TECHNISCHE UNIVERSITÄT DARMSTADT

UPDATED REDUCED MAGNETIC VECTOR POTENTIAL METHOD FOR SUPERCONDUCTING MAGNETS

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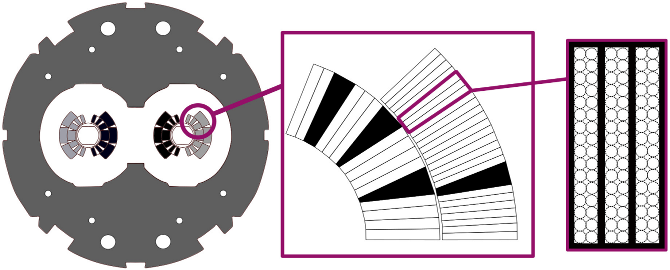
12.09.2024

SUPERCONDUCTING ACCELERATOR MAGNET = MULTI-SCALE PROBLEM



(a) (b) (c)

- **Problem:** Meshing the coil in detail within a finite-element method is too expensive!
- **Idea:** Do not resolve the coils/cables/wires in the FE mesh itself, but as a separate set of threads & compute their contribution by Biot-Savart's law

ANSATZ

We want to solve:

$$\nabla \times \left(\nu \nabla \times \vec{A}\right) = \vec{J} \qquad \text{in } V,$$
$$\vec{n} \times \vec{A} = 0 \qquad \text{on } \partial V.$$

Reduced magnetic vector potential (RMVP) ansatz:

Split the magnetic vector potential (MVP):

$$\vec{A}(\vec{r}, t) = \underbrace{\vec{A}_\mathrm{s}(\vec{r})}_{\text{source MVP}} + \underbrace{\vec{A}_\mathrm{r}(\vec{r})}_{\text{reaction MVP}}$$

Compute a Biot-Savart integral for each thread $\mathcal{L}'$:

$$\vec{A}_\mathrm{s}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{\mathcal{L}'} \frac{I}{|\vec{r} - \vec{r}'|} \, \mathrm{d}\vec{r}'$$

Compute $\vec{A}_\mathrm{r}$ using the finite-element method:

$$\vec{A}_\mathrm{r}(\vec{r}) \approx \sum_{j=1}^{N_\mathrm{edge}} \widehat{a}_{\mathrm{s},j} \, \vec{w}_j(\vec{r})$$

STANDARD RMVP FORMULATION

BY OSZKÁR BÍRÓ 1990, CHRISTIAN PAUL 1997

1. Evaluate the source MVP $\vec{A}_\mathrm{s}$ via Biot-Savart for each point $\vec{r} \in V$.
2. Solve a boundary value problem for the reaction MVP $\vec{A}_\mathrm{r}$.
3. Compose total MVP: $\vec{A} = \vec{A}_\mathrm{s} + \vec{A}_\mathrm{r}$ in V.

$\rightarrow$ Can get computationally expensive (V very large)!

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$\rightarrow$ Can get computationally expensive (V very large)!

Can we reduce the evaluation domain of the Biot-Savart integrals, thus accelerating the method?

OUTLINE

Updated RMVP formulation

↓

Performance study

↓

Extension to superconducting effects

↓

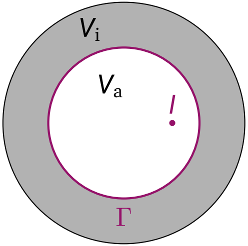
Summary & Outlook

OUTLINE

- **Updated RMVP formulation**
- Performance study
- Extension to superconducting effects
- Summary & Outlook

UPDATED RMVP FORMULATION: DERIVATION

BY LAURA D'ANGELO, DOMINIK MOLL, HERBERT DE GERSEM ET AL. 2024



Domain decomposition: V is decomposed into

- V_a: air domain with source currents
- V_i: source-free domain, typically iron yoke
- Γ: interface surface between V_a and V_i

Goal: Evaluate Biot-Savart only on V_a, as typically $|V_\mathrm{a}| \ll |V_\mathrm{i}|$.

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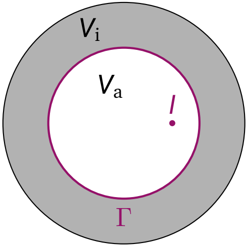

Goal: Have $\vec{A}_\mathrm{s}$ only in V_a.

Express the boundary value problem for V_a and V_i separately and introduce the source MVP $\vec{A}_\mathrm{s}$ in V_a:

$$\begin{aligned}
\nabla \times \left(\nu_0 \nabla \times \left(\vec{A}_\mathrm{a} \qquad \right)\right) &= 0 && \text{in } V_\mathrm{a},\\
\nabla \times \left(\nu \nabla \times \vec{A}_\mathrm{i}\right) &= 0 && \text{in } V_\mathrm{i},\\
\vec{n} \times \vec{A}_\mathrm{a} &= \vec{n} \times \vec{A}_\mathrm{i} && \text{at } \Gamma,\\
\vec{n} \times \vec{H}_\mathrm{a} &= \vec{n} \times \vec{H}_\mathrm{i} && \text{at } \Gamma,\\
\vec{n} \times \vec{A}_\mathrm{i} &= 0 && \text{at } \partial V.
\end{aligned}$$

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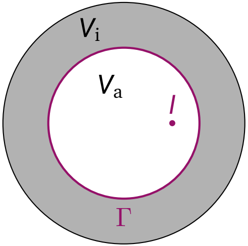

Goal: Have $\vec{A}_s$ only in V_a.

Express the boundary value problem for V_a and V_i separately and introduce the source MVP $\vec{A}_s$ in V_a:

$$\begin{aligned}
\nabla \times \left(\nu_0 \nabla \times \left(\vec{A}_a - \vec{A}_s\right)\right) &= 0 && \text{in } V_a,\\
\nabla \times \left(\nu \nabla \times \vec{A}_i\right) &= 0 && \text{in } V_i,\\
\vec{n} \times \vec{A}_a &= \vec{n} \times \vec{A}_i && \text{at } \Gamma,\\
\vec{n} \times \vec{H}_a &= \vec{n} \times \vec{H}_i && \text{at } \Gamma,\\
\vec{n} \times \vec{A}_i &= 0 && \text{at } \partial V.
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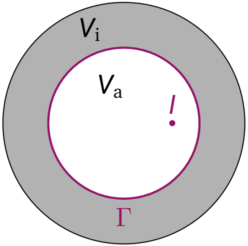

Goal: Have $\vec{A}_\mathrm{s}$ only in V_a.

Express the boundary value problem for V_a and V_i separately and introduce the source MVP $\vec{A}_\mathrm{s}$ in V_a:

$$\begin{aligned}
\nabla \times \left(\nu_0 \nabla \times \left(\vec{A}_\mathrm{a} - \vec{A}_\mathrm{s}\right)\right) &= 0 && \text{in } V_\mathrm{a},\\
\nabla \times \left(\nu \nabla \times \vec{A}_\mathrm{i}\right) &= 0 && \text{in } V_\mathrm{i},\\
\vec{n} \times \vec{A}_\mathrm{a} &= \vec{n} \times \vec{A}_\mathrm{i} && \text{at } \Gamma,\\
\vec{n} \times \vec{H}_\mathrm{a} &= \vec{n} \times \vec{H}_\mathrm{i} && \text{at } \Gamma,\\
\vec{n} \times \vec{A}_\mathrm{i} &= 0 && \text{at } \partial V.
\end{aligned}$$

$\rightarrow$ **Problem**: The separate solutions $\vec{A}_\mathrm{a} - \vec{A}_\mathrm{s}$ in V_a and $\vec{A}_\mathrm{i}$ in V_i are not continuous at Γ!

UPDATED RMVP FORMULATION: DERIVATION

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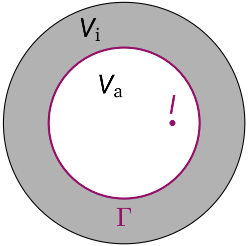

Solution: Enforce continuity by introducing the image MVP $\vec{A}_\mathrm{m}$ with $\vec{n} \times \vec{A}_\mathrm{m} = -\vec{n} \times \vec{A}_\mathrm{s}$ on Γ,

$$\begin{aligned} \nabla \times \left(\nu_0 \nabla \times (\vec{A}_\mathrm{a} - \vec{A}_\mathrm{s} - \vec{A}_\mathrm{m})\right) &= 0 && \text{in } V_\mathrm{a}, \\ \nabla \times \left(\nu \nabla \times \vec{A}_\mathrm{i}\right) &= 0 && \text{in } V_\mathrm{i}, \\ \vec{n} \times \vec{A}_\mathrm{a} &= \vec{n} \times \vec{A}_\mathrm{i} && \text{at } \Gamma, \\ \vec{n} \times \vec{H}_\mathrm{a} &= \vec{n} \times \vec{H}_\mathrm{i} && \text{at } \Gamma, \\ \vec{n} \times \vec{A}_\mathrm{i} &= 0 && \text{at } \partial V. \end{aligned}$$

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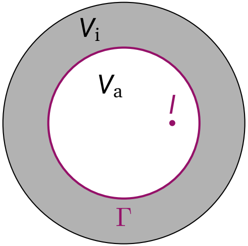

Solution: Enforce continuity by introducing the image MVP $\vec{A}_\mathrm{m}$ with $\vec{n} \times \vec{A}_\mathrm{m} = -\vec{n} \times \vec{A}_\mathrm{s}$ on Γ, and add zero:

$$\nabla \times \left(\nu_0 \nabla \times (\vec{A}_\mathrm{a} - \vec{A}_\mathrm{s} - \vec{A}_\mathrm{m})\right) = 0 \qquad \text{in } V_\mathrm{a},$$

$$\nabla \times \left(\nu \nabla \times \vec{A}_\mathrm{i}\right) = 0 \qquad \text{in } V_\mathrm{i},$$

$$-\underbrace{\vec{n} \times (\vec{A}_\mathrm{s} + \vec{A}_\mathrm{m})}_{=0} + \vec{n} \times \vec{A}_\mathrm{a} = \vec{n} \times \vec{A}_\mathrm{i} \qquad \text{at } \Gamma,$$

$$\underbrace{-\vec{n} \times (\vec{H}_\mathrm{s} + \vec{H}_\mathrm{m}) + \vec{n} \times (\vec{H}_\mathrm{s} + \vec{H}_\mathrm{m})}_{=0} + \vec{n} \times \vec{H}_\mathrm{a} = \vec{n} \times \vec{H}_\mathrm{i} \qquad \text{at } \Gamma,$$

$$\vec{n} \times \vec{A}_\mathrm{i} = 0 \qquad \text{at } \partial V.$$

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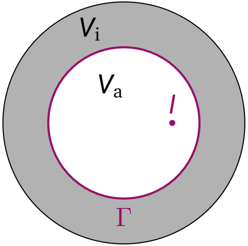

Summarize with $\vec{A}_a' = \vec{A}_a - \vec{A}_s - \vec{A}_m$:

$$
\begin{aligned}
\nabla \times \left(\nu_0 \nabla \times \vec{A}_a'\right) &= 0 && \text{in } V_a,\\
\nabla \times \left(\nu \nabla \times \vec{A}_i\right) &= 0 && \text{in } V_i,\\
\vec{n} \times \vec{A}_a' &= \vec{n} \times \vec{A}_i && \text{at } \Gamma,\\
\vec{n} \times \vec{H}_a' - \vec{n} \times \vec{H}_i &= \vec{n} \times (\vec{H}_s + \vec{H}_m) && \text{at } \Gamma,\\
\vec{n} \times \vec{A}_i &= 0 && \text{at } \partial V.
\end{aligned}
$$

- Sub-domain solutions $\vec{A}_a'$ and $\vec{A}_i$ are tangentially continuous at Γ.
- Jump of $\vec{H}$ can be interpreted as a surface current density $\vec{K}_g = \vec{n} \times (\vec{H}_s + \vec{H}_m)$.

UPDATED RMVP FORMULATION: DERIVATION

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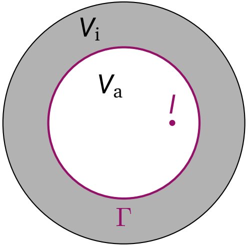

Summarize with $\vec{A}'_\mathrm{a} = \vec{A}_\mathrm{a} - \vec{A}_\mathrm{s} - \vec{A}_\mathrm{m}$:

$$
\begin{aligned}
\nabla \times \left(\nu_0 \nabla \times \vec{A}_\mathrm{g}\right) &= 0 && \text{in } V_\mathrm{a}, \\
\nabla \times \left(\nu \nabla \times \vec{A}_\mathrm{g}\right) &= 0 && \text{in } V_\mathrm{i}, \\
\vec{n} \times \vec{A}_\mathrm{g} &= \vec{n} \times \vec{A}_\mathrm{g} && \text{at } \Gamma, \\
\vec{n} \times \vec{H}_\mathrm{g} - \vec{n} \times \vec{H}_\mathrm{g} &= \vec{n} \times (\vec{H}_\mathrm{s} + \vec{H}_\mathrm{m}) && \text{at } \Gamma, \\
\vec{n} \times \vec{A}_\mathrm{g} &= 0 && \text{at } \partial V.
\end{aligned}
$$

- Sub-domain solutions $\vec{A}'_\mathrm{a}$ and $\vec{A}_\mathrm{i}$ are tangentially continuous at Γ.
- Jump of $\vec{H}$ can be interpreted as a surface current density $\vec{K}_\mathrm{g} = \vec{n} \times (\vec{H}_\mathrm{s} + \vec{H}_\mathrm{m})$.

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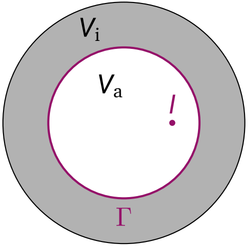


Finally, substitute

$$\vec{A}_\mathrm{g} = \begin{cases} \vec{A}'_\mathrm{a} & \text{in } V_\mathrm{a}, \\ \vec{A}_\mathrm{i} & \text{in } V_\mathrm{i}, \end{cases}$$

to obtain a single domain problem:

$$\nabla \times \left(\nu \nabla \times \vec{A}_\mathrm{g}\right) = \vec{K}_\mathrm{g}\delta_\Gamma \qquad \text{in } V,$$

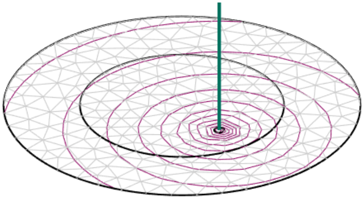
$$\vec{n} \times \vec{A}_\mathrm{g} = 0 \qquad \text{at } \partial V.$$

UPDATED RMVP FORMULATION: RECIPE

BY LAURA D'ANGELO, DOMINIK MOLL, HERBERT DE GERSEM ET AL. 2024

1. Evaluate the source MVP $\vec{A}_s$ via Biot-Savart only at the interface $\Gamma = \partial V_a$...
   ...and on every point of interest $\vec{r} \in V_a$.

$\rightarrow$ Huge improvement in computational efficiency!



source MVP $\vec{A}_s$ (everywhere) and source current

UPDATED RMVP FORMULATION: RECIPE

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2. Find the image MVP $\vec{A}_\mathrm{m} \in H(\mathrm{curl}; V_\mathrm{a})$, $\vec{n} \times \vec{H}_\mathrm{m} \in H^{-1/2}(\mathrm{curl}; \Gamma)$, s.t.

$$\left(\nu_0 \nabla \times \vec{A}_\mathrm{m}, \nabla \times \vec{A}'_\mathrm{m}\right)_{V_\mathrm{a}} + \left(\vec{n} \times \vec{H}_\mathrm{m}, \vec{A}'_\mathrm{m}\right)_\Gamma = 0 \qquad \forall \vec{A}'_\mathrm{m} \in H(\mathrm{curl}; V_\mathrm{a}),$$

$$\left(\vec{A}_\mathrm{m}, \vec{n} \times \vec{H}'_\mathrm{m}\right)_\Gamma + \left(\vec{A}_\mathrm{s}, \vec{n} \times \vec{H}'_\mathrm{m}\right)_\Gamma = 0 \qquad \forall \vec{n} \times \vec{H}'_\mathrm{m} \in H^{-1/2}(\mathrm{curl}; \Gamma).$$

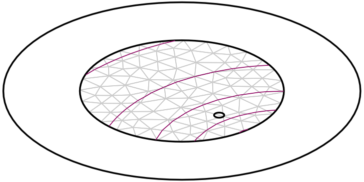

image MVP $\vec{A}_\mathrm{m}$ in V_a

UPDATED RMVP FORMULATION: RECIPE

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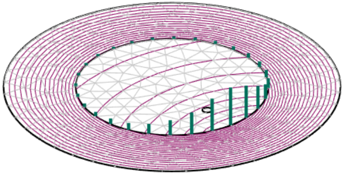
3. Find the reaction MVP $\vec{A}_\mathrm{g} \in H_0(\mathrm{curl}; V)$ s.t.

$$\left(\nu\nabla \times \vec{A}_\mathrm{g}, \nabla \times \vec{A}'_\mathrm{g}\right)_V = \left(\vec{n} \times \vec{H}_\mathrm{s}, \vec{A}'_\mathrm{g}\right)_\Gamma + \left(\vec{n} \times \vec{H}_\mathrm{m}, \vec{A}'_\mathrm{g}\right)_\Gamma \quad \forall \vec{A}'_\mathrm{g} \in H_0(\mathrm{curl}; V).$$



reaction MVP $\vec{A}_\mathrm{g}$ in V,

surface current density $\vec{K}_\mathrm{g} = \vec{n} \times (\vec{H}_\mathrm{s} + \vec{H}_\mathrm{m})$ on Γ

UPDATED RMVP FORMULATION: RECIPE

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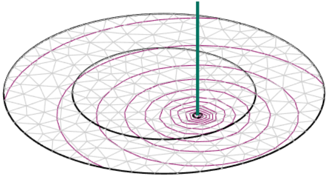


source MVP $\vec{A}_\mathrm{s}$

+

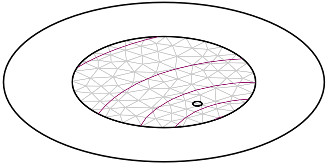
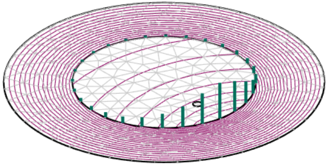


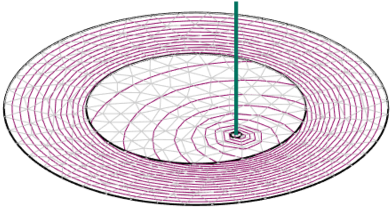
image MVP $\vec{A}_\mathrm{m}$

+



reaction MVP $\vec{A}_\mathrm{g}$

=



4. Compose final solution:

$$\vec{A} = \vec{A}_\mathrm{s} + \vec{A}_\mathrm{m} + \vec{A}_\mathrm{g} \quad \text{in } V_\mathrm{a},$$
$$\vec{A} = \vec{A}_\mathrm{g} \quad \text{in } V_\mathrm{i}.$$

OUTLINE

Updated RMVP formulation

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Performance study

↓

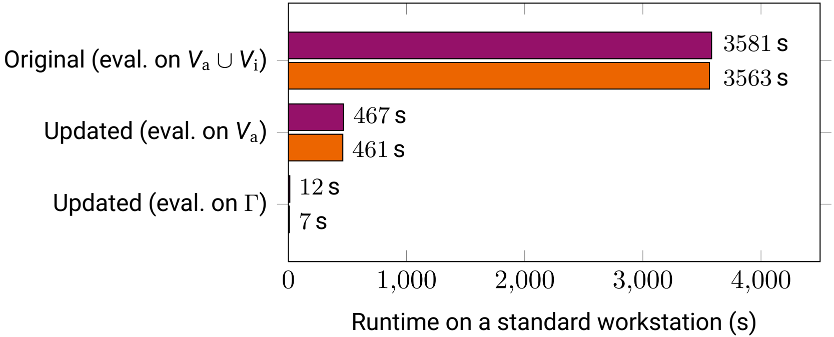
Extension to superconducting effects

↓

Summary & Outlook

PERFORMANCE COMPARISON

BENCHMARK MODEL WITH > 100,000 DOF AND 18 WIRES



1. **Biot-Savart** computation is dominant $\Rightarrow$ Parallelize and/or use other known techniques
2. **Updated RMVP** formulation by far superior than standard one

OUTLINE

Updated RMVP formulation

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Performance study

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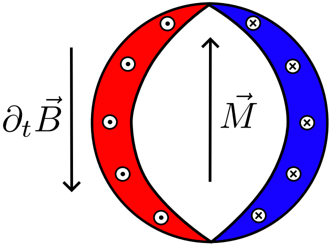
Extension to superconducting effects

↓

Summary & Outlook

SUPERCONDUCTOR MAGNETIZATION



Time-varying external magnetic flux density $\vec{B}$

$\downarrow$ induces $\downarrow$

screening currents / magnetization

$\downarrow$ shielding $\downarrow$

the superconductor's interior (diamagnetic behavior)

- This magnetization effect has to be considered in the RMVP formulation.
- Can be done by adding two artificial opposed wires... but we don't want to do that.
- **Note**: The screening current distribution in the wire gives rise to a dipole field.

DIPOLE MAGNETIC MOMENT

Biot-Savart:

$$\vec{A}_\mathrm{s}(\vec{r}) = \frac{\mu_0}{4\pi} \int\limits_{\mathcal{V}'} \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} \, \mathrm{d}V'$$

Multipole expansion:

$$\frac{1}{|\vec{r} - \vec{r}'|} = \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\vec{r}' \cdot \nabla\right)^n \frac{1}{|\vec{r}|}$$

DIPOLE MAGNETIC MOMENT

Biot-Savart:

$$\vec{A}_s(\vec{r}) = \frac{\mu_0}{4\pi} \int_{\mathcal{V}'} \frac{\vec{J}(\vec{r}')}{|\vec{r} - \vec{r}'|} \,\mathrm{d}V'$$

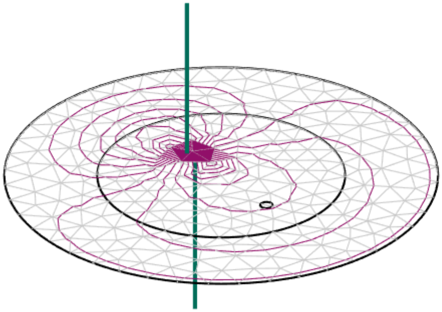
Multipole expansion:

$$\frac{1}{|\vec{r} - \vec{r}'|} = \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\vec{r}' \cdot \nabla\right)^n \frac{1}{|\vec{r}|}$$

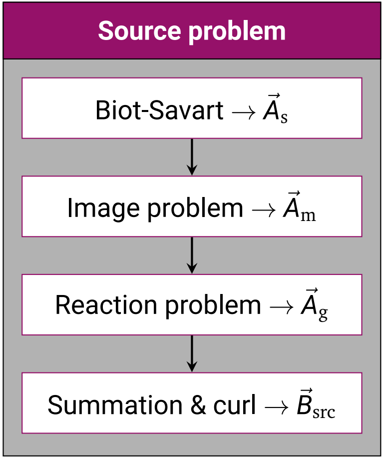
$n = 1$ (dipole):

$$\vec{A}_{\mathrm{dp}}(\vec{r}) = -\frac{\mu_0}{4\pi} \int_{\mathcal{V}'} \vec{J}(\vec{r}') \frac{\vec{r} \cdot \vec{r}'}{|\vec{r}|^3} \,\mathrm{d}V'$$

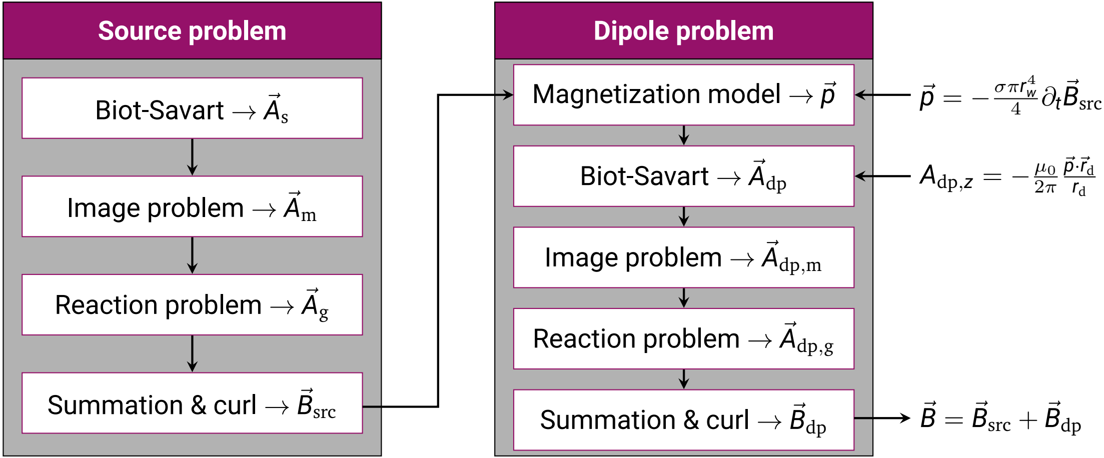
↪ field generated by a dipole moment $\vec{p}$ at $\vec{r}'$ or two opposite wires around $\vec{r}'$



EXTENDED RMVP METHOD SCHEME

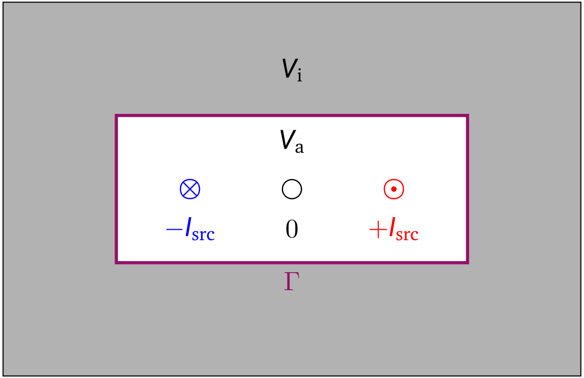


EXTENDED RMVP METHOD SCHEME

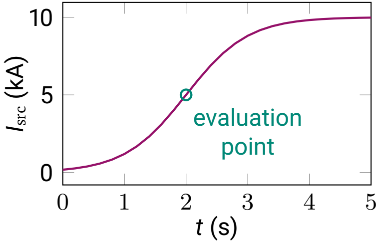


DIPOLE RMVP TEST MODEL: SETUP

Racetrack coil with central currentless wire:



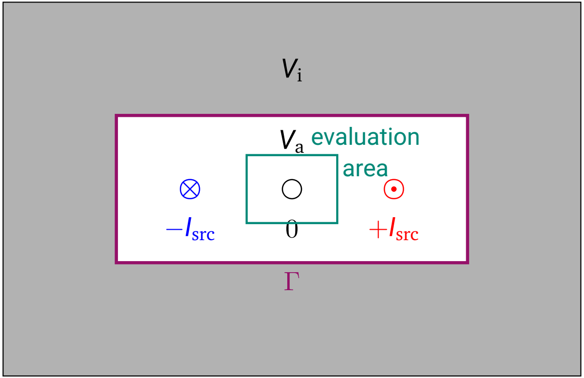
Source current excitation:



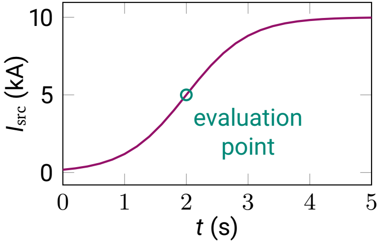
Expectation: Diamagnetic behavior of the central wire

DIPOLE RMVP TEST MODEL: SETUP

Racetrack coil with central currentless wire:

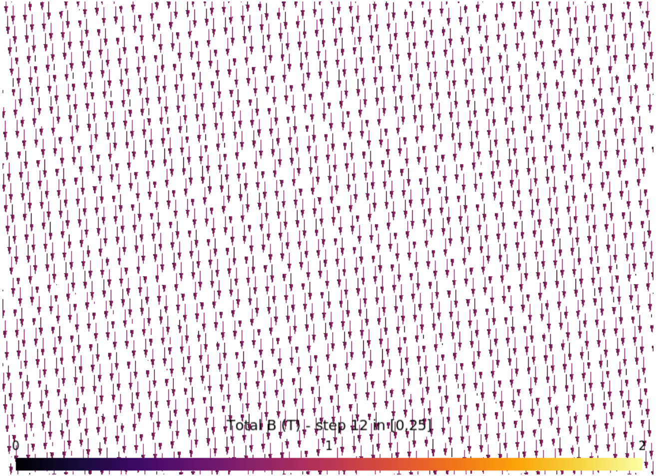


Source current excitation:



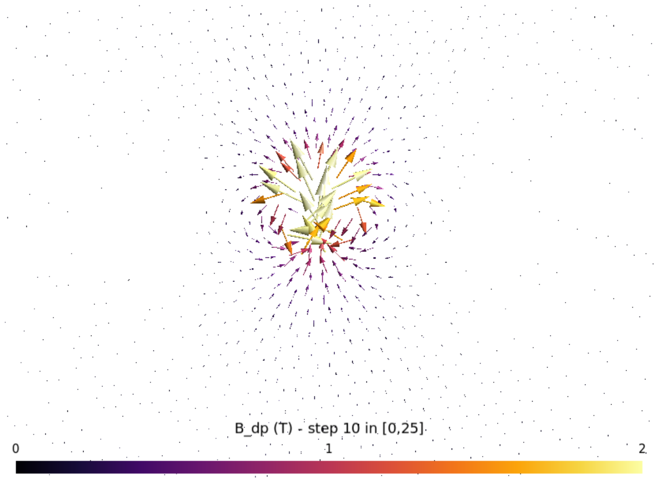
Expectation: Diamagnetic behavior of the central wire

DIPOLE RMVP TEST MODEL: SIMULATION



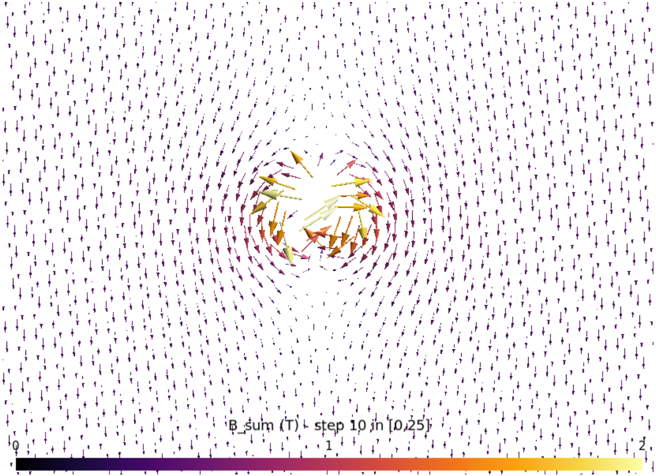
- Magnetic flux density $\vec{B}_{\mathrm{src}}$ as solution of the source problem
- Generated by coil winding carrying a source current I_{src}

DIPOLE RMVP TEST MODEL: SIMULATION



- Magnetic flux density $\vec{B}_{\mathrm{dp}}$ as solution of the dipole problem
- Generated by magnetic dipole moment $\vec{p}$

DIPOLE RMVP TEST MODEL: SIMULATION



- Total magnetic flux density $\vec{B} = \vec{B}_{\mathrm{src}} + \vec{B}_{\mathrm{dp}}$
- Diamagnetic behavior ☺

OUTLINE

Updated RMVP formulation

↓

Performance study

↓

Extension to superconducting effects

↓

Summary & Outlook

SUMMARY

Updated RMVP method:

- RMVP ansatz: No explicit meshing of wires in the FE mesh
- Update: Evaluate Biot-Savart's law only on V_a at most
- High efficiency gain compared to original RMVP approach

↪ QR code to paper

Extended RMVP method:

- Consider screening currents by a dipole moment without adding artificial geometrical wires
- Test simulation was successful

OUTLOOK

- Higher-order magnetic moments to consider magnetization and eddy current effects
- Adapt RMVP approach for high-temperature superconducting coils (tapes!)