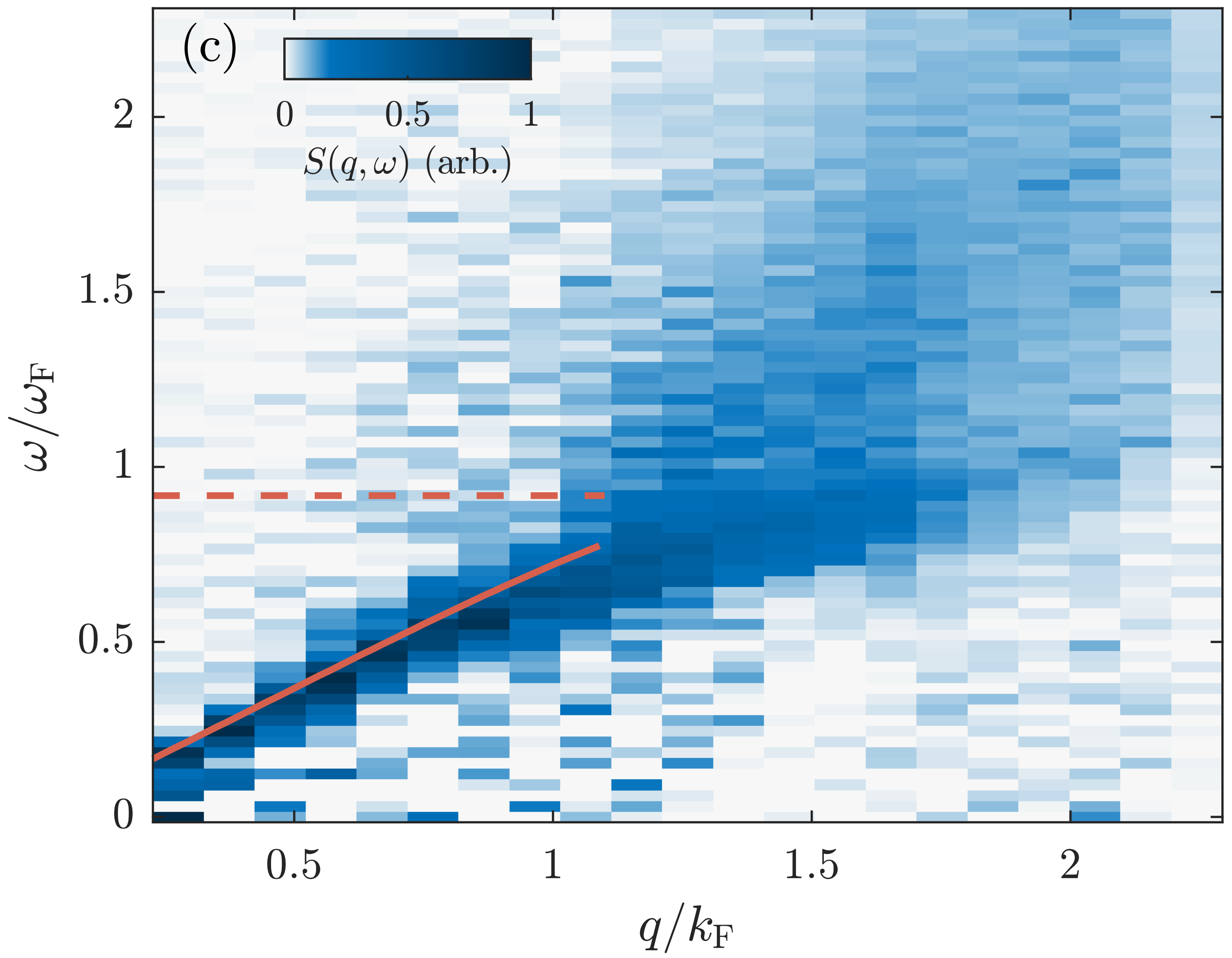
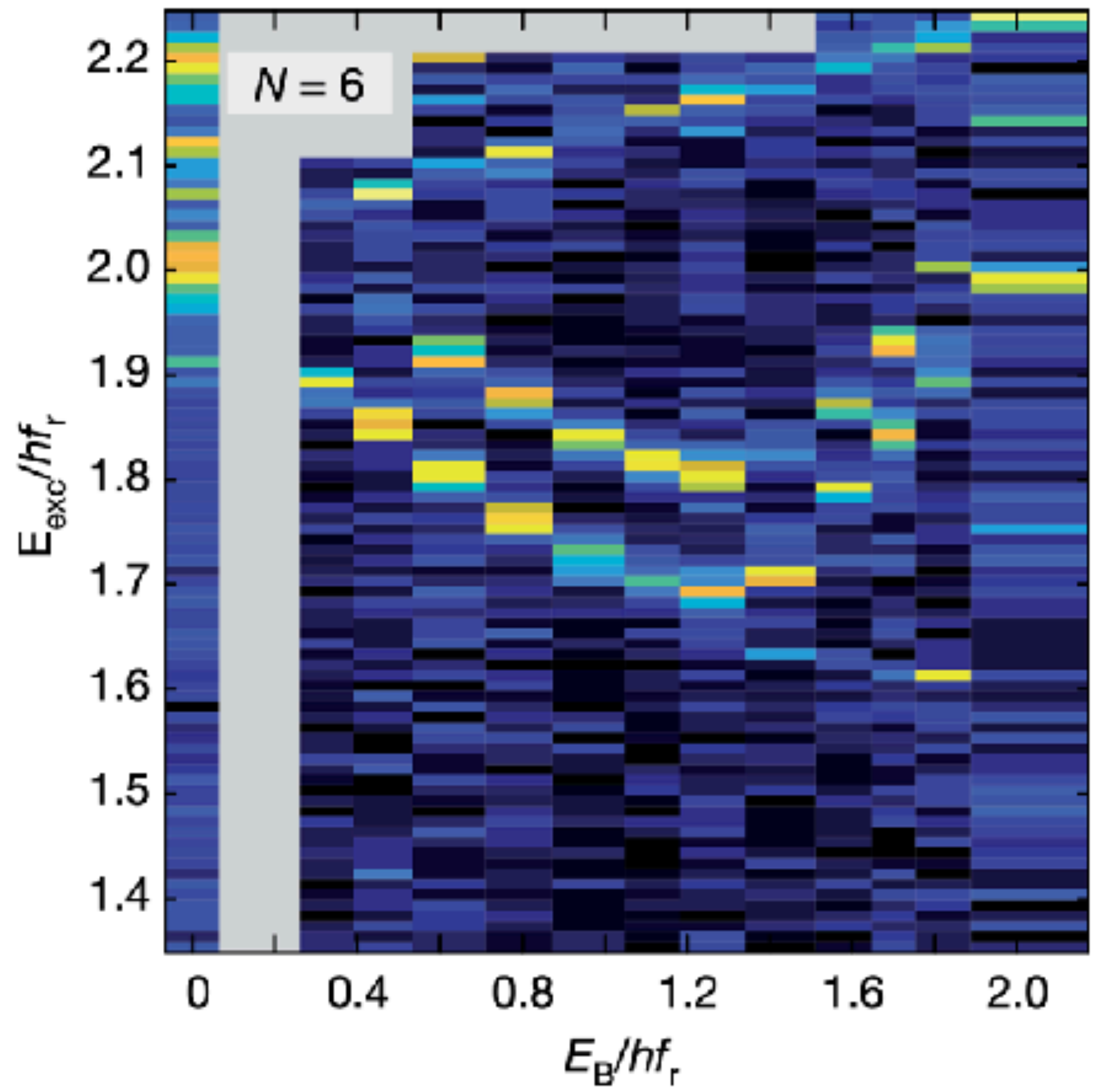
Collective phenomena with ultracold atoms: history and perspectives





Georg M. Bruun
Aarhus University

Heidelberg March 18, 2024

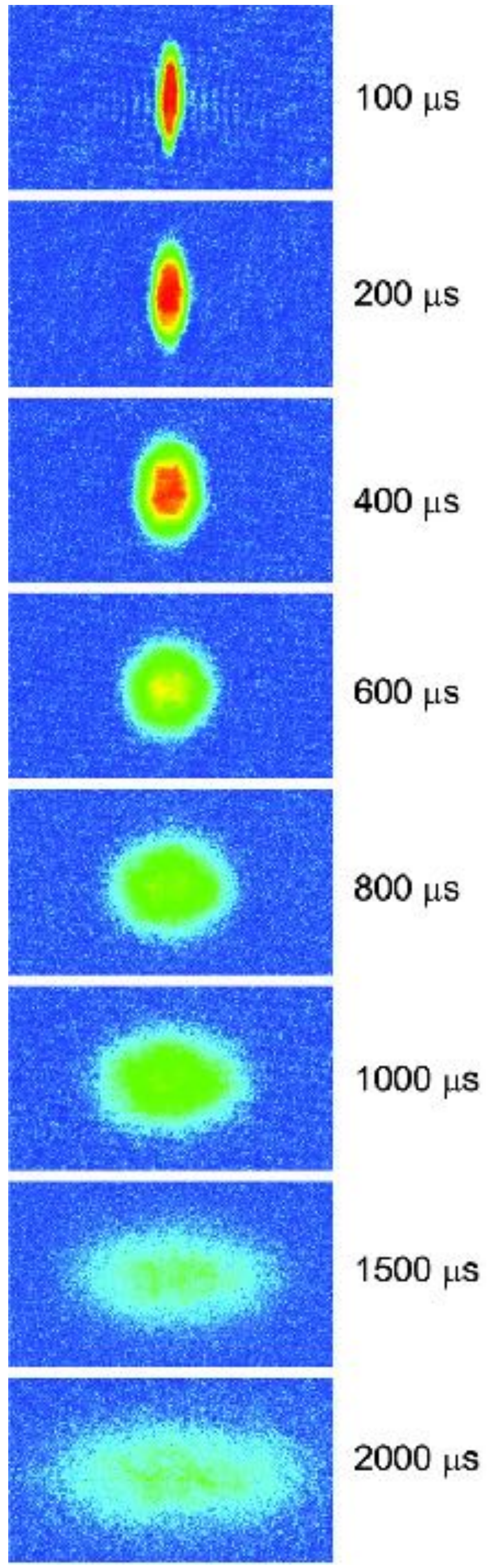
Outline

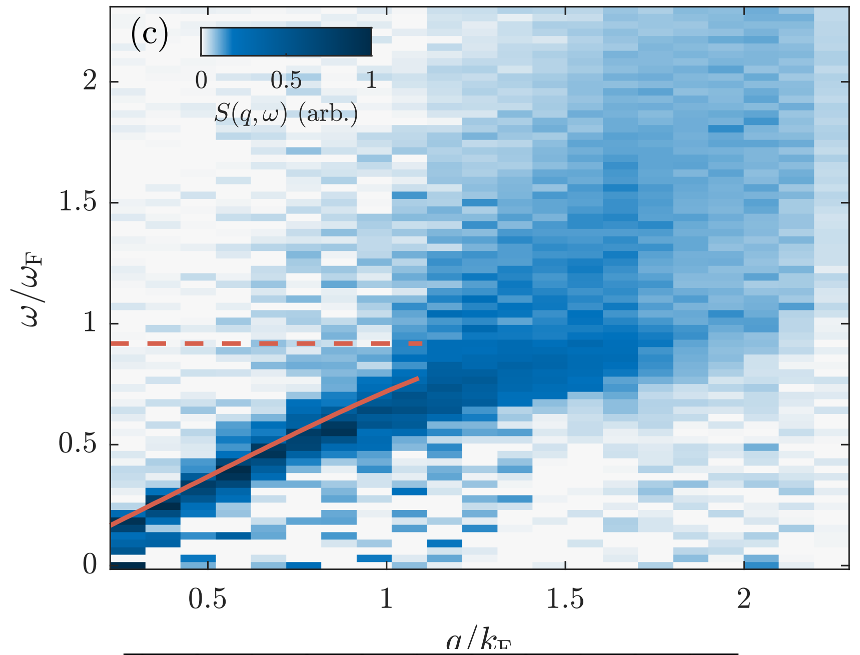
- Emergence of macroscopic dynamics: Hydrodynamics

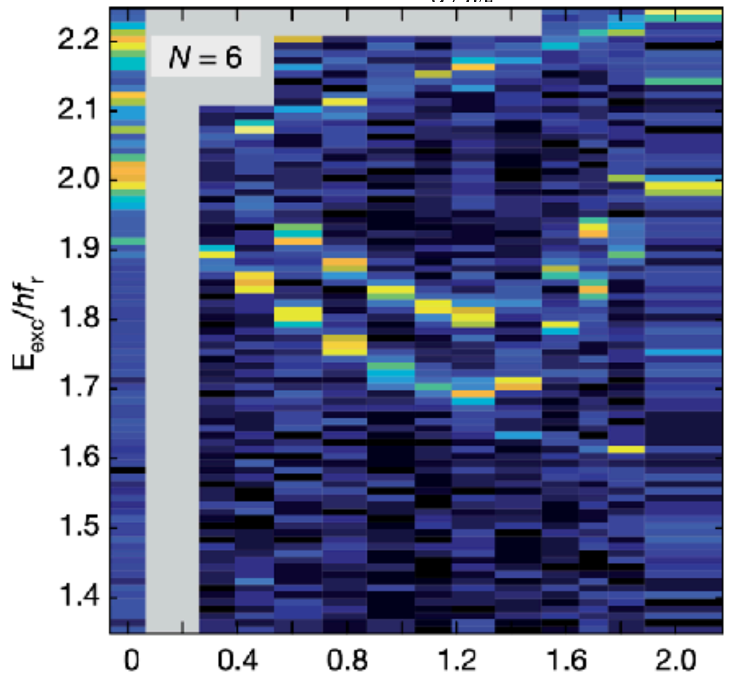
- Expansion and collective modes of unitary Fermi gas

- Goldstone mode in homogeneous Fermi gas

- Pairing and collective modes in a 2D trap

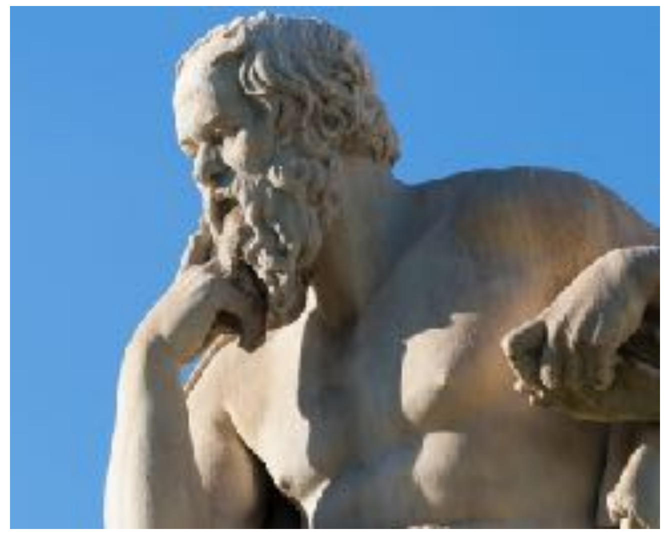






Emergence of macroscopic dynamics

Sorites paradox:

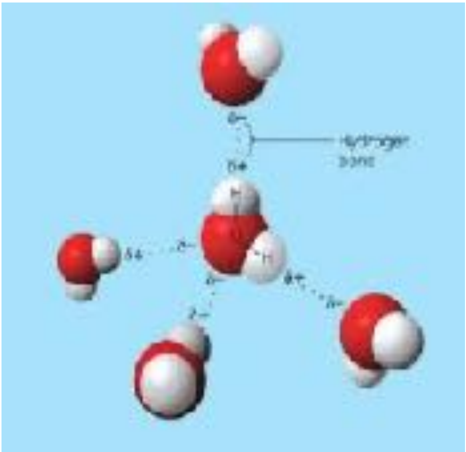


Grain of sand

Pile of sand

P. W. Anderson:
"*More is different*"

Water molecules



?

Water, ice, vapor

Thermodynamic limit:
Effective theories

Pressure P
Temperature T

Hydrodynamics

Slowly varying perturbations $\omega\tau \ll 1$ or $l_{mf}/\lambda \ll 1$

Response determined by conservation laws

- Continuity: $\partial_t \rho + \nabla(\rho \mathbf{v}) = 0$
- Euler: $\partial_t(\rho v_i) = -\partial_k \Pi_{ik}$

$$\Pi_{ik} = P\delta_{ik} + \rho v_i v_k - \eta\left(\partial_i v_k - \partial_k v_i - \frac{2}{3}\delta_{ik}\nabla\cdot\mathbf{v}\right) - \zeta\delta_{ik}\nabla\cdot\mathbf{v}$$

Ideal fluid

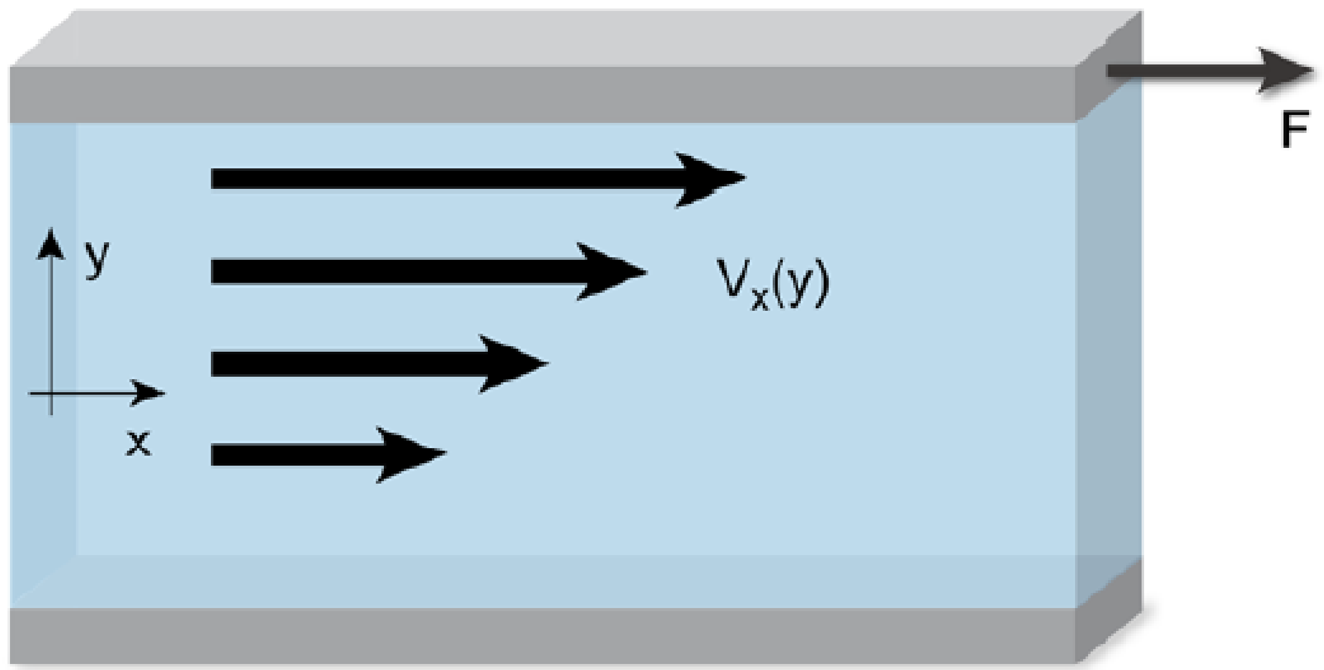
Viscous fluid

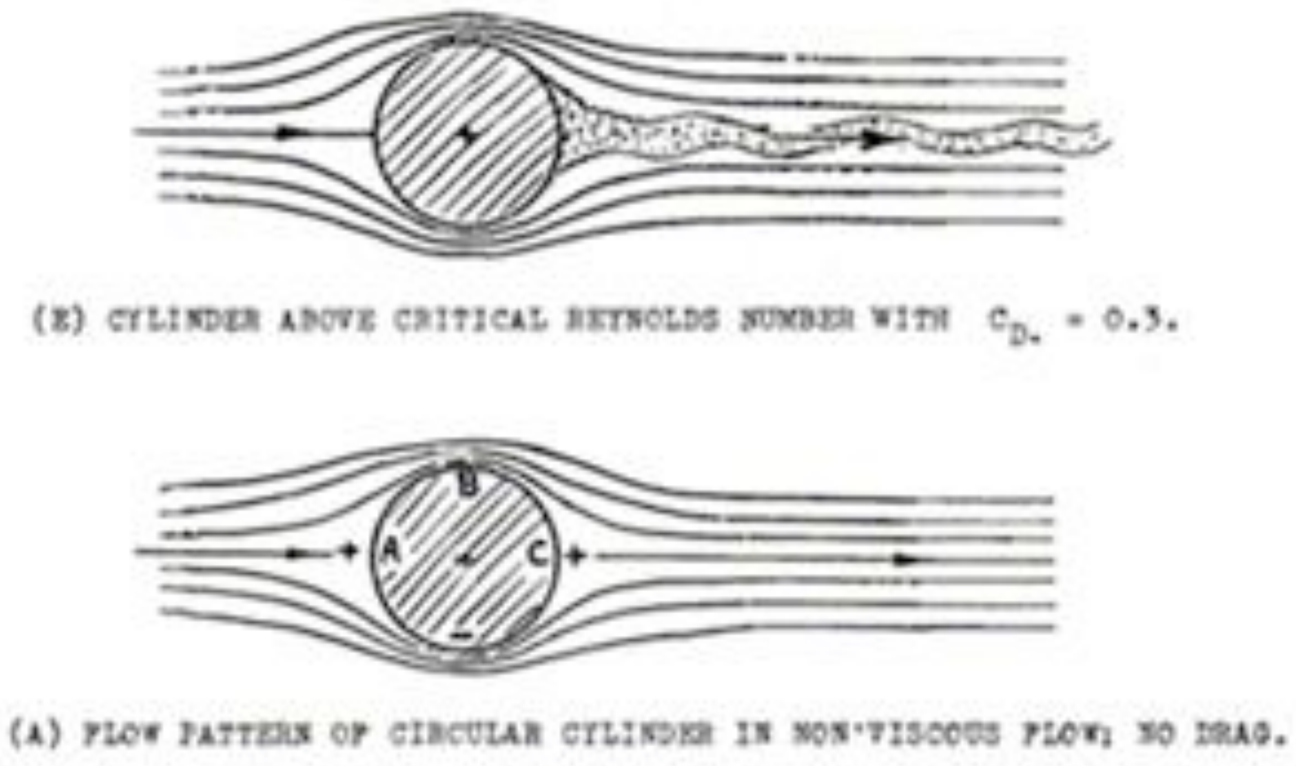
Shear viscosity

Bulk viscosity

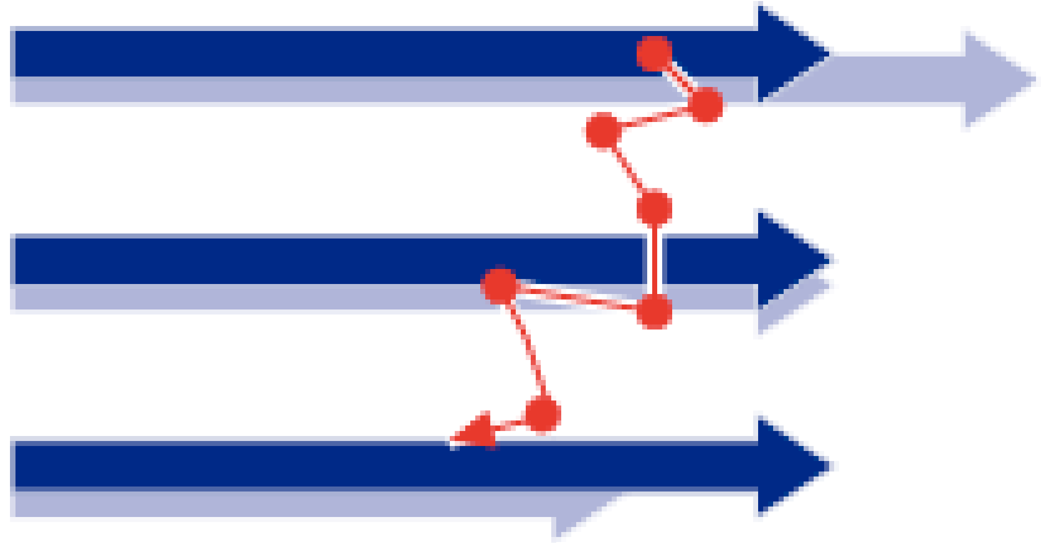
Friction

$$\frac{F}{A} = \eta \partial_y v_x$$





Kinetic picture



Low collision rate → Large viscosity

High collision rate → Small viscosity

$\Delta x \Delta p \geq \hbar/2$

Minimum: $\eta \sim n p l_{mf} \gtrsim \hbar n$

$$s \sim n k_B$$

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

"Perfect fluid"

- Strong interactions
- Quantum system

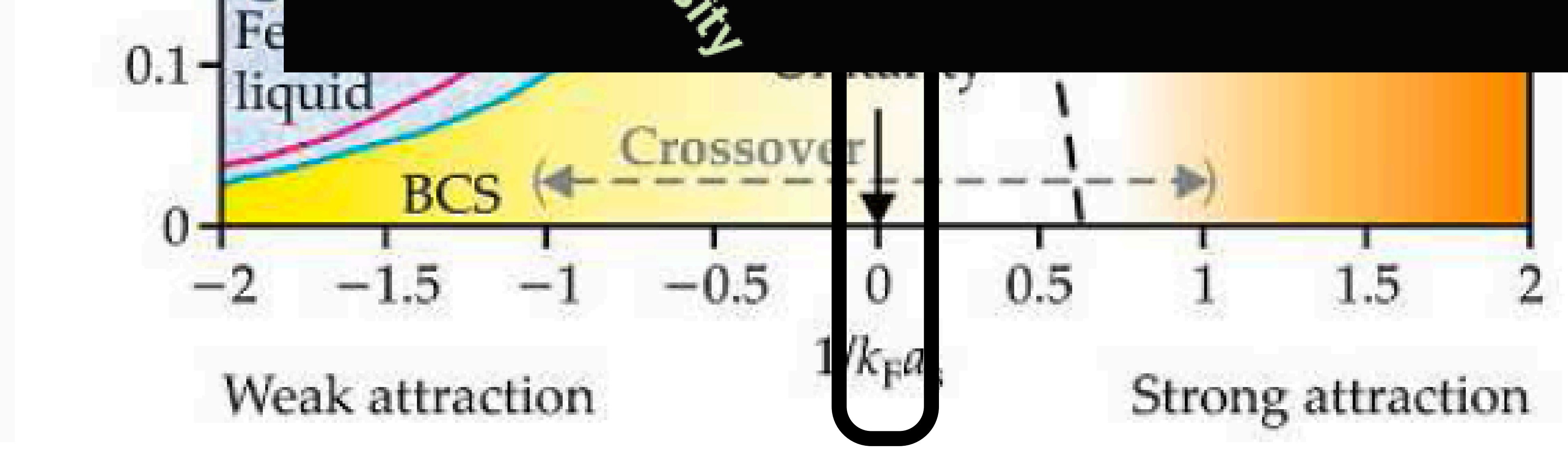
Kovtun *et al.* PRL **94**, 111601 (2005)

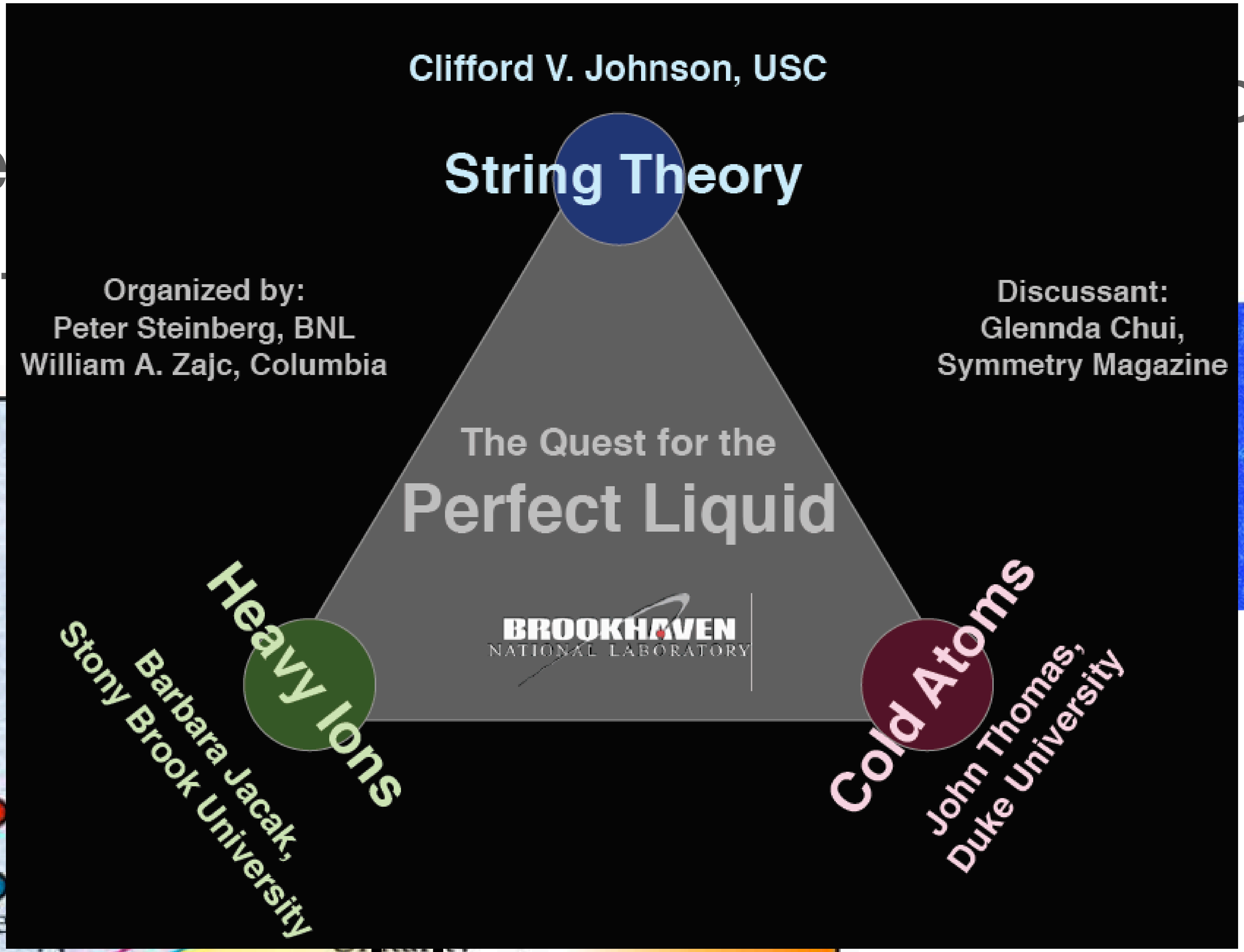
Expansion & collective modes of Fermi gas

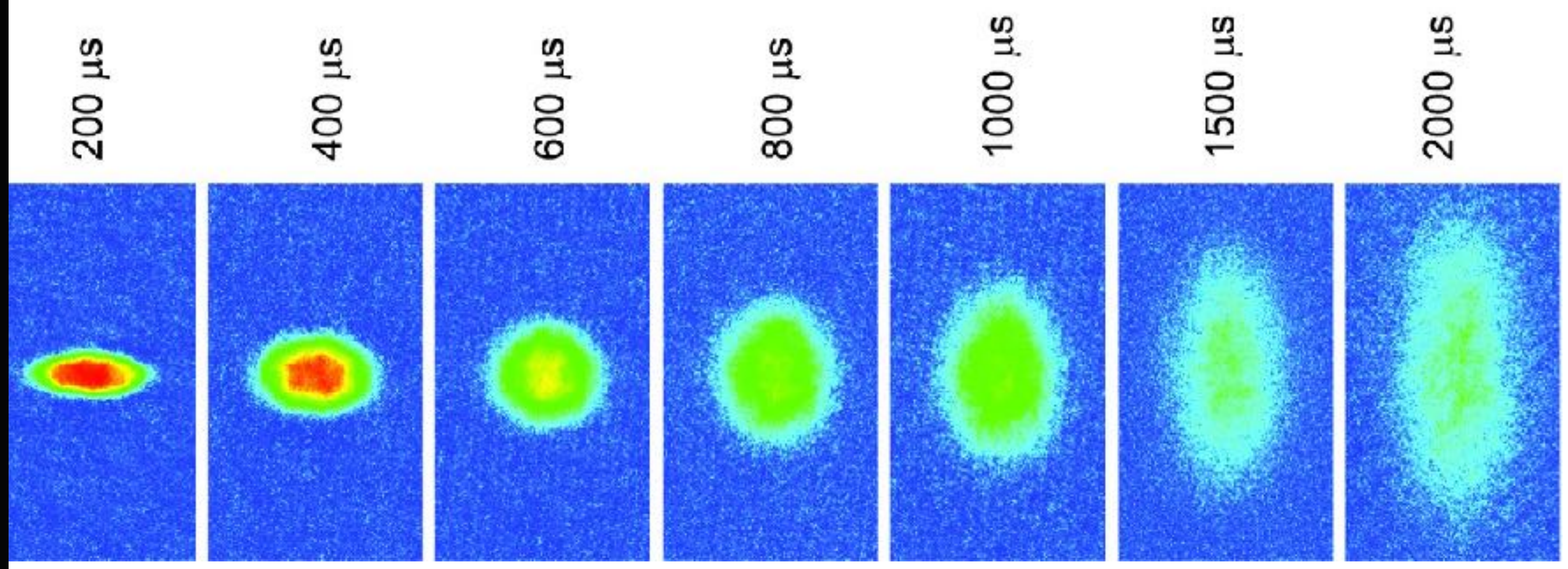
Fe

a

odynamic expansion $\rho\partial_t \mathbf{u} = -\nabla P$



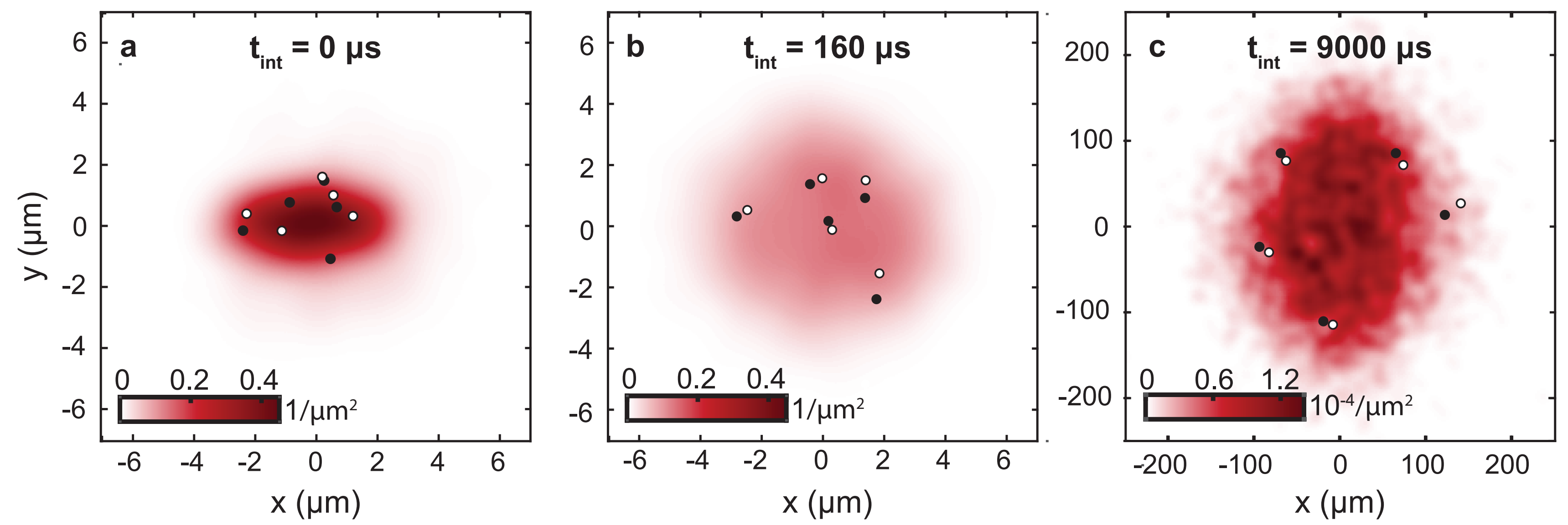




Duke group: Science **298**, 2179 (2002)

10000 thinner than air

10 particles



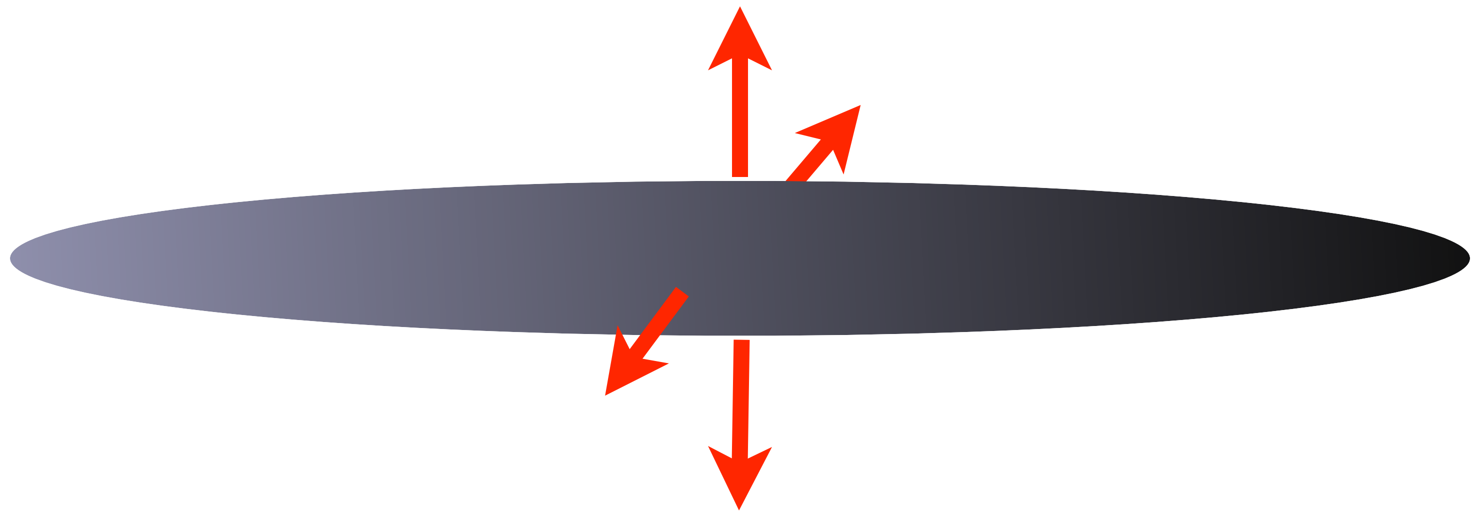
S. Brandstetter *et al.*, arXiv:2308.09699

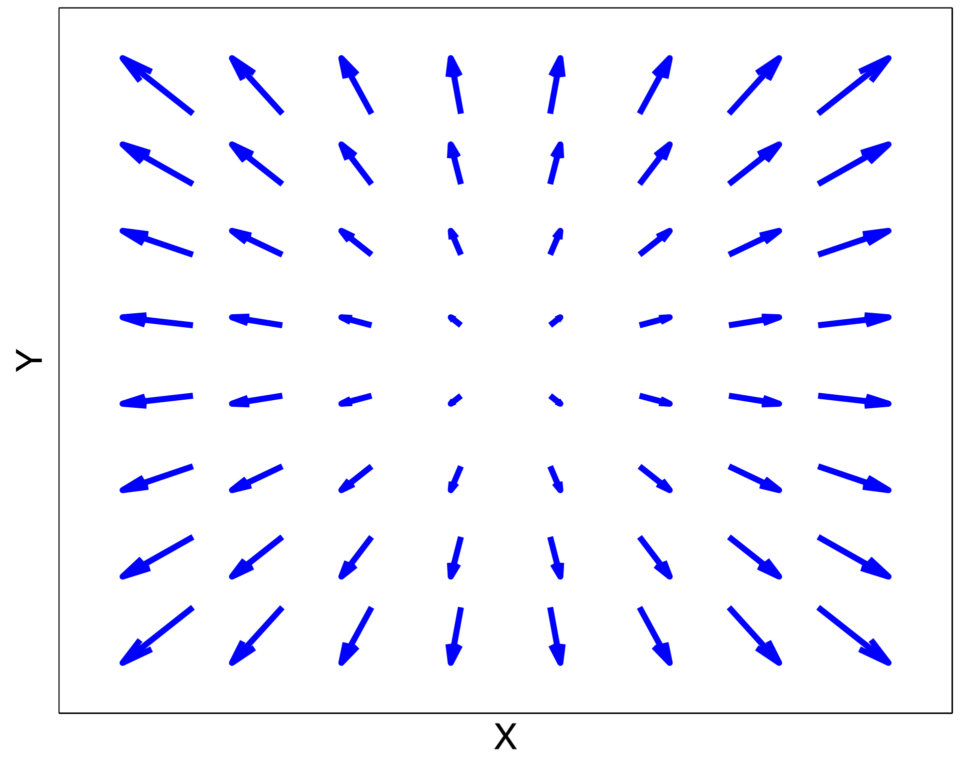
Collective modes (Grimm group)

$$V(x, y, z) = \frac{m}{2}\omega_\perp^2 (x^2 + y^2 + \lambda^2 z^2)$$

$$\lambda \ll 1$$

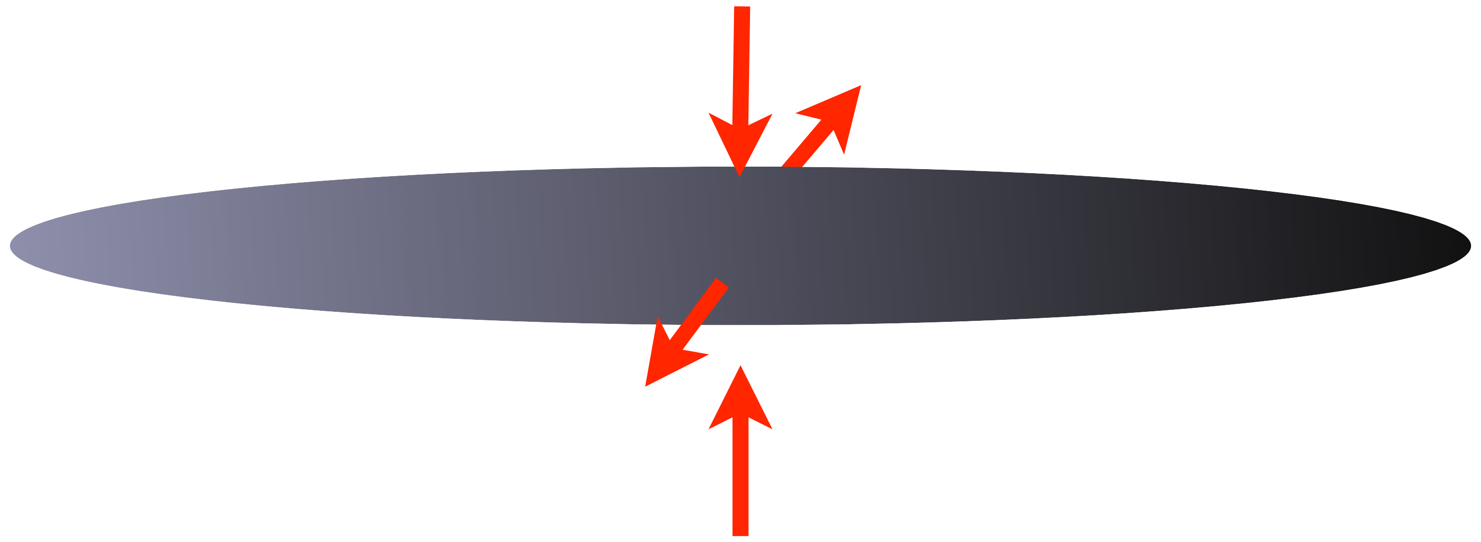
Breathing mode

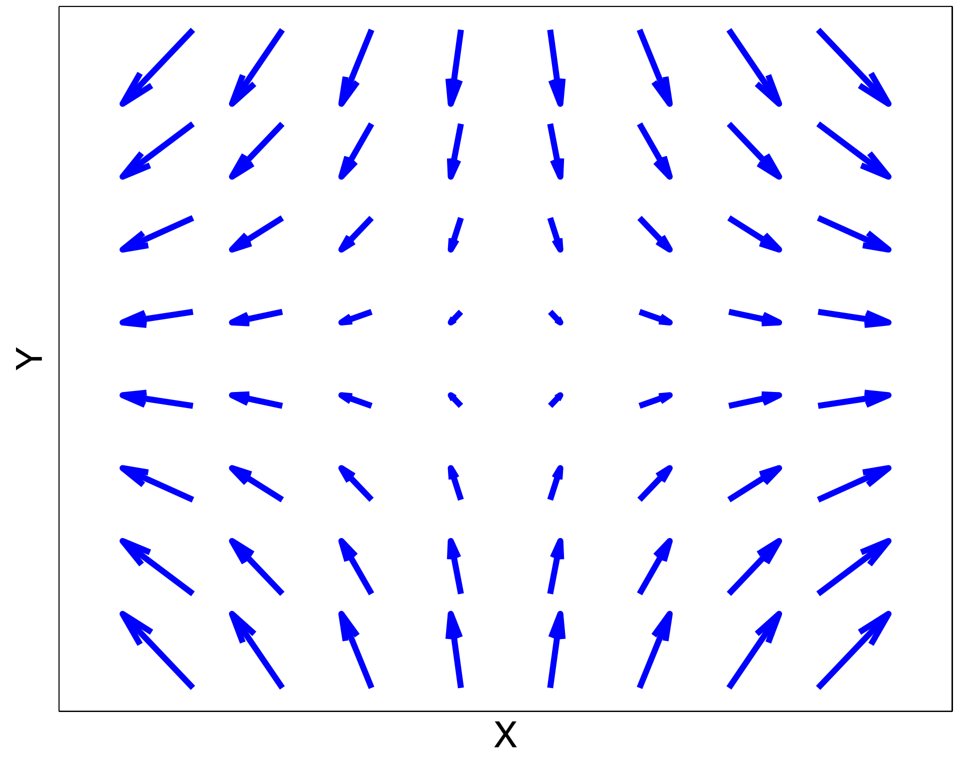




$$\mathbf{v} \propto \begin{pmatrix} x \\ y \end{pmatrix}$$

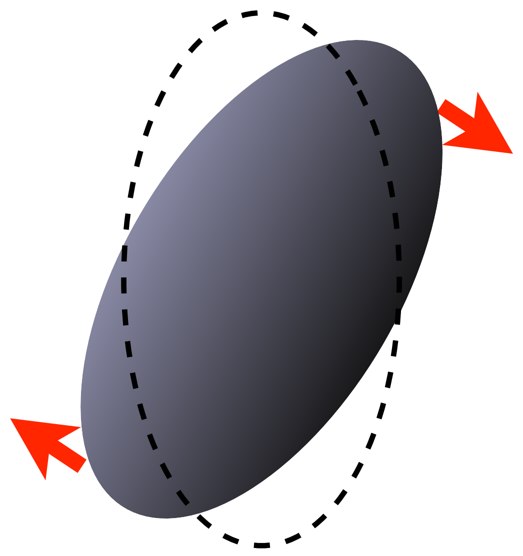
Quadrupole mode

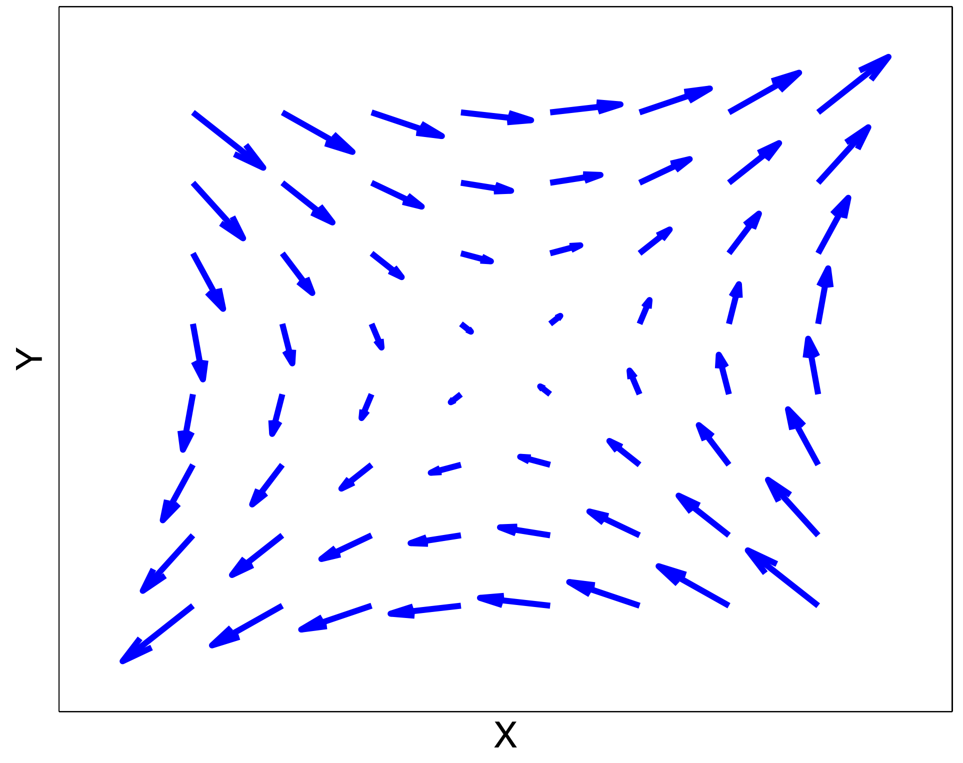




$$\mathbf{v} \propto \begin{pmatrix} x \\ -y \end{pmatrix}$$

Scissors mode



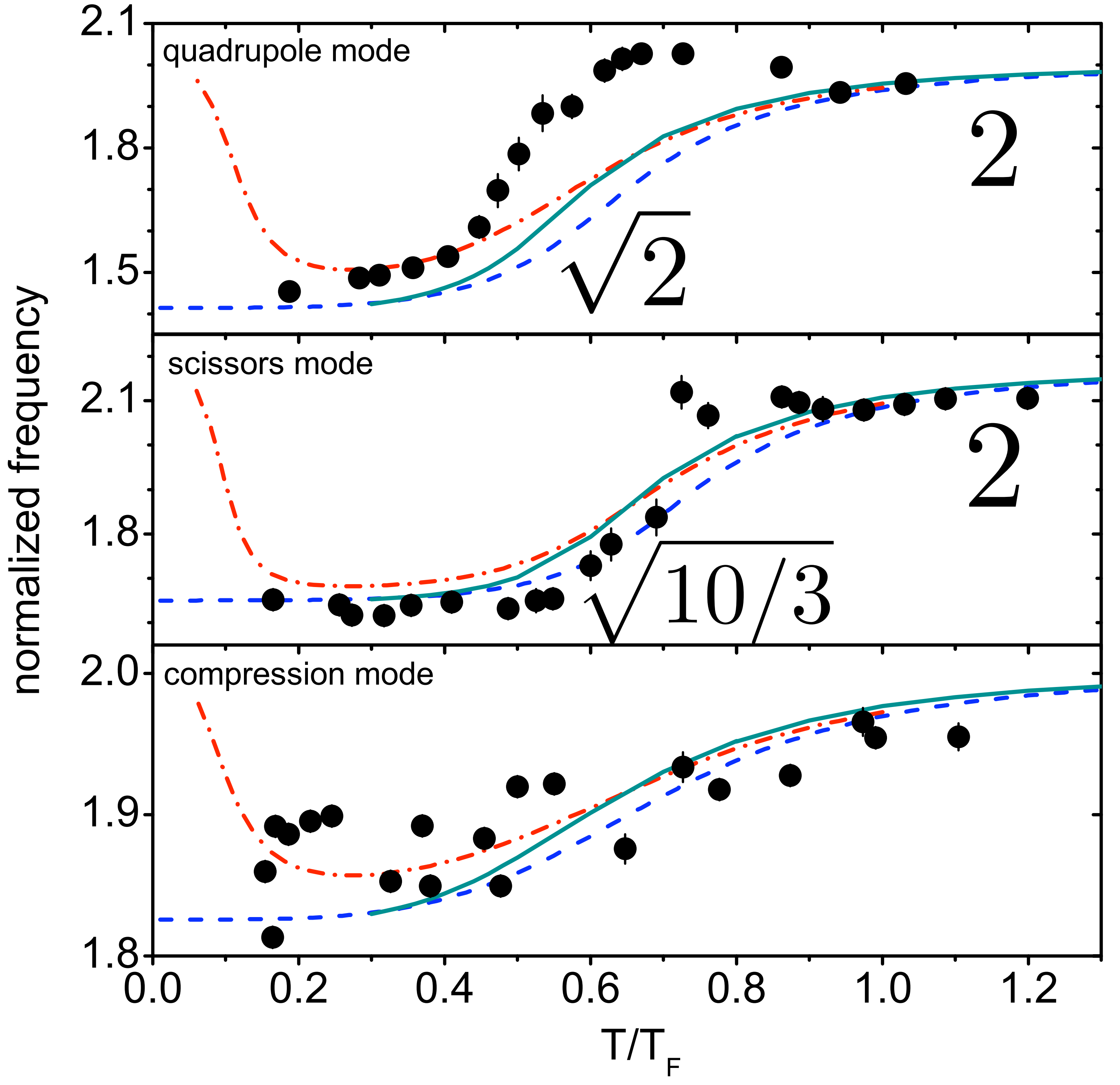


$$\mathbf{v} \propto \begin{pmatrix} y \\ x \end{pmatrix}$$

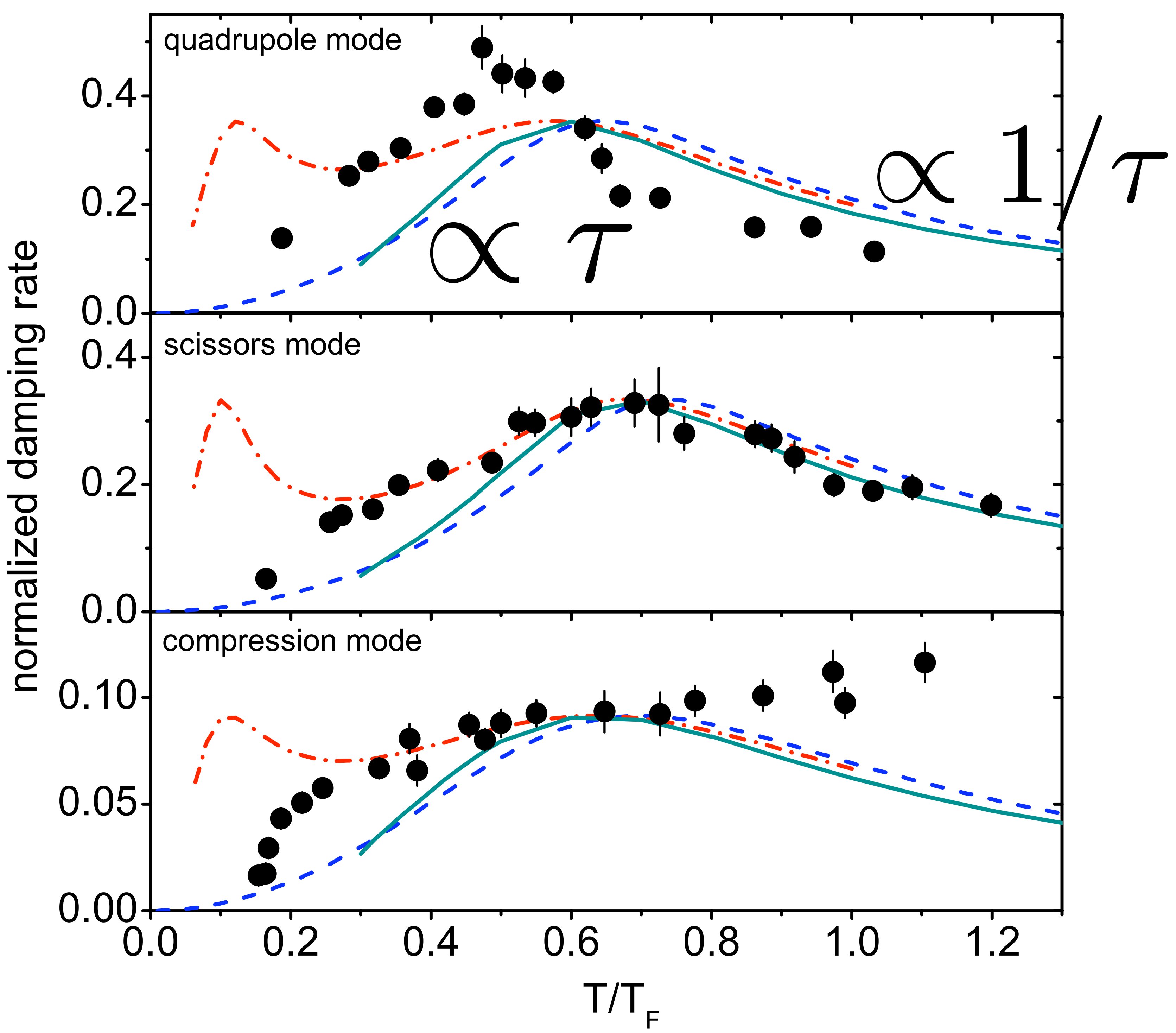
Boltzmann equation $$\frac{\partial f}{\partial t} + \dot{\mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{r}} + \dot{\mathbf{p}} \cdot \frac{\partial f}{\partial \mathbf{p}} = -I[f]$$

Linearize $f = f^0 + \delta f$ $$\delta f(\mathbf{r}, \mathbf{p}, t) = f^0(\mathbf{r}, \mathbf{p})[1 - f^0(\mathbf{r}, \mathbf{p})]\Phi(\mathbf{r}, \mathbf{p}, t)$$

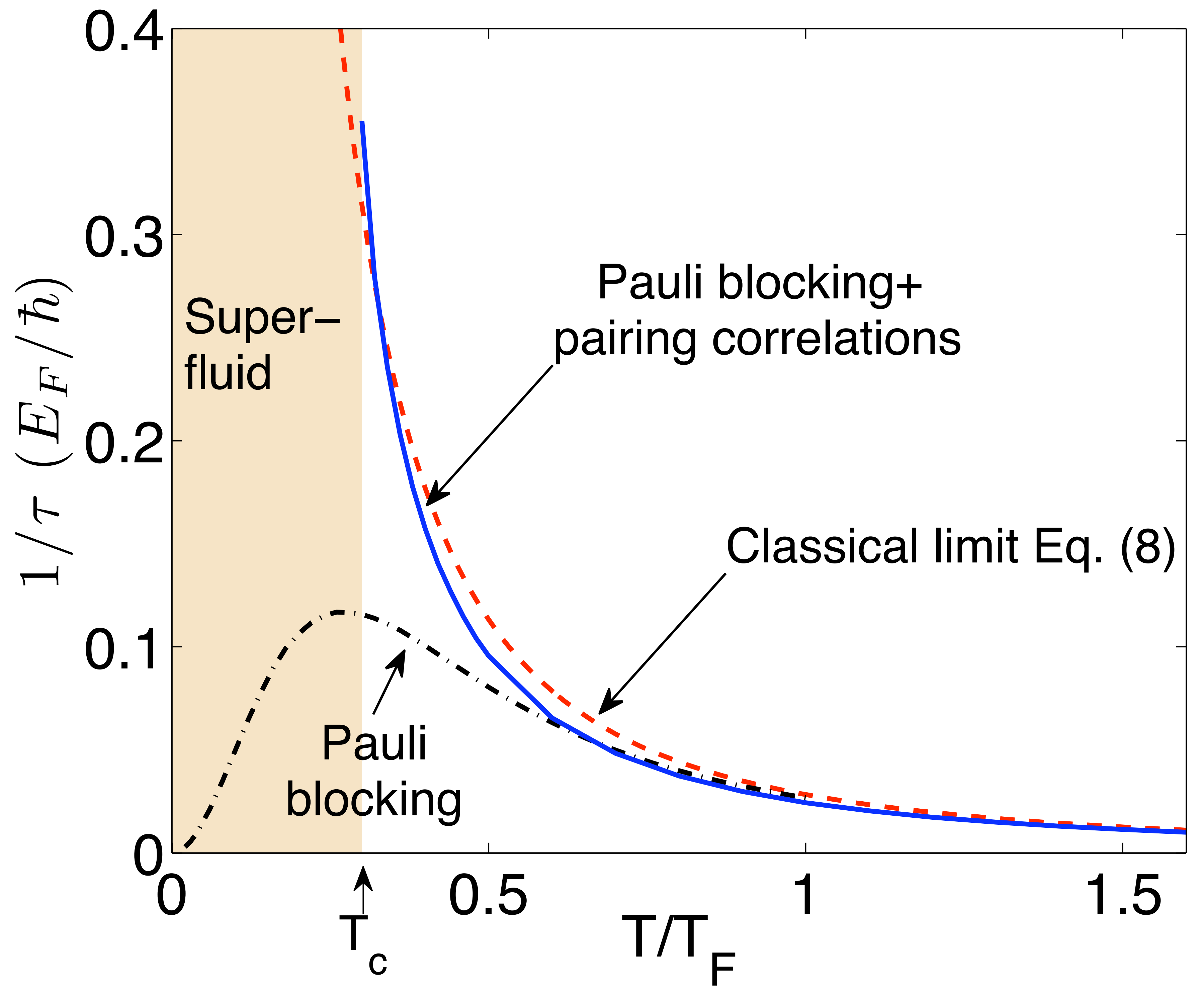
Frequency



Damping



Viscous relaxation rate



S. Riedl *et al.*, PRA **78**, 053609 (2008)

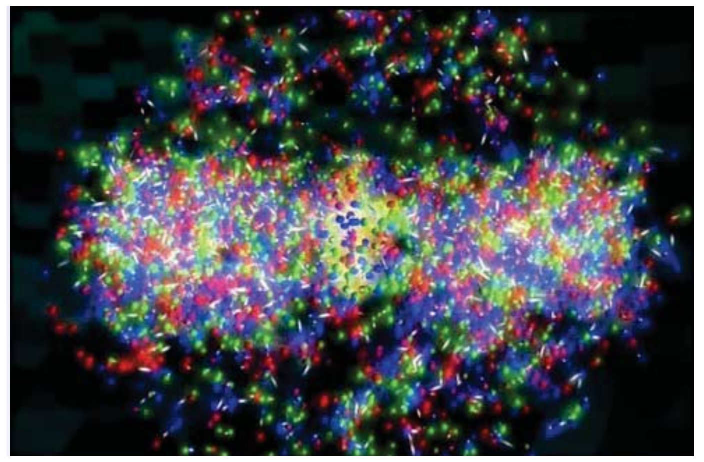
“Perfect fluid”

G. Rupak, T. Schäfer, PRA **76** 053607 (2007)

GMB and H. Smith PRA **75**, 043612 (2007)

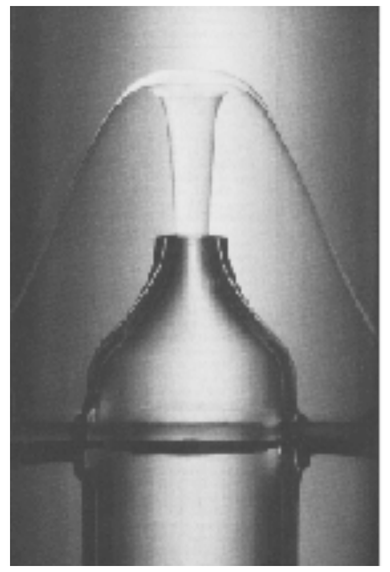
$T \approx 10^{12}$K

QGP $\eta/s \lesssim 0.4$

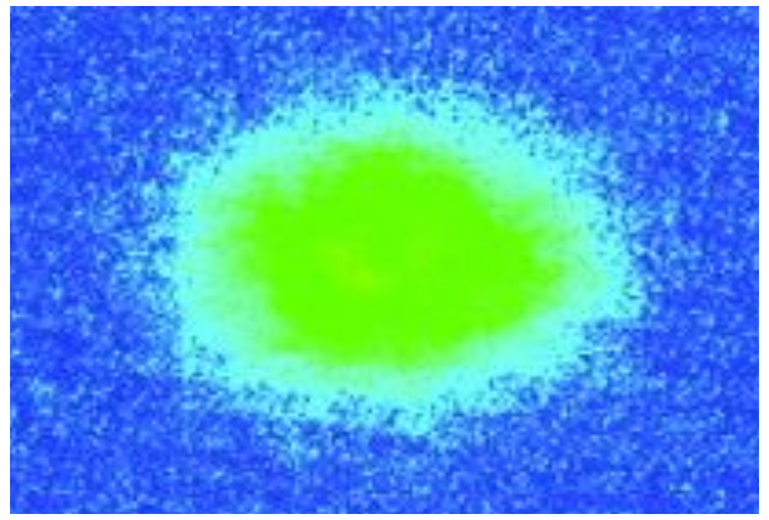


$T \approx 1$K

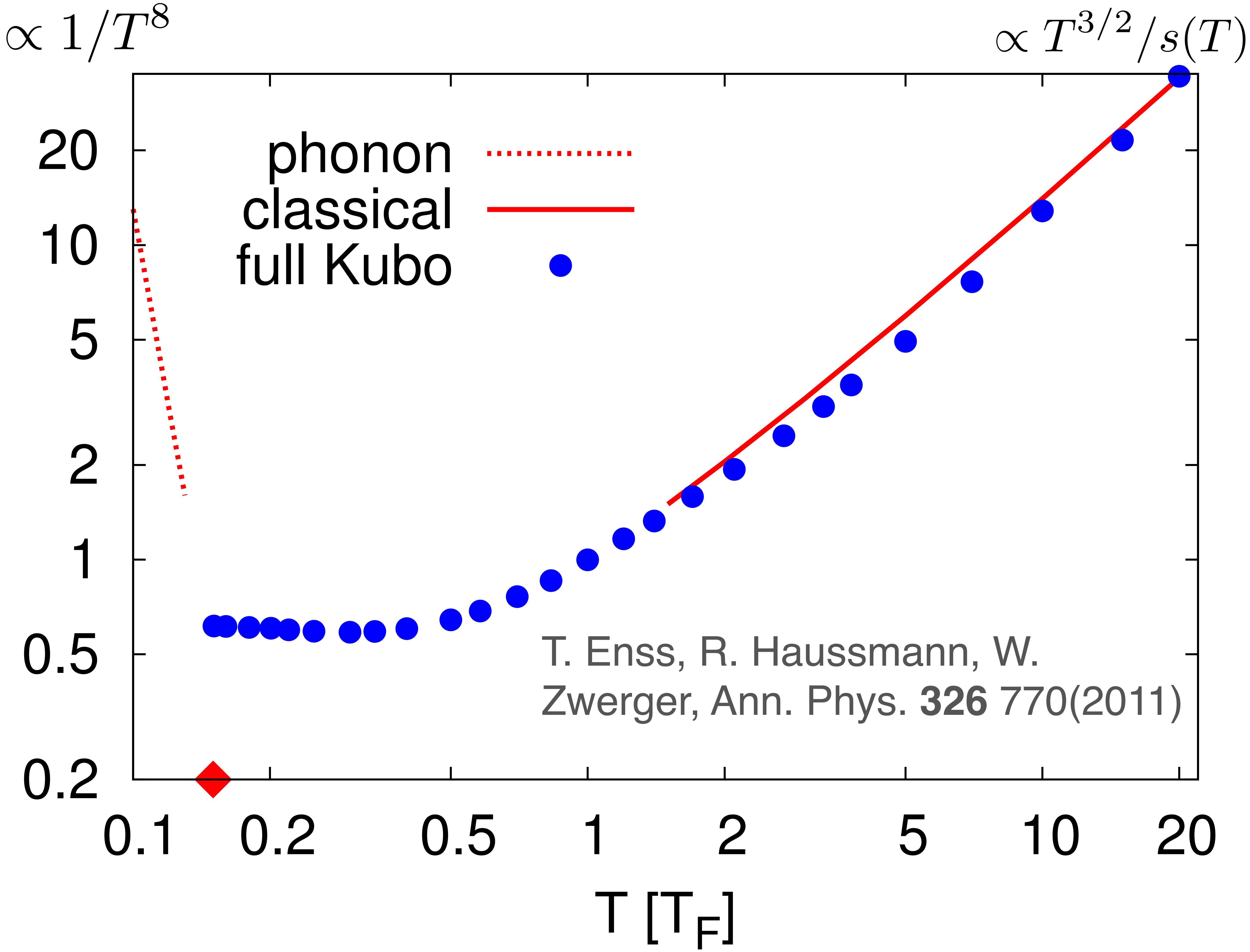
^{4}He $\eta/s \simeq 0.7$



$T \approx 10^{-7}$K



Fermi gas $\eta/s \lesssim 0.5$



M. Bluhm and T. Schäfer, PRL **116** 115301 (2016)

T. Schäfer and D. Teaney 2009 RPP **72** 126001 (2009)

Superfluid hydrodynamics

Superfluid wave function $\Delta(\mathbf{r}) = |\Delta(\mathbf{r})| e^{i\theta(r)}$

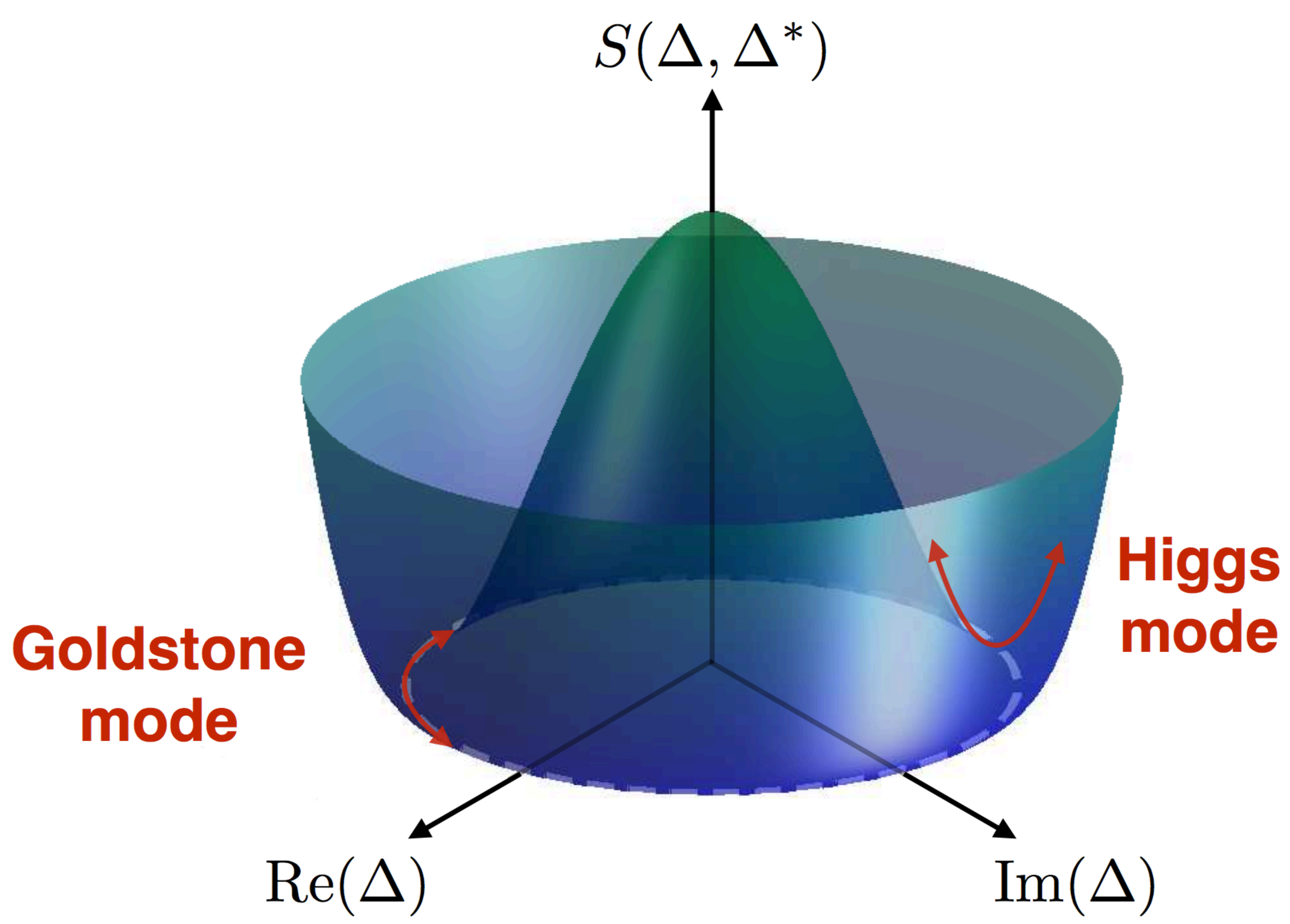
Phase fluctuations (Goldstone modes) low energy degrees of freedom

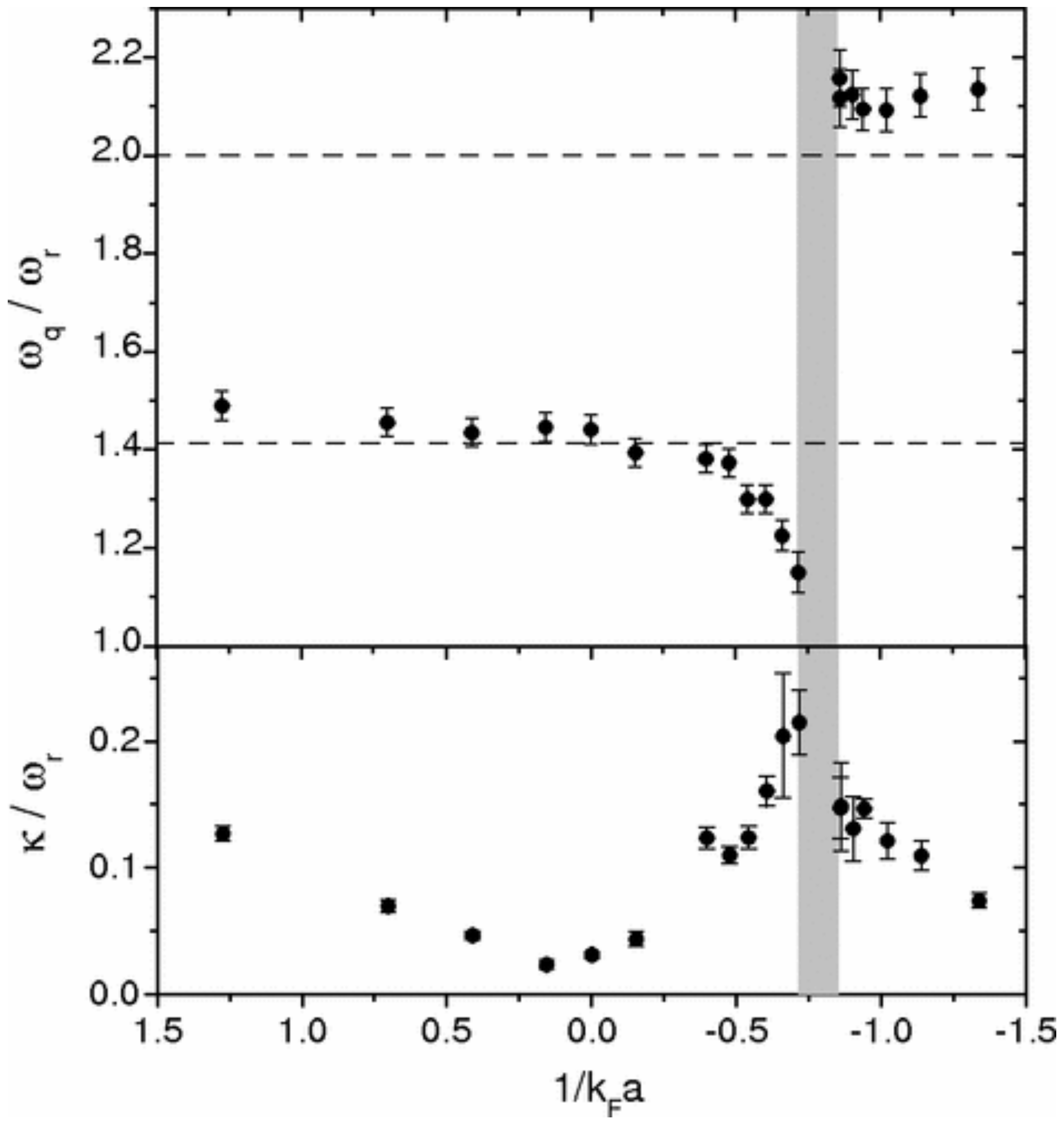
Hydrodynamic (two-fluid) equations with irrotational supercurrent $\mathbf{v}_s \propto \nabla\theta$

Collective mode spectrum identical to that of collisional hydrodynamics

Seen in trapped Fermi gas at low T

Grimm group, PRA **76**, 033610 (2007)

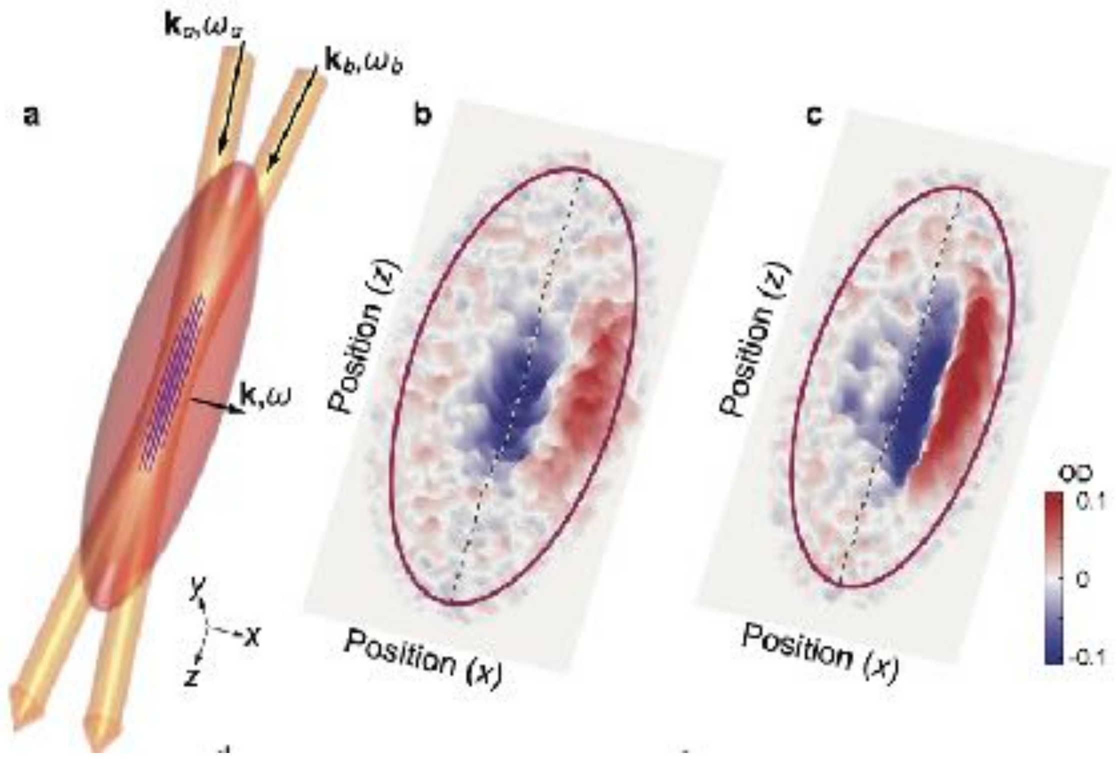




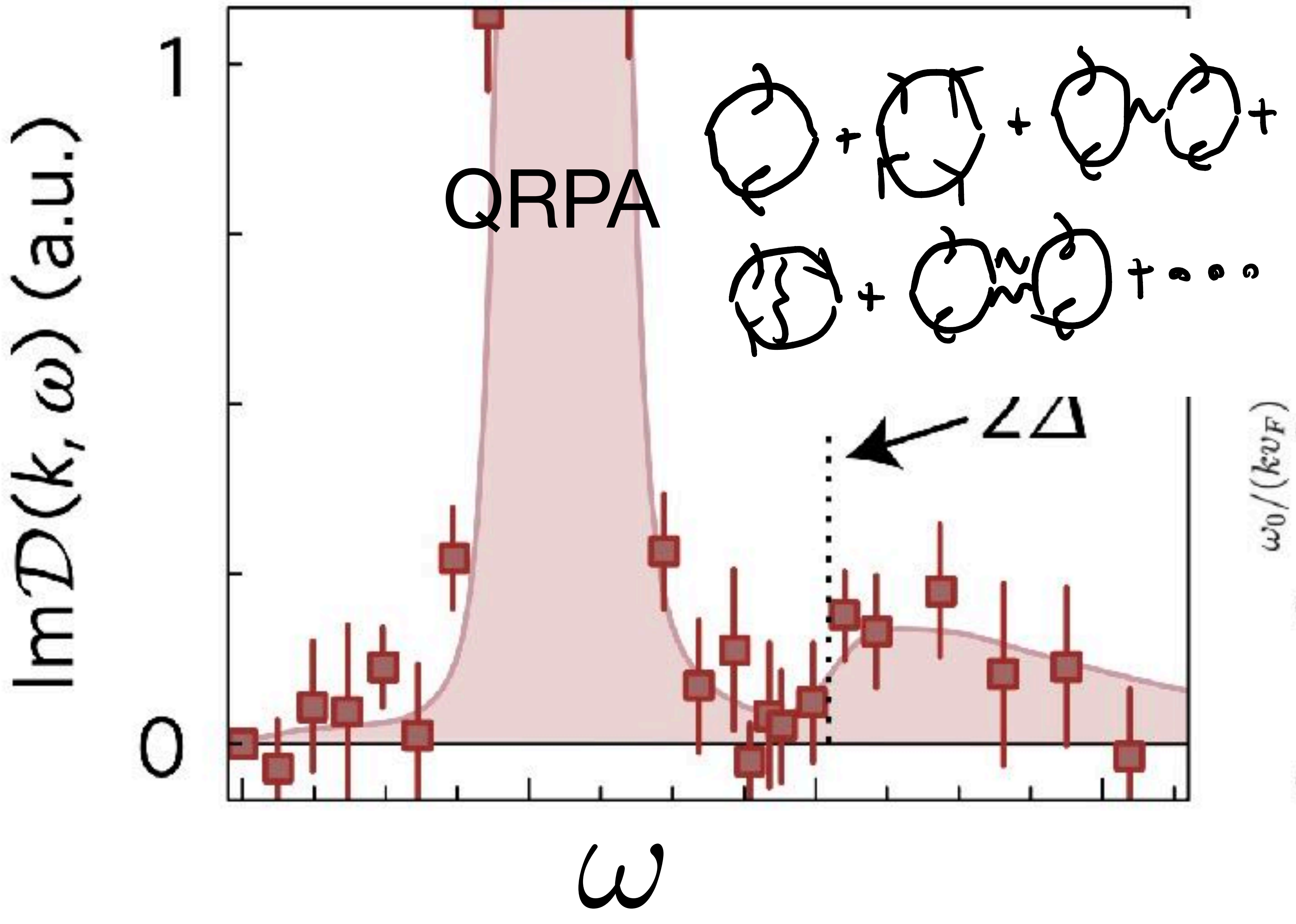
Goldstone mode for homogeneous gases

$$\Delta(\mathbf{r}, t) = |\Delta| e^{i\theta(r,t)}$$

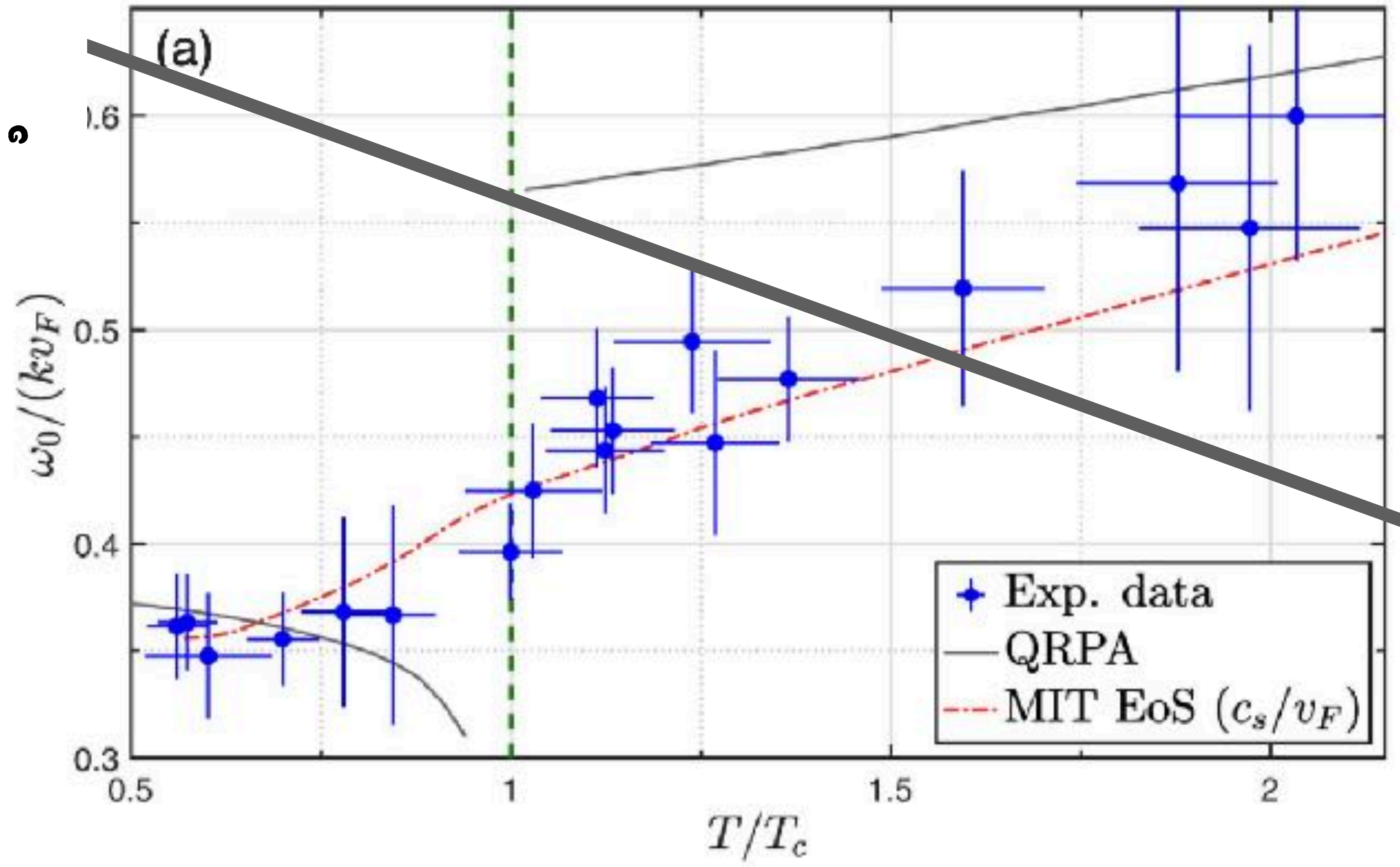
Bragg spectroscopy

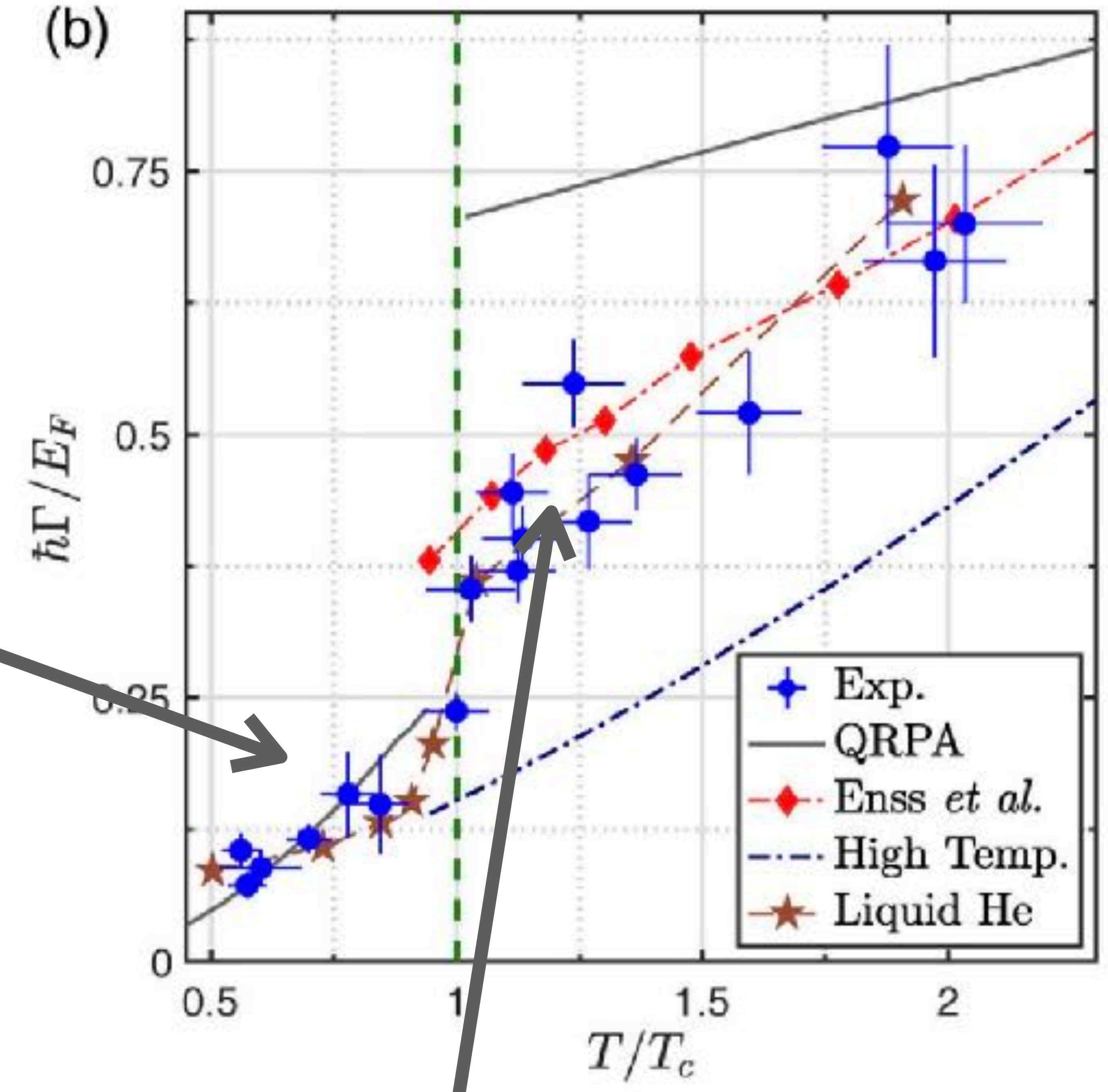


$k \simeq k_F/2$



T-dependence





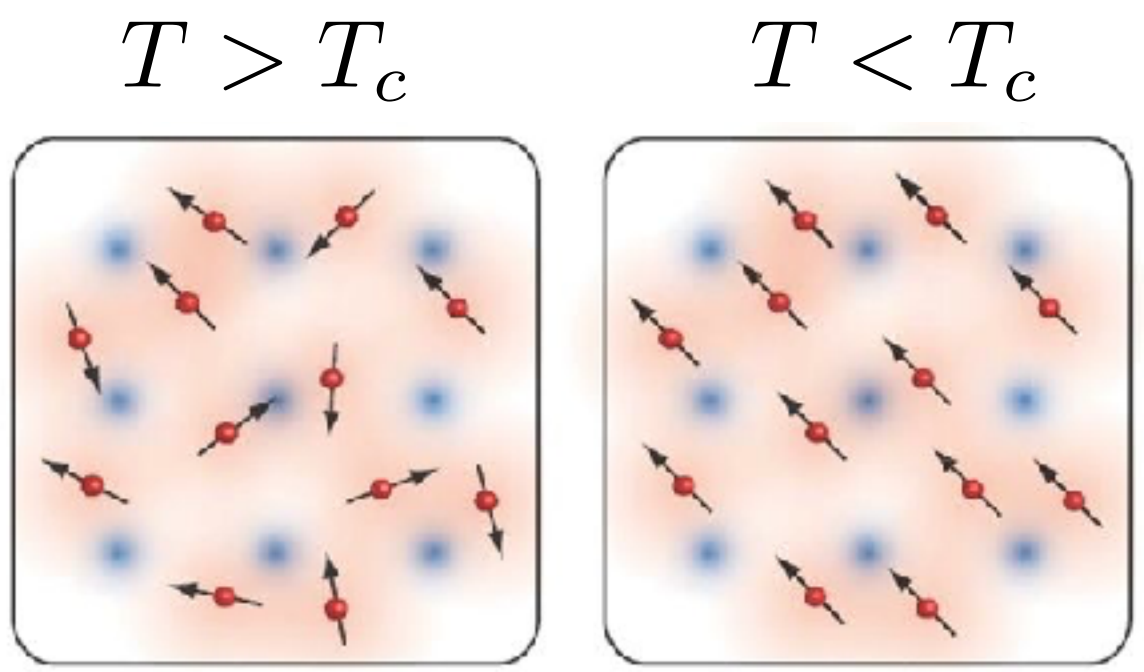
Vale group, Phys. Rev. Lett. **124**, 150401 (2020);
Nat. Phys. **13**, 943 (2017)

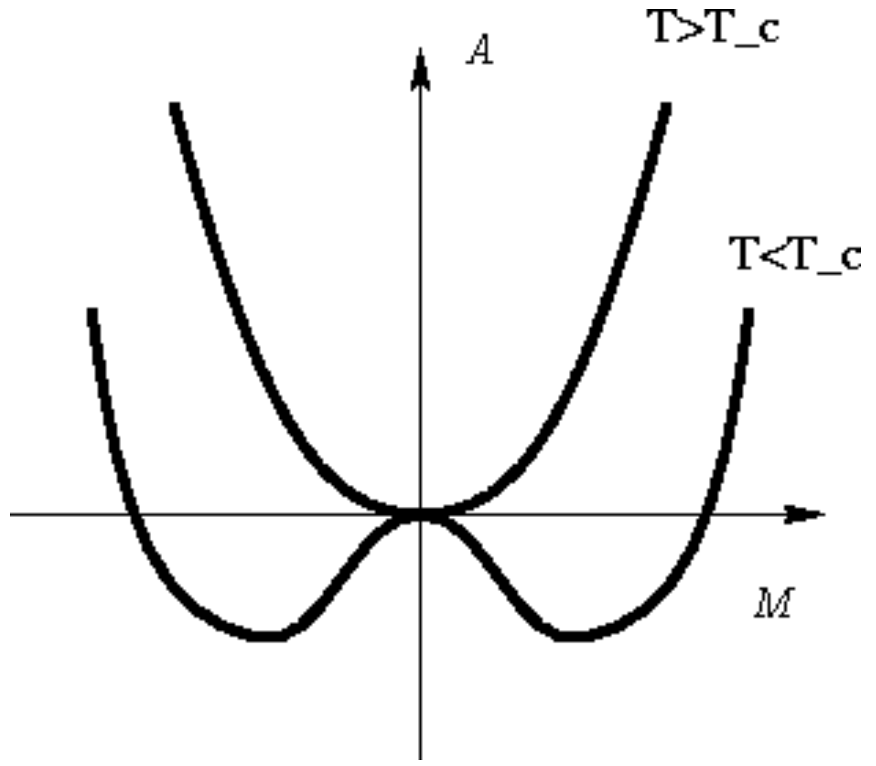
Viscous damping $\propto \eta$

Moritz group, Phys. Rev. Lett. **128**, 100401 (2022)

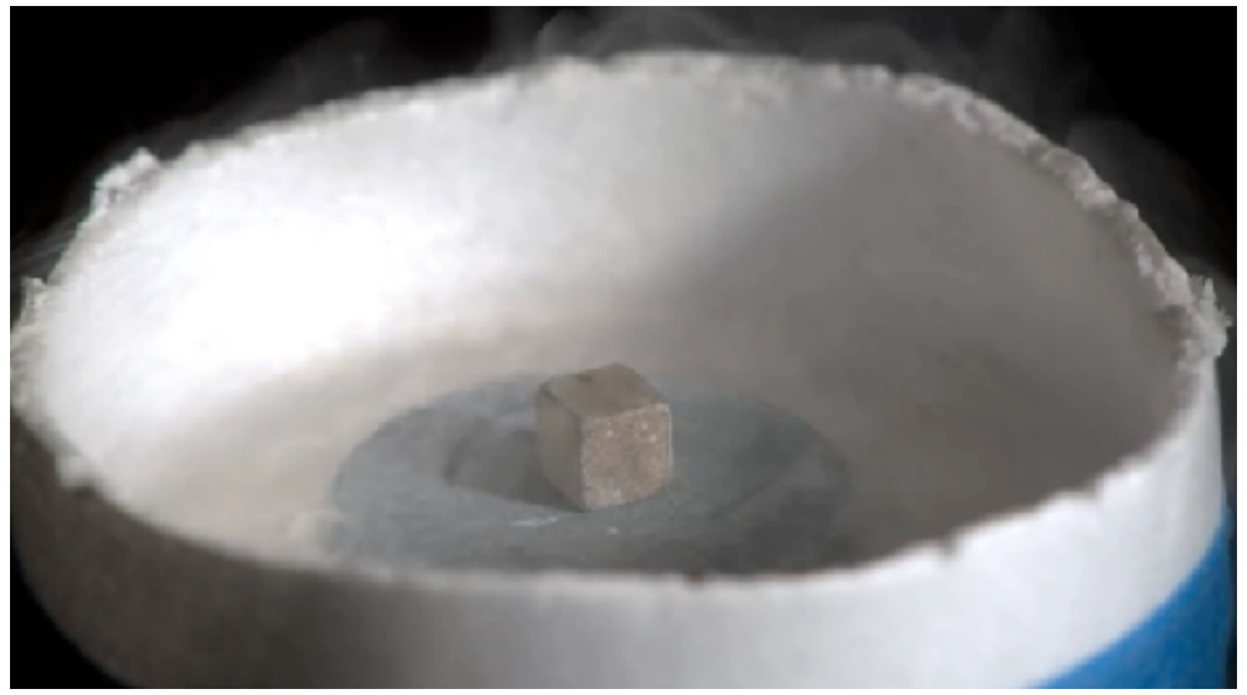
Spontaneous broken symmetry

Ferromagnet

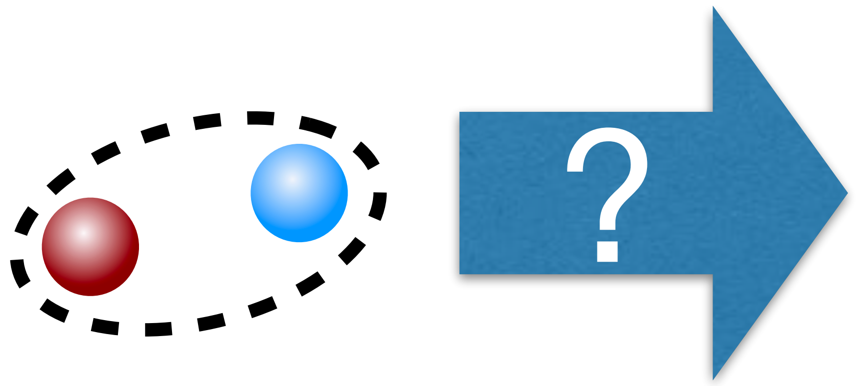


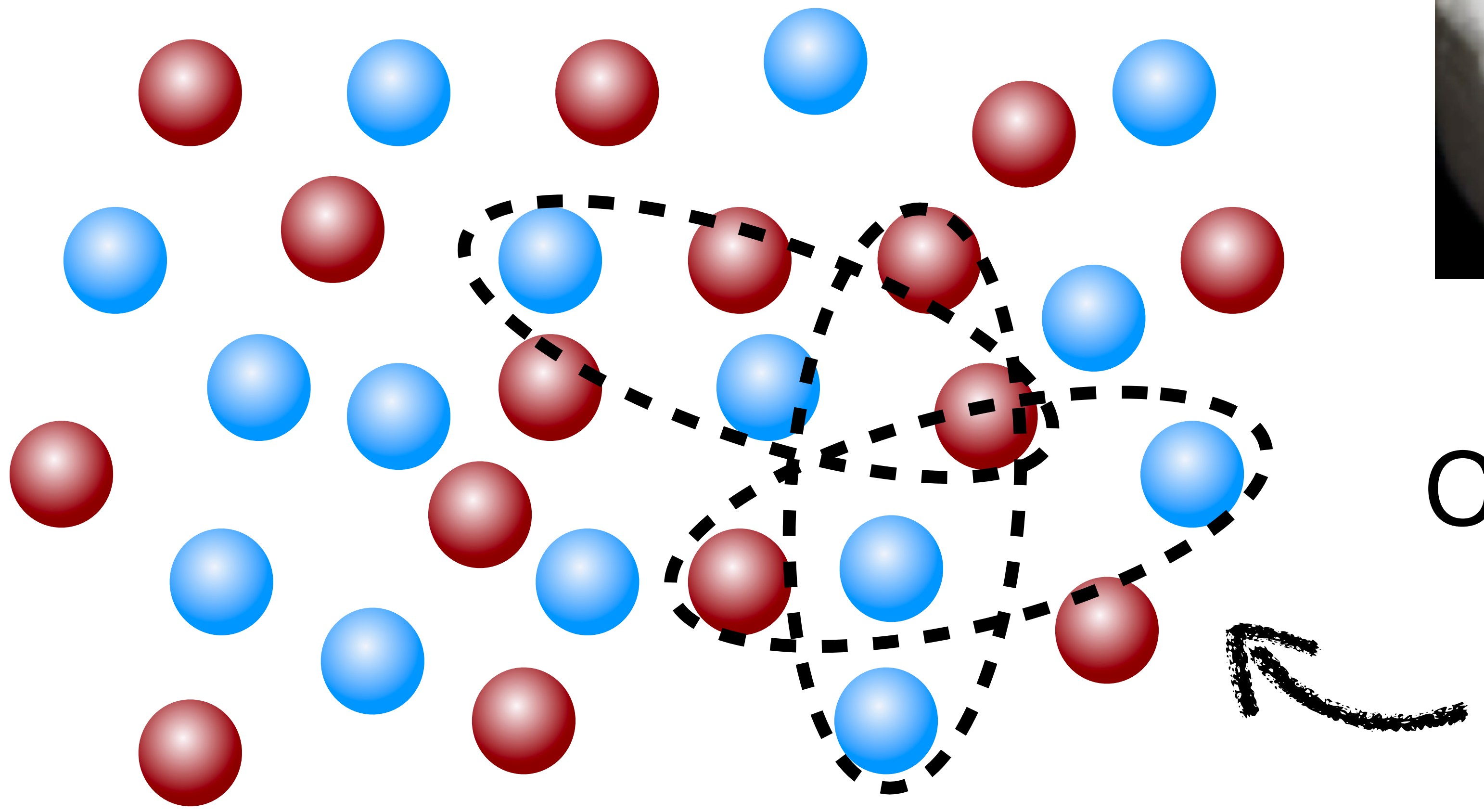


Superconductor



Attraction between two fermions



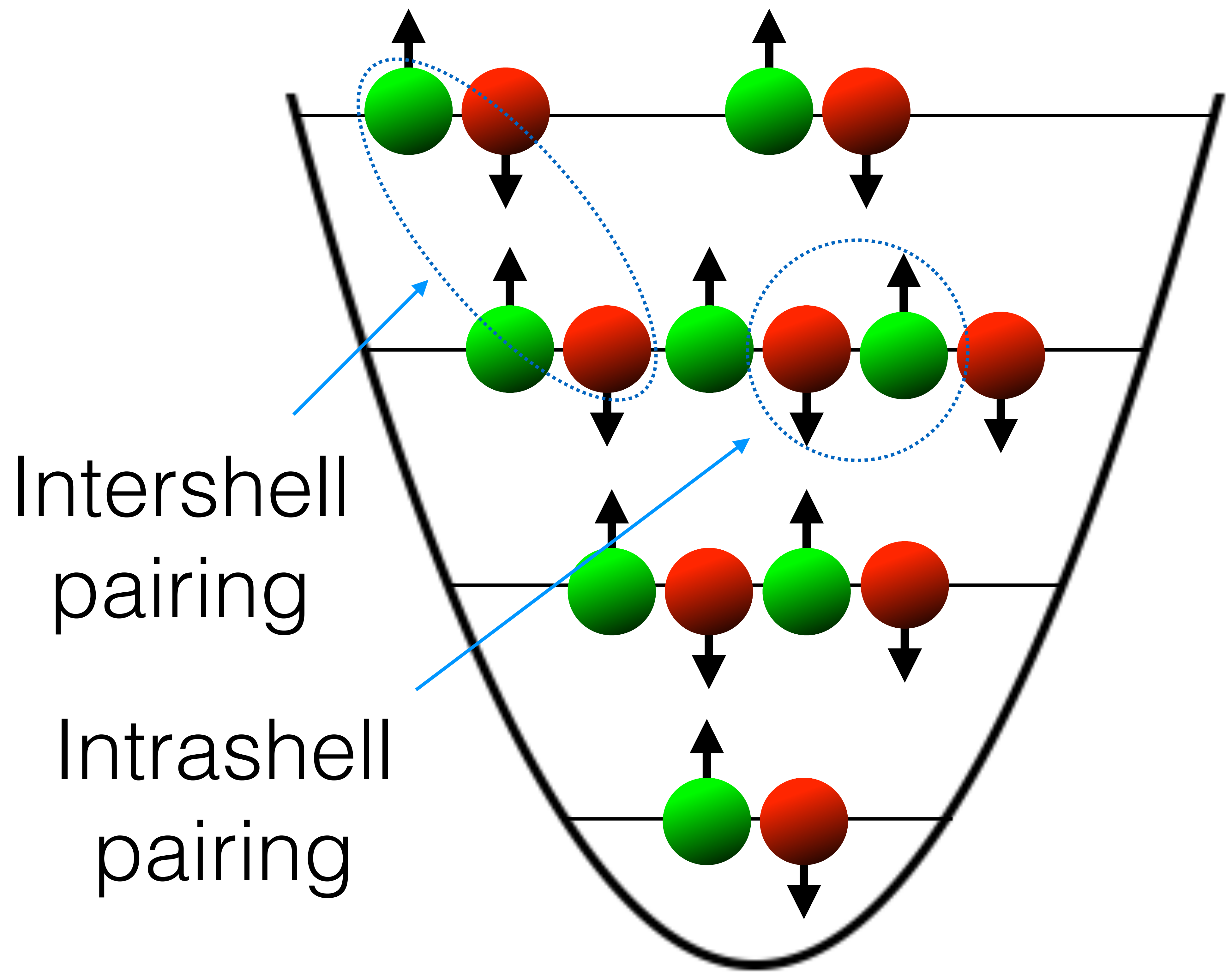


Cooper pairs for T<T_c

$$\Delta(\mathbf{r}) = |\Delta(\mathbf{r})| e^{i\theta(\mathbf{r})}$$

Pairing and collective modes in a 2D trap

Two-component attractive Fermi gas in 2D trap



<u>Usual case: Δ≫ℏω</u>

Intershell pairing dominates

Cooper pairs ≪ cloud

<u>Microtraps: Δ≪ℏω</u>

Cooper pairs~cloud

Intrashell (n,m,↑)↔(n,-m,↓) pairing dominates

- Open shell case

Always pairing for T=0

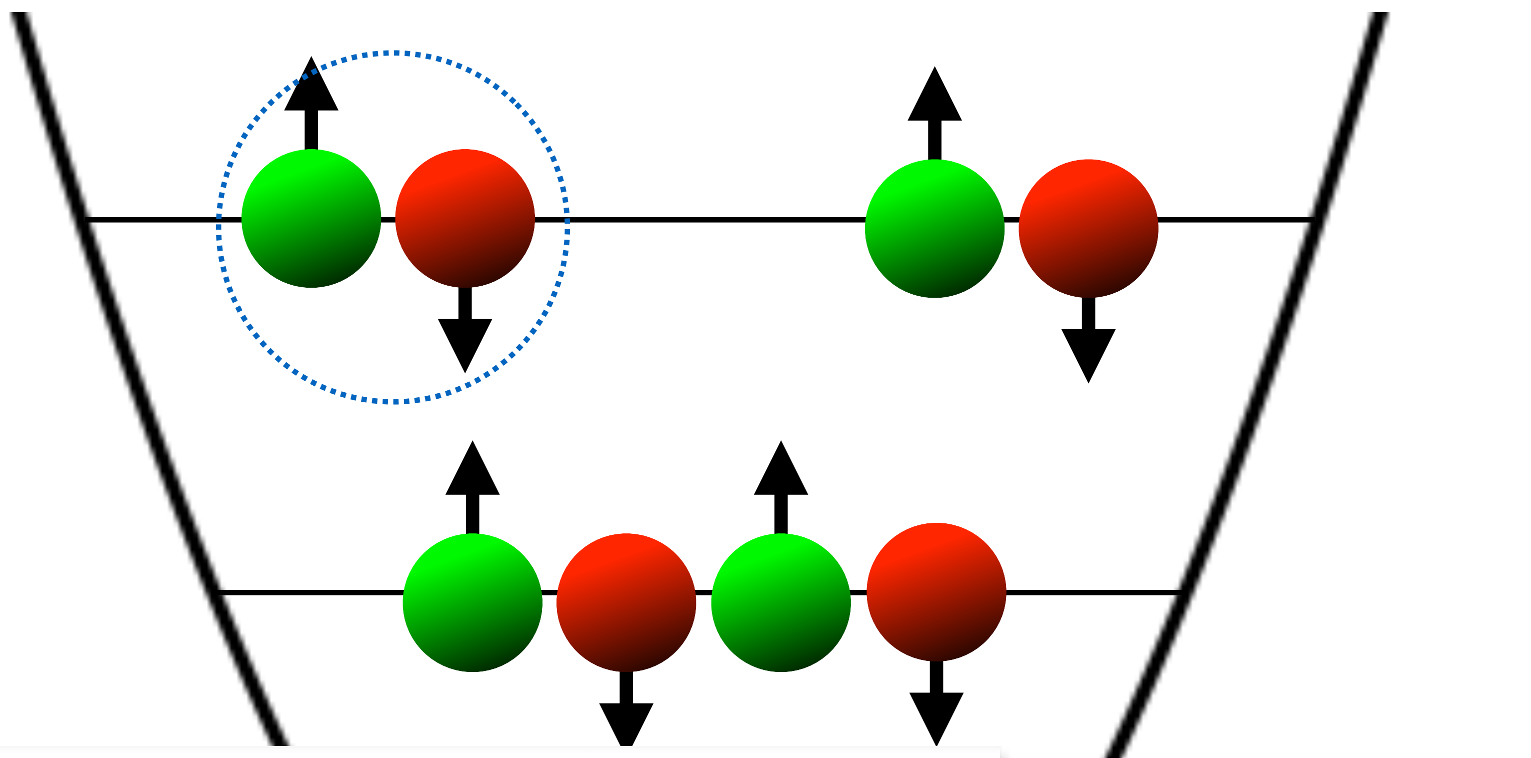
ϵ_B

Δ_n

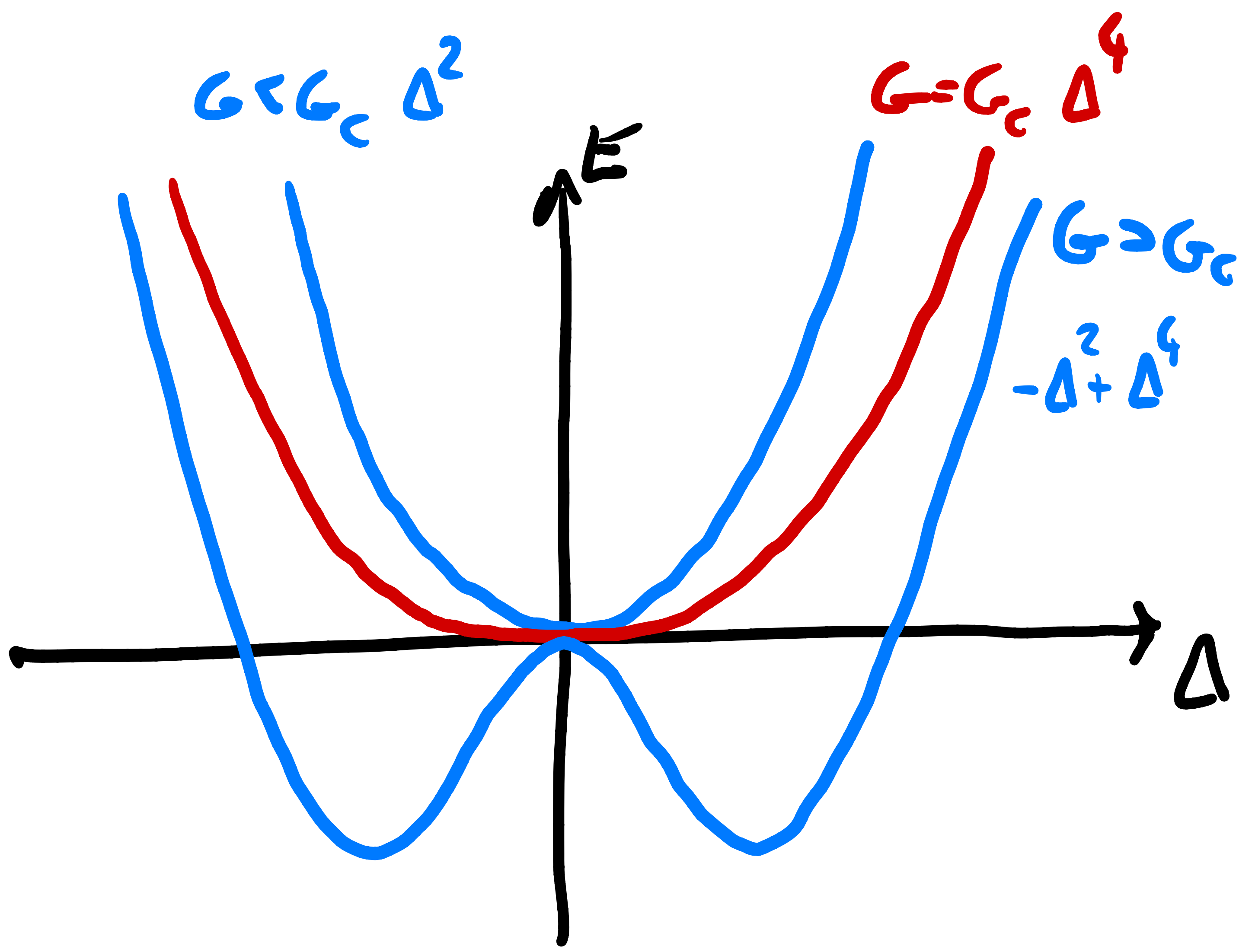
Euler

- Clo

Onl



Higgs mode must go "soft" at Gc

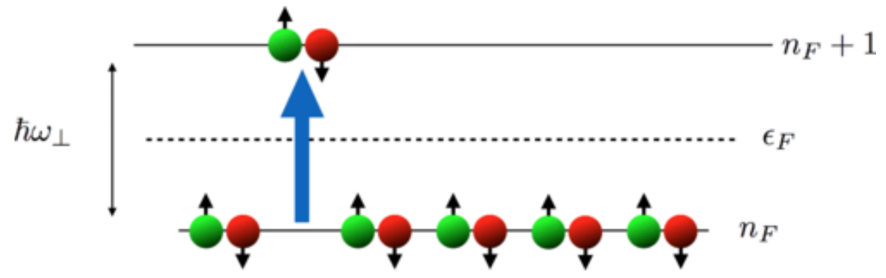


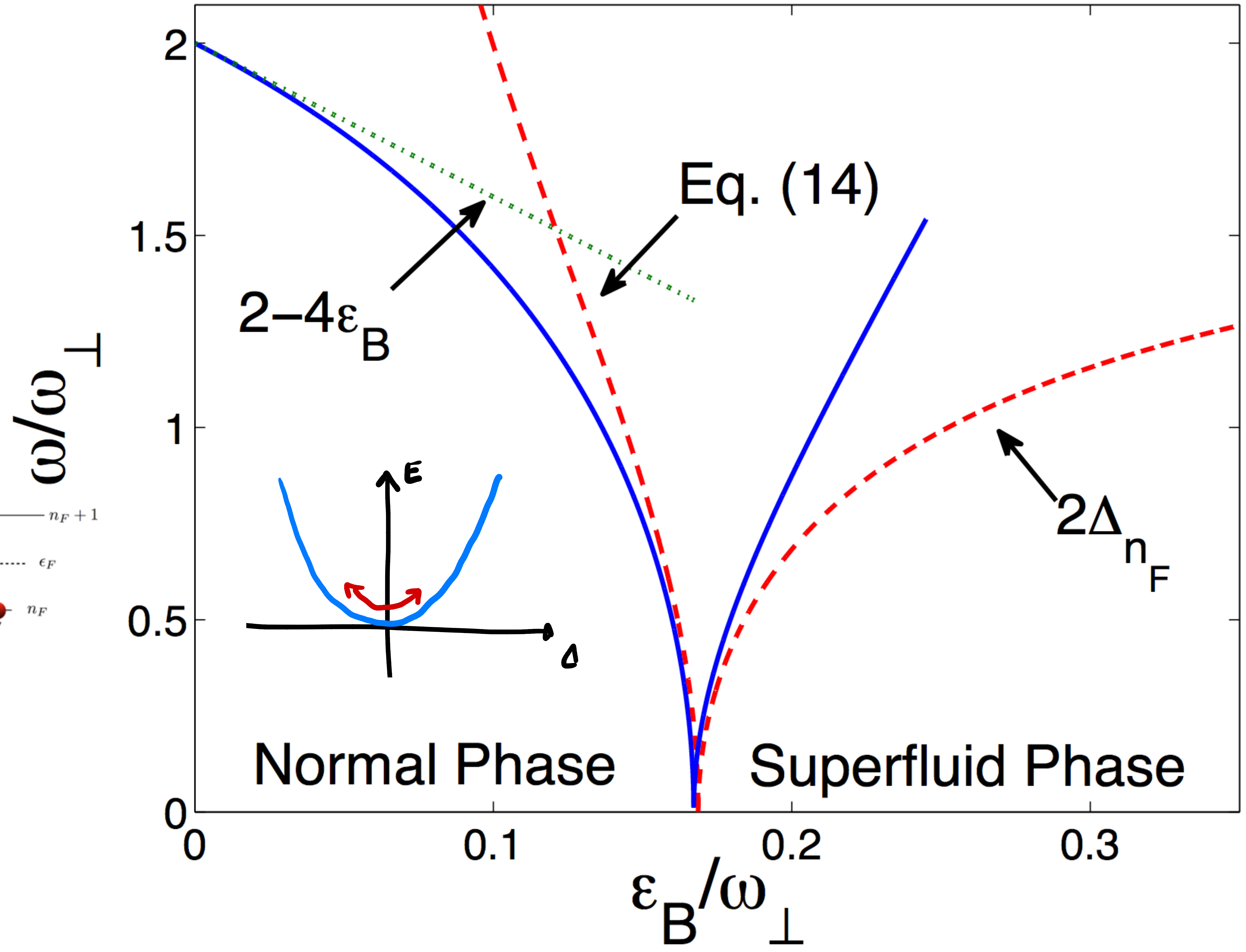
$$\frac{\epsilon_B^c}{\omega_\perp} = \frac{B(n_F)}{2\xi(2)}\left[\sqrt{1+4\xi(2)/B(n_F)^2} - 1\right] \qquad \Delta_{n_F} = \frac{\omega_\perp}{\sqrt{7\xi(3)}}\sqrt{\frac{\omega_\perp}{\epsilon_B^c} - \frac{\omega_\perp}{\epsilon_B} + \xi(2)\left(\frac{\epsilon_B}{\omega_\perp} - \frac{\epsilon_B^c}{\omega_\perp}\right)}$$

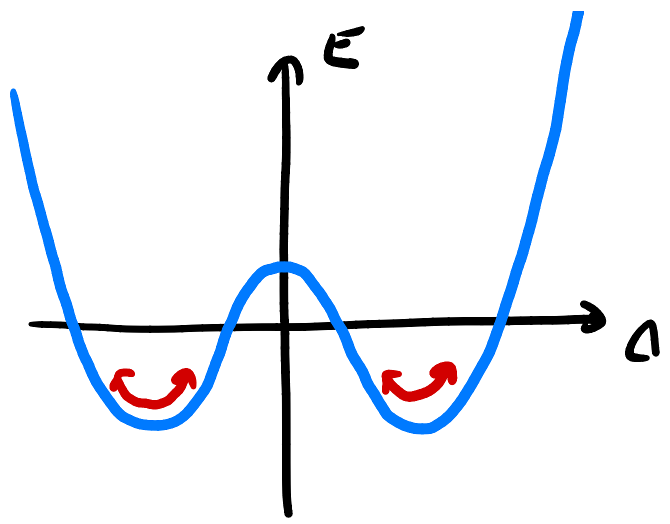
Riemann zeta fn ζ(z)

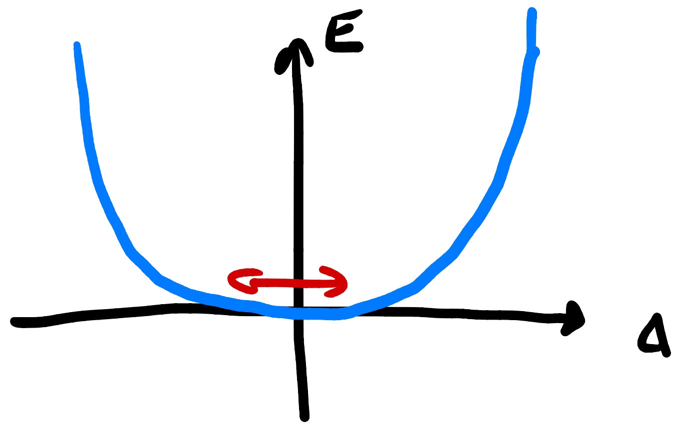
GMB, Phys. Rev. A **90**, 023621 (2014)

Closed Shell









GMB, Phys. Rev. A **90**, 023621 (2014)

Few-body limit

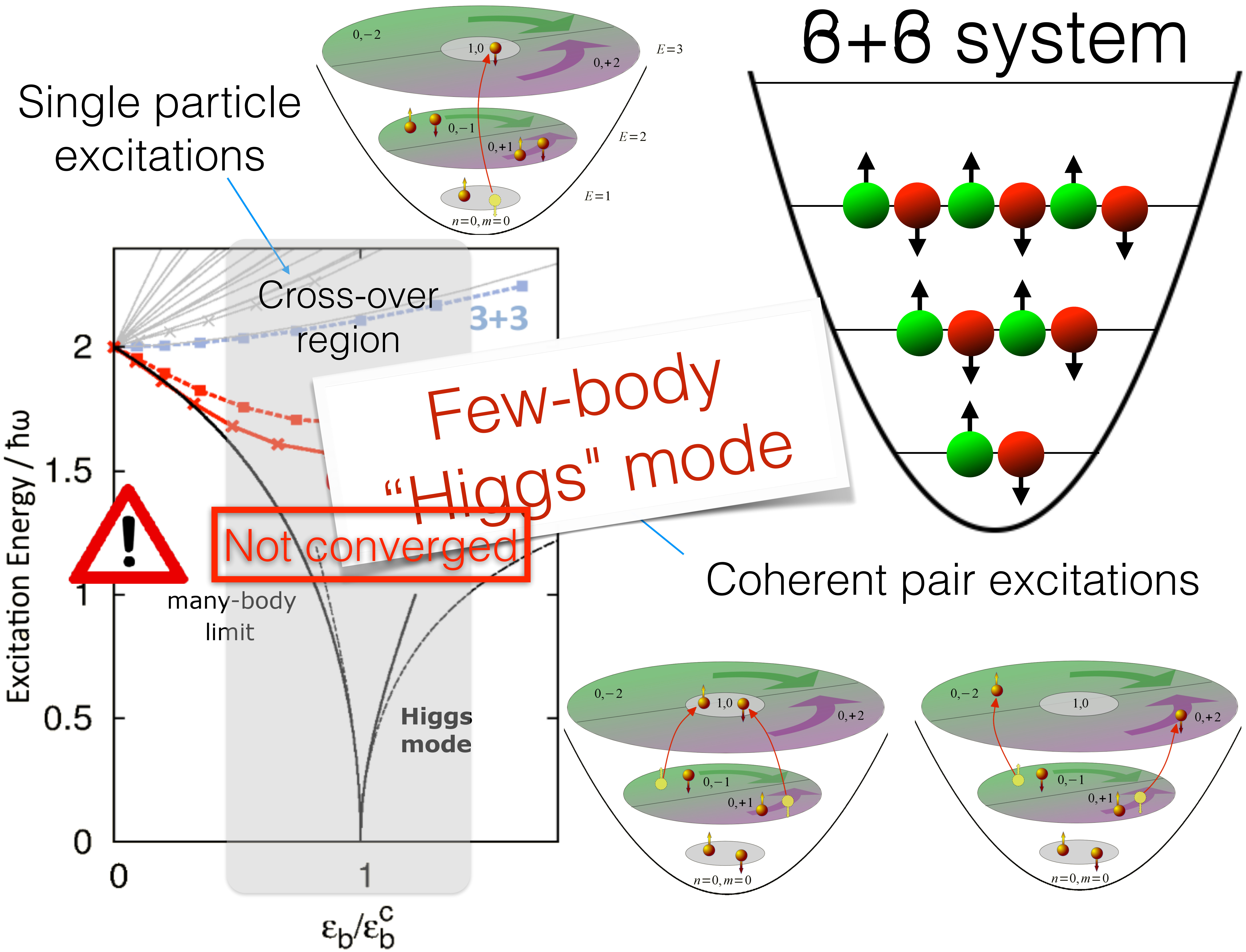
- Two-component attractive Fermi gas in microtrap (2D)

$$\hat{H} = \sum_{i=1}^{N} \left(-\frac{\hbar^2 \nabla_i^2}{2m} + \frac{1}{2} m\omega^2 \mathbf{r}_i^2 \right) + g \sum_{k,l} \delta(\mathbf{r}_k - \mathbf{r}_l)$$

- Exact diagonalisation by expanding in 2D Harmonic oscillator states. Up to ~10^7 states

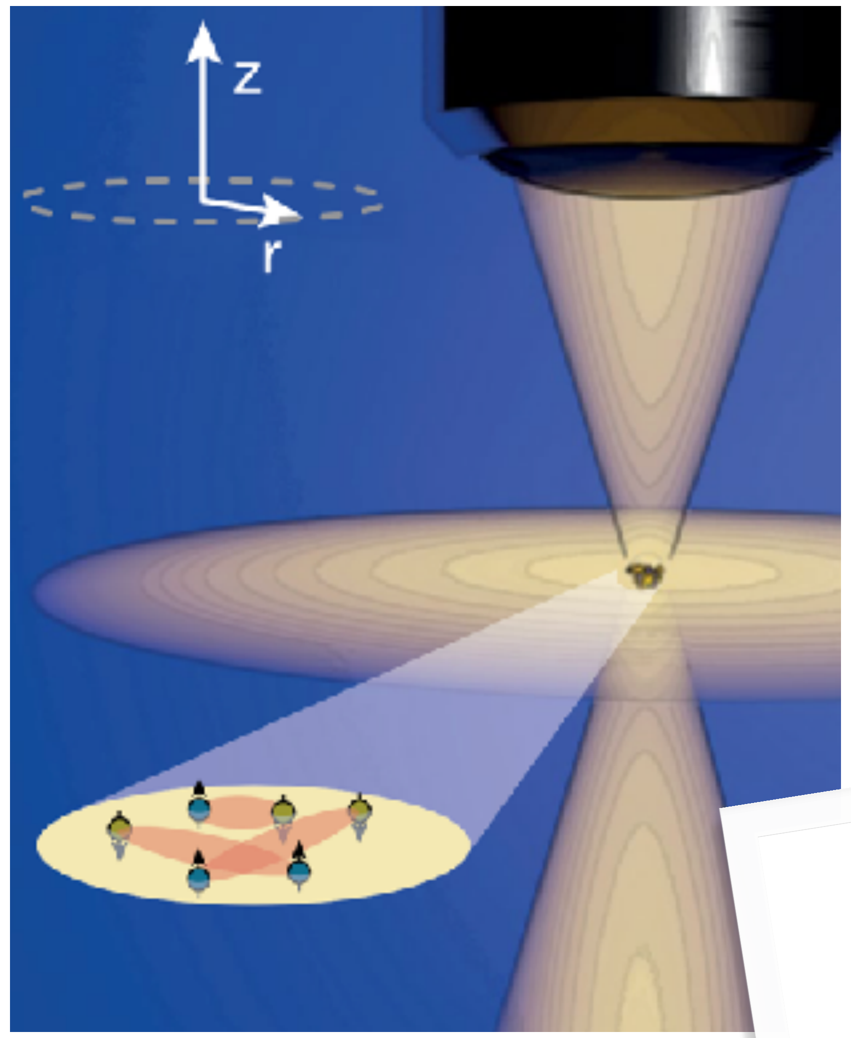
- Divergence from δ(r)-interaction eliminated by expressing energies in terms of 2-body binding energy ε_b: $$E_2 = 2\hbar\omega - 2\epsilon_b$$

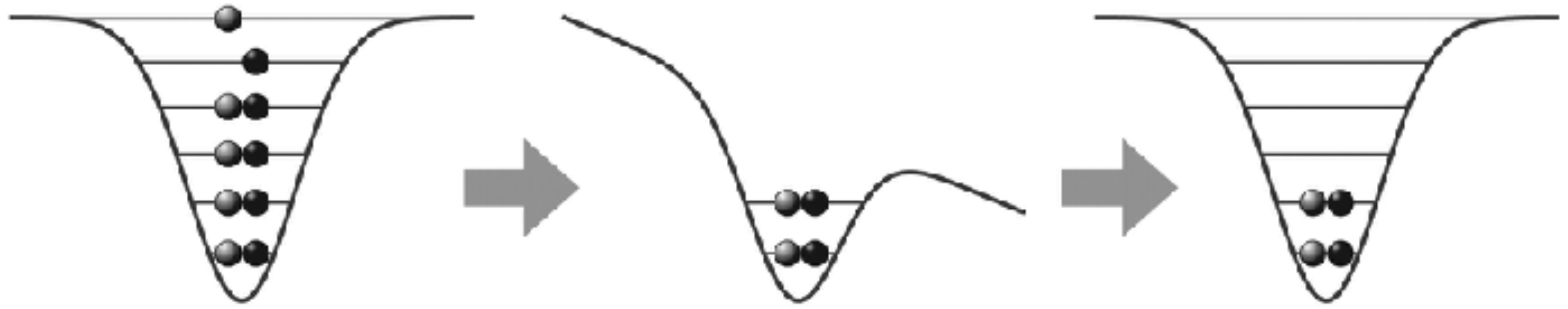
J. Bjerlin, S. M. Reimann, and GMB, Phys. Rev. Lett. **116**, 155302 (2016)



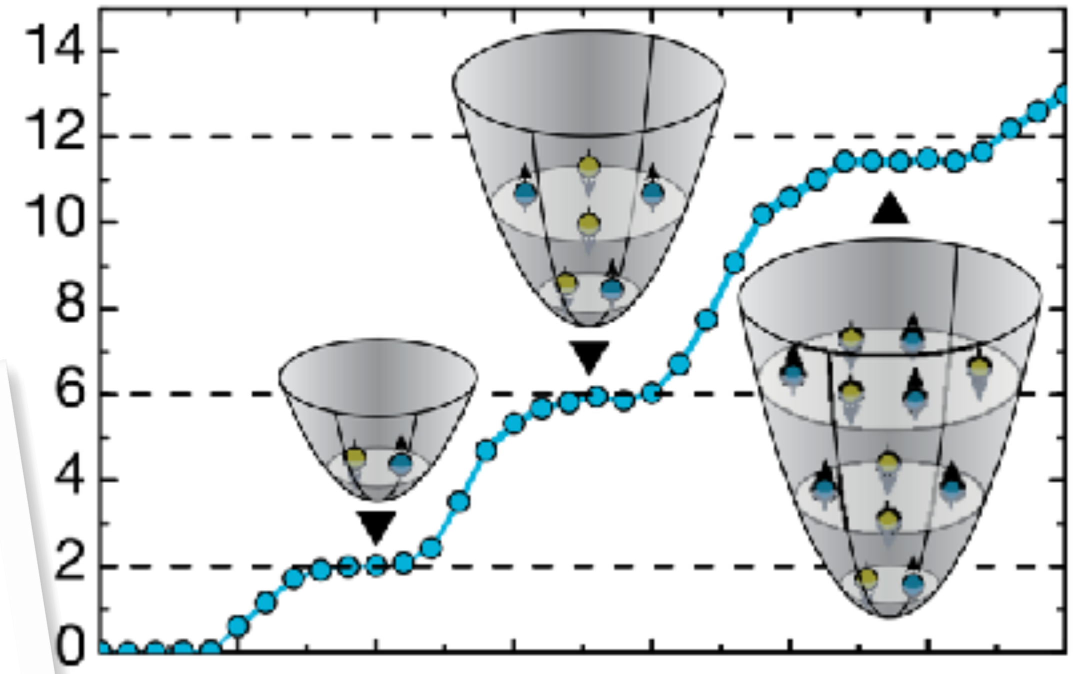
6+6 system
Single particle excitations
Cross-over region
3+3
Few-body
“Higgs" mode
Not converged
many-body limit
Higgs mode
Coherent pair excitations
Excitation Energy / ħω
εb/εbc
0
0.5
1
1.5
2
0
1
E=3
E=2
E=1
n=0, m=0
1,0
0,−2
0,+2
0,−1
0,+1

Experiment

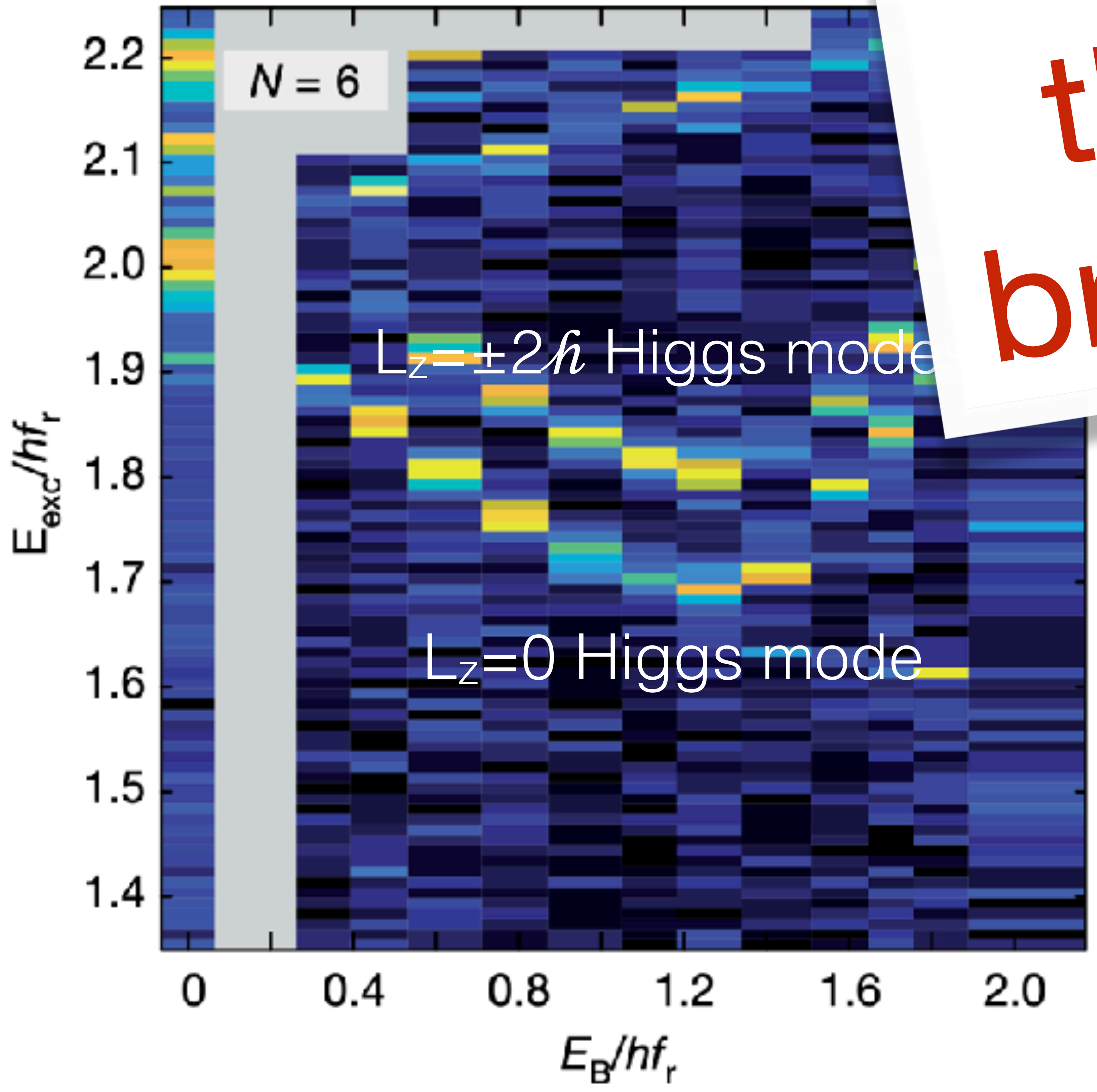


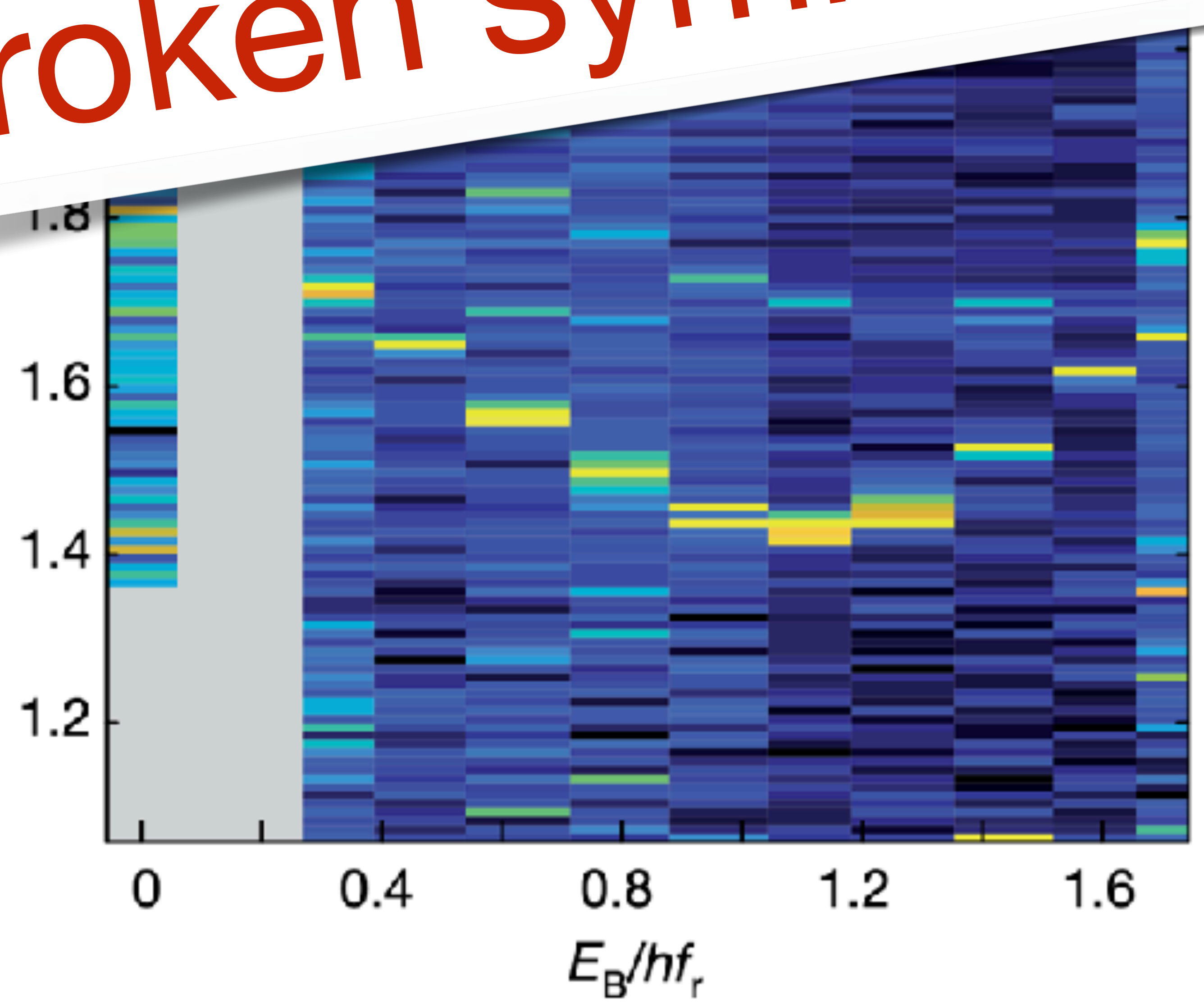


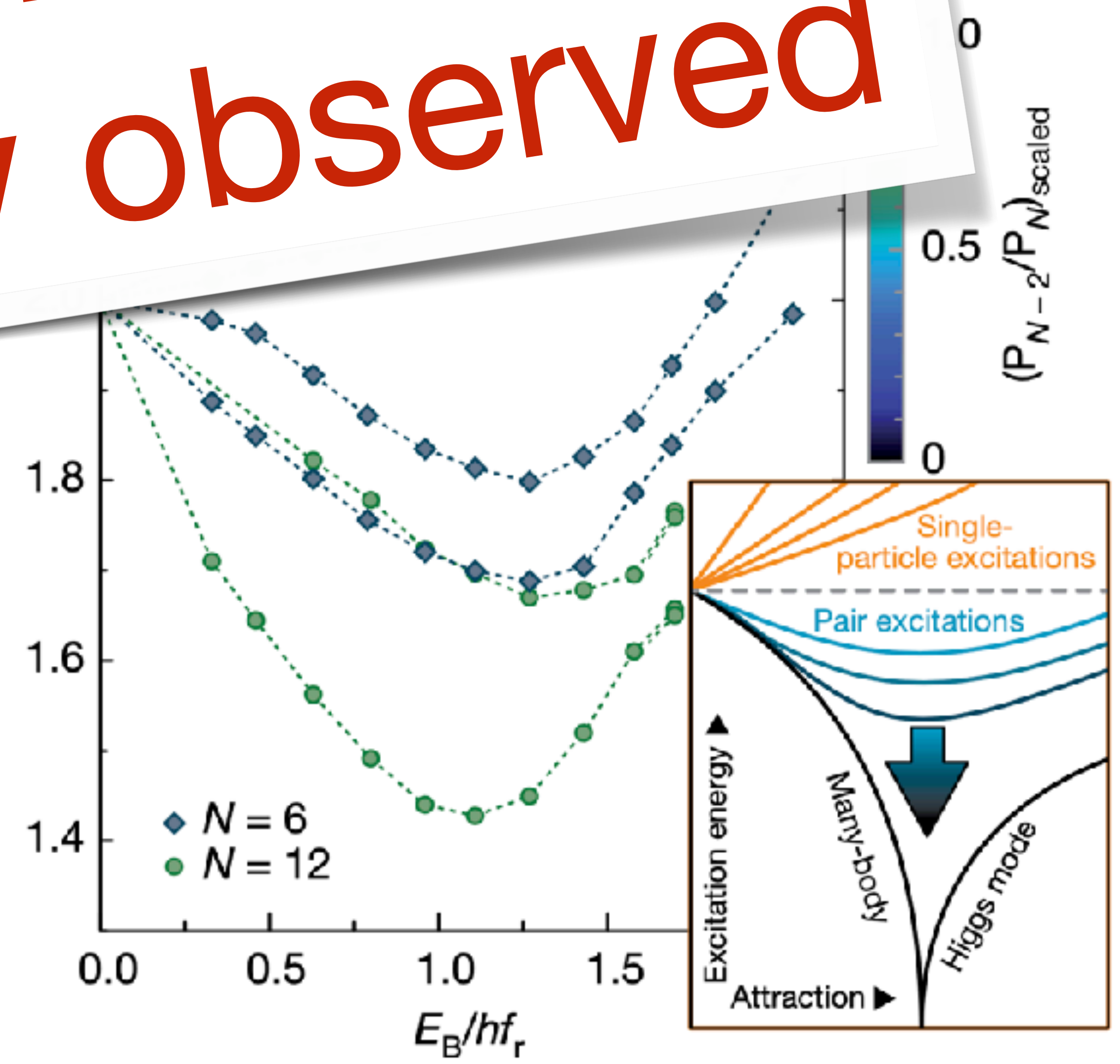
Deterministic loading ...



Approach to thermodynamic limit and broken symmetry observed





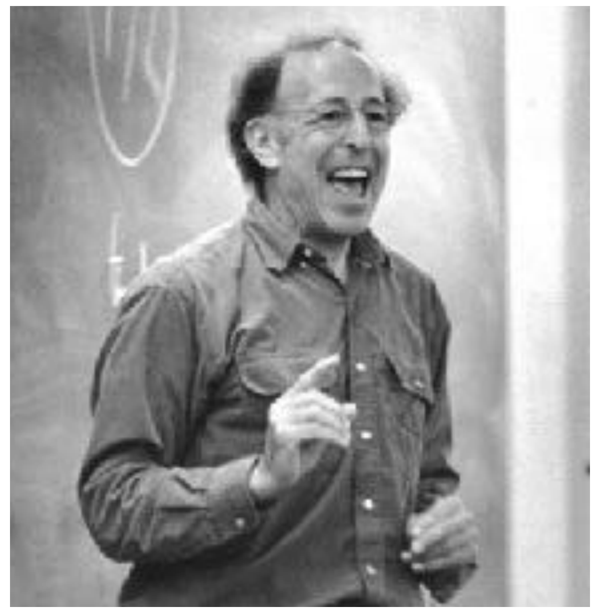


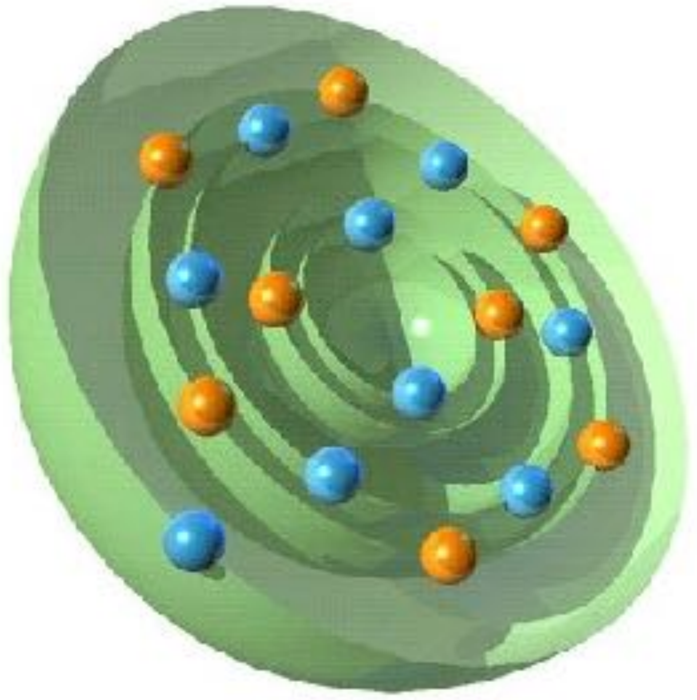
Higgs mode deepens with increasing particle number

L. Bayha *et al.*, Nature **587**, 583 (2020)

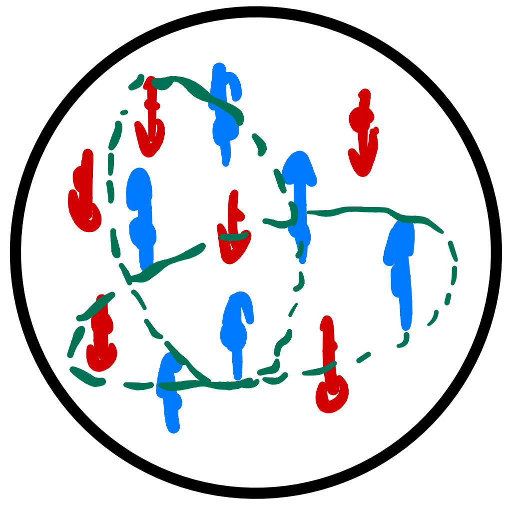
Perspectives

QRPA in spherical trap: Emergence of Goldstone modes



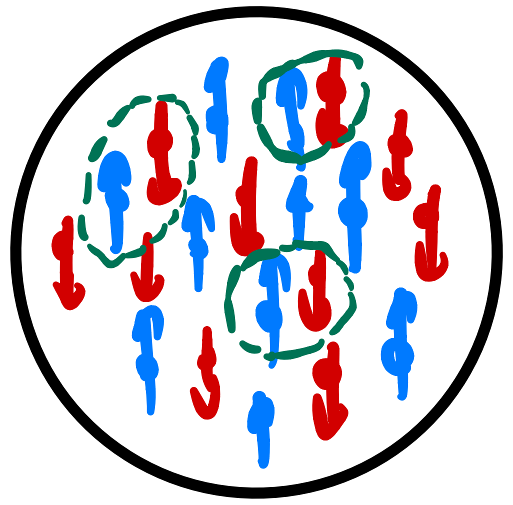


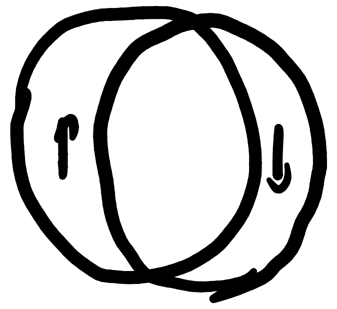
$\Delta << \hbar\omega_c$

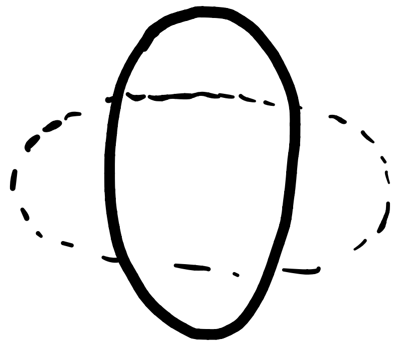


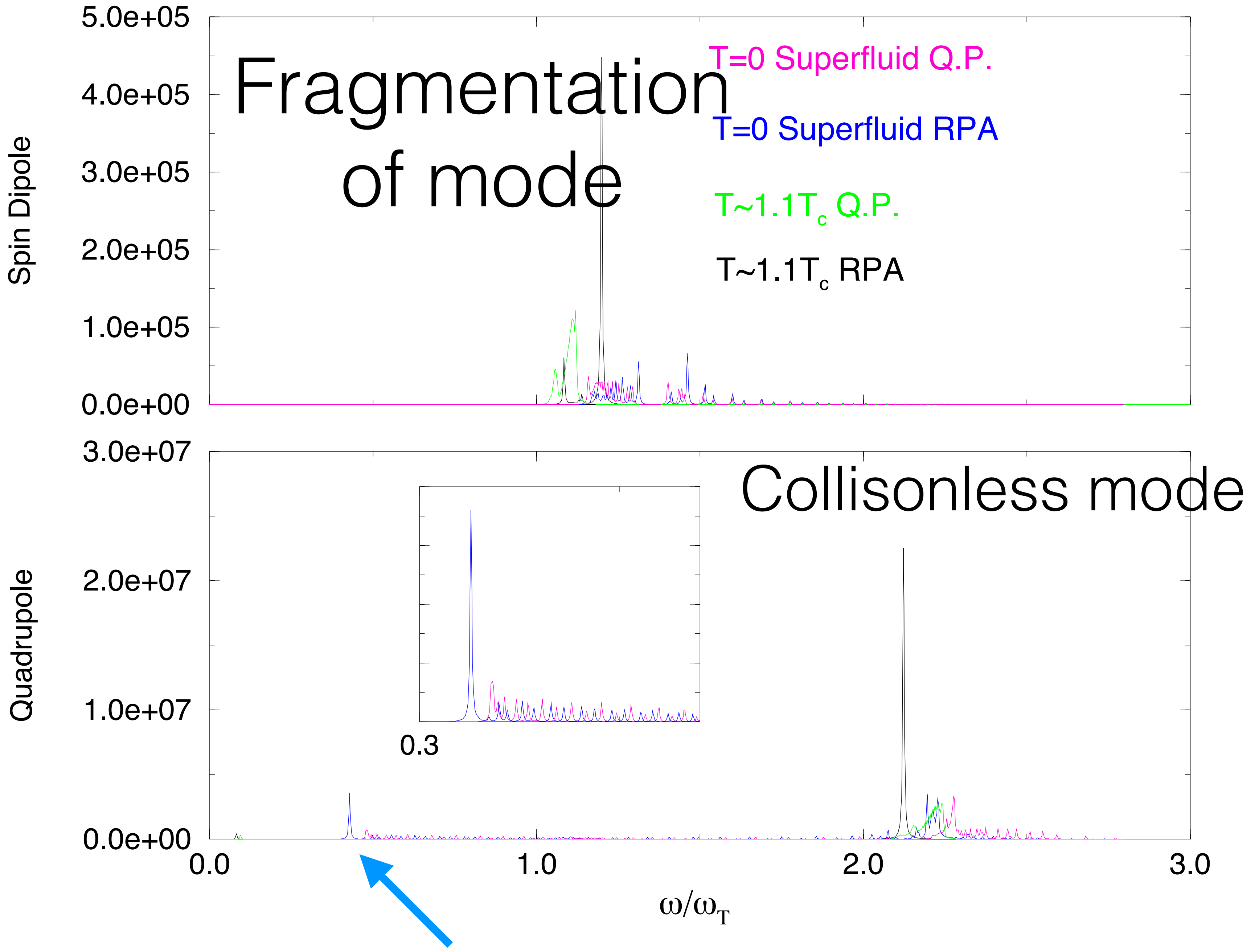
$N_\uparrow + N_\downarrow \sim 10^4$

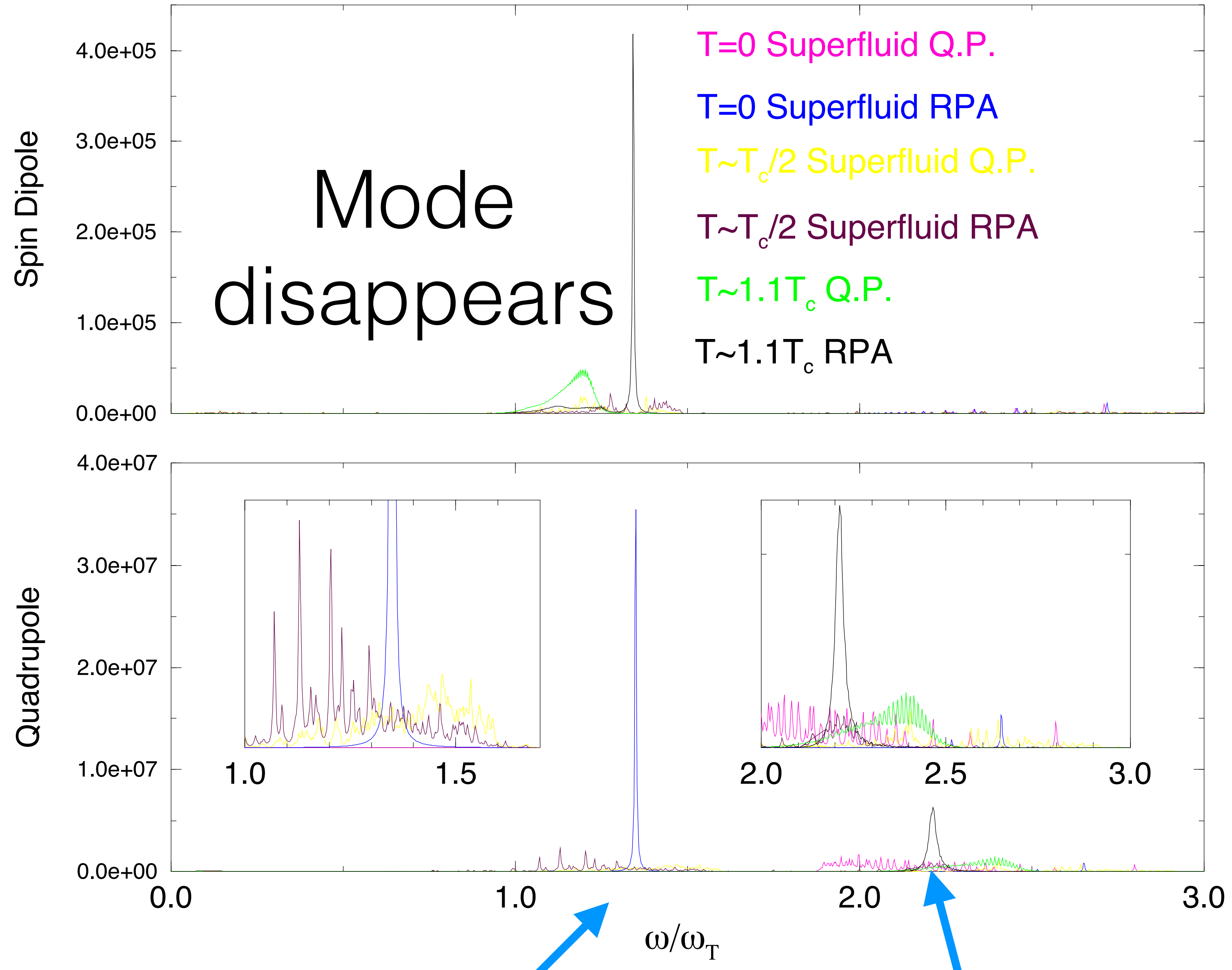
$\Delta >> \hbar\omega$
$\xi << R$











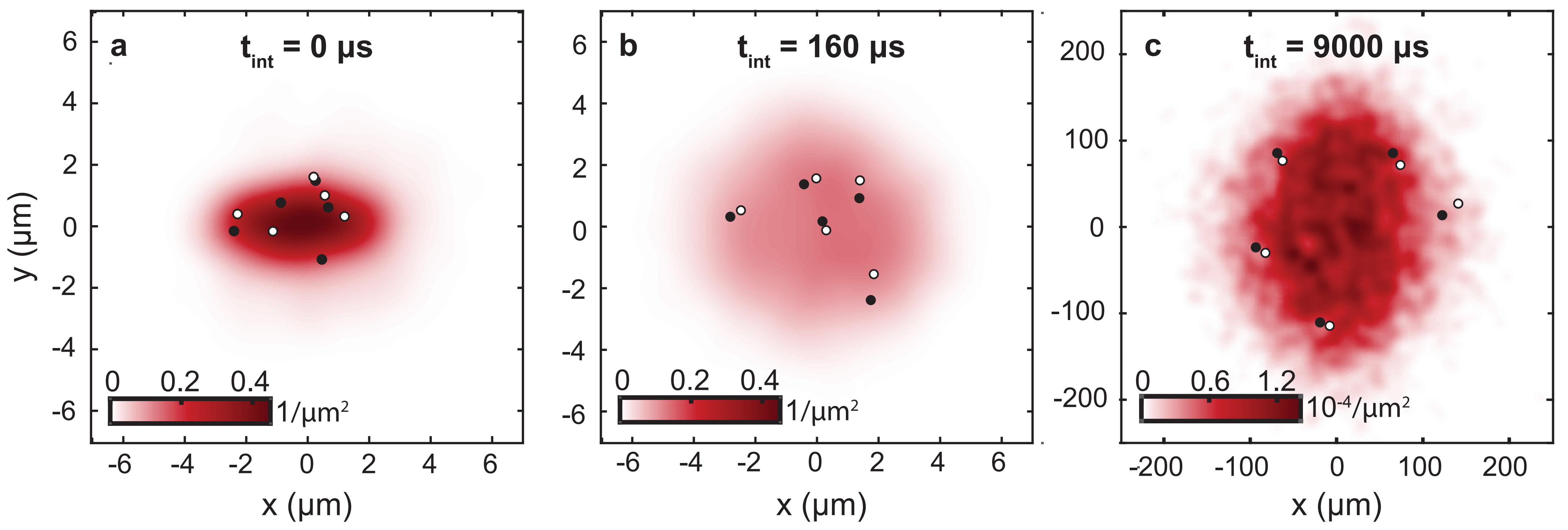
Pair-breaking mode~2Δ

Goldstone mode $\sim \sqrt{2}\omega$

Collisionless mode $\sim 2\omega$

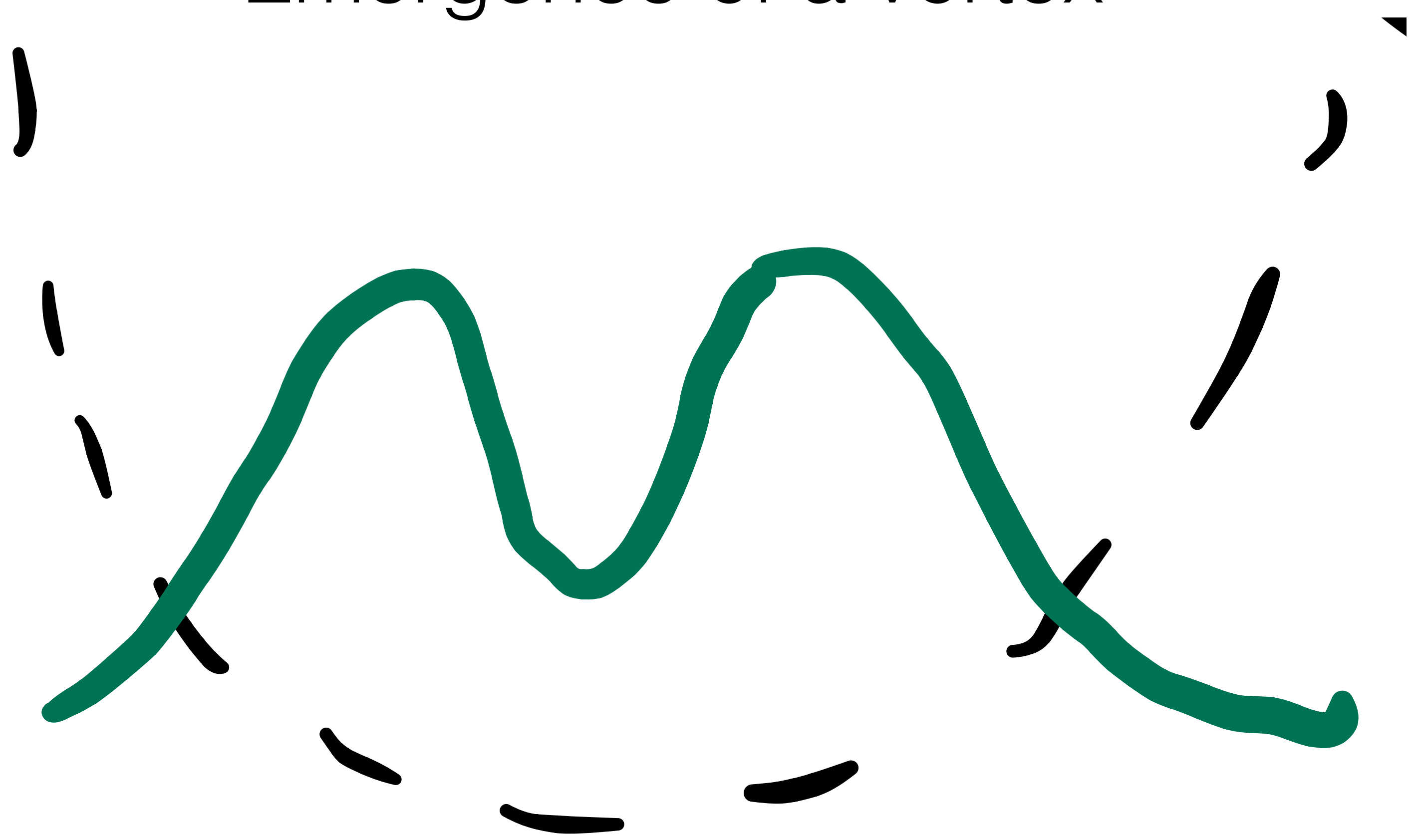
GMB and B. Mottelson, Phys. Rev. Lett. **87** 270403 (2001)

Collisional hydrodynamics & viscosity



Rotation and superfluidity

Emergence of a vortex



Theory bridging few-body ↔ many-body