



Quantum Field Simulation with a Bose-Einstein Condensate: From Cosmology to Spontaneous Pattern Formation

Cosmic
scale factor
 $a(t)$

Time t

Acoustic metric
 $ds^2 = -dt^2 + \frac{1}{c_s^2} (dr^2 + r^2 d\Omega^2)$

BEC density contrast δ_c

no ramp

$\gamma = 0.5$

Illustration: Sebastian Stapelberg (cosmos picture)



STRUCTURES
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Nikolas Liebster

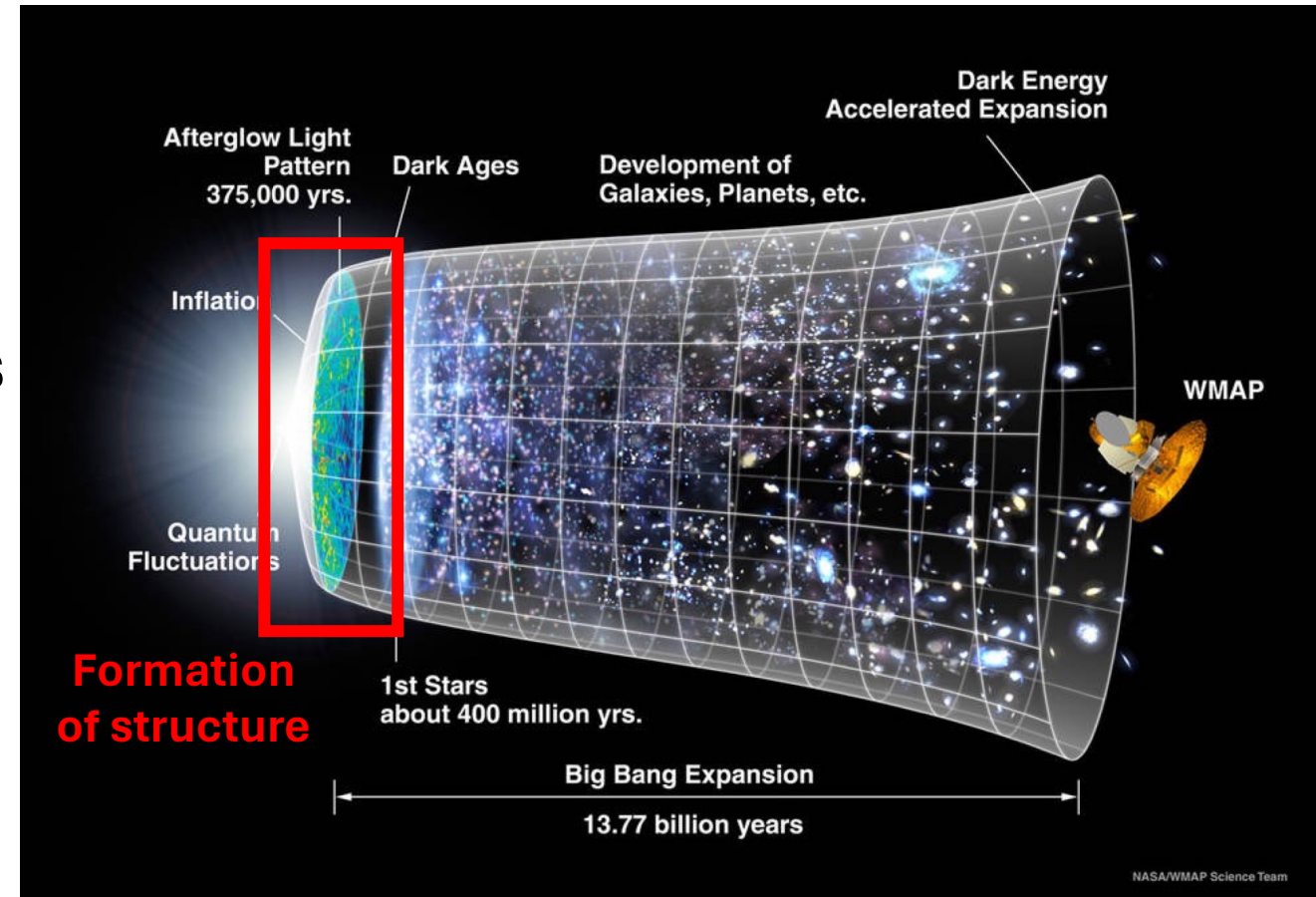
Synthetic
Quantum
Systems

SynQS

Cosmology and Quantum

- Dynamics of early universe can explain large scale structure
- Theoretical challenge
- Quantum simulator can address some questions

Want to simulate dynamics of quantum field on an expanding, curved spacetime



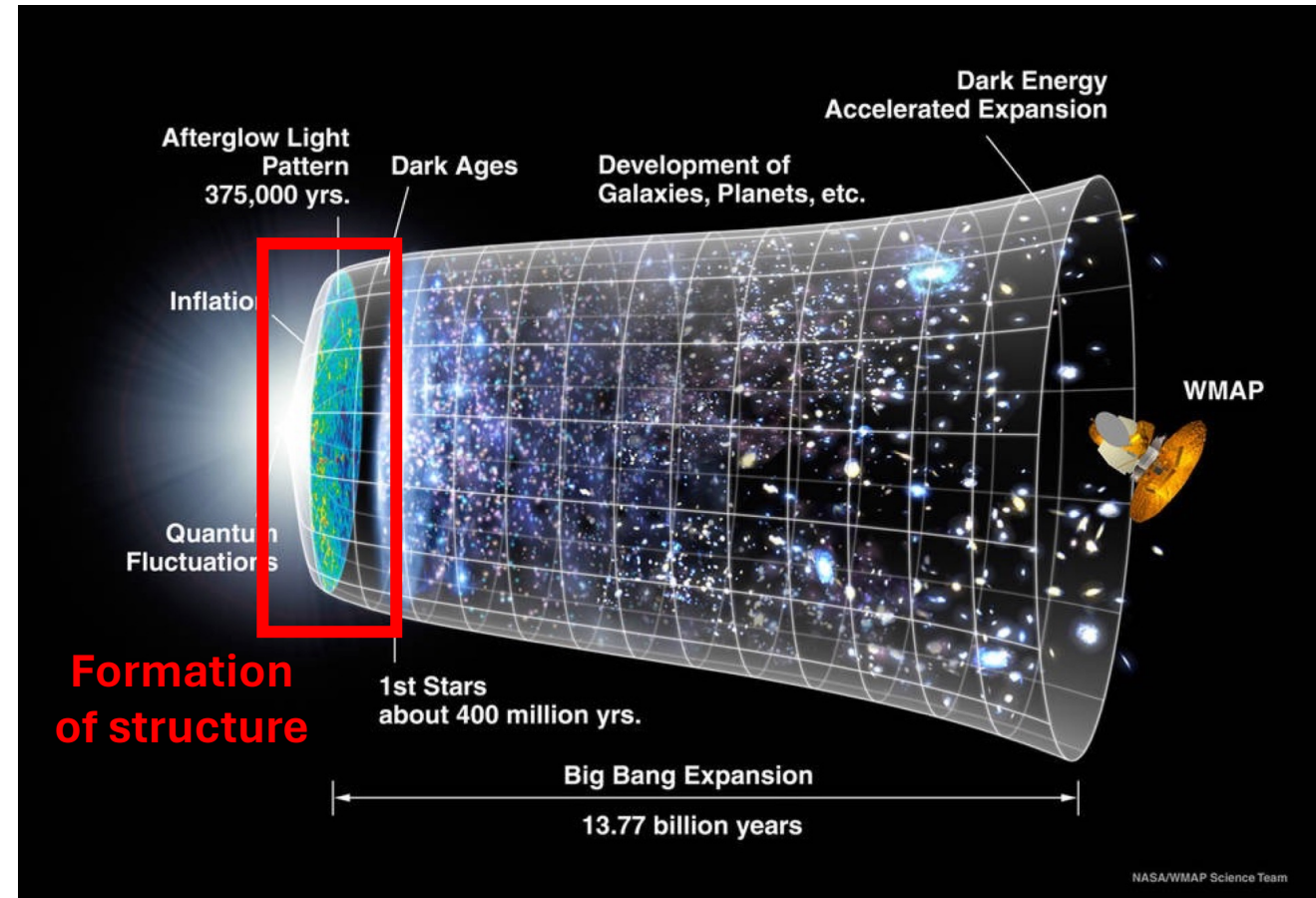
NASA/WMAP Science Team

Friedmann-Lemaître-Robertson-Walker (FLRW) Metric

$$ds^2 = -dt^2 + a^2(t) \left(\frac{du^2}{1 - \kappa u^2} + u^2 d\varphi^2 \right)$$

Time → **Time-dependent scale factor**

Space → **Spatial curvature**



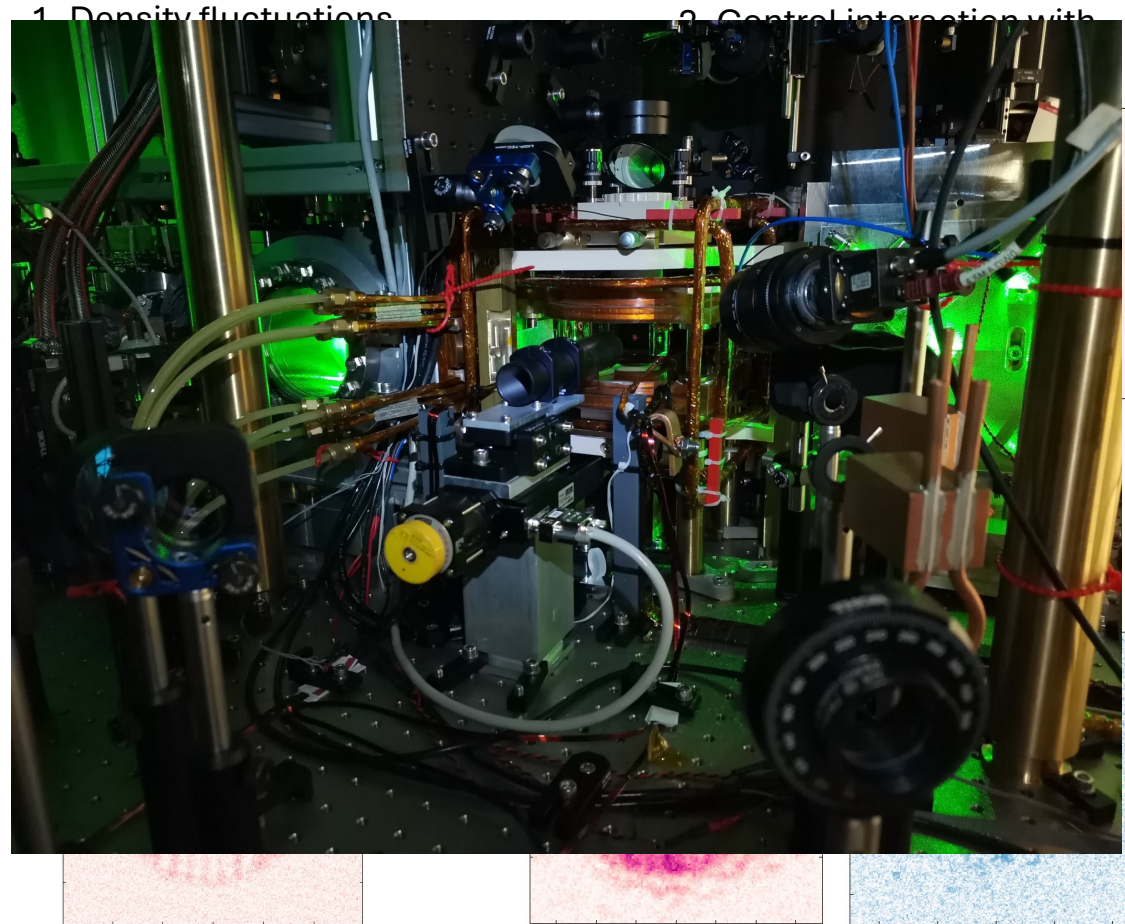
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Now we need a quantum field...

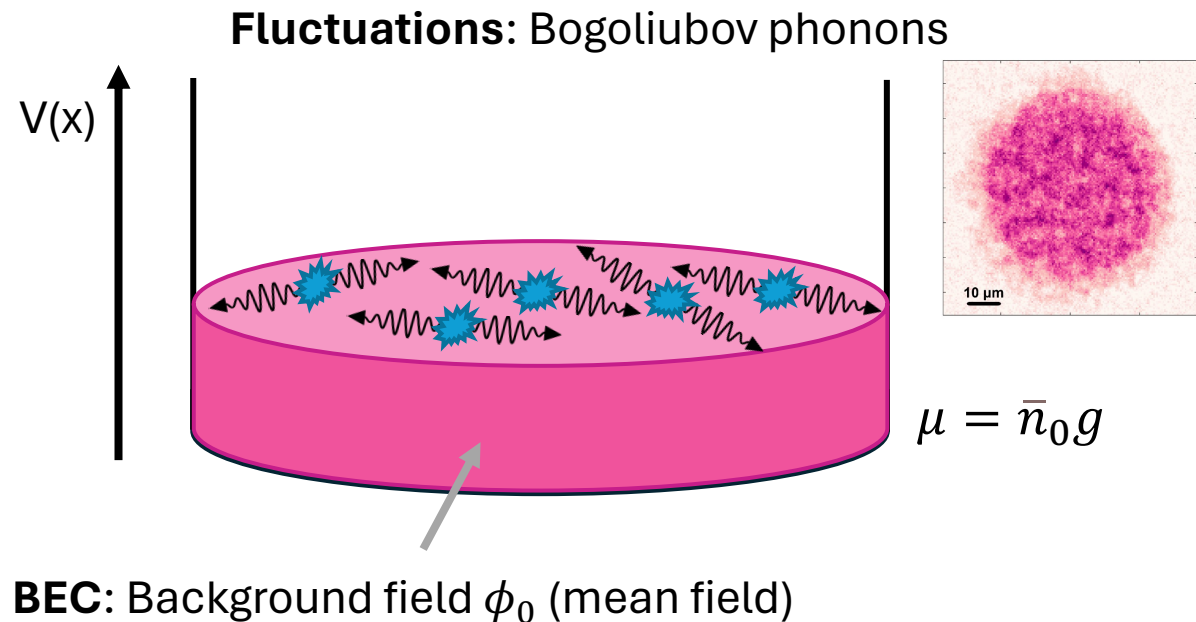
Quantum Field Simulator Wish List

- Quantum field with the correct physical properties
- Ability to drive dynamics through metric
- Good (local) control of the system
- Readout mechanism

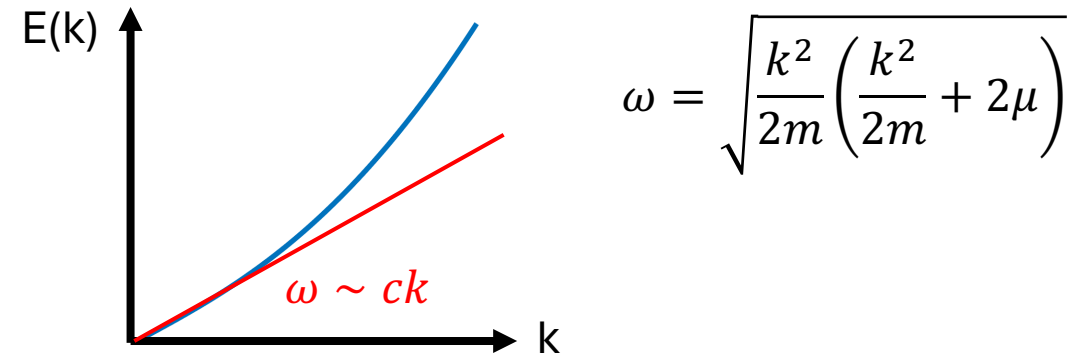
Bose-Einstein Condensate is a great platform!



Features of phonons



Bogoliubov dispersion relation:



$$c = \sqrt{\frac{n_0(r)g(t)}{m}}$$

n_0 : Background density
 g : Interaction strength

FLRW

$$ds^2 = -dt^2 + a^2(t) \left(\frac{du^2}{1 - \kappa u^2} + u^2 d\varphi^2 \right)$$



Acoustic metric in BEC

$$ds^2 = -dt^2 + \frac{1}{c^2} (dr^2 + r^2 d\varphi^2)$$

Time and space dependent speed of sound implements an FLRW metric for phonons!

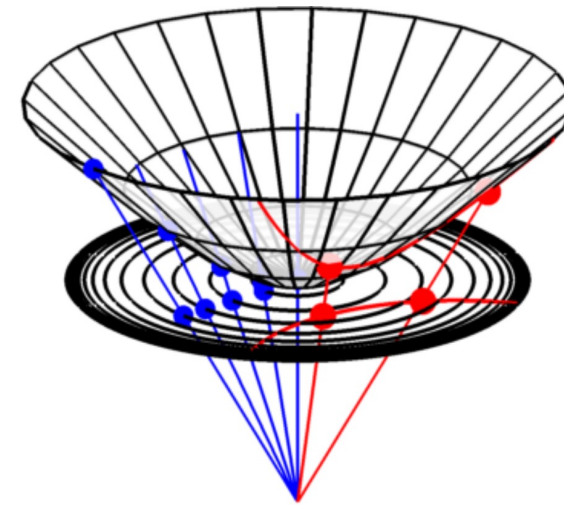
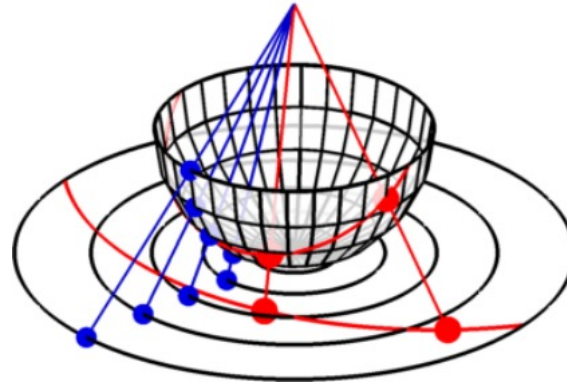
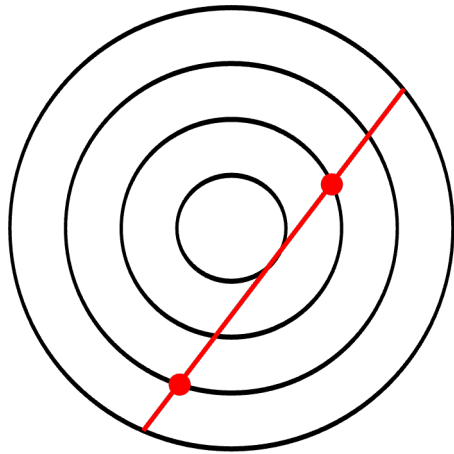
Curved Spacetimes in a BEC - Curvature

Flat $\kappa = 0$

Spherical $\kappa > 0$

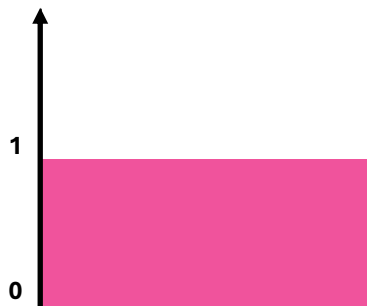
Hyperbolic $\kappa < 0$

Curved Space:

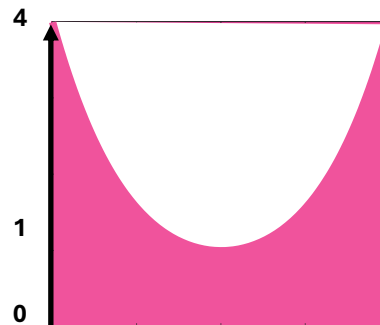


Density:

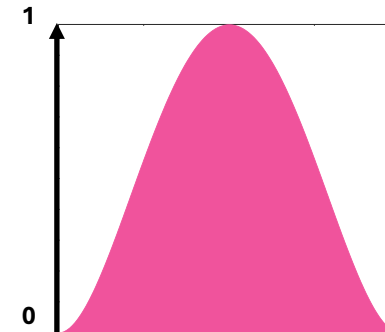
$$n_0(r) = \bar{n}_0$$



$$n_0(r) = \bar{n}_0 \left(1 + \frac{r^2}{R^2}\right)^2$$



$$n_0(r) = \bar{n}_0 \left(1 - \frac{r^2}{R^2}\right)^2$$



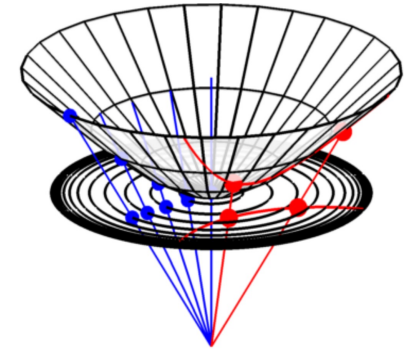
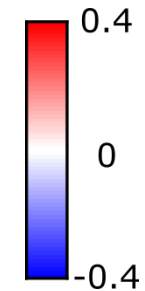
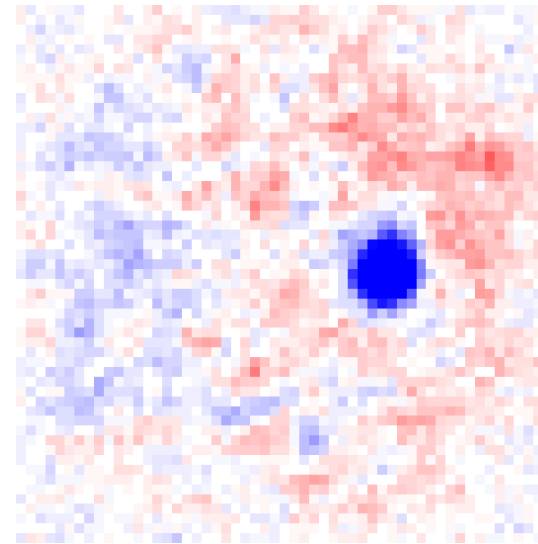
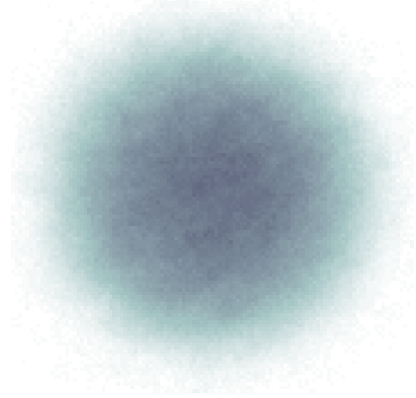
C. Viermann, M. Sparn, NL... Oberthaler, *Nature* **611**, 260-264 (2022)

Phonon Trajectories in Curved Spacetimes

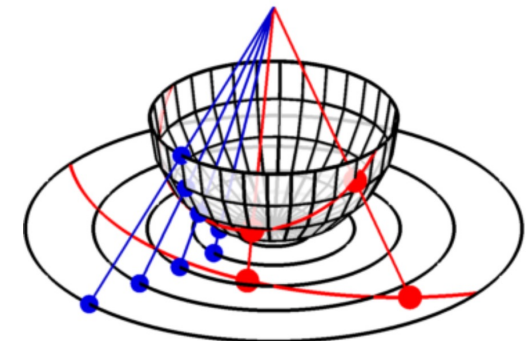
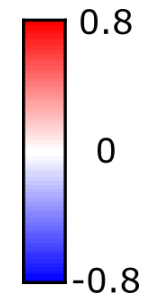
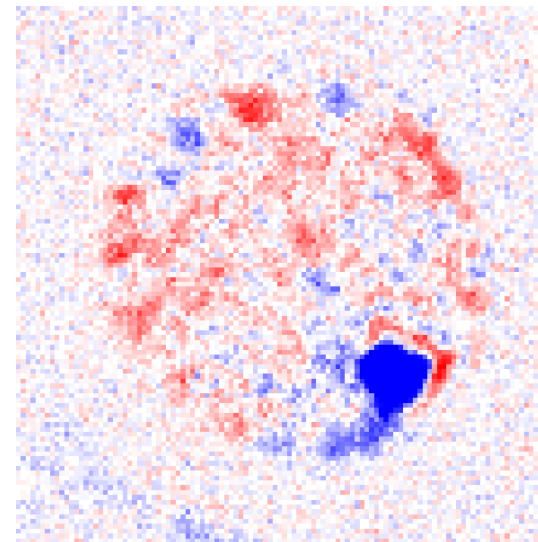
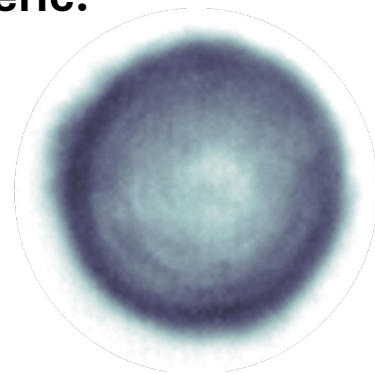
Wave packet trajectories:

- Initial density dip with blue-detuned laser beam
- Turn off beam
- Observe wave packet travelling outwards

Hyperbolic:

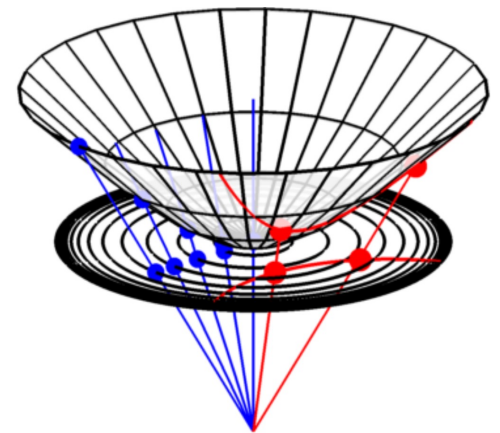
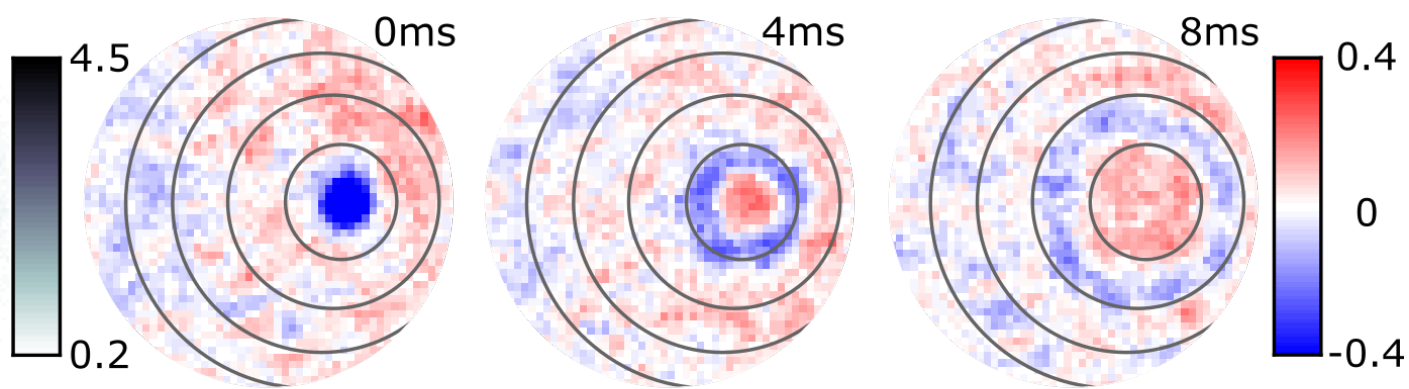
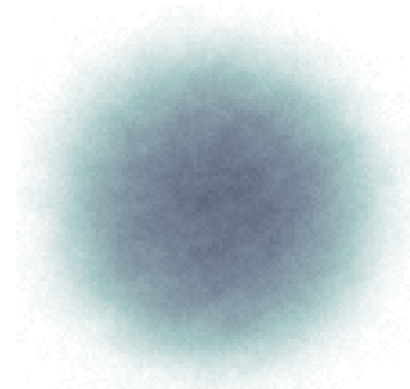


Spheric:

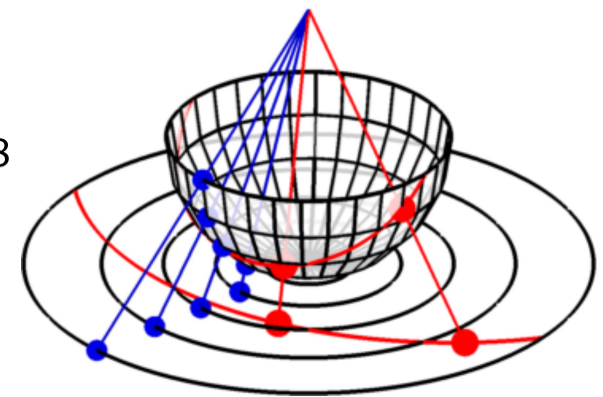
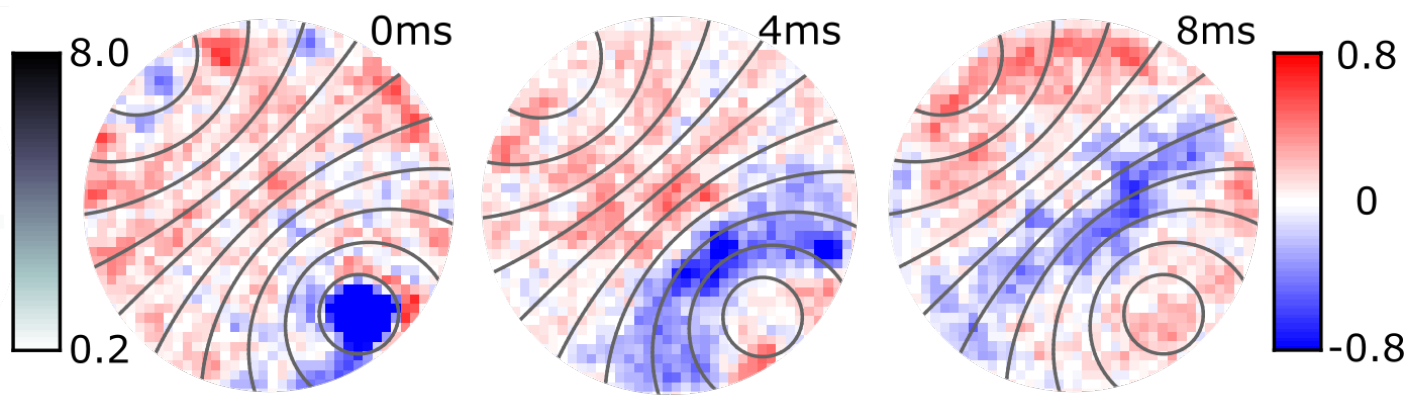
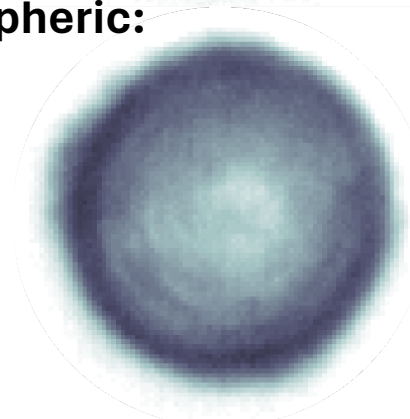


Phonon Trajectories in Curved Spacetimes

Hyperbolic:



Spheric:



Implementing Expansion

FLRW Metric:

Time

Space

$$ds^2 = -dt^2 + a^2(t) \left(\frac{du^2}{1 - \kappa u^2} + u^2 d\varphi^2 \right)$$

time-dependent
scale factor

spatial curvature

In an **expanding spacetime**,
QFT predicts
particle production

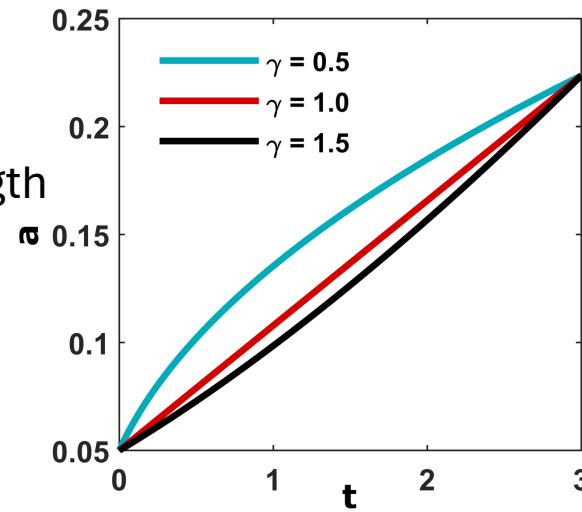
Power-law expansions:

$$a(t) = Qt^\gamma$$

$$a(t) \sim \frac{1}{g^{1/2}(t)}$$

interaction strength

$$c = \sqrt{\frac{n_0(r)g(t)}{m}}$$

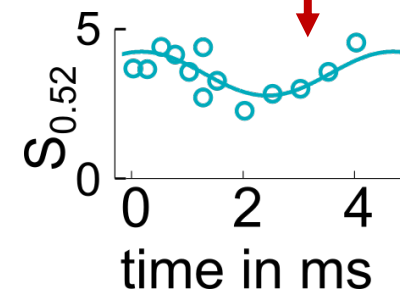
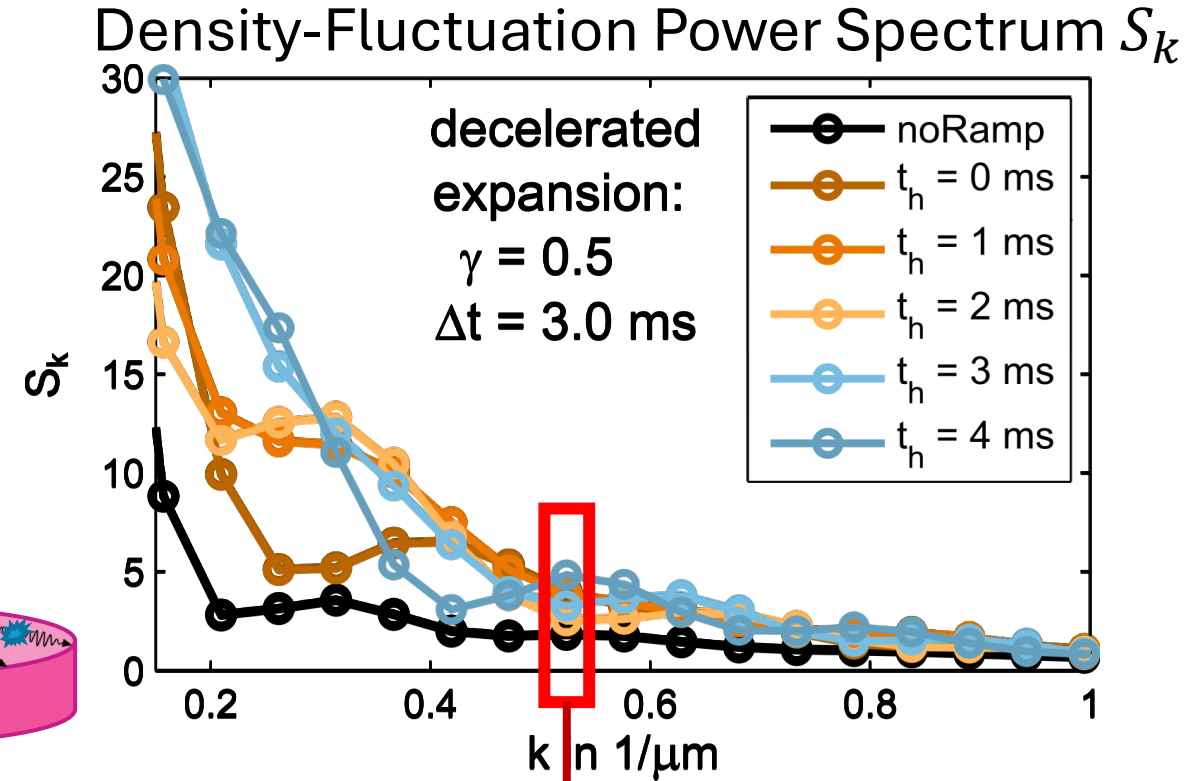
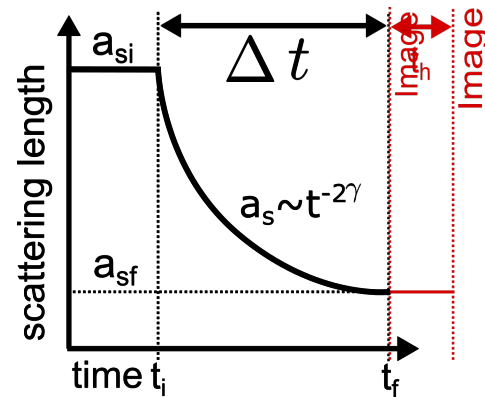
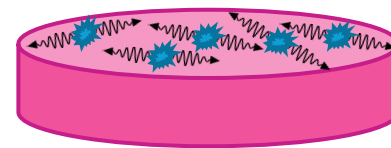
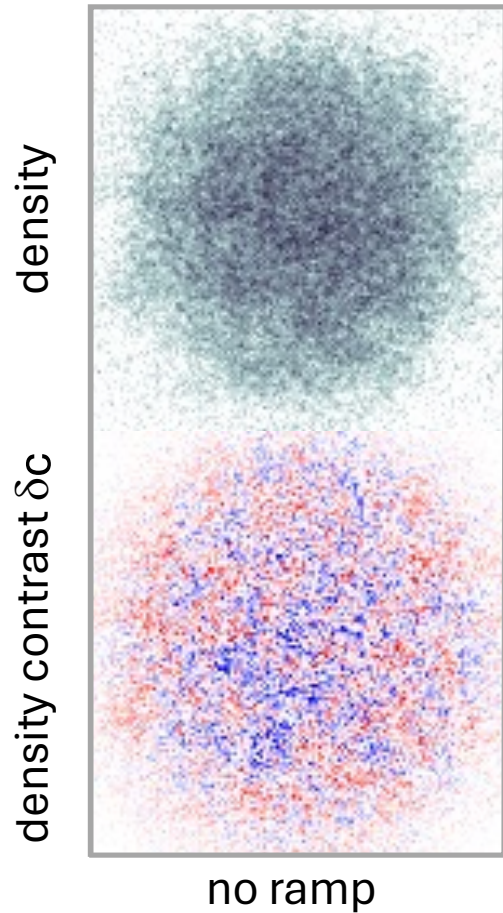


Tolosa-Simeón et al., *Phys. Rev. A* **106**, 033313 (2022)

Viermann et al., *Nature* **611**, 260-264 (2022)

Particle Production in Expanding Spacetimes

Power-law ramps: $a(t) = Qt^\gamma$



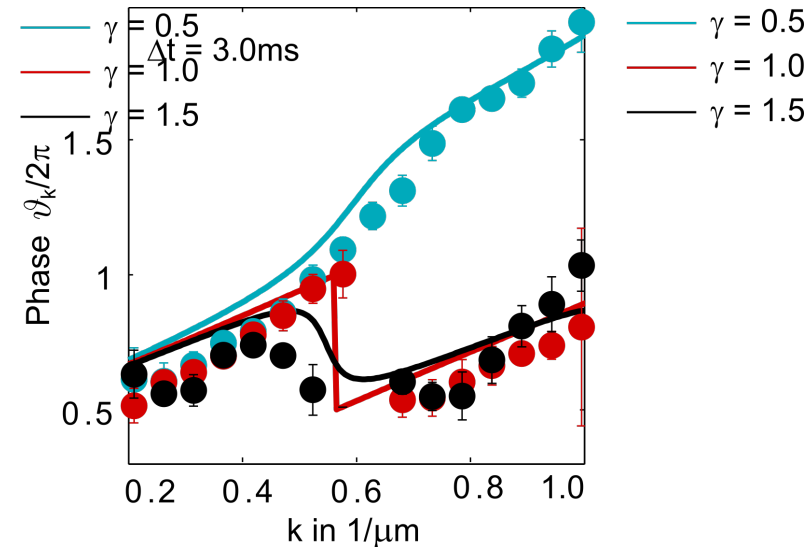
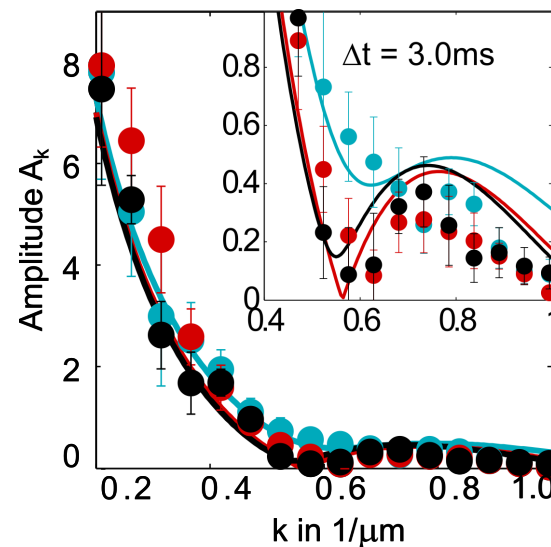
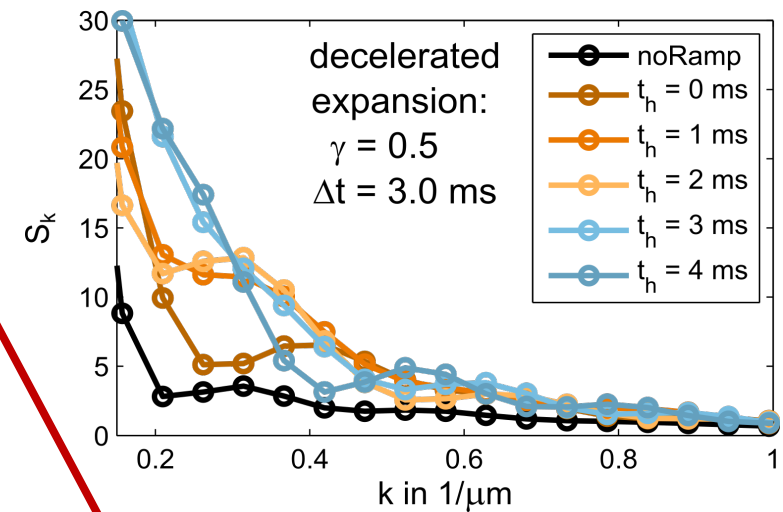
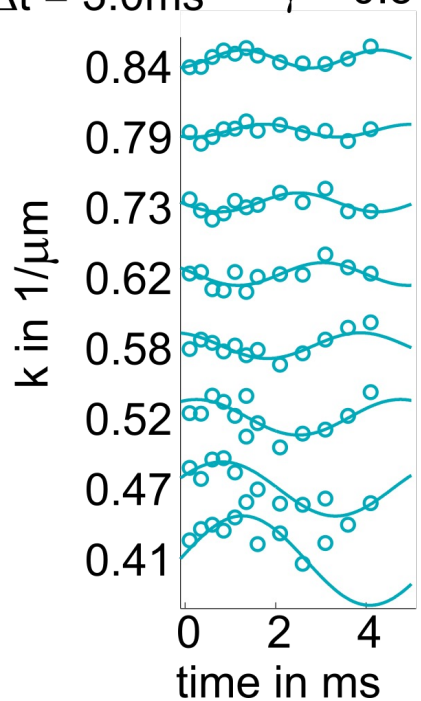
Hung et al. *Science* **341**, 1213–1215 (2013)
Viermann et al., *Nature* **611**, 260–264 (2022)

Time-Dependent Spectra

Time Evolution of Spectrum: $S_k(t) = \frac{1}{2} + N_k + A_k \cos(2\omega_k t + \vartheta_k)$

(decelerated)

$\Delta t = 3.0\text{ms}$ $\gamma = 0.5$



Tolosa-Simeón et al., *Phys. Rev. A* **106**, 033313 (2022)

Viermann et al., *Nature* **611**, 260-264 (2022)

The team!



Celia
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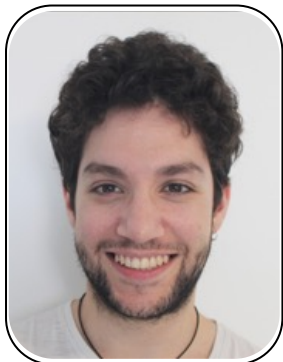
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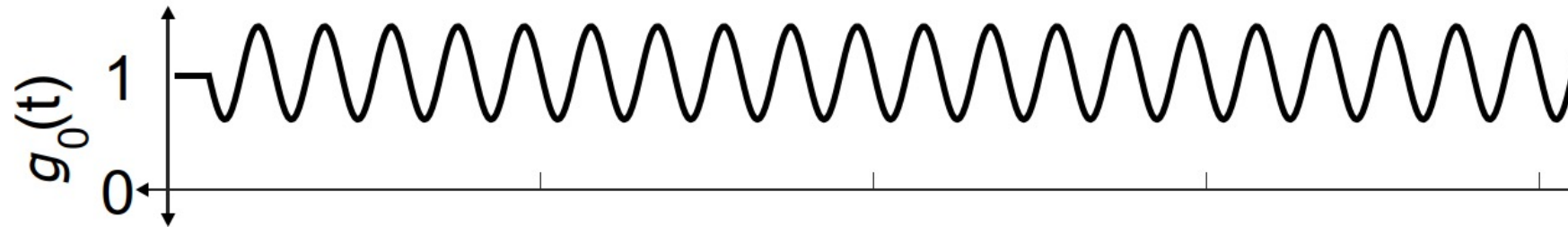
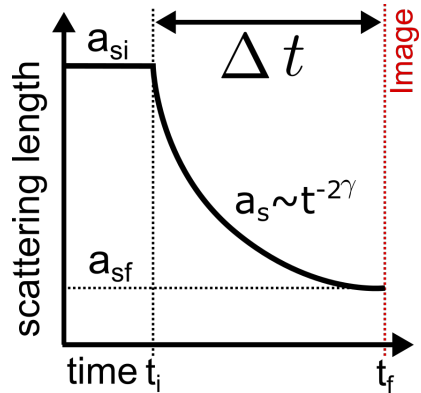


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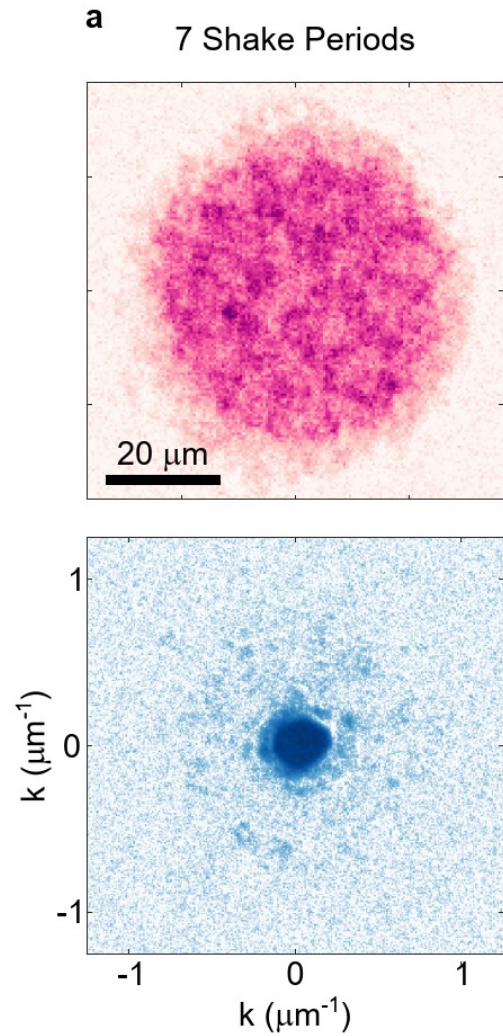
Synthetic
Quantum
Systems



What if we drive interaction periodically?

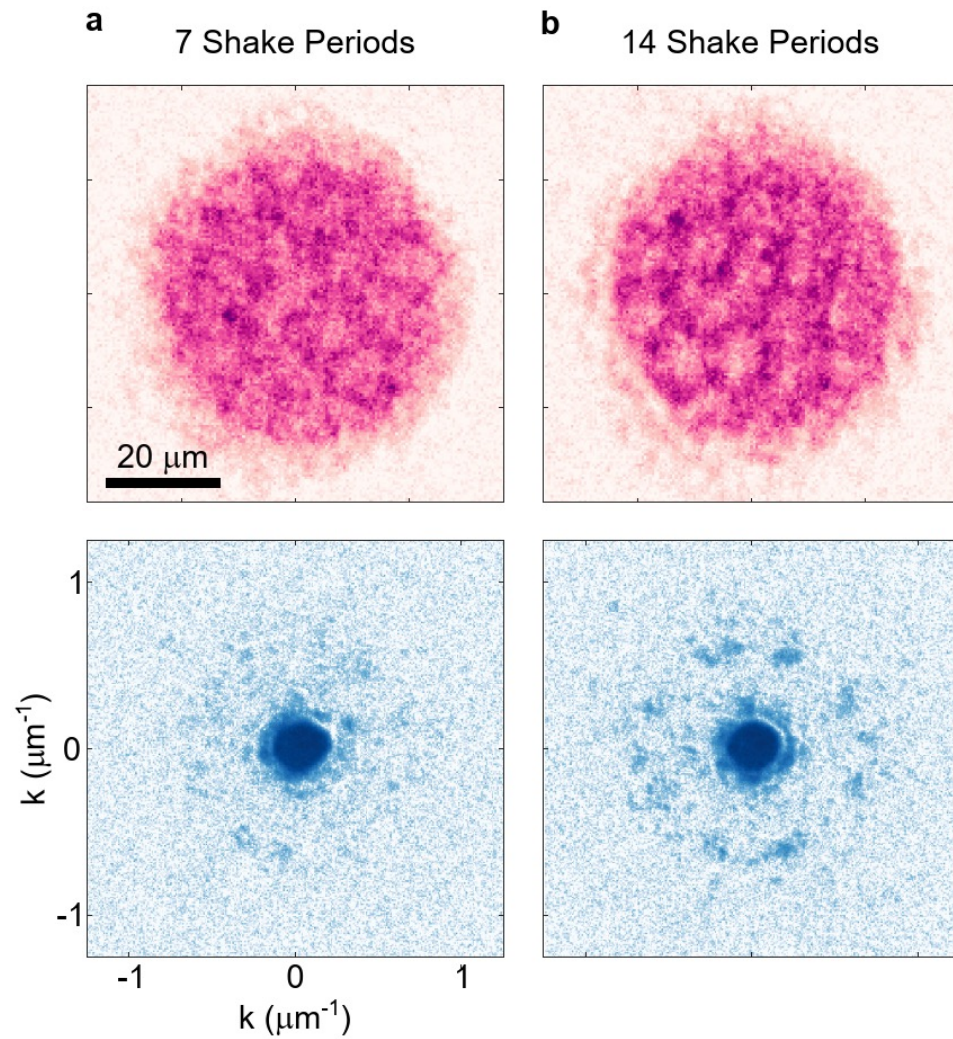


Emergence of Crystalline Structure



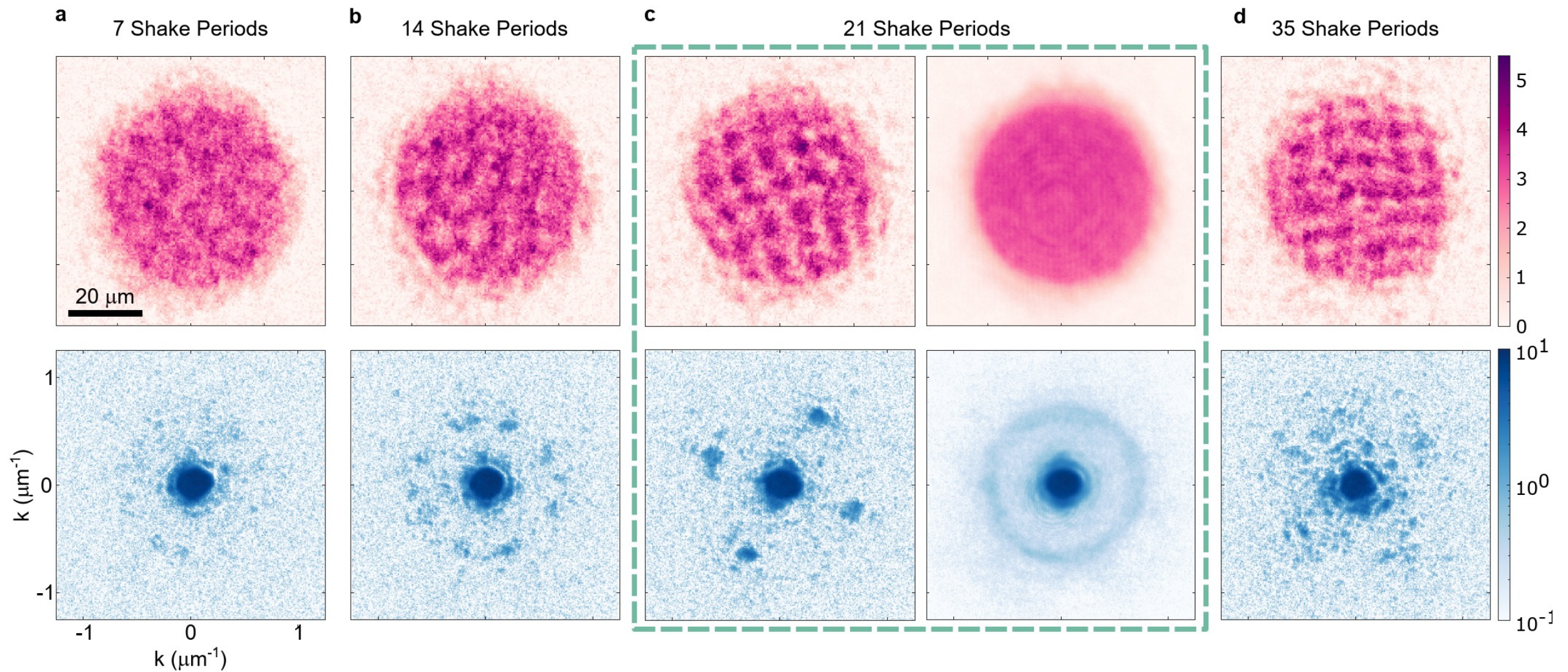
Exp: NL, et al. arXiv, 2309.03792

Emergence of Crystalline Structure



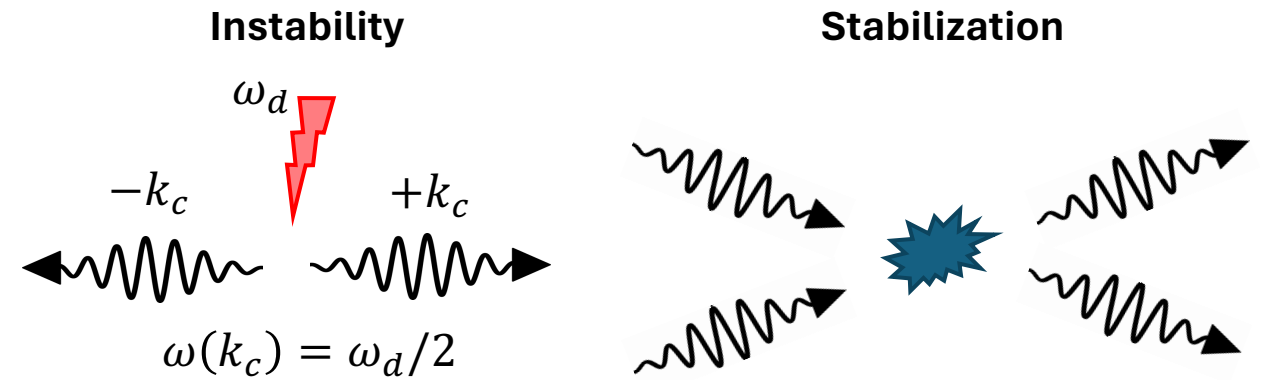
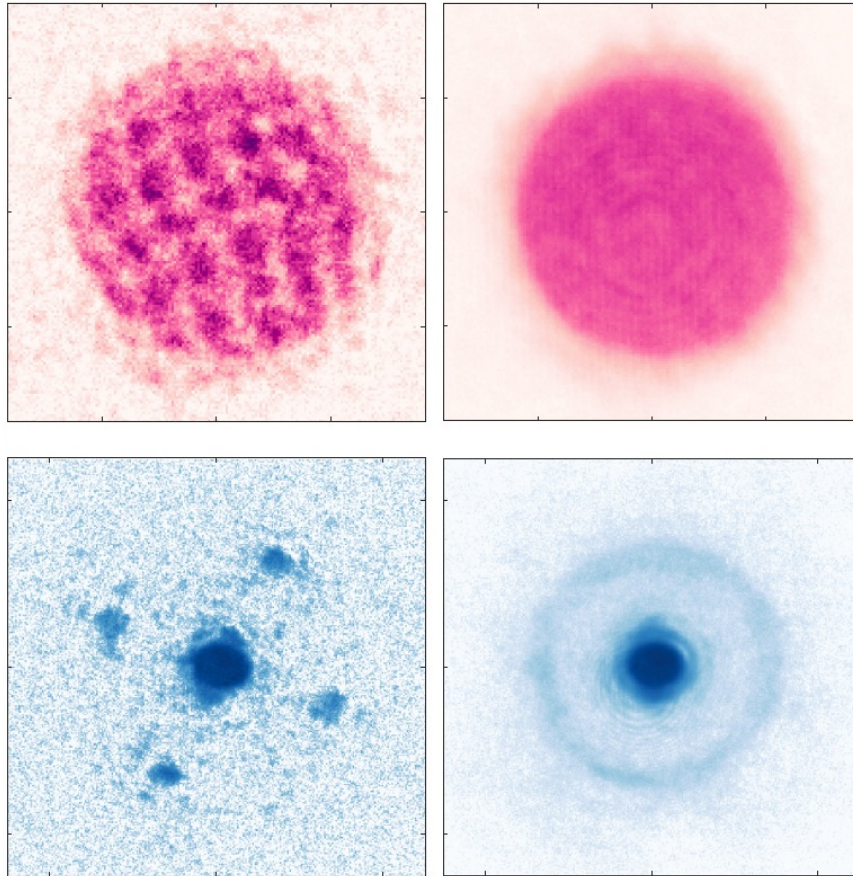
Exp: NL, et al. arXiv, 2309.03792

Emergence of Crystalline Structure



Exp: NL, et al. arXiv, 2309.03792

Emergence of Crystalline State



The team!



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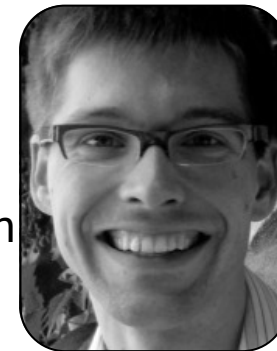
Markus
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Keisuke
Fujii



Sarah
Görlitz



Tilman
Enss



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Conclusion

