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Modeling of intrabeam scattering in electron injectors

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Introduction

- Uncorrelated energy spread in x-ray FELs

  - FEL brightness: $$B = \frac{2I}{\varepsilon_{n_x}\varepsilon_{n_y}\sigma_\gamma}$$

    peak current

    transverse emittance

    uncorrelated (slice) energy spread (SES)

Lower SES → higher brightness electron beams

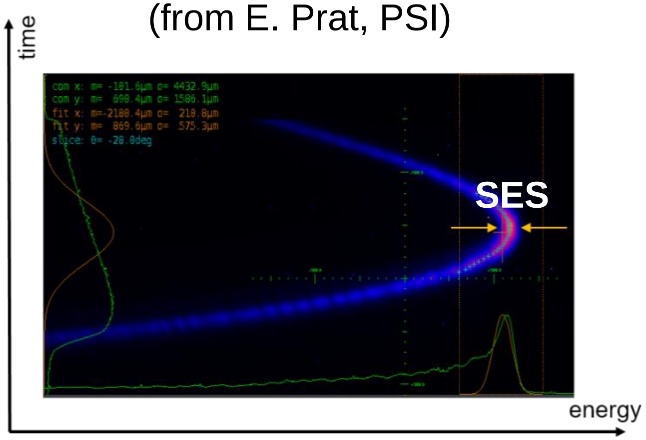
  - For FEL performance: $\frac{\sigma_\gamma}{\gamma} < \rho$ ← **FEL parameter**
  - Compression schemes for short bunches require small energy spread
  - Critical interaction between SES and micro-bunching instability (MBI) via Landau damping

Introduction

- **Intrabeam scattering (IBS) and SES**
  - Recent measurements in FEL injectors:

Facility	Q (pC)	E (MeV)	L (m)	SES (keV)
SwissFEL	200	320	110	6
EuXFEL	250	130	45	4
PITZ	250	20	20	2

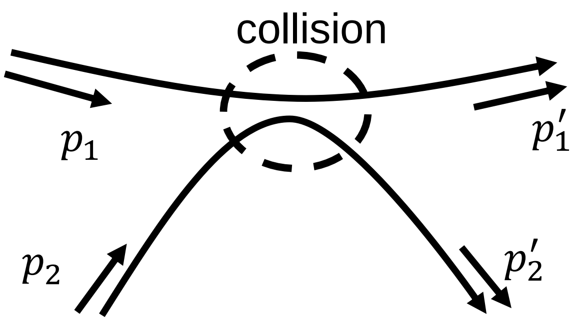
(from E. Prat, PSI)



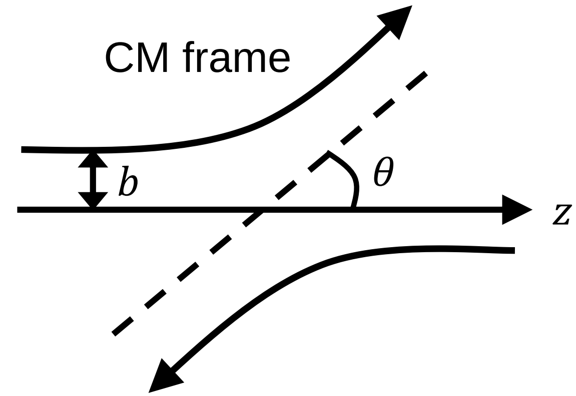
  - Numerical simulations suggest SES ~ 0.5-1 keV
  - **The SES growth is due to IBS**: Di Mitri et al., *Experimental evidence of intrabeam scattering in a free-electron laser driver,* (2020)
  - Analytical models cannot describe the SES growth in the injector section due to the highly nonlinear beam dynamics there

IBS Theory

- IBS = Short-range Coulomb collisions within the bunch
  - Non-collective effect: standard space-charge tracking solvers not sufficient
  - Single collision events cannot be resolved by time step
  - For a simple binary collision:



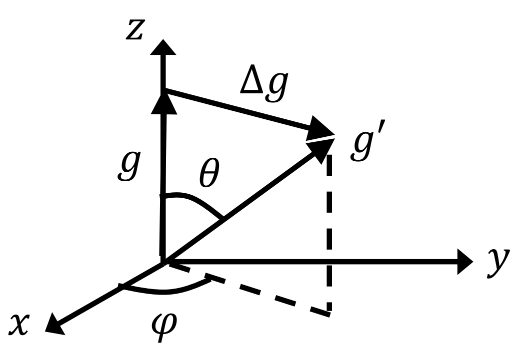
$$\tan\left(\frac{\theta}{2}\right) = \frac{me^2}{2\pi\varepsilon_0 g^2 b}$$



relative momenta

$$g = p_1 - p_2$$

$$g' = p_1' - p_2' \quad |g| = |g'|$$



post-collision momenta

$$p_1' = p_1 + \Delta g/2$$

$$p_2' = p_2 - \Delta g/2$$

IBS Theory

- Piwinski model (1974 → 2017)
  - Relative momentum change (lab frame):

$$\left(\frac{\delta p_1}{p}\right)_s = \frac{1}{2}\left[\gamma\frac{g_\perp}{p}\cos\phi sin\theta + \frac{g}{p}(cos\theta - 1)\right], \qquad \left(\frac{\delta p_1}{p}\right)_x = \cdots, \qquad \left(\frac{\delta p_1}{p}\right)_y = \cdots$$

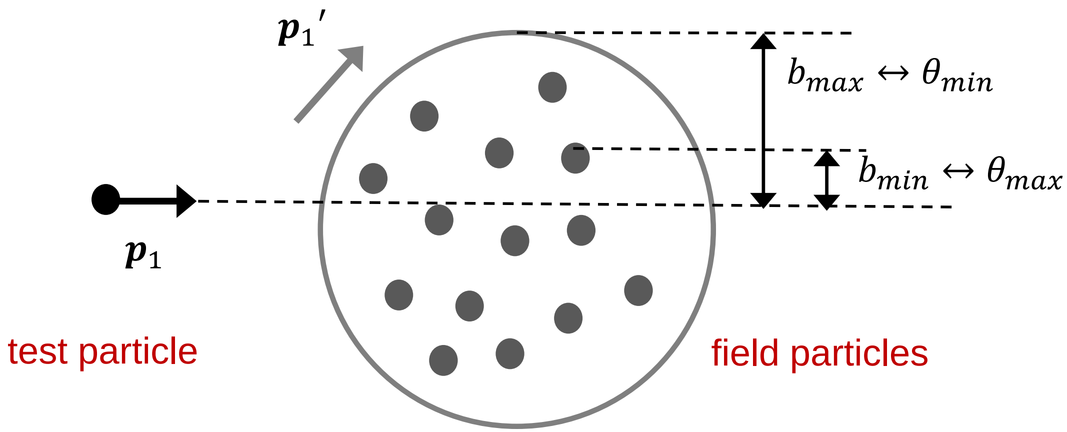
  - Relative energy (long. invariant): $H = \left(\frac{g}{p}\right)^2 \Rightarrow \delta H_1 = 2\frac{g}{p}\frac{\delta p_1}{p} + \left(\frac{\delta p_1}{p}\right)^2$
  - Relative energy change in a bunch (Piwinski, 1974):

$$\left\langle \frac{d}{dt}\frac{\langle H\rangle}{\gamma^2}\right\rangle = \int d^3p_2 d^3r_2 d^3p_1 d^3r_1 \underbrace{f_1(r_1,p_1)}_{\text{Gaussian bunch}} f_1(r_2,p_2)\, g \int_0^{2\pi} d\phi \int_{\theta_{min}}^{\theta_{max}} d\theta \frac{d\sigma(g,\theta,\phi)}{d\Omega}\left(\frac{\delta H_1}{\gamma^2}\right)$$

  - High energy approximation: $\boxed{\sigma_\gamma \frac{d\sigma_\gamma}{dz} = \frac{r_e^2 N_b \Lambda_c}{8\varepsilon_x^n \sigma_x \sigma_z}}$ (Bane, 2002)

Monte-Carlo collision schemes

- Cumulative scattering angle distribution



$$\frac{d\sigma(g,\theta)}{d\Omega} = \frac{r_e^2}{16\Delta\beta^4}\frac{1}{\sin^4(\frac{\theta}{2})}$$

Variance of scattering angle for single collision event:

$$\langle\theta^2\rangle = \frac{\int_{\theta_{min}}^{\theta_{max}} d\theta \sin(\theta)\theta^2 \frac{d\sigma(g,\theta)}{d\Omega}}{\int_{\theta_{min}}^{\theta_{max}} d\theta \sin(\theta) \frac{d\sigma(g,\theta)}{d\Omega}} = 2\theta_{min}^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right)$$

Coulomb log.: $$\Lambda_c = \ln\left(\frac{\theta_{max}}{\theta_{min}}\right) \approx \ln\left(\frac{b_{max}}{b_{min}}\right)$$

with: $b_{min} \backsimeq \frac{r_e}{2(\Delta\beta)^2}$, $b_{max} \backsimeq \lambda_D$ (?)

Monte-Carlo collision schemes

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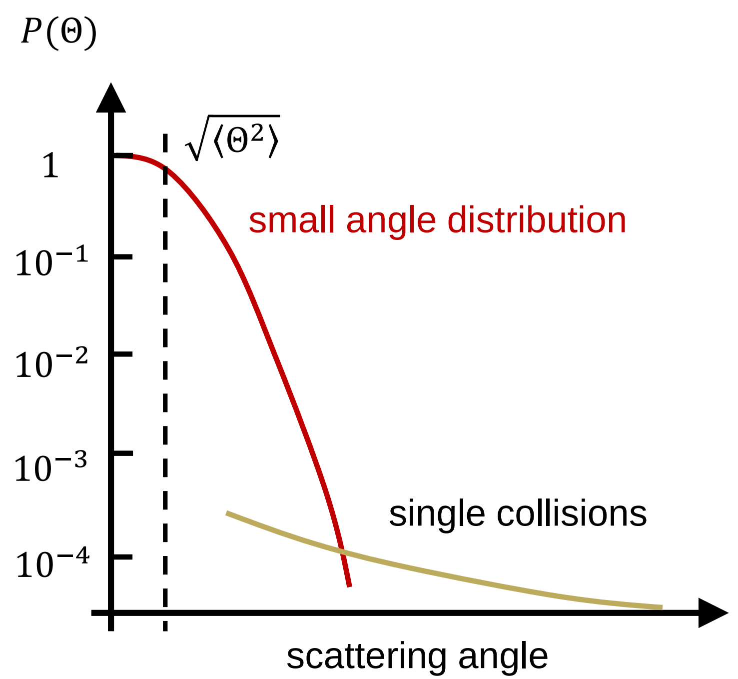
- Cumulative scattering angle distribution
  - For large number of (small angle) collisions:
    - Distribution is Gaussian
    - For N collisions:

$$\langle \Theta^2 \rangle = N \langle \theta^2 \rangle \quad \text{with:} \quad N = 2\pi b_{max}^2 n g \Delta t$$

  - But:
    - Distribution is not Gaussian for ‘small’ N
    - Flat tail at large angles (rare events)
    - Rutherford formula is valid for small angles only
    - Value of Λ_c is ambiguous in practical calculations

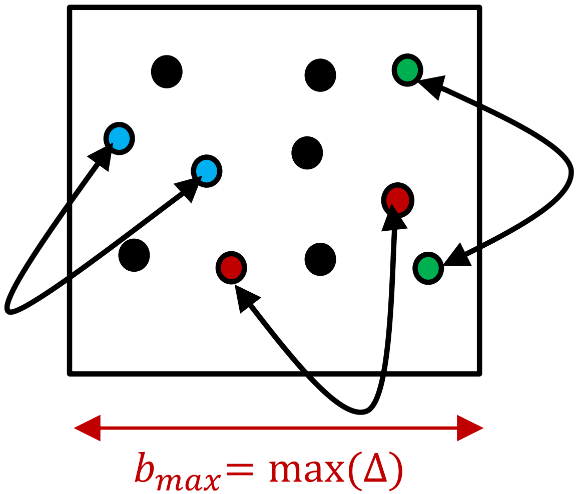


(*modified from Jacksons’ book*)

Monte-Carlo collision schemes

- Takizuka & Abe (1977)
  - Transform momenta in the bunch rest frame
  - Compute charge density in each mesh cell
  - Pair particles randomly within each cell
  - For each pair:
    - Produce a random scattering angle from normal distribution:

$$\langle \Theta^2 \rangle = 2\, N\, \theta_{min}^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right)$$



    - Rotate Δg in the CM-frame with (θ, φ) with $\varphi \in [0, 2\pi]$
    - Compute scattered momenta of the two particles, $p' = p \pm \frac{1}{2}\Delta g$
  - Transform momenta back to the lab frame
  - Proceed with space-charge calculations to the next time step

Monte-Carlo collision schemes

- Nanbu (1994)
  - Transform momenta in the bunch rest frame
  - Compute charge density in each mesh cell
  - Pair particles randomly within each cell
  - For each pair:
    - Solve: $\coth(A) - A^{-1} = \exp(-2\langle\Theta^2\rangle)$
    - Generate a scattering angle from distribution:

      $\theta(u) = \arccos\{\log[\exp(-\mathrm{A}) + 2\,\mathrm{u}\,\sinh(\mathrm{A})]\},\ \mathrm{u} \in [0,1]$
    - Rotate Δg in the CM-frame with (θ, φ) with $\varphi \in [0,2\pi]$
    - Compute scattered momenta of the two particles, $p' = p \pm \frac{1}{2}\Delta g$
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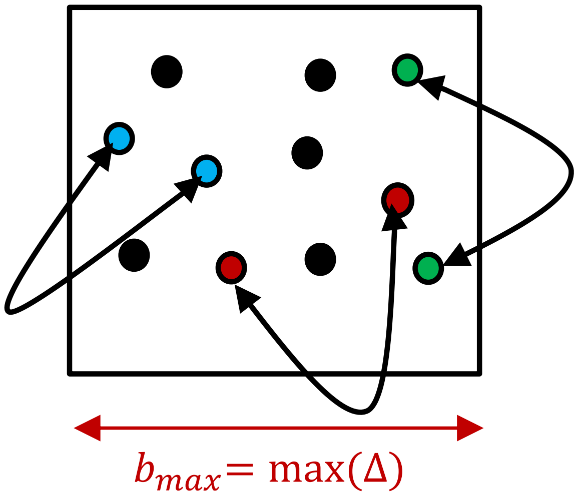
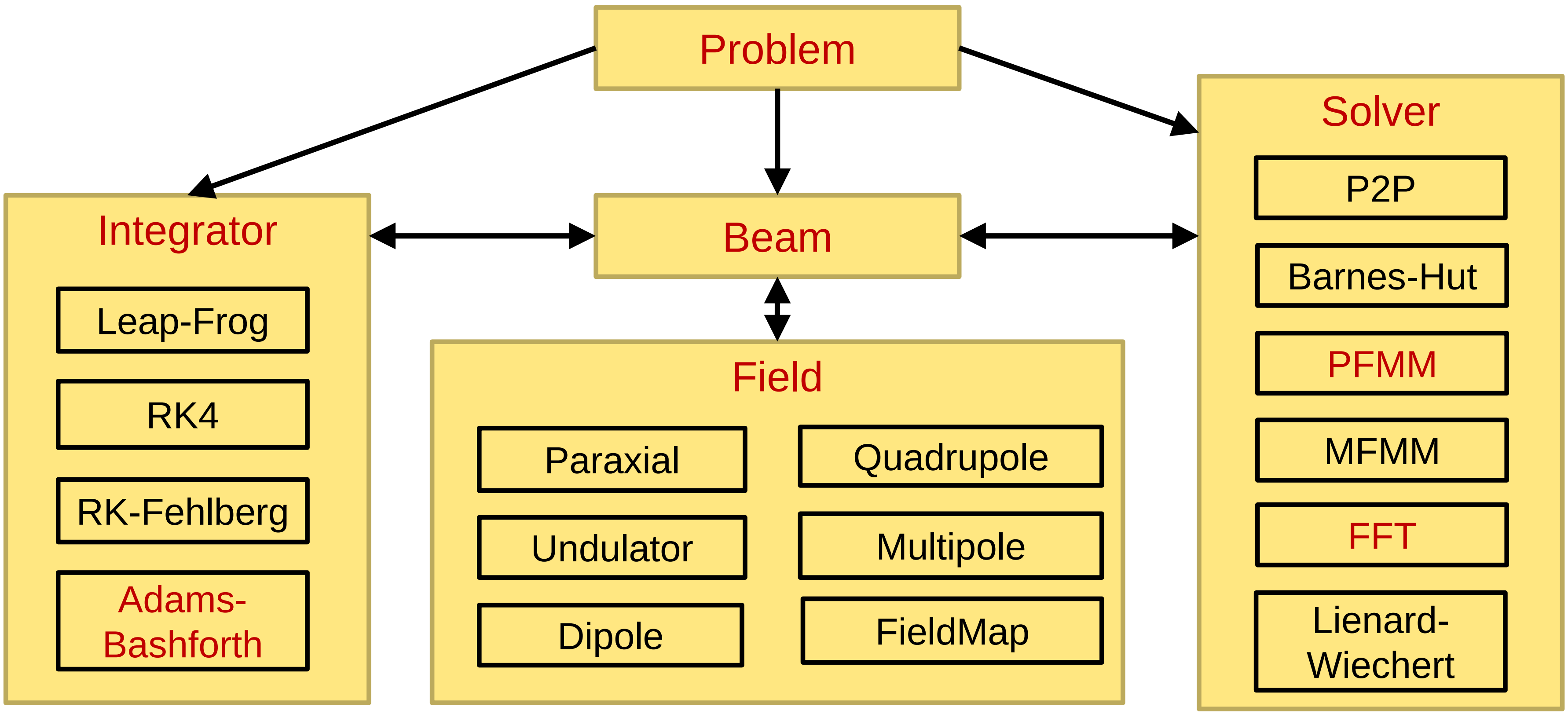


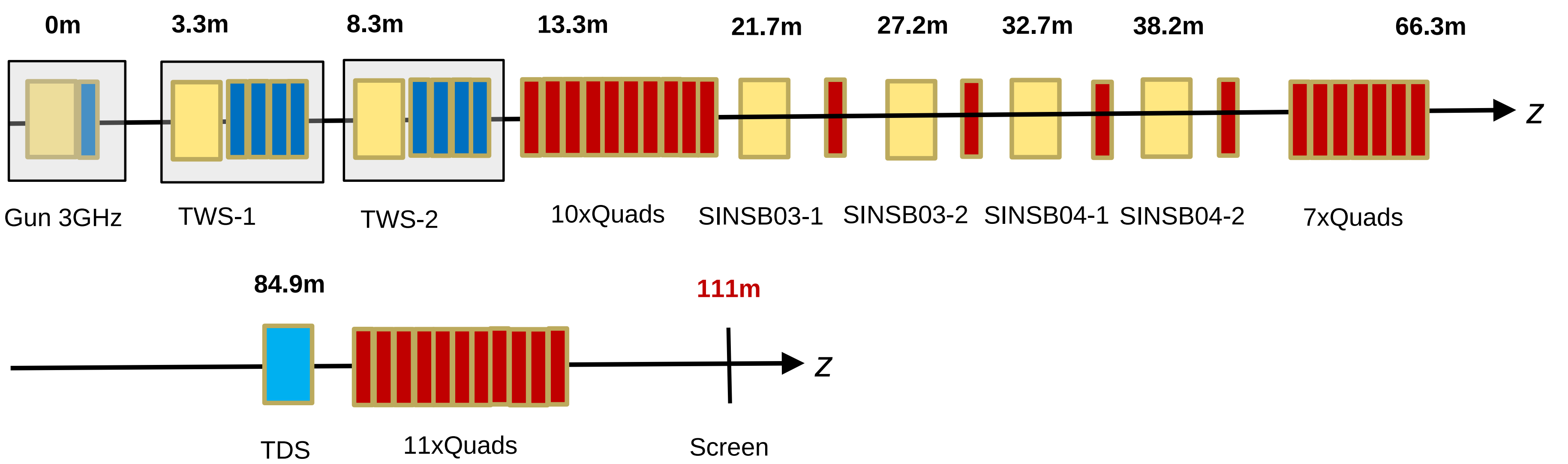
The Reptil code

- RElativistc Particle Tracking for Injectors & Linacs



Simulations for the SwissFEL

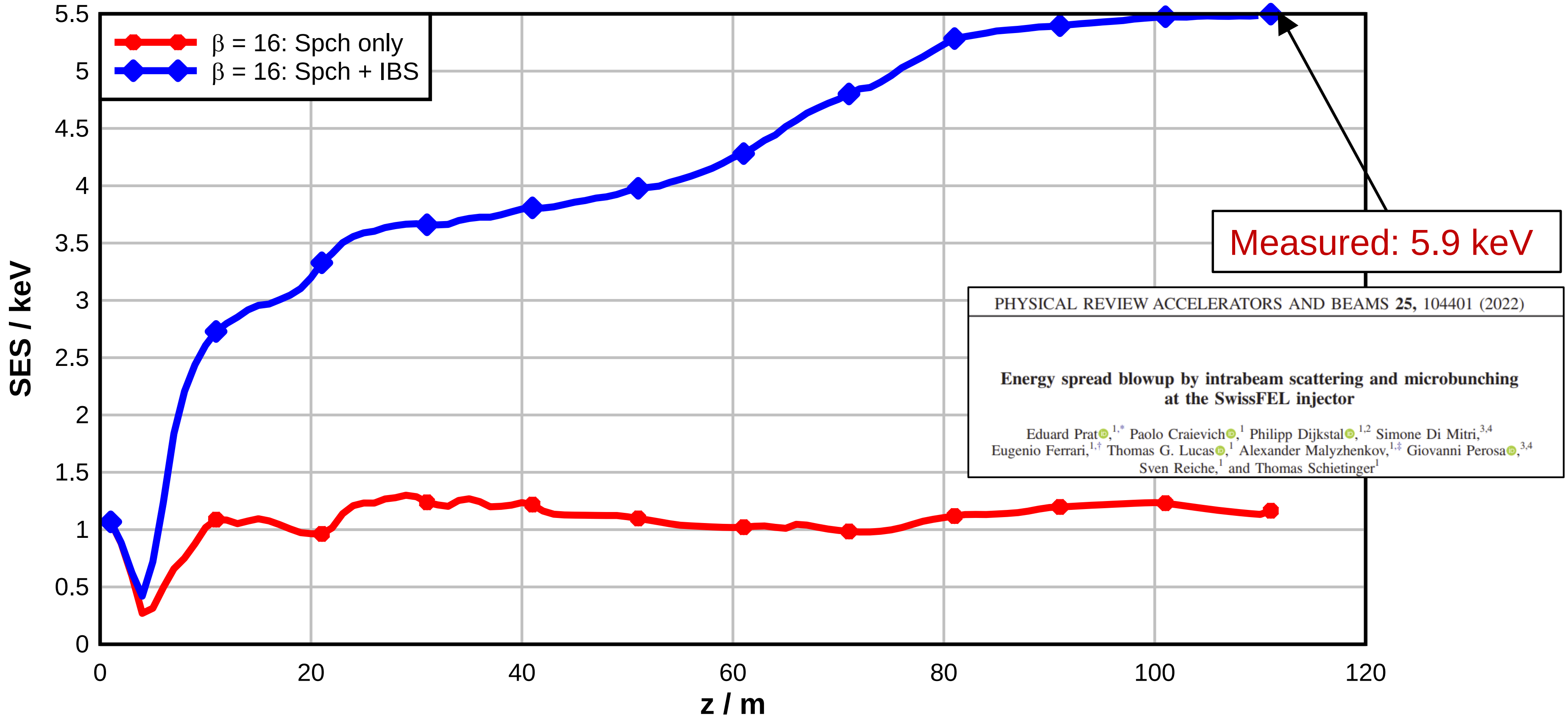
- The SwissFEL injector ine



- Final energy: 320 MeV
- Bunch charge: 10…200 pC (nominal 200 pC)
- Average β-function: ~16 m (nominal)
- Bunch compressor and laser heater switched off and $R_{56} = 0$

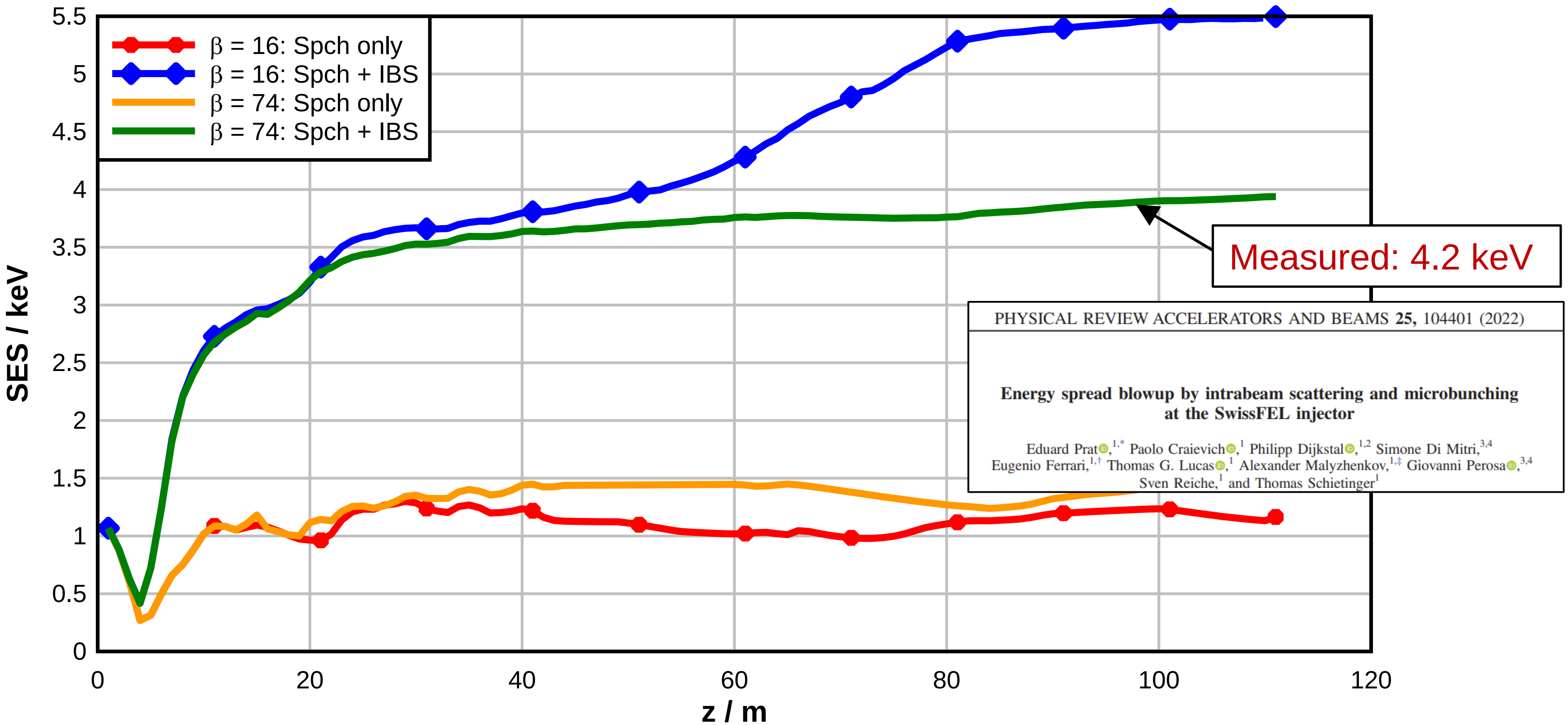
Simulations for the SwissFEL

- Slice energy spread (nominal configuration)



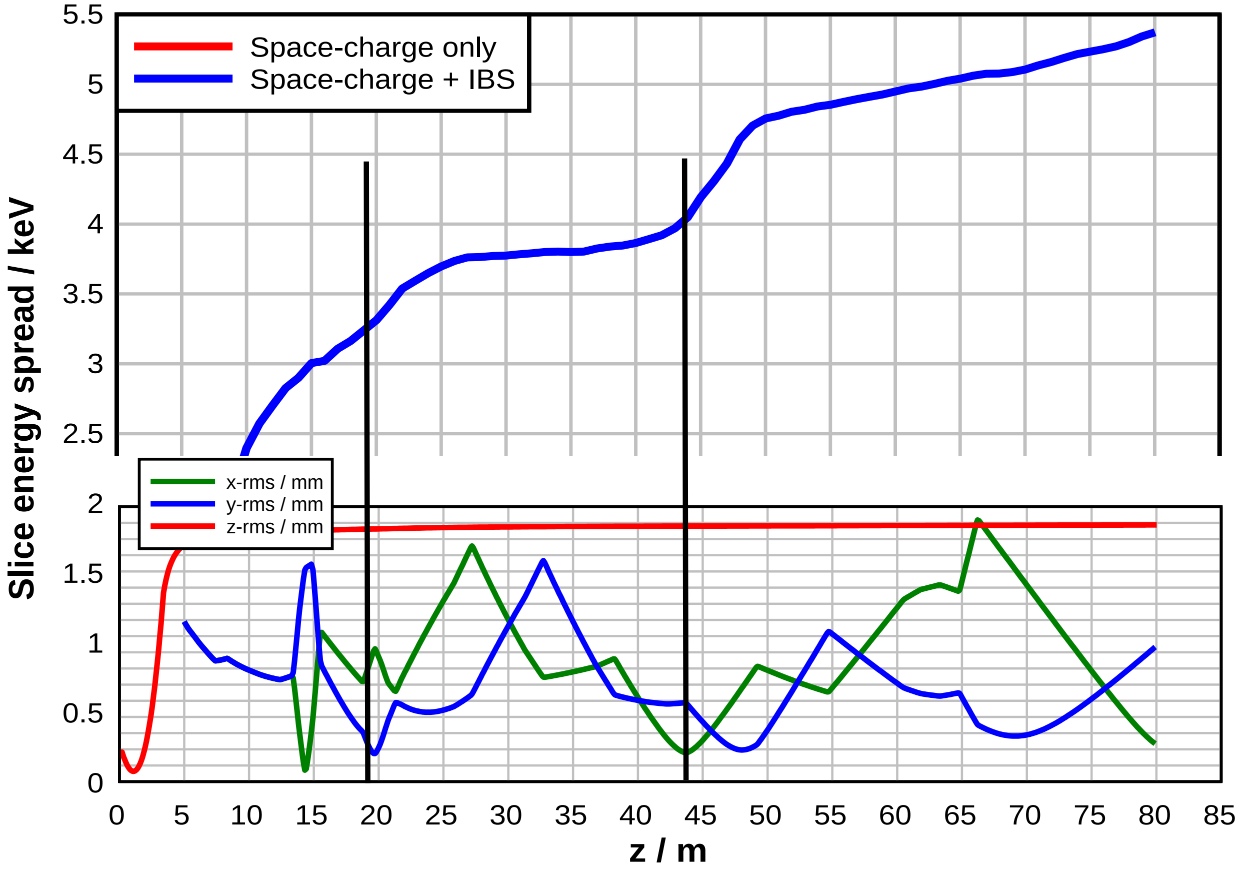
Simulations for the SwissFEL

- Slice energy spread (“large“ optics)



Simulations for the SwissFEL

- Effect of lattice optics



$$\sigma_\gamma \frac{d\sigma_\gamma}{dz} = \frac{r_e^2 N_b \Lambda_c}{8\varepsilon_x^n \sigma_x \sigma_z}$$

(K. Bane, 2002)

Simulations for the SwissFEL

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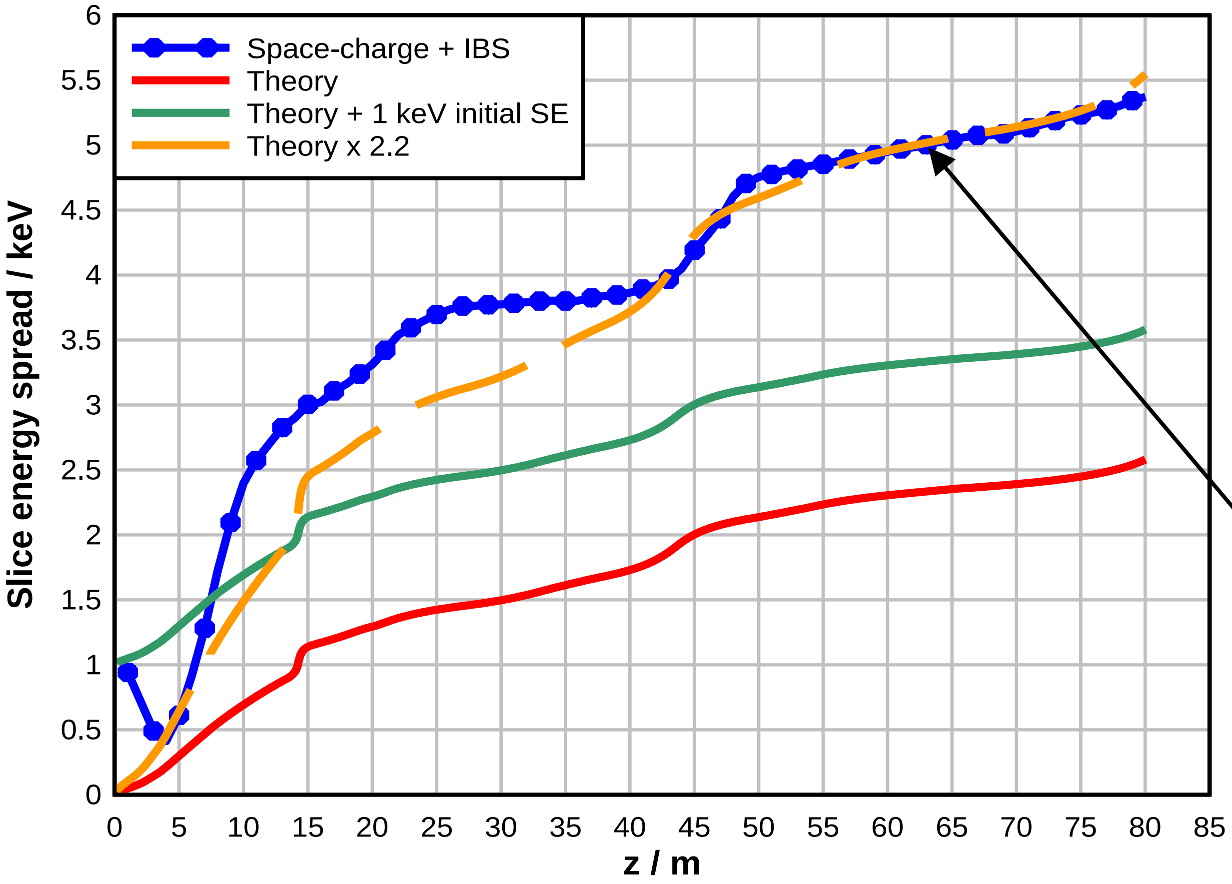
- Comparison with analytical model



$$\sigma_\gamma^2(z) = \sigma_{\gamma 0}^2 + \frac{r_e^2 N_b \Lambda_c}{4\varepsilon_x^n} \int_{z_0}^{z} \frac{r_e^2}{\sigma_x \sigma_z}$$

Initial SE (cathode) does not explain discrepancy

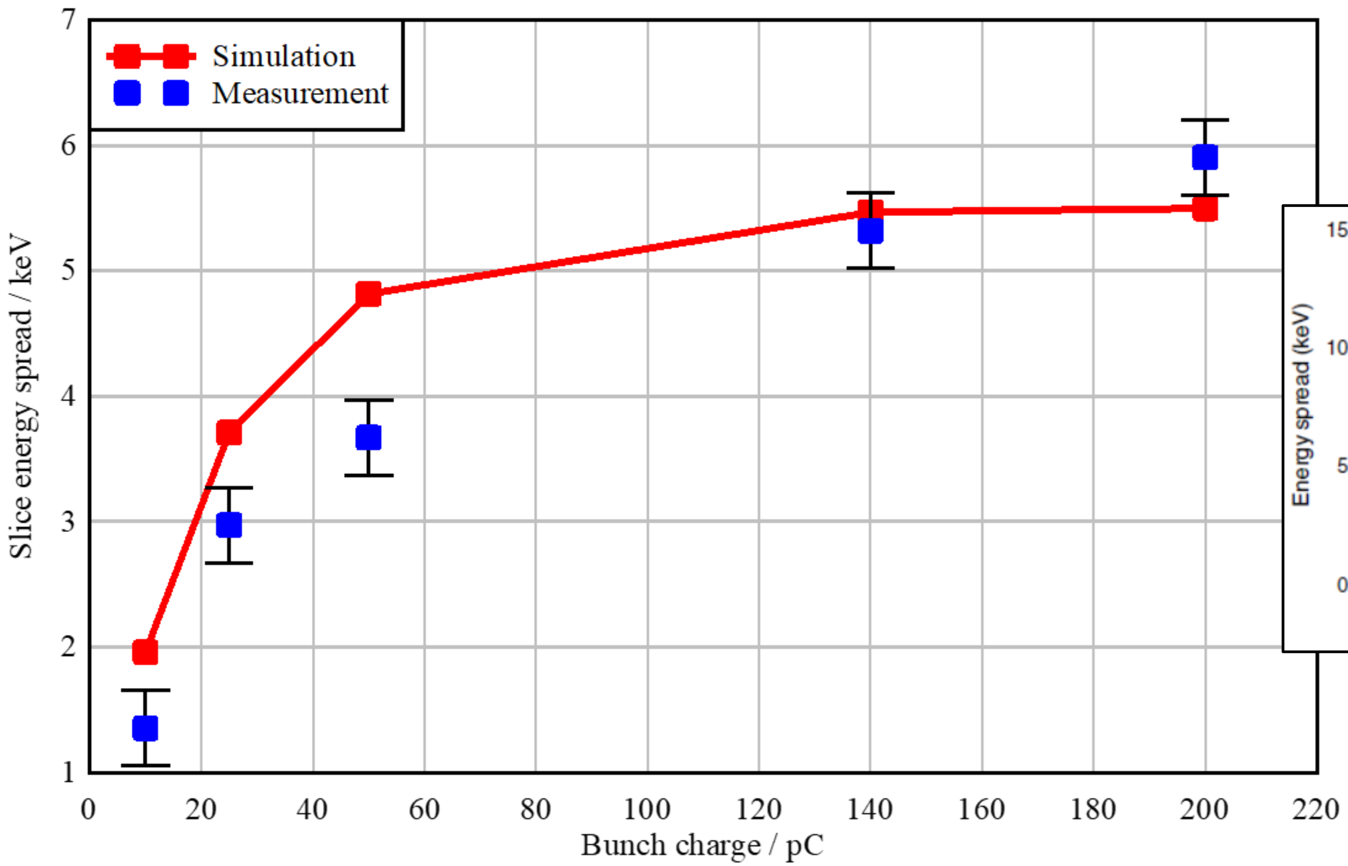
There is **a factor of roughly 2.2** between simulation and theory*

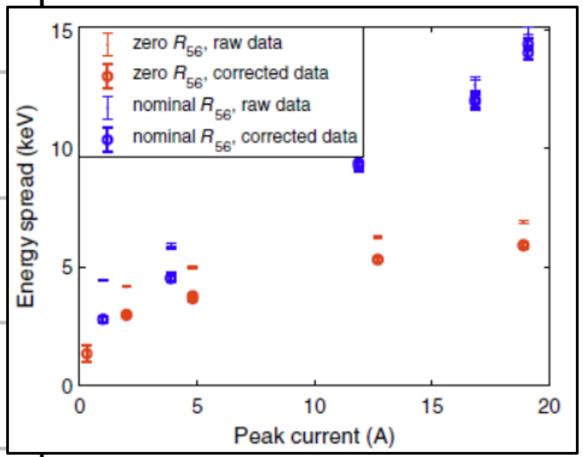
Similar IBS growth rates in the high energy section

*PRAB 25, 104401 (2022)

Simulations for the SwissFEL

- Charge scan





*With permission from Prat et al.

Summary & Conclusions

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- The IBS effect on uncorrelated energy spread remains a challenging topic
  - Several experiments at different labs predict important IBS effects in FELs
  - Available IBS theory not practicable for FEL-injectors
- Simulation approaches (and codes) exist
  - Most promising so far: classical Monte Carlo collision models
  - Other techniques exist…
- Self-consistent simulations with the Reptil code
  - Very good agreement for the SwissFEL injector
  - Ongoing work for the European XFEL
  - Open question: interaction between IBS and MBI in dispersive sections

Thank you very much
for your attention