Gravitational Scattering of Compact Bodies from Worldline Quantum Field Theory

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Period of PhD: 10/20 — 07/23
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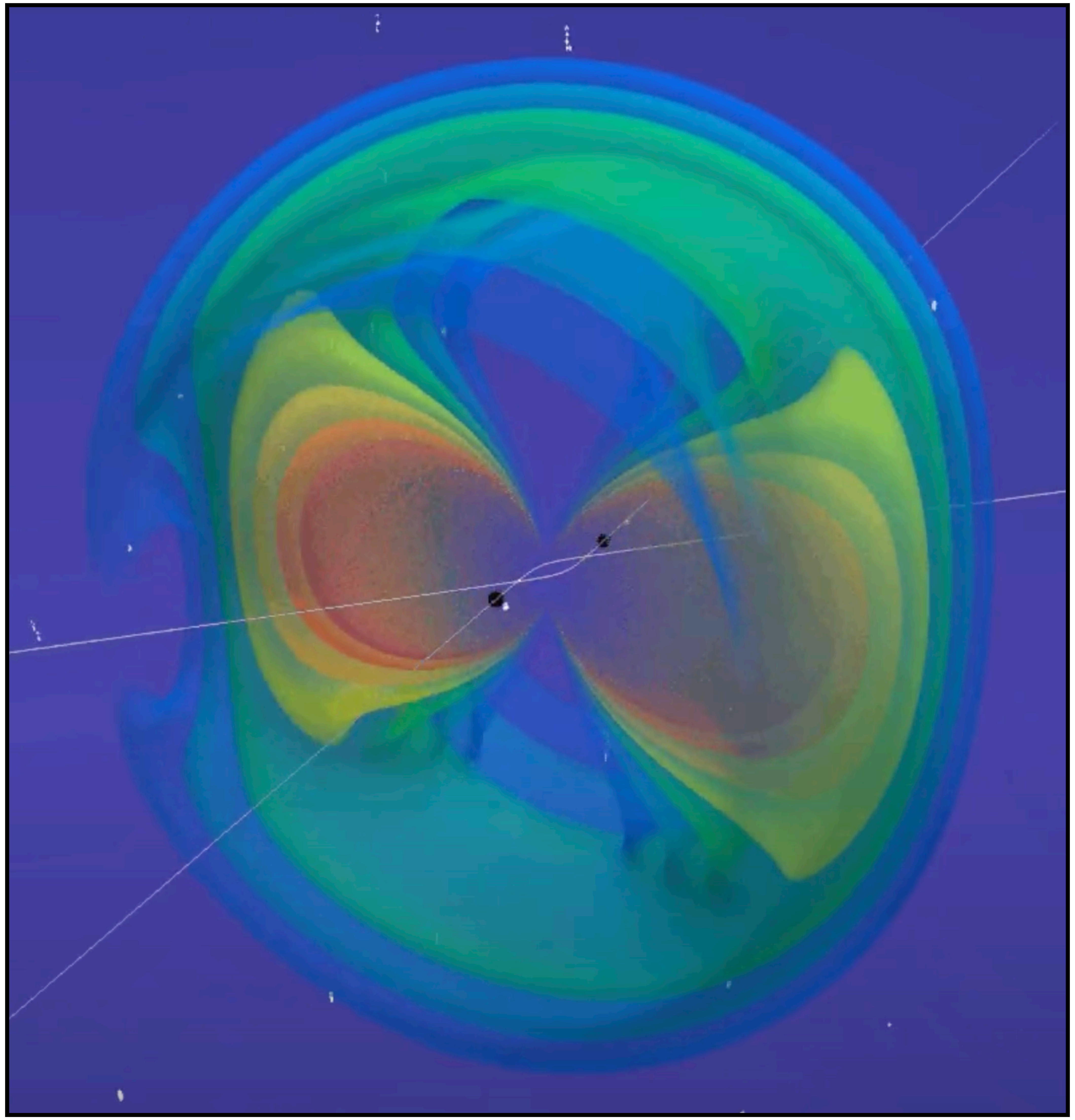

Credit: Oluwadamilola Babayemi

RTG 2575: Rethinking Quantum Field Theory

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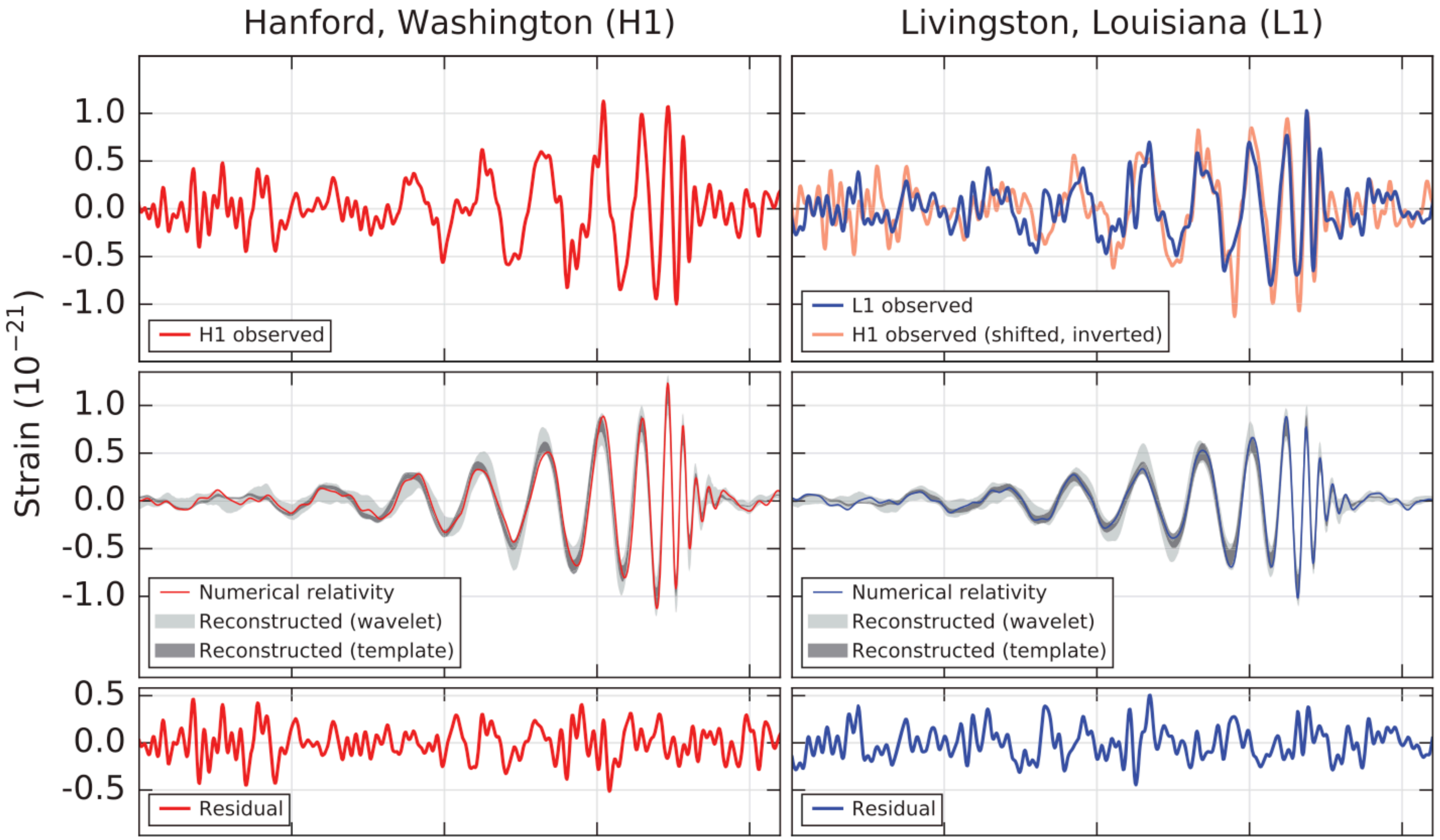
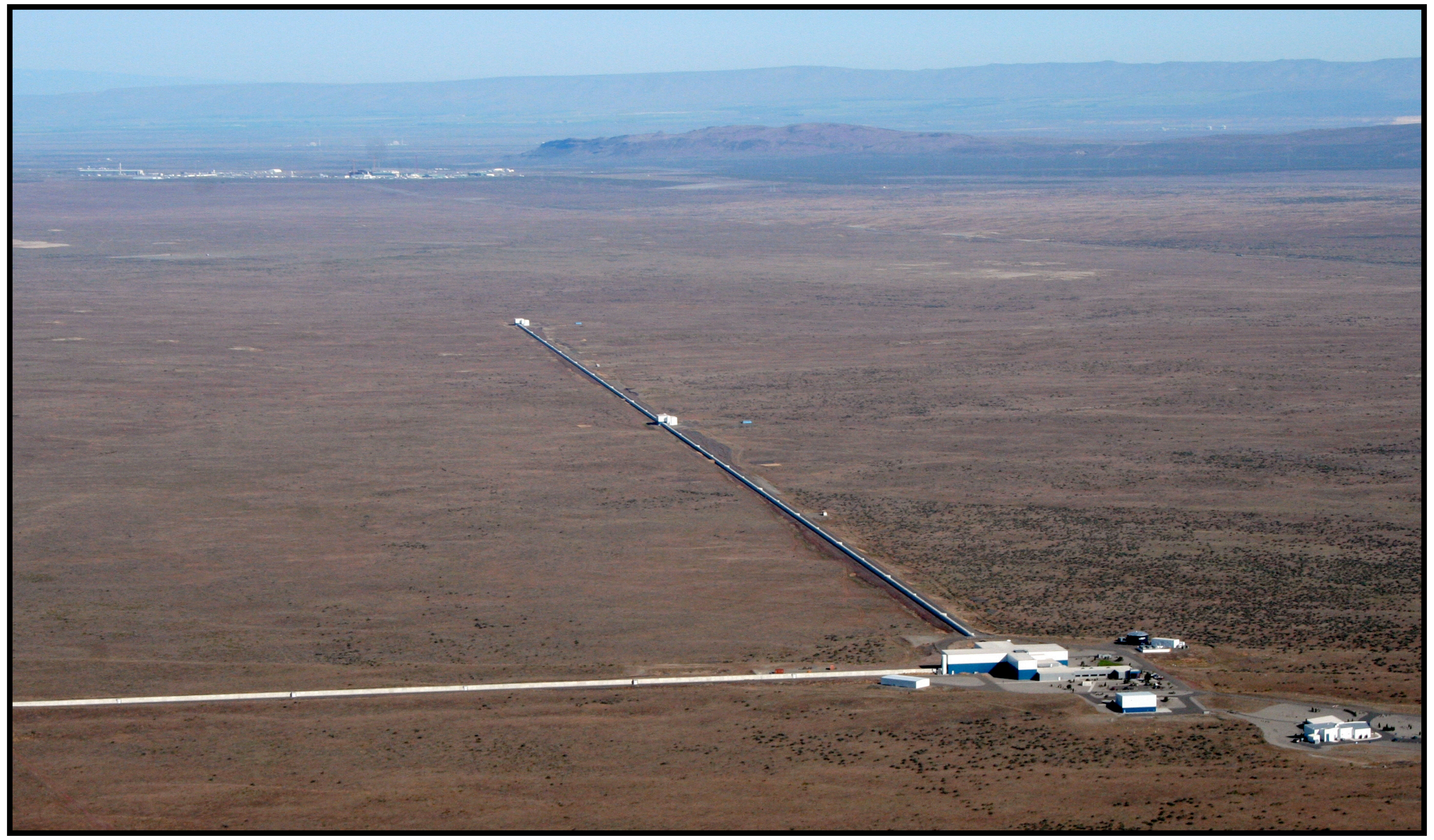
Gravitational wave physics

- In 2015, first direct observation of gravitational waves by the LIGO detectors
- In the future, new detectors promise increased precision which must be matched by theory computations

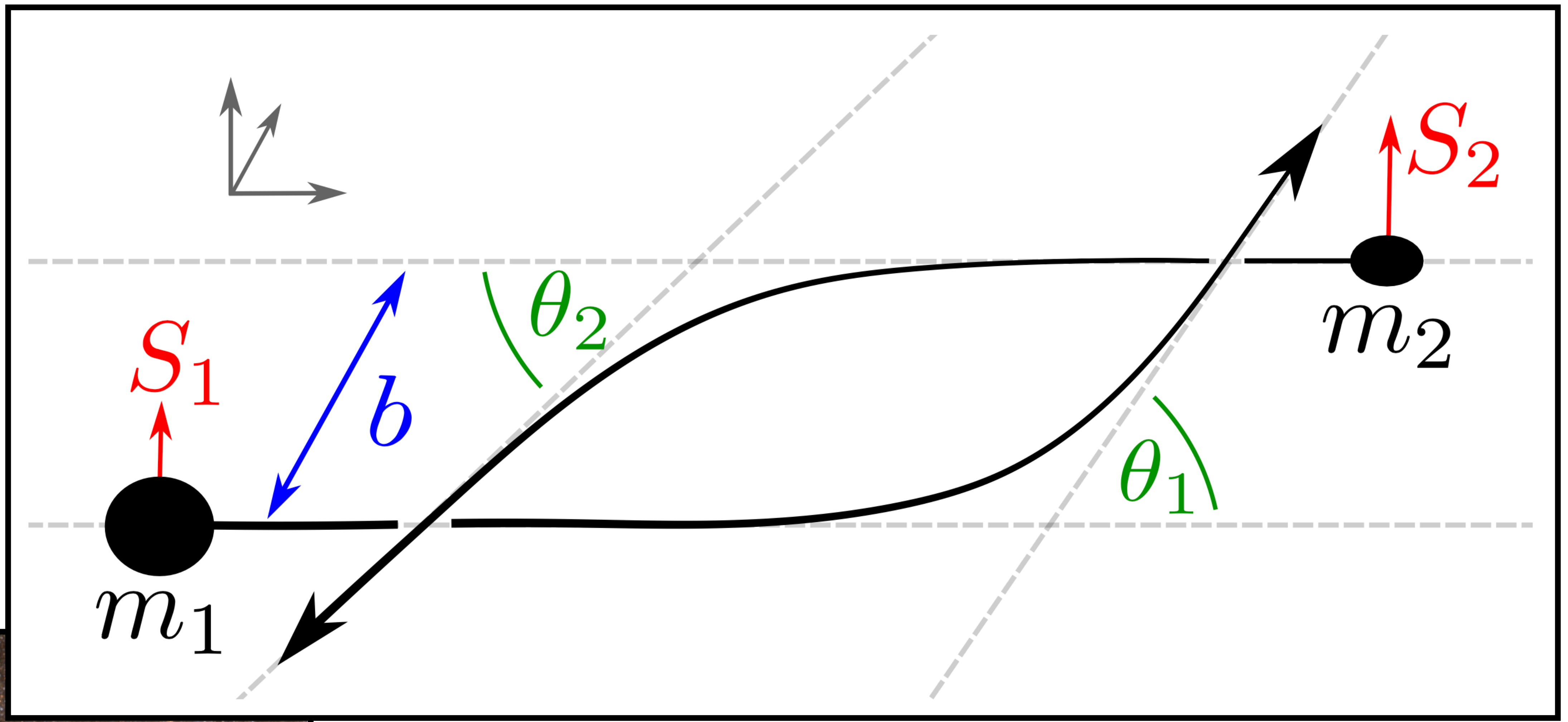


Gravitational strains of GW150914 from [PRL 116, 061102 (2016)]



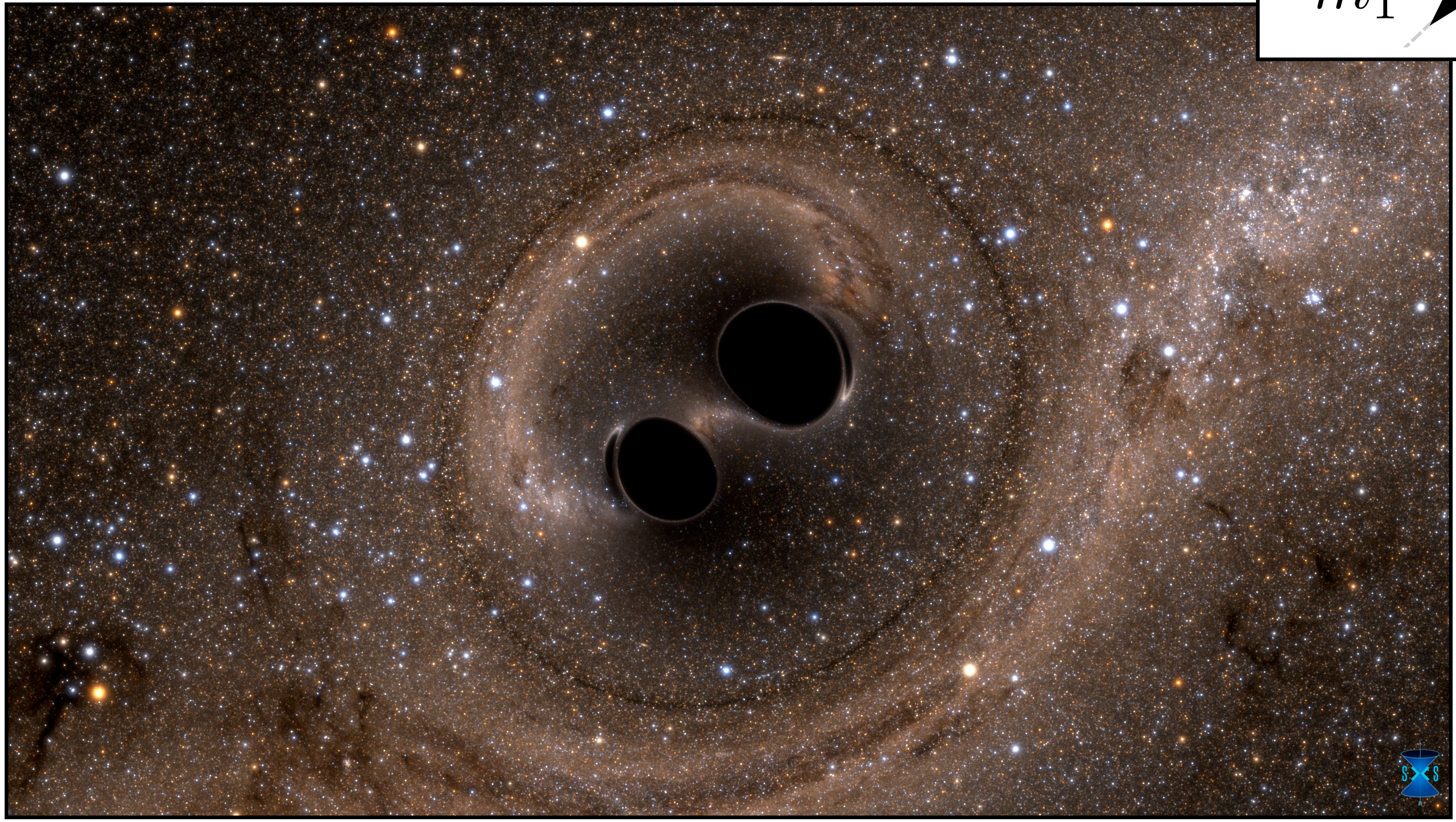
The LIGO Hanford observatory (credit: LIGO Lab/Caltech/MIT)

General relativistic dynamics



(Unbound) Scattering

(Bound) Compact binary mergers



Credit: SXS, the Simulating eXtreme Spacetimes (SXS) project

Compact bodies

Black holes

Neutron stars

Einstein,
Kovacs, Thorne, Crowley,
Damour,
…

(Classical) General relativity (GR)

Feynman,
DeWitt,
Donoghue,
…

(Effective) Quantum gravity

Worldline effective field theory

Goldberger, Rothstein,
Levi, Steinhoff,
Dlapa, Kälin, Liu, Porto,
Saketh, Vines, Buonanno,
Mougiakakos, Riva, Vernizzi, Wong,
…

Scattering amplitudes

Bern, Cheung, Roiban, Shen, Solon, Zeng,
Bjerrum-Bohr, Damgaard, Planté, Vanhove,
Di Vecchia, Heissenberg, Russo, Veneziano,
Kosower, Maybee, O'Connell,
…

Worldline quantum field theory (WQFT)

GUJ, Mogull, Plefka, Sauer, Steinhoff,
Comberiati, Shi,
Bastianelli, de la Cruz,
Wang

List of publications

[1] G. U. Jakobsen, G. Mogull, J. Plefka and J. Steinhoff, “*Classical Gravitational Bremsstrahlung from a Worldline Quantum Field Theory*”, Phys. Rev. Lett. 126 (2021) 201103

[2] G. U. Jakobsen, G. Mogull, J. Plefka and J. Steinhoff, “*Gravitational Bremsstrahlung and Hidden Supersymmetry of Spinning Bodies*”, Phys. Rev. Lett. 128 (2022) 011101

[3] G. U. Jakobsen, G. Mogull, J. Plefka and J. Steinhoff, *“SUSY in the Sky with Gravitons”,* JHEP 01 (2022) 027

[4] G. U. Jakobsen and G. Mogull, “*Conservative and Radiative Dynamics of Spinning Bodies at Third Post-Minkowskian Order Using Worldline Quantum Field Theory*”, Phys. Rev. Lett. 128 (2022) 141102

[5] G. U. Jakobsen, G. Mogull, J. Plefka and B. Sauer, “*All Things Retarded: Radiation-Reaction in Worldline Quantum Field Theory*”, JHEP 10 (2022) 128

[6] G. U. Jakobsen and G. Mogull, “*Linear Response, Hamiltonian and Radiative Spinning Two-Body Dynamics*”, Phys. Rev. D 107 (2023) 044033

Outline

- Worldline Quantum Field Theory (WQFT) formalism
- Worldline observables at two-loop order:
  - Quantum Field Theory (QFT) integration techniques
  - Perturbative results at two loops and quadratic in spins
- Conclusions and outlook

Worldline quantum field theory (WQFT)

- Effective field theory of compact bodies as point particles
- Path integral on worldline and gravitational fields
- Tree diagrams → classical physics

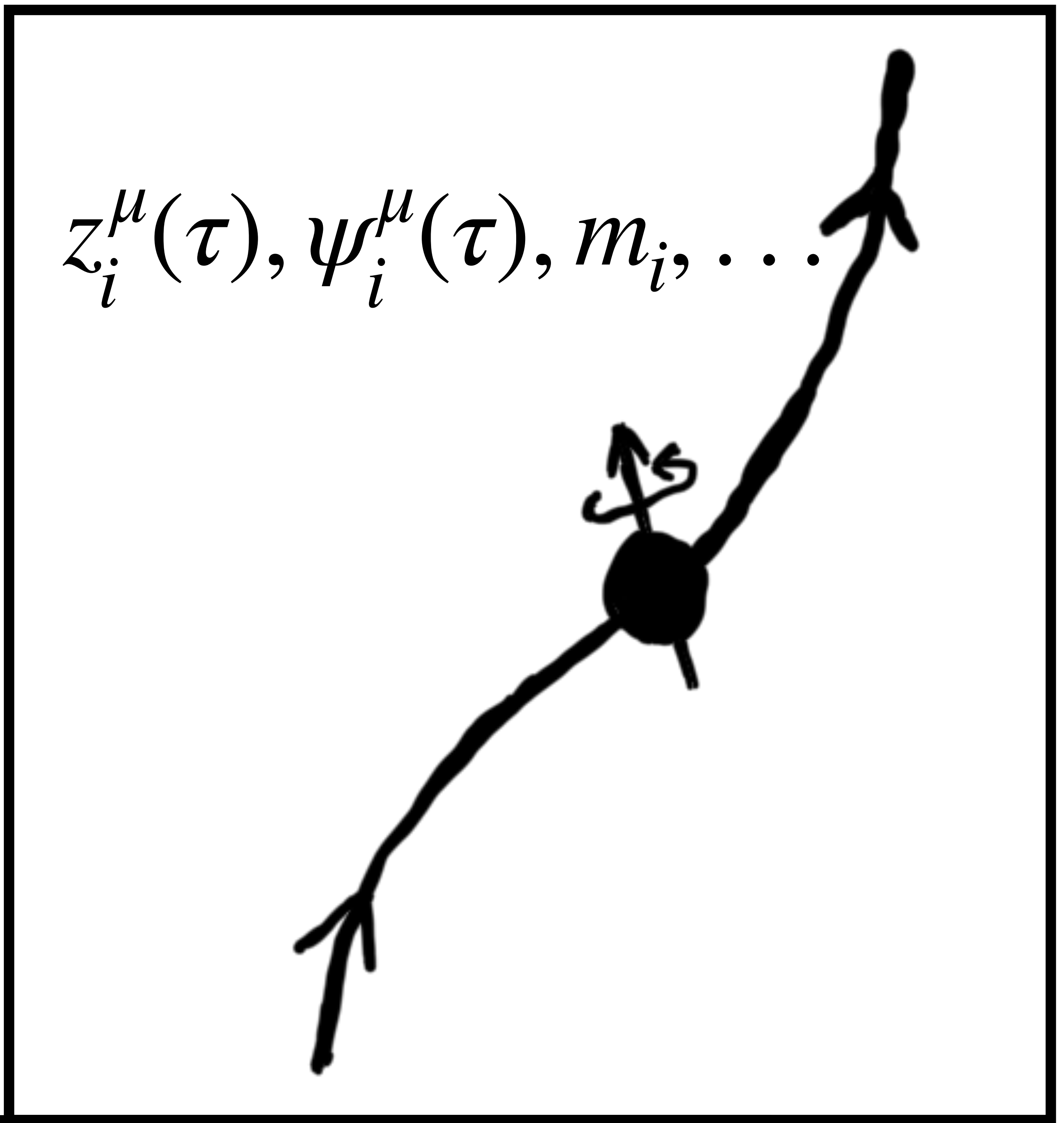
$$\langle X \rangle = \int \mathcal{D}[w(\tau), g_{\mu\nu}(x)] X e^{-iS/\hbar}$$

$$X = g_{\mu\nu}(k^\sigma) \quad \rightarrow \quad k^2 \epsilon^\mu \epsilon^\nu \langle g_{\mu\nu}(k^\sigma) \rangle \Big|_{k^2 \to 0}$$

$$X = w(\omega) \quad \rightarrow \quad \omega^2 \langle w(\omega) \rangle \Big|_{\omega \to 0}$$

Spinning point particles [2,3,6]

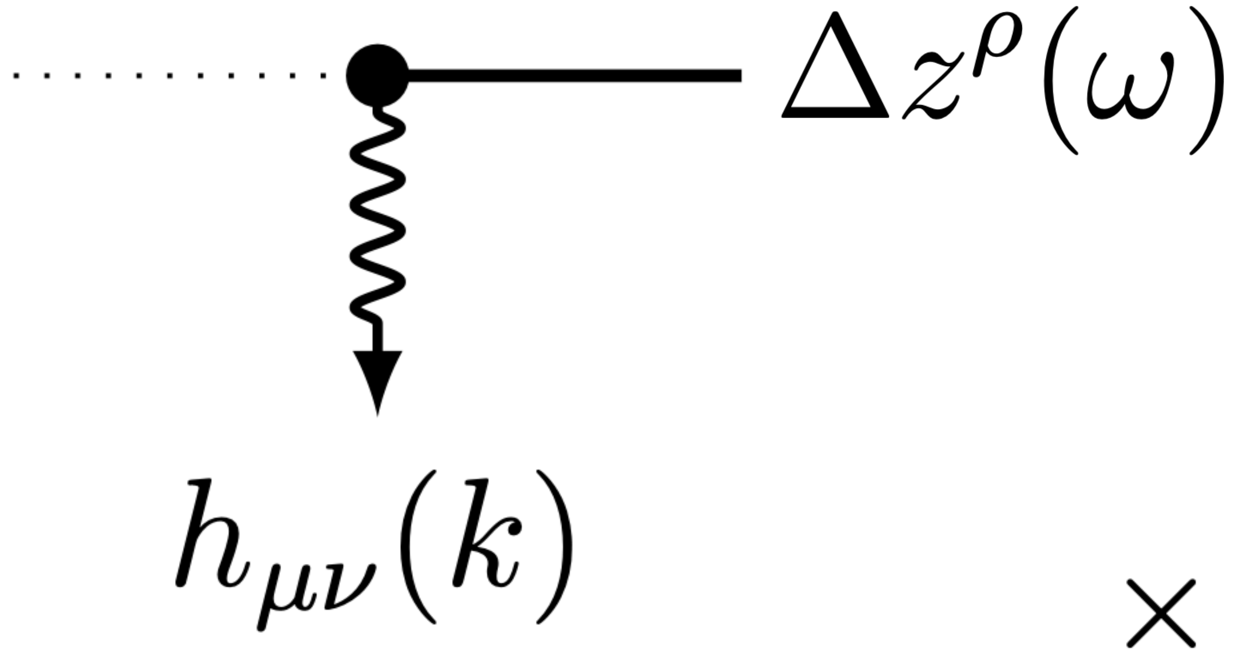
- Worldline trajectory $z_i^\mu(\tau)$ and Grassmann field $\psi_i^\mu(\tau)$
- Physical spin vector $S_i^\mu = -\, i m_i (\psi_i \times \psi_i)^\mu$
- Supersymmetry with $\delta z_i^\mu \sim \eta \psi_i^\mu$ and $\delta \psi_i^\mu \sim \eta \dot{z}_i^\mu$



$$S_i = -\frac{m_i}{2}\int d\tau\, g_{\mu\nu}\Big[\dot{z}_i^\mu\, \dot{z}_i^\nu + i\psi_i^\mu \frac{D\psi_i^\nu}{d\tau} + \frac{1}{8m_i} R_{\mu\nu\alpha\beta} S_i^{\mu\nu} S_i^{\alpha\beta} + \frac{C_{E,i}}{2m_i} R_{\alpha\mu\beta\nu} \dot{z}_i^\mu \dot{z}_i^\nu S_i^{\alpha\rho} S_{i\ \rho}^{\beta} + \ldots\Big]$$

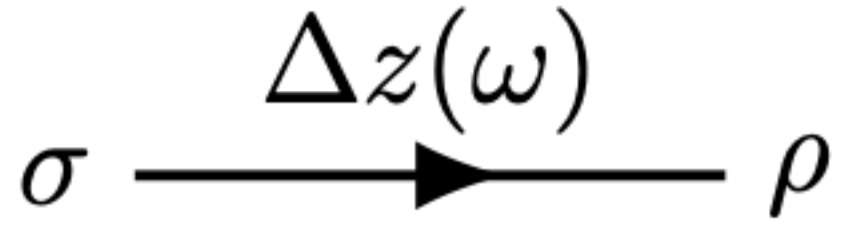
WQFT Feynman rules

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \sqrt{G_N} h_{\mu\nu}(x)$$
$$z^\mu(\tau) = b^\mu + \tau v^\mu + \Delta z^\mu(\tau)$$
$$\psi^\mu(\tau) = \Psi^\mu(\tau) + \Delta\psi^\mu(\tau)$$

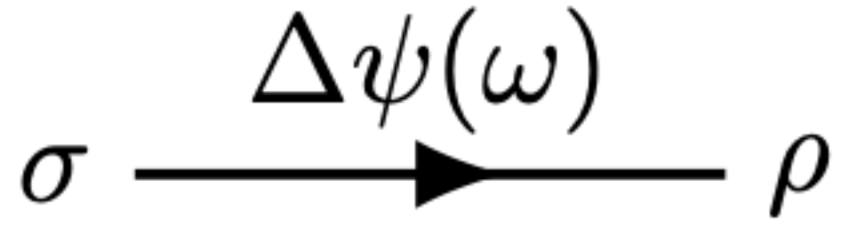


$$= \frac{m\kappa}{2} e^{ik\cdot b} \delta\!\!\!{}^-(k\cdot v + \omega) \times \left(k_\rho v^\mu v^\nu + 2\omega v^{(\mu}\delta^{\nu)}_\rho - i\frac{1}{m}(k\cdot S)^{(\mu}\left(k_\rho v^{\nu)} + \omega\delta^{\nu)}_\rho\right) + \ldots\right)$$

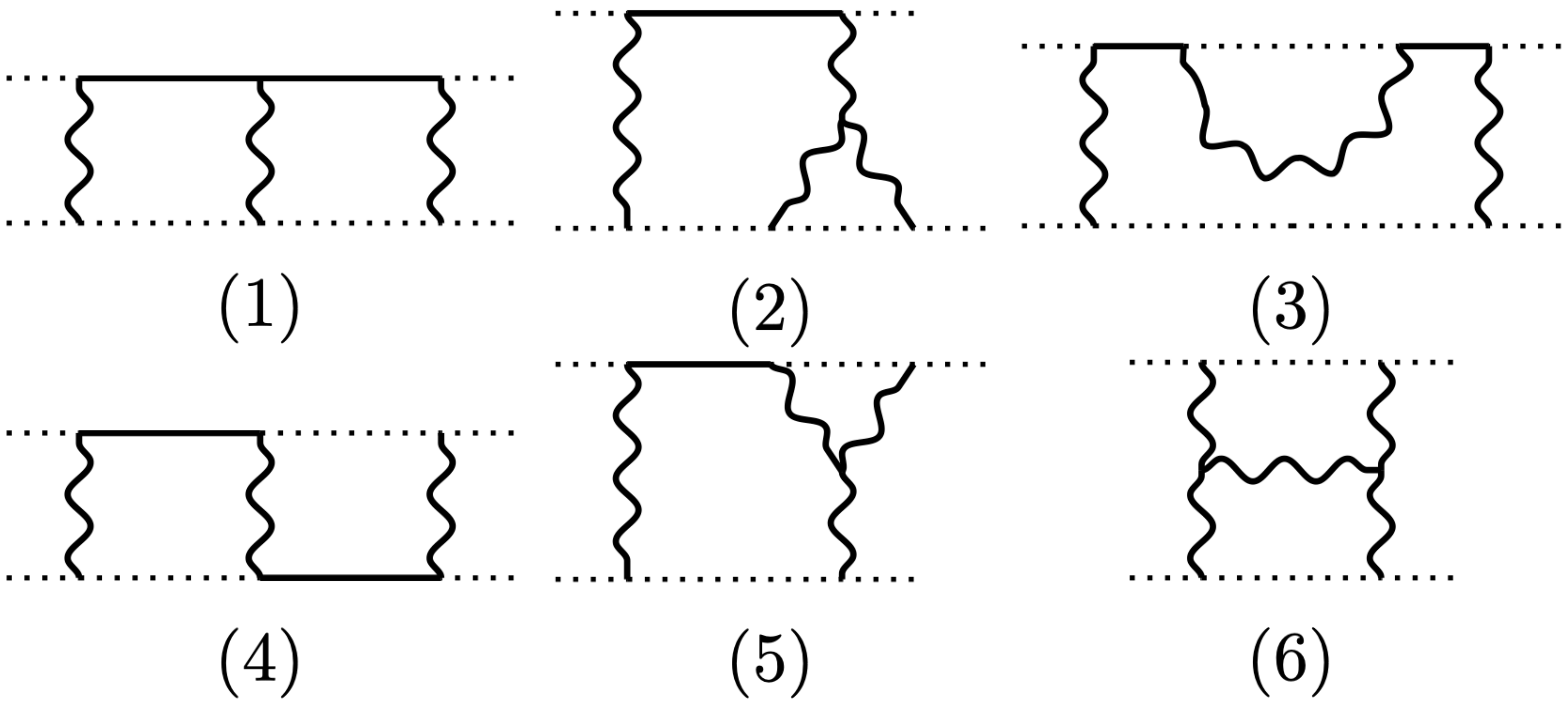
$$= i\frac{\mathcal{P}^{-1}_{\alpha\beta\mu\nu}}{(k^0 + i\epsilon)^2 - \boldsymbol{k}^2}\ ,$$



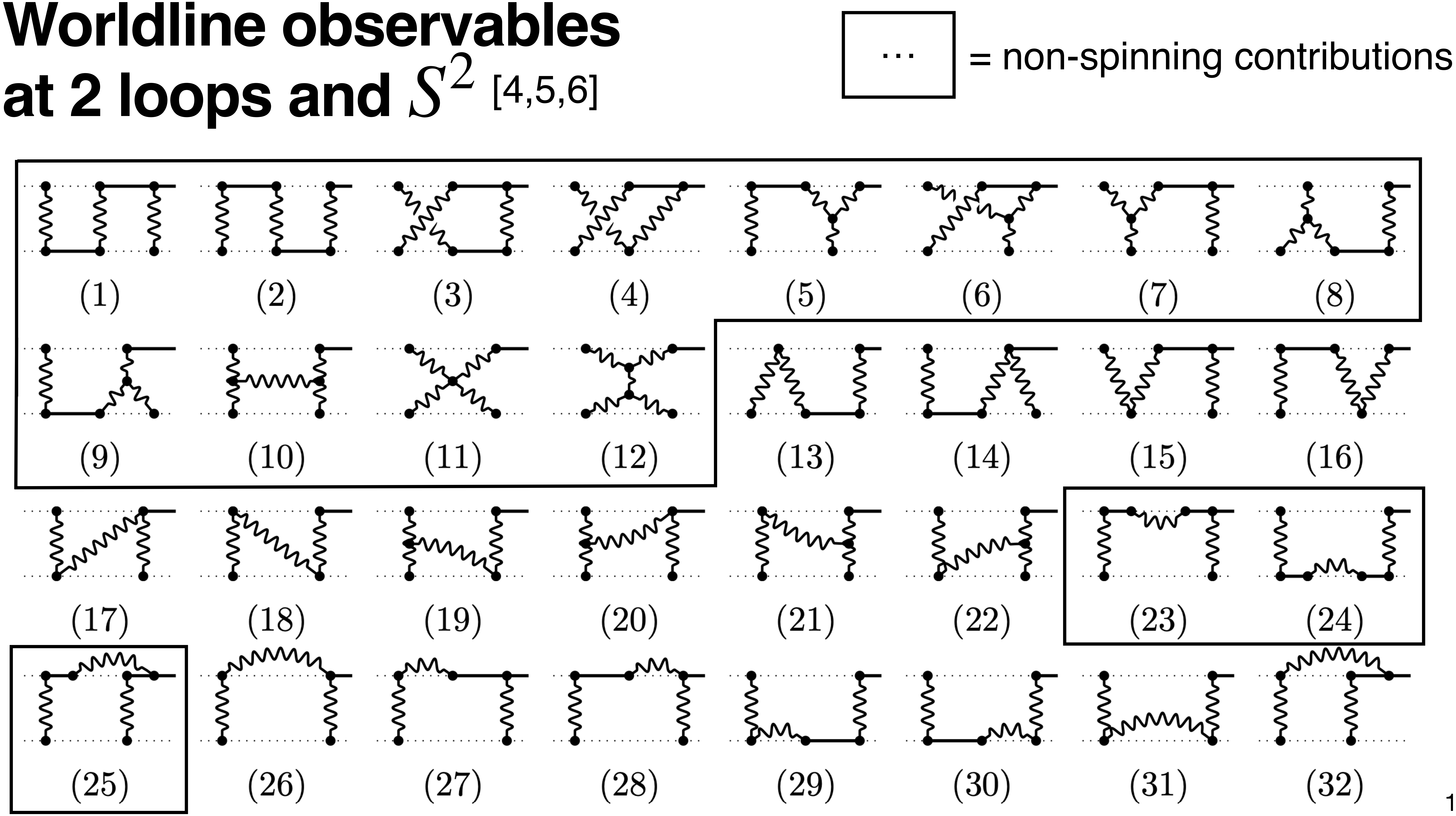
$$= -\frac{i\eta_{\rho\sigma}}{m}\frac{1}{(\omega + i\epsilon)^2}\ ,$$



$$= -\frac{i\eta_{\rho\sigma}}{m}\frac{1}{\omega + i\epsilon}\ .$$



Worldline observables at 2 loops and S^2 [4,5,6]



QFT loop integration

- Tensor reduction
- Integration-by-parts reduction
- Differential Equations
- Method of Regions

$$\int d^{d-1}\boldsymbol{\ell}_1 d^{d-1}\boldsymbol{\ell}_2 \frac{\boldsymbol{\ell}_1^{i_1}\ldots\boldsymbol{\ell}_1^{i_m}\boldsymbol{\ell}_2^{j_1}\ldots\boldsymbol{\ell}_2^{j_l}}{D_1^{n_1}D_2^{n_2}D_3^{n_3}D_4^{n_4}D_5^{n_5}D_6^{n_6}D_7^{n_7}}$$

$$D_1 = \boldsymbol{\ell}_1\cdot\hat{\boldsymbol{e}} \qquad D_2 = \boldsymbol{\ell}_1^2 \qquad D_3 = (\boldsymbol{\ell}_1 - \hat{\boldsymbol{q}})^2$$

$$D_4 = \boldsymbol{\ell}_2\cdot\hat{\boldsymbol{e}} \qquad D_5 = \boldsymbol{\ell}_2^2 \qquad D_6 = (\boldsymbol{\ell}_2 - \hat{\boldsymbol{q}})^2$$

$$D_7 = (\boldsymbol{\ell}_1 - \boldsymbol{\ell}_2)^2 - 2(\gamma-1)\boldsymbol{\ell}_1\cdot\hat{\boldsymbol{e}}\,\boldsymbol{\ell}_2\cdot\hat{\boldsymbol{e}}$$

$$\hat{\boldsymbol{e}}^2 = \hat{\boldsymbol{q}}^2 = 1$$

$$\hat{\boldsymbol{e}}\cdot\hat{\boldsymbol{q}} = 0$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Post-Minkowskian spinning worldline observables at two loops [4,6]

- Complete post-Minkowskian conservative and dissipative results for Δp_i^μ and ΔS_i^μ at two loops and $\mathcal{O}(S^2)$.
- Two-body Hamiltonian $H(\mathbf{x}, \mathbf{p}, \mathbf{S_i})$ for generic spins and bound motion.
- Post-Newtonian (PN) expansion provides first independent check of NNLO PN spinning Hamiltonian [Levi, Steinhoff: 1506.05794].
- State-of-the-art, partly verified by other groups [Alessio, Di Vecchia: 2203.13272; Cordero, Kraus, Lin, Ruf, Zeng: 2205.07357; Riva, Vernizzi, Wong: 2205.15295]

Scattering Angle

$$\theta_{\text{cons}} = \Gamma \sum_{n,m} \left(\frac{G_N M}{b} \right)^n \frac{1}{b^m} \theta_{\text{cons}}^{(n,m)}$$

$$\theta_{\text{cons}}^{(3,0)} = 2\frac{64\gamma^6 - 120\gamma^4 + 60\gamma^2 - 5}{3(\gamma^2-1)^3}\Gamma^2 - \frac{8\nu\gamma(14\gamma^2+25)}{3(\gamma^2-1)} - 8\nu\frac{4\gamma^4 - 12\gamma^2 - 3}{(\gamma^2-1)^{3/2}}\text{arccosh}\gamma \ ,$$

$$\theta_{\text{cons}}^{(3,1)} = 2\gamma\frac{16\gamma^4 - 20\gamma^2 + 5}{(\gamma^2-1)^{5/2}}(5\Gamma^2 s_+ - \delta s_-) - 4\nu s_+ \left(\frac{44\gamma^4 + 100\gamma^2 + 41}{(\gamma^2-1)^{3/2}} + 12\gamma\frac{(\gamma^2-6)(2\gamma^2+1)}{(\gamma^2-1)^2}\text{arccosh}\gamma \right)$$

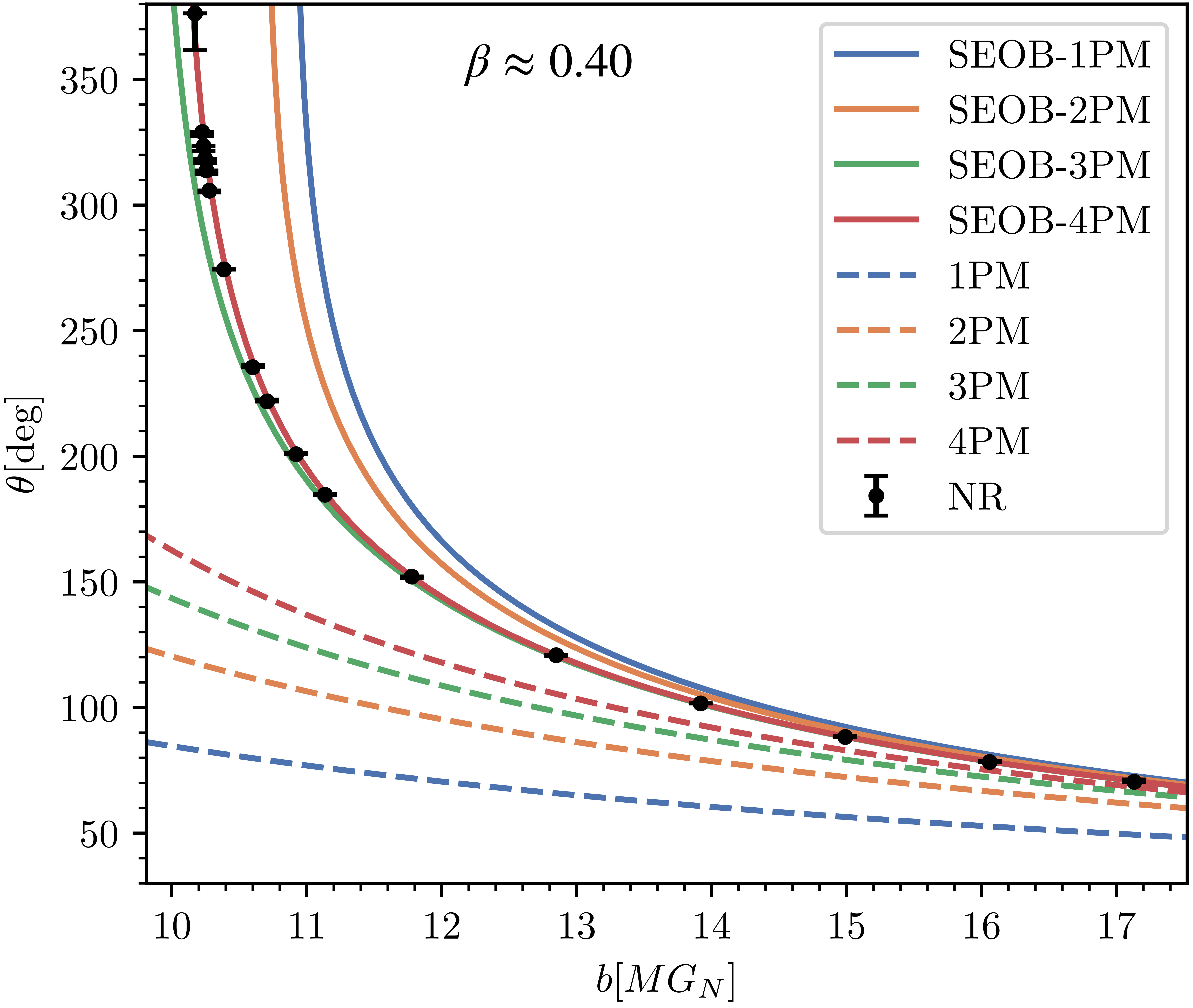
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \ , \ \beta = \text{relativistic velocity} \ ,$$

$$\Gamma^2 = 1 + 2\nu(\gamma - 1) \ ,$$

$$\nu = \frac{m_1 m_2}{M^2} \ , \ \delta = \frac{m_1 - m_2}{M} \ , \ M = m_1 + m_2 \ , \ m_i = \text{masses} \ ,$$

$$s_\pm = s_1 \pm s_2 \ , \ s_i = \text{spin lengths}$$

Outlook



3-loop scattering with spin and tidal effects:

2306.01714 **GUJ**, Mogull, Plefka, Sauer, Xu
2308.11514 **GUJ**, Mogull, Plefka, Sauer
2312.00719 **GUJ**, Mogull, Plefka, Sauer

Electromagnetic **self-force** with spin and polarization effects:

2311.04151 **GUJ**

Post-Minkowskian effective-one-body (EOB) **resummations**:

2402.12342 Buonanno, **GUJ**, Mogull

Conclusions

- The WQFT is an efficient and flexible formalism for computing gravitational scattering observables
- Spin and dissipative effects are naturally incorporated into the WQFT
- Advanced integration techniques allow for high orders in perturbation theory
- State-of-the-art results at two loops and S^2