 \sum_{\vec{x}} \left\langle J_k^{\rm e.m.}(\vec{x},t) J_k^{\rm e.m.}(0)\right\rangle$$

($\tilde{K}(t)$: known analytically)

- No reliance on experimental data, except for simple input quantities → scale setting, calibration
- Not sensitive to exclusive hadronic channels

Challenges

- Exponentially increasing statistical noise as $t \to \infty$
- Correct for finite-volume effects
- Control discretisation effects ("lattice artefacts")
- Include isospin-breaking corrections



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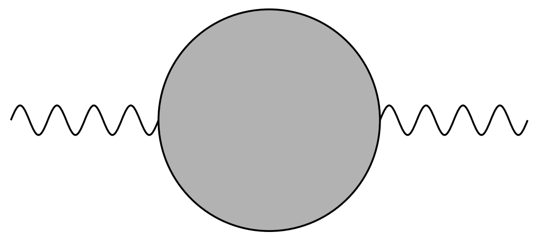

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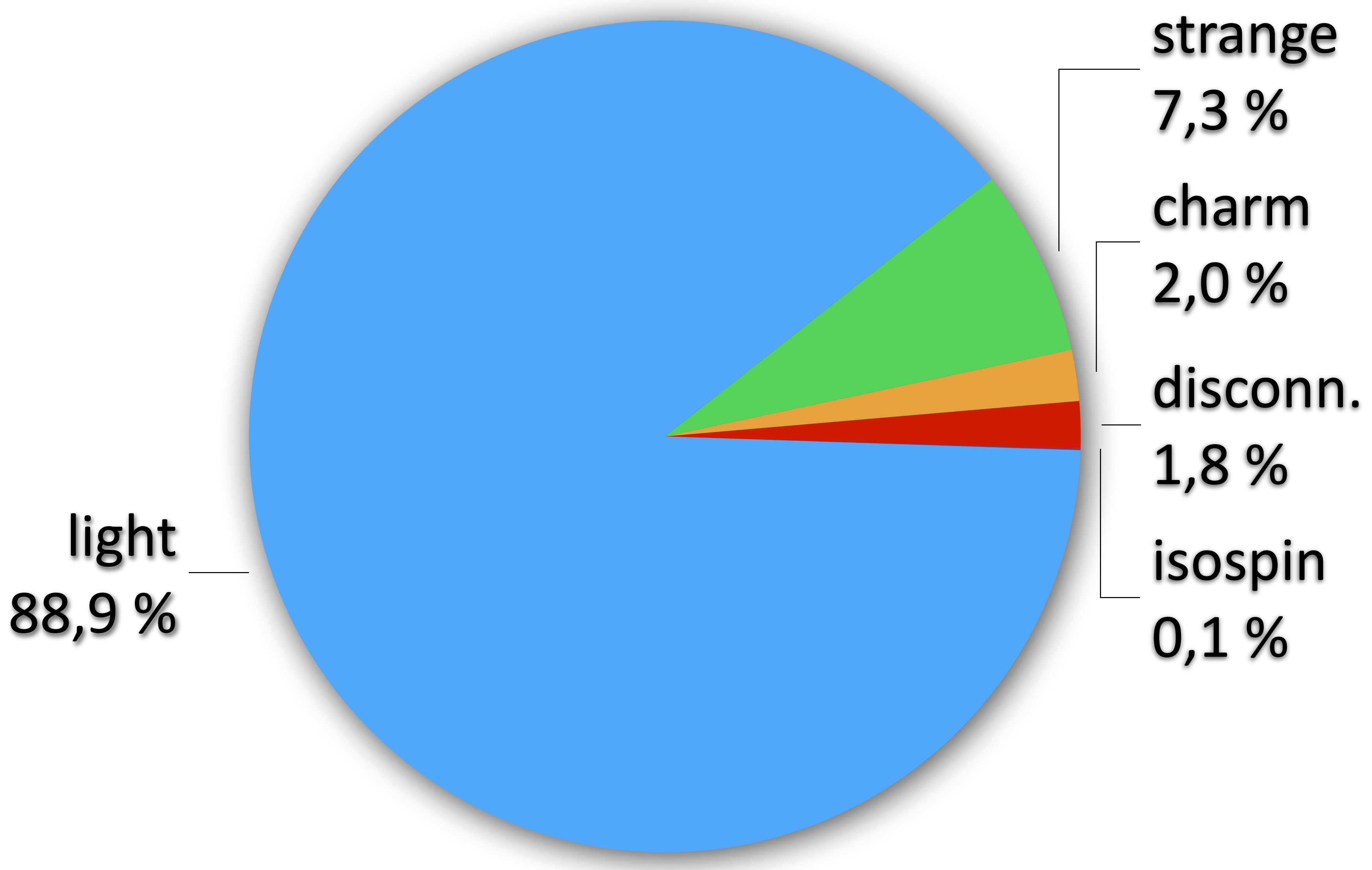
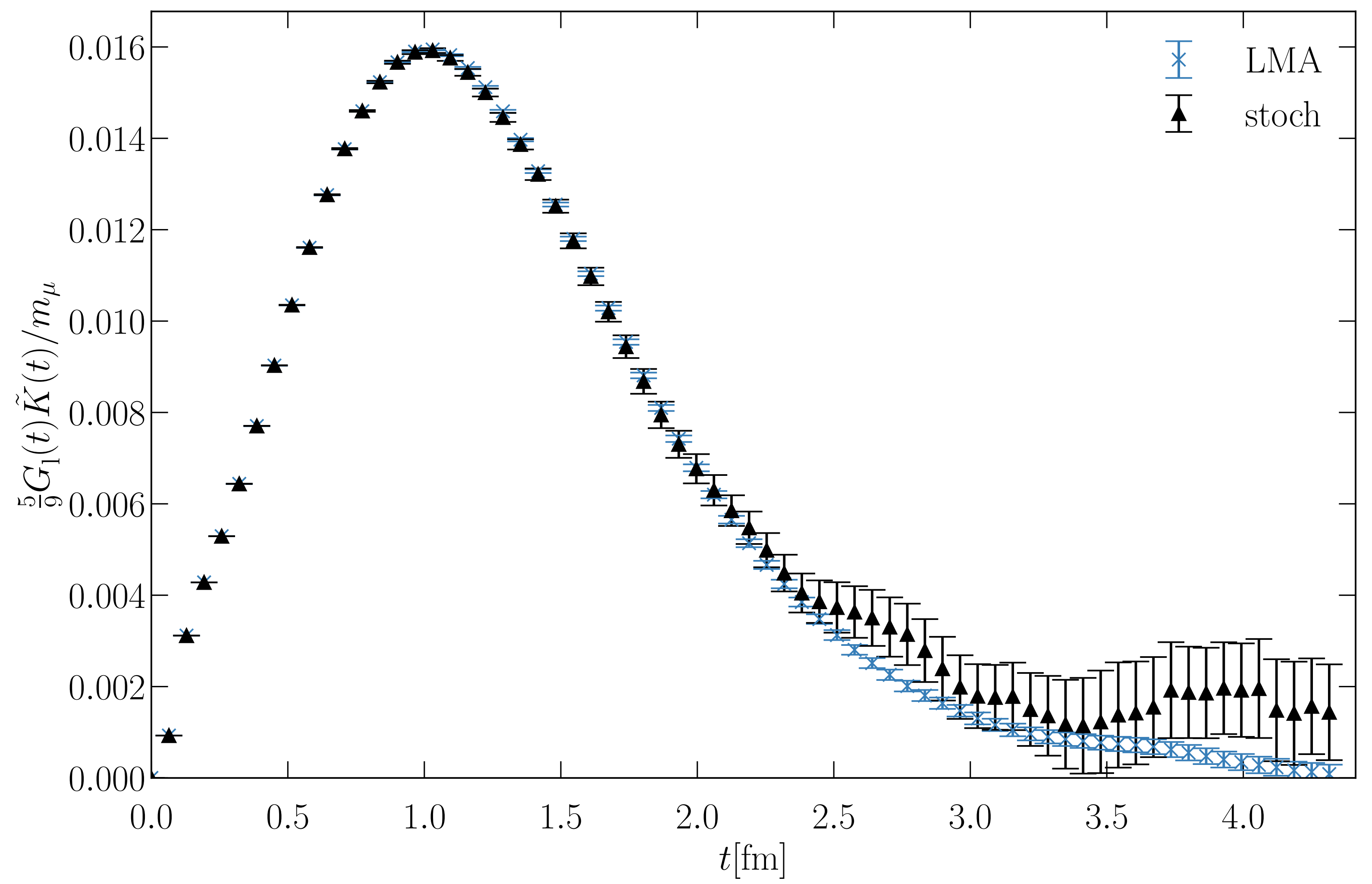
Light-quark connected contribution dominates



Controlling the long-distance tail of $G(t)$

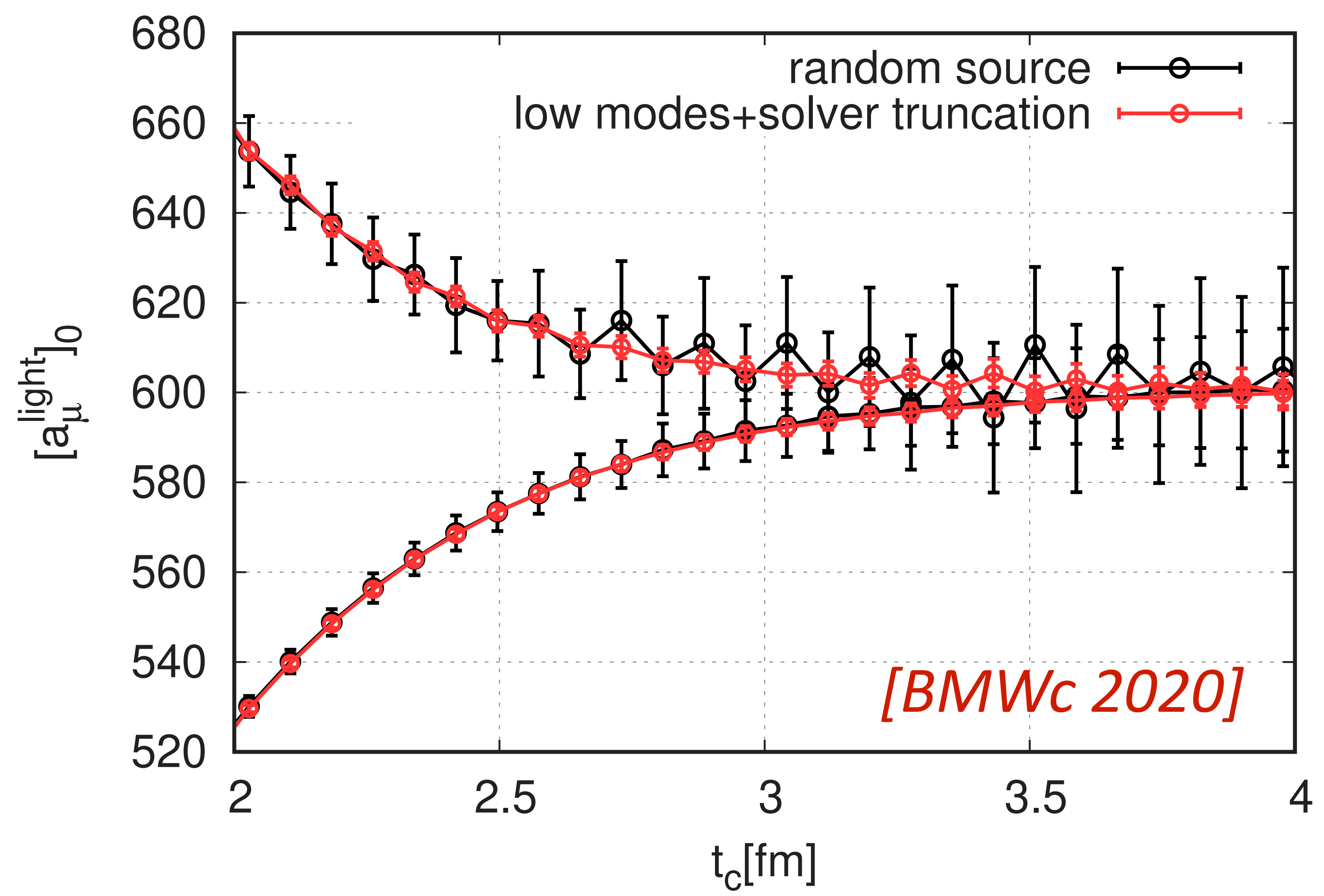
- Long-distance tail of the light quark contribution to $G(t)$: limiting factor for overall statistical precision
- Correlator dominated by isovector two-pion contribution

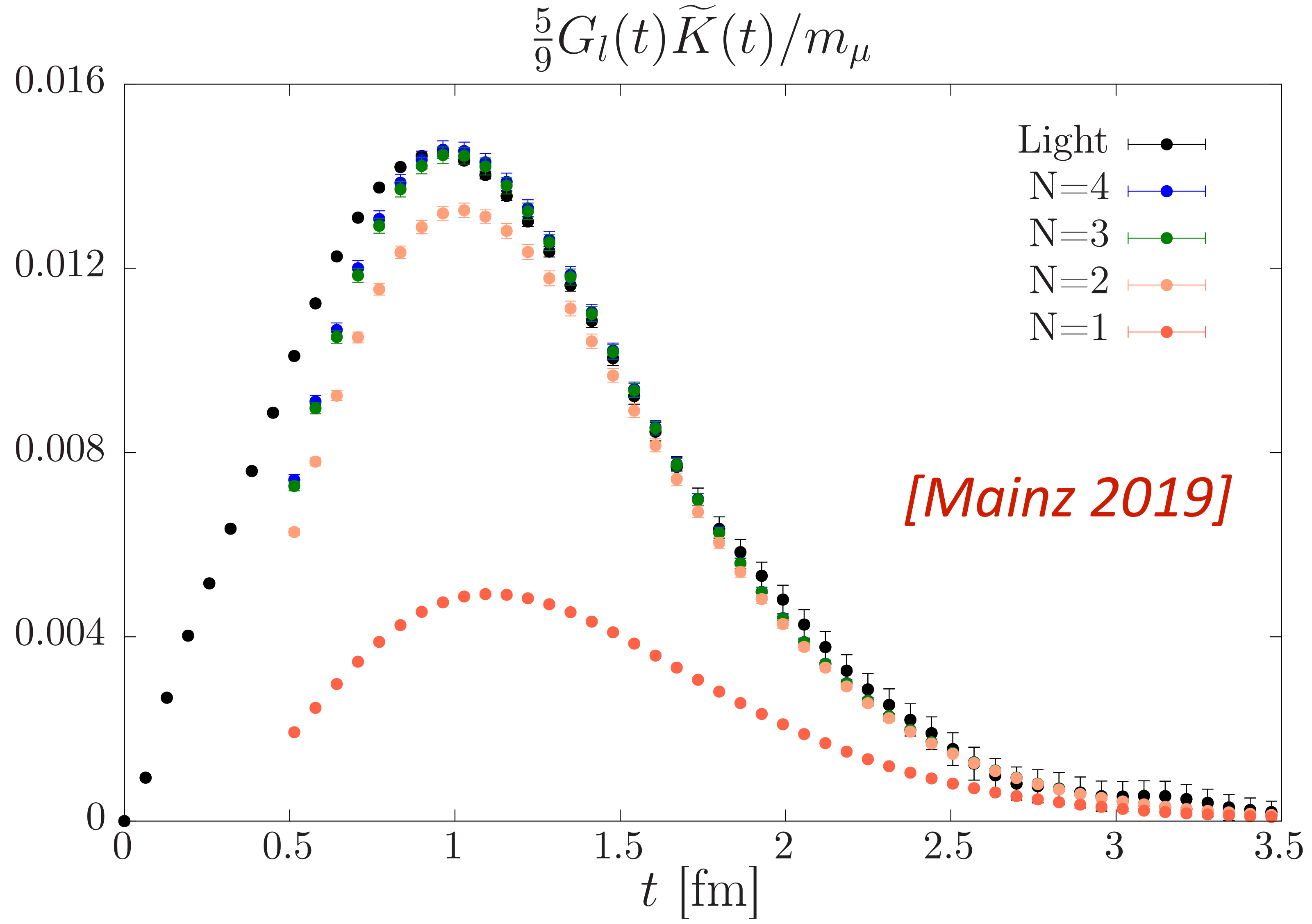


Strategies:

- Dedicated calculations of the spectrum in isovector channel and/or pion form factor $F_\pi(\omega)$
- "Bounding method":

$$0 \leq G(t) \leq G(t_c)\,\frac{G^{\pi\pi}(t)}{G^{\pi\pi}(t_c)}$$





- Noise-reduction methods: AMA, LMA, truncated solver
- Machine Learning

Common discretisations of the quark action

Computational cost depends significantly on the chosen discretisation

"Fermion doubling problem"

Domain wall /overlap quarks:

- no doublers; chiral symmetry breaking exponentially small
- live in five dimensions (dwf)
- evaluate sign function of "conventional" action (ovlp)
- used by: RBC/UKQCD, χQCD,…

Rooted staggered quarks:

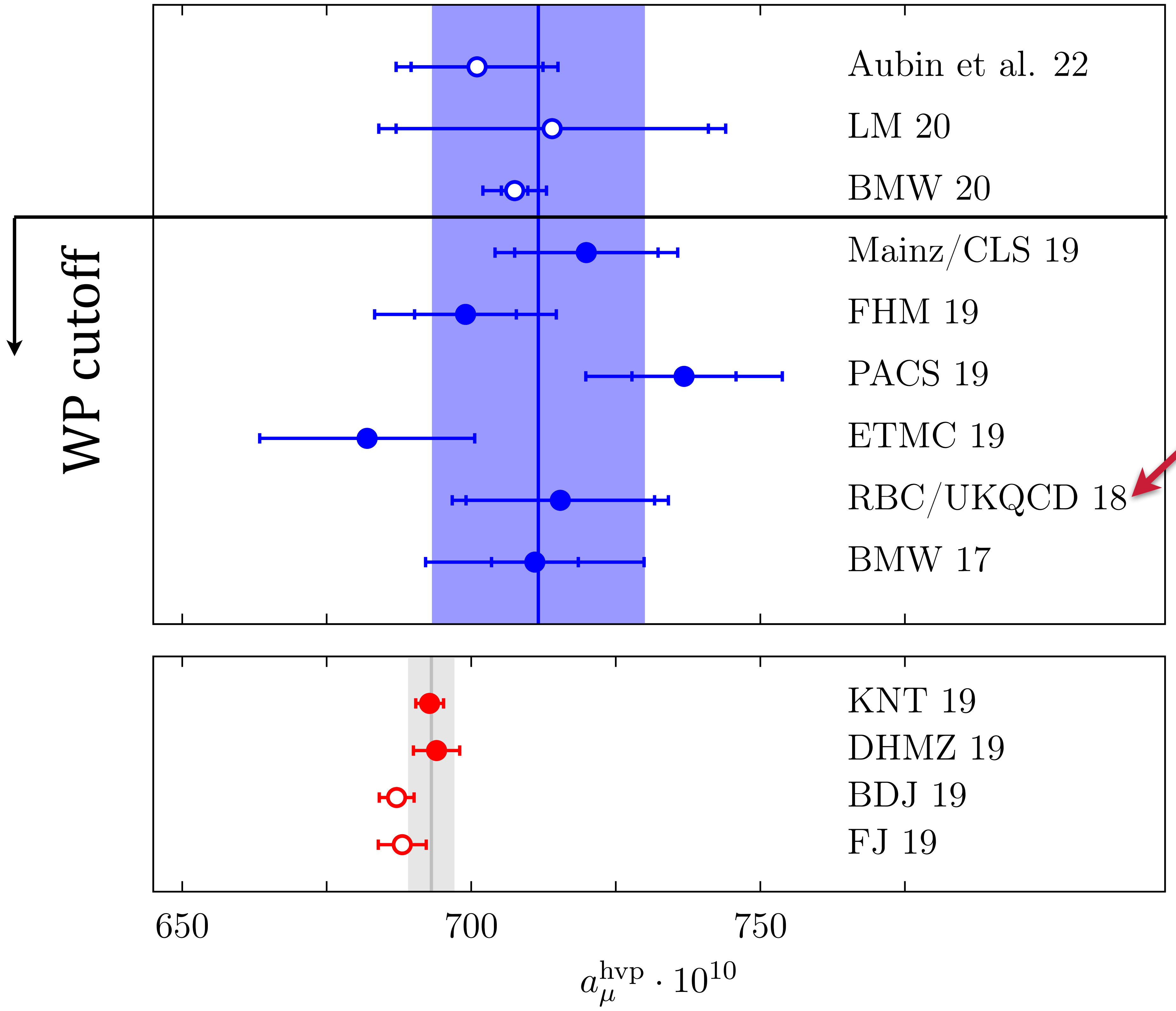
- remnant fermion doublers — "tastes"
- correct analytically for taste-induced lattice artefacts
- used by:
  BMW, Fermilab-HPQCD-MILC, ABGP,…

Wilson quarks:

- no doublers; chiral symmetry broken explicitly
- "exceptional configurations":
  negative eigenvalues of Wilson-Dirac operator
- used by: Mainz/CLS, ETM, PACS

computational cost

HVP in Lattice QCD



RBC/UKQCD *[Blum et al., Phys. Rev. Lett. 121 (2018) 022003]*

- Domain wall fermions
- Two ensembles: $a = 0.114,\ 0.084\ \mathrm{fm}$ at $m_\pi^{\rm phys}$
- Leading isospin-breaking corrections included
- Naive continuum extrapol'n in a^2 including estimated a^4-term

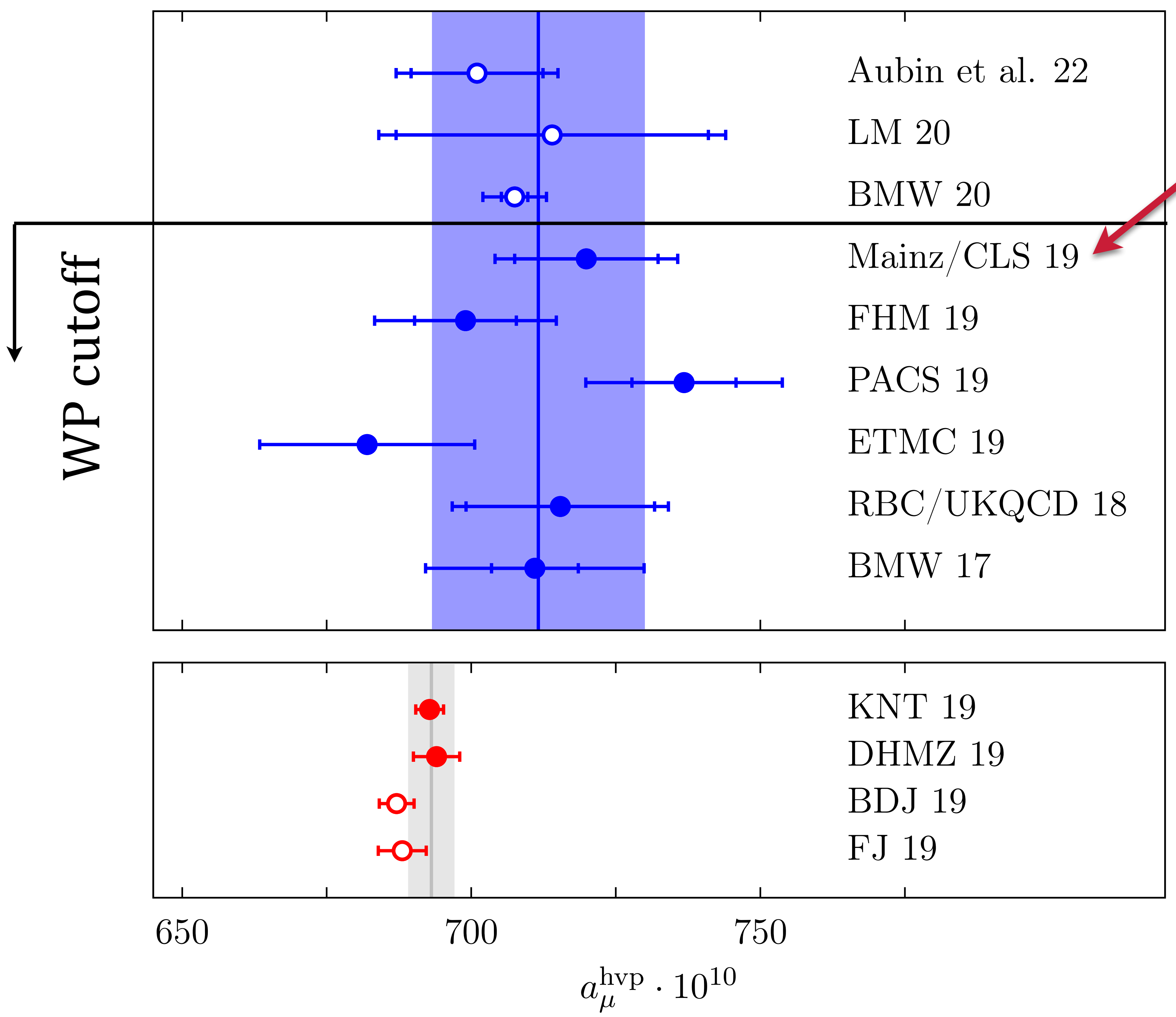
$$a_\mu^{\rm hvp,\,LO} = (715.4 \pm 16.3 \pm 9.2) \cdot 10^{-10} \quad [2.6\%]$$

White Paper:

R-ratio: $a_\mu^{\rm hvp,\,LO} = (693.1 \pm 4.0) \cdot 10^{-10}$ [0.6%]

LQCD: $a_\mu^{\rm hvp,\,LO} = (711.6 \pm 18.4) \cdot 10^{-10}$ [2.6%]

HVP in Lattice QCD



White Paper:

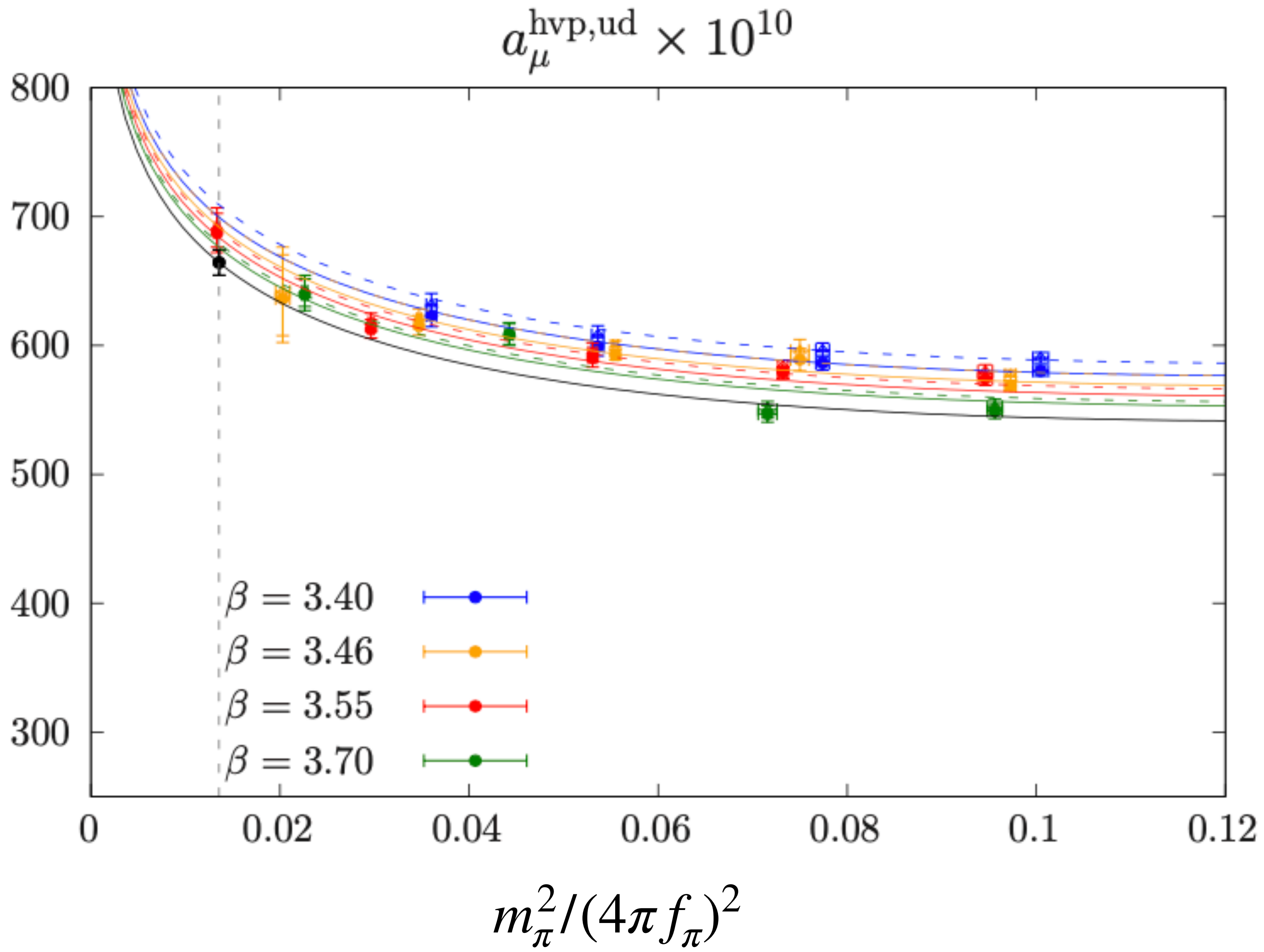
R-ratio: $a_\mu^{\rm hvp,\,LO} = (693.1 \pm 4.0) \cdot 10^{-10}$ [0.6%]

LQCD: $a_\mu^{\rm hvp,\,LO} = (711.6 \pm 18.4) \cdot 10^{-10}$ [2.6%]

Mainz/CLS

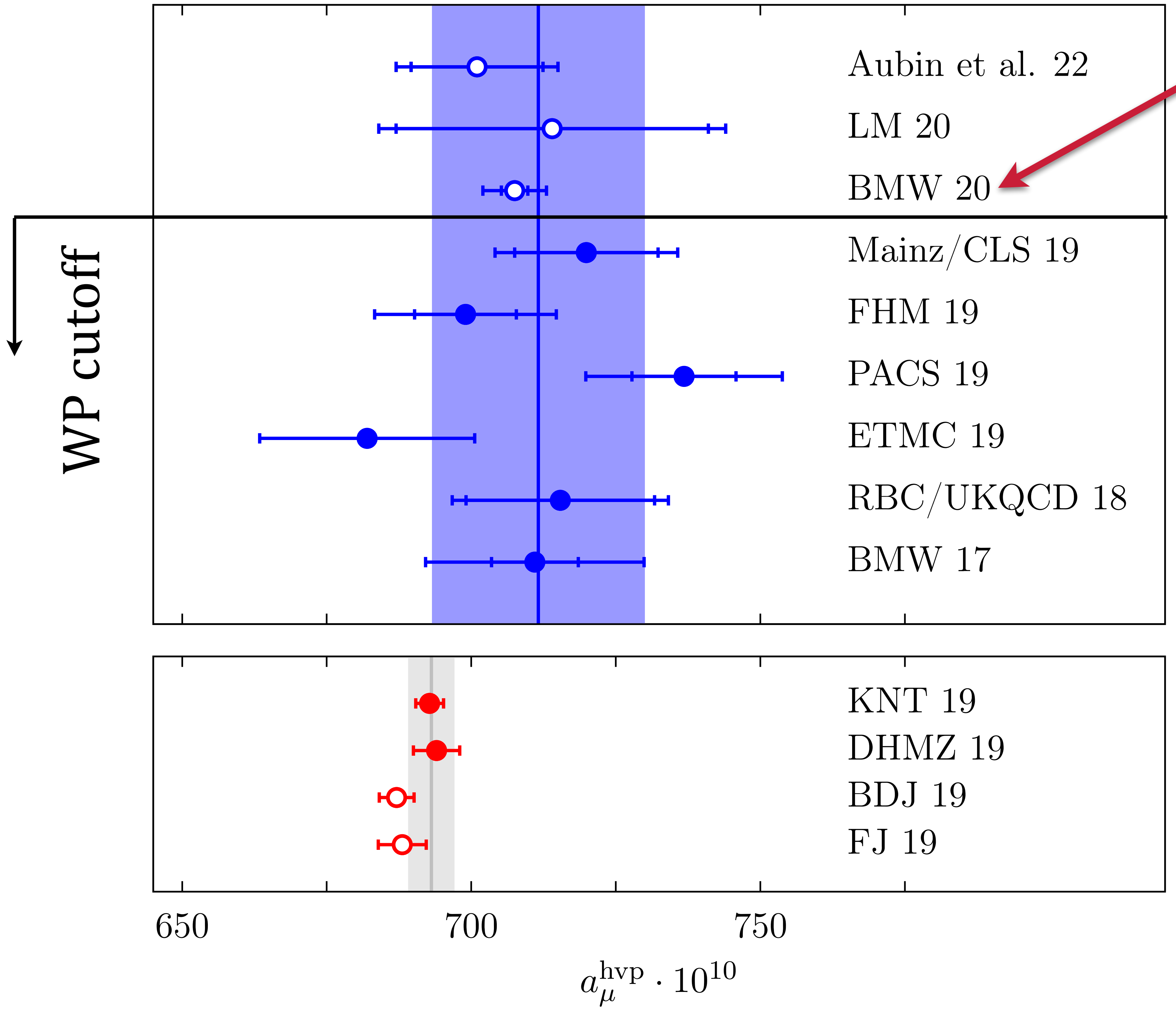
[Gérardin et al., Phys. Rev. D 100 (2019) 014510]

- $\mathrm{O}(a)$ improved Wilson fermions
- Four lattice spacings: $a = 0.085 - 0.050\,\mathrm{fm}$
- Pion masses $m_\pi = 130 - 420\,\mathrm{MeV}$
- Isospin-breaking correction by ETMC added to error
- Simultaneous chiral and continuum extrapolation



$a_\mu^{\rm hvp,\,LO} = (720.0 \pm 12.6 \pm 9.9) \cdot 10^{-10}$ [2.2%]

HVP in Lattice QCD



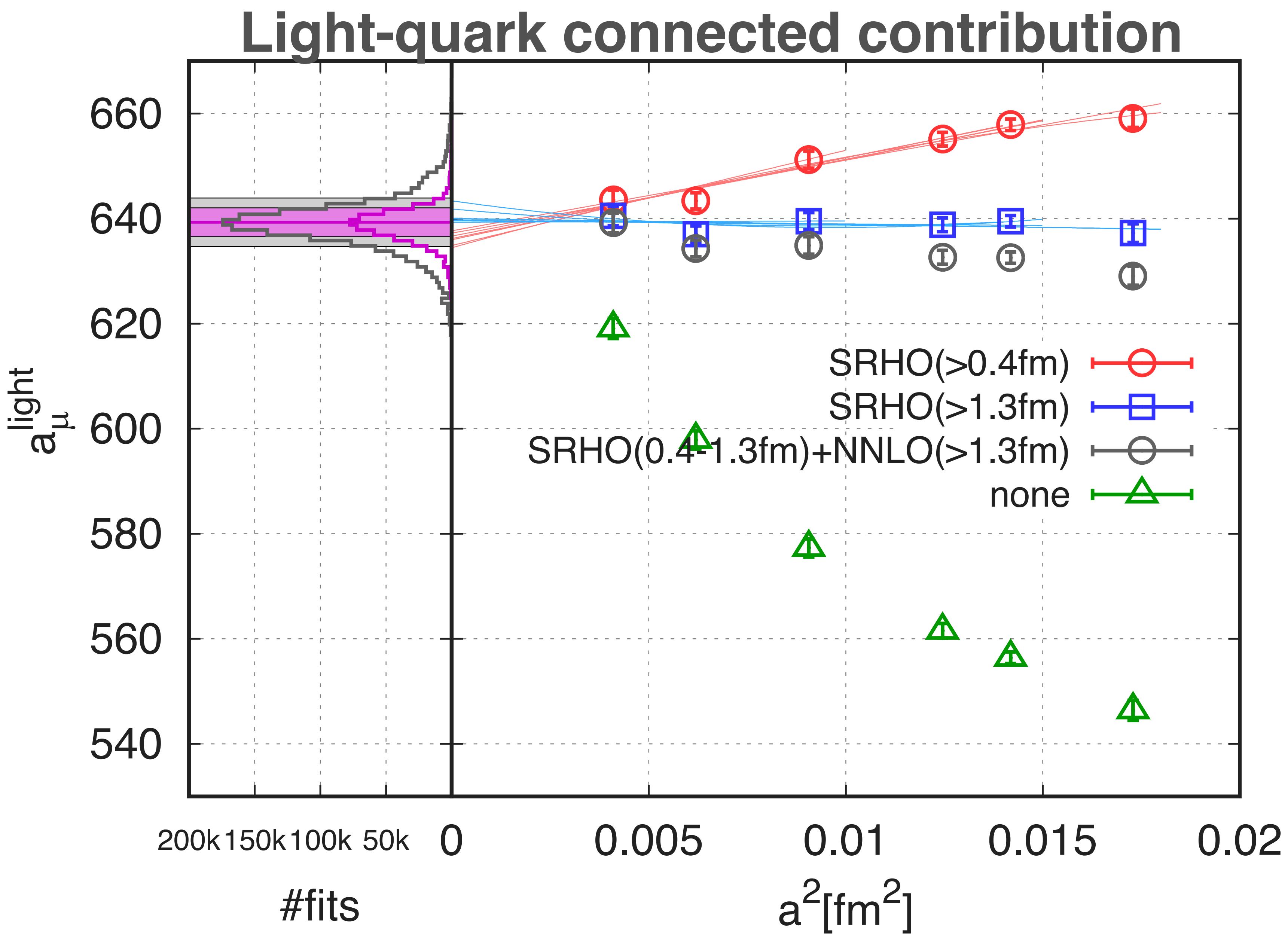
White Paper:

R-ratio: $a_\mu^{\rm hvp,\,LO} = (693.1 \pm 4.0) \cdot 10^{-10}$ [0.6%]

LQCD: $a_\mu^{\rm hvp,\,LO} = (711.6 \pm 18.4) \cdot 10^{-10}$ [2.6%]

BMWc [Borsányi et al., Nature 593 (2021) 7857]

- Rooted staggered fermions
- Six lattice spacings: $a = 0.132 - 0.064\,\text{fm}$
- Physical pion mass throughout
- Correct for taste-breaking before continuum extrapol'n
- Final result selected from distribution of different fits



$a_\mu^{\rm hvp,\,LO} = (707.5 \pm 2.3 \pm 5.0) \cdot 10^{-10}$ [0.8%]

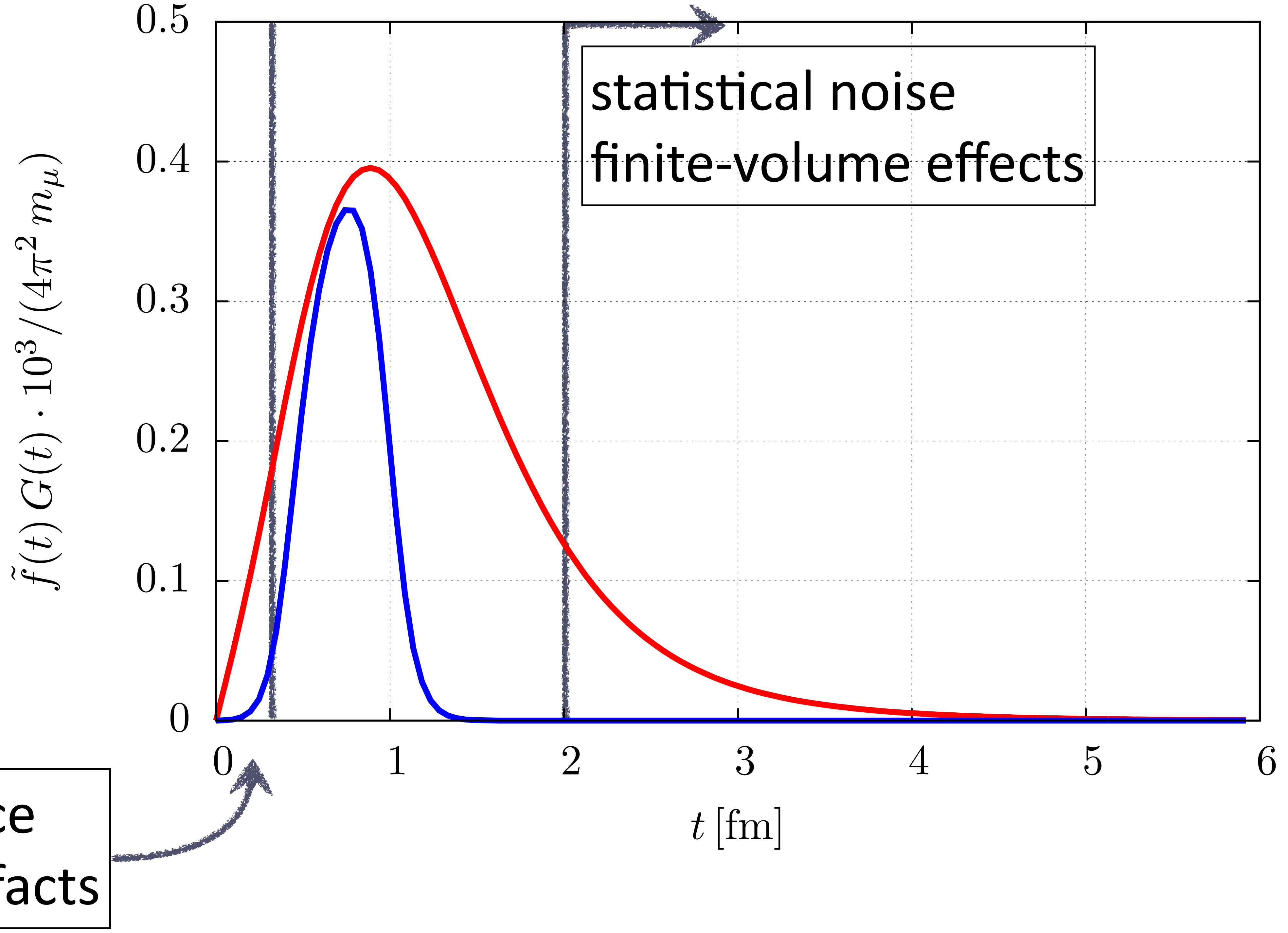
Window observables

[Blum et al., Phys. Rev. Lett. 121 (2018) 022003]

Idea: restrict integration to “unproblematic” regions

→ reduce statistical fluctuations and systematic effects

$$a_\mu^{\mathrm{hvp,\,win}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{K}(t)\, G(t)\, W(t;\, t_0,\, t_1)$$



Intermediate-distance window:

$$W^{\mathrm{ID}}(t;\, t_0,\, t_1) = \Theta(t,\, t_0, \Delta) - \Theta(t,\, t_1, \Delta)$$

$$\Theta(t,\, t', \Delta) = \tfrac{1}{2}\left[1 + \tanh(t - t')/\Delta\right]$$

$$t_0 = 0.4\,\mathrm{fm},\;\; t_1 = 1.0\,\mathrm{fm},\;\; \Delta = 0.15\,\mathrm{fm}$$

- Finite-volume correction reduced to 0.25%
- Uncertainty dominated by statistics

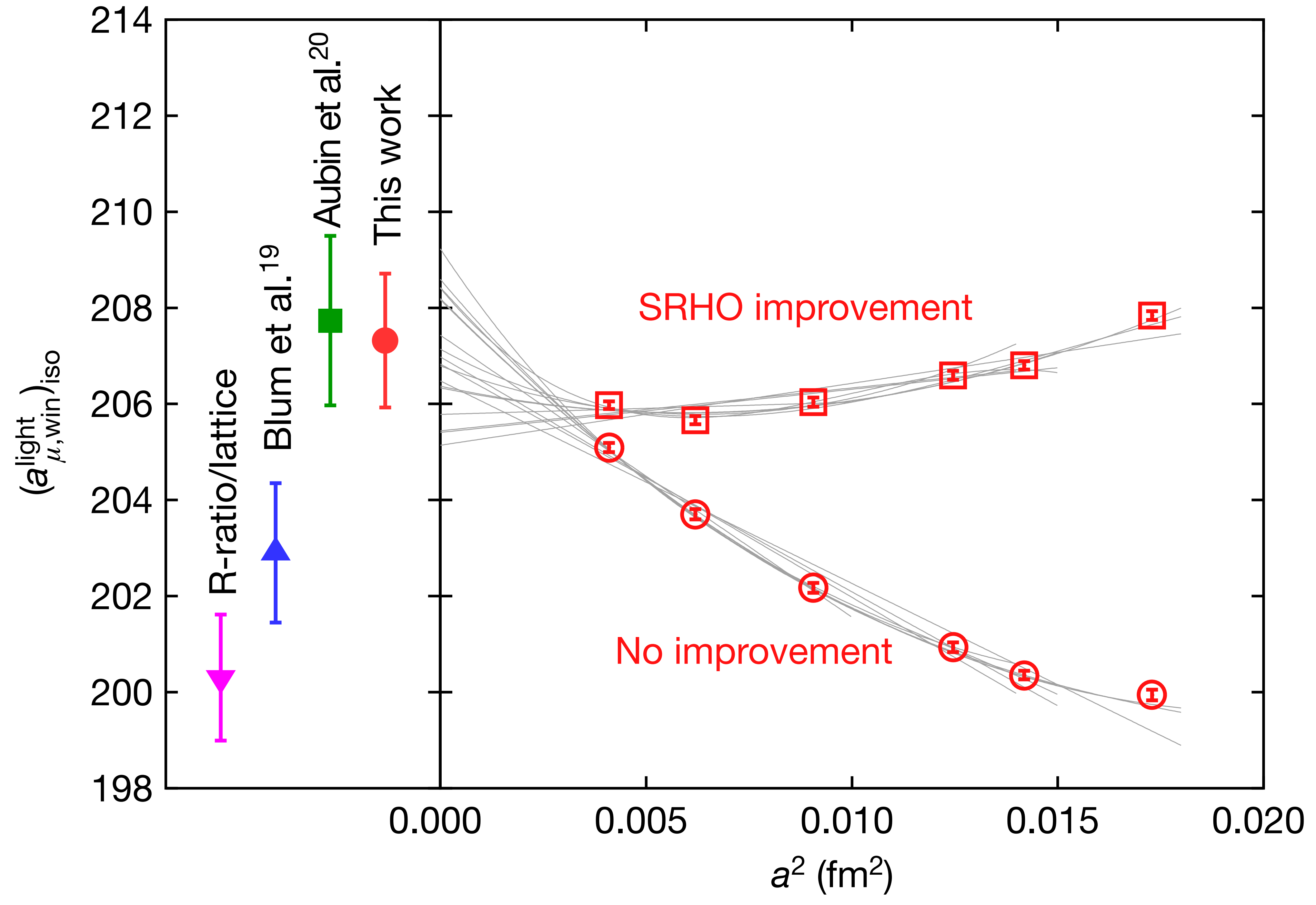
→ Benchmark quantity for sub-contribution of HVP

Data-driven approach: $a_\mu^{\mathrm{win}} = (229.4 \pm 1.4)\cdot 10^{-10}$ [Colangelo et al., Phys Lett B833 (2022) 137313]

(Excluding the 2023 CMD-3 result for $e^+e^- \to \pi^+\pi^-$)

Intermediate window observable in Lattice QCD

BMWc: Rooted staggered quarks



$$a_\mu^{\mathrm{win,ud}} = (207.3 \pm 0.4 \pm 1.3) \cdot 10^{-10}$$

[Borsányi et al., Nature 593 (2021) 7857]

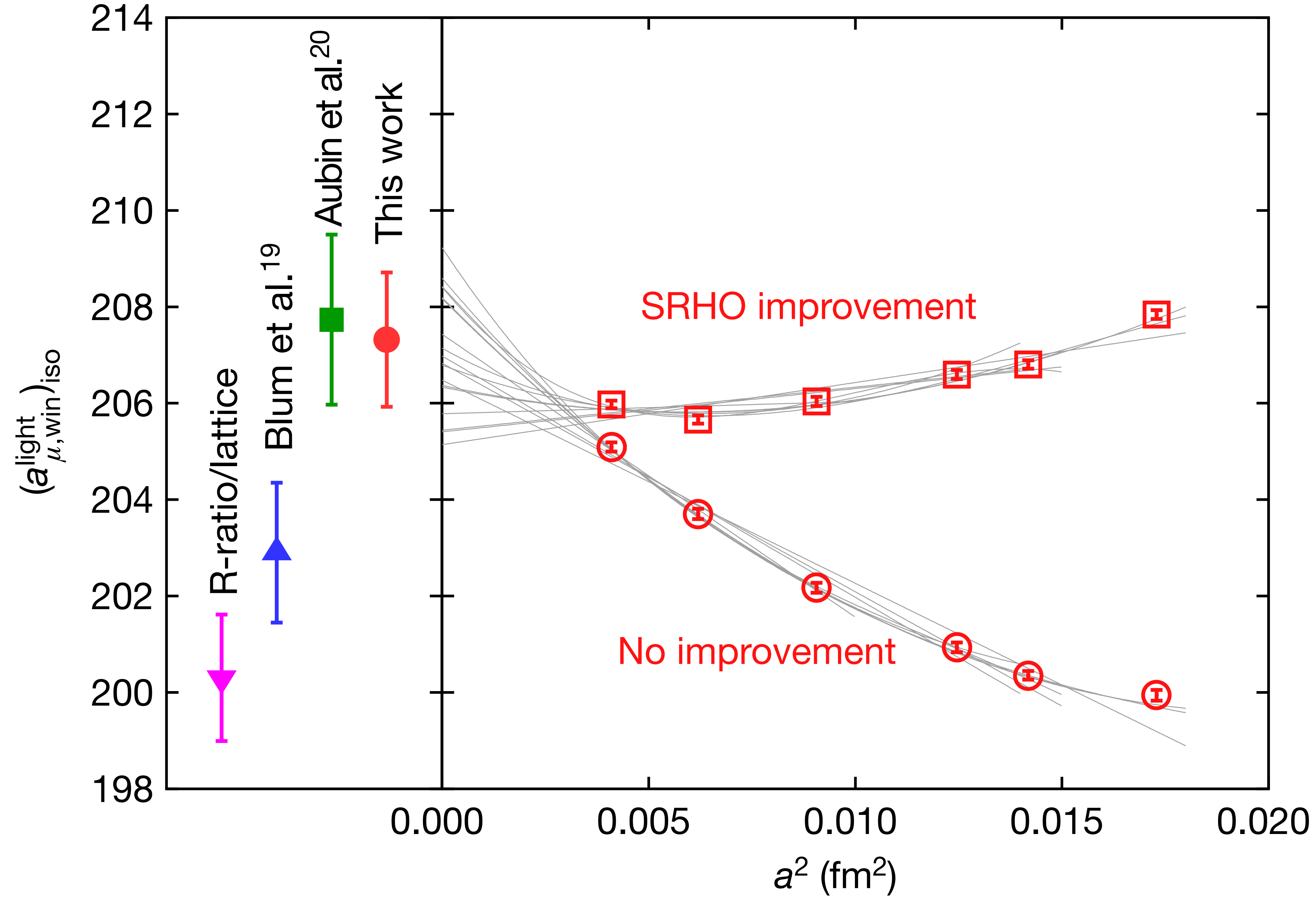
Mainz/CLS: $\mathrm{O}(a)$ improved Wilson quarks

- Extension to six lattice spacings:
  $a = 0.099 - 0.035\,\mathrm{fm}$
- Pion masses $m_\pi = 130 - 420\,\mathrm{MeV}$
- Two discretisations of the vector current: local and conserved
- Simultaneous chiral and continuum extrapolation
- Isospin-breaking correction included

[Cè et al., Phys Rev D106 (2022) 114502]

Intermediate window observable in Lattice QCD

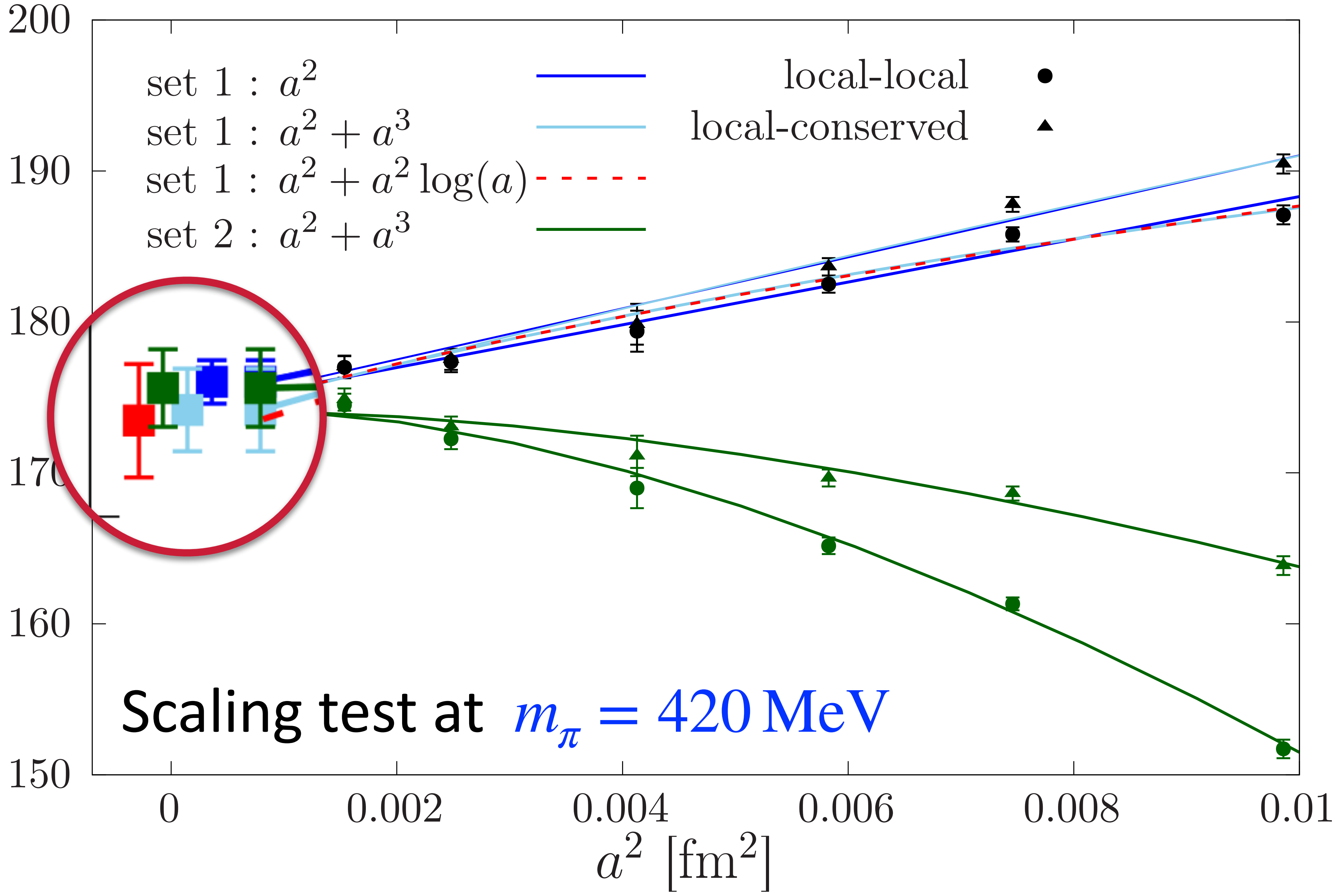
BMWc: Rooted staggered quarks



$$a_\mu^{\mathrm{win,ud}} = (207.3 \pm 0.4 \pm 1.3) \cdot 10^{-10}$$

[Borsányi et al., Nature 593 (2021) 7857]

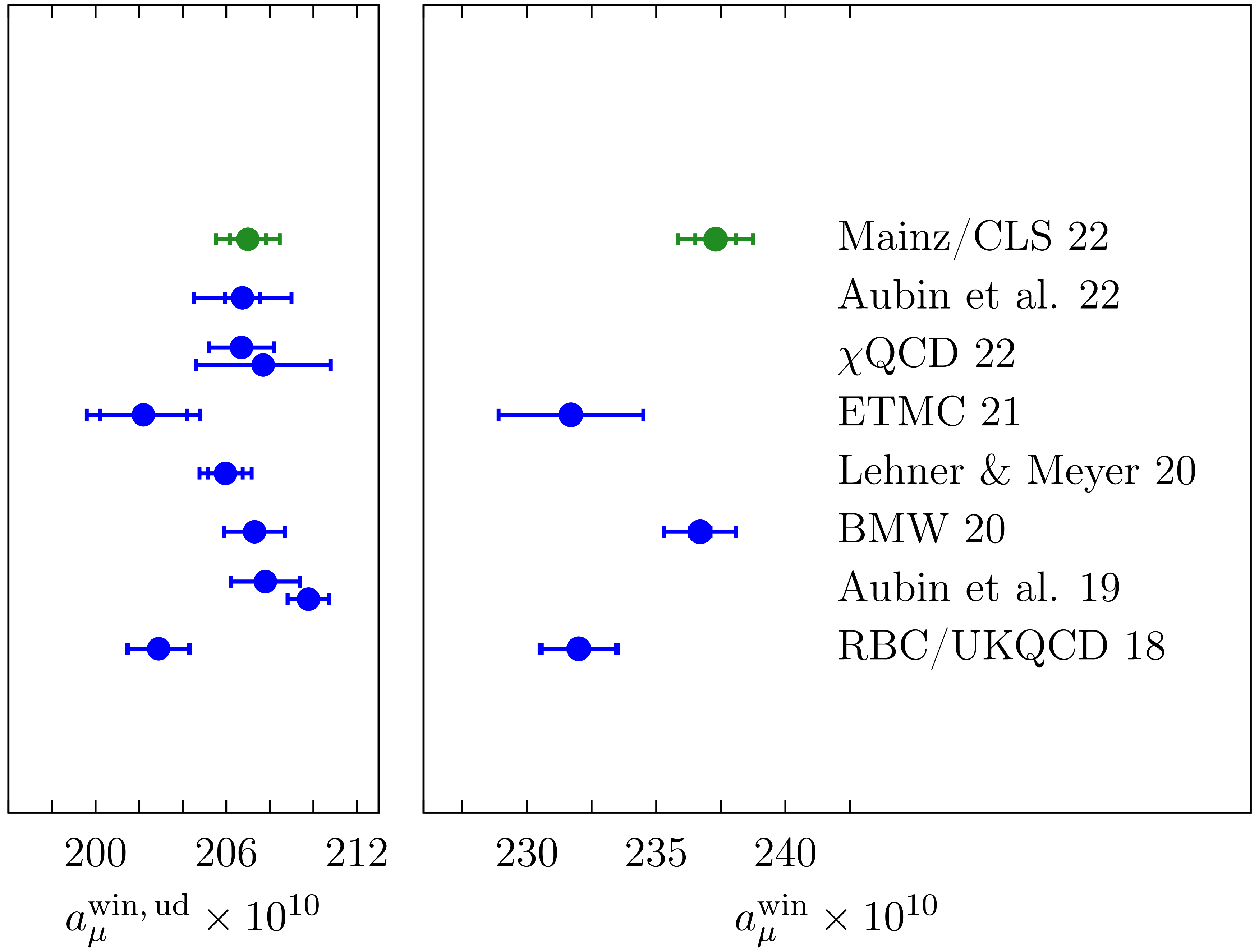
Mainz/CLS: $\mathrm{O}(a)$ improved Wilson quarks



$$a_\mu^{\mathrm{win,ud}} = (207.0 \pm 0.8 \pm 1.2) \cdot 10^{-10}$$

[Cè et al., Phys Rev D106 (2022) 114502]

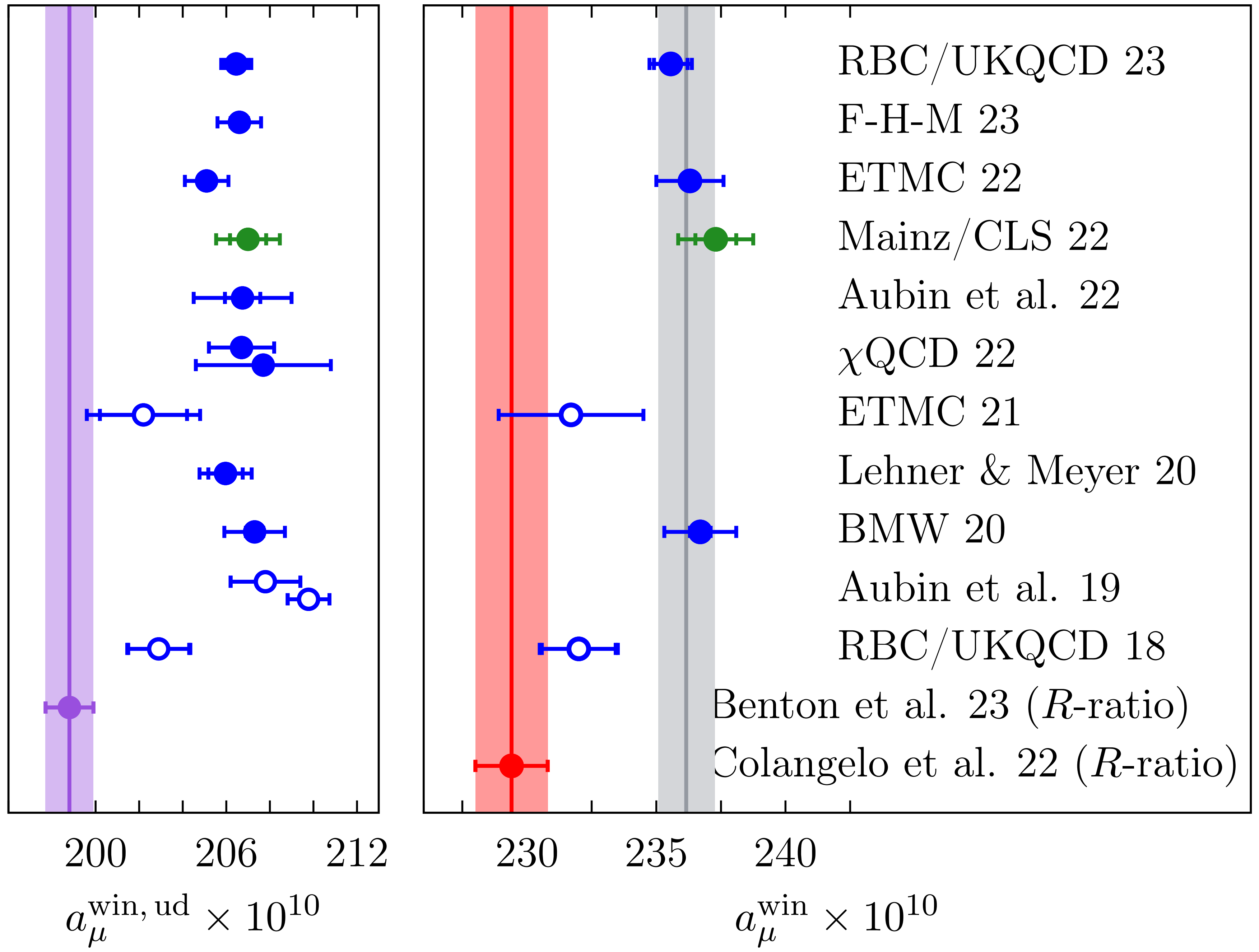
Window observable: Lattice QCD vs. R-ratio



Left: dominant light-quark contribution to a_μ^{win}

Right: including sub-leading contributions

Window observable: Lattice QCD vs. R-ratio



- Dominant light-quark contribution confirmed for wide range of discretisation with sub-percent precision
- Significant tension with results based on the R-ratio*

R-ratio estimate: $a_\mu^{\rm win} = (229.4 \pm 1.4) \cdot 10^{-10}$

Lattice average: $a_\mu^{\rm win} = (236.16 \pm 1.09) \cdot 10^{-10}$

(RBC/UKQCD 23, ETMC 22, Mainz/CLS 22, BMW 20)

[HW, arXiv:2306.04165]

- Tension of 3.8σ in the window observable evaluated from e^+e^- data* and four lattice calculations

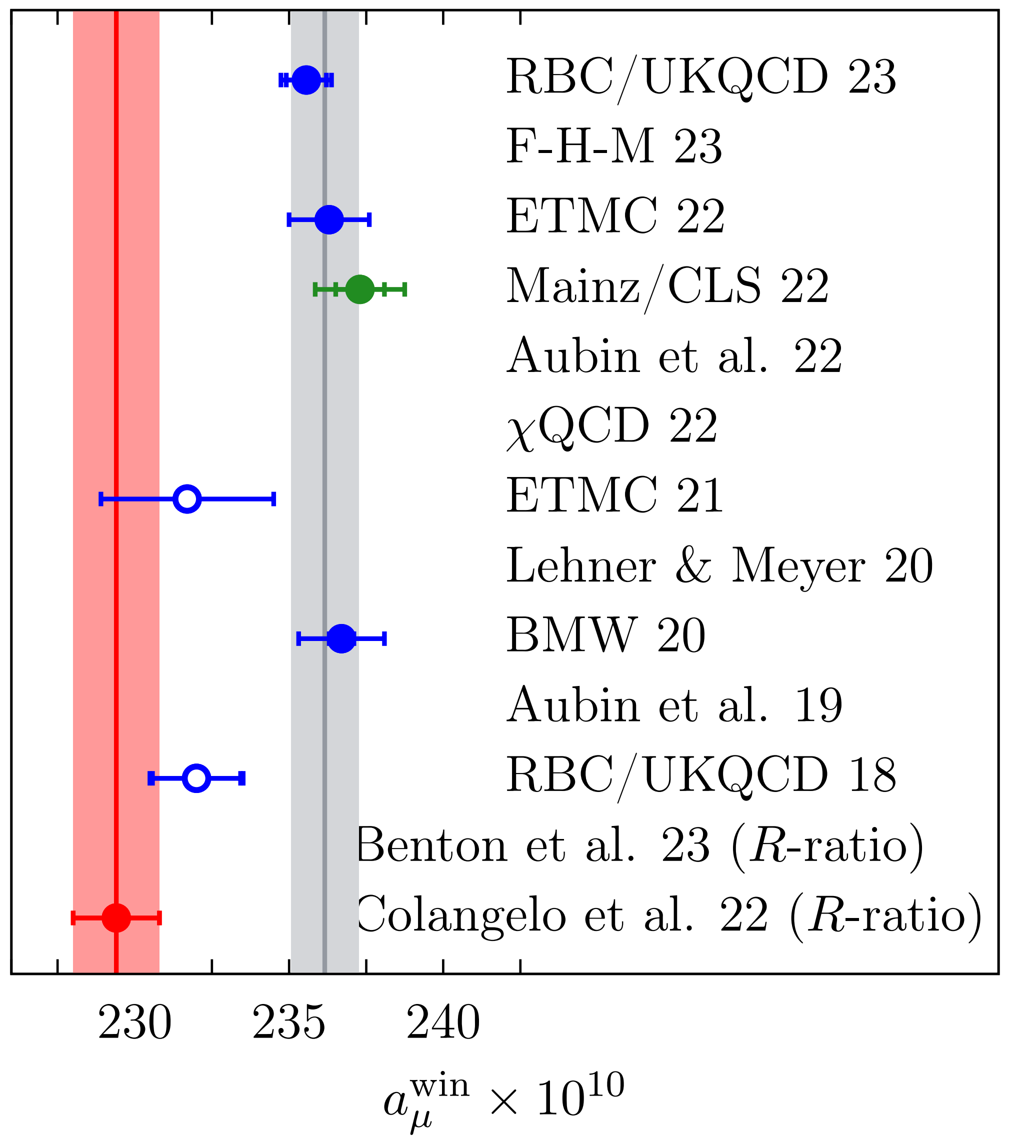
$$a_\mu^{\rm win}\Big|_{\langle \rm lat\rangle} - a_\mu^{\rm win}\Big|_{e^+e^-} = (6.8 \pm 1.8) \cdot 10^{-10} \quad [3.8\,\sigma]$$

- Subtract R-ratio result $a_\mu^{\rm win}\big|_{e^+e^-}$ from WP estimate and replace by lattice average $a_\mu^{\rm win}\big|_{\langle \rm lat\rangle}$:

$$a_\mu^{\rm exp} - a_\mu^{\rm SM}\Big|^{\rm win}_{e^+e^- \to \langle \rm lat\rangle} = (18.1 \pm 4.8) \cdot 10^{-10} \quad [3.8\,\sigma]$$

*excluding the CMD-3 result

What can we learn from $a_\mu^{\rm win}$?



Primary observable in lattice calculations: vector correlator $G(t)$

$$G(t) \equiv -\frac{a^3}{3}\sum_k\sum_{\vec{x}}\left\langle j_k^{\rm em}(\vec{x},t)\,j_k^{\rm em}(0)\right\rangle = \frac{1}{12\pi^2}\int_{m_{\pi^0}^2}^{\infty} d(\sqrt{s})\,R(s)^{\rm lat}\,s\,{\rm e}^{-\sqrt{s}t}$$

$a_\mu^{\rm win}\big|_{\rm lat} > a_\mu^{\rm win}\big|_{e^+e^-}$ implies that $R(s)^{\rm lat} > R(s)^{e^+e^-}$ in some interval of $\sqrt{s}$

Energy interval $600 \leq \sqrt{s} \leq 900\,{\rm MeV}$ contributes the same fraction to $a_\mu^{\rm hvp}$ and $a_\mu^{\rm win}$

$\sqrt{s}$ interval	$a_\mu^{\rm hvp}$	$(a_\mu^{\rm hvp})^{\rm SD}$	$(a_\mu^{\rm hvp})^{\rm ID}$	$(a_\mu^{\rm hvp})^{\rm LD}$	$\bar{\Pi}(1\ {\rm GeV}^2)$
Below 0.6 GeV	15.5	1.5	5.5	23.5	8.2
0.6 to 0.9 GeV	58.3	23.1	54.9	65.4	52.6
Above 0.9 GeV	26.2	75.4	39.6	11.1	39.2
Total	100.0	100.0	100.0	100.0	100.0

[Cè et al., Phys Rev D106 (2022) 114502]

What can we learn from $a_\mu^{\rm win}$?

- Phenomenological model for R-ratio predicts [Mainz/CLS, Cè et al., Phys Rev D 106 (2022) 114502]

$$\sqrt{s} = 600 - 900\,\text{MeV:} \qquad \frac{R(s)^{\rm lat}}{R(s)^{e^+e^-}} = 1+\epsilon \quad \Rightarrow \quad \frac{(a_\mu^{\rm hvp})^{\rm lat}}{(a_\mu^{\rm hvp})^{e^+e^-}} \approx \frac{(a_\mu^{\rm win})^{\rm lat}}{(a_\mu^{\rm win})^{e^+e^-}} = 1+0.6\epsilon$$

- Lattice average vs. R-ratio: $(a_\mu^{\rm win})^{\rm lat}/(a_\mu^{\rm win})^{e^+e^-} = 1.030(8)$

  $\Rightarrow$ $R(s)^{\rm lat}$ is enhanced by 5% relative to $R(s)^{e^+e^-}$ for $\sqrt{s} = 600 - 900\,\text{MeV}$

- If confirmed, it would imply that BMW's estimate might be too low….

Similar conclusions

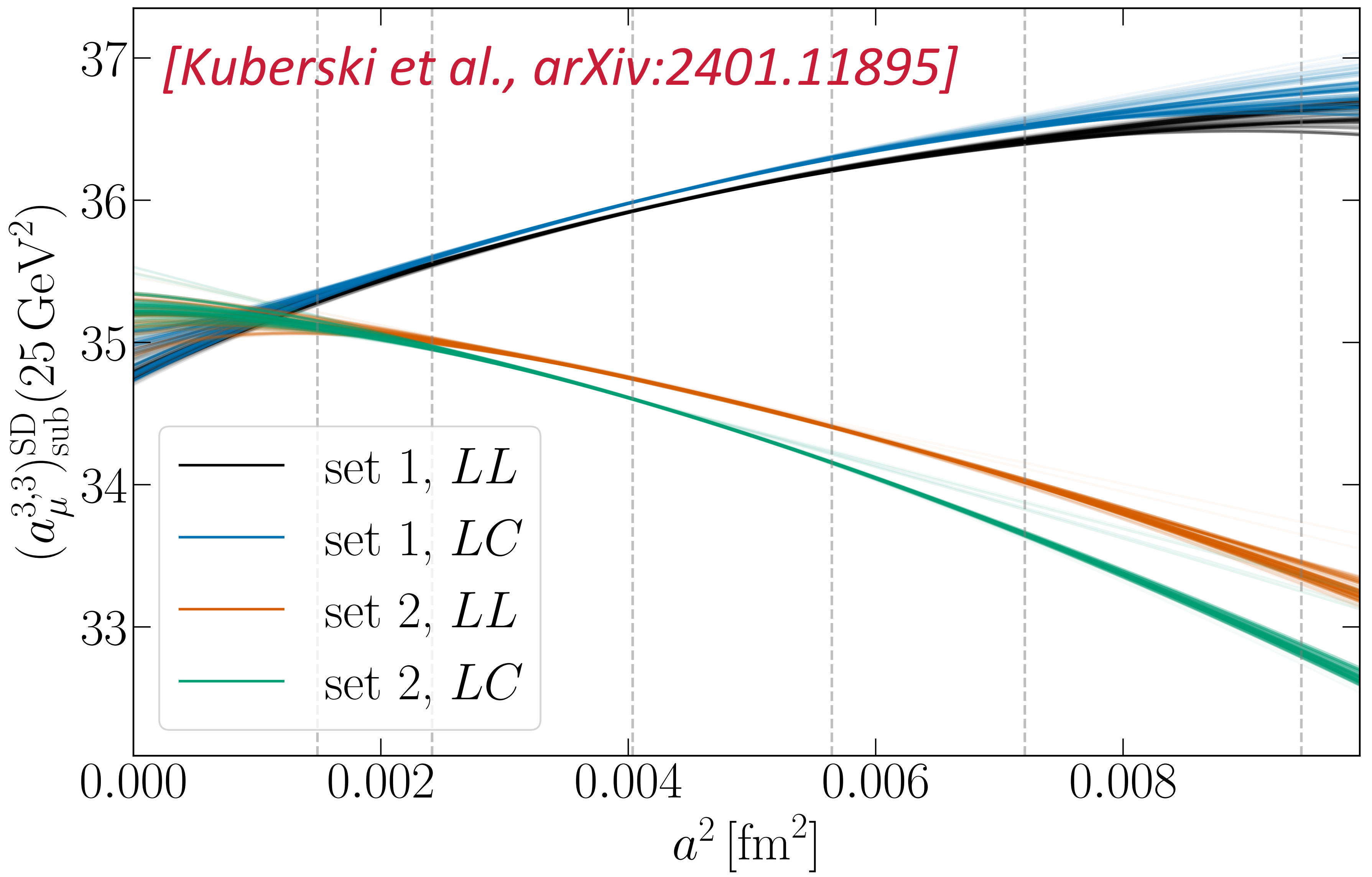
- Dispersive treatment of pion form factor [Colangelo, Hoferichter, Stoffer, PLB 814 (2021) 136073]
- "Energy-smeared" R-ratio from lattice data [ETMC, Alexandrou et al., PRL 130 (2023) 241901]

More windows….

Short-distance window:

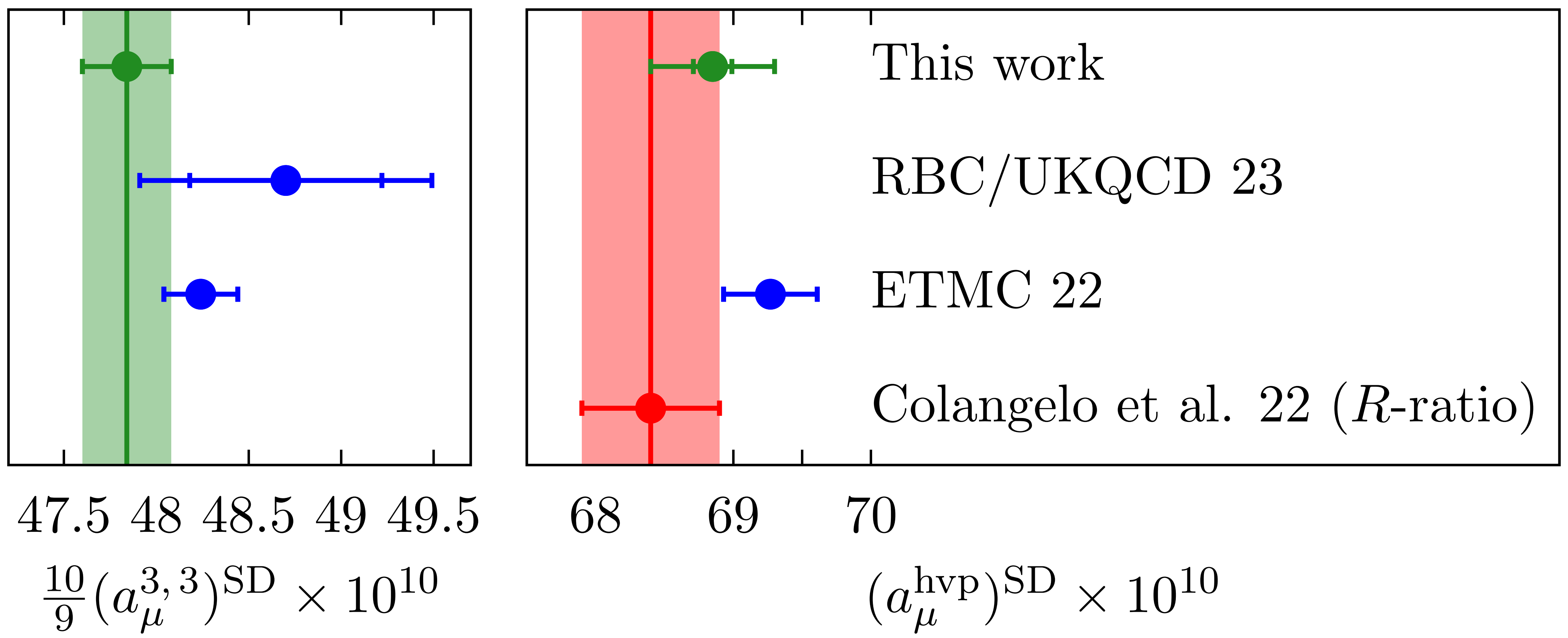
- Finite-volume correction negligible
- Uncertainty dominated by control over lattice artefacts

$$(a_\mu^{\rm win})^{\rm SD} = (68.85 \pm 0.15 \pm 0.42) \times 10^{-10}$$



Hadronic model:

- 5% enhancement of $R(s)^{\rm lat}$ for $0.6\,{\rm GeV} \leq \sqrt{s} \leq 0.9\,{\rm GeV}$ increases $(a_\mu^{\rm win})^{\rm SD}$ by $+1 \times 10^{-10}$
- Expectation confirmed by lattice calculations



Hadronic running of electromagnetic coupling

Electromagnetic coupling is energy-dependent:

$$\alpha^{-1} = 137.035\,999\ldots \qquad \alpha(q^2) = \frac{\alpha}{1-\Delta\alpha(q^2)} \qquad \alpha^{-1}(M_Z^2) = 127.951 \pm 0.009$$

Correlation between $a_\mu^{\rm hvp}$ and the hadronic running of $\Delta\alpha_{\rm had}$:

$$\Delta\alpha_{\rm had}(q^2) = -\frac{\alpha\, q^2}{3\pi} \,\mathcal{P}\!\!\int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R(s)}{s(s-q^2)}\,, \qquad a_\mu^{\rm hvp} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R(s)\,\hat{K}(s)}{s^2}$$

Euclidean momenta

$\Delta\alpha_{\rm had}(-Q^2)$ accessible in lattice QCD via the same correlator $G(t)$ with a different kernel function:

$$\Delta\alpha_{\rm had}(-Q^2) = \frac{\alpha}{\pi}\frac{1}{Q^2}\int_0^\infty dt\, G(t)\left[Q^2t^2 - 4\sin^2\left(\tfrac{1}{2}Q^2t^2\right)\right]$$

Hadronic running at Z-pole: $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ $\rightarrow$ key quantity in global electroweak fit

Evaluation of $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ and comparison with EW precision data

Adler function approach, aka. "Euclidean split technique"

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = \Delta\alpha_{\rm had}^{(5)}(-Q_0^2) \quad \leftarrow \text{ lattice QCD}$$

$$+[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)] \quad \leftarrow \text{ perturbative Adler function}$$

$$+[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)] \quad \leftarrow \text{ pQCD}$$

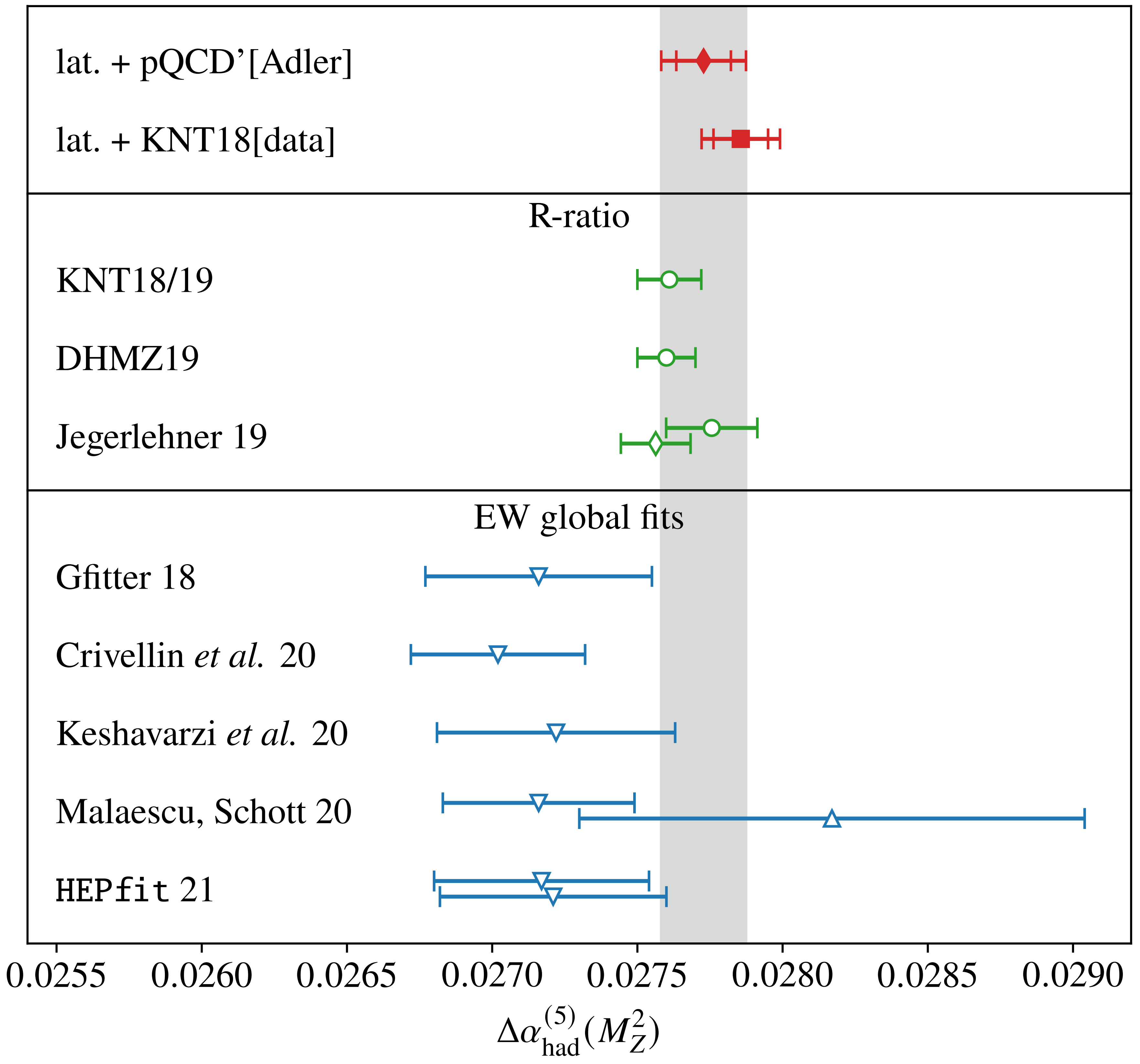
Evaluation of $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ and comparison with EW precision data

Adler function approach, aka. "Euclidean split technique"

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)$$
$$+[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-Q_0^2)]$$
$$+[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)]$$

$$\Rightarrow \quad \Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.027\,73(9)_{\rm lat}(2)_{\rm btm}(12)_{\rm pQCD}$$

[Mainz/CLS, Cè et al., JHEP 08 (2022) 220, arXiv:2203.08676]



- No inconsistency with global electroweak fit!

Standard Model can accommodate a larger value for a_μ without contradicting electroweak precision data

Summary and outlook

Hadron physics holds the key in the quest for new physics via $(g-2)_\mu$

No straightforward interpretation of the Fermilab E989 experiment

Discrepant determinations of the HVP contribution:

- Tensions between lattice QCD and e^+e^- hadronic cross sections*
- Tension in $\pi^+\pi^-$channel between BaBar vs. KLOE and CMD-3 vs. all other results

Analyses / re-analyses of e^+e^- data in progress: BaBar, BESIII, CMD-3, KLOE

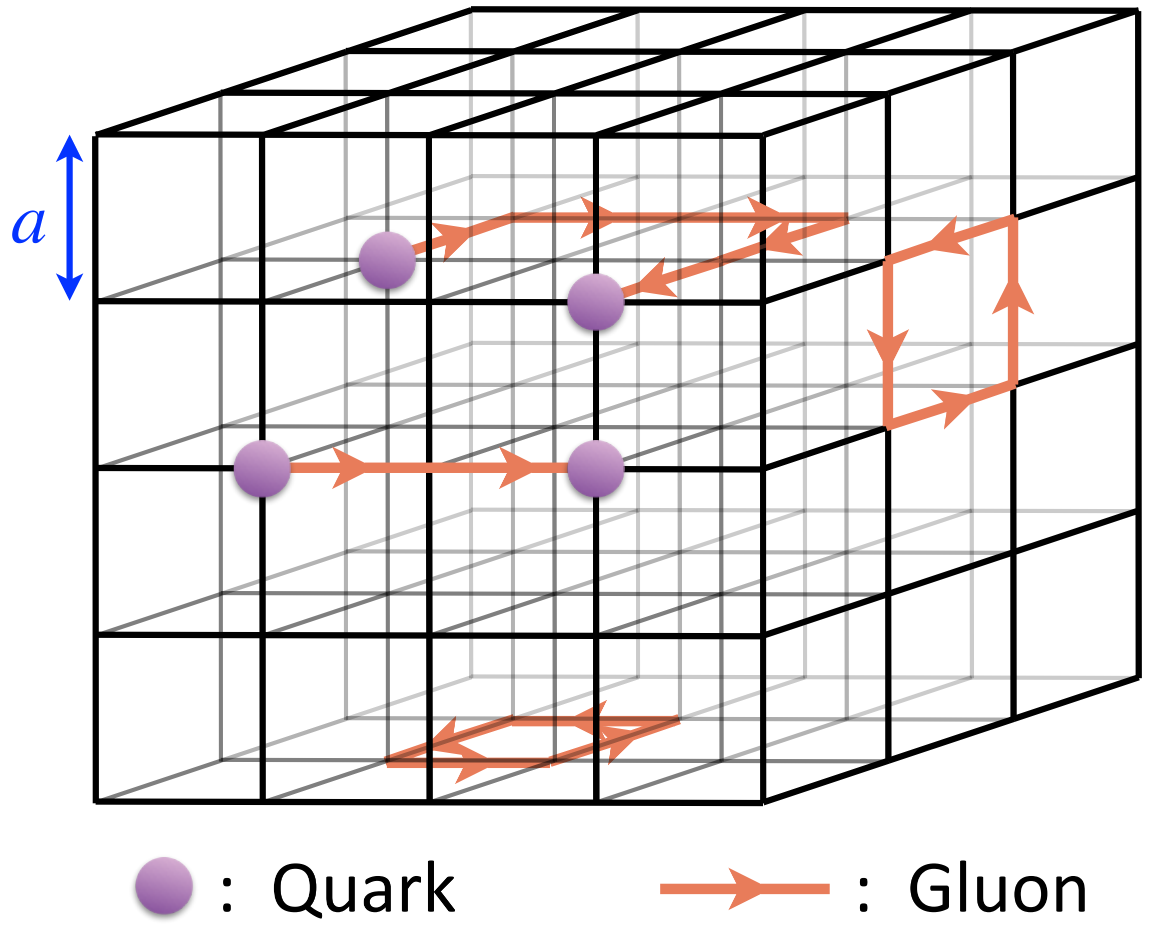
Lattice QCD to produce more results for HVP contribution with sub-percent precision

Experimental measurement of the HVP contribution by MUonE experiment

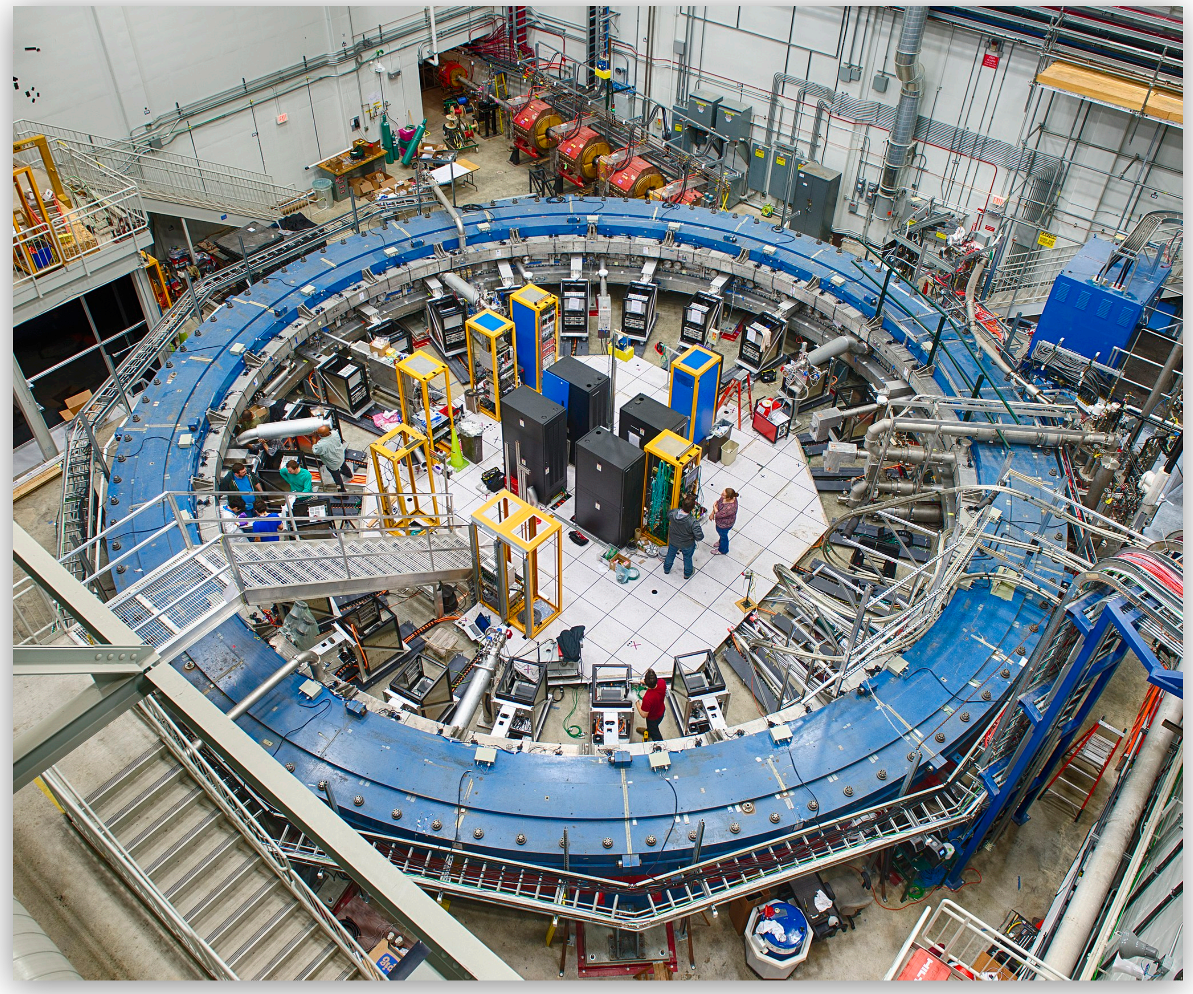
Fermilab E989 to analyse data from Runs 4–6

Update of White Paper expected by $\approx$ Dec 2024

*pre-2023



It’s not magnitude that matters
but significance!



Thank you!

Backup

QED contributions to a_μ

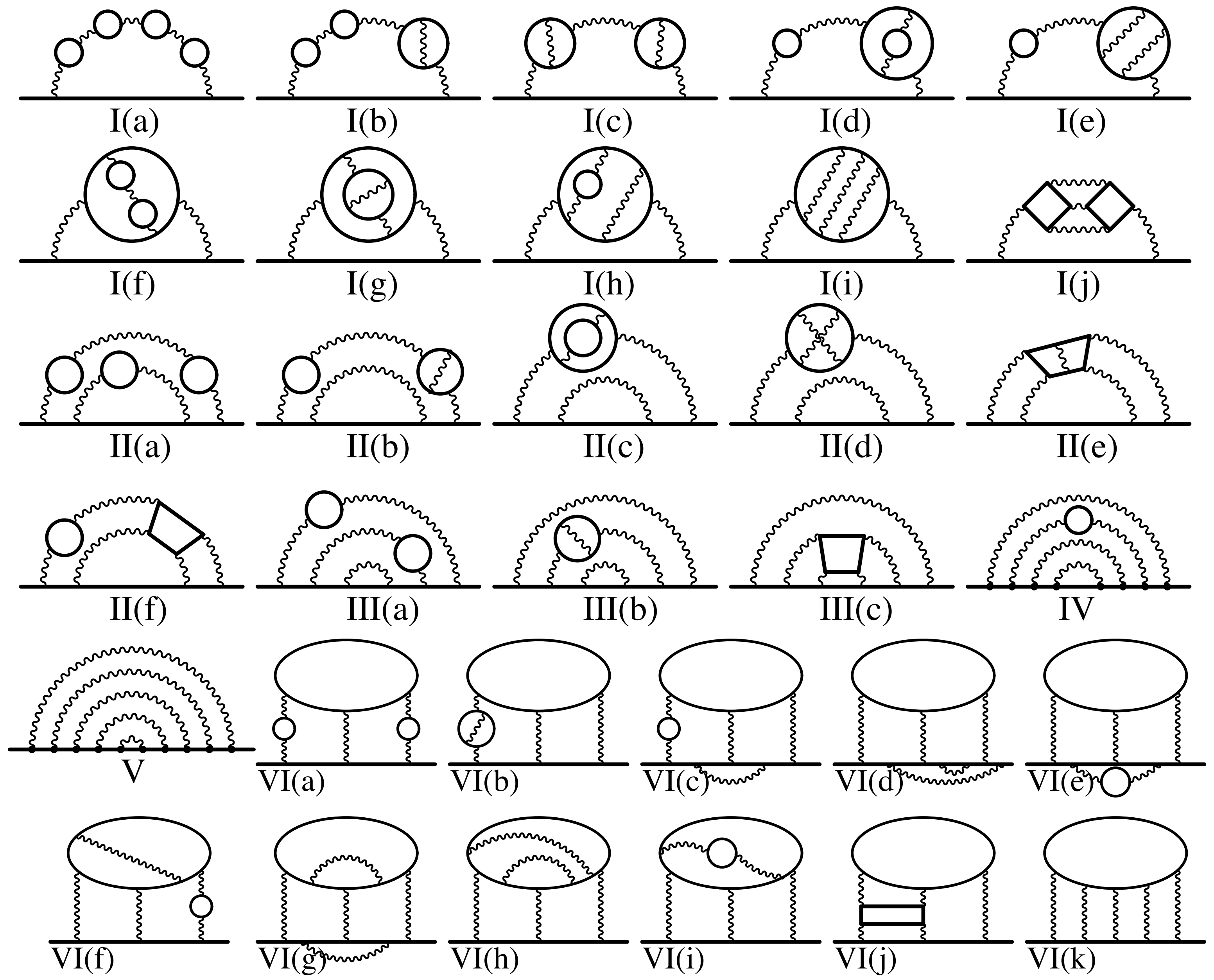
QED contribution has been worked out to in perturbation theory to 5-loop order:

PRL **109,** 111808 (2012) PHYSICAL REVIEW LETTERS week ending 14 SEPTEMBER 2012

Complete Tenth-Order QED Contribution to the Muon $g - 2$

Tatsumi Aoyama,[1,2] Masashi Hayakawa,[3,2] Toichiro Kinoshita,[4,2] and Makiko Nio[2]
[1]*Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya, 464-8602, Japan*
[2]*Nishina Center, RIKEN, Wako, Japan 351-0198*
[3]*Department of Physics, Nagoya University, Nagoya, Japan 464-8602*
[4]*Laboratory for Elementary Particle Physics, Cornell University, Ithaca, New York 14853, USA*
(Received 24 May 2012; published 13 September 2012)

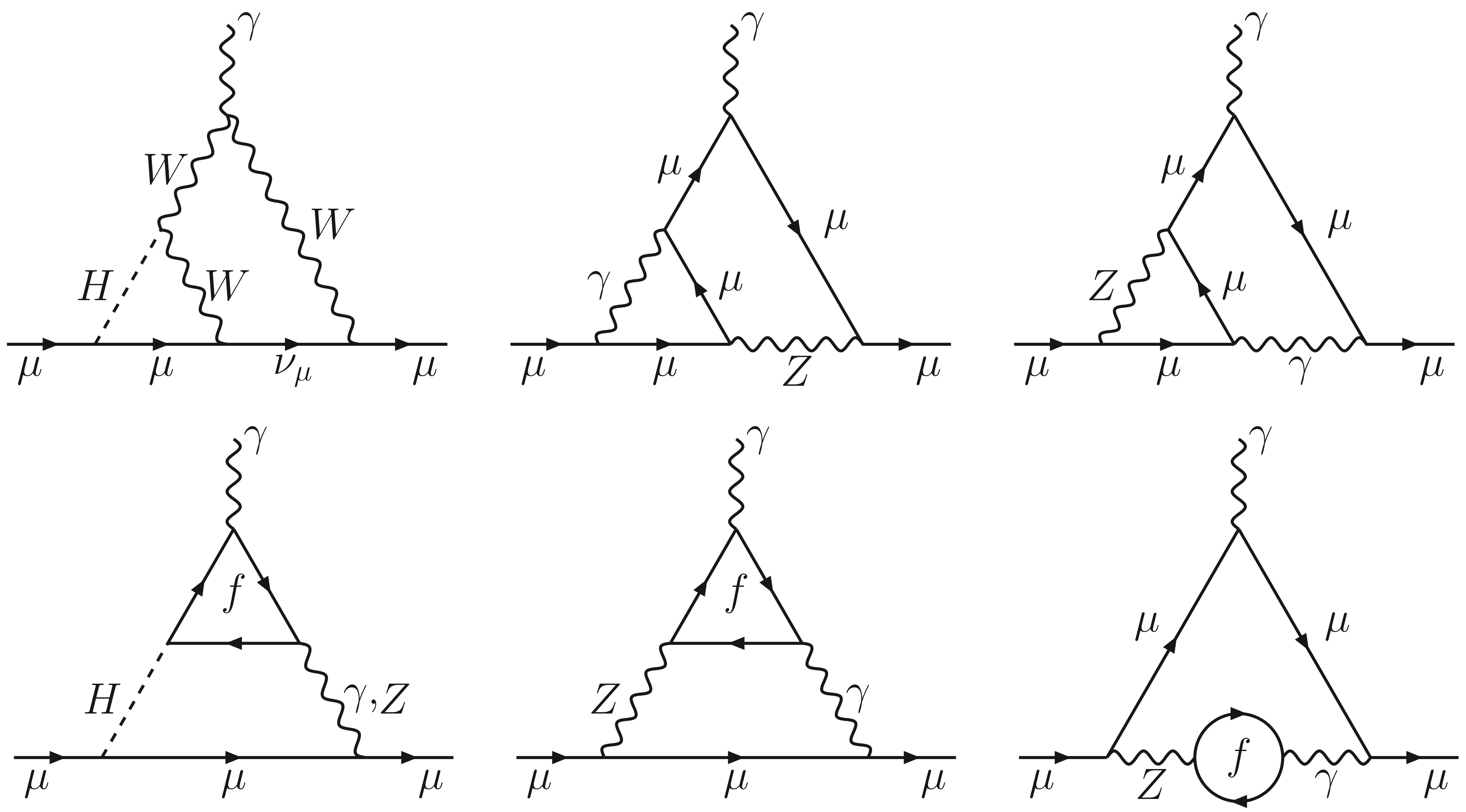
SM	116 591 810	100 %	#diagrams
QED(tot)	116 584 718.931	99,9939 %	
2	116 140 973.321	99,6133 %	1
4	413 217.626	0,3544 %	9
6	30 141.902	0,0259 %	72
8	381.004	0,0003 %	891
10	5.078	$4\cdot10^{-6}$ %	12672



Electroweak contributions to a_μ

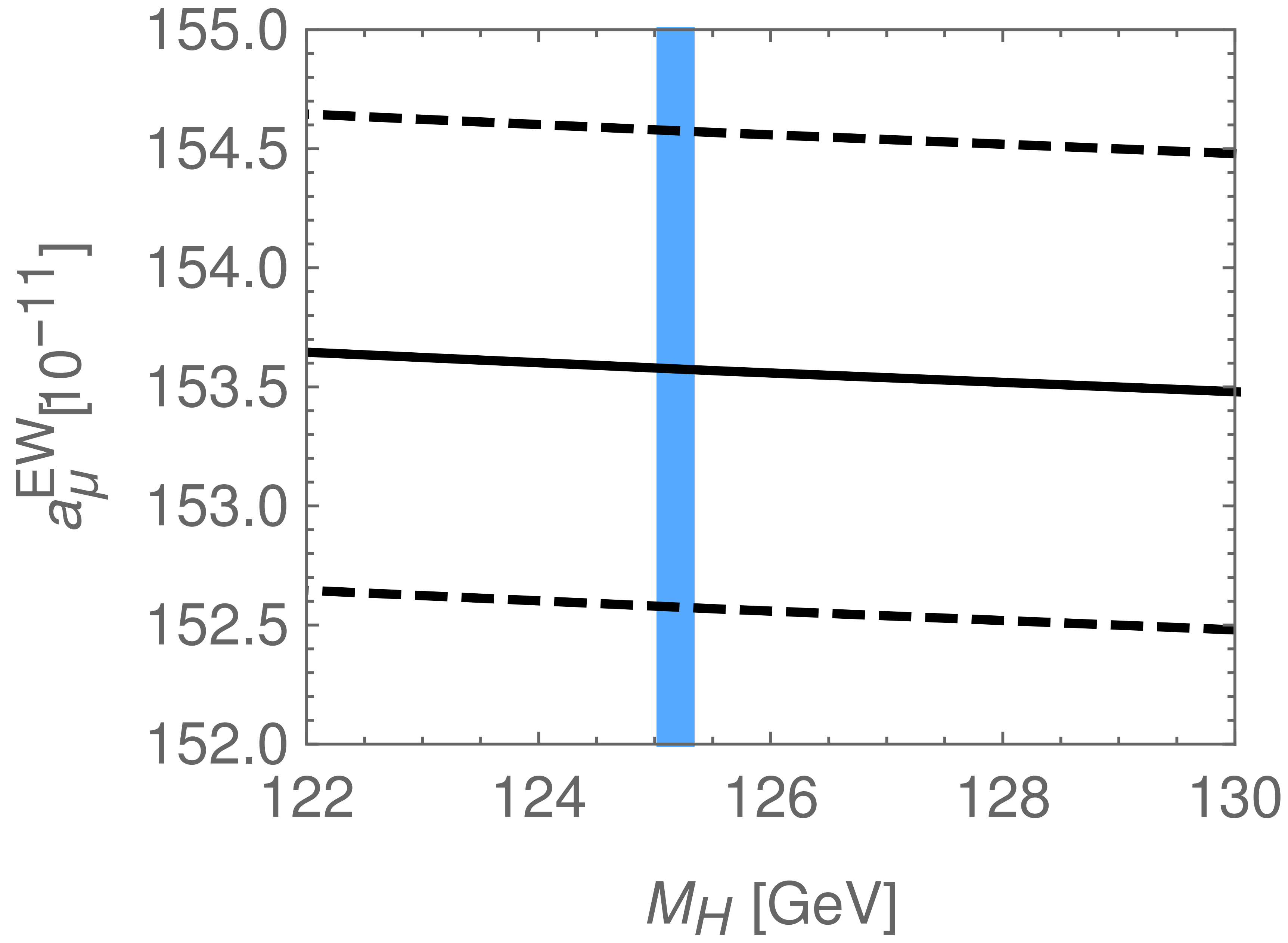
Weak contributions known to leading three-loop order

Sample two-loop diagrams:



$$a_\mu^{\text{weak}} = (153.6 \pm 1.0) \times 10^{-11}$$

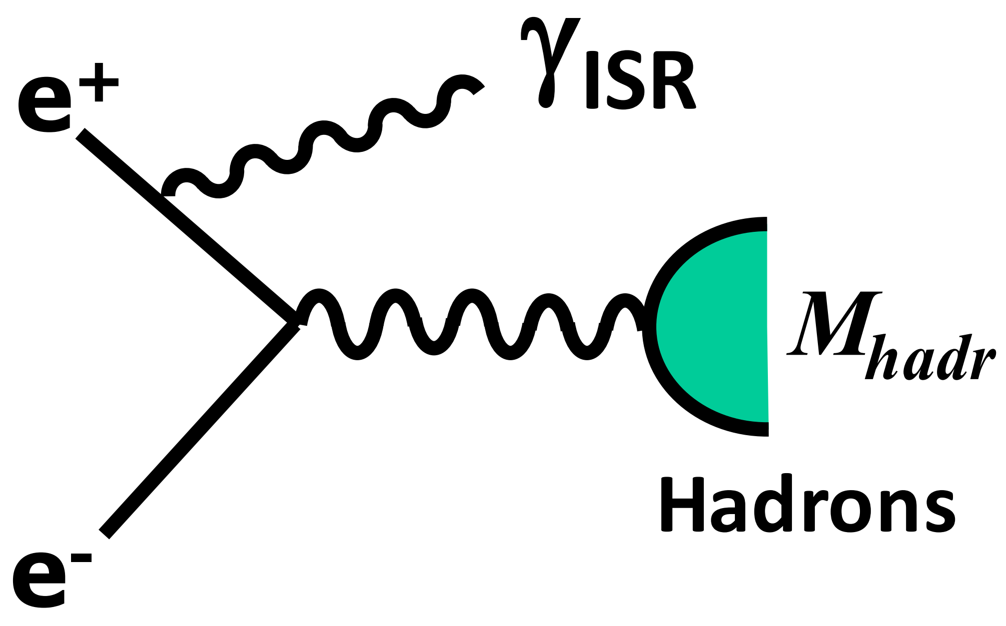
Dependence on the Higgs mass



[Gnendiger et al., arXiv:1306.5546]

[Aoyama et al., Phys. Rep. 887 (2020) 1, arXiv:2006.04822]

Hadronic cross section data and their analysis



Cross section data collected either via energy scan (VEPP-2000, …) or using the ISR technique (BaBar, CLEO, KLOE, BESIII,…)

Determine the ISR luminosity:

$$\frac{d\sigma_{\rm ISR}(\sqrt{s'})}{d\sqrt{s'}} = \frac{2\sqrt{s'}}{s} W(s, E_\gamma, \theta_\gamma)\,\sigma(\sqrt{s'})$$

W : radiator function

Monte Carlo event generators for $e^+e^- \to \text{hadrons}(\gamma)$, e.g. BABAYAGA, PHOKHARA

Several variants of ISR analyses:

- Tagged vs. untagged ISR photon
- Measured radiator function vs. event generator

Other issues: particle ID and π-μ separation

Hadronic vacuum polarisation: Data-driven approach

Decade-long effort to measure e^+e^- cross sections

$$a_\mu^{\rm hvp,\,LO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds\, \frac{R_{\rm had}(s)\hat{K}(s)}{s^2}$$

$\sqrt{s} \lesssim 2\,\mathrm{GeV}$: sum of exclusive channels

$\sqrt{s} > 2\,\mathrm{GeV}$: inclusive channels, narrow resonances, perturbative QCD

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\rm HVP,\ LO}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\rm DV+QCD}$	692.8(2.4)	1.2

[Aoyama et al., Phys. Rep. 887 (2020) 1, arXiv:2006.04822]

White Paper estimate of the HVP contribution

Merging procedure

- Determine $a_\mu^{\rm hvp,\,LO}$ as the sum of simple averages of individual hadronic channels (DHMZ, KNT, CHHKS below 1 GeV), including correlations in the data
- Experimental and theoretical uncertainties: use maximum error estimate in each channel (except $\pi^+\pi^-$) of either DHMZ or KNT
- Extra systematic uncertainty of $\Delta \equiv \frac{1}{2}\,|{\rm DHMZ} - {\rm KNT}|$ in each channel, except $\pi^+\pi^-$; For $\pi^+\pi^-$: use maximum of Δ and tension between BaBar and KLOE
- Add uncertainty associated with duality violations and perturbative QCD

$$a_\mu^{\rm hvp,\,LO} = 693.1(2.8)_{\rm exp}(2.8)_{\rm syst}(0.7)_{\rm DV+QCD} \times 10^{-10} = 693.1(4.0) \times 10^{-10} \qquad [0.6\%]$$