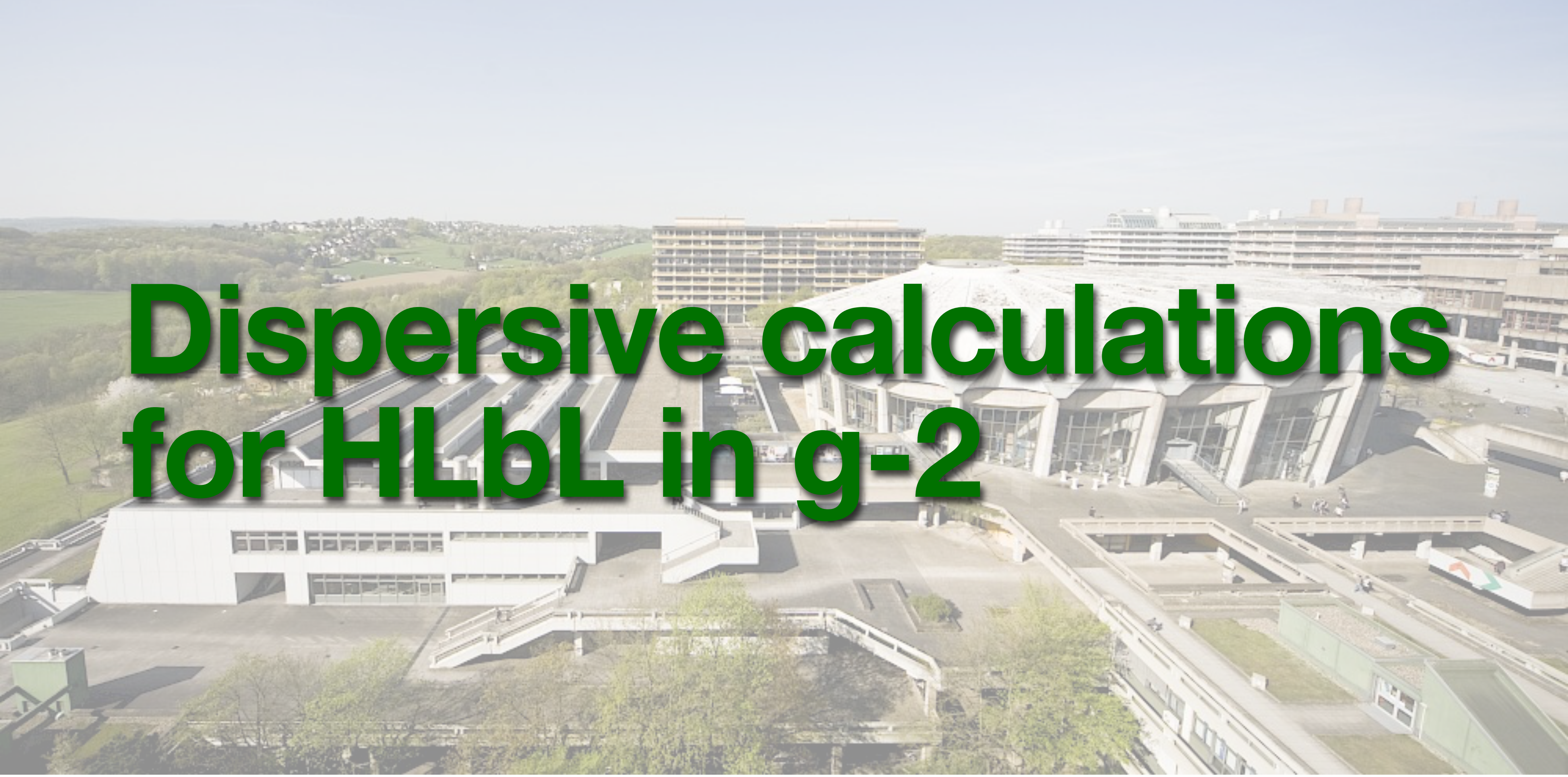
Dispersive calculations for HLbL in g-2



Oleksandra Deineka

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DFG Deutsche Forschungsgemeinschaft
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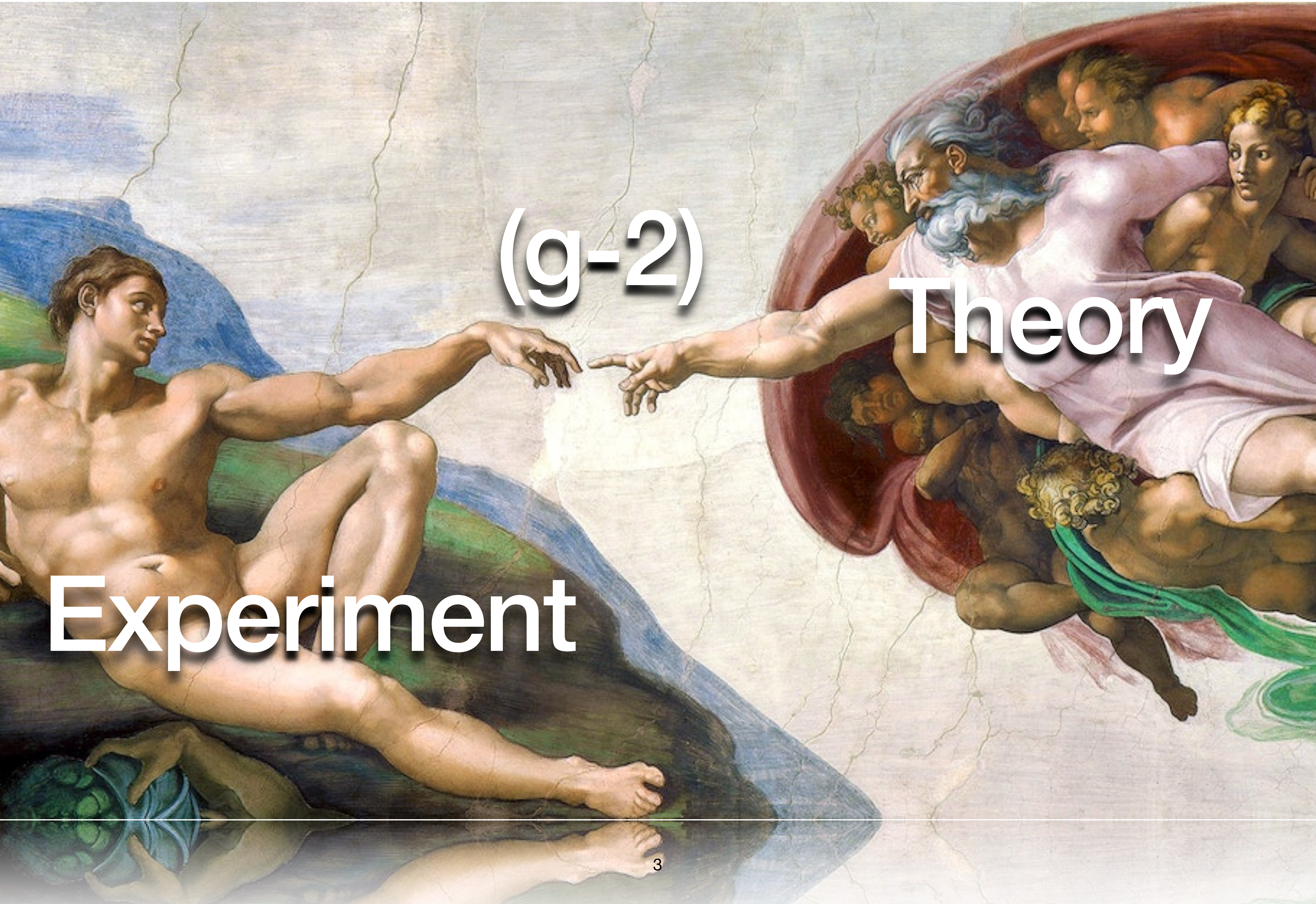
Outline

- Introduction and motivation
  - History of achieved experimental accuracy
  - Contributions from theory
- Current theoretical challenges
  - Hadronic Vacuum Polarisation
  - Hadronic Light-by-Light scattering
- Examples of HLbL contributions
  - Two-photon fusion reactions
  - $f_0(980)$ and $a_0(980)$ in g-2

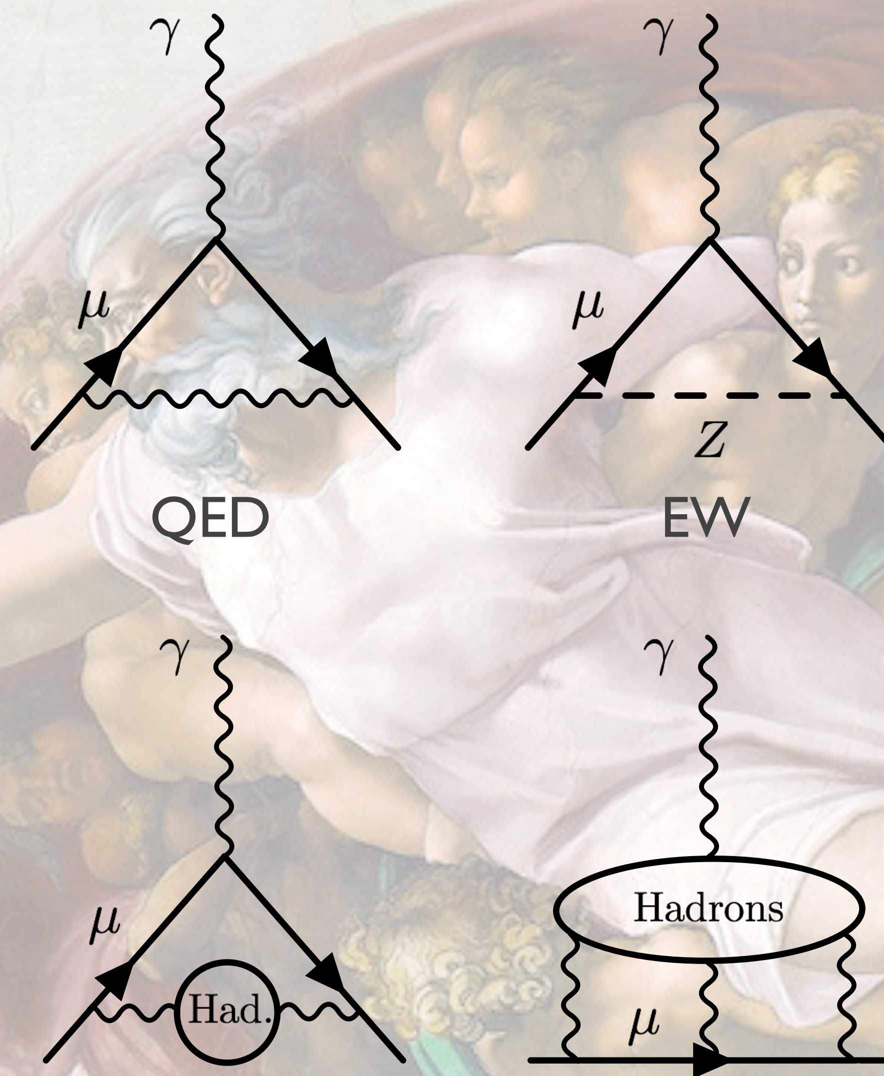
The problem



The problem

$$a_\mu^{exp} = 116592059(22) \times 10^{-11}$$
$$a_\mu^{SM} = 116591810(43) \times 10^{-11}$$

$5.1\,\sigma$ difference



HVP
Hadronic vacuum polarisation

HLbL
Hadronic light-by-light scattering

Brookhaven National Laboratory

Fermilab

What is actually g-2?

Magnetic moment of the lepton

$$\vec{\mu}_l = g_l \frac{e}{2m} \vec{S}$$

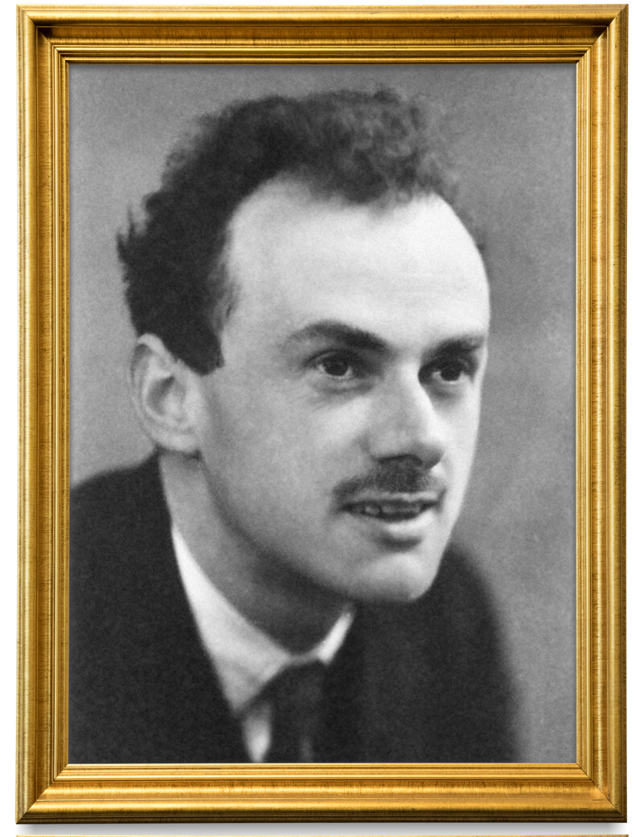
g_l - gyromagnetic ratio

Anomalous part: deviation from Dirac's value

$$a_l = \frac{g_l - 2}{2}$$

If only we could measure it precisely on experiment to test our theoretical understanding…

It turns out we can!



Dirac, 1928:

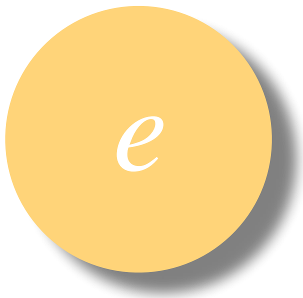
$g = 2$

Schwinger, 1948:

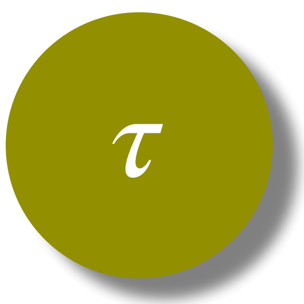
$$a_e = \frac{\alpha}{2\pi}$$

but there is **more**!

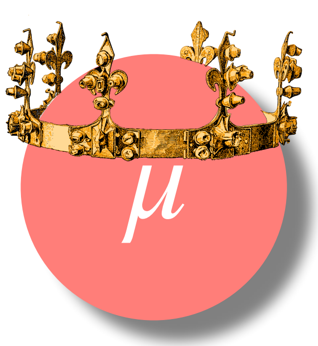
Why muons?



- \+ The most precise measurements ~ 0.24 ppb
- – SM calculation is sensitive to the measured value of α
  $\Delta a_e = -1.7\sigma, -2.5\sigma, +1.6\sigma$
- – Less sensitive to weak and strong interactions contributions



- \+ Effects of new physics: $a_l^{NP} \sim m_l^2/\Lambda^2$
- – Short life time, poor accuracy of experiment



- \+ $(m_\mu/m_e)^2 \sim 4 \times 10^4$ more sensitive to BSM than electron
- \+ Enhanced hadronic sector contribution compared to electron
- \+ Less sensitive to inconsistencies in measurements of α
- \+ Excellent experimental precision ~ 0.20 ppm

Modern high precision experiments

Polarised muons are injected into magnetic storage ring, where they circulate at the **cyclotron frequency**

$$\vec{\omega}_c = -\frac{e\vec{B}}{m\gamma}$$

Muon **spin precession frequency**

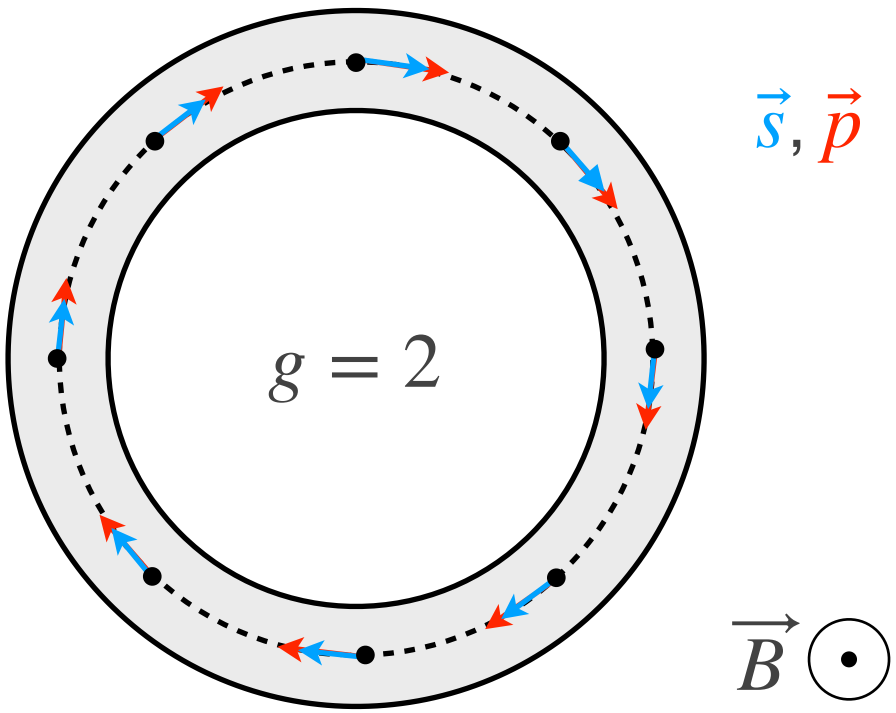
$$\vec{\omega}_s = -\frac{ge\vec{B}}{2m} - (1-\gamma)\frac{e\vec{B}}{m\gamma}$$

Anomalous precession frequency

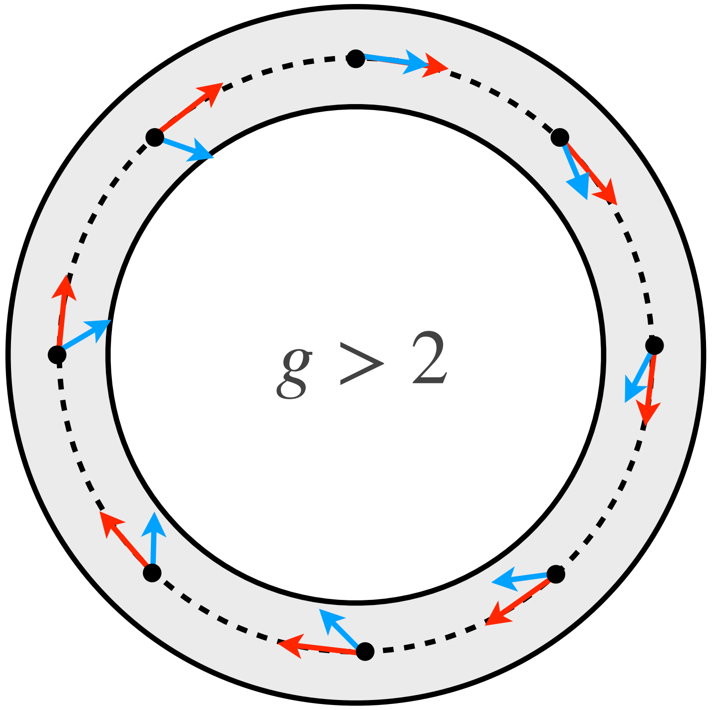
$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -a_\mu \frac{e}{m_\mu}\vec{B}$$

a_μ can be extracted by measuring $\vec{\omega}_a$ and $\vec{B}$

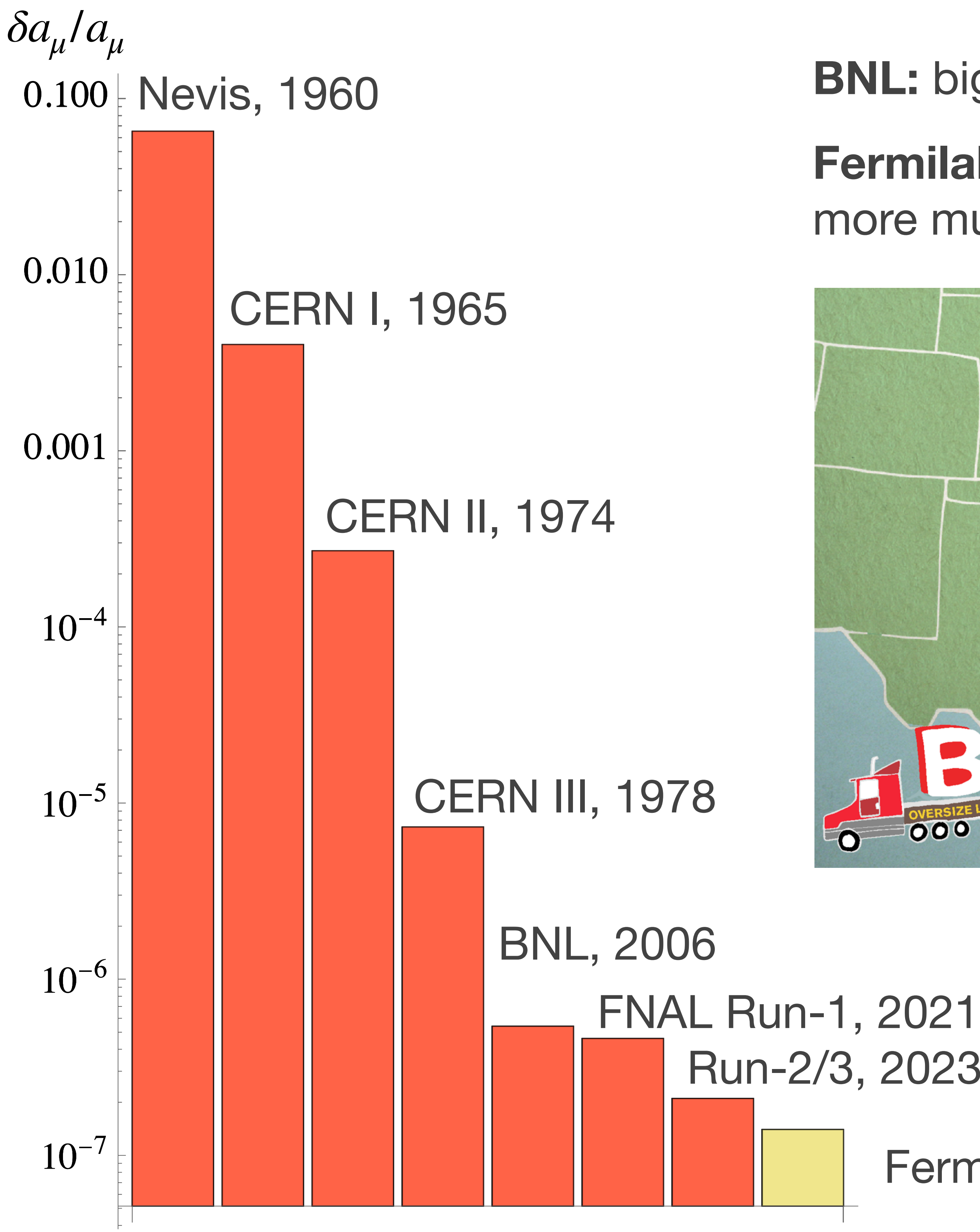
$g = 2 \implies \vec{\omega}_a = 0$



$g \neq 2 \implies \vec{\omega}_a \neq 0$



History of achieved experimental accuracy



BNL: big and powerful magnet

Fermilab: ultra-intense muon beam, 20 times more muons

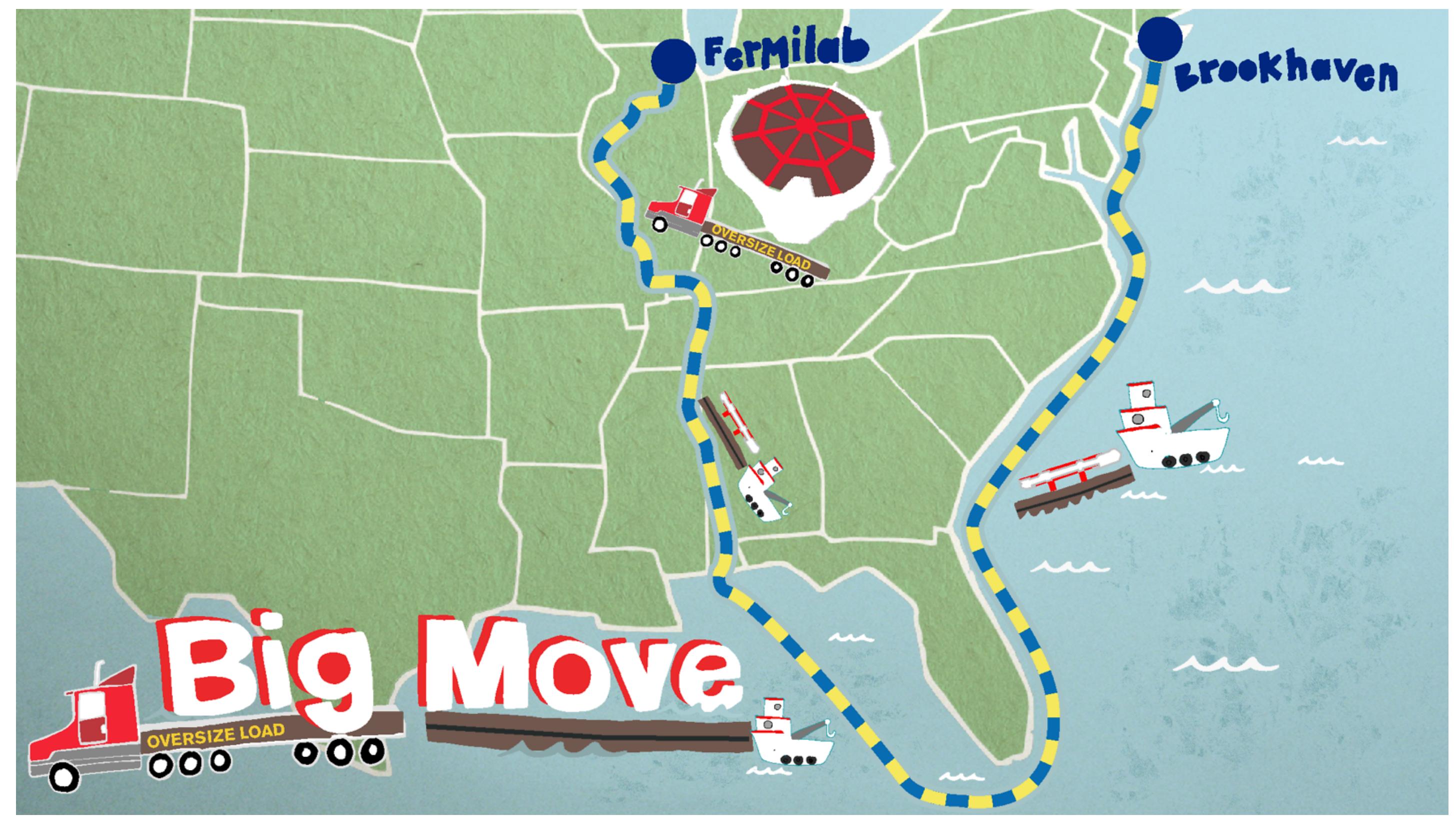
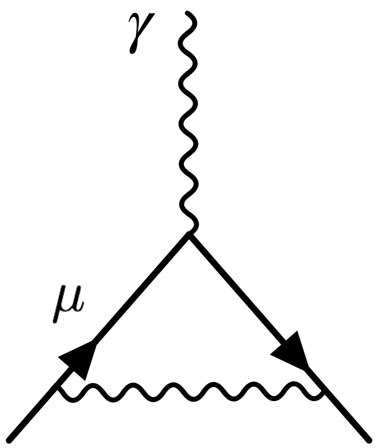


Illustration by Sandbox Studio, Chicago

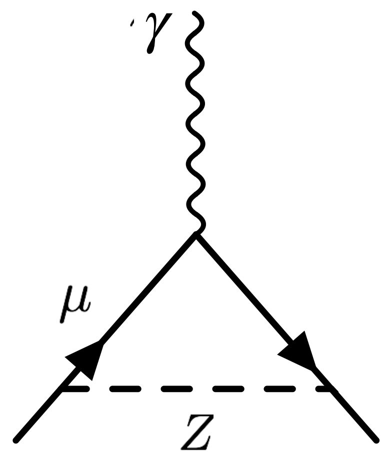
Standard model

Anomalous magnetic moment is determined from the sum of all sectors of the SM

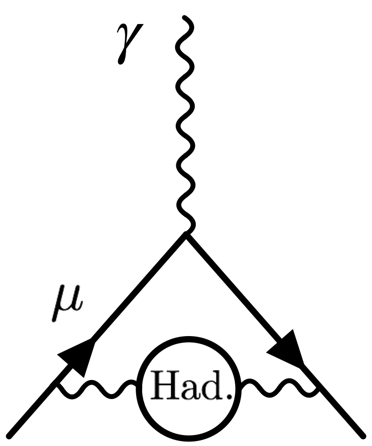
$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{HVP} + a_\mu^{HLbL}$$



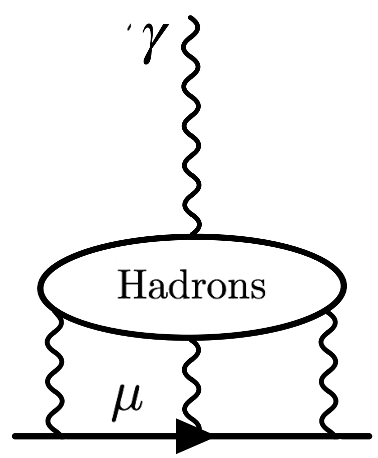
QED



EW



HVP
Hadronic vacuum polarisation

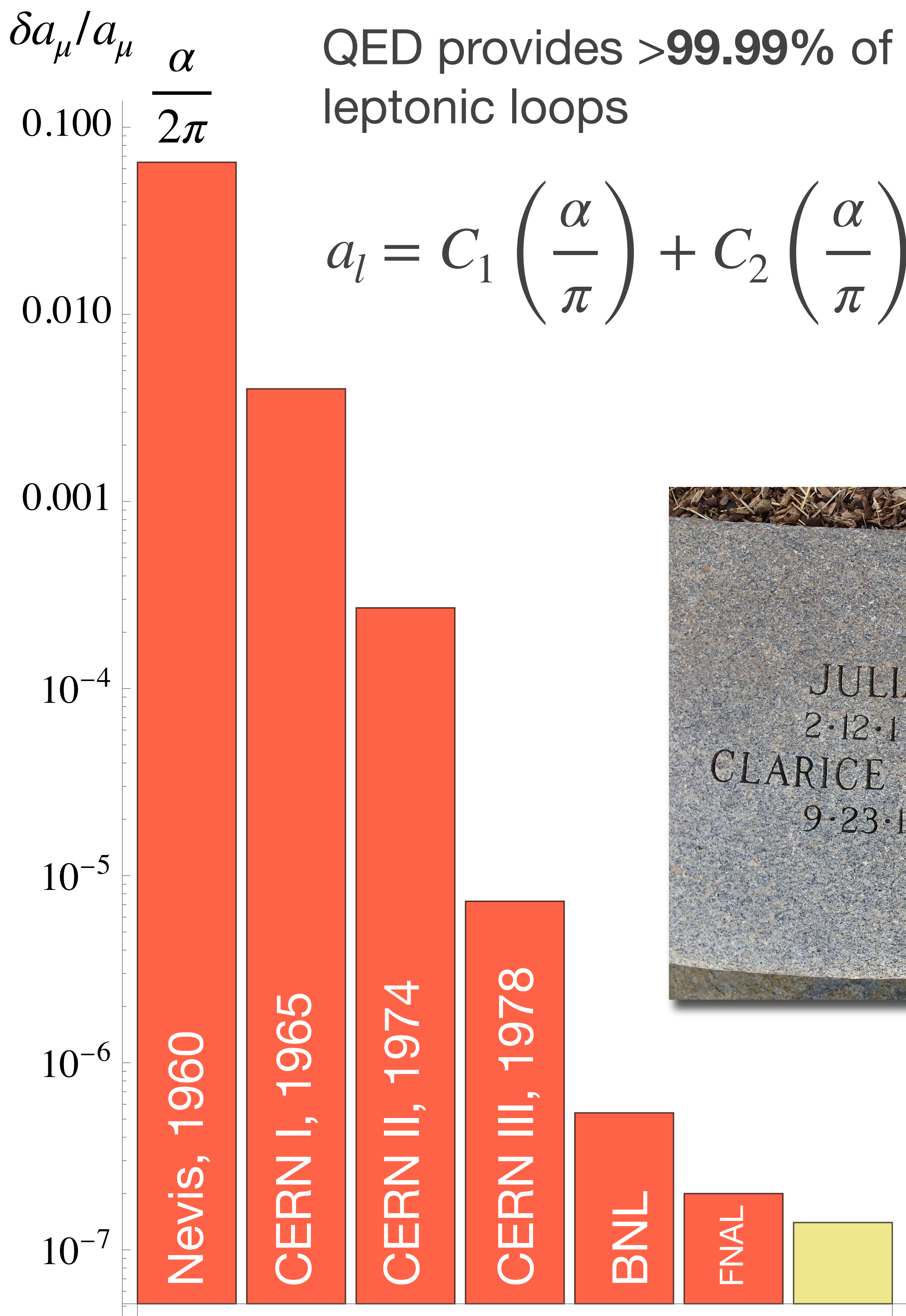


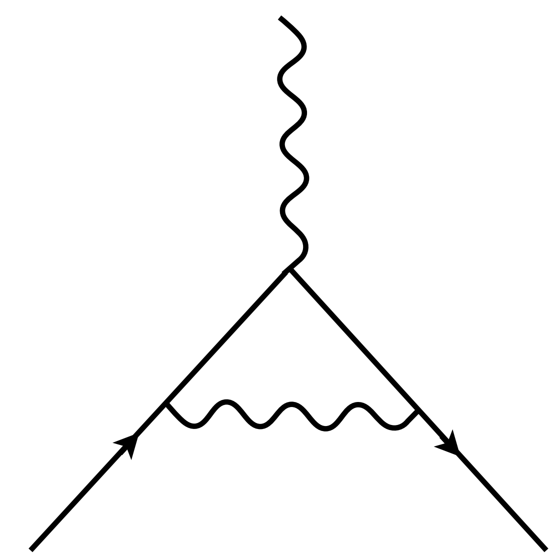
HLbL
Hadronic light-by-light

Standard model: QED

QED provides >**99.99%** of the total value: includes all photonics and leptonic loops

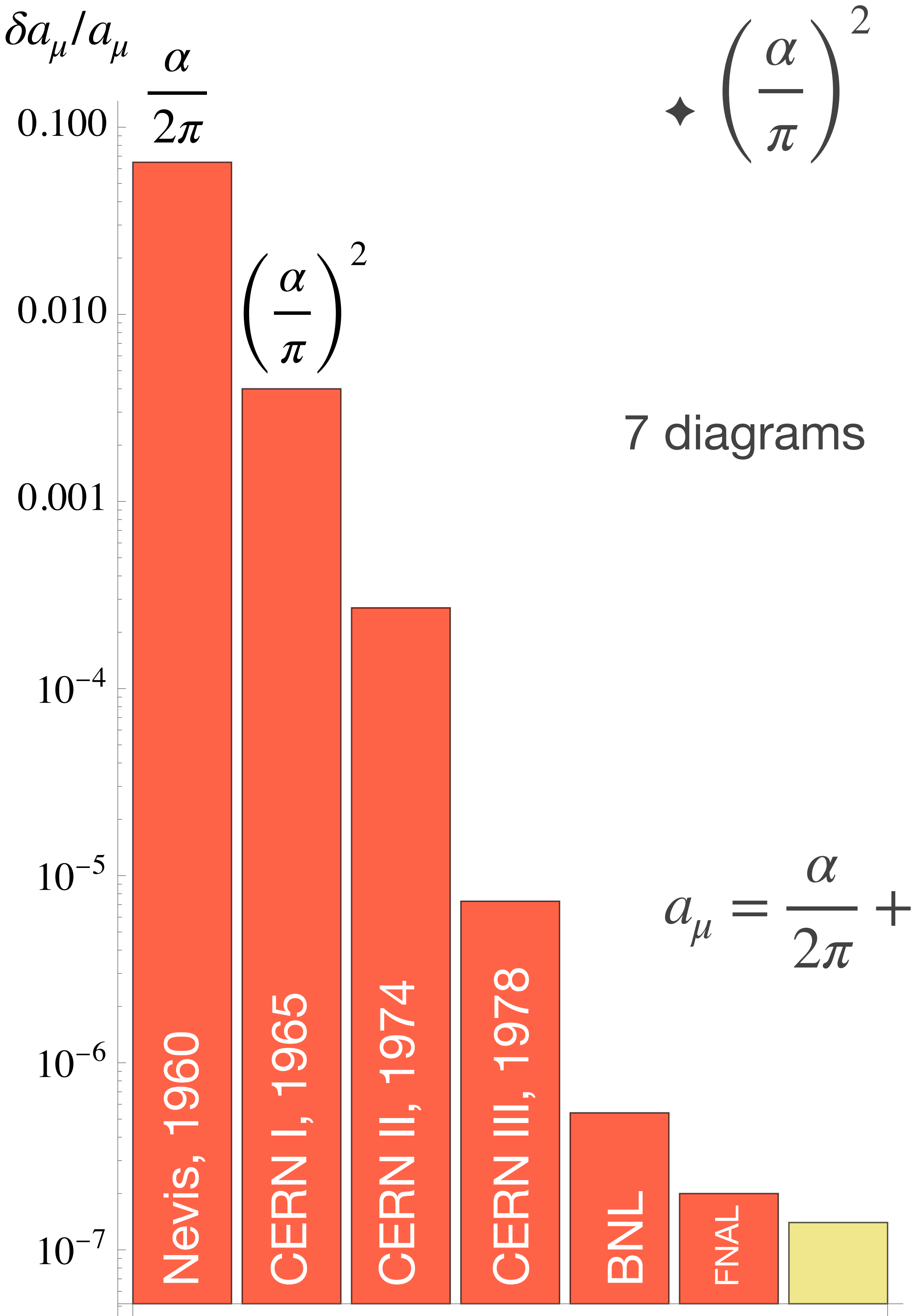
$$a_l = C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^5 + C_5\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$





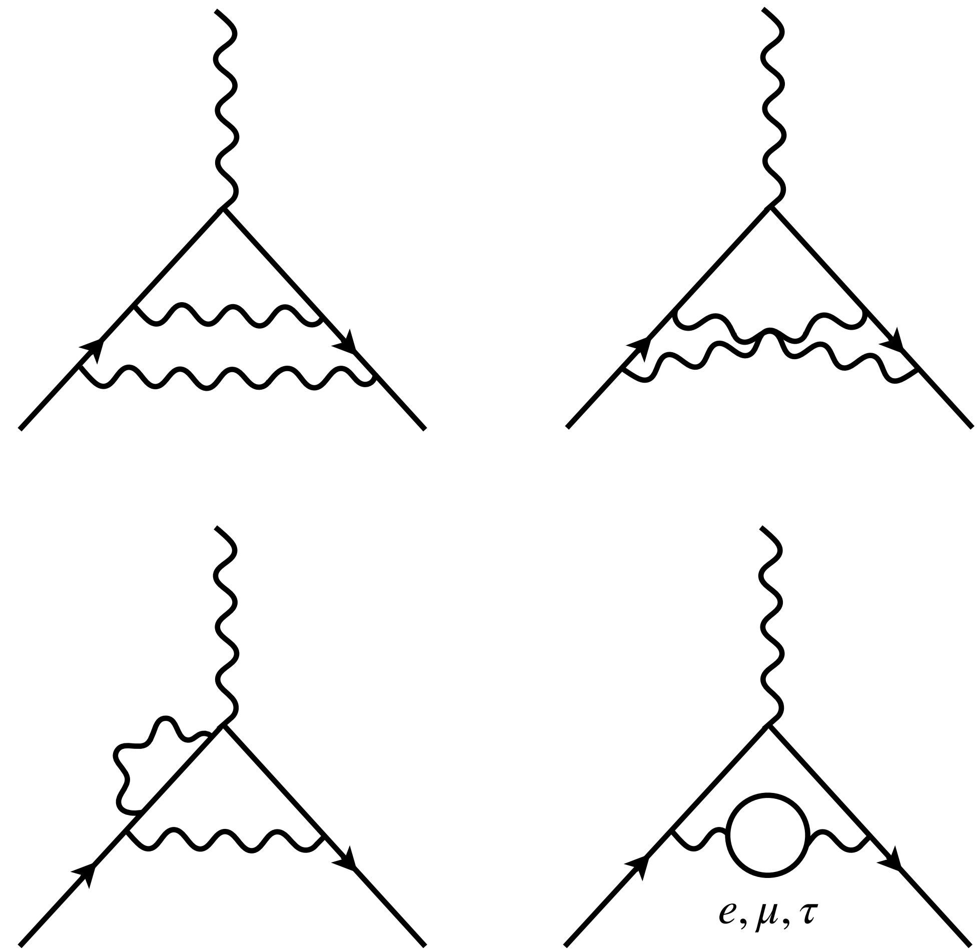
$$a_\mu = \frac{\alpha}{2\pi} \approx 0.00116 = 11\,614\,097 \times 10^{-10}$$

Standard model: QED



$+\left(\frac{\alpha}{\pi}\right)^2$

7 diagrams

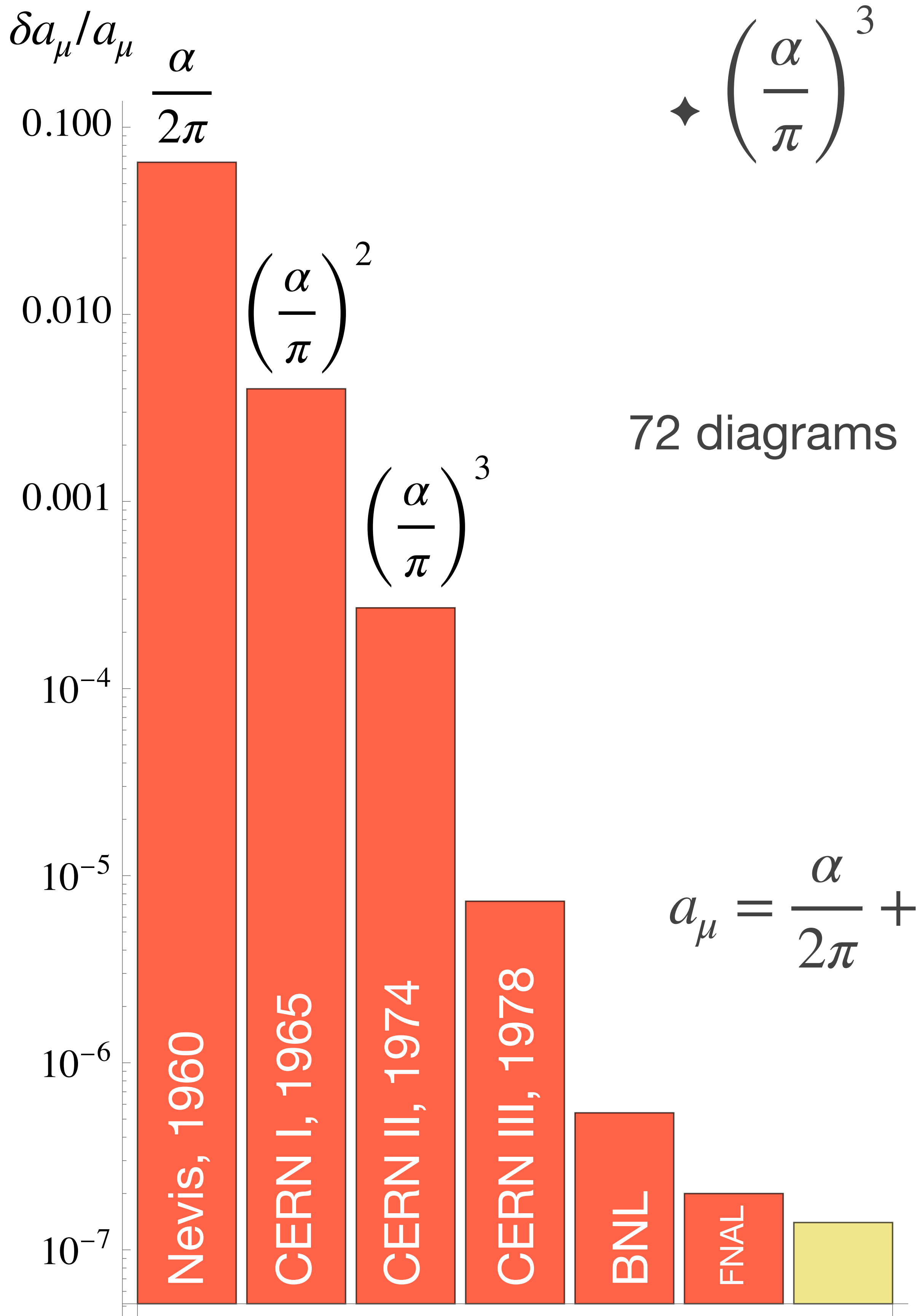


$+ \cdots$

$$a_\mu = \frac{\alpha}{2\pi} + C_2 \left(\frac{\alpha}{\pi}\right)^2 = (11\,614\,097 + 41\,322) \times 10^{-10}$$

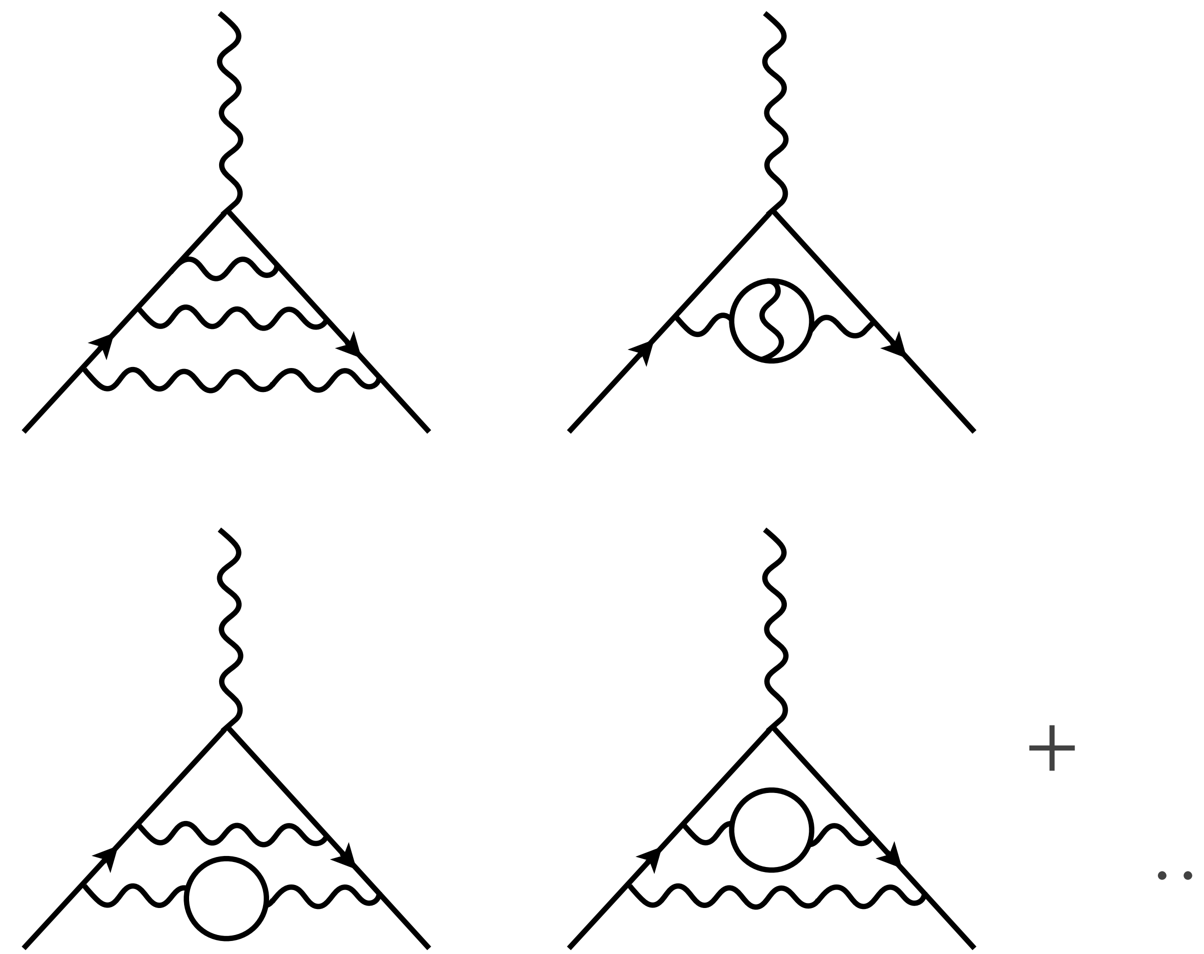
Peterman, Sommerfield (1957)

Standard model: QED



$+ \left(\frac{\alpha}{\pi}\right)^3$

72 diagrams

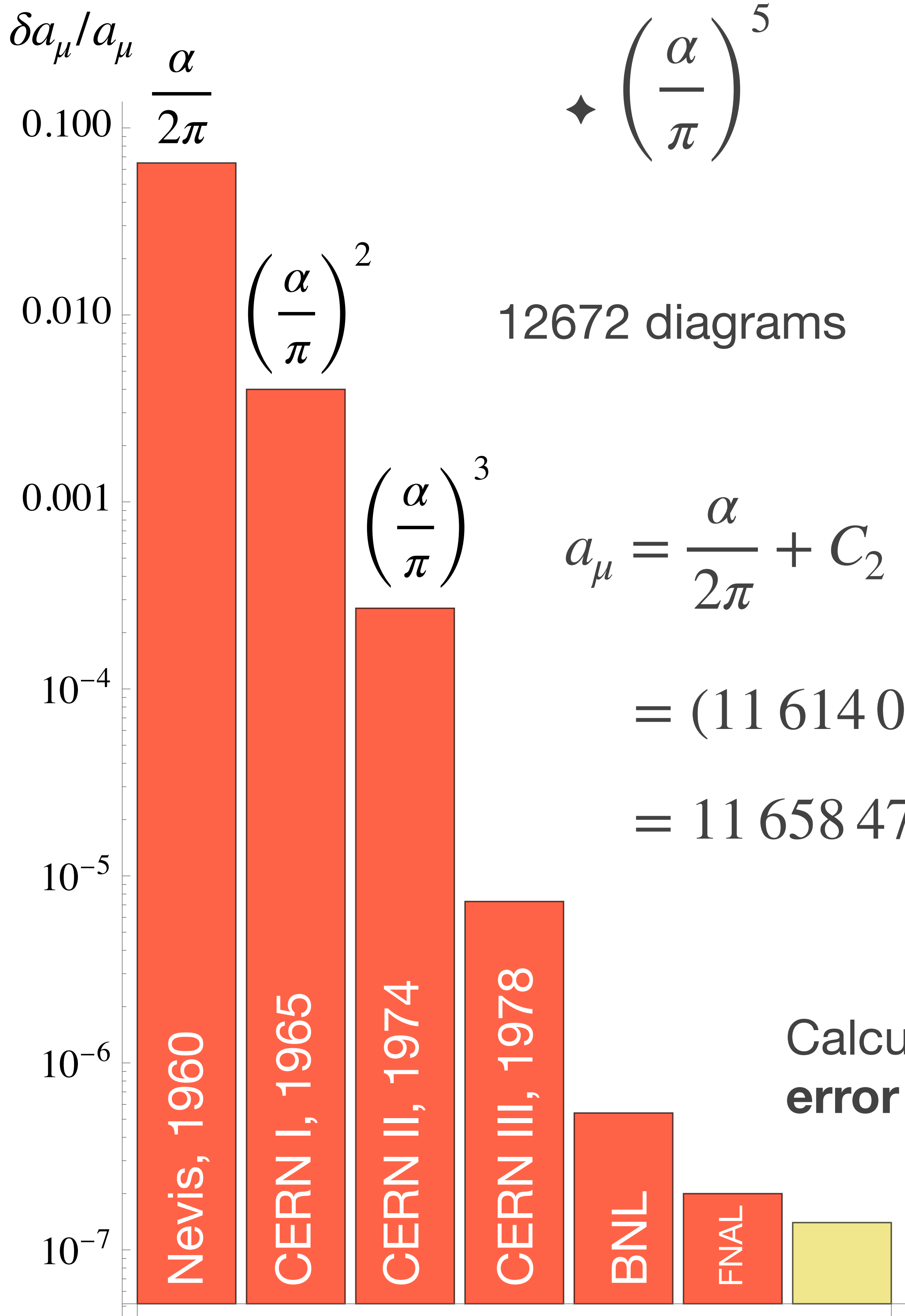


$$a_\mu = \frac{\alpha}{2\pi} + C_2 \left(\frac{\alpha}{\pi}\right)^2 + C_3 \left(\frac{\alpha}{\pi}\right)^3$$

$$= (11\,614\,097 + 41\,322 + 3\,014) \times 10^{-10}$$

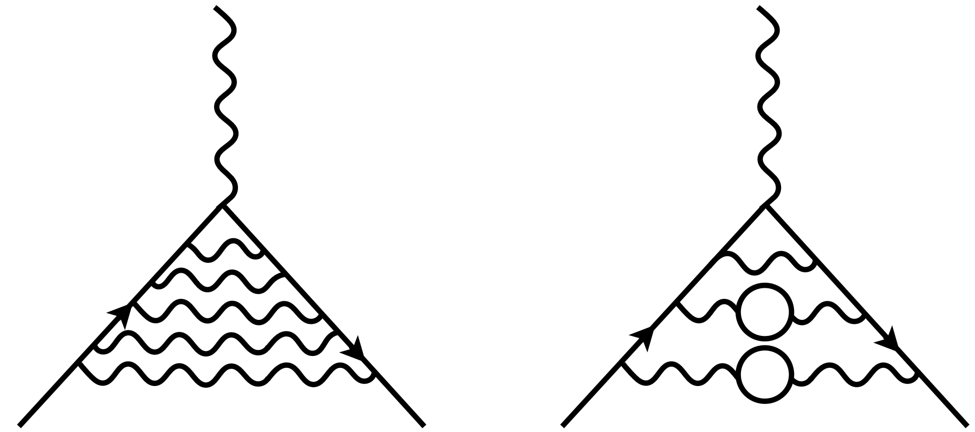
Mignaco, Remiddi (1969)
Aldins, Kinoshita, Brodsky, Dufner (1969)

Standard model: QED



✦ $\left(\frac{\alpha}{\pi}\right)^5$

12672 diagrams



$+ \cdots$

$$a_\mu = \frac{\alpha}{2\pi} + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + C_4\left(\frac{\alpha}{\pi}\right)^4 + C_5\left(\frac{\alpha}{\pi}\right)^5$$

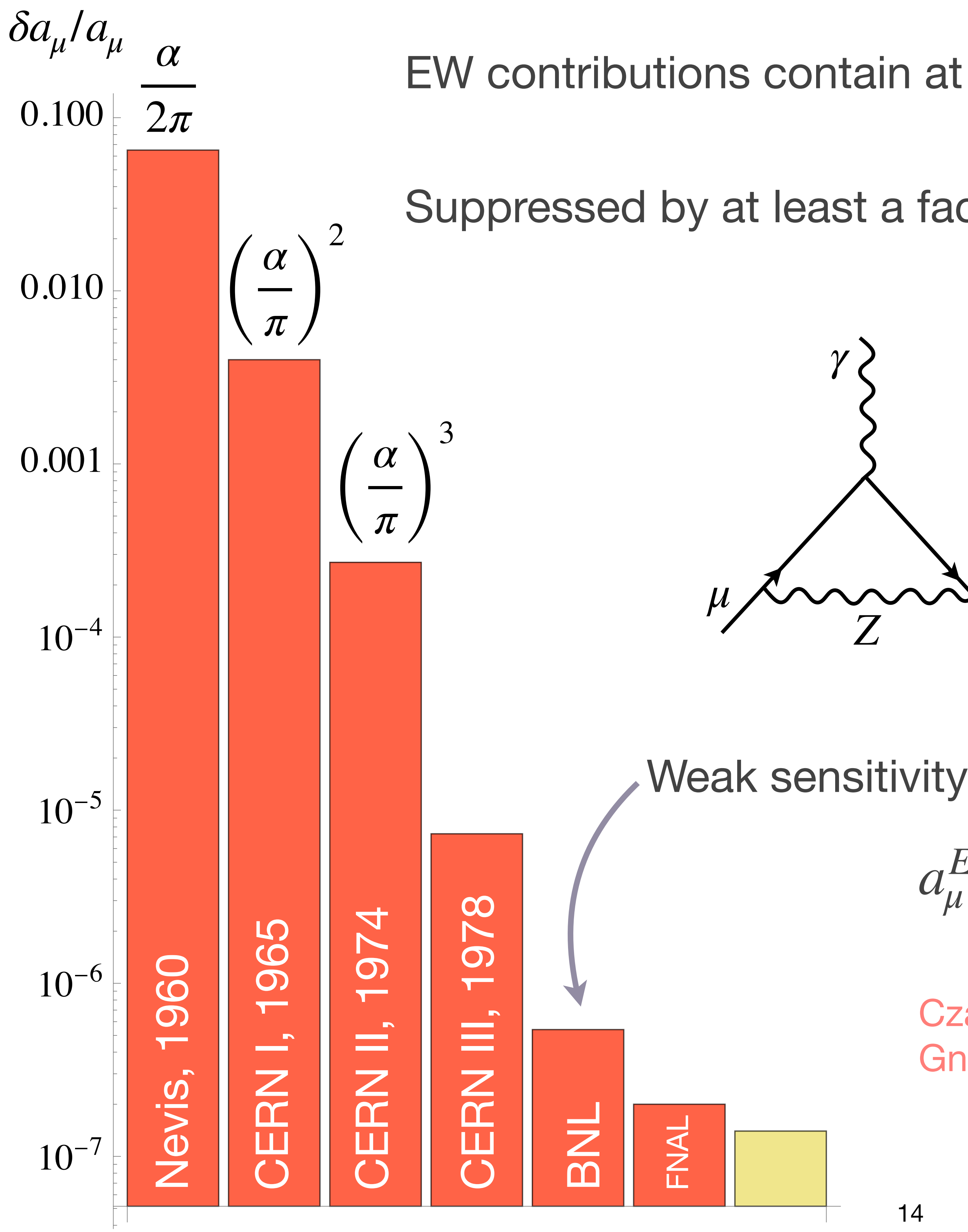
$$= (11\,614\,097 + 41\,322 + 3\,014 + 38 + 0.5) \times 10^{-10}$$

$$= 11\,658\,4718.931(104) \times 10^{-11}$$

Kinoshita et al. (2012)

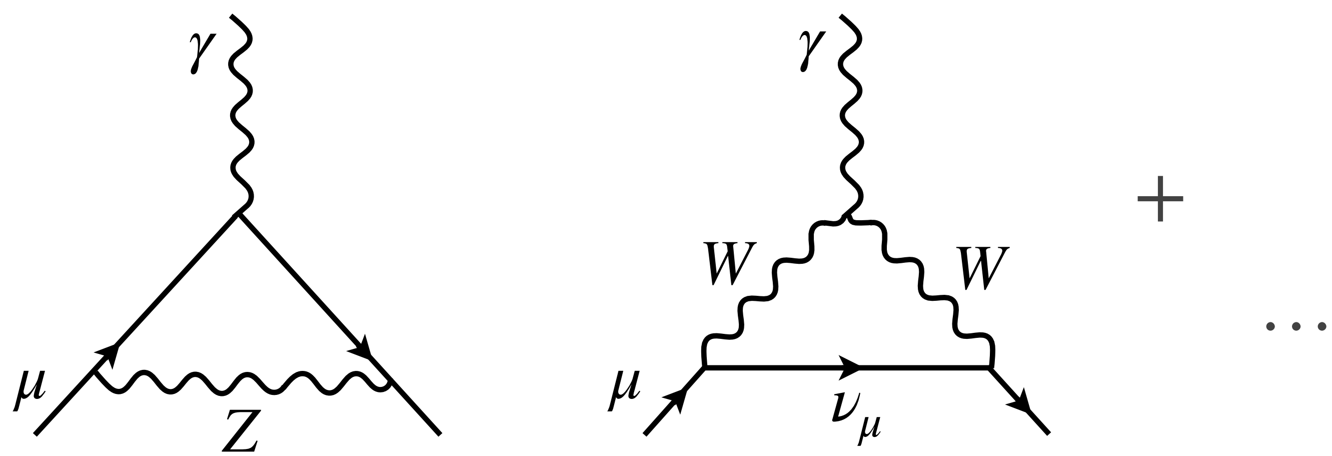
Calculated numerically,
error is well under control

Standard model: electroweak



EW contributions contain at least one of electroweak gauge bosons

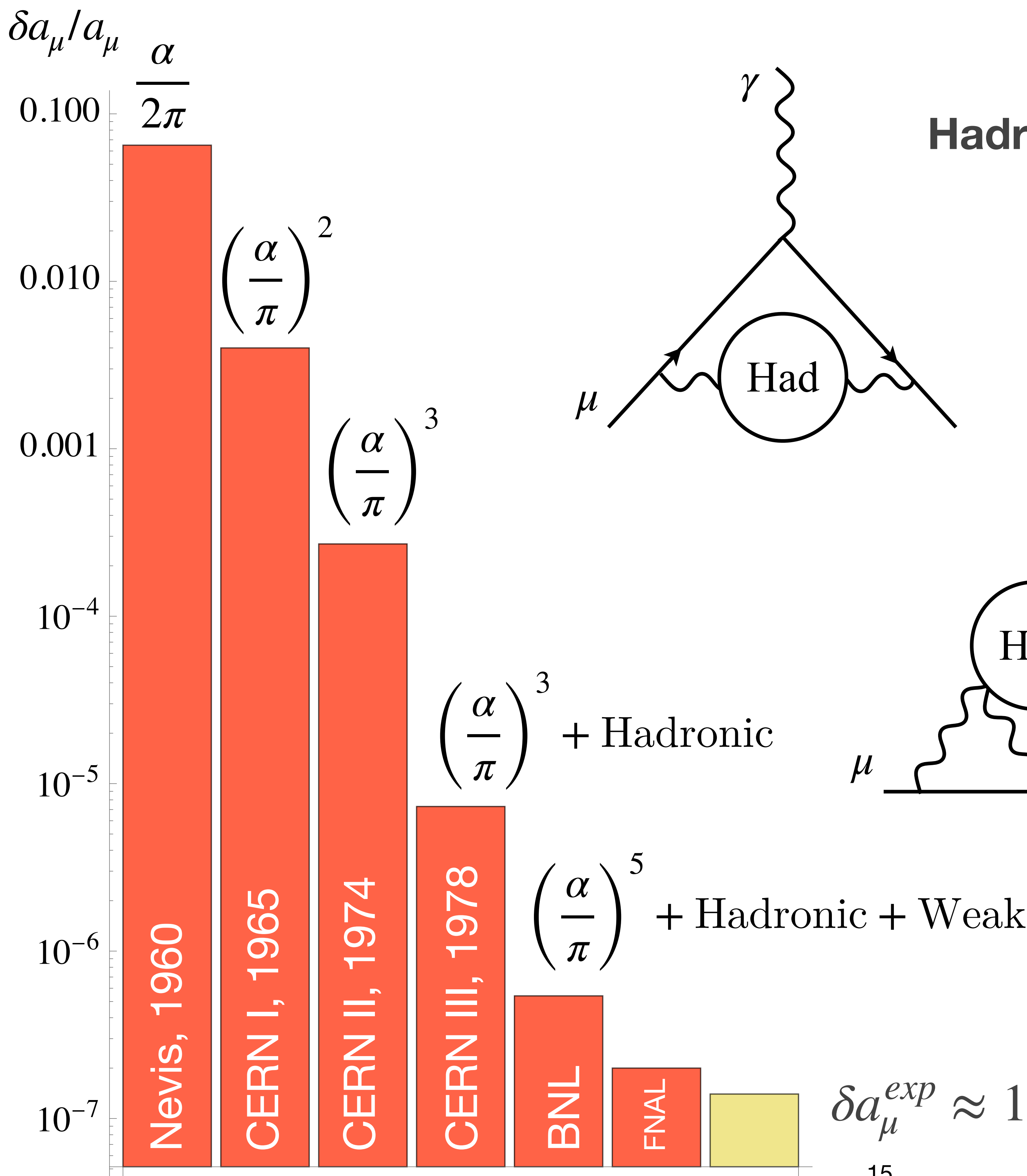
Suppressed by at least a factor of $\frac{\alpha}{\pi}\frac{m_\mu^2}{m_W^2} \simeq 4 \times 10^{-9}$



$$a_\mu^{EW} = 153.6(1.0) \times 10^{-11}$$

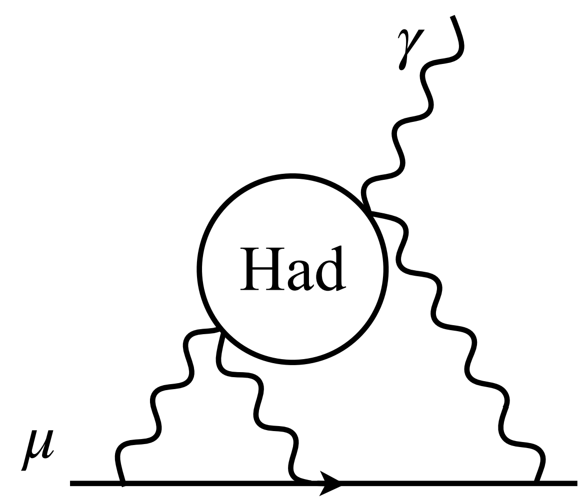
Czarnecki et al. (1998, 2005)
Gnendiger et al. (2013)

Standard model: QCD contributions



Hadronic vacuum polarisation

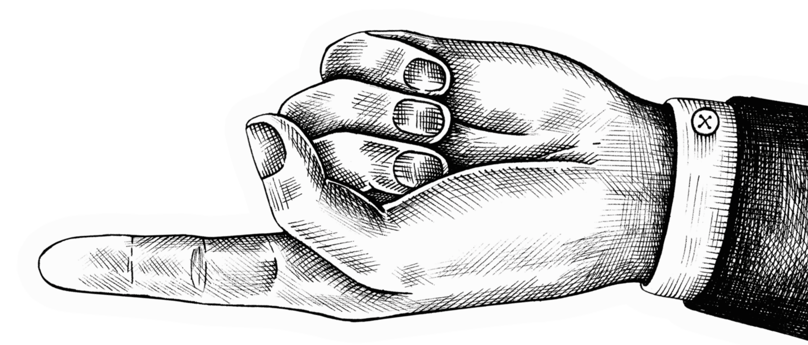
$$a_\mu^{HVP} = 6845(40) \times 10^{-11}$$



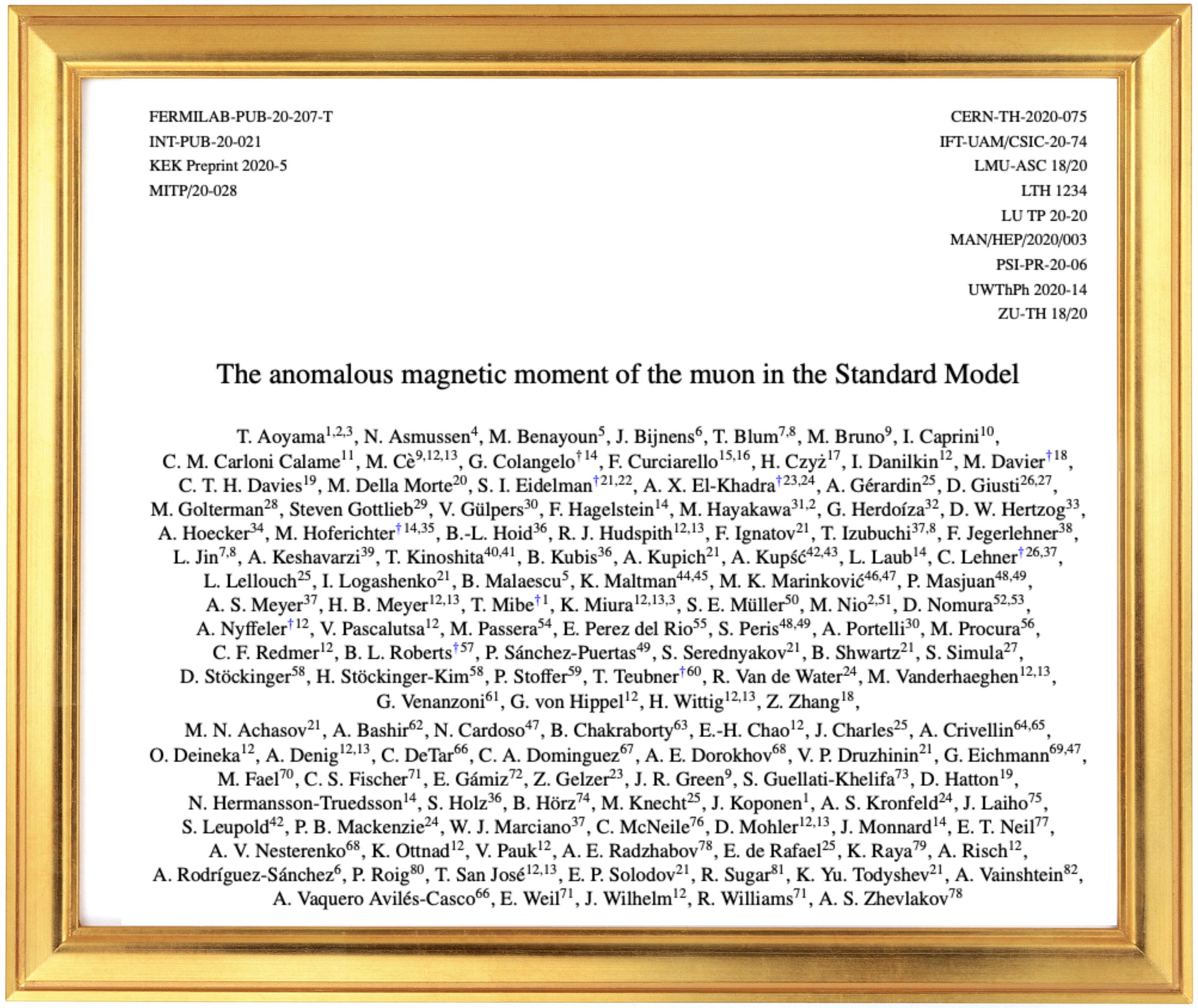
Hadronic light-by-light scattering

$$a_\mu^{HLbL} = 92(18) \times 10^{-11}$$

$$\delta a_\mu^{exp} \approx 16 \times 10^{-11}$$



Theory vs experiment



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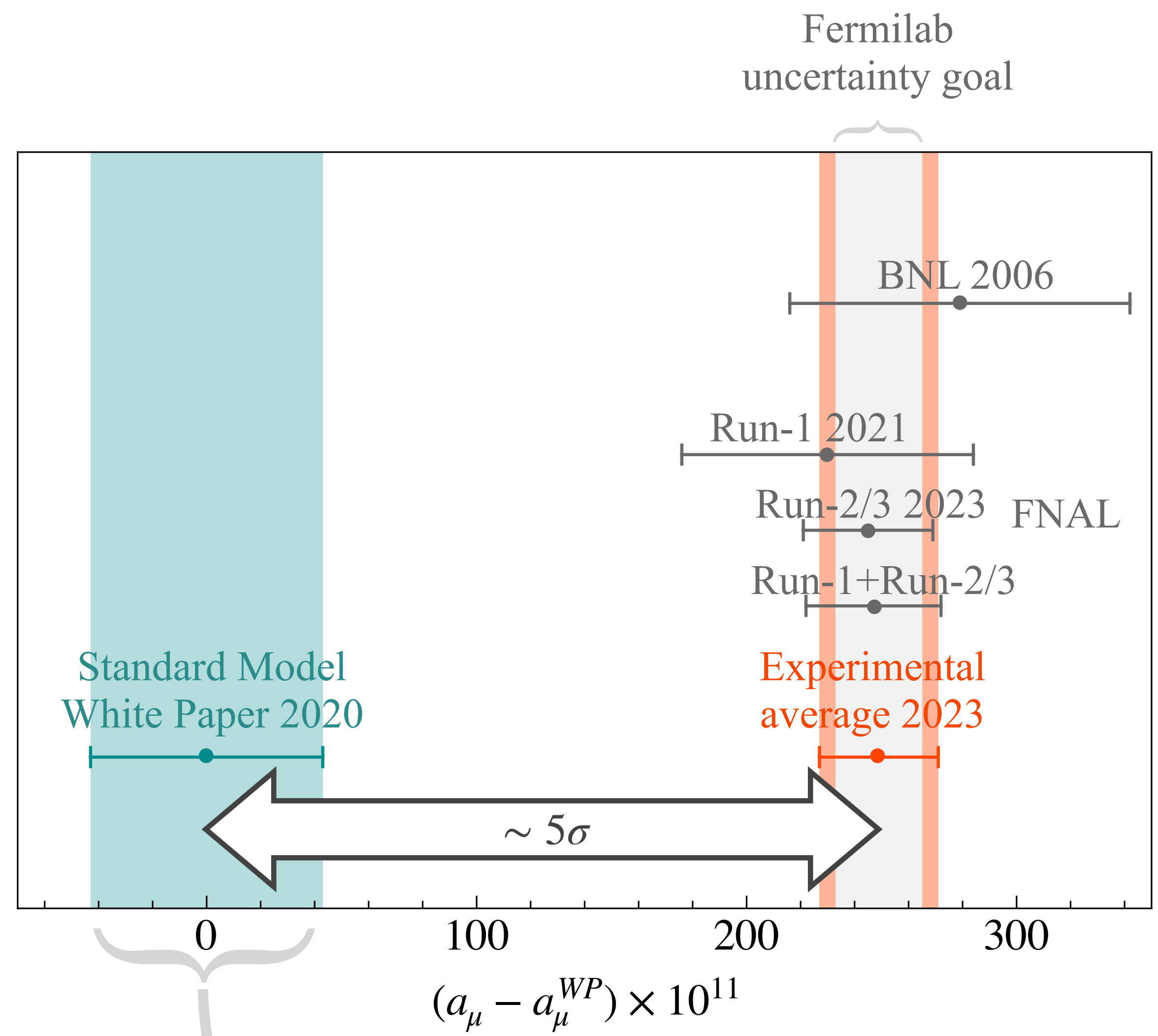
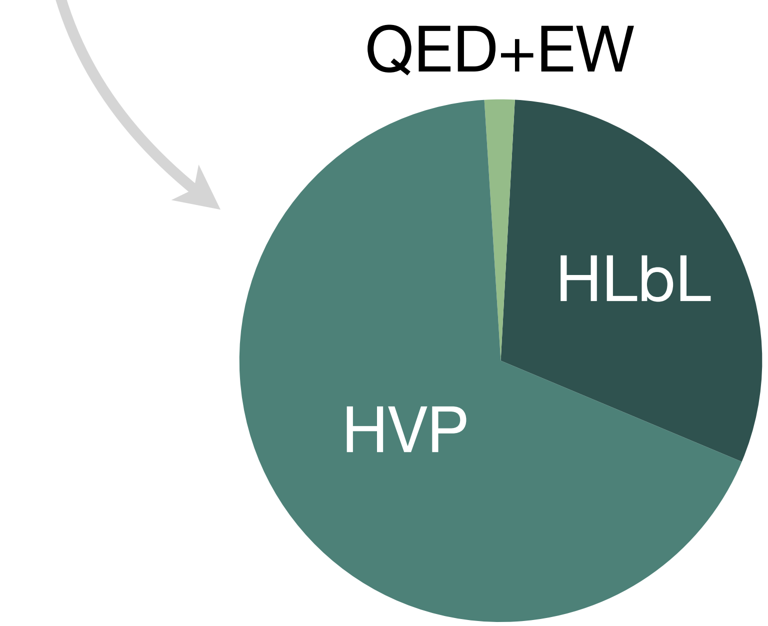
LMU-ASC 18/20
LTH 1234
LU TP 20-20
MAN/HEP/2020/003
PSI-PR-20-06
UWThPh 2020-14
ZU-TH 18/20

The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama[1,2,3], N. Asmussen[4], M. Benayoun[5], J. Bijnens[6], T. Blum[7,8], M. Bruno[9], I. Caprini[10], C. M. Carloni Calame[11], M. Cè[9,12,13], G. Colangelo[†14], F. Curciarello[15,16], H. Czyż[17], I. Danilkin[12], M. Davier[†18], C. T. H. Davies[19], M. Della Morte[20], S. I. Eidelman[†21,22], A. X. El-Khadra[†23,24], A. Gérardin[25], D. Giusti[26,27], M. Golterman[28], Steven Gottlieb[29], V. Gülpers[30], F. Hagelstein[14], M. Hayakawa[31,2], G. Herdoíza[32], D. W. Hertzog[33], A. Hoecker[34], M. Hoferichter[†14,35], B.-L. Hoid[36], R. J. Hudspith[12,13], F. Ignatov[21], T. Izubuchi[37,8], F. Jegerlehner[38], L. Jin[7,8], A. Keshavarzi[39], T. Kinoshita[40,41], B. Kubis[36], A. Kupich[21], A. Kupść[42,43], L. Laub[14], C. Lehner[†26,37], L. Lellouch[25], I. Logashenko[21], B. Malaescu[5], K. Maltman[44,45], M. K. Marinković[46,47], P. Masjuan[48,49], A. S. Meyer[37], H. B. Meyer[12,13], T. Mibe[†1], K. Miura[12,13,3], S. E. Müller[50], M. Nio[2,51], D. Nomura[52,53], A. Nyffeler[†12], V. Pascalutsa[12], M. Passera[54], E. Perez del Rio[55], S. Peris[48,49], A. Portelli[30], M. Procura[56], C. F. Redmer[12], B. L. Roberts[†57], P. Sánchez-Puertas[49], S. Serednyakov[21], B. Shwartz[21], S. Simula[27], D. Stöckinger[58], H. Stöckinger-Kim[58], P. Stoffer[59], T. Teubner[†60], R. Van de Water[24], M. Vanderhaeghen[12,13], G. Venanzoni[61], G. von Hippel[12], H. Wittig[12,13], Z. Zhang[18],

M. N. Achasov[21], A. Bashir[62], N. Cardoso[47], B. Chakraborty[63], E.-H. Chao[12], J. Charles[25], A. Crivellin[64,65], O. Deineka[12], A. Denig[12,13], C. DeTar[66], C. A. Dominguez[67], A. E. Dorokhov[68], V. P. Druzhinin[21], G. Eichmann[69,47], M. Fael[70], C. S. Fischer[71], E. Gámiz[72], Z. Gelzer[23], J. R. Green[9], S. Guellati-Khelifa[73], D. Hatton[19], N. Hermansson-Truedsson[14], S. Holz[36], B. Hörz[74], M. Knecht[25], J. Koponen[1], A. S. Kronfeld[24], J. Laiho[75], S. Leupold[42], P. B. Mackenzie[24], W. J. Marciano[37], C. McNeile[76], D. Mohler[12,13], J. Monnard[14], E. T. Neil[77], A. V. Nesterenko[68], K. Ottnad[12], V. Pauk[12], A. E. Radzhabov[78], E. de Rafael[25], K. Raya[79], A. Risch[12], A. Rodríguez-Sánchez[6], P. Roig[80], T. San José[12,13], E. P. Solodov[21], R. Sugar[81], K. Yu. Todyshev[21], A. Vainshtein[82], A. Vaquero Avilés-Casco[66], E. Weil[71], J. Wilhelm[12], R. Williams[71], A. S. Zhevlakov[78]

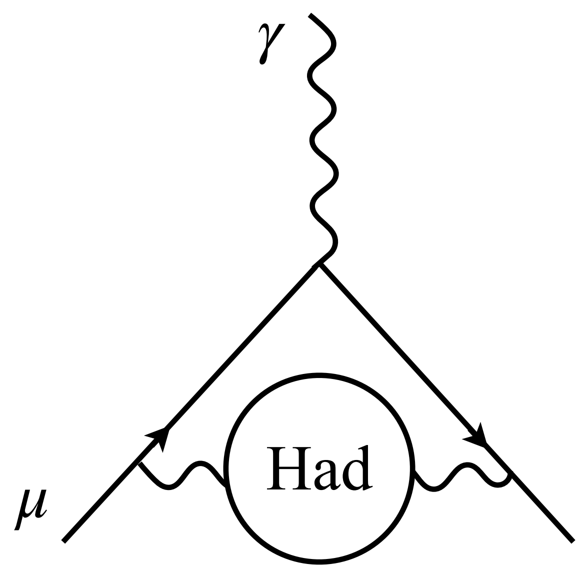




- Uncertainties are **dominated by hadronic contributions**
- Two main principles to evaluate hadronic contributions: **data-driven** and **lattice QCD**

Hadronic vacuum polarisation

Data-driven approaches are using $e^+e^- \to$ hadrons data as input into **dispersion relations** (based on **analyticity** and **unitarity**)



Photon self-energy

$$\Pi^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi(q^2)$$

Analyticity in $s = q^2$ plane allows to write a dispersion integral (Cauchy's theorem)

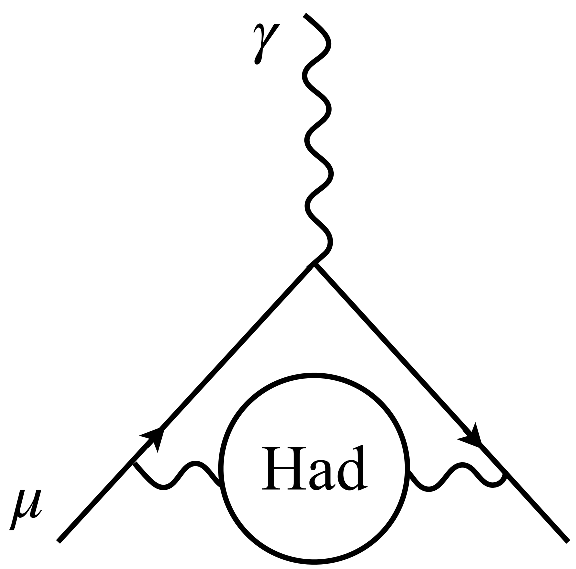
$$\Pi(s) - \Pi(0) = \frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\mathrm{Im}\,\Pi(s')}{s'(s'-s)}$$

Unitarity (optical theorem)

$$\mathrm{Im}\,\Pi(s) \sim \sigma_{tot}(e^+e^- \to \text{anything})$$

Obtain the hadronic contribution if restrict "anything" to hadrons

Hadronic vacuum polarisation



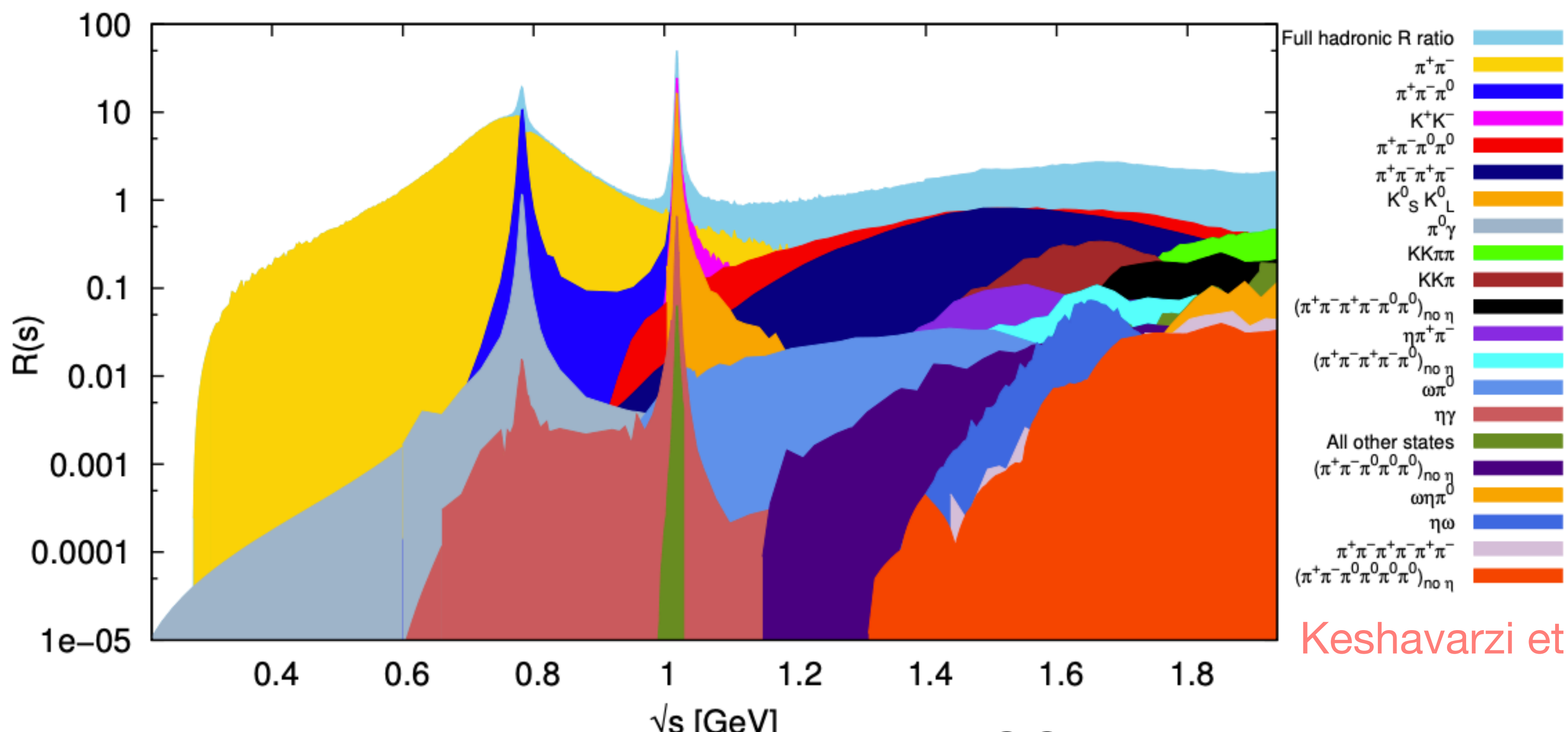
$$a_\mu^{HVP,LO} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

known kernel function

Strong weight at the low-energy part
>70% from $\pi^+\pi^-[\rho(770)]$ channel

$$R(s) = \frac{\sigma^0(e^+e^- \to \text{hadrons}(+\gamma))}{4\pi\alpha^2/(3s)}$$

hadronic R-ratio



Keshavarzi et al. (2018)

+ pQCD

Hadronic vacuum polarisation

NLO and NNLO are determined from similar dispersion integrals and kernel functions

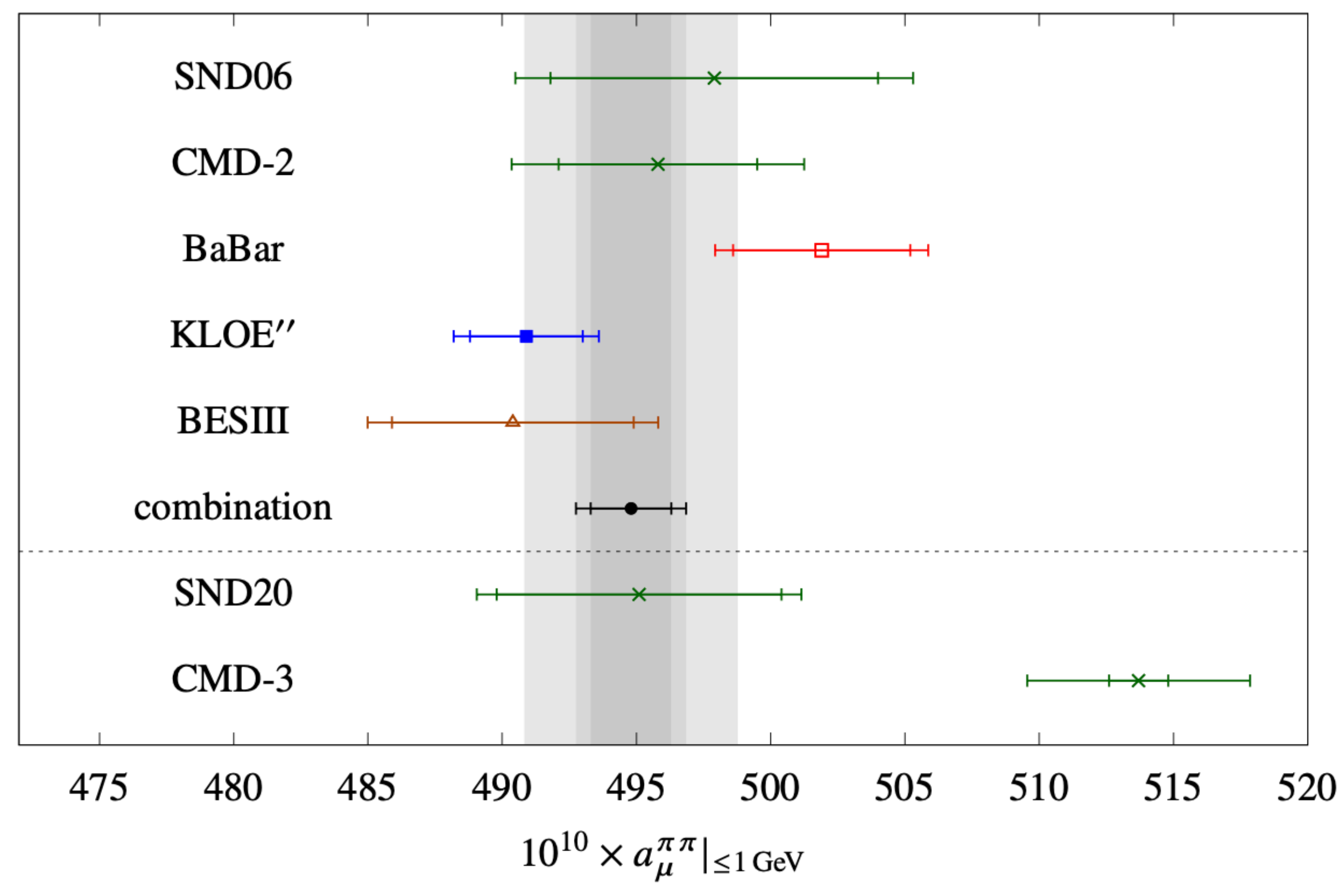
$$a_\mu^{HVP} = a_\mu^{HVP,LO} + a_\mu^{HVP,NLO} + a_\mu^{HVP,NNLO} = 6845(40) \times 10^{-11}$$

Uncertainty is dominated by the total cross section of $e^+e^- \to \pi^+\pi^-$ channel

$$a_\mu^{\pi^+\pi^-} = 5060(34) \times 10^{-11}$$

Tensions for $\sigma(e^+e^- \to \pi^+\pi^-)$ data

- Tensions between BaBar and KLOE
- Discrepancies between CMD-3 and all previous experiments



Colangelo et al. (2023)

No **conceptual** problems with dispersive approach, need to **understand** the tensions

Hadronic vacuum polarisation

HVP contribution can be calculated from **lattice QCD**

No reliance on experimental data

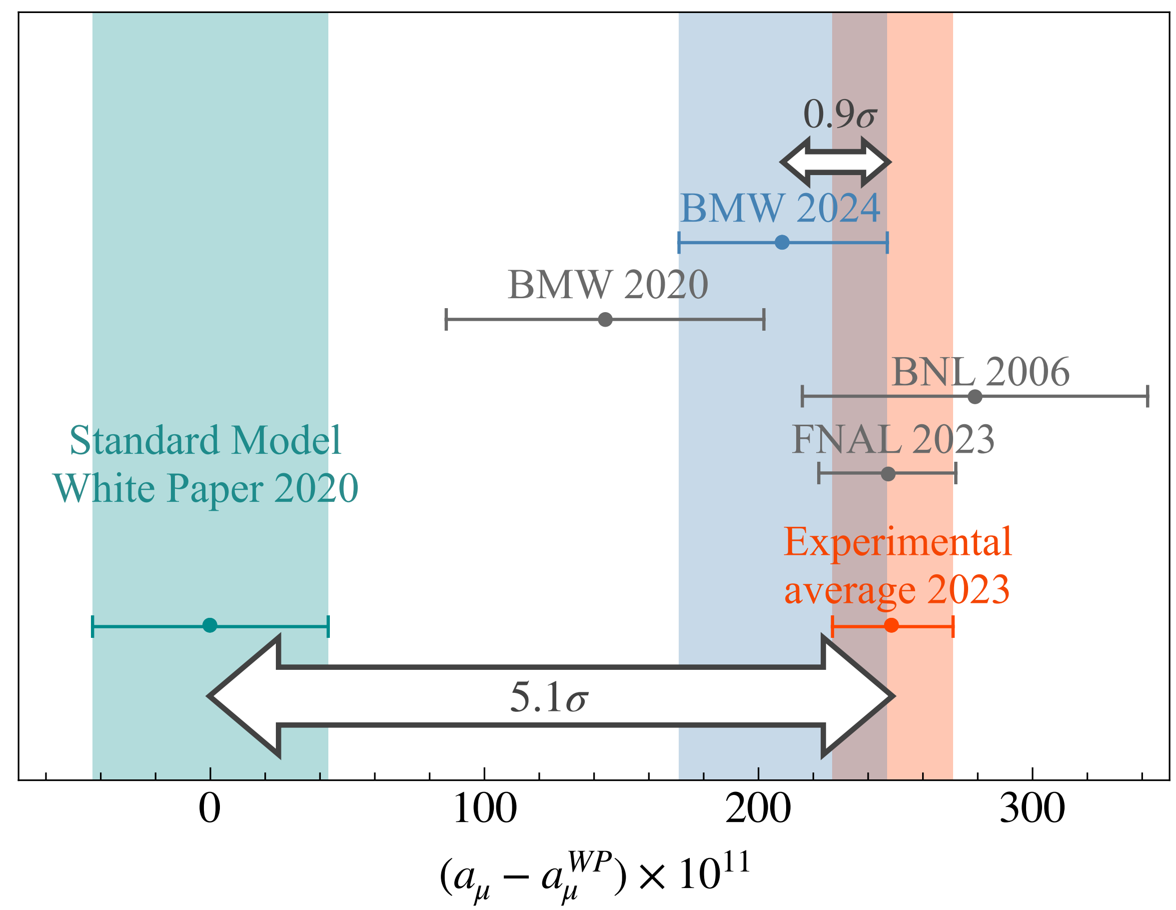
known kernel function

$$a_\mu^{HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt \tilde{K}(t) G(t)$$

correlator of electromagnetic current

$$G(t) = \langle J_\mu(t) J_\nu(0) \rangle$$

- Need more independent checks of BMWc results
- Need to understand the tension with the data driven evaluations of HVP



$$a_\mu^{HVP,LO} = 7116(184) \times 10^{-11} \text{ (WP)}$$

$$a_\mu^{HVP,LO} = 7141(33) \times 10^{-11} \text{ (BMWc)}$$

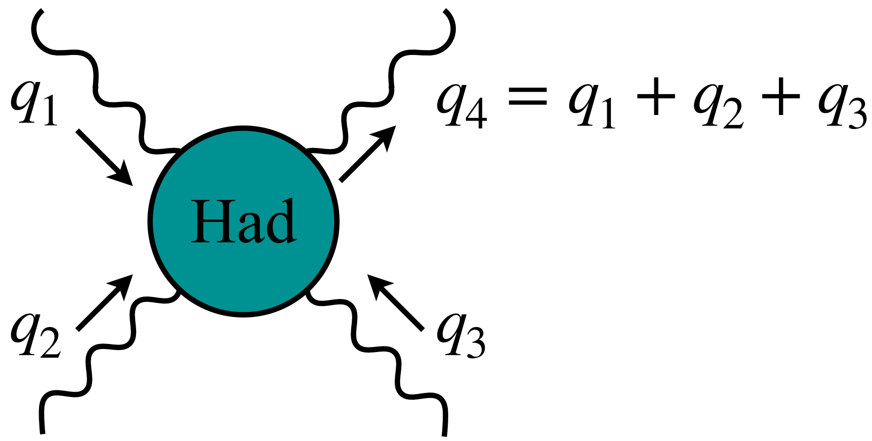
Borsanyi et al. (2020)
Boccaletti et al. (2024)

$$a_\mu^{HVP} = 6845(40) \times 10^{-11} \text{ (WP DR)}$$

Hadronic light-by-light scattering

HLbL contribution is suppressed by a factor of $\left(\frac{\alpha}{\pi}\right)$ compared to HVP

Larger relative uncertainty than HVP (~20%, needs to be <10% to meet the FNAL goal)



- Light-by-light tensor $\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3)$ is much more complicated compared to HVP
- The unitarity relation and data-driven approach is also more complicated

Hadronic light-by-light tensor can be decomposed

$$\Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{138} L_i^{\mu\nu\lambda\sigma} \Xi_i \xrightarrow[\text{no kinematic singularities}]{\text{gauge invariance}} \Pi^{\mu\nu\lambda\sigma} = \sum_{i=1}^{54} T_i^{\mu\nu\lambda\sigma} \Pi_i$$

Bardeen, Tung (1968, 1971)
Tarrach (1975)

Colangelo et. al (2015)

Hadronic light-by-light scattering

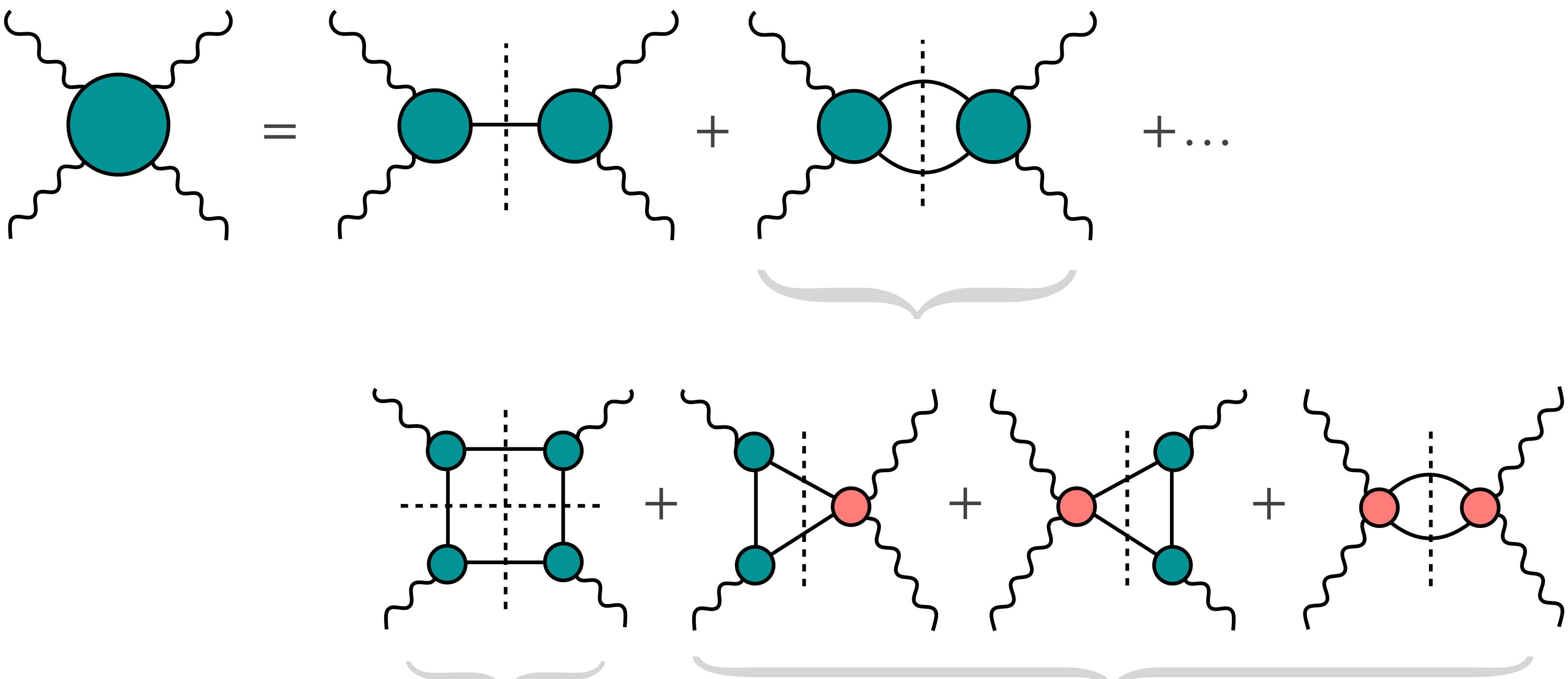
Colangelo et. al (2014-2017)

$$a_\mu^{HLbL} = \frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \sqrt{1-\tau^2}\, Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \bar{\Pi}_i(Q_1, Q_2, \tau)$$

$$Q_i^2 = -q_i^2 \,, Q_3^2 = Q_1^2 + Q_2^2 + 2\tau Q_1 Q_2$$

known kernel functions

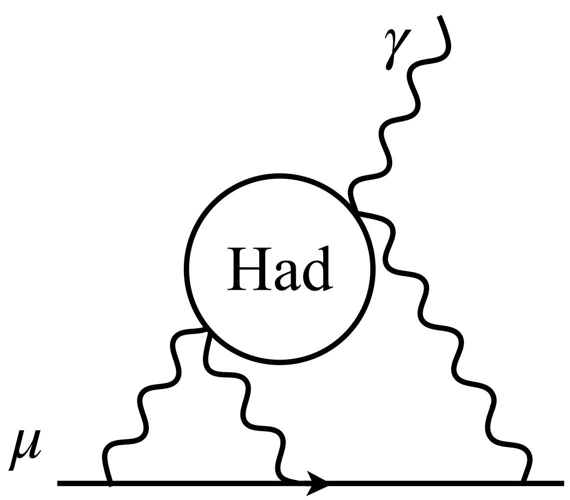
combinations of the scalar functions Π_i

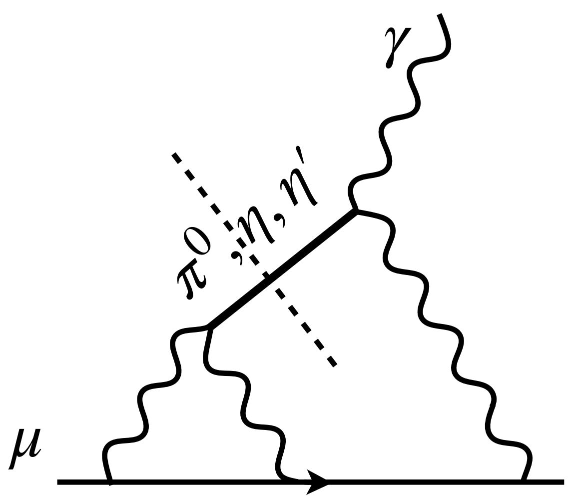


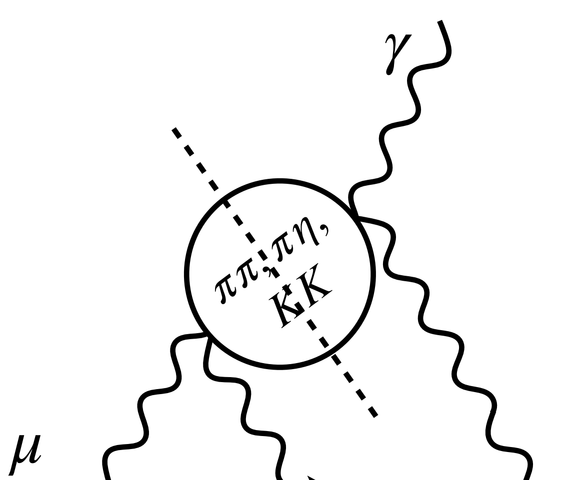
pion/kaon box

rescattering contribution

Experimental input

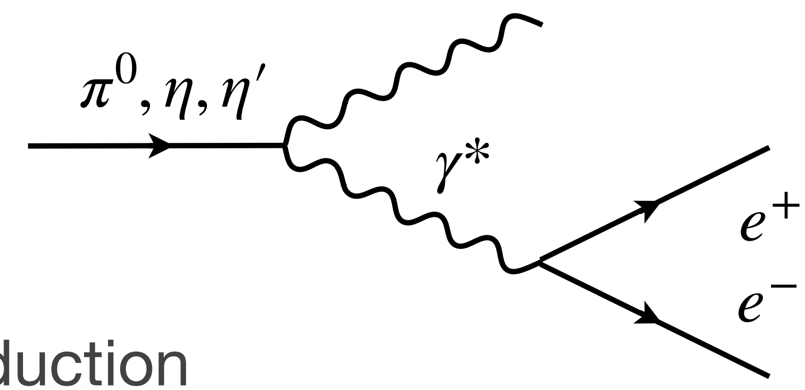




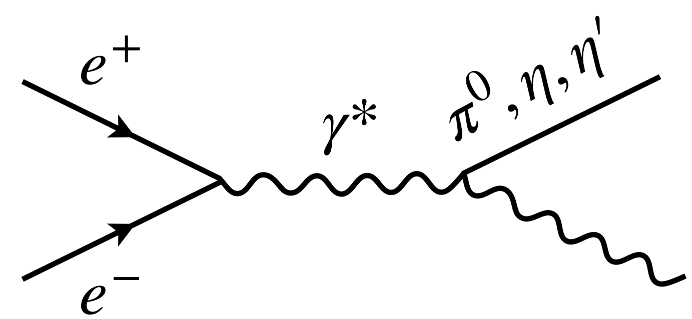


$q^2 > 0$ **timelike** γ^*:

- Dalitz decay

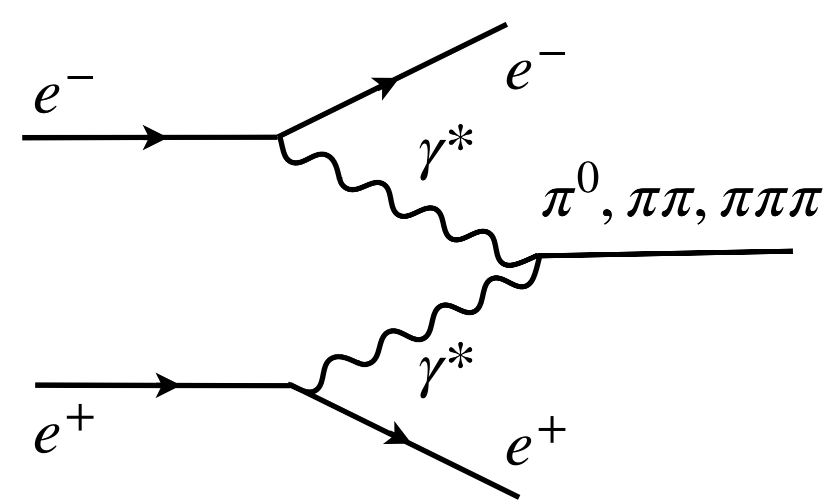


- Radiative production

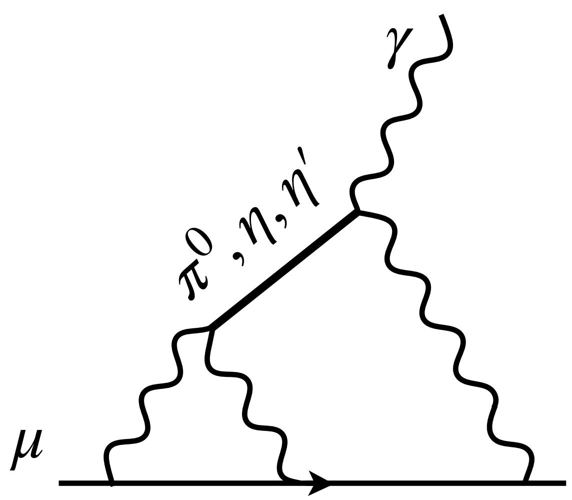


$q^2 < 0$ **spacelike** γ^*

- Two-photon collisions



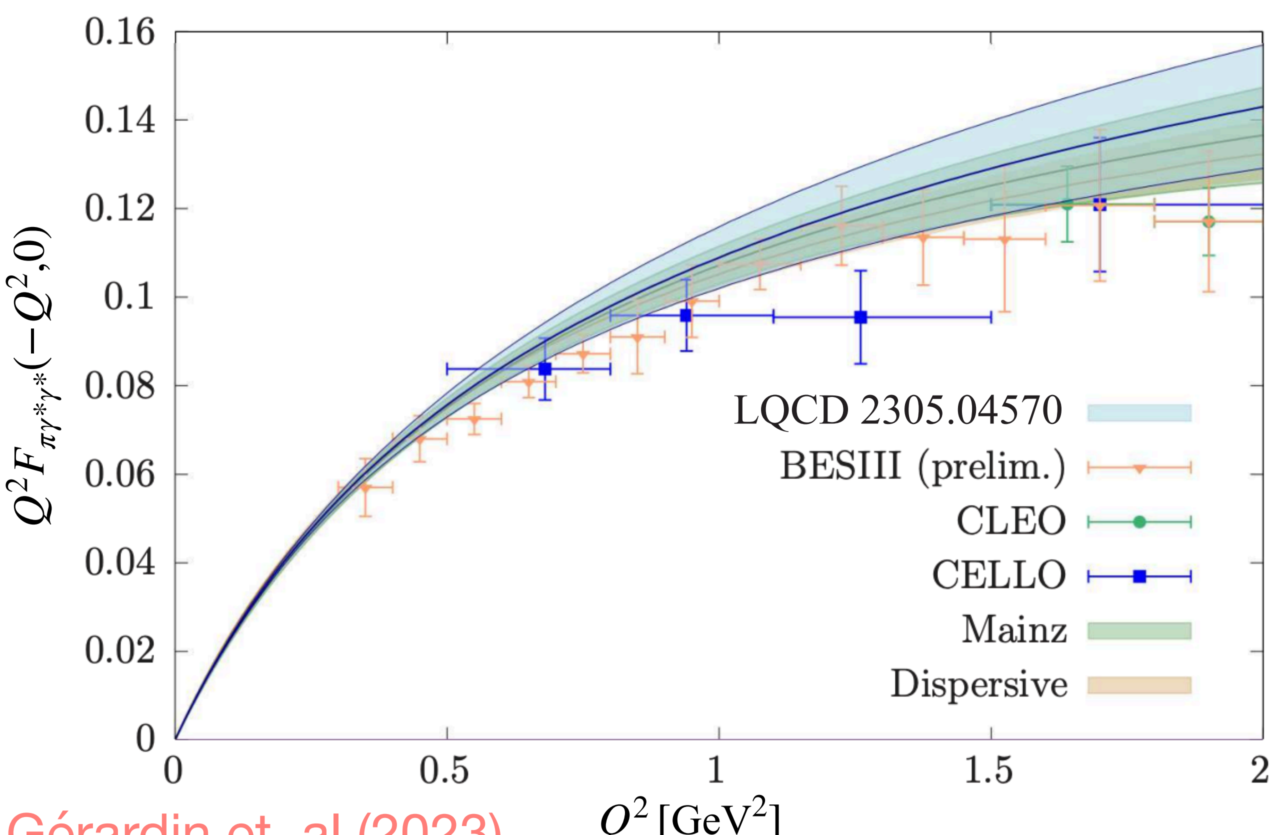
Meson-pole contributions



$$a_\mu = \left(\frac{\alpha}{\pi}\right)^3 \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{+1} d\tau$$
$$\times \{ w_1(Q_1, Q_2, \tau) F_{\pi^0}(Q_1^2, (Q_1 + Q_2)^2) F_{\pi^0}(Q_2^2, 0)$$
$$+ w_2(Q_1, Q_2, \tau) F_{\pi^0}(Q_1^2, Q_2^2) F_{\pi^0}((Q_1 + Q_2)^2, 0) \}$$

weight functions suppress large virtuality contributions

Input: single/double virtual transition form factors (TFF)

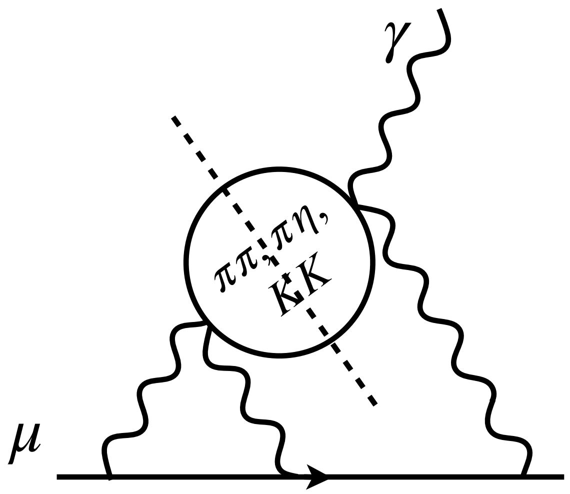


Gérardin et. al (2023)

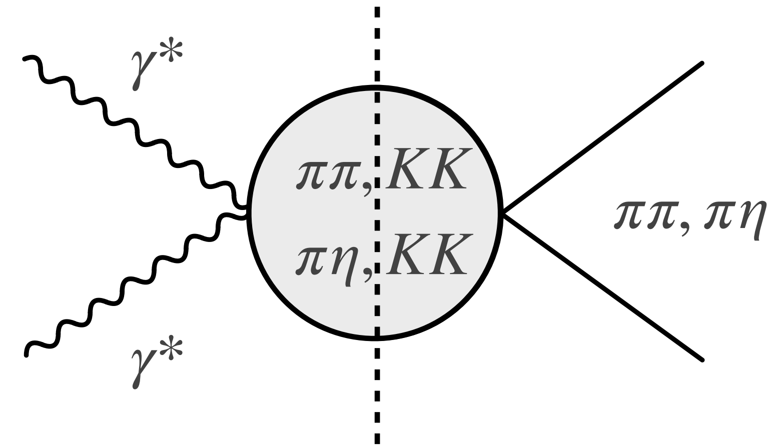
- π^0 TFF is well determined
- $\eta - \eta'$ mixing
- No dispersive analysis available
- Need improvements η, η' TFFs

$$a_\mu^{\pi^0, \eta, \eta' - pole} = 93.8(4.0) \times 10^{-11}$$

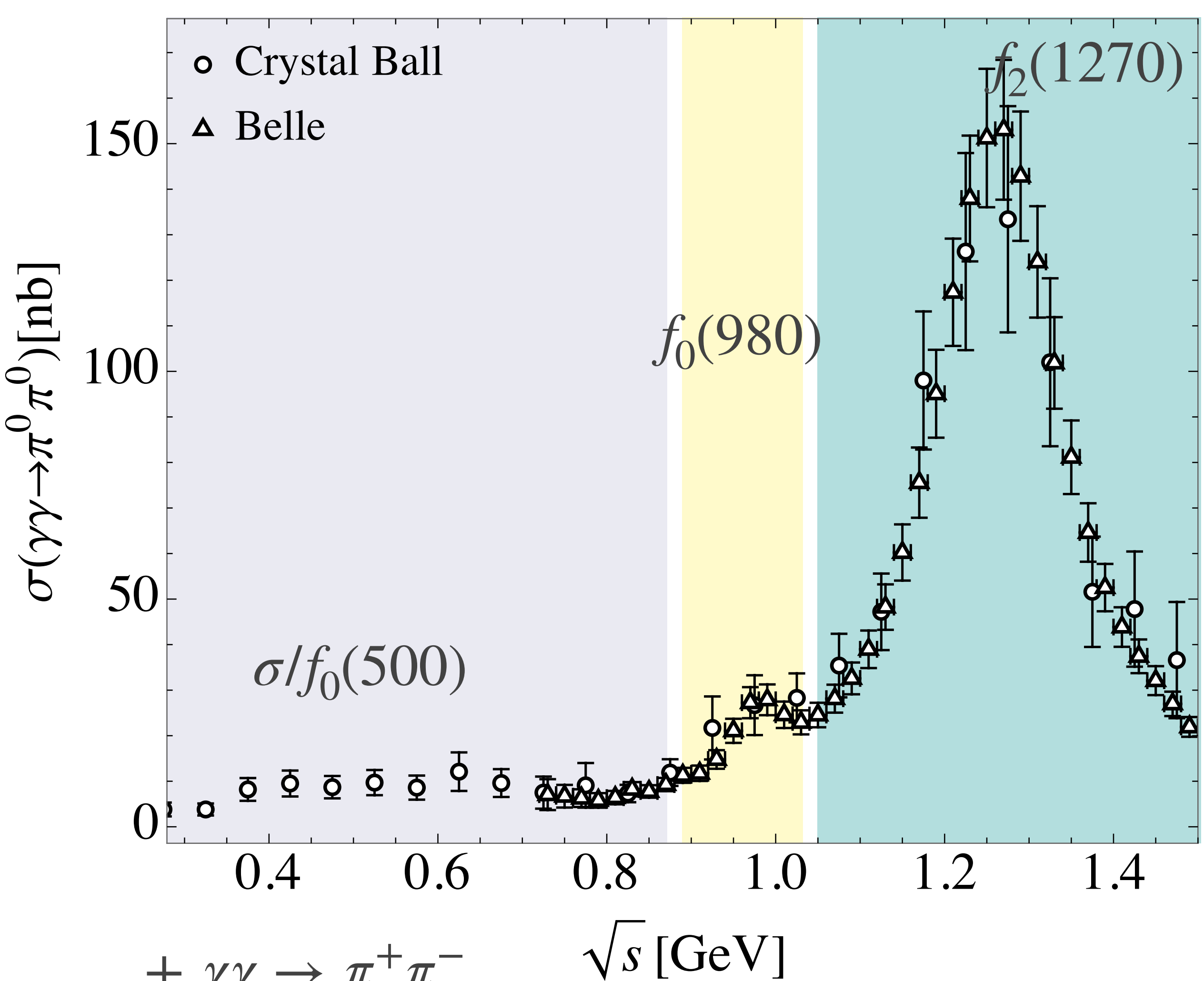
Two pseudoscalar contribution



Important ingredients:
$\gamma^*\gamma^* \to \pi\pi, \pi\eta, \ldots$
for spacelike γ^*

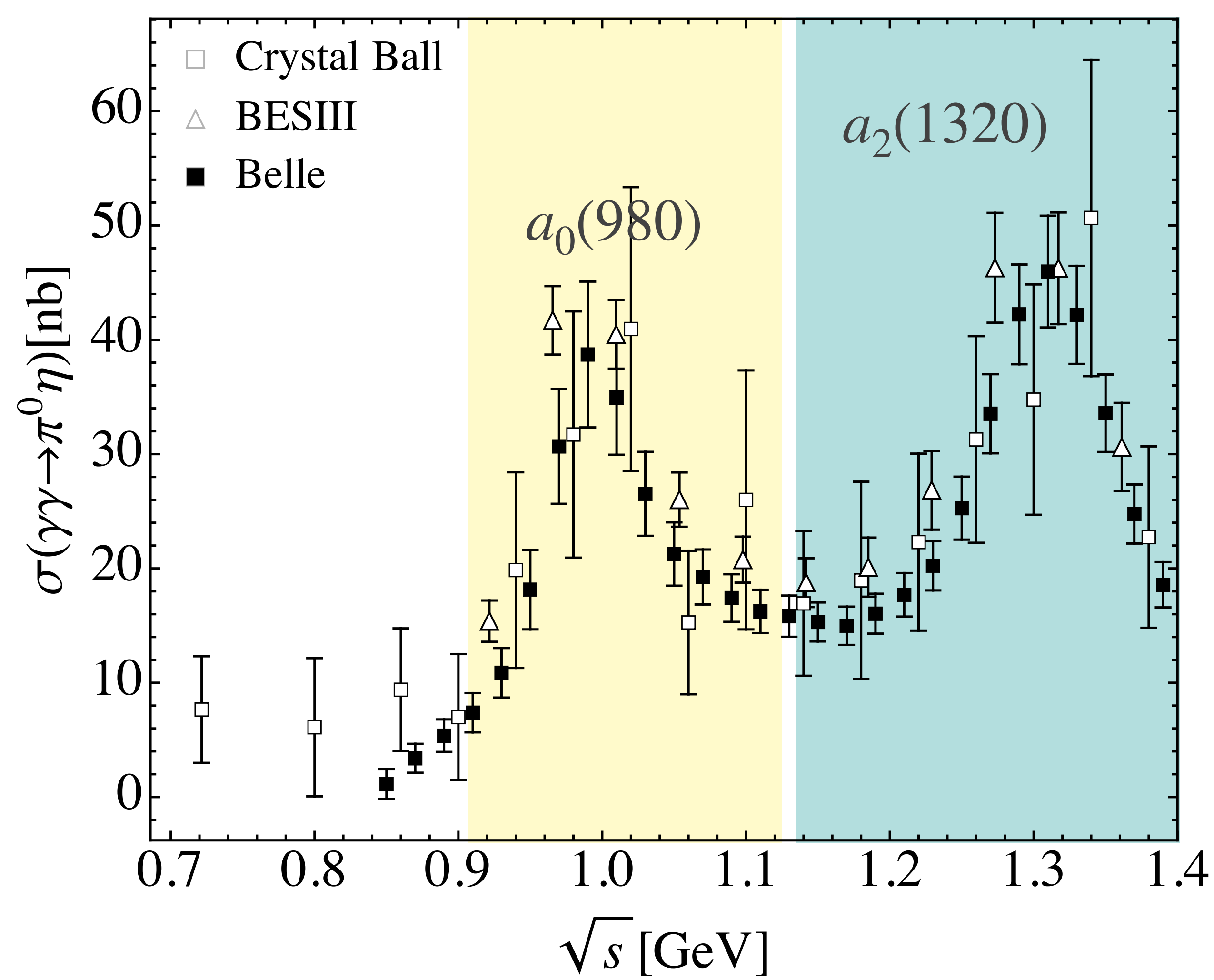


$\gamma\gamma \to \pi^0\pi^0$

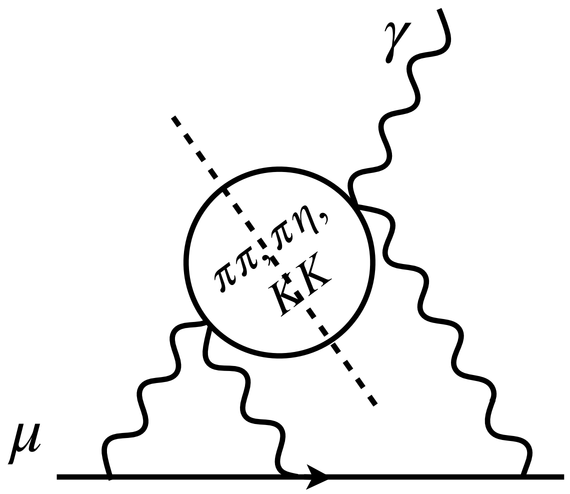


$+\ \gamma\gamma \to \pi^+\pi^-$

$\gamma\gamma \to \pi^0\eta$



Two pseudoscalar contribution



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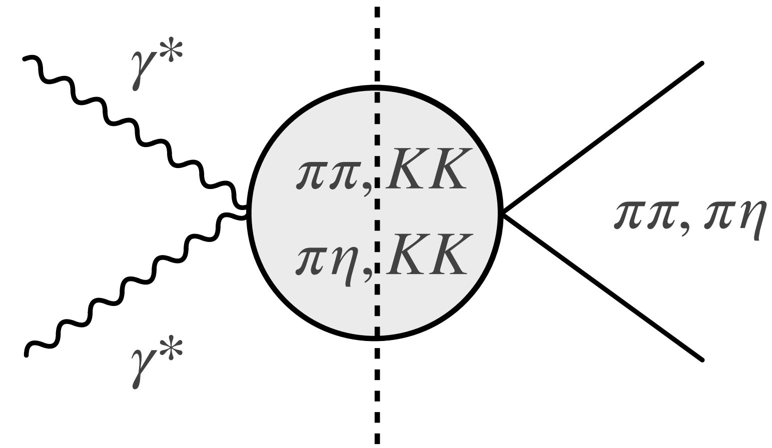


$$a_\mu^{HLbL} = \frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \sqrt{1-\tau^2} Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \bar{\Pi}_i(Q_1, Q_2, \tau)$$

$\bar{\Pi}_i$ for the rescattering contribution in the S-wave

Colangelo et. al (2017)

$$\bar{\Pi}_i^{J=0} \sim \frac{1}{\pi} \int_{s_{th}}^\infty ds' \frac{1}{\lambda_{12}(s')(s'-q_3^2)^2} \left(f(s') \mathrm{Im}\bar{h}^{(0)}_{++,++}(s') - g(s') \mathrm{Im}\bar{h}^{(0)}_{00,++}(s') \right)$$

+crossed

helicity amplitudes
$\gamma^*\gamma^* \to \gamma^*\gamma^*$

$\gamma^*\gamma^* \to \pi\pi$
$\gamma^*\gamma^* \to \pi\eta$

$\gamma^*\gamma^* \to KK$

Unitarity $\mathrm{Im}\bar{h}^{(0)}_{\lambda_1\lambda_2,\lambda_3\lambda_4}(s) = \bar{h}^{(0)}_{\lambda_1\lambda_2}(s)\rho_{\pi\pi/\pi\eta}(s)\bar{h}^{(0)*}_{\lambda_3\lambda_4}(s) + \bar{k}^{(0)}_{\lambda_1\lambda_2}(s)\rho_{KK}(s)\bar{k}^{(0)*}_{\lambda_3\lambda_4}(s)$

phase-space factor

Dispersion relation

S-wave amplitudes free from kinematic constraints

$$\bar{h}^{(0)}_{i=1,2} = \frac{\bar{h}^{(0)}_{++} \mp Q_1 Q_2 \bar{h}^{(0)}_{00}}{s - s^{(\mp)}_{\text{kin}}}, \quad s^{(\pm)}_{\text{kin}} \equiv -(Q_1 \pm Q_2)^2$$

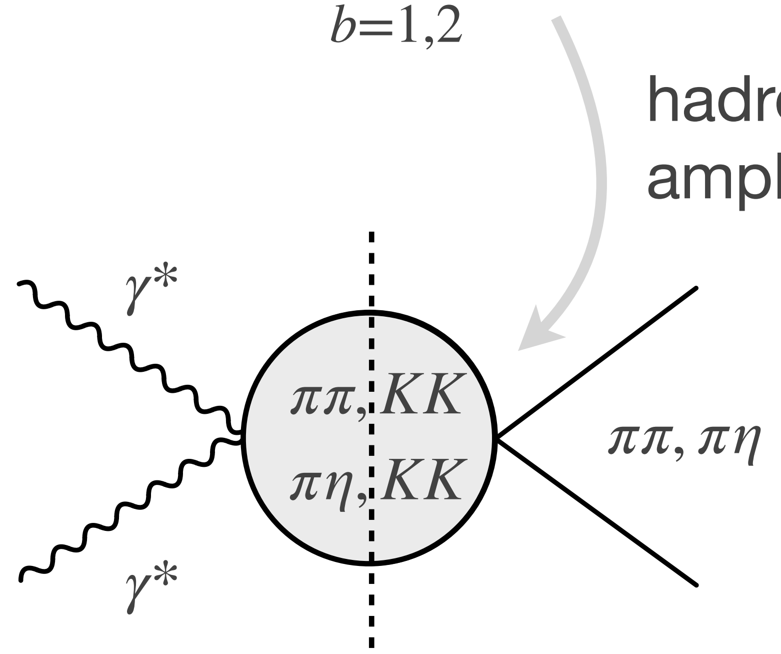
Can write a **dispersion relation**

$$\bar{h}^J_i(s) = \int_{-\infty}^{s_L} \frac{ds'}{\pi} \frac{\text{Disc}\, \bar{h}^{(J)}_i(s')}{s' - s} + \int_{s_{th}}^{\infty} \frac{ds'}{\pi} \frac{\text{Disc}\, \bar{h}^{(J)}_i(s')}{s' - s}$$

Coupled-channel unitarity

$$\text{Disc}\, h^{(J)}_{i,a}(s) = \sum_{b=1,2} t^{(J)*}_{ab}(s)\, \rho_b(s)\, h^{(J)}_{i,b}(s)$$

hadronic scattering amplitude



Hadronic input

Unitarity relation for the hadronic amplitude

$$\text{Disc } t_{ab}(s) = \sum_c t_{ac}(s)\rho_c(s)t^*_{cb}(s)$$

Once-subtracted dispersion relation

$$t_{ab}(s) = U_{ab}(s) + \frac{s}{\pi}\sum_c \int_{s_{thr}}^{\infty} \frac{ds'}{s'} \frac{\text{Disc } t_{ab}(s')}{s'-s}$$

Can be solved by means of **N/D ansatz**

contributions from the right-hand cuts

$$t_{ab}(s) = \sum_c D^{-1}_{ac}(s)\, N_{cb}(s)$$

contributions from the left-hand cuts

Chew, Mandelstam (1960)
Luming (1964)
Johnson, Warnock (1981)

Conformal mapping expansion for hadronic lhc

Gasparyan, Lutz (2010)

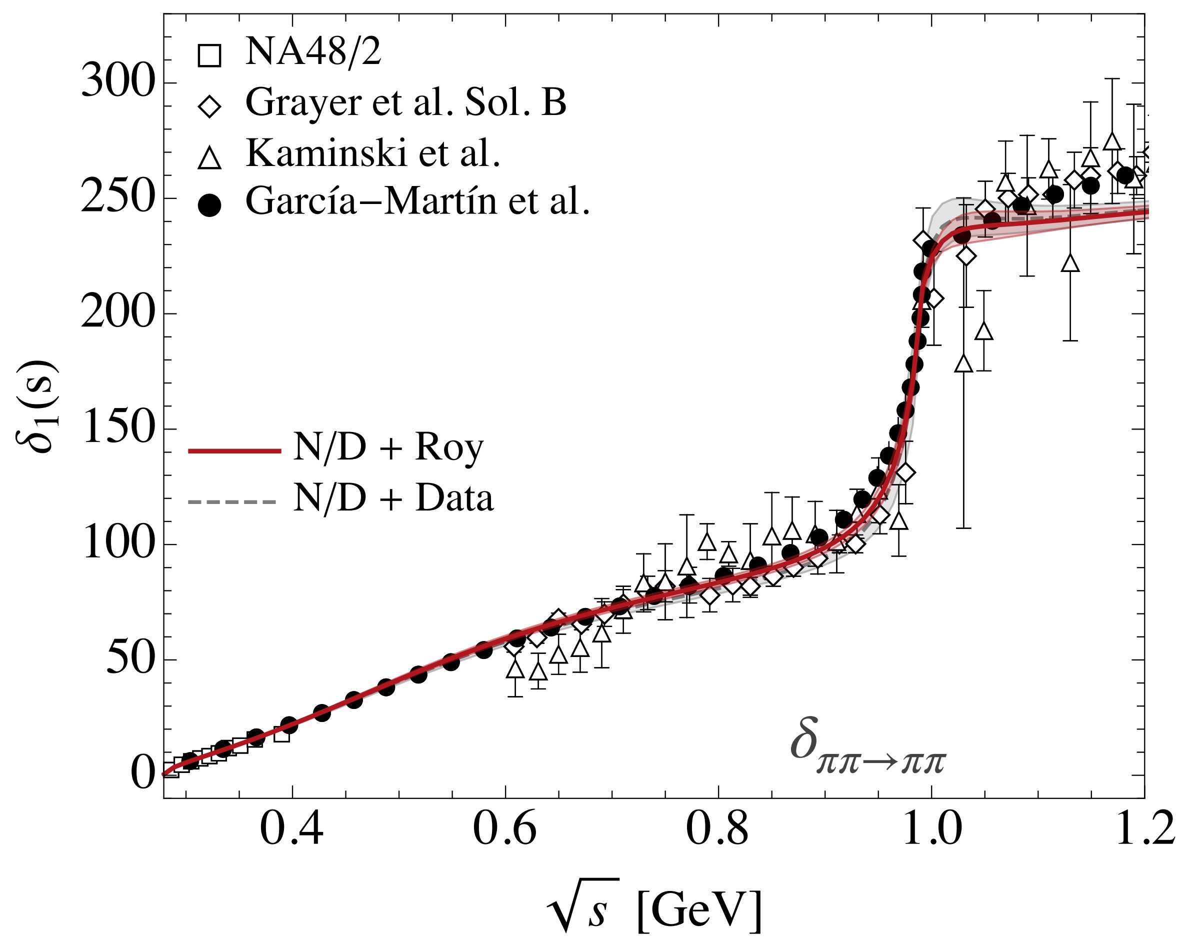
$$U(s) = \sum_{n=0}^{\infty} C_n(\xi(s))^n$$

Hadronic input

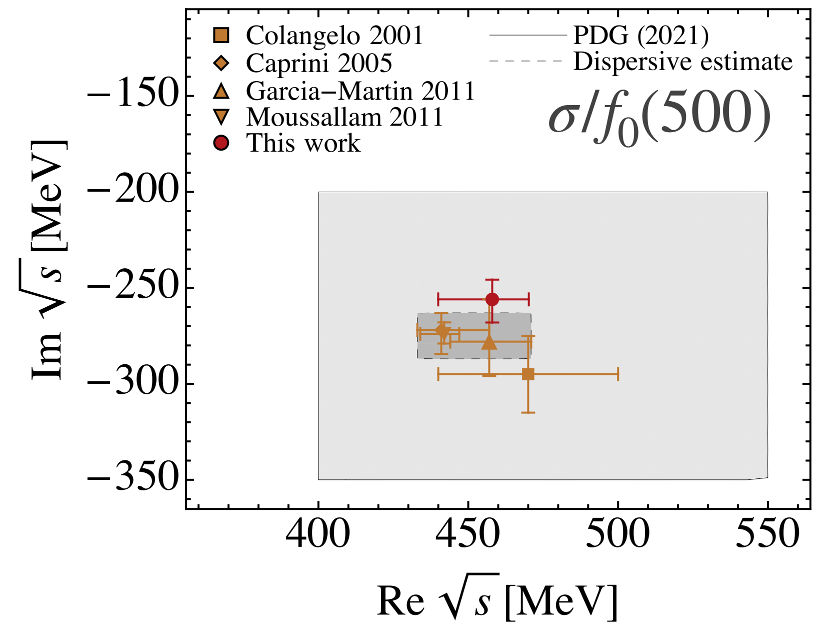
$$U(s) = \sum_{n=0}^{\infty} C_n (\xi(s))^n$$

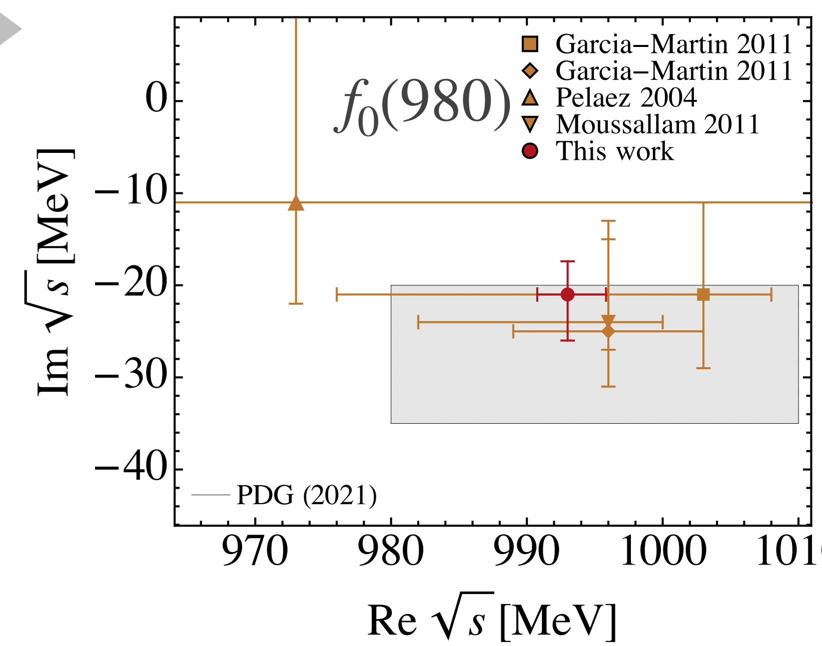
coefficients fitted to the data

$\{\pi\pi, KK\}$: fit to the hadronic data/Roy analysis



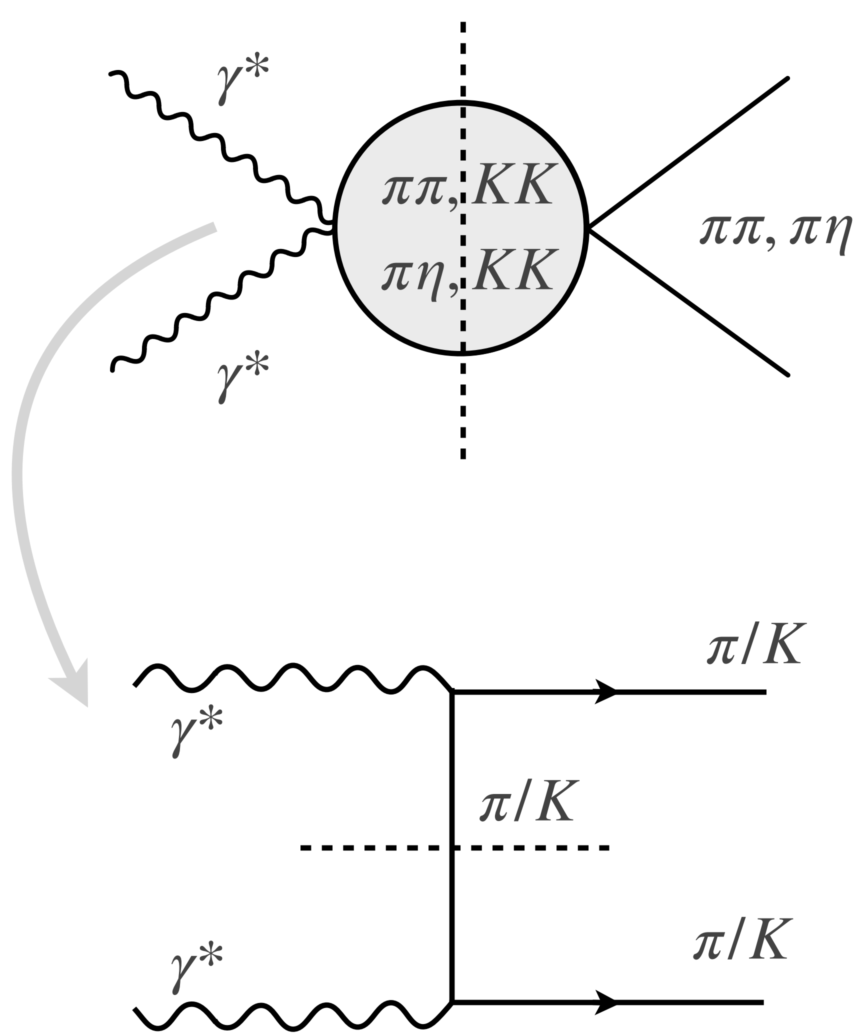
Danilkin, D., Vanderhaeghen (2020)





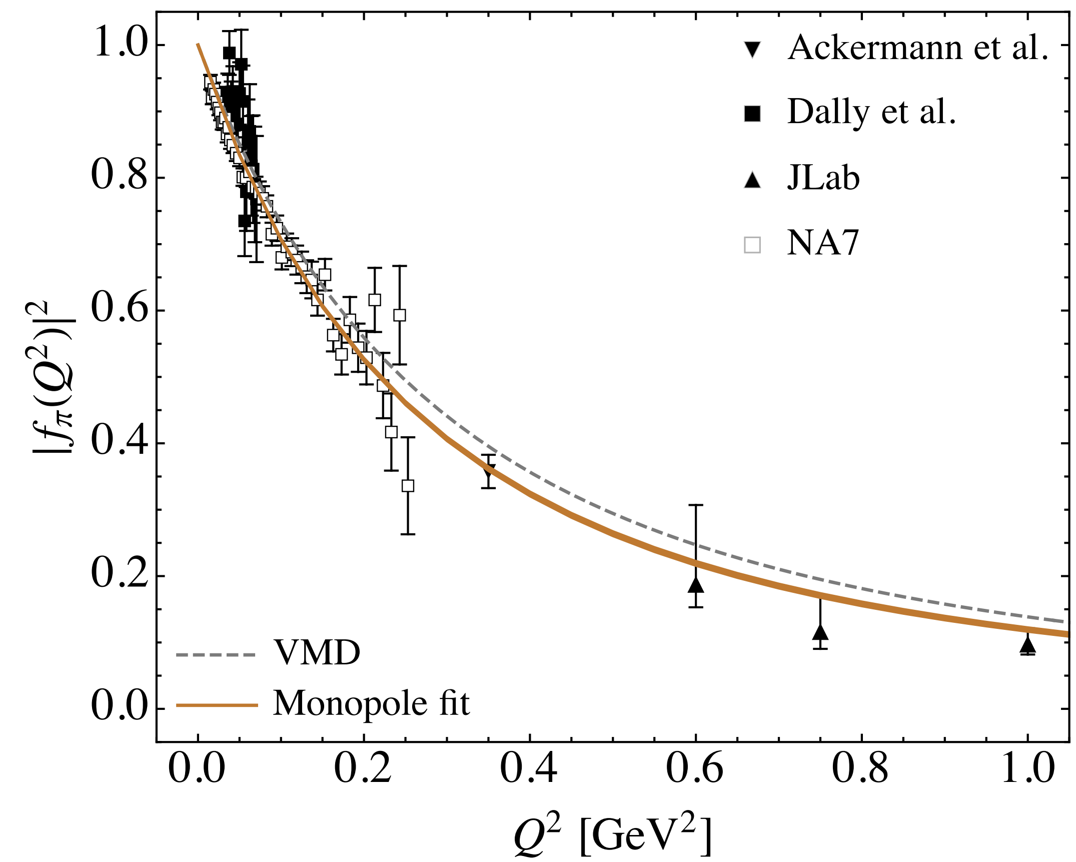
$\{\pi\eta, KK\}$: **no hadronic data available**, coefficients C_n fitted to the cross-section data on $\gamma\gamma \to \pi^0\eta$, $\gamma\gamma \to K_s K_s$

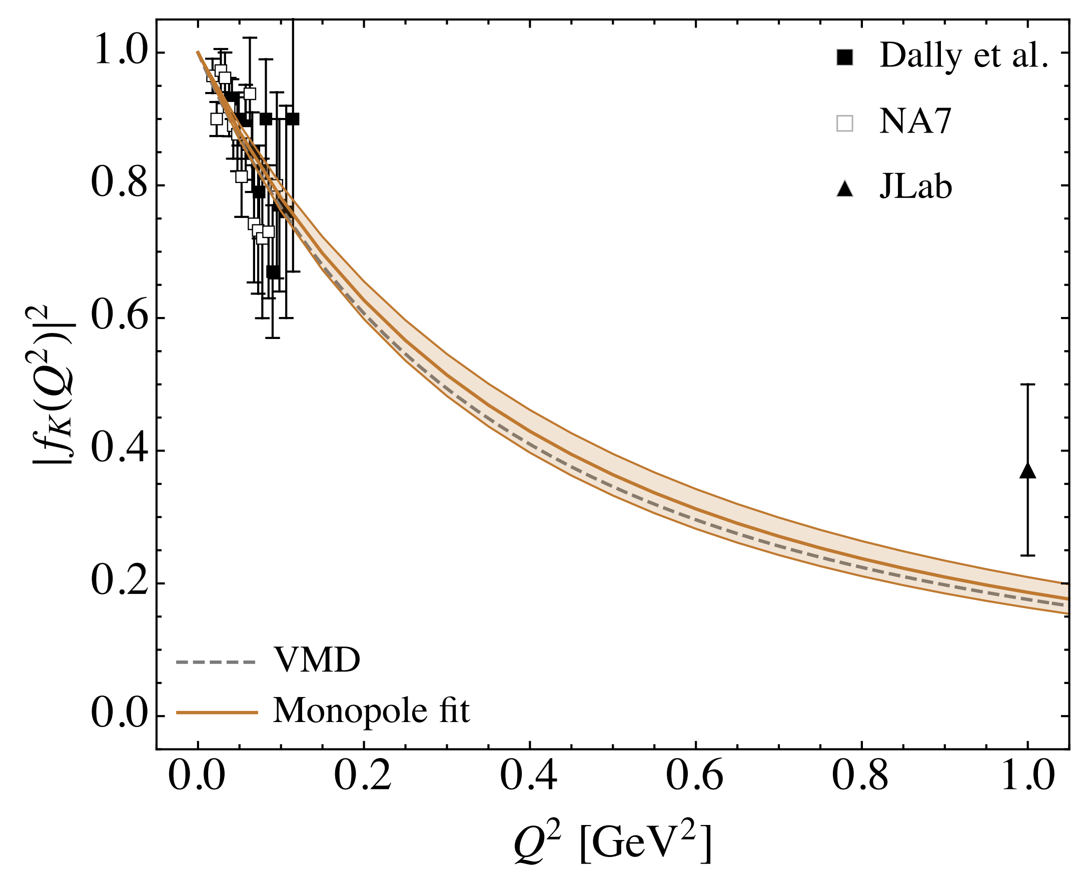
$\gamma\gamma$ left-hand cuts



For the S-wave use **Born lhc** only

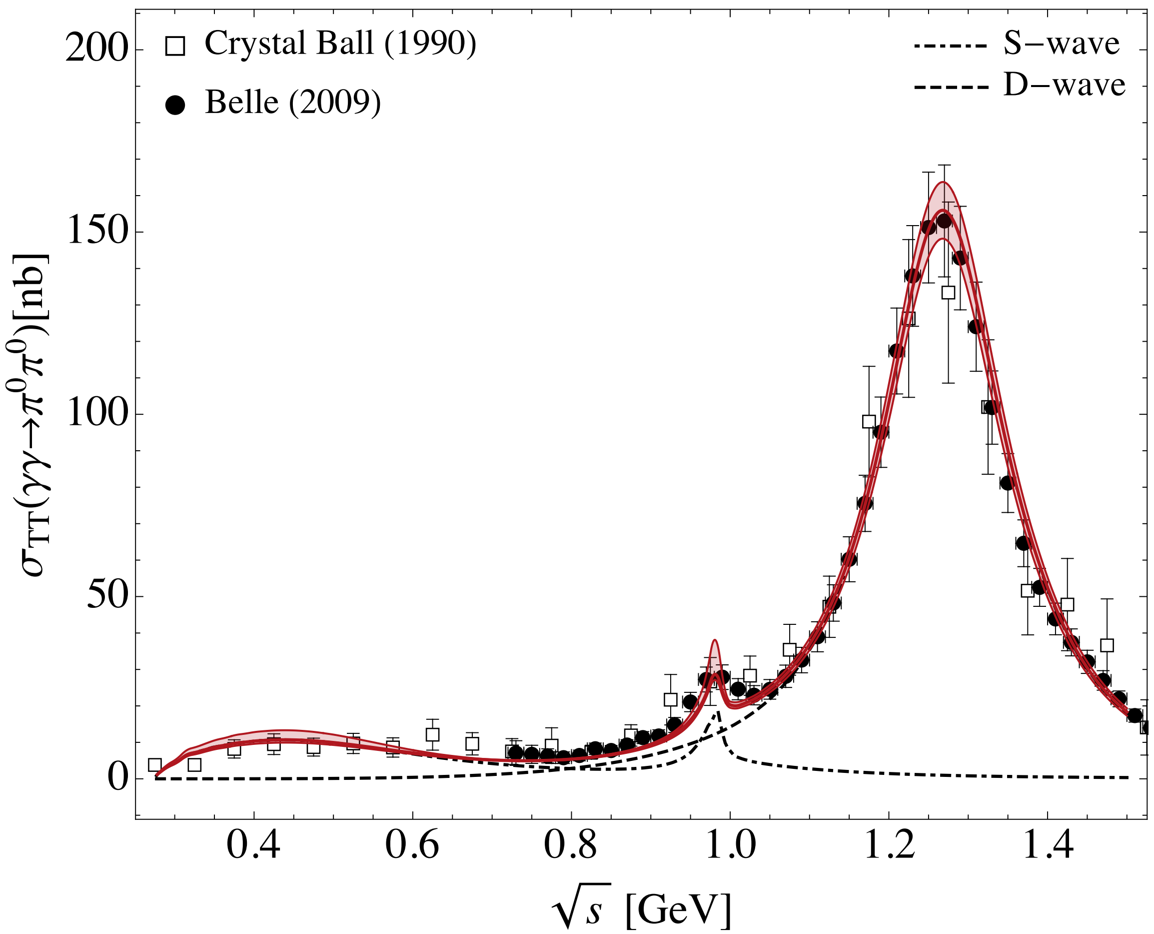
The generalization to the case of off-shell photons require knowledge of electromagnetic pion/kaon form factors



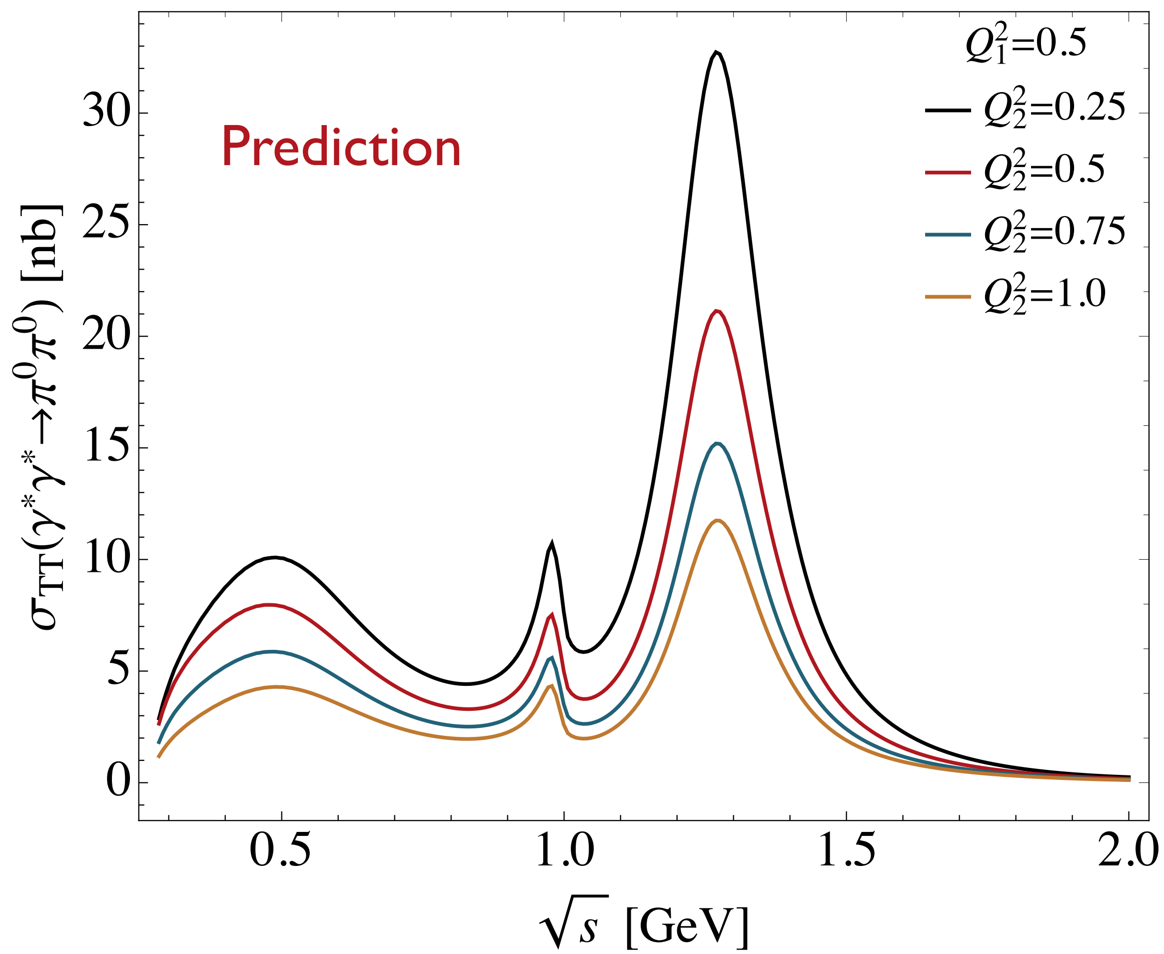


Results for $f_0(500) + f_0(980)$

$\gamma\gamma \to \pi^0\pi^0$



$\gamma^*\gamma^* \to \pi^0\pi^0$



Prediction for $\gamma\gamma^* \to \pi\pi$ needs to be validated with upcoming BESIII data

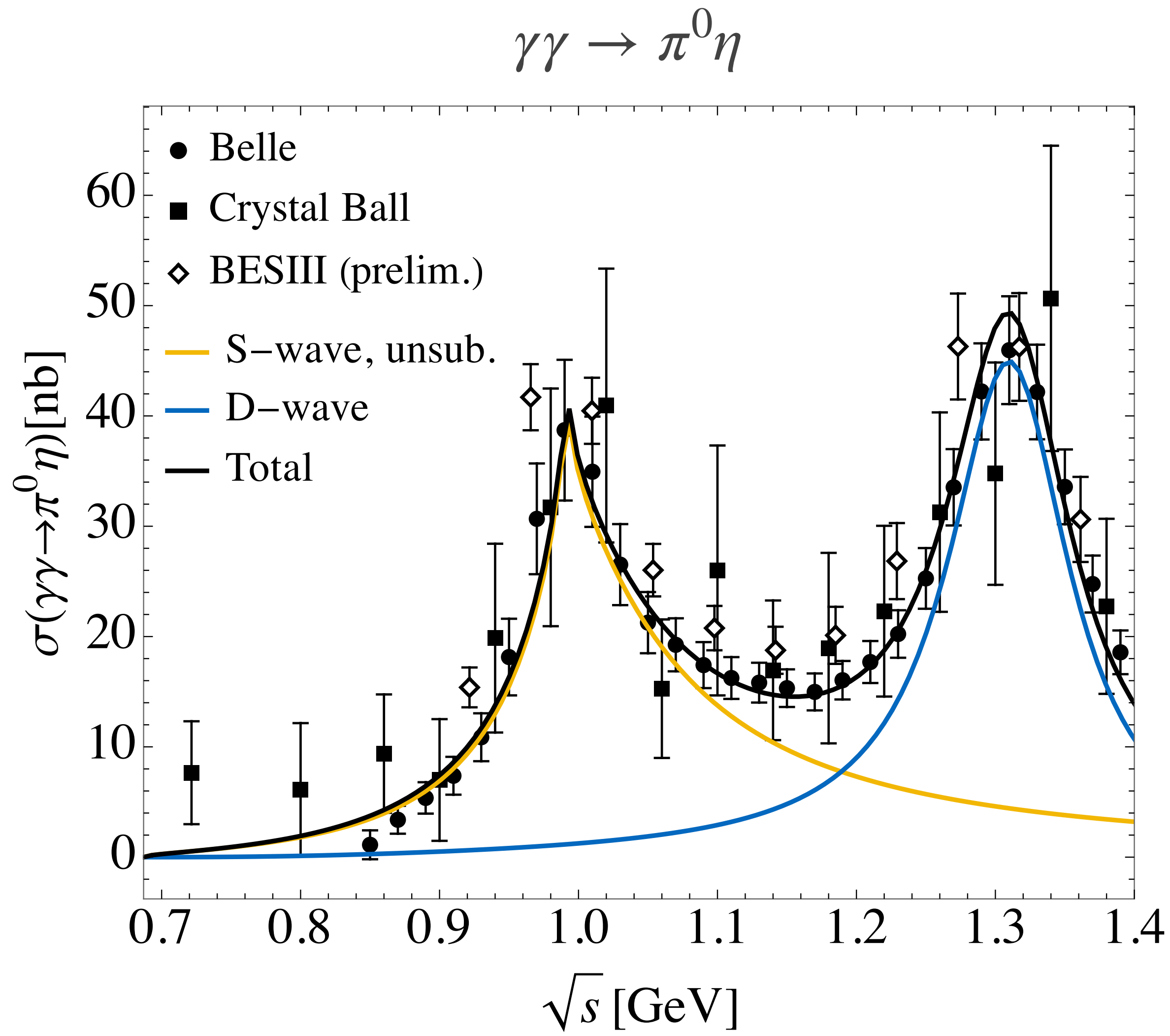
For $I = 0$, the contributions from $f_0(500) + f_0(980)$:

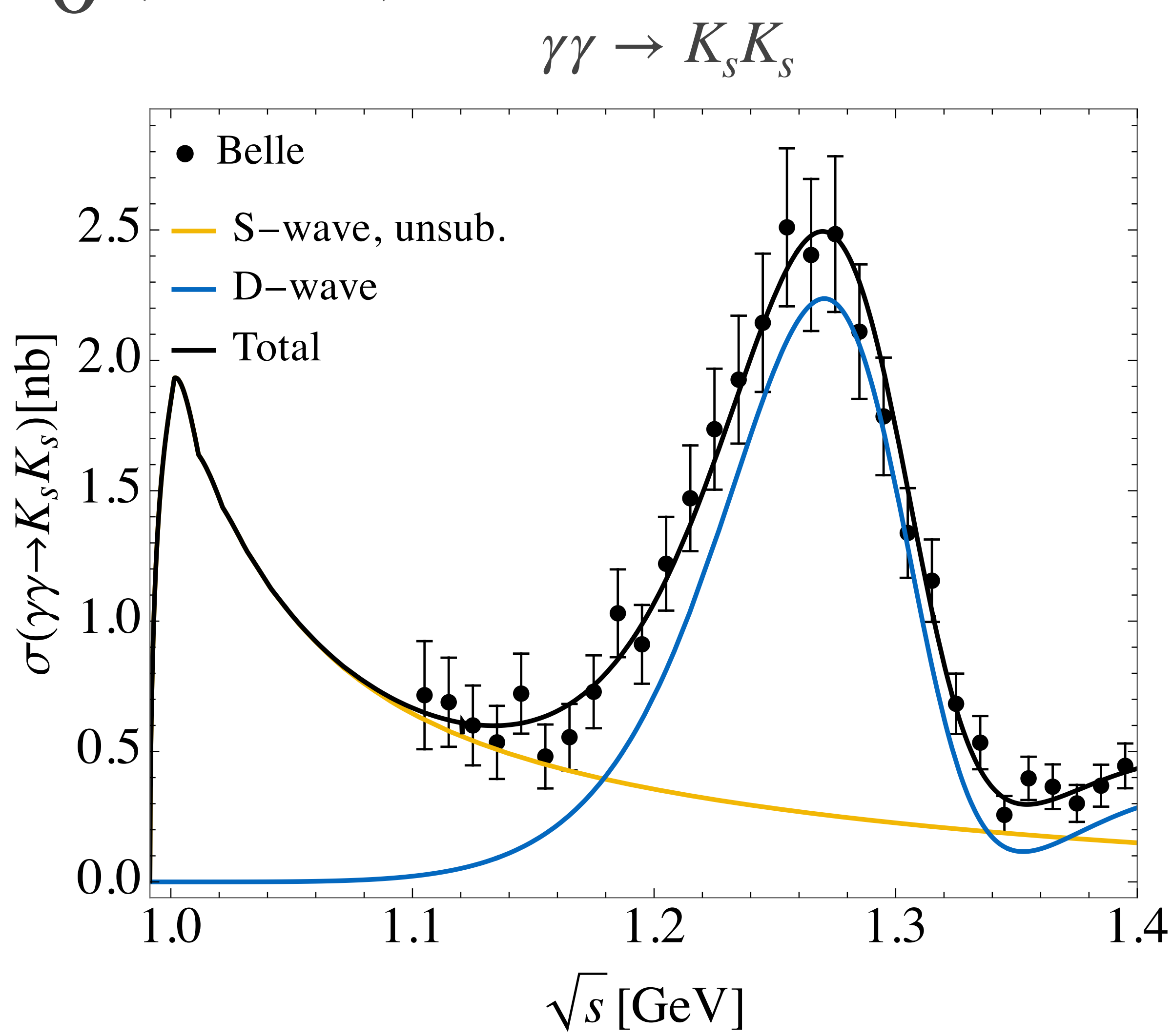
$a_\mu^{HLbL}[S\text{-wave}, I = 0]_{resc.} = -9.8(1) \times 10^{-11}$

$a_\mu^{HLbL}[f_0(980)]_{resc.} = -0.2(1) \times 10^{-11}$

Colangelo et al. (2014-2017)
Danilkin, Hofferichter, Stoffer (2021)

Preliminary results for $a_0(980)$



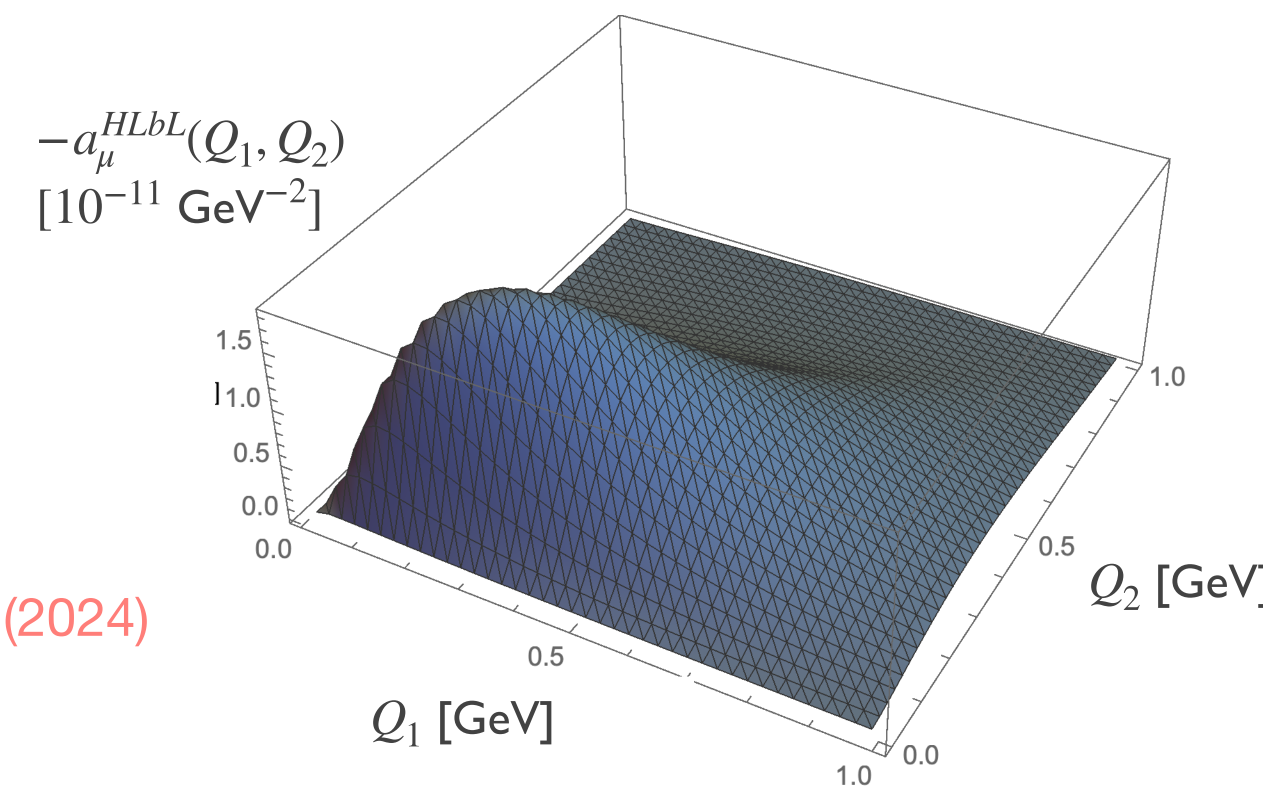


Need BESIII data for $\gamma\gamma \to K^+K^-$ as an additional input

For $I = 1$, the contribution from $a_0(980)$

$$a_\mu^{HLbL}[a_0(980)]_{resc.} = -0.46(2) \times 10^{-11}$$

D., Danilkin, Vanderhaeghen (2024)



Current status of HLbL

$$a_\mu^{HLbL} = 92(19) \times 10^{-11}$$

pseudoscalar poles	$93.8(4.0)$
pion box	$-15.9(2)$
S-wave $\pi\pi$ rescattering	$-8(1)$
kaon box	$-0.5(1)$

well determined contributions

Scalars+tensors $\gtrsim 1$ GeV	$\sim -1(3)$
axial vectors	$\sim 6(6)$
short distance	$\sim 15(10)$
heavy quarks	$\sim 3(1)$

major source of uncertainty

Lattice: $a_\mu^{HLbL} = 109.6(15.9) \times 10^{-11}$ Chao et al. (2021, 2022)

$= 124.7(14.9) \times 10^{-11}$ Blum et al. (2023)

Summary and outlook

- Experiment
  - New Fermilab results are expected very soon
  - More experiments on the way

- Lattice
  - Convincing case for HVP
  - Becomes competitive for HLbL

- Theory
  - Uncertainties are dominated by hadronic contributions
  - Need to understand tensions in HVP
  - Need to reduce the uncertainty in HLbL

Thank you for attention!