TECHNISCHE UNIVERSITÄT DARMSTADT

CALCULATING THE TRANSVERSE SHUNT IMPEDANCE FROM EIGENMODE RESULTS

ICAP 2024

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MOTIVATION

Section 1

MOTIVATION

UPGRADE TO PETRA IV

Active planning process of the upgrade

PETRA III ⟶ PETRA IV

Goal - 4th generation light source:

- Low emittance
- High beam current
- Long beam lifetime
- Stable particle acceleration and storage

Challenges:

- Toucheck effect
- Intrabeam scattering

Solution - Bunch lengthening:

- Active 3rd harmonic cavity

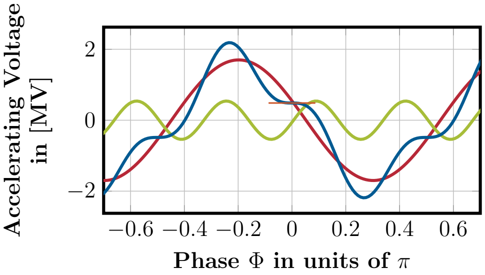
MOTIVATION

ACTIVE 3rd HARMONIC CAVITY

$$V(t) = V_1 \cos(\omega_{RF}\, t + \Phi_1) + V_2 \cos(3\,\omega_{RF}\, t + \Phi_2)$$

Requirements of the 3rd harmonic cavity

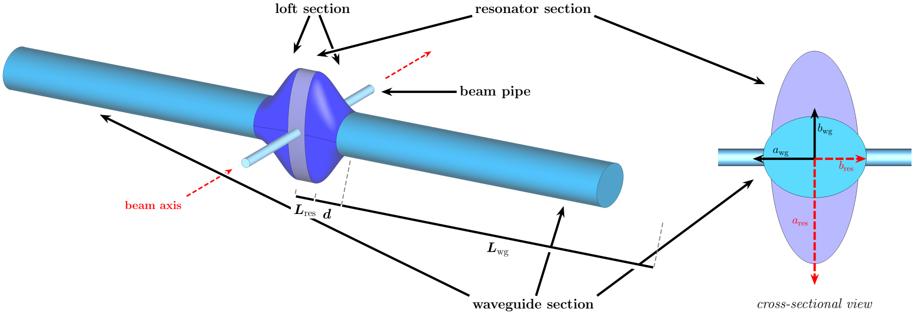
- No phase dependency of the voltage
- Inexpensive and simple manufacturing
- Mitigation of higher order modes (HOM)



THE SINGLE MODE CAVITY

Section 2

THE SINGLE MODE CAVITY



[1]

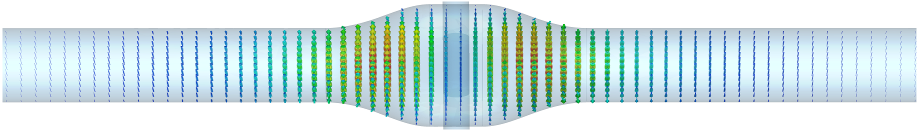
- Resonator Section: resonant frequency, $f_{res} = f_1$
  - Desired accelerating mode resonates around the beam axis
- Waveguide Section: Connected to damper to attenuate HOMs
  - Cutoff frequency between resonant mode and next higher, $f_1 \ll f_c \lesssim f_2$

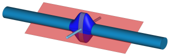
[1] Kronshorst et al.: *Design of a single mode 3rd harmonic cavity for PETRA IV*, Preprint IPAC'24, 10.18429/JACoW-IPAC2024-TUPG52

THE SINGLE MODE CAVITY

UNDESIRED HIGHER ORDER MODE

$\mathbf{qTE_{112,even}}$
$f_{13} = 2.2499\,\mathrm{GHz}$
$Q_0 = 259206.4$





- Not all HOM couple to the waveguide section
- These modes have to be studied
  - Either their influence is negligible
  - Or their occurrence has to be suppressed

How to assess the different transverse modes?

$\Rightarrow$ **Through the kick factor $k_\perp$ and shunt impedance $R_{S,n,\perp}$**

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CALCULATION METHODS FOR THE TSI

Section 3

CALCULATION METHODS FOR THE TSI

AND WHAT IT IS

- 3 different approaches to obtain the transverse shunt impedance

frequency domain impedance solver	time domain wakefield solver	eigenmode solver

- It gauges the interaction of the particle beam and the cavity wall in transverse direction
- Relation to the kick factor
  - $k_{n,\perp} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}\omega \frac{\omega_{r,n}}{\omega} \frac{\vec{R}_{\mathrm{S,n,\perp}}}{1+jQ\left(\frac{\omega}{\omega_{r,n}} - \frac{\omega_{r,n}}{\omega}\right)} \mathrm{e}^{-\omega^2\sigma^2}$ [2, 3]
- Panofsky-Wenzel theorem [4]
  - $\vec{p} = \frac{q}{c} \int_0^l \mathrm{d}z \, [\vec{E} + c\vec{e}_z \times \vec{B}] \, e^{j\omega \frac{z}{c}}$
  - $\frac{\partial}{\partial t} \vec{p}_\perp = -c \, \nabla_\perp p_\parallel$

[2] Mosnier: *Analyse de la stabilite de faisceau dans un accelerateur lineaire...*, Nucl. Instruments and Methods in Ph. Research, 1987

[3] Zotter, Kheifets: *Impedances and wakes in high-energy particle accelerators*, 2000, World Scientific

[4] Panofsky, Wenzel: *Some Considerations Concerning the Transverse Deflection of Charged Particles in Radio-Frequency Fields* , Review of Scientific Instrtuments 1956

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CALCULATION METHODS FOR THE TSI

frequency domain impedance solver

time domain wakefield solver

eigenmode solver

to solve

$$\nabla \times \nabla \times \underline{\vec{E}} - k_0^2 \underline{\vec{E}} = -j k_0 Z_0 \underline{\vec{J}}(\vec{r}_1^{\perp}, \omega)$$

$$\underline{Z}_{||}(\omega, \vec{r}_2^{\perp}) = -\frac{1}{q_1\, q_2} \int_0^l \mathrm{d}z \underline{\vec{E}}(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, z, \omega) \cdot \underline{\vec{J}}_s^*(\vec{r}_2^{\perp}) \quad [5]$$

Panofsky-Wenzel theorem

$$\vec{R}_{\mathrm{S,n},\perp} = \underline{\vec{Z}}_{\perp}(\omega_{r,n}, \vec{r}_2^{\perp}) = \frac{c}{\omega_{r,n}} \nabla_{\perp} \underline{Z}_{||}(\omega_{r,n}, \vec{r}_2^{\perp})$$

[5] Quetscher, Gjonaj: *Impedance computation for large accelerator strucures using a domain decomposition method*, Preprint IPAC’24, 10.18429/JACoW-IPAC2024-THPC62

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CALCULATION METHODS FOR THE TSI

frequency domain impedance solver | time domain wakefield solver | eigenmode solver

to solve

$$\nabla \times \nabla \times \underline{\vec{E}} - k_0^2 \underline{\vec{E}} = -jk_0 Z_0 \underline{\vec{J}}(\vec{r}_1^{\perp}, \omega)$$

$$\underline{Z}_{\|}(\omega, \vec{r}_2^{\perp}) = -\frac{1}{q_1\, q_2} \int_0^l \mathrm{d}z\, \underline{\vec{E}}(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, z, \omega) \cdot \underline{\vec{J}}_s^{*}(\vec{r}_2^{\perp}) \qquad [5]$$

not implemented in CST

Panofsky-Wenzel theorem

$$\vec{R}_{\mathrm{S,n},\perp} = \underline{\vec{Z}}_{\perp}(\omega_{r,n}, \vec{r}_2^{\perp}) = \frac{c}{\omega_{r,n}} \nabla_{\perp} \underline{Z}_{\|}(\omega_{r,n}, \vec{r}_2^{\perp})$$

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CALCULATION METHODS FOR THE TSI

frequency domain impedance solver

time domain wakefield solver

eigenmode solver

↓ to solve

$$\frac{\partial}{\partial t}\vec{B} = -\nabla \times \vec{E}$$
$$\frac{\partial}{\partial t}\vec{E} = \frac{1}{c^2}\nabla \times \vec{B} - \frac{1}{\varepsilon_0}\vec{J}(\vec{r}_1^{\perp}, t)$$

↓

$$\vec{w}(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, s) = \frac{1}{q}\int_{-\infty}^{\infty} \mathrm{d}z \left[\vec{E}\left(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, z, t\right) + c\vec{e}_z \times \vec{B}\left(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, z, t\right)\right] \quad [6]$$

↓ Fourier transform

$$\vec{R}_{\mathrm{S,n},\perp} = \underline{\vec{Z}}_{\perp}(\omega_{r,n}, \vec{r}_2^{\perp}) = \mathcal{F}\{\vec{w}_{\perp}(\vec{r}_1^{\perp}, \vec{r}_2^{\perp}, s)\}$$

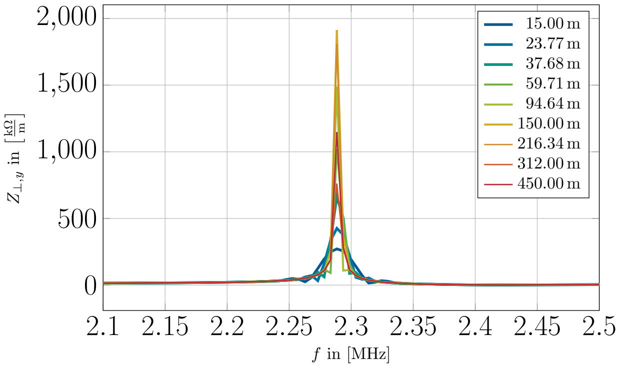
[6] Weiland, Wanzenberg: *Wake fields and impedances*, Frontiers of Particle Beams 1992, 10.1007/3-540-55250-2_26

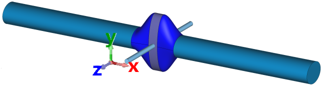
CALCULATION METHODS FOR THE TSI

frequency domain impedance solver

time domain wakefield solver

eigenmode solver





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CALCULATION METHODS FOR THE TSI

frequency domain impedance solver

time domain wakefield solver

eigenmode solver

to solve

$$\nabla \times \nabla \times \underline{\vec{E}} - \underline{\vec{k}}^2 \, \underline{\vec{E}} = 0$$

$$\underline{V}_{\mathrm{acc},\parallel} = \int_0^l \mathrm{d}z \, \underline{\vec{E}}_n \, e^{-j\,\omega_{r,n} \frac{z}{c}}$$

$$R_{\mathrm{S,n},\parallel} = \frac{|\underline{V}_{\mathrm{acc}\,\parallel}|^2}{2 P_d}$$

Panofsky-Wenzel

$$\underline{E}_z \approx \frac{\partial}{\partial r} \underline{E}_z \, r$$

$$\vec{R}_{\mathrm{S,n},\perp} = \frac{|\nabla_\perp \underline{V}_{\mathrm{acc},\parallel}|^2}{2\,k\,P_d} \quad [2]$$

CALCULATION METHODS FOR THE TSI

frequency domain impedance solver

time domain wakefield solver

eigenmode solver

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CALCULATION METHODS FOR THE TSI

frequency domain impedance solver | time domain wakefield solver | **eigenmode solver**

$$\nabla \times \nabla \times \underline{\vec{E}} - \underline{\vec{k}}^2 \underline{\vec{E}} = 0$$

$$\underline{V}_{\mathrm{acc},\parallel} = \int_0^l \mathrm{d}z \underline{\vec{E}}_n \, e^{-j\,\omega_{r,n} \frac{z}{c}}$$

$$\vec{R}_{\mathrm{S,n},\perp} = \frac{|\nabla_\perp \underline{V}_{\mathrm{acc},\parallel}|^2}{2\,k\,P_d} \qquad [2]$$

beam excitation: $\underline{\vec{J}}(\vec{r}_\perp, z, \omega_{r,n}) = I_0 \delta(\vec{r}_\perp) e^{jk_n z} \vec{e}_z$

Poynting's theorem: $P_n = \int \mathrm{d}V \, \underline{\vec{J}} \cdot \hat{\underline{\vec{E}}}$

steady state $\Rightarrow P_n = \hat{P}_{d,n} = |a_n|^2 \, P_{d,n}$

$$a_n = \frac{I_0}{P_{d,n}} \underline{V}^*_{\mathrm{acc}\parallel} \qquad [7]$$

$$\underline{V}_{\mathrm{acc}\perp} = \int_0^l dz e^{jk_n z} \left[\underline{\vec{E}} + c\vec{e}_z \times \underline{\vec{B}}\right]_\perp$$

$$\vec{R}_{\mathrm{S,n},\perp} = |\underline{\vec{Z}}_\perp(\omega_{r,n})| = \frac{|\hat{\underline{V}}_{\mathrm{acc},\perp}|}{I_0} = \frac{1}{\vec{r}_\perp} \frac{|\underline{V}^*_{\mathrm{acc}\parallel}|\,|\underline{V}_{\mathrm{acc}\perp}|}{2\,P_{d,n}}$$

[7] Quetscher, Gjonaj: *unpublished*

GAUGING THE CST EXPORT

Section 4

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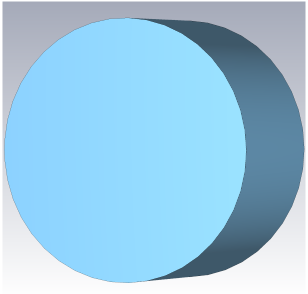
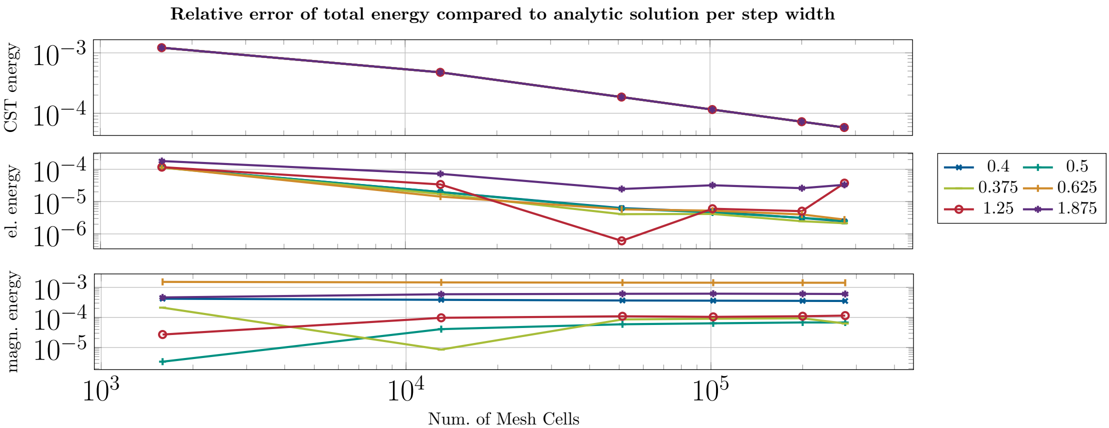
GAUGING THE CST EXPORT

- Gauging the transverse shunt impedance calculation method necessitates investigating the CST export error
- Toy model: circular cylindrical cavity
  - For the TM_{110}-mode
  - Analytically solvable



GAUGING THE CST EXPORT

ENERGY OF CYLINDRICAL CAVITY



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GAUGING THE CST EXPORT

LONGITUDINAL AND TRANSVERSE FIELD INTEGRALS CLOSE TO BEAM AXIS

- Does this quality hold for values close to the cavity center?
  - Field amplitudes are smaller $\rightarrow$ possibly higher inaccuracy
- Investigation of longitudinal and transverse voltage for $x_{\text{offset}} = 5\,\text{mm}$

rel. error compared to analytical value	longitudinal voltage	transverse voltage
preconditioned meshgrid	$5.188\,18 \times 10^{-5}$	$8.244\,978 \times 10^{-5}$
free meshgrid	$5.188\,18 \times 10^{-5}$	$8.244\,978 \times 10^{-5}$

- At least for this export no deviation can be observed

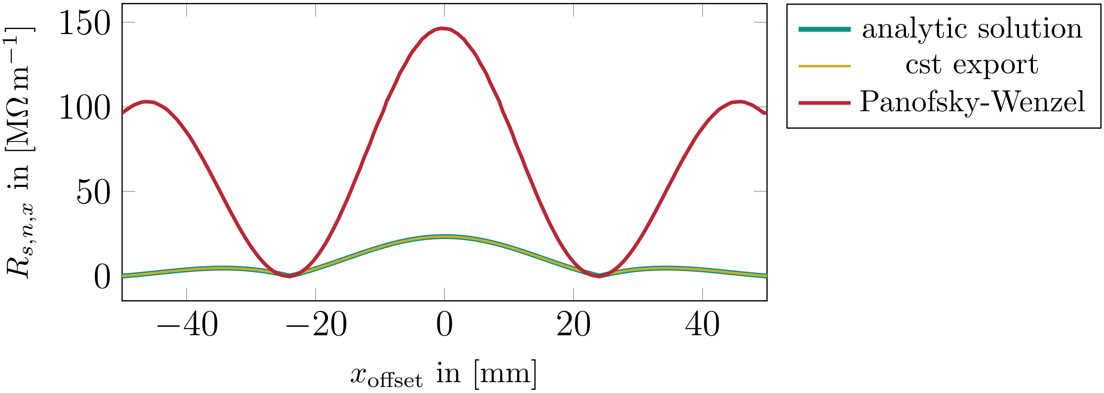
$\Rightarrow$ The meshgrid does not need preconditioning to the integration axis

EVALUATION OF THE EM ANSATZ

Section 5

EVALUATION OF THE EM ANSATZ

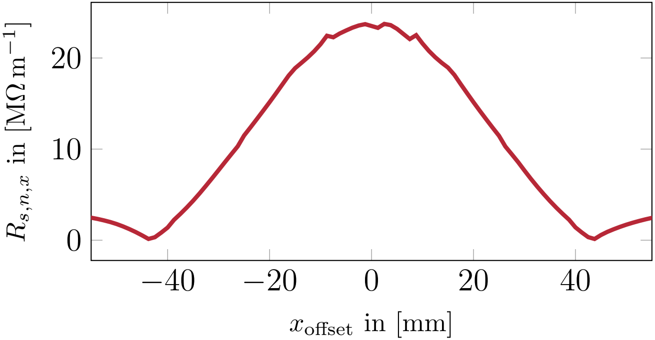
FOR THE TM_{110}-MODE



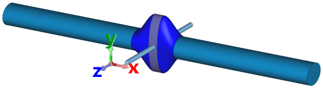
APPLICATION TO THE SINGLE MODE CAVITY

Section 6

APPLICATION TO THE SINGLE MODE CAVITY



- $\text{qTE}_{112,\text{even}}$-Mode
- $f_{13} = 2.2499\,\text{GHz}$



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CONCLUSION/OUTLOOK

Section 7

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CONCLUSION/OUTLOOK

- Conclusion
  - The eigenmode ansatz without any simplifying assumptions seems promising.
  - The discrepancy with the usually used function derived with Panofsky-Wenzel is concerning.
- Outlook
  - Investigation of difference for the two eigenmode methods
  - Comparison with frequency domain simulation
  - Investigate the radial dependency
  - **Use methodology to gauge HOM of cavity and further optimize it**