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Fachgebiet BP Beschleunigerphysik

SIMULATION OF DRIVEN PLASMA MODES IN PENNING-MALMBERG TRAPS

2024-10-04

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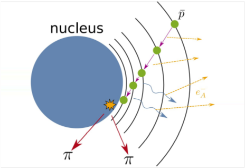
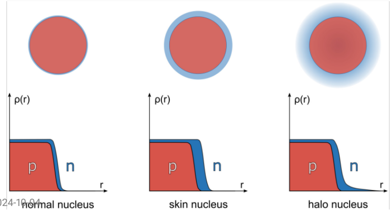
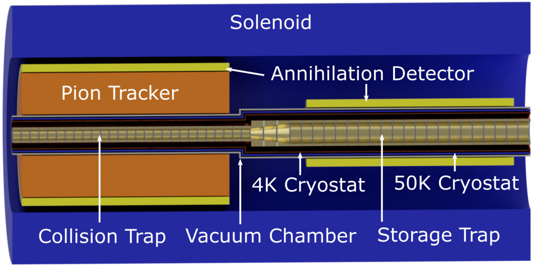
2 Model for Rotating Wall
3 PIC code for Plasma Simulations
4 Alternatives
5 Conclusion

STABLE ANTIPROTON PLASMA FOR THE PUMA EXPERIMENT

⇒ antiProton Unstable Matter Annihilation







⇒ Transport $\bar{p}$ to the rare isotopes

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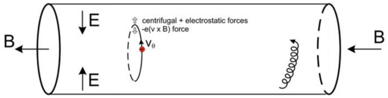
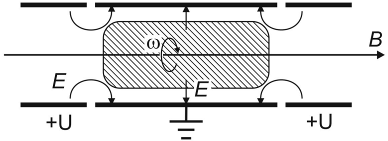
PENNING-MALMBERG-TRAP





Particles in Plasma Column:

- cyclotron motion around B_z: $v_{x,y} = v_\perp \cdot \exp(\pm i\omega_c t + i\delta_{x,y})$
- space charge field: $E_r(r) = -(m\,r/2\,q) \cdot \omega_p$ for $r \leq r_p$

$\Rightarrow$ constant angular rotation: $\omega_r^- = v_{theta}^-/r \approx \omega_p^2/2\omega_c \equiv \omega_{E\times B}$

with plasma frequency $\omega_p^2 = n_q q^2/m$ and $\omega_c = qB_z/m$

Penning-Malmberg Trap:

- Radial confinement by axial magnetic field
- Axial confinement by electrostatic potential

source: F. Chen, Introduction to Plasma Physics and Controlled Fusion. Springer, 2016 (top) A. Piel, Plasma Physics, An Introduction to Laboratory, Space, and Fusion Plasmas. Springer, 2010 (bottom)

DENSITY CONTROL BY ROTATING WALL TECHNIQUE

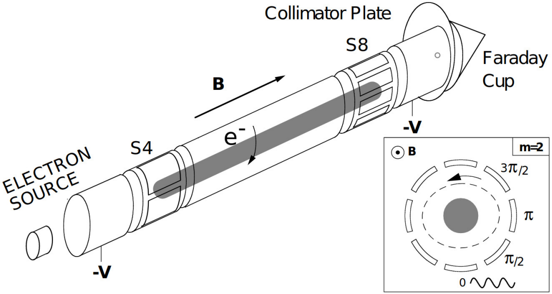


image source: Danielson et. al., Phys.Rev.Lett. 94, 035001 (2005)

! Field asymmetries or neutral background gases can cause a radial expansion

⇒ Rotating Wall Technique:

- Rotating electrostatic multipole field around trap axis
- Increase of plasma rotation ⇒ Increase of density
- Segments of a ring electrode driven by sinusoidal voltages
- Phase difference between segments determines multipole order of RW drive m_θ

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SCIENTIFIC GOALS

1. response of multi-species non-neutral plasma column to external field
2. compression rates by external rotating fields

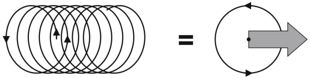
→ parameter study on experimental parameters like L,Δz, n_0, $T_{particle}$, ω_{RW}, m_{RW}

MODEL ASSUMPTIONS

- collisionless weakly coupled ideal plasma
- strong axial magnetic field: $\vec{B} = B_0 \hat{e}_z$

\+ clear frequency ordering $\omega_\mathrm{c} \gg \omega_\mathrm{z} > \omega_\mathrm{r}$

→ guiding center approximation



- assume (quasi) electrostatic field

→ $\vec{v}_{\mathrm{E}\times\mathrm{B}} = \vec{E} \times \vec{B} / B^2$ with $\vec{E} = -\nabla\phi$

- cold, nonrelativistic plasma

→ $\omega_{\mathrm{E}\times\mathrm{B}} = \omega_\mathrm{r}$

- long plasma column

\+ neglect finite length effects

→ infinitely long column with periodicity length L

- perfect conducting trap wall: $\phi_{sc}(r = r_\mathrm{wall}) = 0$

image source: A. Piel, Plasma Physics, An Introduction to Laboratory, Space, and Fusion Plasmas. Springer, 2010

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SYSTEM OF EQUATIONS

Distribution function in the guiding-center approximation:

$$\frac{\partial f_j(r,\theta,z,v_z,t)}{\partial t} + \frac{\vec{E}_\perp \times \hat{e}_z}{B_0} \nabla_\perp f_j(r,\theta,z,v_z,t) + v_z \frac{\partial f_j(r,\theta,z,v_z,t)}{\partial z} + \frac{q}{m} E_\parallel \frac{\partial f_j(r,\theta,z,v_z,t)}{\partial v_z} = 0$$

E-field by space charge and rotating wall field

$$\vec{E}(r,\theta,z,t) = -\nabla(\phi_{sc} + \phi_{RW})$$

$$\nabla^2 \phi_{sc}(r,\theta,z,t) = \frac{q}{\epsilon_0} \sum_j n_j(r,\theta,z,t)$$

boundary conditions

$$f_j(r > r_{\mathrm{wall}}, \theta, z, v_z, t) = 0$$

$$\phi_{sc}(r = r_{\mathrm{wall}}) = 0$$

$$f_j(r,\theta,z=0,v_z,t) = f_j(r,\theta,z=L,v_z,t)$$

$$\phi_{sc}(r,\theta,z=0) = \phi_{sc}(r,\theta,z=L)$$

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REQUIREMENTS FOR NUMERICAL STUDIES

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- Compression by rotating wall:
  - relevant mechanism: Landau resonance
  - compression is a weak effect:
    relative change of the central density during one characteristic rotation of the plasma column is approximately 10^{-6} (estimated from experiments)

→ high number of particles

→ very low numerical noise

→ fast calculation to get results in reasonable time

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DETAILS OF PIC SIMULATION CODE

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- rectangular grid:
  - Dirichlet b.c. in transversal direction
  - periodic b.c. in longitudinal direction
- Self-field calculation:
  - embedded boundary ($\phi_{sc}(r = r_{wall}) = 0$)
  - finite difference
- trap wall at $r = r_{wall}$ absorbing for particles
- time stepping: low storage Runge-Kutta (direct solver)

- tracking of 3D guiding center position and longitudinal velocity:

$$\frac{dx}{dt} = \frac{E_y}{B_0} \, , \quad \frac{dy}{dt} = -\frac{E_x}{B_0} \, , \quad \frac{dz}{dt} = v_z$$

$$\frac{dv_z}{dt} = \frac{q}{m} E_z$$

- Input parameters: $\hat{f}_0(\vec{x}, v_z, t = 0)$, B_0, r_{wall}, q/m, L, ϕ_{ext}

→ usage of existing code: GEMPIC - (arXiv:1609.03053)
  - modified for the drift-kinetic approximation by K. Kormann
  - parallel computation and suited for large data structure

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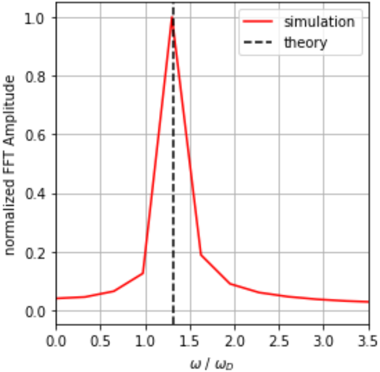
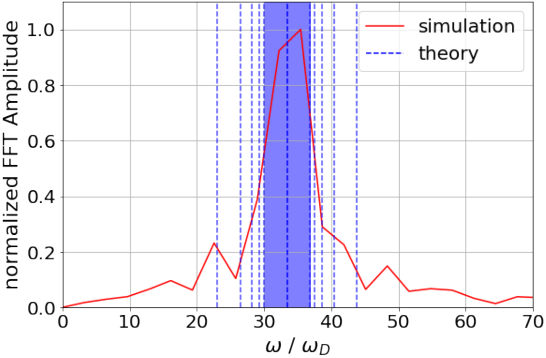
VERIFICATION - MODE FREQUENCIES

- computation of perturbation mode frequencies ✓

flute perturbations ($k_z = 0$)



plasma filled wave guide



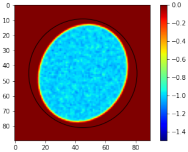
VERIFICATION - COMPRESSION BY MODE PERTURBATION

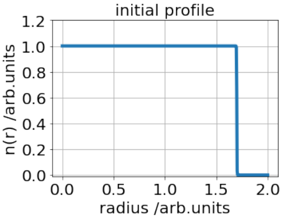
- compression rate* for perturbation $\hat{\phi}_0(r)e^{i(l\theta+kz-\omega t)}$:

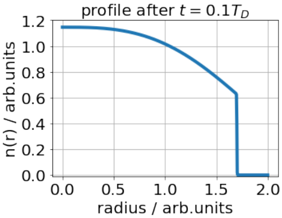
$$\Gamma = \frac{\pi q n_0 l |\phi|^2}{2mBr}\left[\frac{\partial \hat{f}_0}{\partial v_z} - \frac{l}{kr\omega_c n_0}\frac{\partial n_0}{\partial r}\hat{f}_0\right]_{v_z=(\omega-l\omega_r)/k}$$

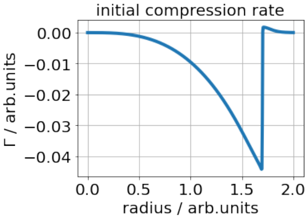
initialized perturbation disintegrate into many mode very fast

idea: drive perturbation by an external field



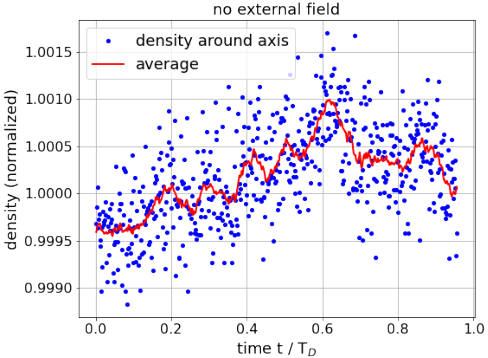


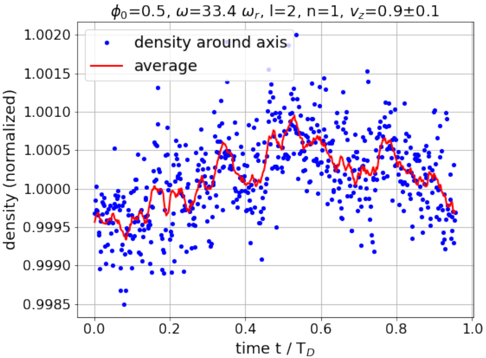




*source: Kiwamoto et. al. Plasmas 12, 094501 (2005)

VERIFICATION - COMPRESSION BY MODE PERTURBATION





no compression observable for reasonable computation times

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ALTERNATIVE APPROACH

Spectral Method for 3D space charge calculation

$\rightarrow$ Fourier-Bessel-decomposition of ρ and ϕ

$$\rho^{lmn} = \frac{Q}{\pi r_{wall}^2 J_m'^2(\gamma_{lm} r_{wall})} \sum_{j=1}^{N} \exp(im\theta_j) J_m(\gamma_{lm} r_j) \exp(inz_j/L)$$

$$J_m(\gamma_{lm} r_{wall}) = 0 \ , \quad \left(k^2 - \gamma_{lm}\right) \phi^{lmn} = \frac{\rho^{lmn}}{\epsilon_0}$$

$$\phi(r_j, \theta_j, z_j) = \sum_{l=1}^{N_l} \sum_{m=-N_m/2}^{N_m/2} \sum_{n=-N_n/2}^{N_n/2} \phi^{lmn} \exp(-im\theta_j) J_m(\gamma_{lm} r_j) \exp(inz_j/L)$$

Properties:

- suppress higher order potential perturbation
- Complexity of $\mathcal{O}(N \times N_m \times N_l \times N_n)$

$\rightarrow$ expectation: only low mode numbers of interest

- cylinder coordinates allow direct readout of relevant values

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Summary PIC code for plasma simulations in Penning-Malmberg traps

- Scientific goal: study of plasma mode response and compression rates of rotating wall drive
- Methods: PIC code for tracking particles in the drift-kinetic approximation
- Status Quo: mode frequencies calculated correctly, plasma response to external field
- Challenges for simulation of compression effect:
  - high number of particles needed for Landau resonance
  - destabilization by high perturbation modes
  - reducing unstable grow but keeping reasonable computation time

Alternative Approach

- 3D spectral Poisson solver:
  - natural low pass filter for field perturbations
  - expectation: good approximation of E-field for low mode numbers