HOMOGENIZATION OF HTS MAGNET COILS USING THE FOIL CONDUCTOR MODEL

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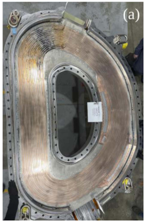
Motivation

- Large scale applications of HTS REBCO tapes contain coils with a large number turns
- Example: The SPARC TFMC with 16 pancake coils **256 turns** each

Z. S. Hartwig et al., The SPARC Toroidal Field Model Coil Program, IEEE Trans. Appl. Super. 2024.



Hartwig et al. 2024

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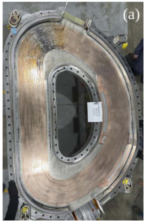
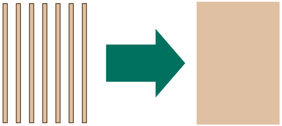


Hartwig et al. 2024

- Full spatial resolution often prohibitively expensive

  ⇒ **Need to reduce computational complexity**

  ⇒ **This work: FEM with homogenization**



Motivation

- HTS coils are topologically similar to foil windings in NC applications, e.g., power transformers
- Modeling techniques for foil windings can be extended for HTS coils
- **Insulated** coils are examined

siemens-energy.com

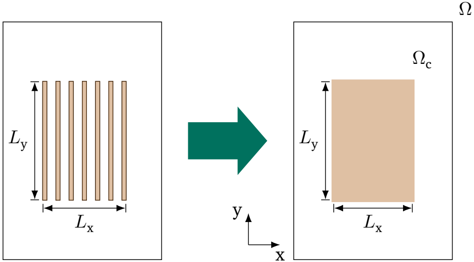
H. De Gersem and K. Hameyer, A finite element model for foil winding simulation, IEEE Trans. Magn. 2001.

P. Dular and C. Geuzaine, Spatially dependent global quantities associated with 2-D and 3-D magnetic vector potential formulations for foil winding modeling, IEEE Trans. Magn. 2002.

Overview

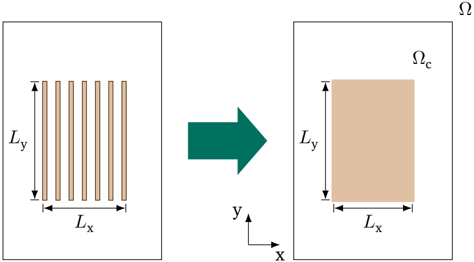
Overview

Homogenization of HTS coils



- Details of the geometry are washed away, local effects are not captured
- Engineering current density defined $J_{c,eng} = \lambda J_c$
- Anisotropic σ or ρ needed in 3-D
- Only HTS layers are considered

Homogenization of HTS coils



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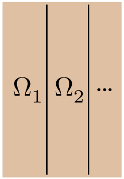
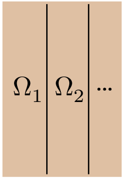
Problem: How to impose I and preserve correct $\vec{J}$?

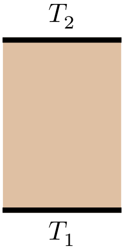
Imposing a current



- Current constraint for each subregion Ω_n
- Implemented, e.g., with $\vec{H}-$ and $\vec{J}-\vec{A}$-formulations

Imposing a current





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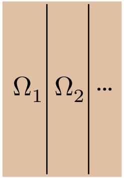
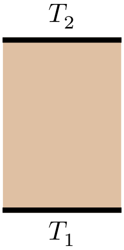
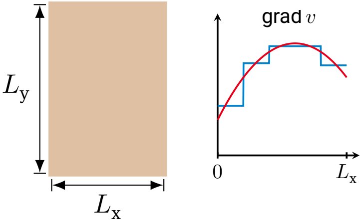
- Setting $\vec{T}$ at the boundaries

Imposing a current







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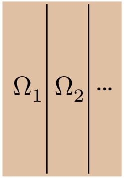
- Setting $\vec{T}$ at the boundaries

- One current constraint
- 1-D discretization of grad v

$$\mathrm{grad}\, v = \sum_j u_j p_j(x)$$

Imposing a current



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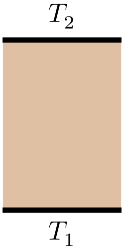
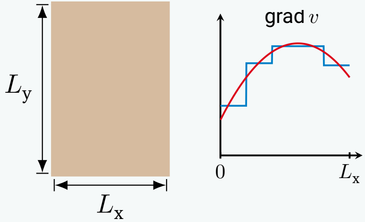


- Setting $\vec{T}$ at the boundaries



- One current constraint
- 1-D discretization of grad v

$$\mathrm{grad}\, v = \sum_j u_j p_j(x)$$

Foil conductor model

- Current imposed by modifying $V = \mathrm{grad}\, v$

$\Rightarrow$ The approach is applicable to $\vec{B}$-conforming formulations

1. Classical choice: the $\vec{A} - V$-formulation

- The power law needs to be expressed for $\sigma(\|\vec{E}\|)$

$$\sigma(\|\vec{E}\|) = \frac{J_{\mathrm{c,eng}}}{E_{\mathrm{c}}} \left(\epsilon_\sigma + \left(\frac{\|\vec{E}\|}{E_{\mathrm{c}}} \right)^{(n-1)/n} \right)^{-1}$$

- The regularization term ϵ_σ is needed for numerical stability
- **Numerical problems expected**

J. Dular et al., Finite-element formulations for systems with high-temperature superconductors, IEEE Trans. Appl. Super. 2020.

Foil conductor model

2. Mixed $\vec{J}-\vec{A}-V$-formulation
   - Power law can be written in terms of $\rho(\|\vec{J}\|)$

$$\rho(\|\vec{J}\|) = \frac{E_\mathrm{c}}{J_\mathrm{c,eng}} \left(\frac{\|\vec{J}\|}{J_\mathrm{c}} \right)^{n-1},$$

   - Regularization is not needed
   - Basis functions need to be chosen correctly to avoid spurious oscillations, e.g., 2nd order elements for $\vec{A}$ and 0th order for $\vec{J}$ in Ω_c

J. Dular et al., Finite-element formulations for systems with high-temperature superconductors, IEEE Trans. Appl. Super. 2020.

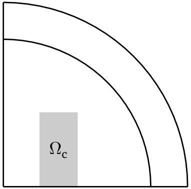
S. Wang et al., Numerical calculations of high temperature superconductors with the J-A formulation, Supercond. Sci. Technol. 2023.

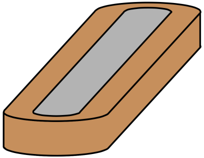
Overview

Numerical results

- All models are implemented with **GetDP** and **Gmsh**
- $\vec{A} - V$- and $\vec{J} - \vec{A} - V$-formulated models verified against a reference model
- Let us examine two simulation examples:
  1. A single racetrack coil
  2. Stack of racetrack coils

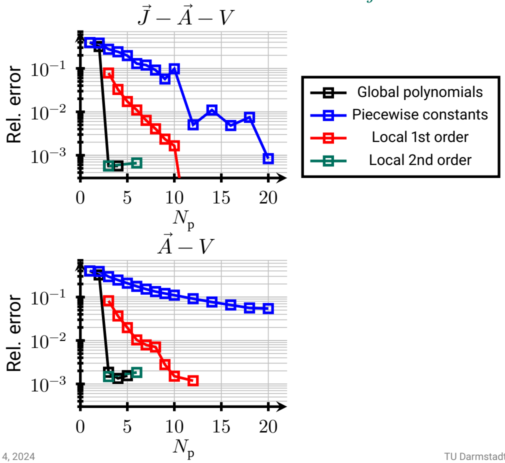
N	20
λ	0.01
J_c	10^{10} A/m^2
n	25
d_{HTS}	1 µm
d_{tape}	100 µm
w	12 mm





Numerical results - one coil

$$\hat{I} = 0.8 I_c\,,\; f = 50\,\mathrm{Hz}\,,\quad \mathrm{grad}\, v = \sum_j u_j p_j(x)$$



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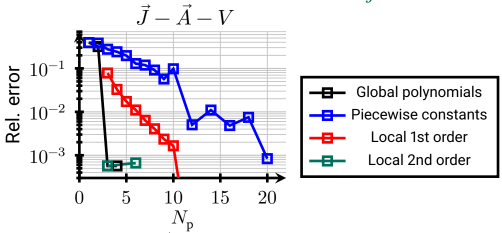
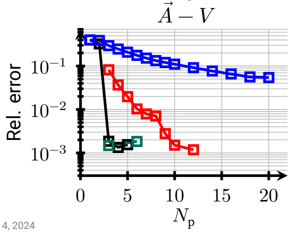
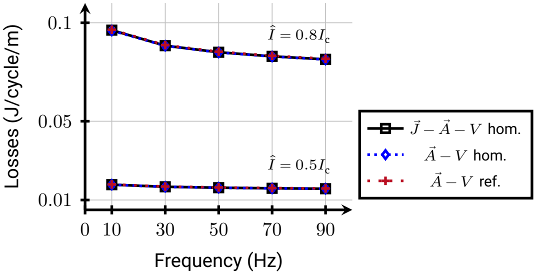




3rd order global polynomial chosen for V

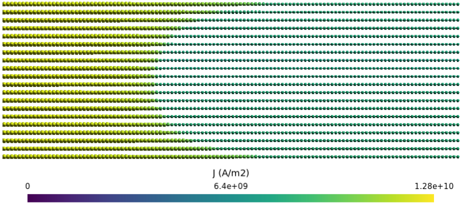


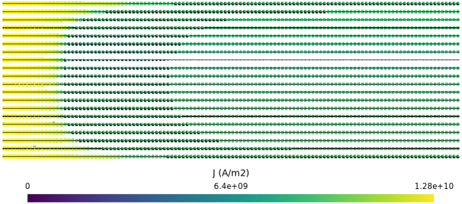
Good agreement in losses for multiple choices of basis functions for V

Numerical results - one coil

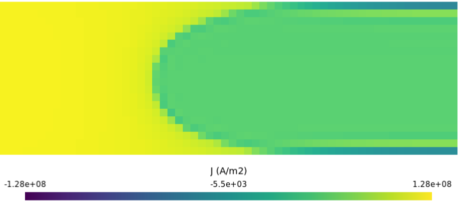
$\vec{J}\,(t = 0.25T)$ | $\vec{J}\,(t = 0.5T)$

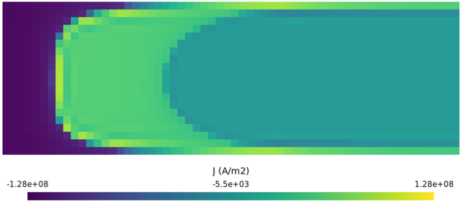
Reference model





$\vec{J} - \vec{A} - V$
hom. model





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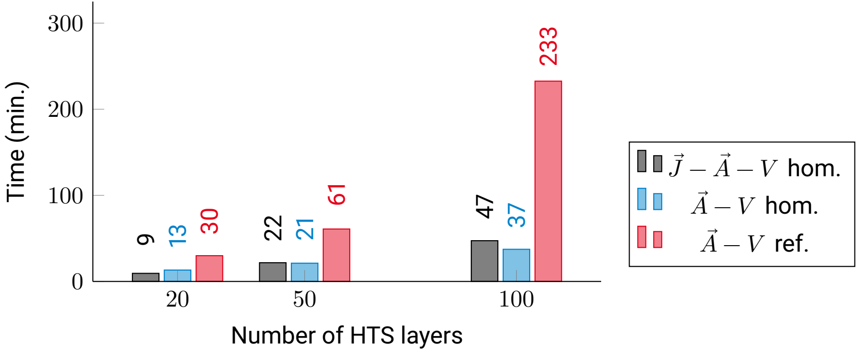
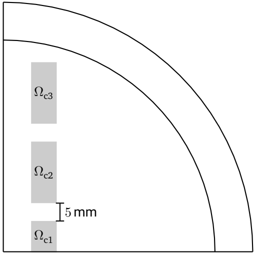
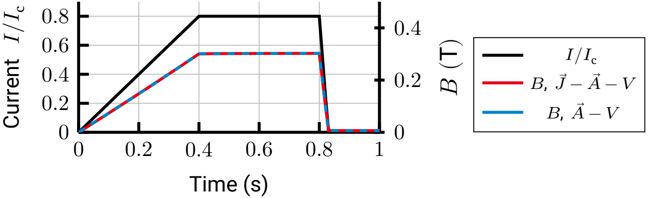


The $\vec{J} - \vec{A} - V$ homogenized model has the best numerical performance

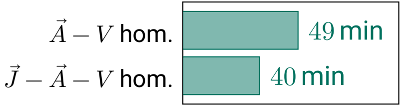
Numerical results - stack of coils

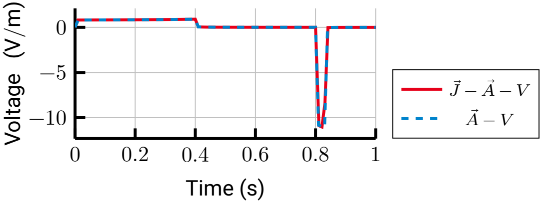
5 racetrack coils with 50 turns each





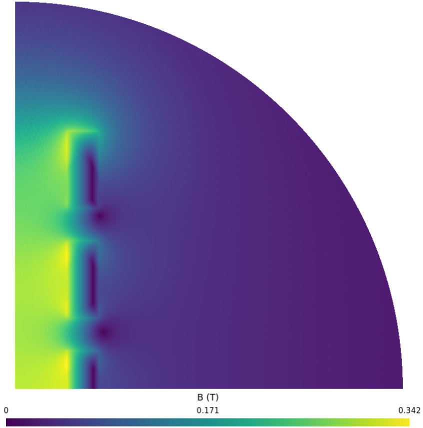
Simulation times





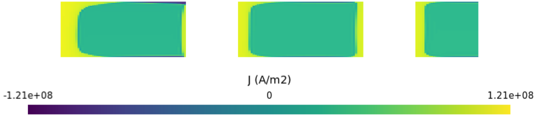
Numerical results - stack of coils

$B(t = 0.5\,\mathrm{s})$ $\vec{J} - \vec{A} - V$ hom. model

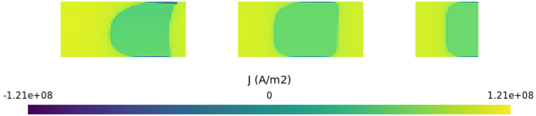


$\vec{J} - \vec{A} - V$ hom. model

$J(t = 0.2\,\mathrm{s})$



$J(t = 0.5\,\mathrm{s})$



Overview

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- The coil current can be imposed through an 1-D discretization of $\operatorname{grad} v$
- A speedup is obtained without significant loss of accuracy
- $\vec{J} - \vec{A} - V$-formulated model has better numerical properties than the $\vec{A} - V$-formulated model

Thank you for your attention!

The work of Elias Paakkunainen is supported by the Graduate School CE within Computational Engineering at the Technical University of Darmstadt.

Weak Formulation (2-D)

- $\vec{A} - V$-formulation:

$$\left(\nu \operatorname{curl} \vec{A},\, \operatorname{curl} \vec{A}^{\,\prime}\right)_{\Omega} + \left(\sigma \partial_t \vec{A},\, \vec{A}^{\,\prime}\right)_{\Omega_\mathrm{c}} + \left(\sigma \Phi \vec{e}_\mathrm{z},\, \vec{A}^{\,\prime}\right)_{\Omega_\mathrm{c}} = 0$$

$$\left(\sigma \partial_t \vec{A},\, \Phi^{\prime} \vec{e}_\mathrm{z}\right)_{\Omega_\mathrm{c}} + \left(\sigma \Phi \vec{e}_\mathrm{z},\, \Phi^{\prime} \vec{e}_\mathrm{z}\right)_{\Omega_\mathrm{c}} + \int_{L_\mathrm{x}} \frac{I}{d} \Phi^{\prime} \, dx = 0,$$

- $\vec{J} - \vec{A} - V$-formulation:

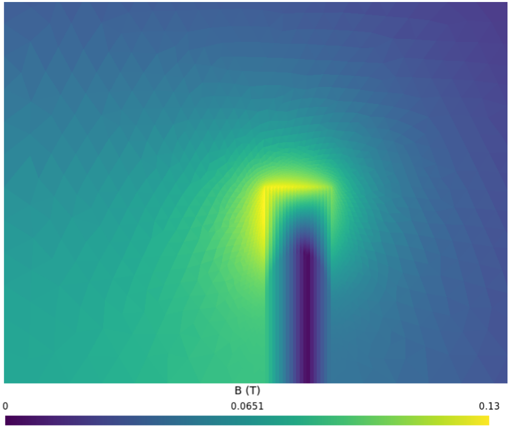
$$\left(\nu \operatorname{curl} \vec{A},\, \operatorname{curl} \vec{A}^{\,\prime}\right)_{\Omega} - \left(\vec{J},\, \vec{A}^{\,\prime}\right)_{\Omega_\mathrm{c}} = 0$$

$$\left(\vec{J}, \Phi^{\prime} \vec{e}_\mathrm{z}\right)_{\Omega_\mathrm{c}} - \int_{L_\mathrm{x}} \frac{I}{d} \Phi^{\prime} \, dx = 0$$

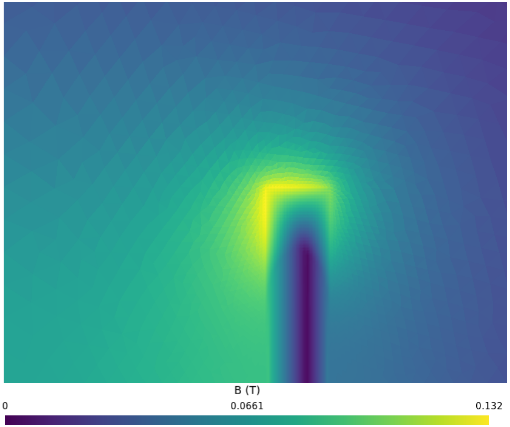
$$\left(\rho \vec{J},\, \vec{J}^{\prime}\right)_{\Omega_\mathrm{c}} + \left(\partial_t \vec{A},\, \vec{J}^{\prime}\right)_{\Omega_\mathrm{c}} + \left(\Phi \vec{e}_\mathrm{z},\, \vec{J}^{\prime}\right)_{\Omega_\mathrm{c}} = 0,$$

Numerical results - one coil

Reference model $B(t = 0.25T)$



$\vec{J} - \vec{A} - V$ hom. model $B(t = 0.25T)$



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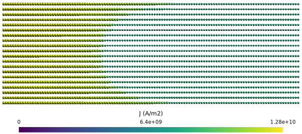
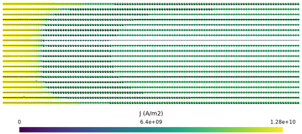
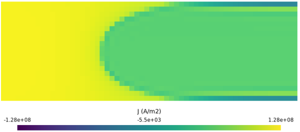
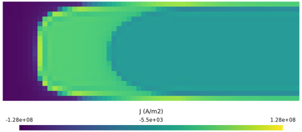




$\vec{J} - \vec{A} - V$ hom. model





$\vec{J} - \vec{A} - V$ bulk