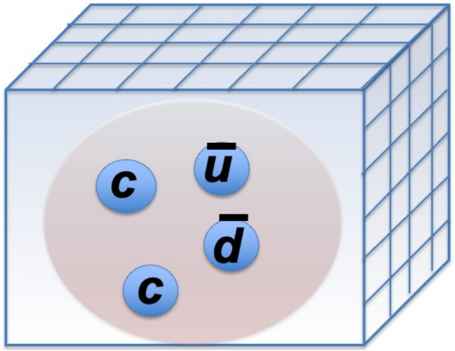
Doubly heavy tetraquarks from lattice QCD



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School on Modern Techniques in Hadron spectroscopy
From quarks and gluons to hadrons and nuclei
Bochum, July 24th, 2024

”Seminar”:

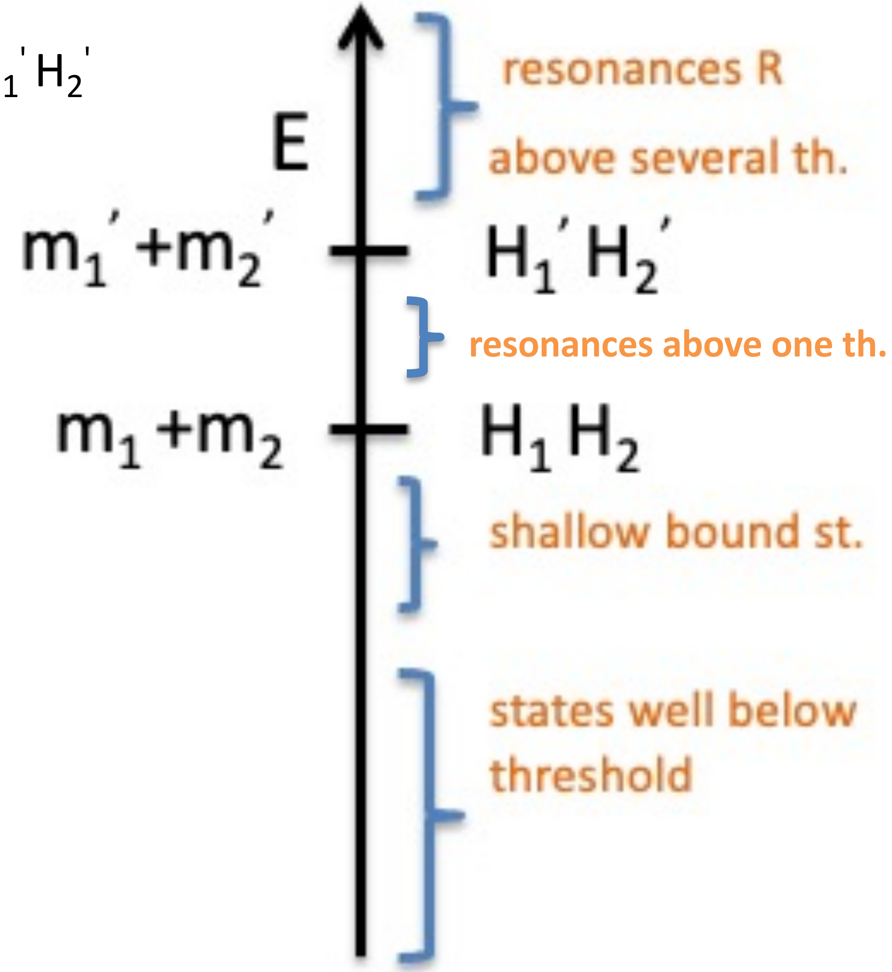
- examples chosen for pedagogical purpose
- not review of all existing results

How difficult it is to theoreticaly study a given hadron ab-initio?

strong, ~~EW~~

$R \rightarrow H_1 H_2 , H_1' H_2'$

$R \rightarrow H_1 H_2$



$$\left.\right\} \quad E_n(P=0) = m$$

How difficult it is to theoreticaly study a given hadron ab-initio?

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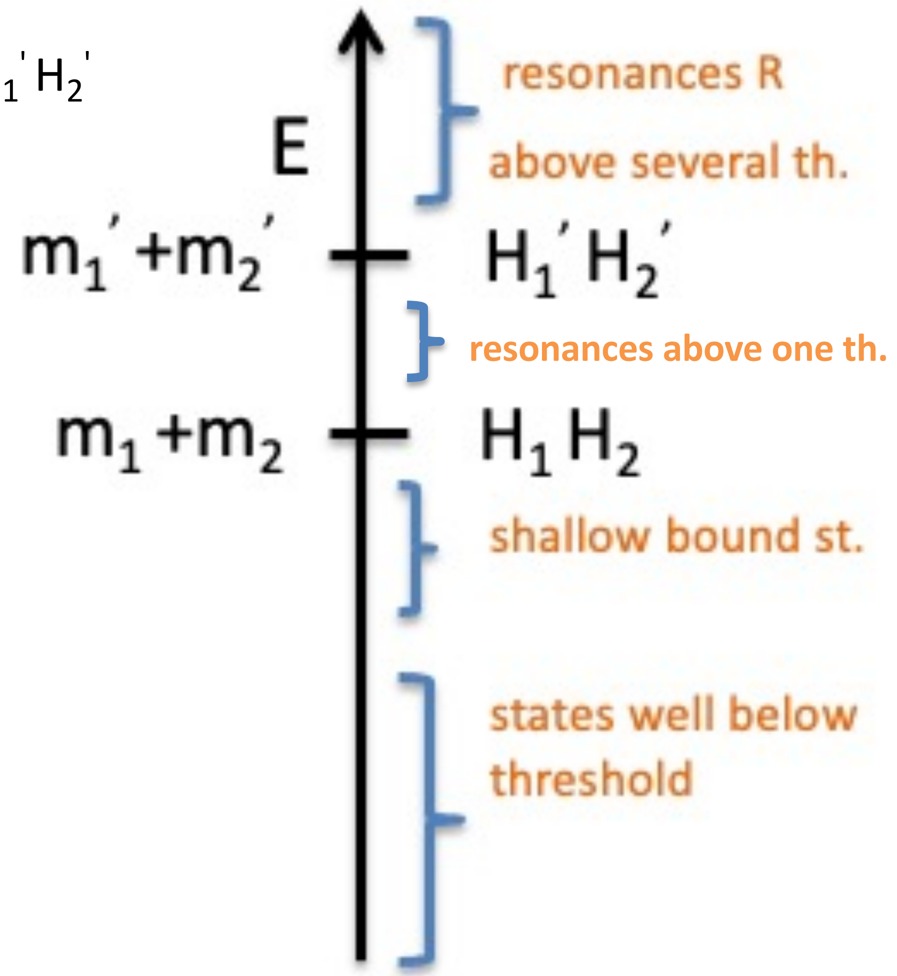
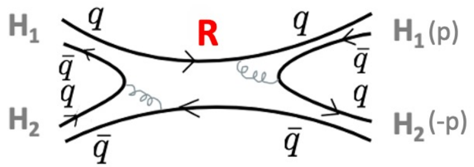




T(E): scattering amplitude

How difficult it is to theoreticaly study a given hadron ab-initio?

strong, E~~W~~

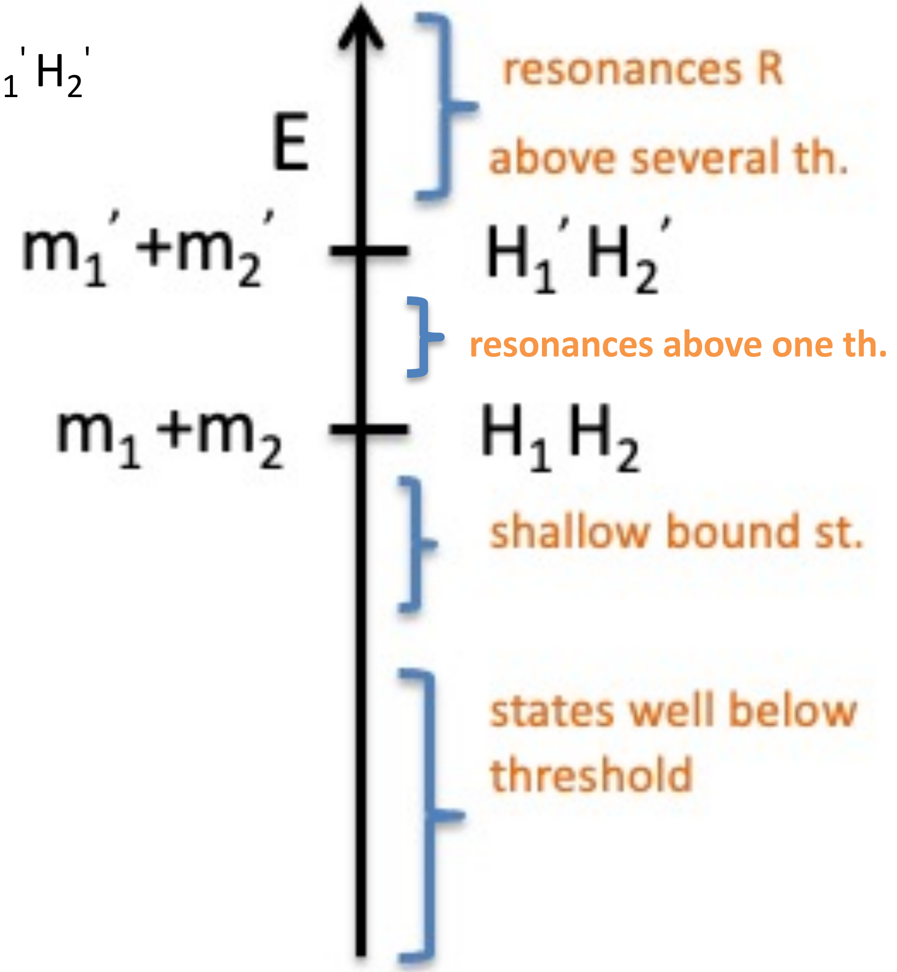
$R \to H_1 H_2,\ H_1' H_2', \ldots$

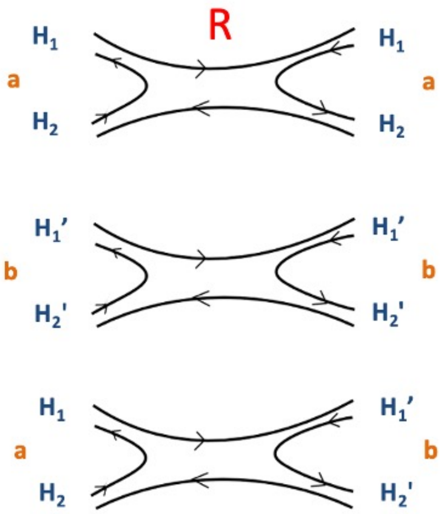
channel a : $H_1 H_2$
channel b : $H_1' H_2'$

$$T(E) = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} \text{a -> a} & \text{a -> b} \\ \left[\begin{array}{cc} T_{aa}(E) & T_{ab}(E) \\ T_{ab}(E) & T_{bb}(E) \end{array}\right] & \\ \text{b->a} & \text{b -> b} \end{array}$$

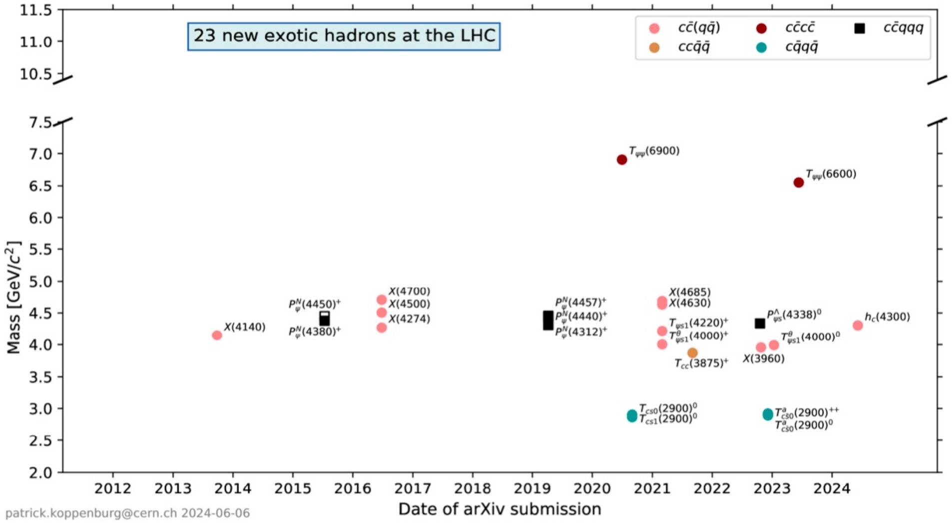
$R \to H_1 H_2,\ H_1' H_2'$

$R \to H_1 H_2$



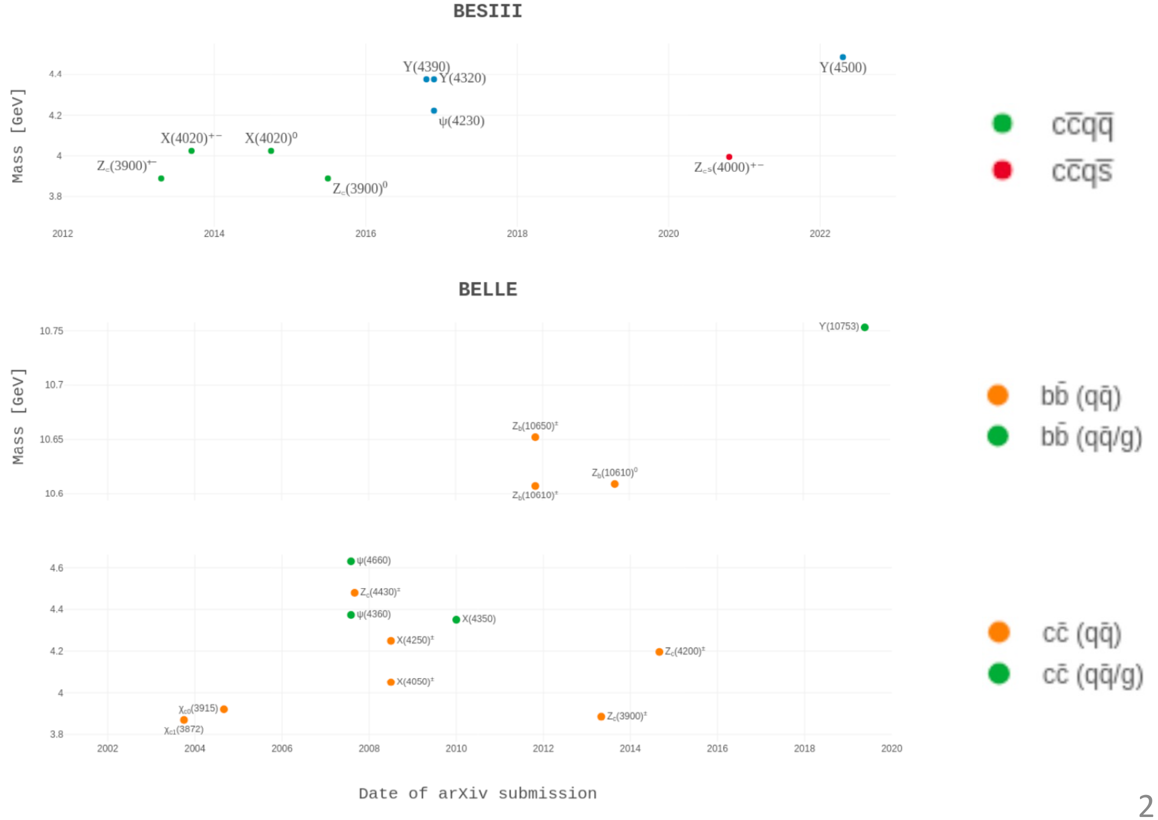


Discovered exotic hadrons contain heavy quarks



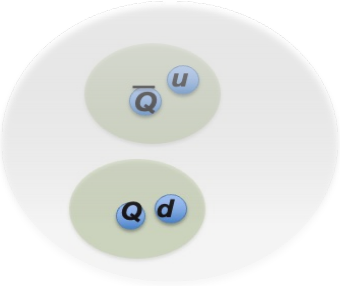
patrick.koppenburg@cern.ch 2024-06-06

ExoticHub



Simplistic argument: for a given V:

heavier particles are easier to bind



$$\hat{H} = \frac{\hat{p}^2}{2m_r} + V$$

Tetraquarks

doubly heavy tetraquarks vs quarkonium-like states Zc, Zb, X(3872)

$$QQ\bar{q}\bar{q}$$

$$\bar{Q}Q\bar{q}q$$

Q=c,b

q=u,d

$$QQ\bar{q}\bar{q} \to (\bar{q}Q)\ (\bar{q}Q)$$

$$\bar{Q}Q\bar{q}q \to (\bar{Q}q)\ (\bar{q}Q)$$

$$\to (\bar{Q}Q)\ (\bar{q}q)$$

lower

lying

example

$$cc\bar{d}\bar{u} \to DD^*$$

Tcc

$$\bar{c}c\bar{d}u \to D\bar{D}^*,\ J/\psi\pi,\ \eta_c\rho$$

Zc

“easier” theoretically

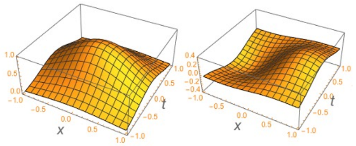
more difficult experimentally

QCD: $\mathcal{L}_{QCD} = \frac{1}{4}G_a^{\mu\nu}G_a^{\mu\nu} + \bar{q}i\gamma_\mu(\partial^\mu + ig_sG_a^\mu T^a)q - m_q\bar{q}q$ $g_s \cancel{\ll} 1$ at hadronic energy scale

Lattice QCD

Lectures by Bulava and Jackura

$$\langle C\rangle = \int DG\, Dq\, D\bar{q}\, C\, e^{-S_{QCD}/\hbar}$$



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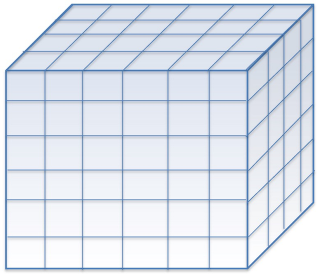


Main quantity extracted: finite-volume eigen-energies E_n $\hat{H}|n\rangle = E_n|n\rangle$

often “non-precision” studies:

single a, $m_{u/d} > m_{u/d}^{phy}$, $m_\pi > 140\ \mathrm{MeV}$

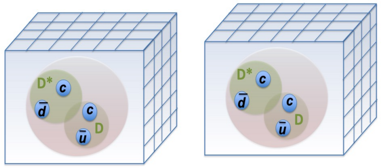
$\sum_n |n\rangle\langle n|$

$e^{-iE_nt_M}$

Eucledian time

$$\boxed{C^{2pt}_{ij}(t) = \langle 0|\mathcal{O}_i(t)\ \mathcal{O}_j^+(0)|0\rangle = \sum_n \langle 0|\mathcal{O}_i|n\rangle\ e^{-E_n t_E}\langle n|\mathcal{O}_j^+|0\rangle}$$

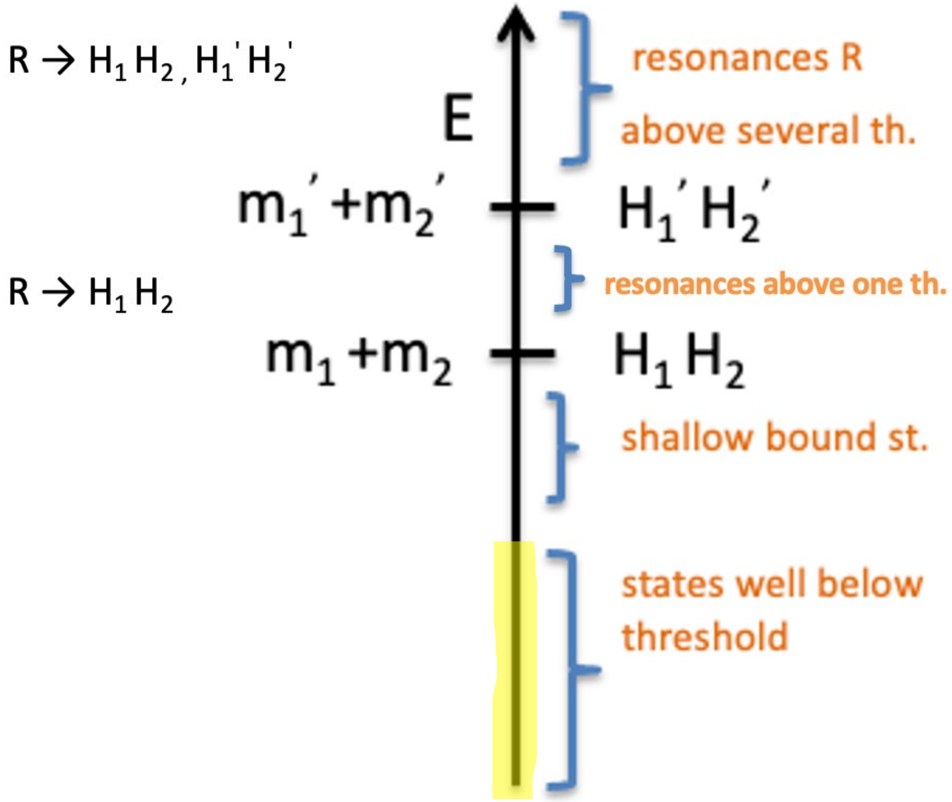
t, J^P, I t=0, J^P, I



$$\mathcal{O} = \mathcal{O}(q,G)$$

All results in this talk will be based on E_n :

- for strongly stable state well below threshold : $E_n(P=0) = m$
- resonances (Luscher’s relation) $E_n^{cm} \rightarrow T(E_n^{cm})$
- static potentials: $E_n \rightarrow V(r)$ not covered in this talk



$R \to H_1 H_2, H_1' H_2'$

E

resonances R above several th.

$m_1' + m_2'$ $H_1' H_2'$

$R \to H_1 H_2$

resonances above one th.

$m_1 + m_2$ $H_1 H_2$

shallow bound st.

states well below threshold

$QQ'\bar{q}\bar{q}'$ well below threshold

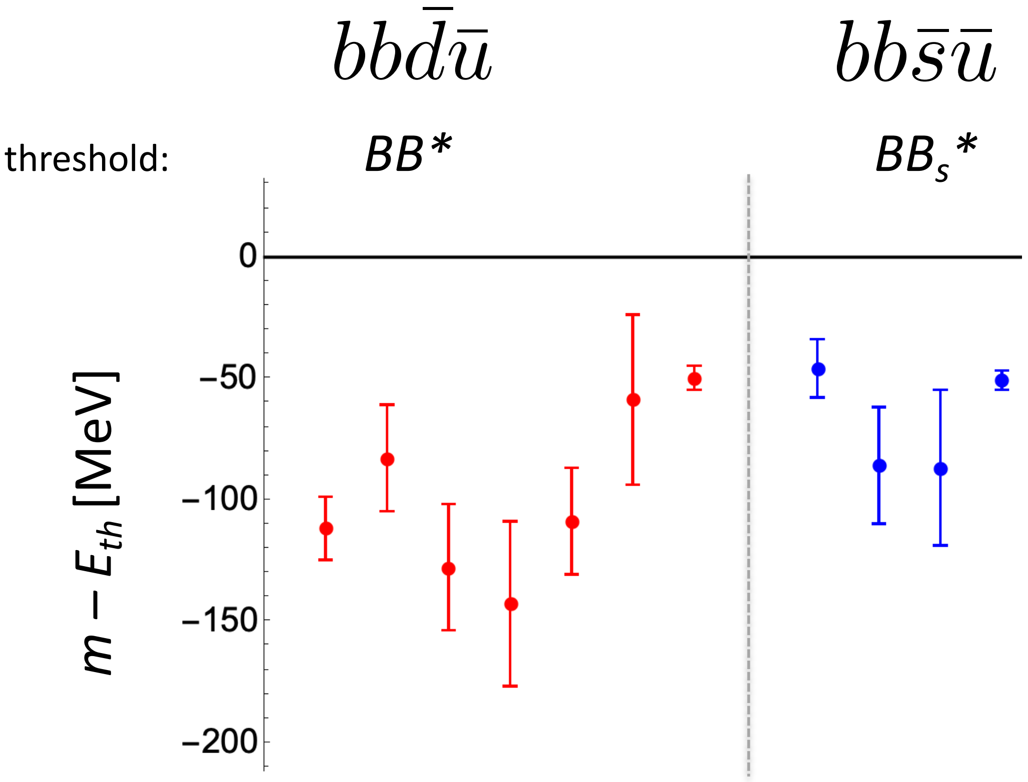
$$E_n(P=0) \;=\; m$$

Doubly bottom tetraquarks

not found in exp, difficult to find

$I=0,\ J^P=1^+$

$bb\bar{d}\bar{u}$ $\quad$ $bb\bar{s}\bar{u}$



$$O = (\bar{u}\gamma_5 b)\ (\bar{d}\gamma_i b) + .. = BB^*$$
$$[b\Gamma_1 b]_{\bar{3}_c} [\bar{u}\Gamma_2 \bar{d}]_{3_c}$$
$$\ldots$$

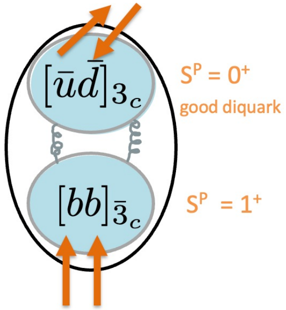
from left to right (lattice QCD)

Hudspith, Mohler, 2303.17295
HALQCD, 2306.03565 (cosidering coupling with B*B*)
Leskovec, Meinel, Pflaumer, Wagner, 1904.04197
Junnarkar, Mathur, Padmanth, 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021 PosLat)
Bicudo, Wagner et al. 1612.02758, static potentials
Brown, Orginost, 1210.1953, static potentials

Hudspith, Mohler, 2303.17295
Meinel, Pflaumer, Wagner, 2205.13982
Junnarkar, Mathur, Padmanth 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021, PosLat)

there are even more recent results ..

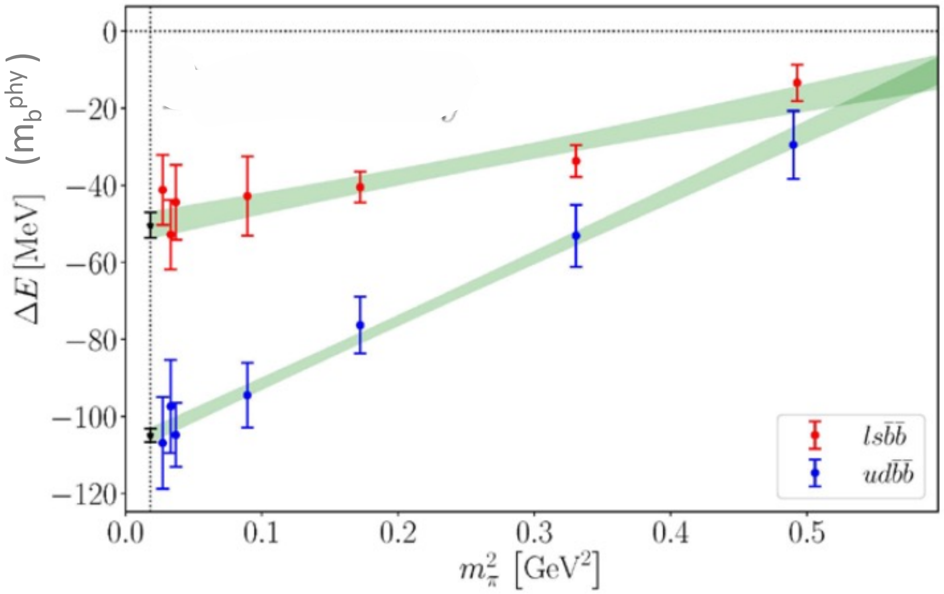
likely dominant
(B and B* to close
in BB* molecule
with binding ~0.1 GeV)



Doubly bottom tetraquarks

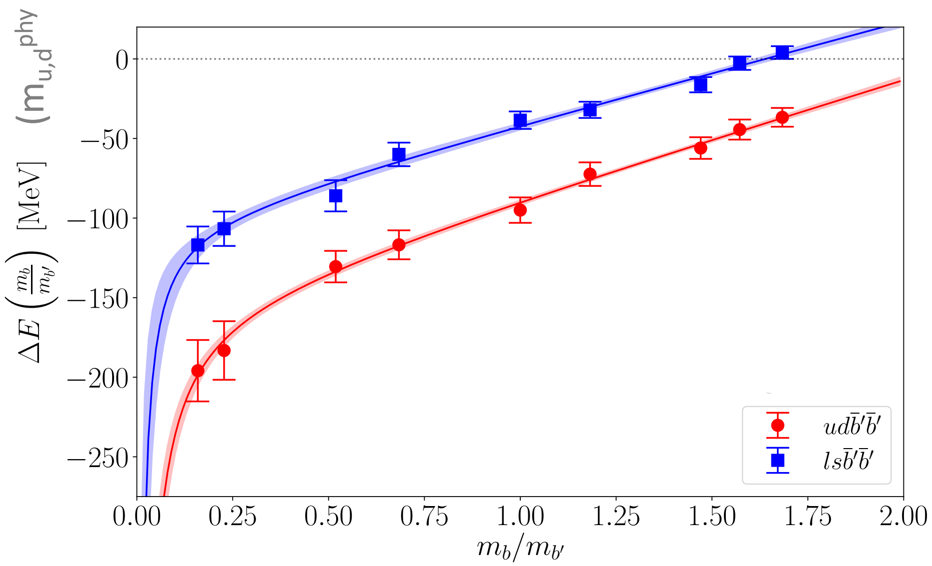
$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$

lattice: dependence on m_b and $m_{u,d}$



$m_{u,d}$ increases →

Colquhoun, Francis, Hudspith, Maltman, Lewis
1810.10550 , *PoS* LATTICE2021 (2022) 144



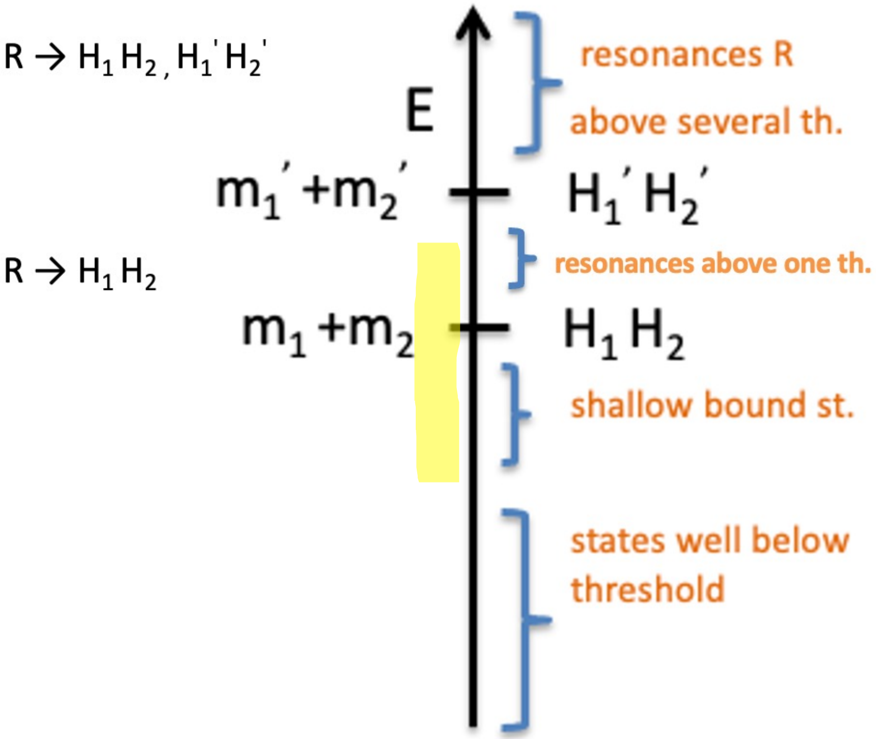
$m_{b'}$ decreases →

Other $QQ'\bar{q}\bar{q}'$ and J^P : $bc\bar{q}\bar{q}'$, $cc\bar{q}\bar{q}'$

Theoretically expected near or above threshold

States near or above threshold have to be identified from scattering T(E): next Section

$QQ'\bar{q}\bar{q}'$ from one-channel scattering



$R \to H_1 H_2 , H_1' H_2'$

E

$m_1' + m_2'$ — $H_1' H_2'$

resonances R above several th.

$R \to H_1 H_2$

resonances above one th.

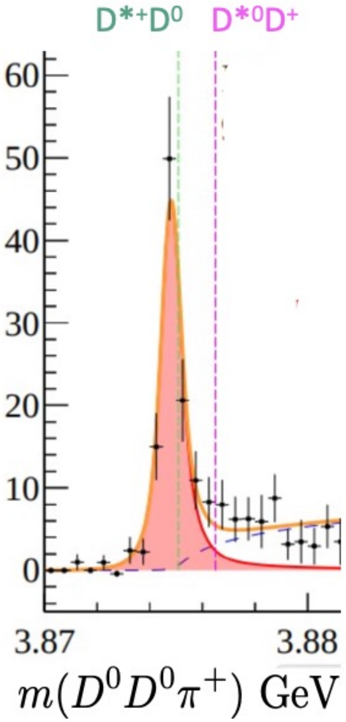
$m_1 + m_2$ — $H_1 H_2$

shallow bound st.

states well below threshold

example

LHCb 2021: Tcc

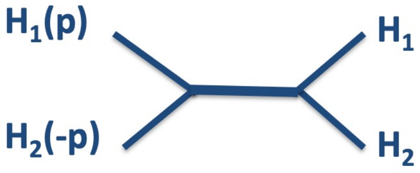


One-channel scattering : Luscher's relation between E and δ(E) , T(E)

$$S(E) = e^{2i\delta(E)} = 1 + 2i\,\frac{2p}{E}\,T(E)$$

$$T_l \propto \frac{1}{p\cot\delta_l - ip}$$

scattering amplitude



$$\mathcal{M}_\ell = \frac{1}{p}\,\frac{1}{\cot\delta_\ell - i} = \frac{1}{p}\,e^{i\delta_\ell}\sin\delta_\ell$$

Luscher's rel.

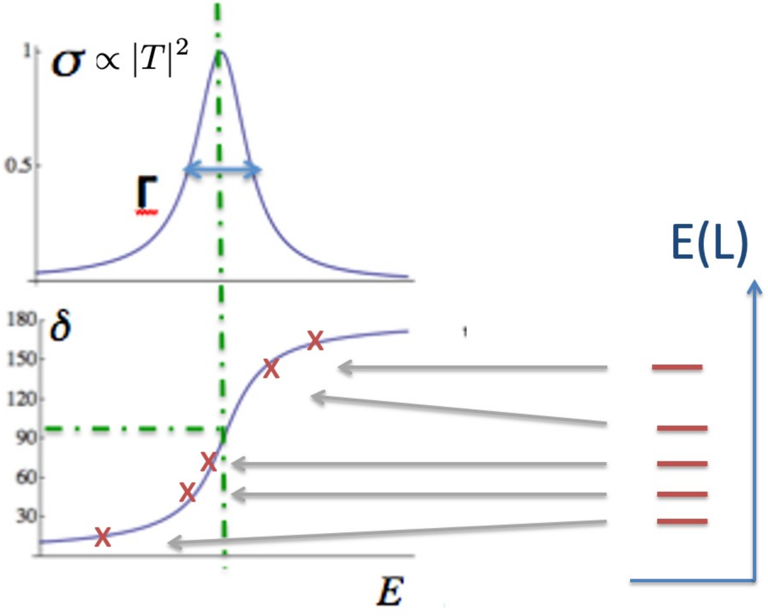
$$\boxed{\det\left[1 + F(P,L)\cdot\mathcal{M}(P)\right] = 0}$$

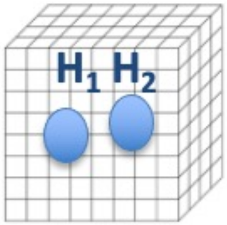
in (ℓ, m_ℓ)-space

$F(P,L) \equiv$ Matrix of known geometric functions

$$F_{\ell' m_\ell', \ell m_\ell}(P,L) \equiv \xi\left[\frac{1}{L^3}\sum_{\vec{k}} - \int\frac{d^3\vec{k}}{(2\pi)^3}\right]\left(\frac{k^*}{q^*}\right)^{\ell'}\frac{Y^*_{\ell' m_\ell'}(\hat{k}^*)\,Y_{\ell m_\ell}(\hat{k}^*)}{2\omega_k 2\omega_{Pk}(E-\omega_k-\omega_{Pk}+i\epsilon)}\left(\frac{k^*}{q^*}\right)^{\ell}$$

lectures by Andrew Jackura



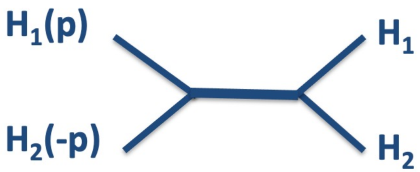


T here is M in Jacura's talk

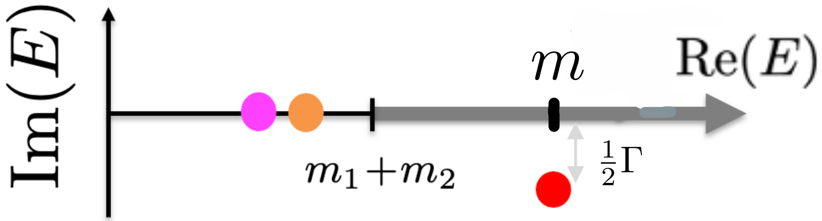
δ(E) , T(E) ← E

Luscher's rel.

One-channel scattering H_1 H_2



E = $\sqrt{m_1^2+p^2}+\sqrt{m_2^2+p^2}$



Virtual bound st.

p = - i |p|, sheet II

Bound st.

p = i |p|, sheet I

Resonance

sheet II

$p^2<0$

$$T(E) \propto \frac{1}{E^2-m^2}$$

$$T(E) \propto \frac{1}{E^2-m^2+iE\Gamma}$$

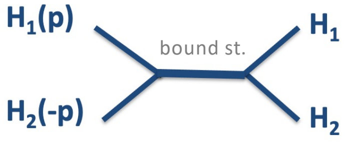
$\sigma(E) \propto |T(E)|^2$

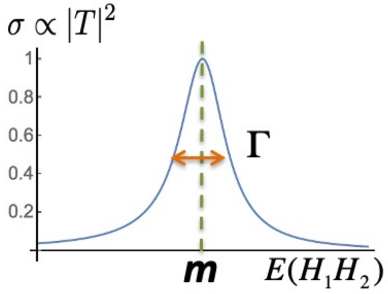
for real E

for complex E

$$T(E) \;\to\; T(E^c)$$

analytic contin.
to complex E





T_{bc} : next exciting discovery from exp ?

$$bc\bar{u}\bar{d}$$

$$I=0,\ J^P=1^+,0^+$$

Alexandrou et al, 2312.02925 PRL

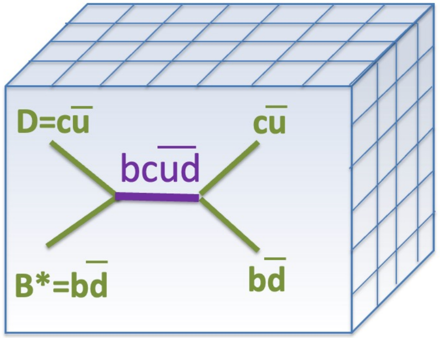
$m_\pi \approx 220MeV$

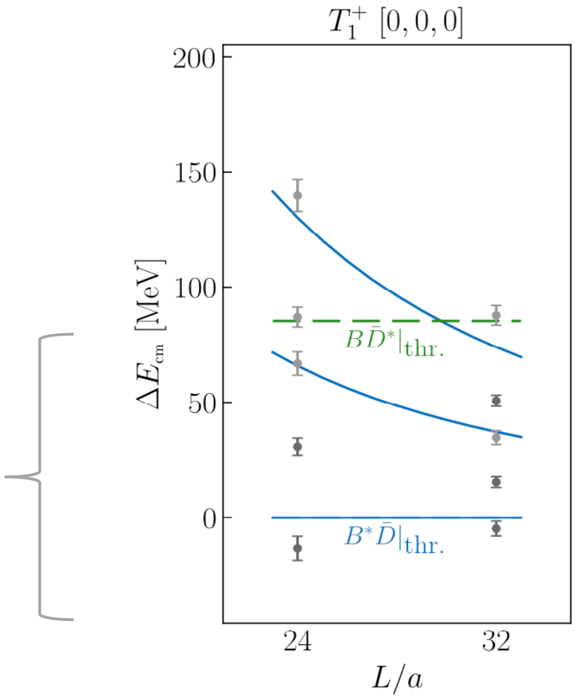
$$O \sim (\bar{u}b)(\bar{d}c),\ \ [bc][\bar{u}\bar{d}]$$

B* D

B D*

$$T_0 \propto \frac{1}{k\cot\delta_0 - ik}$$





lines:

$$E^{n.i.} = \sqrt{m_D^2 + \vec{p}_1^{\,2}} + \sqrt{m_{B^*}^2 + \vec{p}_2^{\,2}}$$

$$\vec{p}_i = \vec{n}_i \frac{2\pi}{L}$$

T_{bc} : next exciting discovery from exp ?

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Alexandrou et al, 2312.02925 PRL

$m_\pi \approx 220 MeV$

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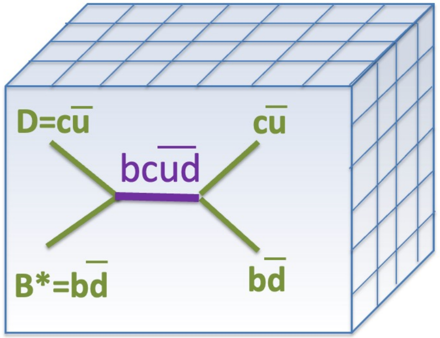
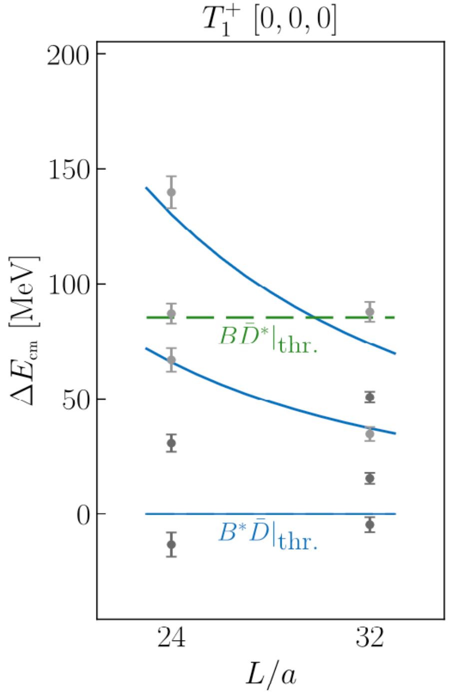
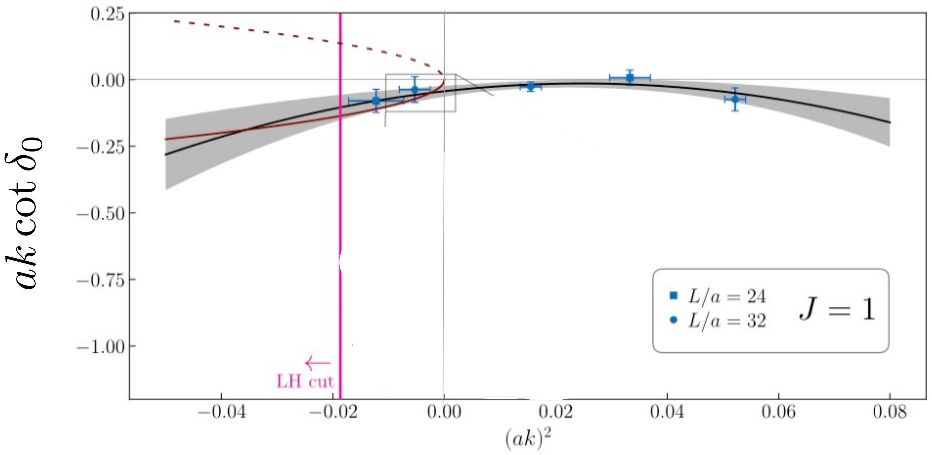


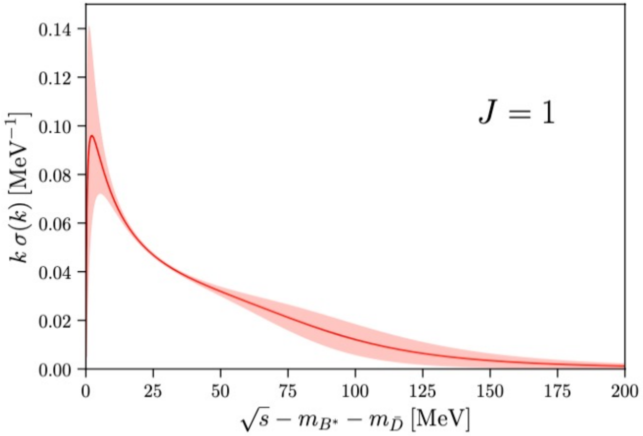


E

Luscher's rel.

δ(E), T(E)





$$m_{T_{bc}} - m_{B^*} - m_D = -2.4^{+2.0}_{-0.7}\ \text{MeV}$$

$$m_R - m_{B^*} - m_D = \quad 67 \pm 24\ \text{MeV} \quad \Gamma_R = 132 \pm 32$$

another study M. Padmanath et al, 2307.1428: also finds a bound state, with deeper binding

T_{cc} from LHCb experiment

$$D^* \to D\pi$$

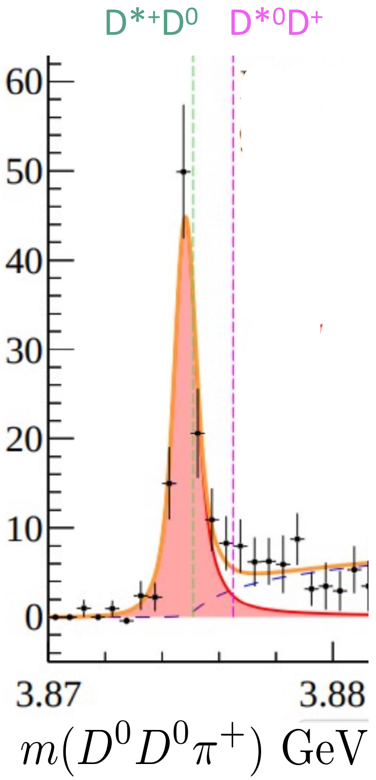
$$m_{\pi^0} \simeq 135 \text{ MeV}$$

$$m_{D^{*+}} - m_{D^+} \simeq 140 \text{ MeV}$$

$cc\bar{d}\bar{u}$

I=0, $J^P=1^+$ (most likely)

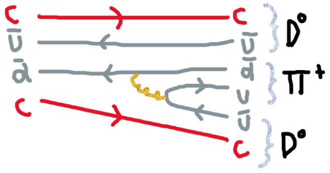
The longest lived exotic hadron ever discovered



$$\delta m = m - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

LHCb 2109.01038, 2109.01056, Nature Physics

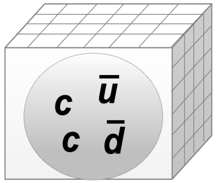


Omitting $D^* \to D\pi, \ T_{cc} \to DD\pi$

T_{cc} would be a bound state

T_{cc} from lattice

all analyzed in 2402.14715, PRD
Collins, Nefediev, Padmanath , SP



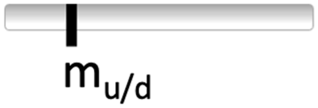
all simulations :

$$m_u = m_d > m_{u,d}^{ph} \qquad D^* \not\to D\pi$$

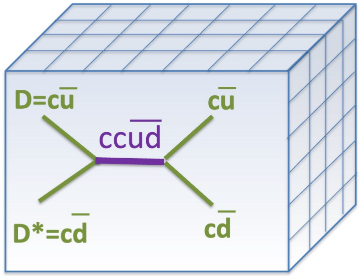
single lattice spacing

(J. Green et al are exploring
several lattice spacings, lat 2023, unpublished)

mc	mpi	L	ensmbles	ref.
five values m_D=1.7–2.4 GeV	280 MeV	~ 2.1, 2.8 fm	CLS Nf=2+1	our, 2402.14715, PRD eigenenergies



mc	mpi	L	ensembles	ref.
~ physical	146 MeV	~ 8 fm	Nf=2+1	HALQCD, 2302.04505, PRL HALQCD potentials
~ physical	280 MeV	~ 2.1, 2.8 fm	Nf=2+1, CLS	our, 2402.14715, PRD eigenenergies
~ physical	348 MeV	~ 2.4 fm	Nf=2	CLQCD, 2206.06186, PLB eigenenergies



recent Hsc 2405.15741
presented at the end

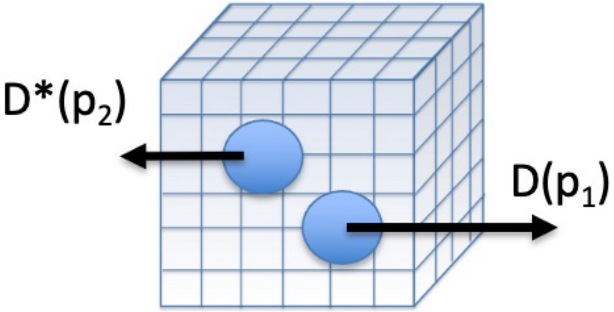
Interpolators and E_n [our simulation, CLQCD]

$I=0,\ J^P=1^+$

$cc\bar{u}\bar{d}$

$\vec{P}=\vec{p}_1+\vec{p}_2$

$|\vec{P}|=0,1$



D(p₁) D*(p₂)

$$\mathcal{O}=(\bar{u}\gamma_5 c)_{\vec{p}_1}\ (\bar{d}\gamma_i c)_{\vec{p}_2}-(\vec{p}_1\leftrightarrow\vec{p}_2)\qquad \vec{p}_{1,2}=\vec{n}_{1,2}\ \frac{2\pi}{L}$$

$(\bar{u}\gamma_5\gamma_t c)_{\vec{p}_1}\ (\bar{d}\gamma_i\gamma_t c)_{\vec{p}_2}$

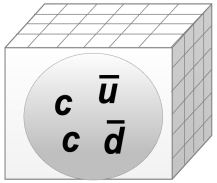
[cc][ud] interloators not employed

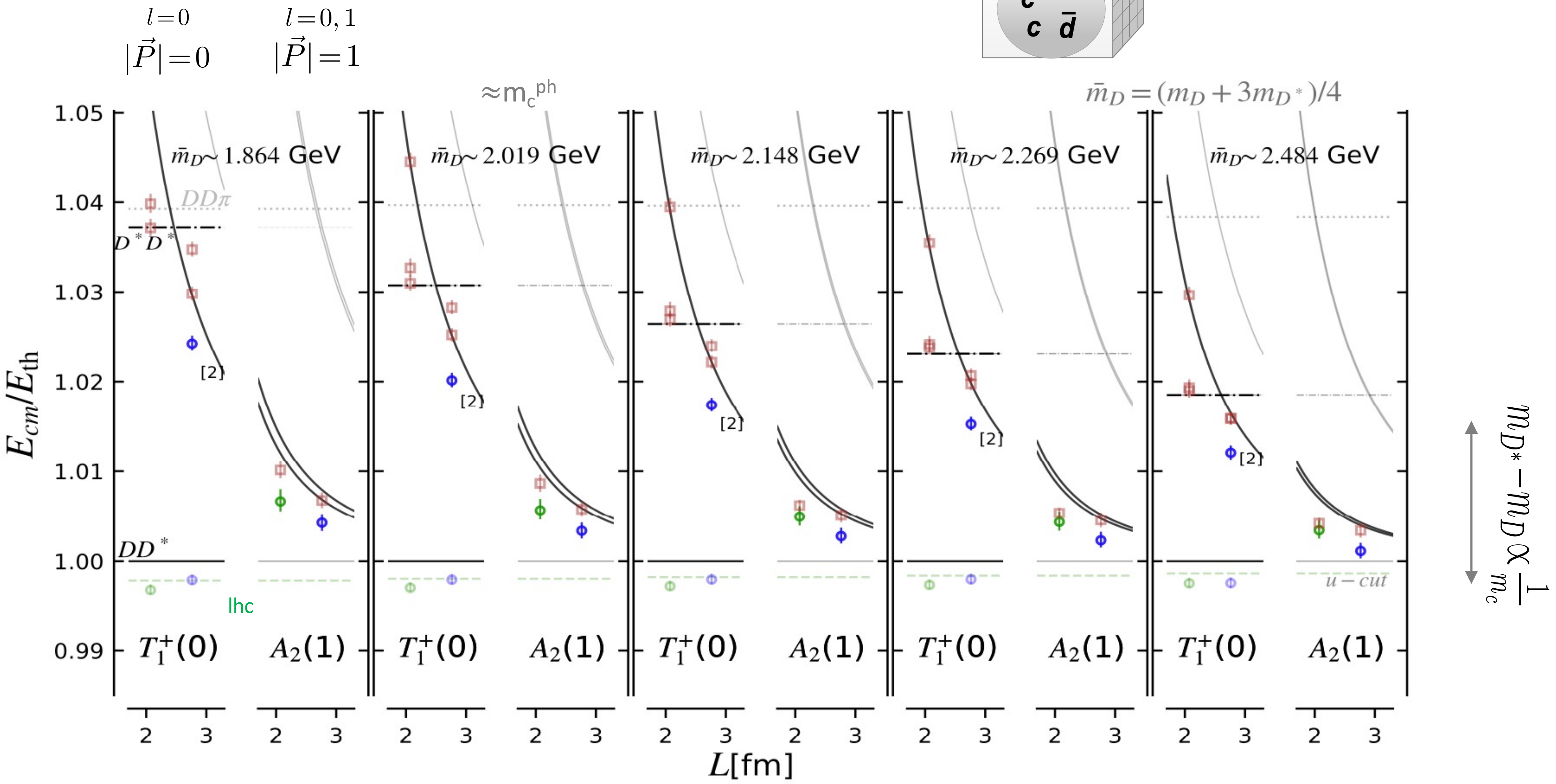
[forthcoming paper with Emmanuel Pacheco and Ivan Vujmilovic]

$$C_{ij}^{2pt}(t)=\langle 0|\,\mathcal{O}_i(t)\ \mathcal{O}_j^{+}(0)\,|0\rangle=\sum_n\ \langle 0|\mathcal{O}_i|n\rangle\ e^{-E_n t_E}\langle n|\mathcal{O}_j^{+}\,|0\rangle$$

E_n using GeVP

T_{cc} : finite-volume eigen-energies





lines

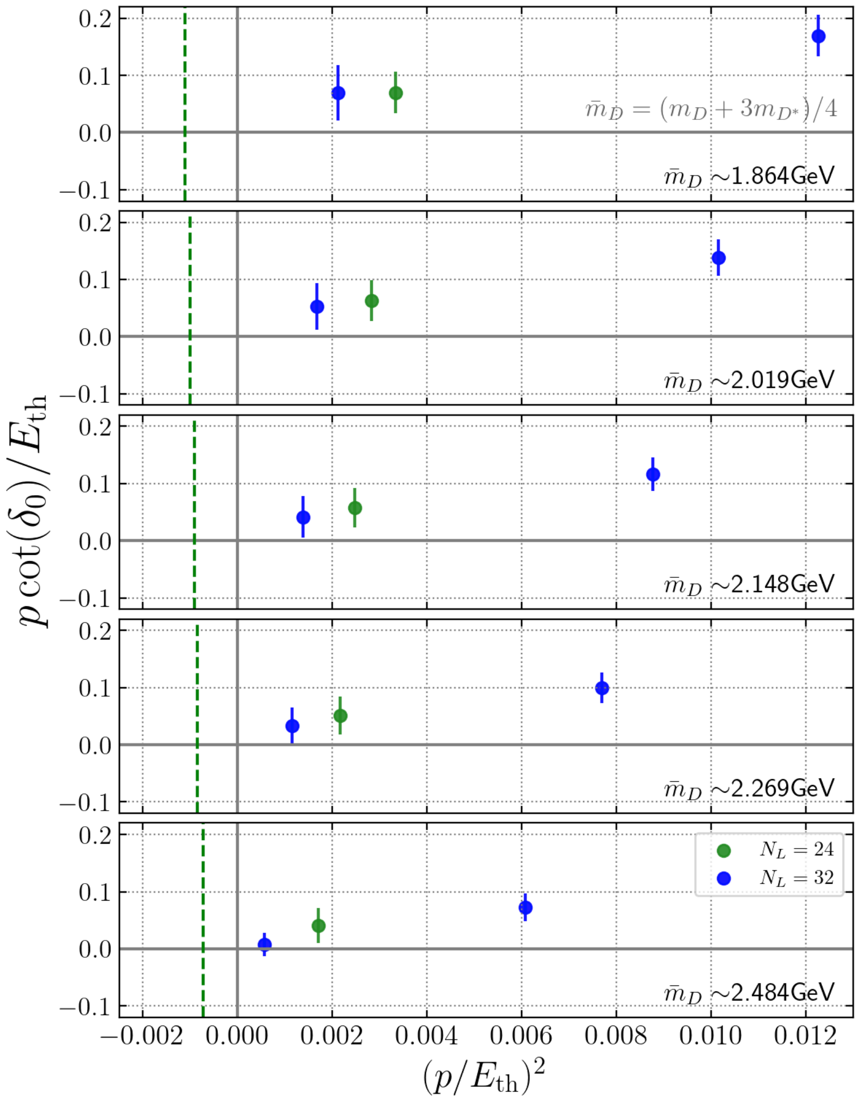
$$E^{n.i.}=\sqrt{m_D^2+\vec{p}_1^{\,2}}+\sqrt{m_{D^*}^2+\vec{p}_2^{\,2}}$$

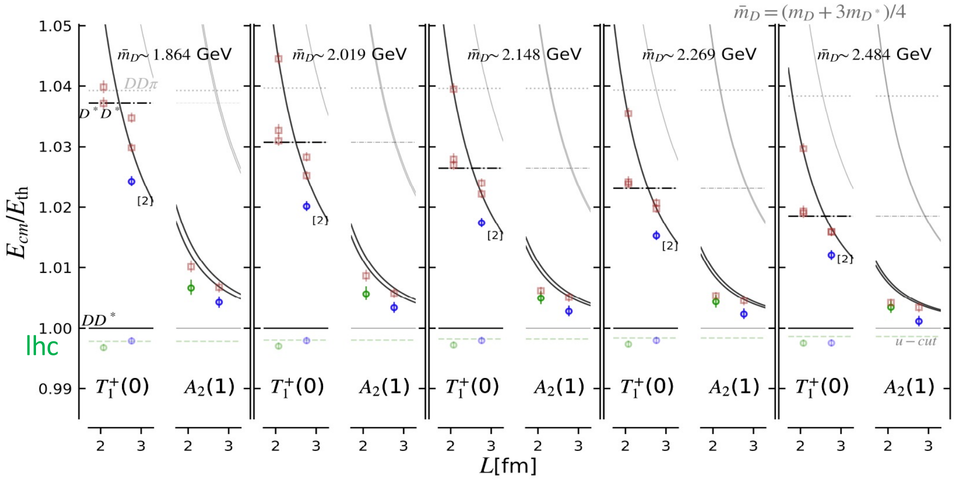
$$\vec{p}_i=\vec{n}_i\frac{2\pi}{L}$$

Collins, Nefediev, Padmanath , SP, 2402.14715, PRD

T_{cc} : scattering amplitude

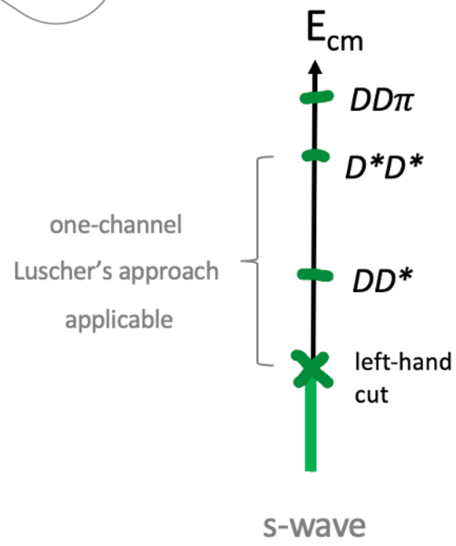
m_c





δ(E)

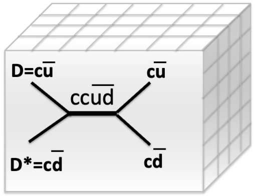
E

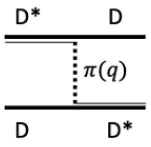


one-channel

Luscher's approach

applicable

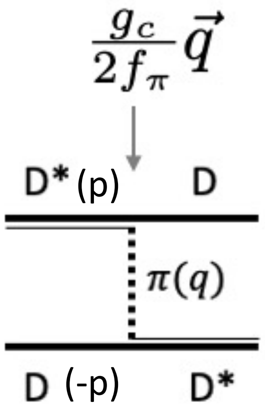




Pion exchange, left-hand cut etc

$$q^2 = q_0^2 - \vec{q}^{\,2} \simeq (m_{D^*} - m_D)^2 - \vec{q}^{\,2}$$

Heavy meson ChPT



$$V_\pi^{cent}(\vec{q}) = \frac{g_c^2}{4f_\pi^2}\,\frac{\vec{q}^{\,2}}{q^2 - m_\pi^2} = \frac{g_c^2}{4f_\pi^2}\left(-1 + \frac{\mu_\pi^2}{\vec{q}^{\,2} + \mu_\pi^2}\right)$$

$$\mu_\pi^2 = m_\pi^2 - (m_{D^*} - m_D)^2$$

$\mathrm{lat}: \mu_\pi^2 > 0$

$\mathrm{ph}: \mu_\pi^2 < 0$

attraction at short distance

slight repulsion at long distance

$-\delta^{(3)}(\vec{r})$

$\frac{\mu_\pi^2}{r} e^{-\mu_\pi r}$

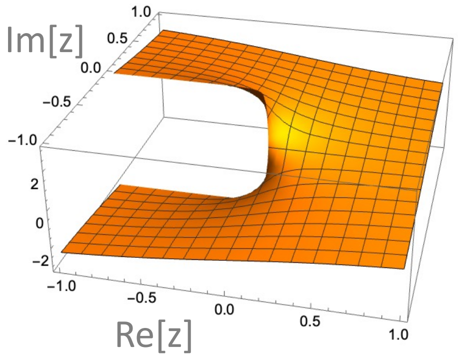
s-wave projection

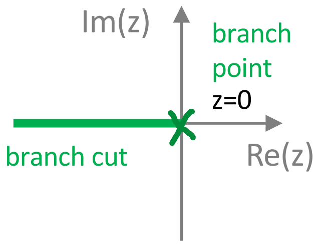
$$V_\pi^S(p,p) \propto \int V_\pi(\vec{q})\ d\cos\theta, \quad \vec{q}^{\,2} = 2p^2(1 - \cos\theta)$$

$$V_\pi^S(p,p) \propto \ln\left(1 + \frac{4p^2}{\mu_\pi^2}\right)$$

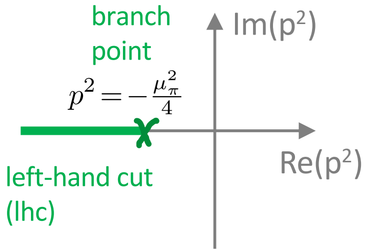
complex p cot δ (Luscher's eq would render it real)

$\mathrm{Im}(\ln z)$





$\ln(z)$



$V_\pi(p^2),\ T_\pi(p^2)$

lhc slightly below DD*, BB*, NN ... th.

T_{cc} analysis based on EFT

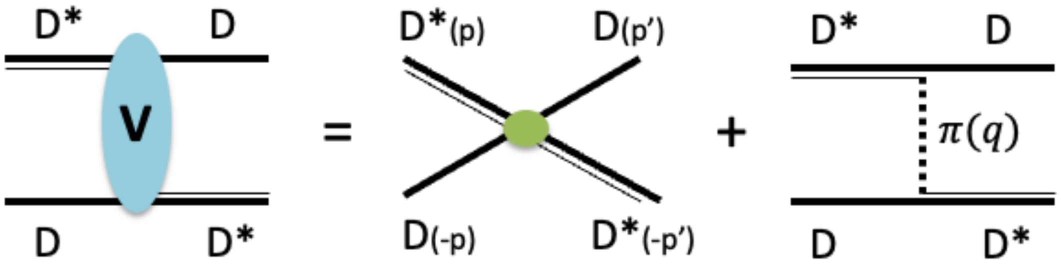
Collins, Nefediev, Padmanath , SP, 2402.14715, PRD

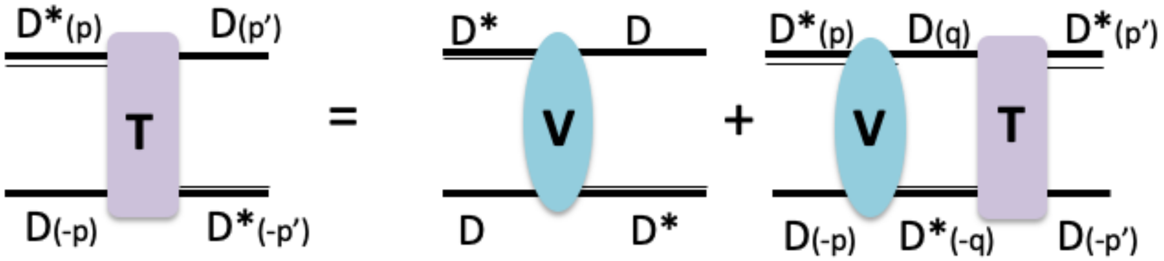
$c_{0,2}$ fitted from lat. data

significant short-distance attraction

$$V_{CT} = 2c_0 + 2c_2(p^2 + p'^2)$$

$$\frac{g_c}{2f_\pi}\vec{q}$$





$$T = V - VGT$$

$$T = \frac{1}{V^{-1} + G}$$

$$T(\boldsymbol{p},\boldsymbol{p}';E) = V(\boldsymbol{p},\boldsymbol{p}') - \int \frac{d^3q}{(2\pi)^3} V(\boldsymbol{p},\boldsymbol{q}) G(\boldsymbol{q};E) T(\boldsymbol{q},\boldsymbol{p}';E)$$

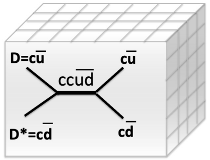
integral
equation

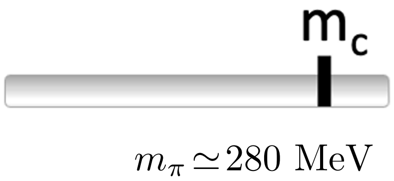
Limann-Schwinger eq.
Bethe-Salpeter eq.

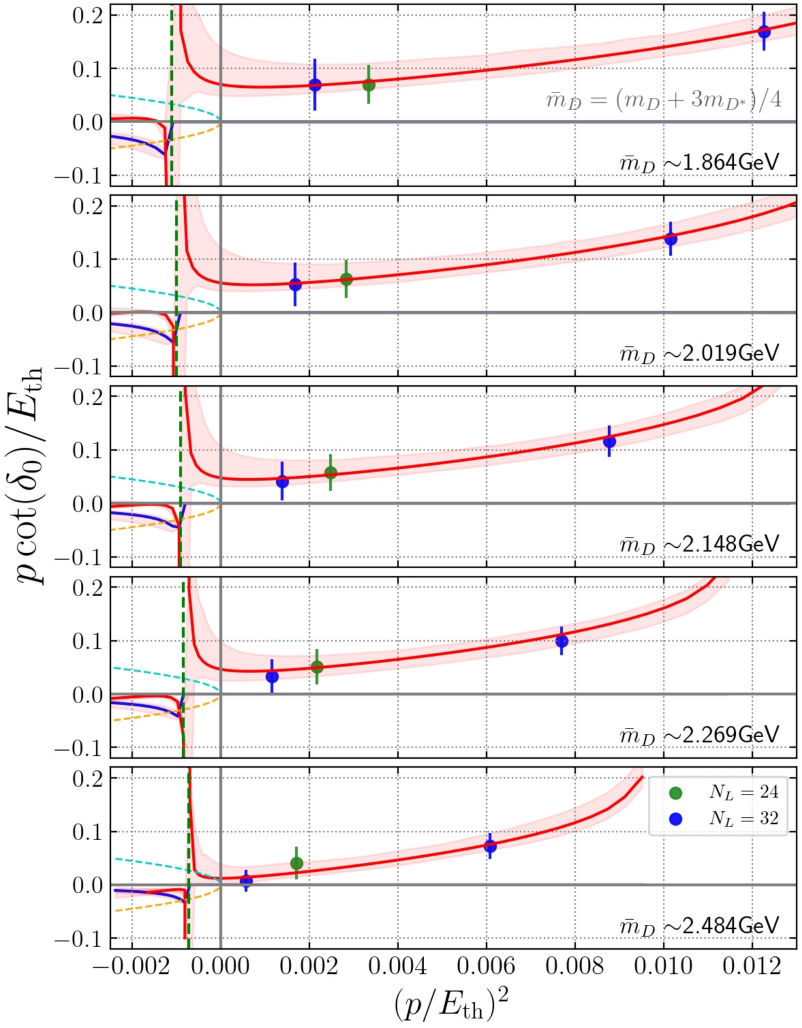
inspired by

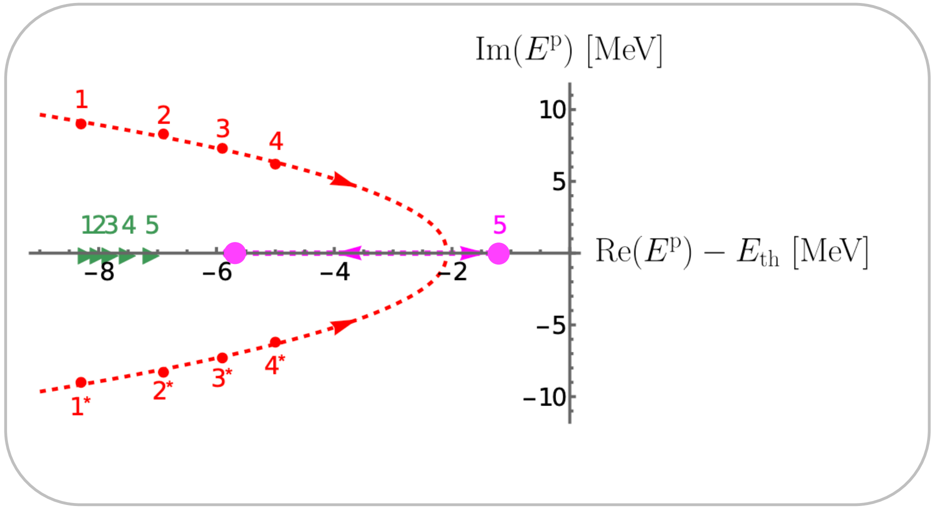
Du, Hanhart, Guo, Nefediev, Filin, et al, PRL 2023, 2303.09441

T_{cc} : scattering amplitude and pole trajectory







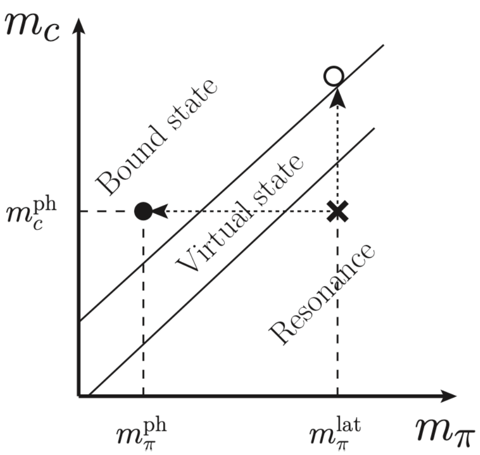


resonance pole

virtual state pole

lhc

arrow: increasing mc



levels below lhc omitted from the fit

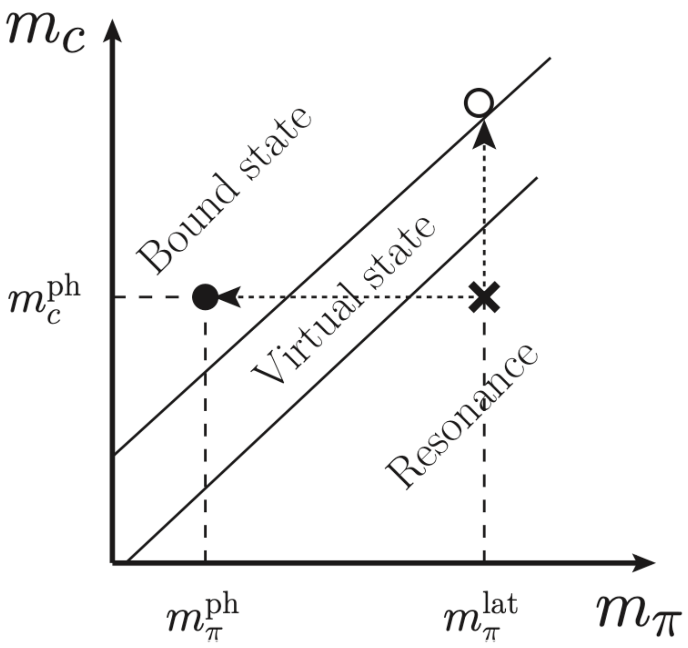
reassuring: plane-wave method incorporates levels below lhc and gets consistent s-wave amplitude Meng, Baru, Epelbaum et al., 2312.01930, PRD

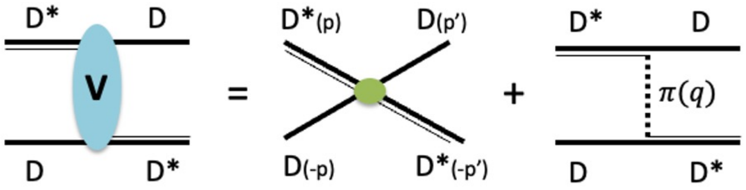
Collins, Nefediev, Padmanath , SP, 2402.14715, PRD

T_{cc} : interpretation

Collins, Nefediev, Padmanath , SP, 2402.14715, PRD

$m_\pi \simeq 280$ MeV



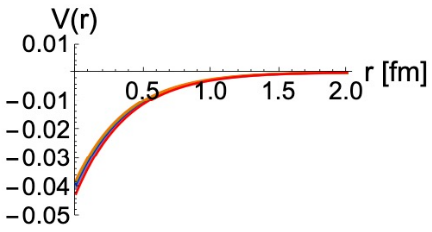


V(r)=FT V(q) at p~const

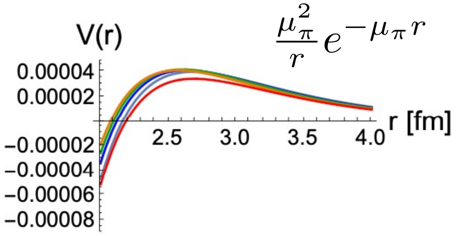
$\mu_\pi^2 = m_\pi^2 - (m_{D^*} - m_D)^2 \quad > 0$

long-range due to one-pion exchange

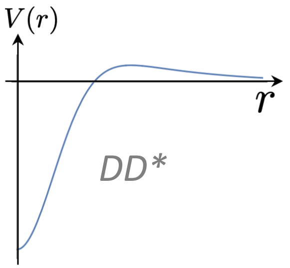
regularized



$\frac{\mu_\pi^2}{r} e^{-\mu_\pi r}$



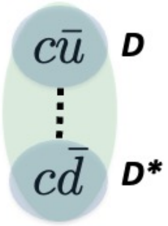
V(r) almost independent on mc



$$H = V + \frac{p^2}{2m_r}$$

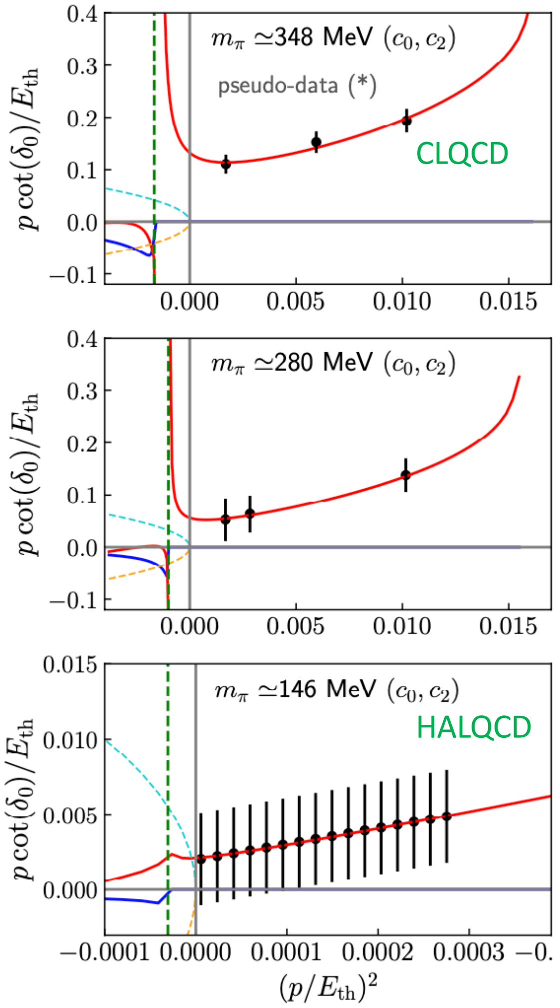
interpretation consistent with

(does not uniqely imply)

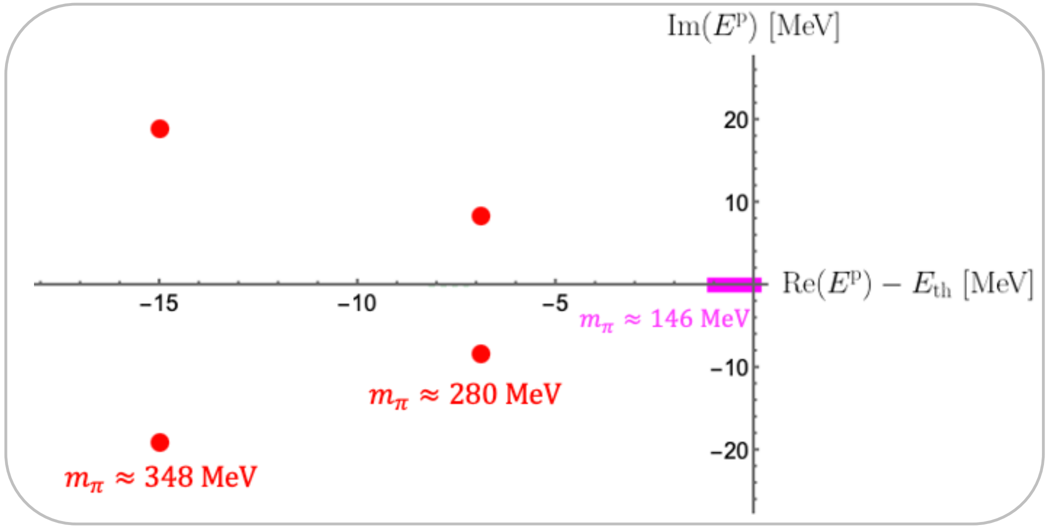


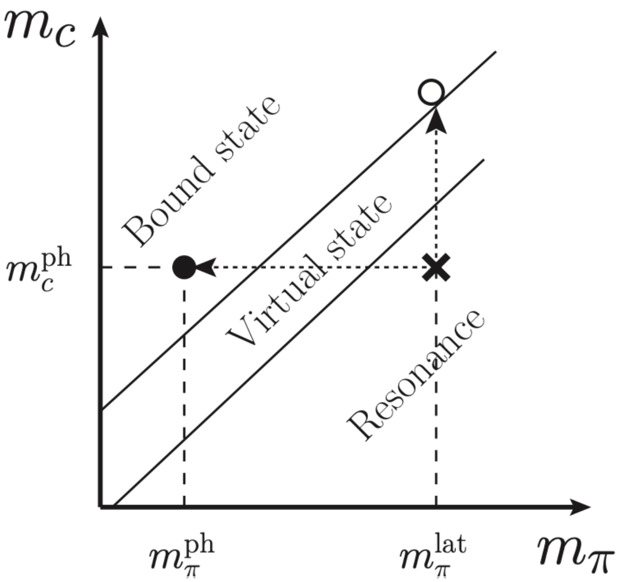
T_{cc}

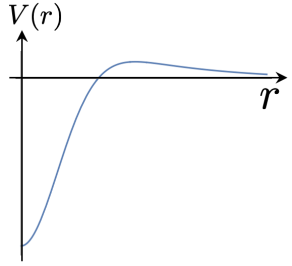
Collins, Nefediev, Padmanath , SP, 2402.14715, PRD

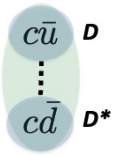


resonance pole virtual state pole









caution: see disclaimers in our paper

errors on pole positions could not be reliably determined

see also: 2407.04649,

Abolnikov, Baru, Epelbaum, Filin, Hanhart, Meng

attraction increases with decreasing mpi

$$H = V + \frac{p^2}{2m_r}$$

$R \to H_1 H_2,\ H_1' H_2', \ldots$

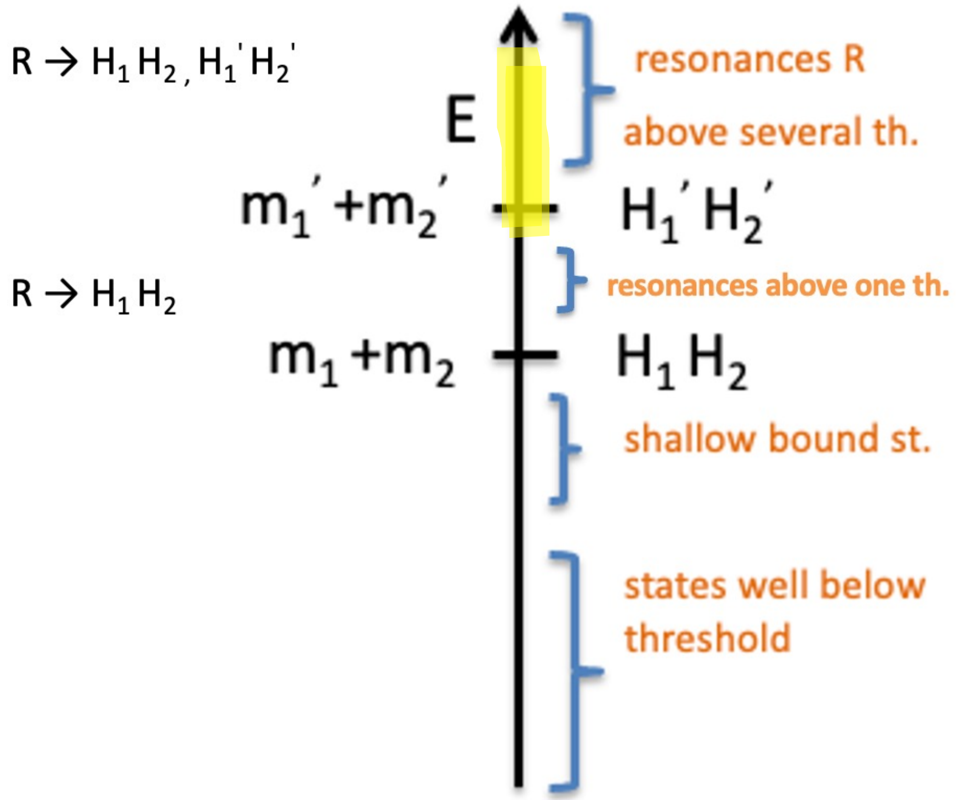
channel $a:\ H_1 H_2$
channel $b:\ H_1' H_2'$

$$T(E)=\begin{array}{c} \\ \\ \\ \end{array}\!\!\begin{array}{cc} \text{a -> a} & \text{a -> b} \\ \left[\begin{array}{c} T_{aa}(E) \\ T_{ab}(E) \end{array}\right. & \left.\begin{array}{c} T_{ab}(E) \\ T_{bb}(E) \end{array}\right] \\ \text{b->a} & \text{b -> b} \end{array}$$

$$\boxed{\det\left[\,1 + F(P,L)\cdot \mathcal{M}(P)\right] = 0}$$

$\uparrow$

in channel space



$R \to H_1 H_2,\ H_1' H_2'$

E

resonances R above several th.

$m_1' + m_2'$ $H_1' H_2'$

resonances above one th.

$R \to H_1 H_2$

$m_1 + m_2$ $H_1 H_2$

shallow bound st.

states well below threshold

$QQ'\bar{q}\bar{q}'$ from coupled-channel scattering

Coupled-channel *DD*-D*D** scattering

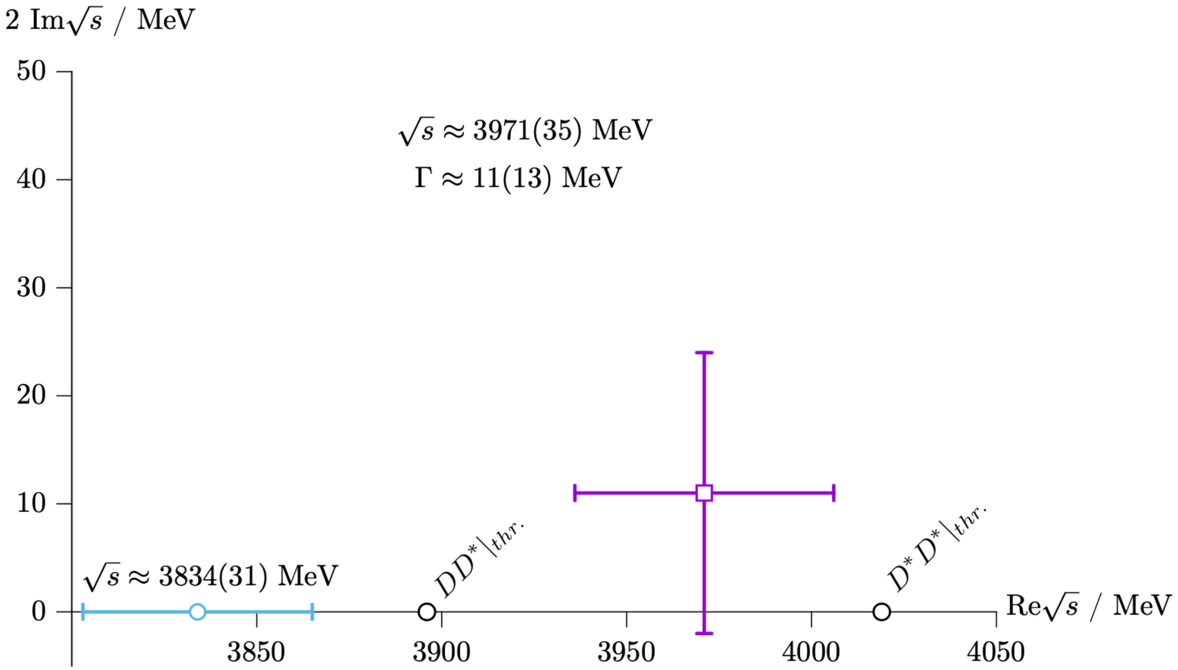
Hadspec 2405.15741

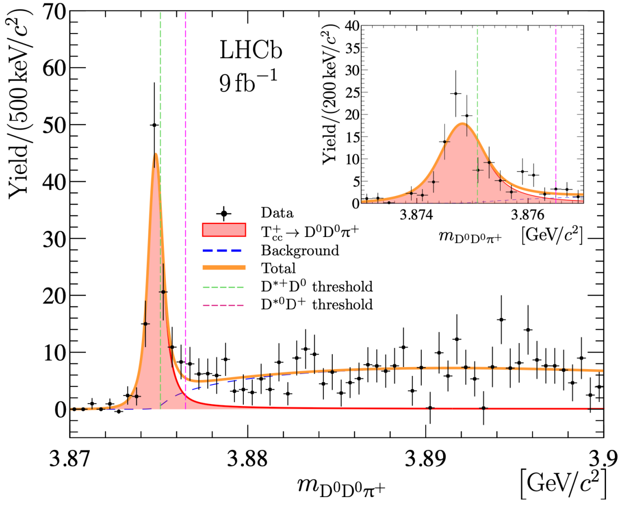
$m_\pi \simeq 391$ MeV

T_{cc} virtual state below DD* threshold (effects from left-hand cut not incorporated)

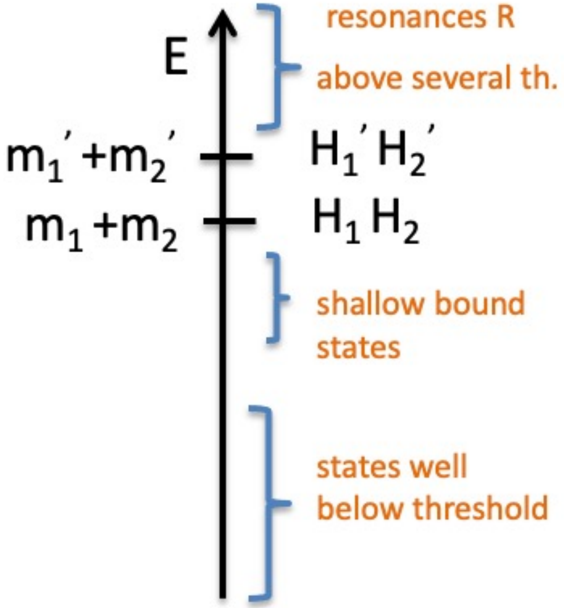
+

T_{cc}' resonance below D*D* threshold : look for it in experiment !





Conclusions



E

resonances R
above several th.

$m_1'+m_2'$ $H_1'H_2'$

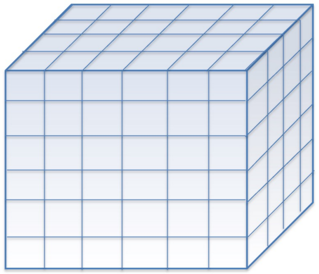
m_1+m_2 H_1H_2

shallow bound
states

states well
below threshold

All presented results are extracted from E_n (except from HALQCD Tcc)

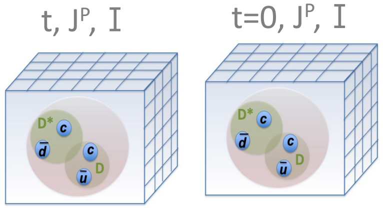
$$\langle C \rangle = \int DG\, Dq\, D\bar{q}\, C\, e^{-S_{QCD}/\hbar}$$



often “non-precision” studies:

single a, $m_{u/d} > m_{u/d}^{phy}$, $m_\pi > 140 \text{ MeV}$

$$C^{2pt}_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^{+}(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle\ e^{-E_n t_E} \langle n | \mathcal{O}_j^{+} | 0 \rangle$$



$$\mathcal{O} = \mathcal{O}(q, G)$$

- for strongly stable state well below threshold : ***$E_n(P=0)$ = m***
- resonances (Luscher’s relation) $$E_n^{cm} \to T(E_n^{cm})$$
- static potentials: $$E_n \to V(r)$$

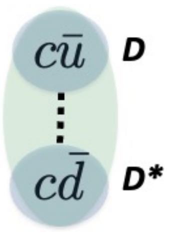
not covered in this talk

Conclusions concerning doubly-heavy tetraquarks

Deeply bound $bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $J^P = 1^+$

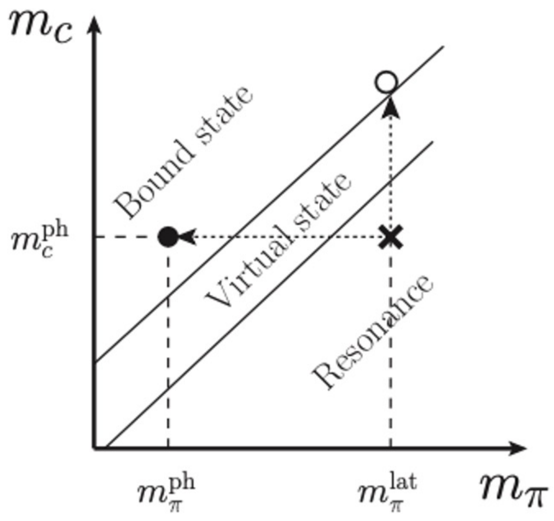
Likely bound, with small binding $bc\bar{u}\bar{d}$ $J^P = 1^+,\ 0^+$

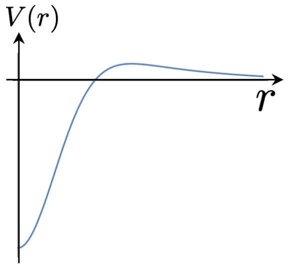
$cc\bar{u}\bar{d}$ $J^P = 1^+$



consistent with

(does not uniqely imply)





V more attractive with decreasing $m_{u/d}$

V almost independent on m_c

$m_\pi > m_\pi^{ph}$ $D^* \not\to D\pi$