A Self-Consistent Model for Wakefield and Space Charge Calculations

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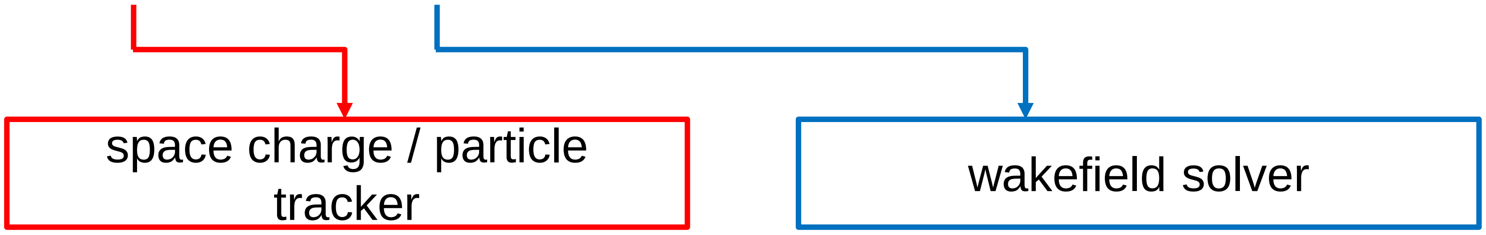
Outline

- Beam dynamics: Solvers @TEMF
- Scattered Field Formulation for Coupled Space Charge and Wakefield Calculations
- Results for traveling wave gun @SwissFEL

Task

- **Solve Maxwell's eqs. + Eq. of Motion:**
  - # Particles, Geometry, multi-scale → ~~Full EM-Particle in cell~~



space charge / particle tracker

- Poisson eq. in Lorentz frame
- Free-space assumption
- − No transient fields

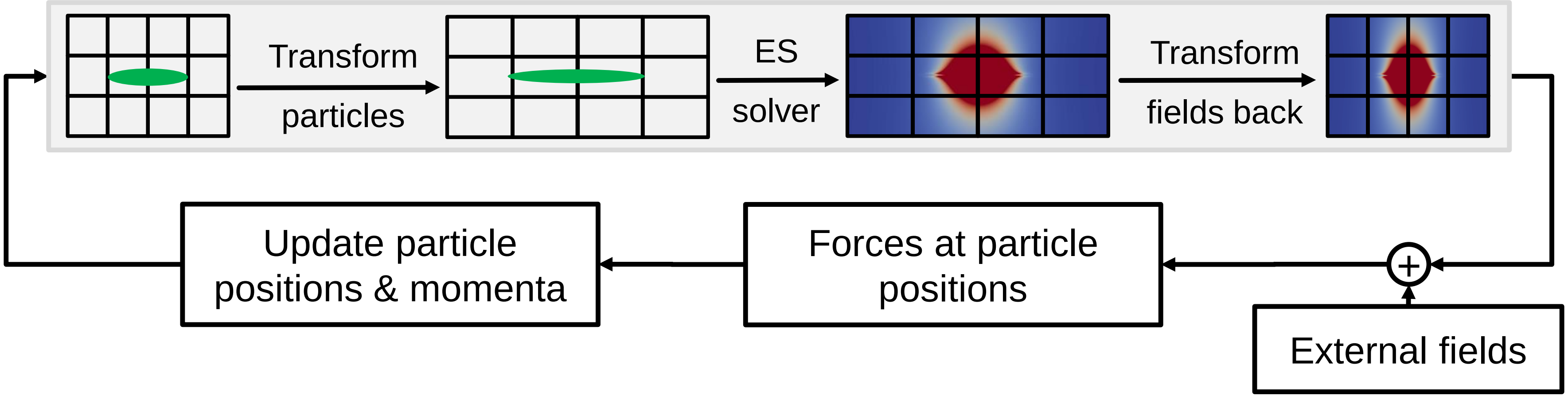
wakefield solver

- EM wave eq.
- Particles => current
- − No intermediate feedback

Solver @TEMF: REPTIL | PBCI

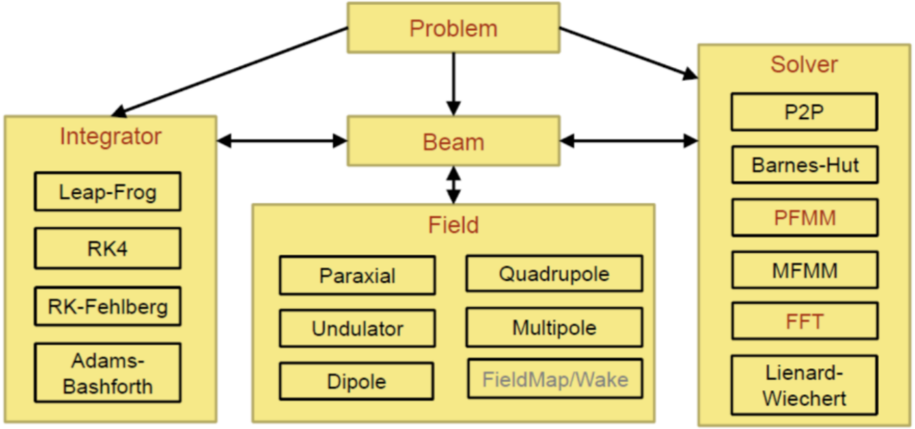
Particle Tracking in REPTIL

- **Solve Space Charge field + Eqs. of motion**
  - Assume particle cloud in free space + nearly-uniform movement
  - Electrostatic field solver in particle’s rest frame



Particle Tracking in REPTIL

- **Relativistic Particle Tracker for Injectors and Linacs (REPTIL)**
  - Nx6D time domain, multi-node & multi-thread
  - Space Charge Field solvers: Grid-based (e.g. 3D-**FFT**) or non-grid (**FMM**, LW)
  - Time integrators (adaptive, symplectic, …)
  - Fieldmaps, optimization engine



Problem

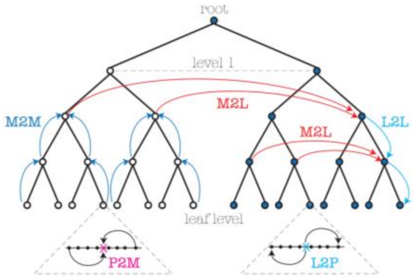
Integrator
Leap-Frog
RK4
RK-Fehlberg
Adams-Bashforth

Beam

Field
Paraxial
Undulator
Dipole
Quadrupole
Multipole
FieldMap/Wake

Solver
P2P
Barnes-Hut
PFMM
MFMM
FFT
Lienard-Wiechert

$$\varphi(x) = \int G(x - x')\varrho(x')d^D x$$



Wakefield Simulation in PBCI

- **Solve Maxwell‘s wave equations in time domain**

current through grid faces

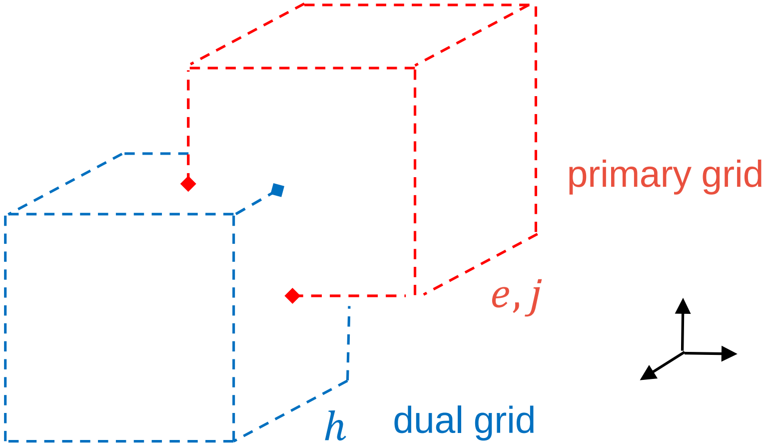
voltages over grid edges

$$\left.\begin{aligned} \frac{d}{dt}E &= \varepsilon^{-1}\,\mathrm{curl}\,H - \varepsilon^{-1}J \\ \frac{d}{dt}H &= -\mu^{-1}\,\mathrm{curl}\,E \end{aligned}\right\} \xrightarrow[\text{FIT / FDTD}]{\text{Discretization}} \frac{d}{dt}\begin{pmatrix}h\\ e\end{pmatrix} = \begin{pmatrix}0 & -M_\mu^{-1}C\\ M_\varepsilon^{-1}C^T & 0\end{pmatrix}\begin{pmatrix}h\\ e\end{pmatrix} - \begin{pmatrix}0\\ M_\varepsilon^{-1}j\end{pmatrix}$$

- Wakefield codes: particle beam is
  1) ultra-relativistic ($v_z = c$) and
  2) rigid ($v_z = \text{const.}$)

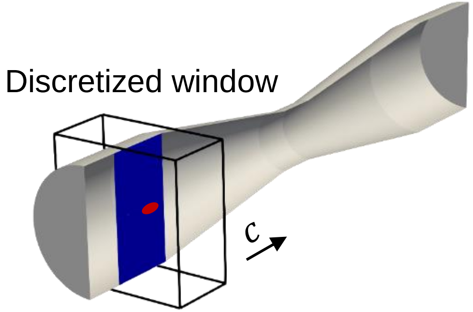
  → prescribed current density j

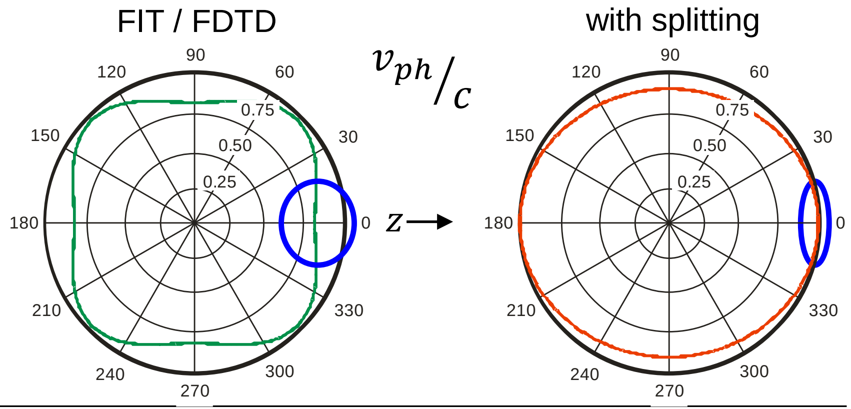
  → kicks (per component)



Wakefield Simulation in PBCI

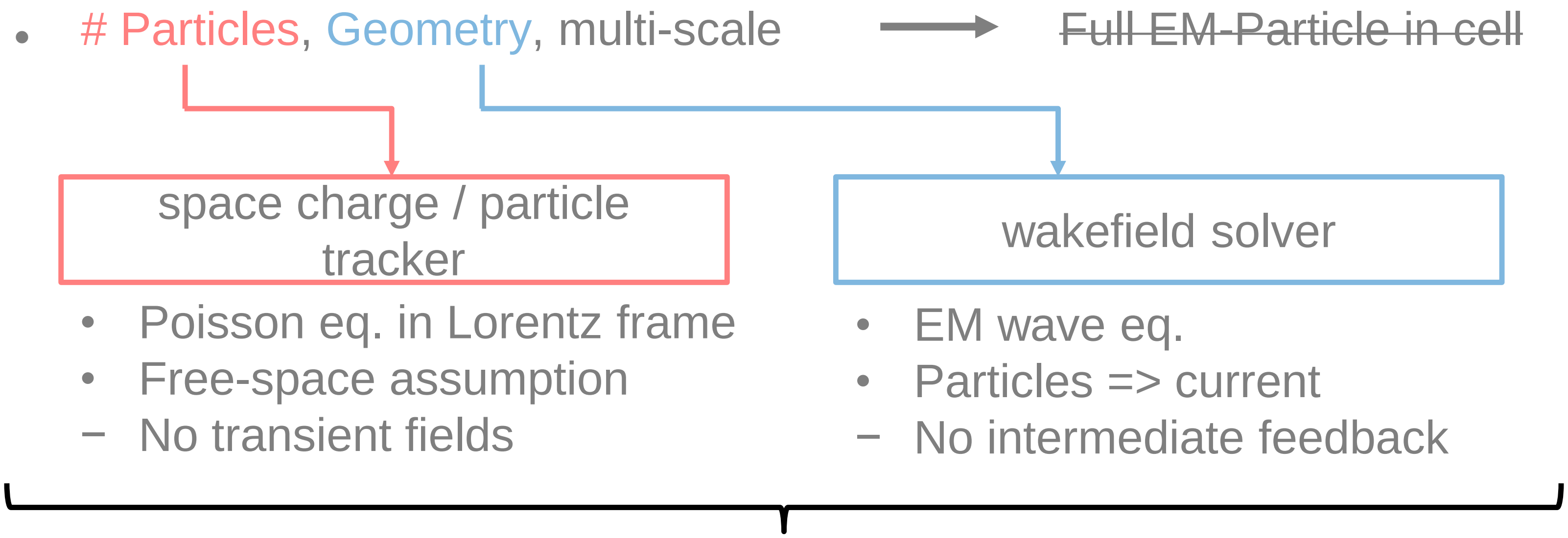
- **Wakefield solver Parallel Beam Cavity Interaction (PBCI)**
  - Especially for short relativistic bunches, long transients
  - 3D time domain, **boundary conformal** FIT / FDTD Maxwell EM-wave solver, multi-node & multi-thread
  - **Moving window**, dispersion-free along z (**operator splitting**), PML, SIBC, conducting material, indirect integration





Task

- **Solve Maxwell's eqs. + Eq. of Motion:**



  - # Particles, Geometry, multi-scale → ~~Full EM-Particle in cell~~

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wakefield solver

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- Particles => current
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Take the best from both?

Scattered Field Formulation

- **Idea: Separate field contributions**

“Incident field”

- Mxw. with beam current

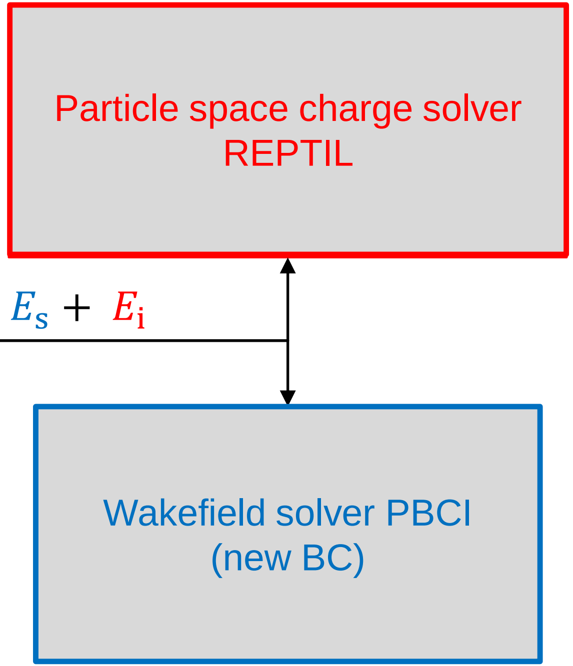
$$\frac{d}{dt} E = \varepsilon^{-1} \operatorname{curl} H - \varepsilon^{-1} J$$

$$\frac{d}{dt} H = -\mu^{-1} \operatorname{curl} E$$

BC: $E_t = 0$ on Γ_{PEC}

“Scattered field”

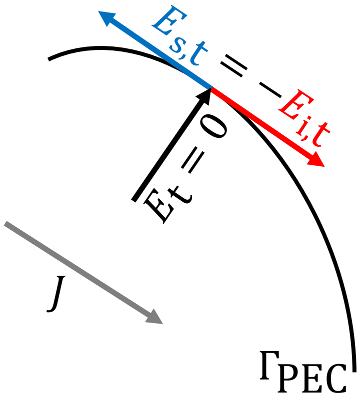
- homogeneous Mxw.
- modified boundary conditions



$E = E_s + E_i$

Particle space charge solver REPTIL

Wakefield solver PBCI (new BC)



$$E_s(0) = E(0) - E_i(0)$$

Set “incident field” to the space charge field of the particles in free-space

Scattered Field Formulation in FIT

- **Realization in FIT – conforming boundaries:**
  - Modification of Faraday's law at PEC boundary

$$\boxed{\frac{d}{dt}\begin{pmatrix} h_{\mathrm{s}} \\ e_{\mathrm{s}} \end{pmatrix} = \begin{pmatrix} 0 & -M_{\mu}^{-1} C \\ M_{\varepsilon}^{-1} C^{T} & 0 \end{pmatrix} \begin{pmatrix} h_{\mathrm{s}} \\ e_{\mathrm{s}} \end{pmatrix} - \begin{pmatrix} M_{\mu}^{-1}\, j_{\mathrm{mag}} \\ 0 \end{pmatrix}}$$

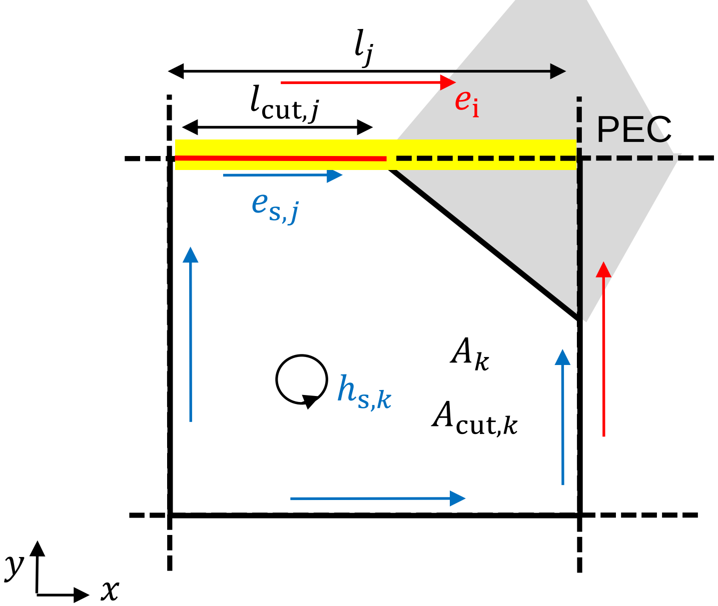
  - Restriction of incident field to conformal lengths / areas

$$\boxed{e_j = e_{\mathrm{s},j} + \frac{l_{\mathrm{cut},j}}{l_j}\, e_{\mathrm{i},j}} \qquad \boxed{b_k = b_{\mathrm{s},k} + \frac{A_{\mathrm{cut},k}}{A_k}\, b_{\mathrm{i},k}}$$

$$\boxed{\Rightarrow j_{\mathrm{mag}} = C\, I_L\, e_{\mathrm{i}} + I_A\, C\, e_{\mathrm{i}}}$$

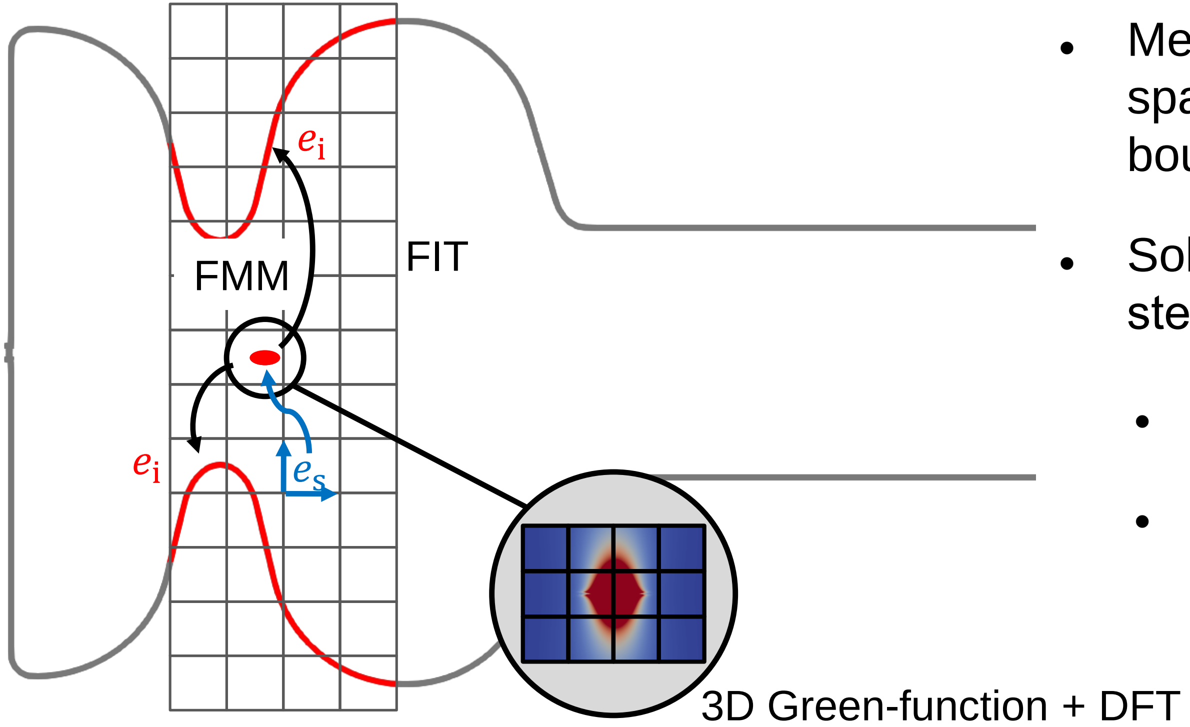
  - Rest of FIT- operators remain the same

$$\boxed{e = e_{\mathrm{s}} + e_{\mathrm{i}}}$$



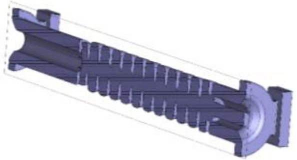
Primary FIT face

Coupling: PBCI + REPTIL



- Mesh-free, fast evaluation of space-charge farfield on boundary: FMM
- Solvers independent (grid, time step, optimization, …)
  - Arbitrary geometry
  - Arbitrary beam dynamics

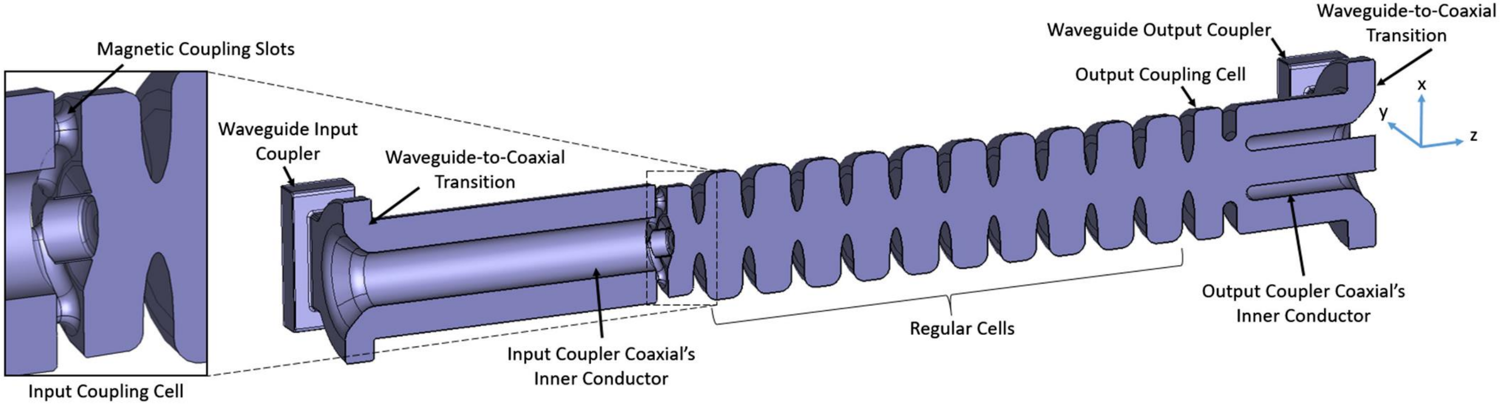
Traveling Wave Gun Model



- 12-cell TW gun under design at SwissFEL (Lucas)
- Narrow, long geometry: 5mm iris radius, ~22cm acceleration path length

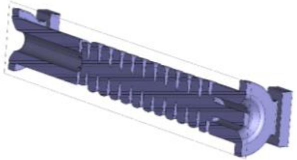
Bunch:

Charge	0.2nC
Length	~0.5mm
Size	~1mm
Energy	13MeV at gun exit

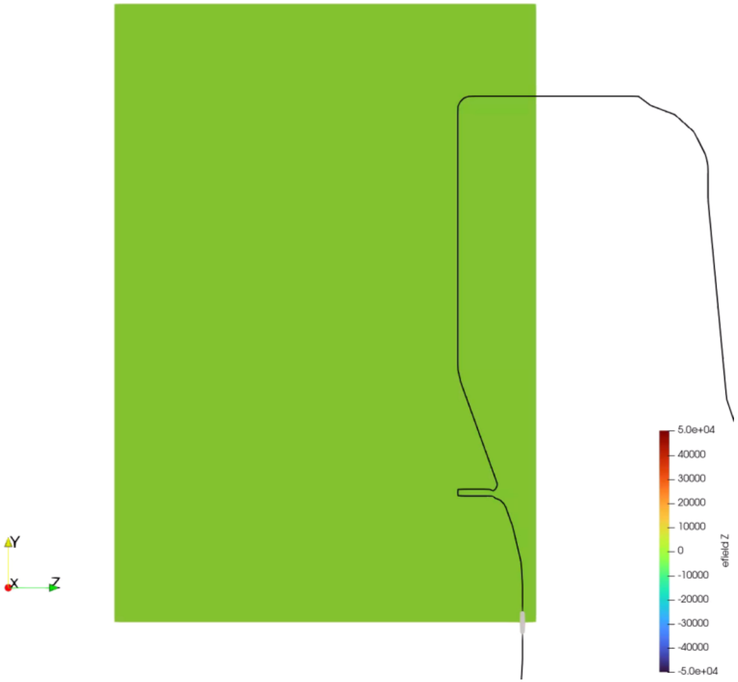


Lucas 2023, DOI: 10.1103/physrevaccelbeams.26.103401

Traveling Wave Gun Model



- 12-cell TW gun under design at SwissFEL (Lucas)
- Narrow, long geometry: 5mm iris radius, ~22cm acceleration path length
- Video: fields build up over time



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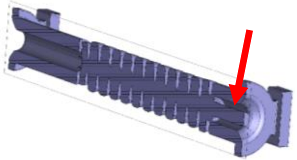
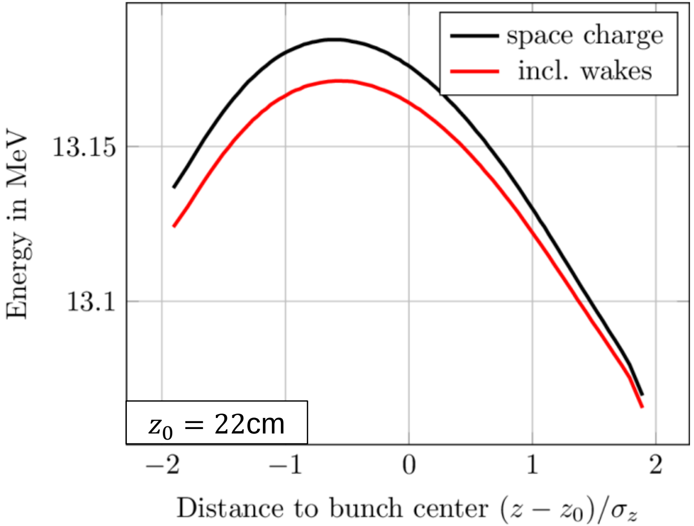
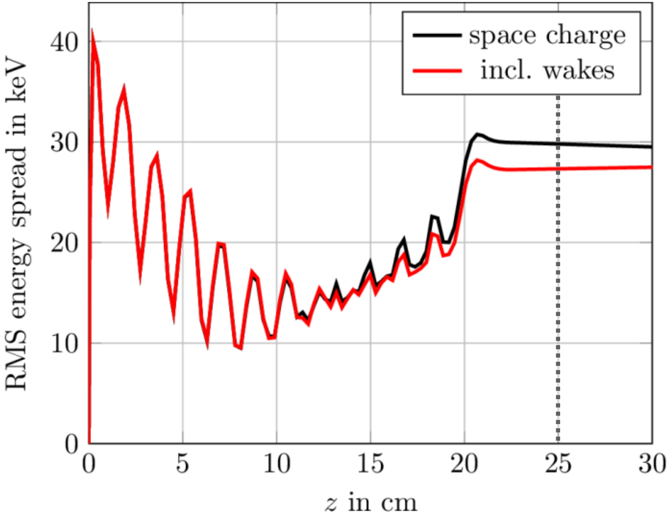
cmp. IPAC'24, DOI: 10.18429/JACoW-IPAC2024-WEPR71

Energy Chirp

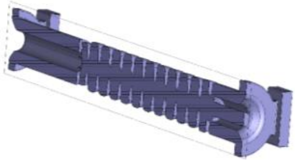


- Wakefields reduce energy chirp in gun
  - Wakes reach tail first
- ~10% RMS energy spread reduction at end of gun





Full Injector Line Simulation



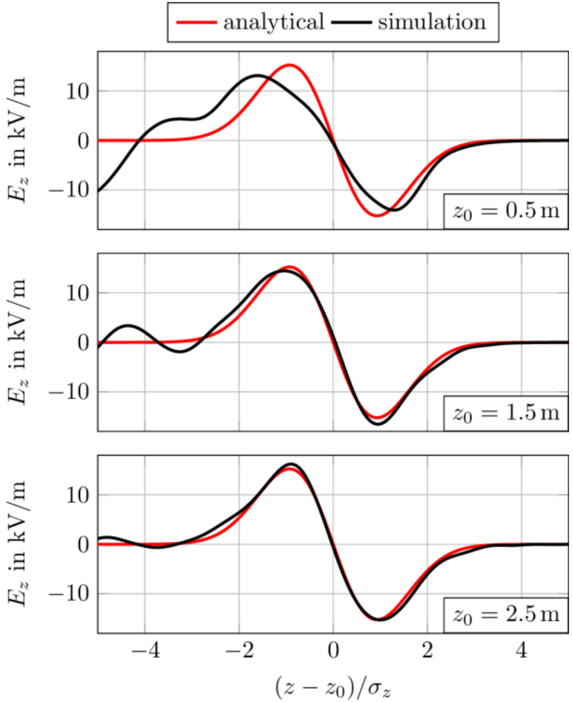
- Field in beam pipe approaches space charge impedance field

  → Weak coupling of wakefields to beam pipe and downstream sections

  → Include wakefields up to first accelerating section, continue with space charge solver only

Space Charge Impedance:

$$E_z(z) = \frac{-Q}{2\pi\varepsilon_0\gamma^2}\Lambda\frac{d\lambda(z)}{dz}$$



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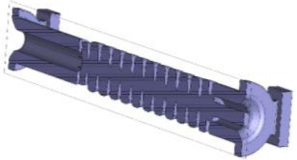


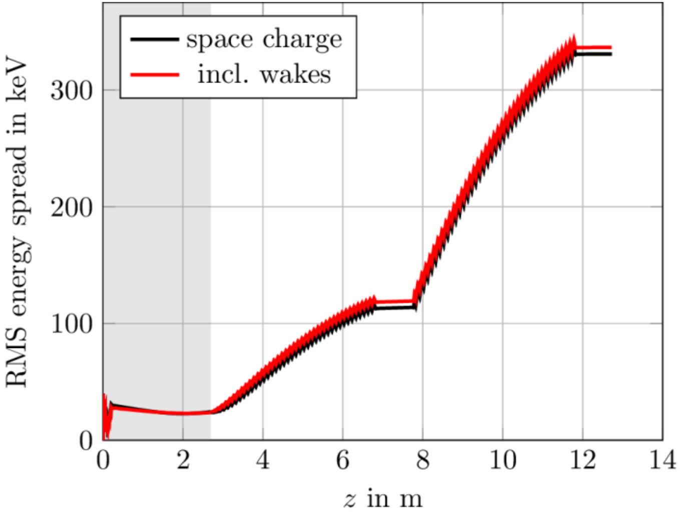
- Field in beam pipe approaches space charge impedance field

  → Weak coupling of wakefields to beam pipe and downstream sections

  → Include wakefields up to first accelerating section, continue with space charge solver only

- Difference in RMS energy spread:

  5.5keV (simulated), 7.1keV (analytical)



Summary

- Coupled Simulations:
  - Space Charge Solver REPTIL
  - Wakefield Solver PBCI
  - Fast Multipole Method
  - Scattered Field Formulation
- Electron Gun:
  - Effect of wakes on energy chirp
  - Limited coupling to downstream section