RUHR UNIVERSITÄT BOCHUM

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Coupled Channel Analysis from the Users Point of View

Modern Techniques in Hadron Spectroscopy

RUB, 15.-27. July 2024

Data Analysis in Hadron Spectroscopy

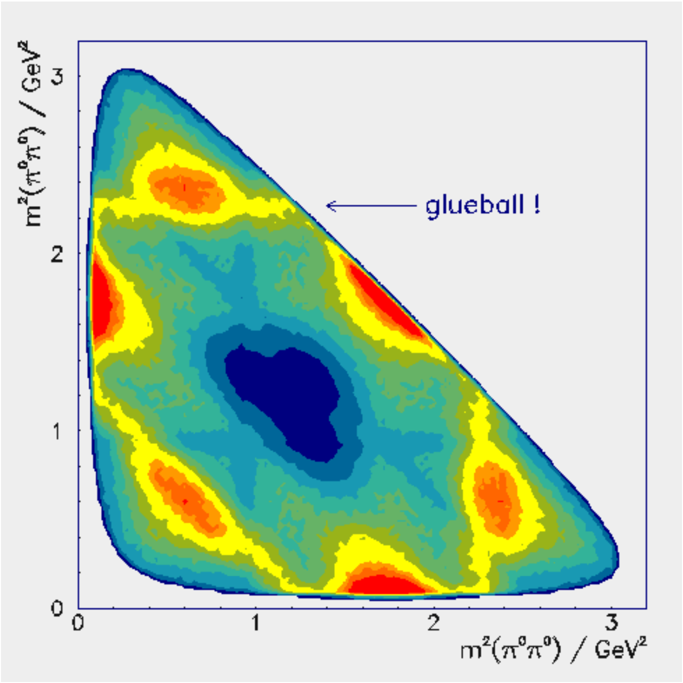
- Find (all) existing resonances and measure precisely their properties
  - unambiguous determination of their quantum numbers
  - accurate measurement of the pole positions (masses and widths)
  - determination of the coupling strengths of the production and decay

- Partial Wave Analysis (PWA) is needed to determine $I^G(J^{PC})$-quantum numbers and to find even new resonances
- Fits in the complete n-dimensional phase-space are in general necessary
  - consideration of the complete decay chain for the initial up to the final state

Quantum Mechanics

Interference effects play an essential role

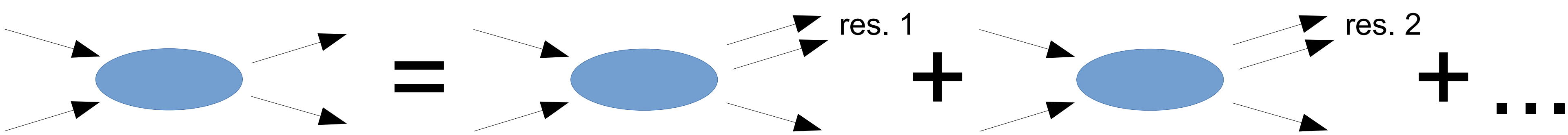
Example: $\bar{p}p \rightarrow \pi^0\pi^0\pi^0$ at rest (Crystal Barrel @ LEAR)



- Reaction chain can be fully described by 2 dimensions → Dalitzplot
- Quantum mechanical effects become clearly visible through interference patterns and are needed to be taken into account in the PWA
- Description of the probability density function (PDF) by quantum mechanical waves is needed

General PWA Strategy

- Extraction of the complete transition amplitude $\mathcal{M}(k_1, \dots, k_N)$
- $|\mathcal{M}(k_1, \dots, k_N)|^2$ accessible
  - (in)coherent sum of the amplitudes of all waves with all individual intermediate resonances



$$\mathcal{M}(k_1, \dots, k_N) \quad = \quad C_1 A_{res1}(k_1, \dots, k_N) \quad + \quad C_2 A_{res2}(k_1, \dots, k_N) \quad + \quad \dots$$

PWA: fitting experimental data to obtain weights (complex amplitudes) C_1, C_2, ...

Unbinned Maximum Likelihood Method

- Dalitzplots can be provided by binned data and thus fitted by a χ^2 - minimization
- Phasespace of the most channels of interest contains more than 2 dimensions and are more complex
- Event-based fits by making use of the unbinned likelihood method

n events — probability at τ_i — efficiency at τ_i

- General likelihood function:

$$L = n! \prod_{i=1}^{n} \frac{\omega_{(}\tau_i, \overrightarrow{x})\epsilon(\tau_i)}{\int \omega_{(}\tau, \overrightarrow{x})\epsilon(\tau)d\tau}$$

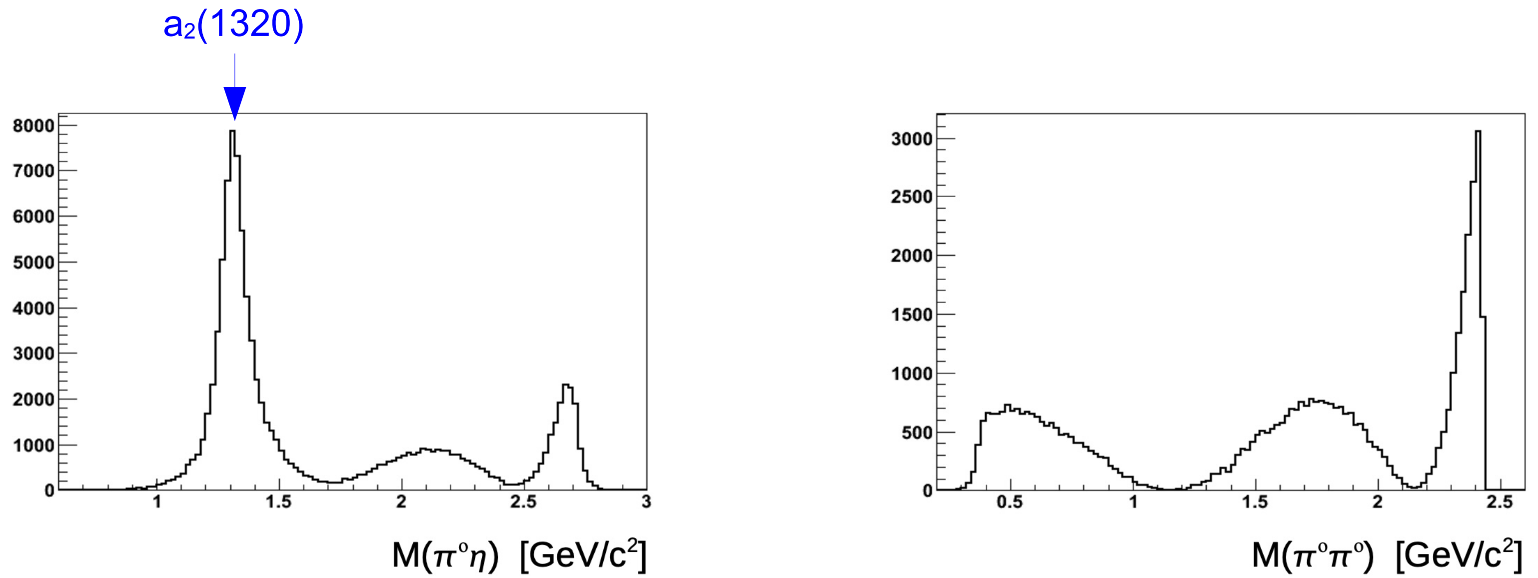
phasespace integral

- Logarithm of likelihood function for PWA
  - nominator: sum over data
  - denominator: sum over reconstructed phasespace distributed events

Why Fits in the n-dimensional Phasespace?

Peaks in the invariant mass spectrum are not necessarily originating from resonances

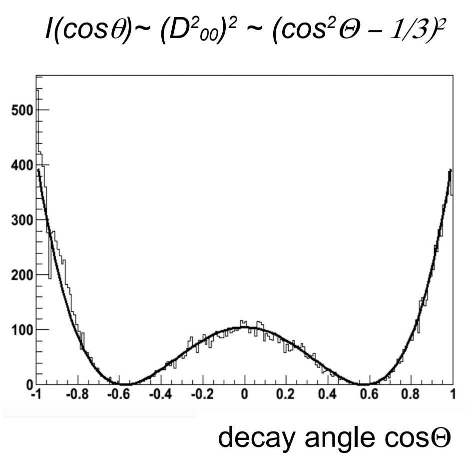
Example: $\eta_c \rightarrow a_2(1320)\,\pi^0 \rightarrow (\pi^0\,\eta)\,\pi^0$

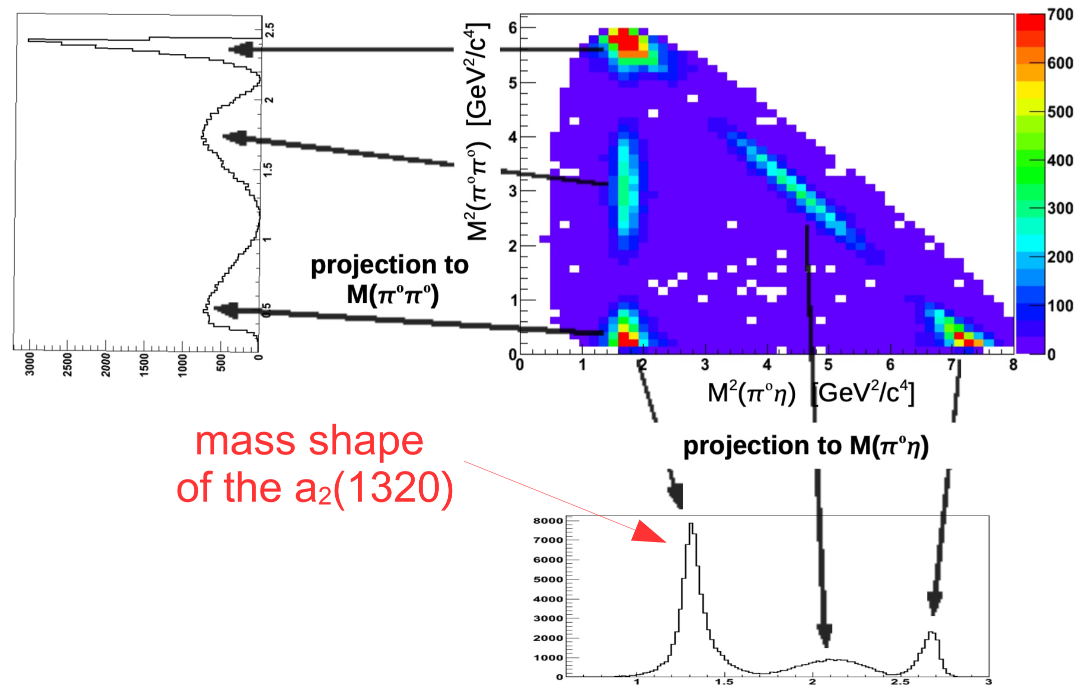


- 2 additional peaks visible in $M(\pi^0\eta)$
- 3 additional peaks visible in $M(\pi^0\pi^0)$

Why Fits in the n-dimensional Phasespace?

- Dalitzplot
  - flat for phasespace distributed events: $|\mathcal{M}(k_1, \ldots, k_N)|^2 = \text{const}$
  - structures (bands) at $m^2_{ab} = m_X$ for resonances X decaying to $X \rightarrow a\, b$
  - density distribution along the band are related to the decay angular distribution of the resonance
- Decay angular distribution of the $a_2(1320)$ in the helicity frame: $D^2_{00} \sim (\cos^2\theta - 1/3)^2$
- Additional peaks are originating from decay angular distribution of the $a_2(1320)$
- Interpretation of the structures in higher dimensional phasespace is much more difficult





Spin Formalisms

- Different spin formalisms on the market
  - needed for the determination of the quantum numbers
  - mainly differ in the choice of the spin quantization
  - all of them have their pros and cons
- Helicity formalism
  - decay characteristics based on Wigner-D rotation matrix $D^{J_X}_{\lambda_{J_X}\,\lambda_a-\lambda_b}(\phi,\theta,-\phi)$
  - easy to use for sequential decays
  - pure helicity amplitudes does not contain information about the angular moments of the decay processes
  - descriptions could be complicated for final state particles with spin (J>0) due to extra-rotations
- Spin-orbit (LS) formalism
  - decay characteristics based on spherical harmonic functions $Y_L^m(\theta,\phi)$
  - easy access to the L-dependent barrier factors
  - simple transformation between helicity and LS-amplitudes

$$F_{\lambda_a\,\lambda_b} = \sum_{LS} \alpha_{LS} \sqrt{\frac{2J_X+1}{2L+1}} < J_a, \lambda_a, J_b, -\lambda_b \big| S, \lambda_a-\lambda_b >< L, 0, S, \lambda_a-\lambda_b \big| J_X, \lambda_a-\lambda_b >$$

Spin Formalisms

- Non-relativistic Tensor formalism (Zemach formalism)
  - only 3-vectors are taken into account
- Relativistic Tensor formalism (Rarita-Schwinger formalism)
  - Lorenz-invariant description using 4-vectors, polarization vectors, orbital momentum tensors, spin-projection tensors, ...
  - choice of any reference frame possible
  - momentum dependent barrier factor (p^L-dependence) is automatically taken into account
  - elegant for final state particles with spin (J>0)
  - difficult and very computationally intensive for large L and S
- Multipole amplitudes
  - suitable choice for radiative decays
  - electric and magnetic multipoles give access to the transition form factors
  - simple transformation between helicity and multipole amplitudes

Dynamics

- Breit-Wigner functions widely used
  - good approximation for isolated resonances appearing in a single channel
  - extracted resonance parameters are not unique and depend on the production and decay process
- More sophisticated descriptions needed for
  - resonances decaying into multiple channels
  - several resonances with the same quantum numbers appearing in the same channel
  - resonances located at thresholds → distortion of the line shape

Approaches with an adequate consideration
of unitarity and analyticity needed
(K-matrix, N/D-method, Two-potential decomposition)

K-Matrix

Aitchison: „Nucl Phys A189 (1972) 417

S.U. Chung, E.Klempt „A Primer on K-matrix Formalism“, BNL Preprint (1995)

- A two body scattering process can be fully described by the S-matrix

$$S = I + 2i\sqrt{\rho}\,T\,\sqrt{\rho}$$

- T-matrix can be expressed by K-matrix: $T = (I - i\,K\,\rho)^{-1}\,K$

- Elements of the K-matrix:

$$K_{ij} = \sum_{\alpha} \frac{g_{\alpha_i} g_{\alpha_j}}{m_{\alpha}^2 - s} + \sum_k c_{kij} s^k$$

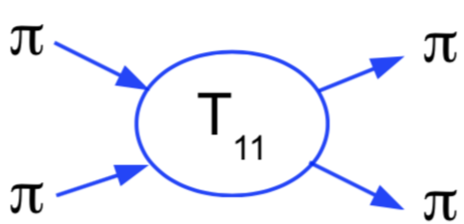
bare coupling to channel i an j of resonance α

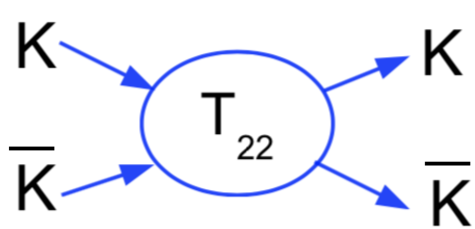
K-matrix element for channel i (initial) and j (final)

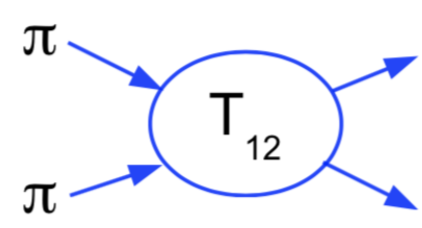
bare mass of resonance α

energy dependent background term

- Example: channel 1: ππ, channel 2: K $\overline{\text{K}}$







K $(I - i\,K\,\rho)^{-1}$

K-Matrix with P-Vector Approach

Aitchison: „The K-Matrix formalism for overlapping resonances“, Nucl Phys A189 (1972) 417

- Generalization of the K-matrix formalism to the case of production of resonances in more complex reactions

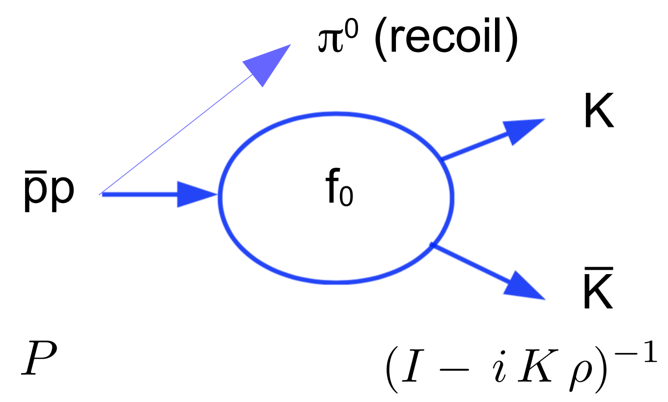
- Dynamical function for P-vector approach: $F = (I - i\,K\,\rho)^{-1}\,P$

bare coupling to the production of resonance α

with: $$P_i = \sum_\alpha \frac{\beta_\alpha g_{\alpha_i}}{m_\alpha^2 - s} + \sum_k c_{ki} s^k$$

same pole structure as for K-matrix

- Example: $\bar{p}p \to f_0\,\pi^0 \to (K\bar{K})\,\pi^0$



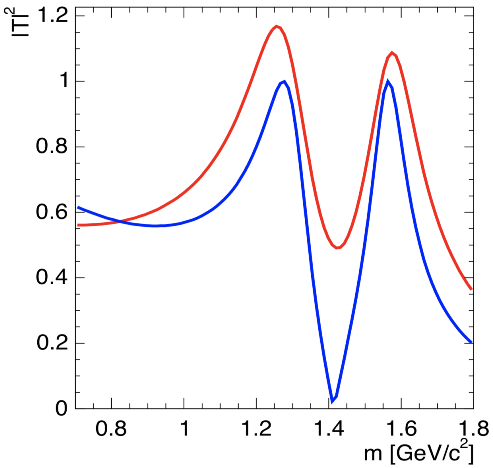
- Parameters in the fit (free or fixed): $g_{\alpha i}$, β_α, m_α, c_{kij}, c_{ki}

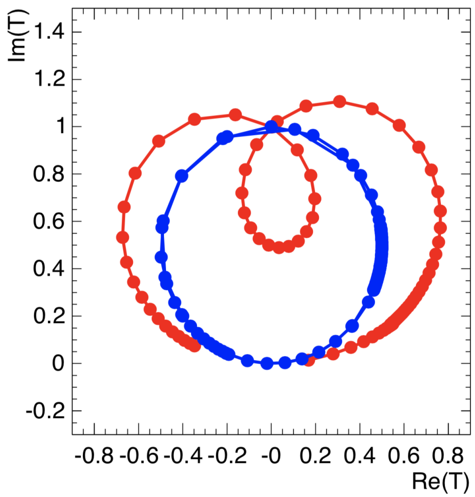
Scattering Process: K-Matrix vs. Breit Wigner

K. Peters
Int.J.Mod.Phys. A21 (2006) 5618-5624
hep-ph/0412069

2 nearby Poles 1 Channel

- m_a= 1.27 GeV/c^2
- m_b= 1.56 GeV/c^2

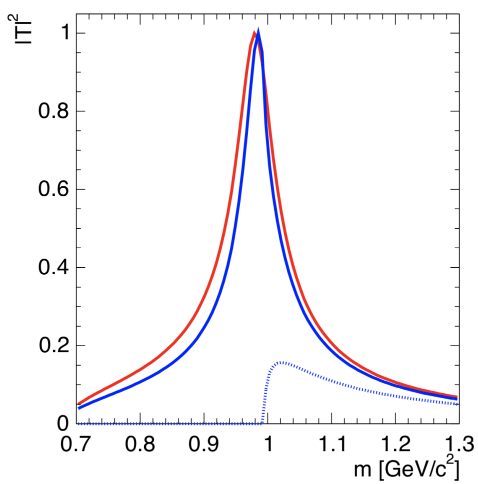


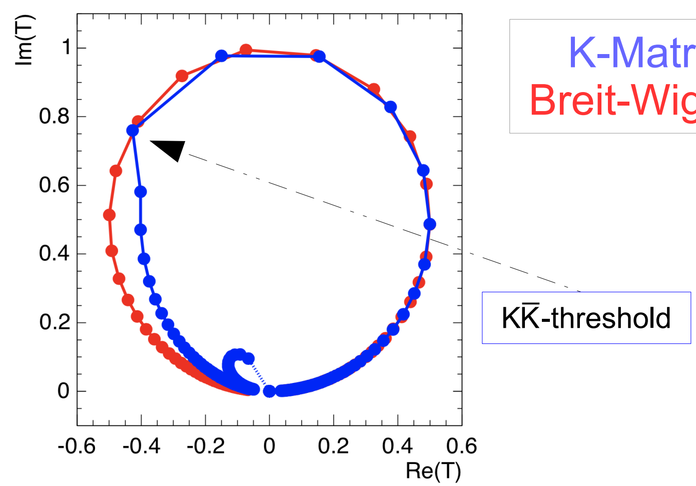


K-Matrix
2 Breit-Wigner

1 Pole 2 Channels

- $a_0(980)$ with coupling to $\eta\,\pi$ and $K\,\overline{K}$

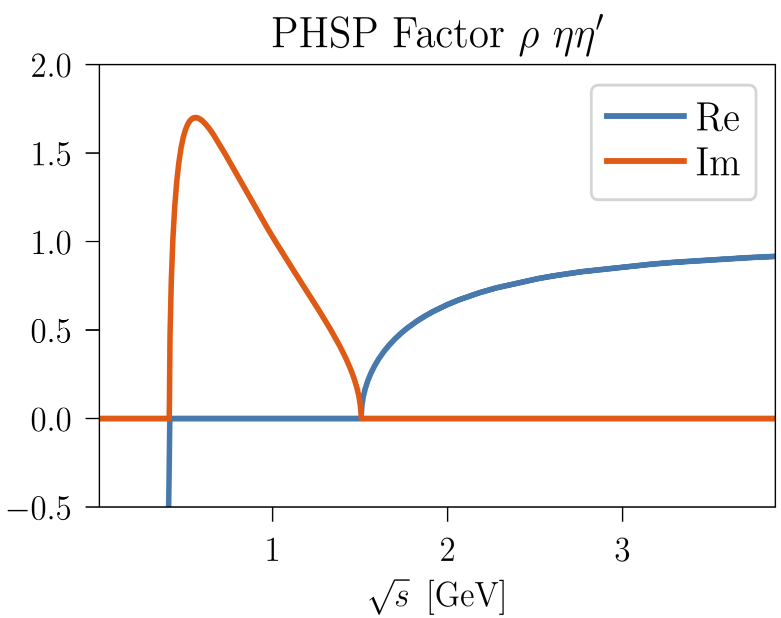


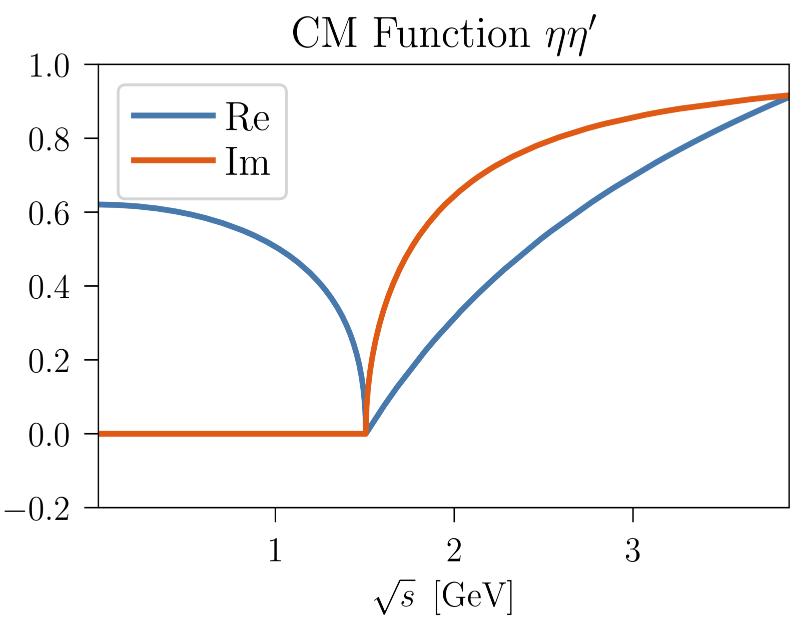


K-Matrix
Breit-Wigner

Analyticity

- Below thresholds proper analytical continuations are needed (e.g. $K\bar{K}$ channel in the region between $\pi\pi$ and $K\bar{K}$ threshold)
- K-matrix with standard phase space factors: $\rho = \sqrt{\left[1 - \left(\frac{m_a + m_b}{m}\right)^2\right] \cdot \left[1 - \left(\frac{m_a - m_b}{m}\right)^2\right]}$
  - violates constraints from analyticity: unphysical cuts for unequal masses
- Proper description with Chew-Mandelstam function from
  - above threshold: $\rho(s)$ = Im(CM(s))
  - $T = (I - i\,K\,\rho)^{-1}\,K$ replaced by $T = (I + K\,CM(s))^{-1}\,K$





Coupled Channel Analysis

- Single channel fits
  - access to only one production mode and one decay channel
  - unitarity cannot be adequately taken into account due to lack of other relevant channels
  - K-matrix difficult to use, as only the coupling strengths to the relevant single channel can be determined
  - Breit-Wigner can often be only used
  - outcome of model- (mass-)independent fits can provide valuable input for coupled channel analyses

- Advantages of coupled channel fits
  - usage of common and unique description of the dynamics possible
  - better description of threshold effects
  - better fulfillment of the conservation of unitarity
  - more constraints due to common amplitudes

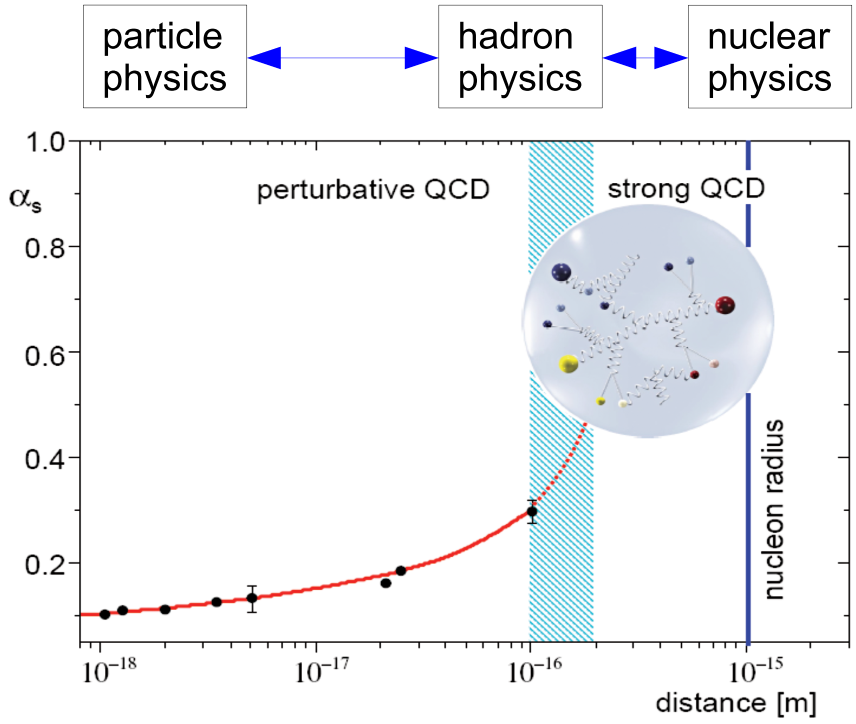
Choice of suitable Channels

- Channels with small number of final state particles
  - less complex due to small dimensions of the phasespace
  - reflections better under control
- (All) decay channels that have significant coupling to the resonances
  - guaranties an adequate consideration of unitarity
  - access to all relevant g-factors
  - access to final state interaction that might occur
- $\pi\pi$- (or $K\pi$-) scattering data
  - process only characterized by elasticity and phase motion
  - good and easy access to the resonances
  - very helpful for the normalization of the g-factors with regard to the unitarity

Examples in the Filed of Light Meson Spectroscopy

- Light mesons are bound states consisting of u-, d- and s-quarks
- Cover the non-perturbative QCD regime
- Description very challenging
  - lattice QCD
  - phenomenological models
- Observation and measurements of the resonance properties very challenging
  - many overlapping resonances with same quantum numbers
  - resonances decay in different channels
  - distinction between conventional $q\bar{q}$-mesons and exotics difficult

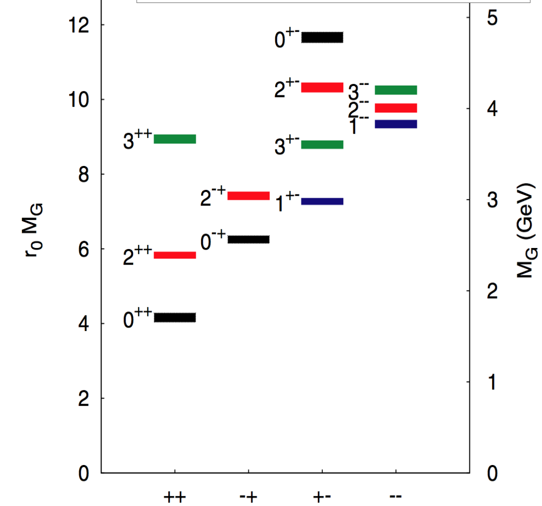
energy dependence of α_s



Research Topic: Glueballs and Hybrids

- A doubtless evidence for exotics are the observation of resonances with spin-exotic quantum numbers which are forbidden for $q\bar{q}$-mesons

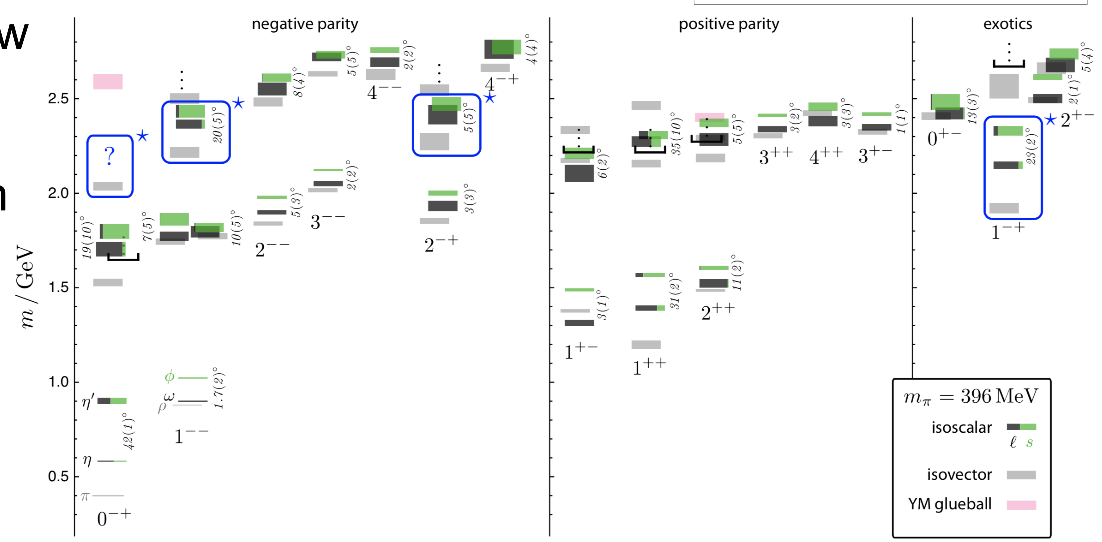
Phys.Rev. D73 (2006) 014516



- LQCD: lightest glueballs with spin-exotic quantum numbers $J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}$ above 4 GeV/c^2

- Glueballs in the light meson mass range only with non exotic quantum numbers $J^{PC} = 0^{++}, 0^{-+}, 2^{++}$ predicted

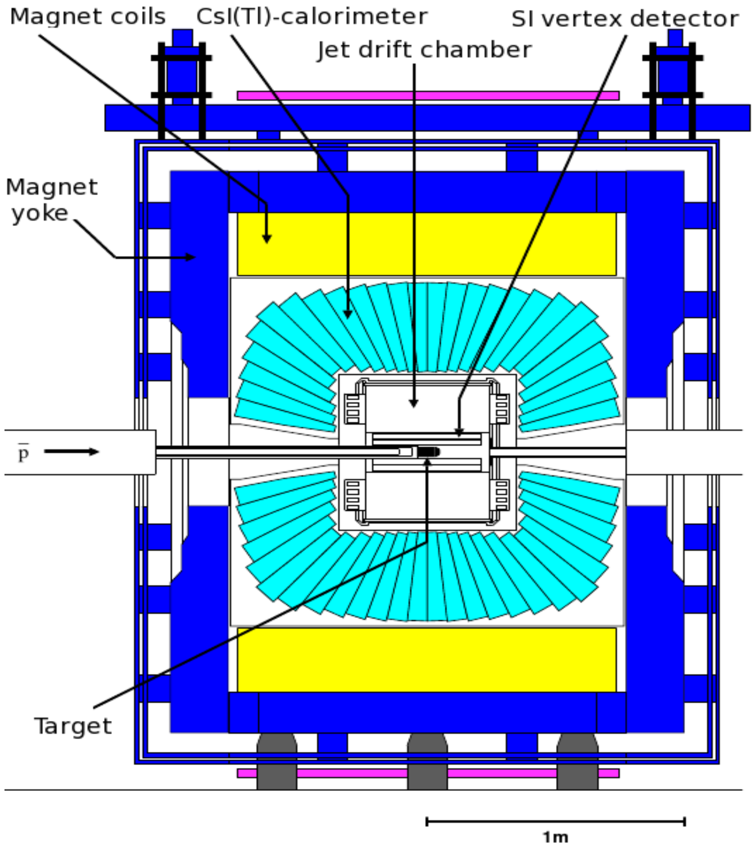
Phys.Rev. D84 (2011) 074023



- Lightest hybrid state expected just below 2 GeV/c^2 with exotic quantum numbers $I^G(J^{PC}) = 1^-(1^{-+})$
  - 2 π_1 candidates below 2 GeV listed in the PDG
  - $\pi_1(1400)$: only observed in the decay to $\pi\eta$
  - $\pi_1(1600)$: observed in several decay channels

PWA with p̄p Data from Crystal Barrel at LEAR

- Fixed target experiment at CERN
- In operation between 1989 and 1996
- p̄p annihilation at rest and in flight
  - highest beam momentum 1.94 GeV/c
- Physics program
  - spectroscopy of light mesons and search for exotic states



Eur. Phys.J. C (2020) 80, 453

Crystal Barrel Collaboration

Coupled channel analysis of $\bar{p}p \rightarrow \pi^0\pi^0\eta, \pi^0\eta\eta$ and $K^+K^-\pi^0$ at 900 MeV/c and of $\pi\pi$-scattering data

$\bar{p}p \to K^+K^-\pi^0$, $\pi^0\pi^0\eta$, $\pi^0\eta\eta$ @ 900 MeV/c

Best Fit Result achieved for

- K-matrix description for
  - f_0 with 5 poles and 5 channels
  - f_2 with 4 poles and 4 channels
  - ρ with 2 poles and 3 channels
  - a_0 and a_2 with 2 poles and 2 channels, each
  - $\pi_1^0 \to \pi^0\eta$ in $\pi^0\pi^0\eta$ with 1 pole and 2 channels
  - $(K\pi)_S$-wave: fixed parameterization from FOCUS-experiment

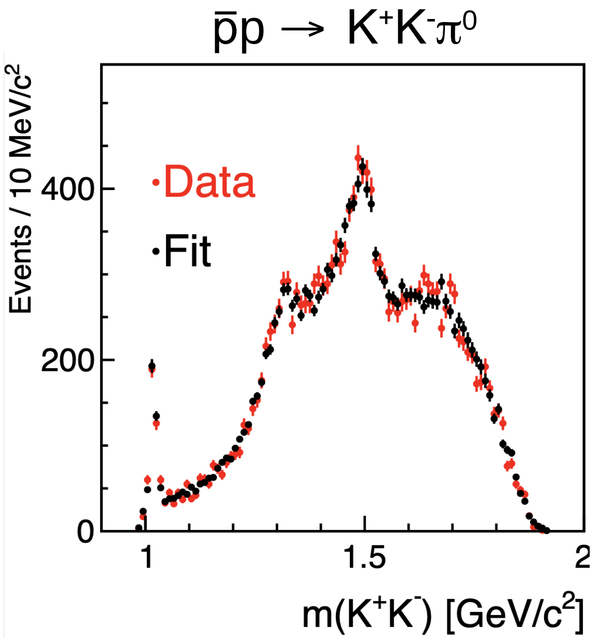
all pole positions and coupling strengths are free parameters

- Breit-Wigner description for
  - $\Phi(1020) \to K^+ K^-$
  - $K^{*\pm}(892) \to K^\pm \pi^0$
- Scattering data are taken into account for $\pi\pi \to \pi\pi$ and $\pi\pi \to K\bar{K}$, $\eta\eta$, $\eta\eta'$

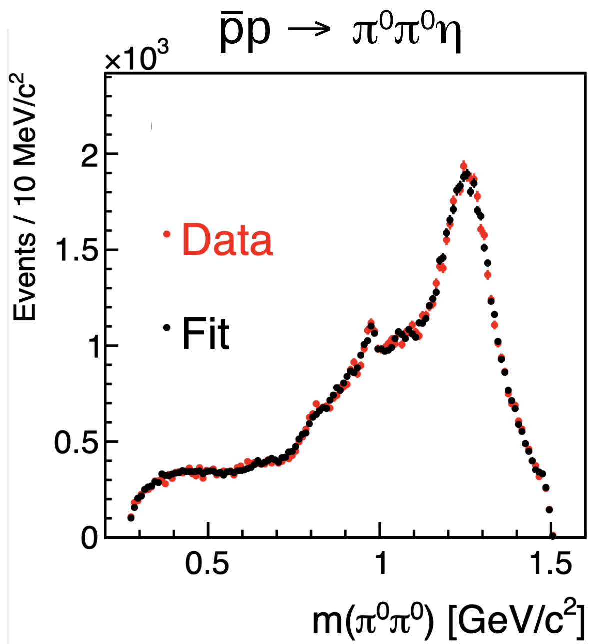
Phys. Rev. D83(2011) 074004
Nucl. Phys B64 (1973) 134-162
Nucl. Phys B100 (1975) 205-224
J. Phys G40 (2013) 043001
Nucl. Phys B64 (1973) 134-162
Nucl. Phys B269 (1986) 485
Nouvo Cimento A80 (1984) 363

$\bar{p}p \to K^+K^-\pi^0$, $\pi^0\pi^0\eta$, $\pi^0\eta\eta$ @ 900 MeV/c

Eur. Phys.J. C (2020) 80, 453

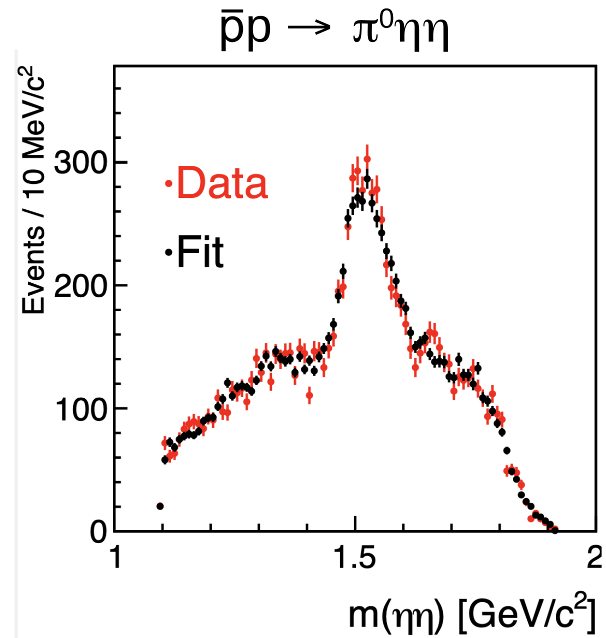


channel	contribution %
K*(892) K	45.0 ±1.3 ±11.0
$\rho\ \pi^0$	17.2 ±1.0 ±4.0
$f_2\ \pi^0$	17.1 ±0.7 ±10.0
$f_0\ \pi^0$	7.4 ±0.3 ±4.1
$(K\pi)_S$ K	6.1 ±0.4 ±4.9
$a_2\ \pi^0$	6.4 ±0.2 ±2.9
$\phi\ \pi^0$	2.5 ±0.3 ±0.3
$a_0\ \pi^0$	6.1 ±0.2 ±2.8
Σ	107.8 ±1.9 ±12.5



channel	contribution %
$f_2\ \eta$	52.3 ±0.8 ±5.0
$a_2\ \pi^0$	33.0 ±0.6 ±2.9
$f_0\ \eta$	10.7 ±0.4 ±1.8
$a_0\ \pi^0$	22.4 ±0.4 ±1.0
$\pi_1\ \pi^0$	16.7± 0.5± 3.0
Σ	135.0 ±1.2 ±8.7

spin-exotic 1^{-+} contribution



channel	contribution %
$f_0\ \pi^0$	23.7 ±1.2 ±2.3
$a_2\ \eta$	18.8 ±1.1 ±5.6
$a_0\ \eta$	28.6 ±1.1 ±7.5
$f_2\ \pi^0$	30.1 ±1.3 ±2.7
Σ	101.2 ±2.4 ±11.7

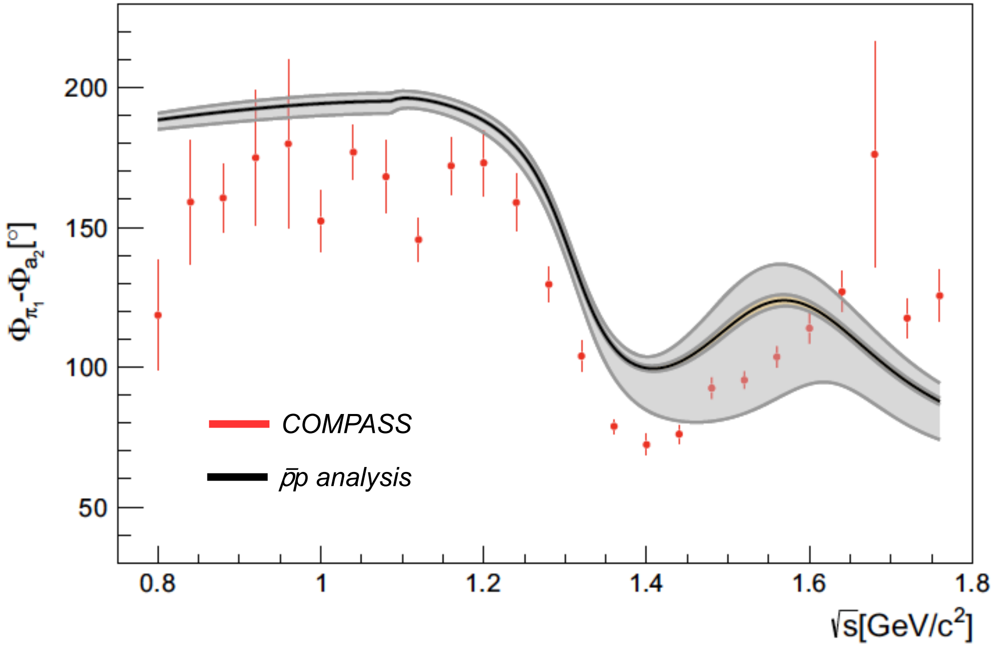
1^{-+} Wave in $\bar{p}p \to \pi^0\pi^0\eta$

- 1^{-+} wave seen in the decay $\pi^0\eta$
- K-matrix description with 1 pole and two channels $\pi\eta$ and $\pi\eta'$
  - no data for $\pi\eta'$ and only used for unitarity
- Phase difference between the π_1 and a_2 wave from $T_{\pi\eta\to\pi\eta}$ in good agreement with COMPASS measurement
- Obtained pole parameters consistent with $\pi_1(1400)$

Phys. Lett. B740 (2015) 303-311
Phys. Lett. B811 (2020) 135913 (erratum)

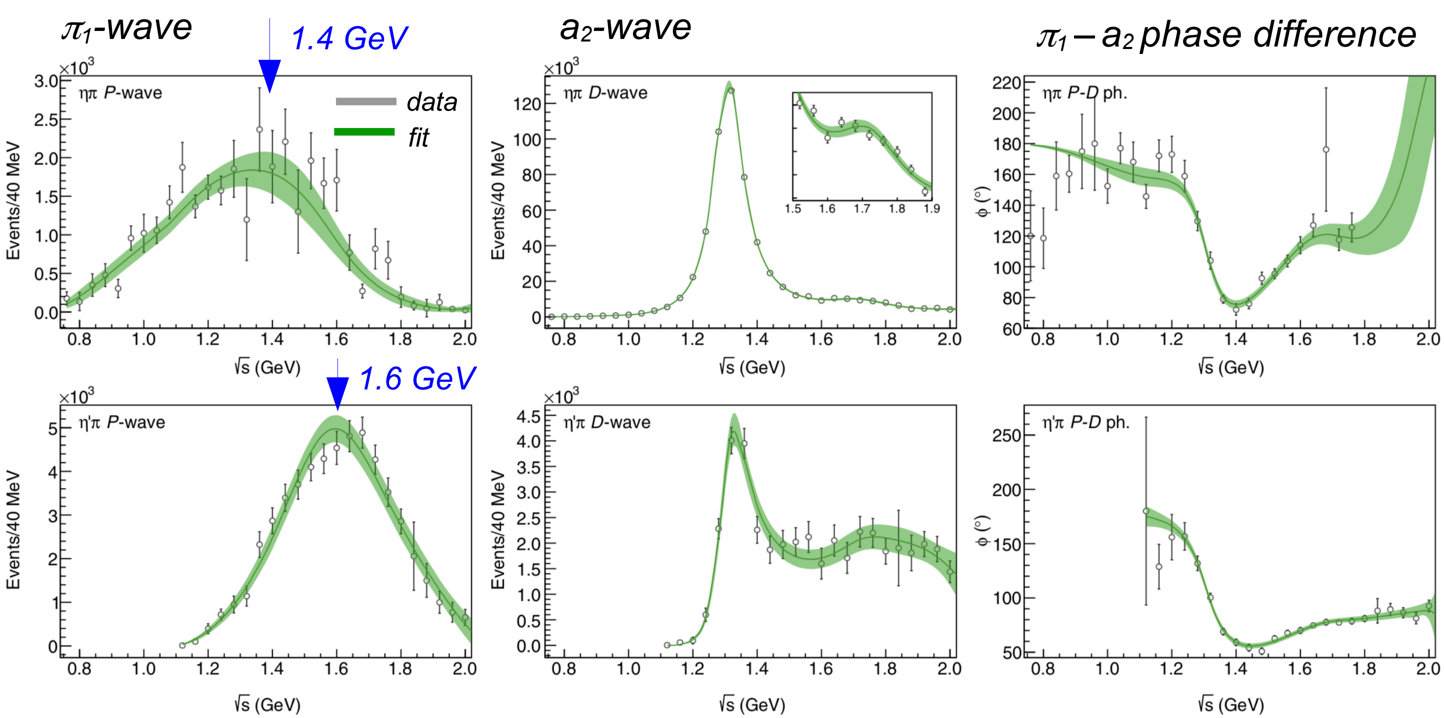
$$(1404.7 \pm 3.5\ (\text{stat.})\ ^{+9.0}_{-17.3}\ (\text{sys.}))\ \text{MeV}/c^2$$

$$(628.3 \pm 27.1\ (\text{stat.})\ ^{+35.8}_{-138.2}\ (\text{sys.}))\ \text{MeV}$$



JPAC Analysis of COMPASS Data

- Coupled channel analysis of the 1^{-+} and 2^{++} wave in $\pi^- p \to \pi^- \eta^{(\prime)} p$
- Enforcing analyticity and unitarity utilizing N/D method
- Mass shapes and phase shifts between 1^{-+} and 2^{++} are considered
- Peak at 1.4 GeV/c² in $\pi\eta$ and 1.6 GeV/c² in $\pi\eta'$ are described by one pole at (1564 ± 24 ± 86) – i(246 ± 27 ± 51) MeV



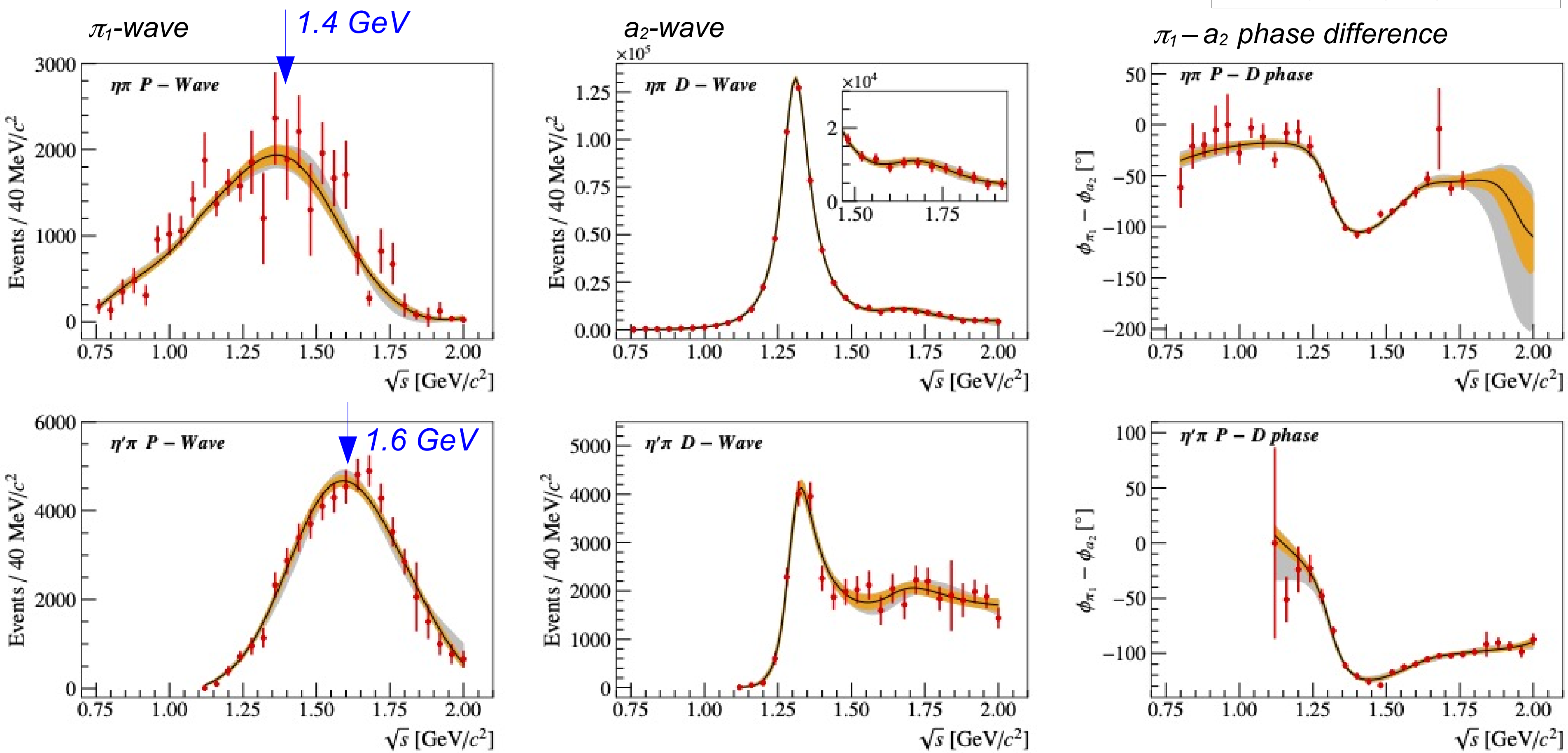
Phys. Rev. Lett. 122 (2019) 4, 042002

200 MeV/c² shift of the peak position between $m_{\pi\eta}$ and $m_{\pi\eta'}$ cannot be described by only one Breit-Wigner function

Coupled Channel Analysis with p̄p, ππ & COMPASS Data

- Extension: simultaneous fit of ππ-scattering data, $\bar{p}p \rightarrow K^+ K^- \pi^0, \pi^0 \pi^0 \eta, \pi^0 \eta \eta$ and $\pi^- p \rightarrow \pi^- \eta^{(\prime)} p$
- Good description with one pole scenario for the 1^{-+} wave using K-matrix
  - confirmation of the JPAC analysis based on N/D-method

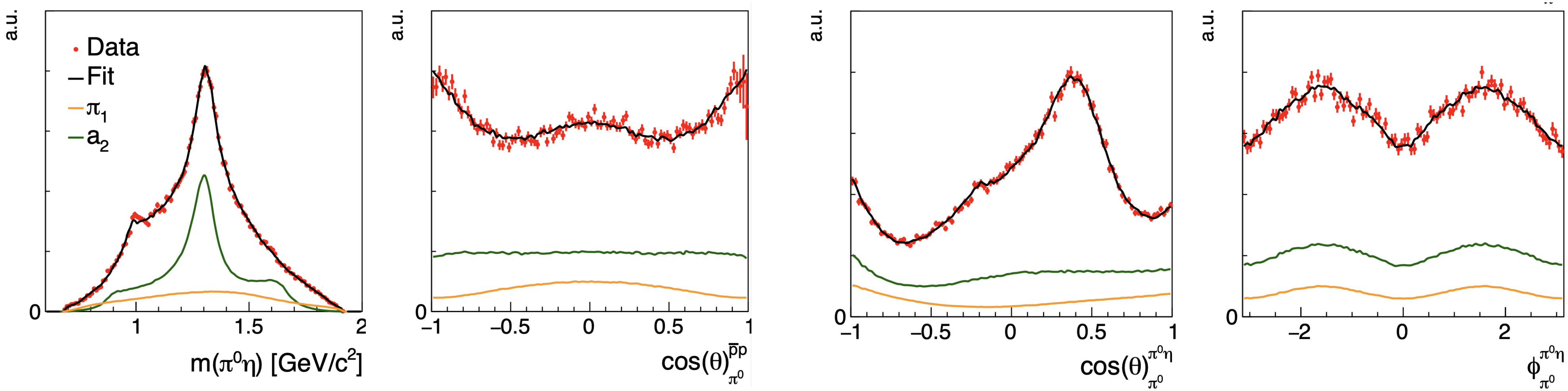
Eur. Phys.J. C (2021) 81, 1056



Coupled Channel Analysis with $\bar{p}p$, $\pi\pi$ & COMPASS Data

$\bar{p}p \to \pi^0 \pi^0 \eta$

Eur. Phys.J. C (2021) 81, 1056



- π_1 mass is moving from 1.4 GeV/c² to 1.6 GeV/c² and consistent with $\pi_1(1600)$ with $\pi\eta'$ data
- Additional decay channel $\pi\eta'$ essential for the proper determination of the π_1 pole position

Table 1 Obtained masses, total widths and ratios of partial widths for the pole of the spin-exotic π_1-wave and for the two poles in the a_2-wave, the $a_2(1320)$ and the $a_2(1700)$. The first uncertainty is the statistical and the second the systematic one

Name	Pole mass (MeV/c^2)	Pole width (MeV)	$\Gamma_{\pi\eta'}/\Gamma_{\pi\eta}$ (%)	$\Gamma_{KK}/\Gamma_{\pi\eta}$ (%)
$a_2(1320)$	$1318.7 \pm 1.9^{+1.3}_{-1.3}$	$107.5 \pm 4.6^{+3.3}_{-1.8}$	$4.6 \pm 1.5^{+7.0}_{-0.6}$	$31 \pm 22^{+9}_{-11}$
$a_2(1700)$	$1686 \pm 22^{+19}_{-7}$	$412 \pm 75^{+64}_{-57}$	$3.5 \pm 4.4^{+6.9}_{-1.2}$	$2.9 \pm 4.0^{+1.1}_{-1.2}$
π_1	$1623 \pm 47^{+24}_{-75}$	$455 \pm 88^{+144}_{-175}$	$554 \pm 110^{+180}_{-27}$	–

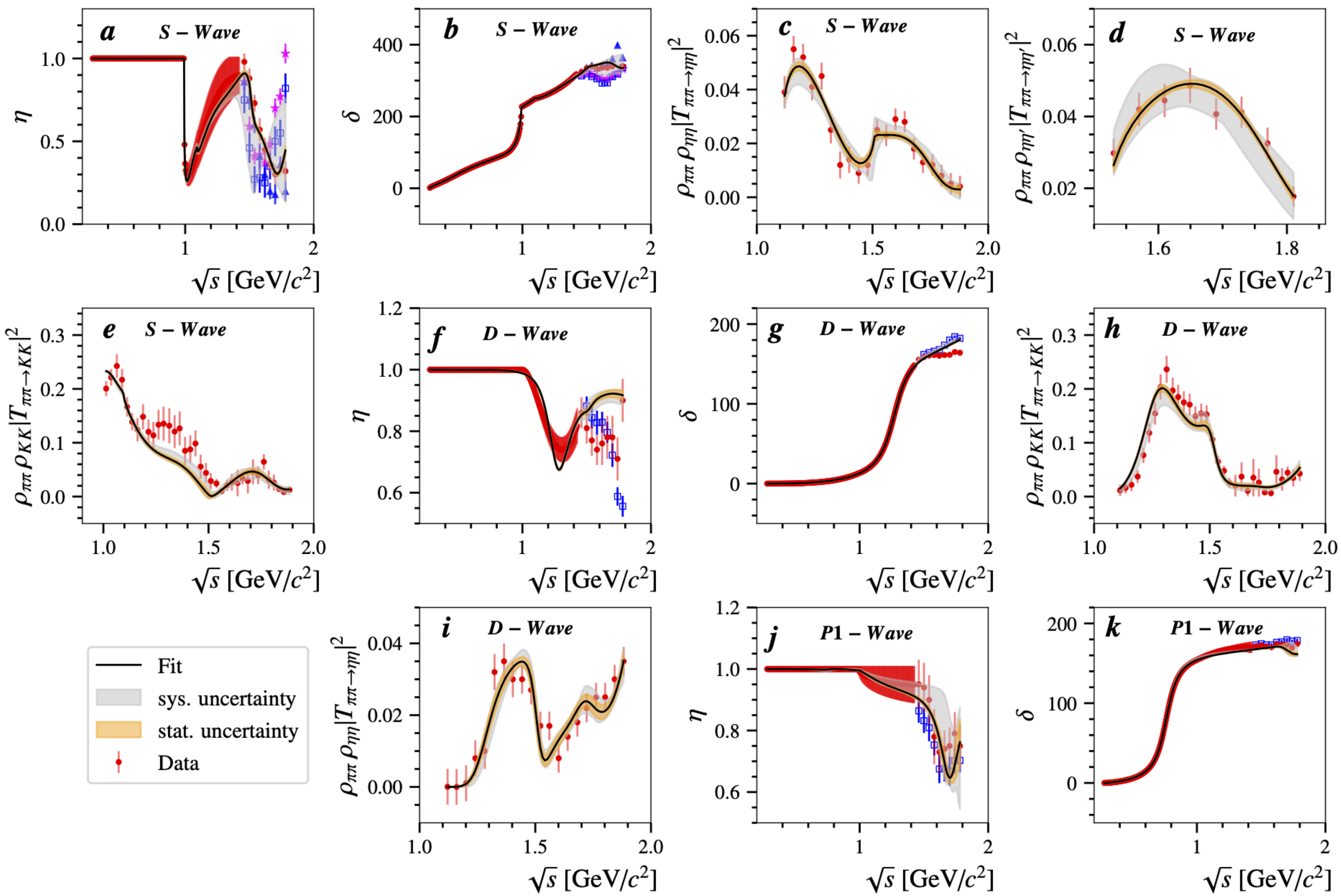
In agreement with LQCD calculations for the lightest hybrid, but uncertainties are large

Phys. Rev. D 103, 05402 (2021)

Coupled Channel Analysis with p̄p, ππ & COMPASS Data

π π scattering data

Eur. Phys.J. C (2021) 81, 1056

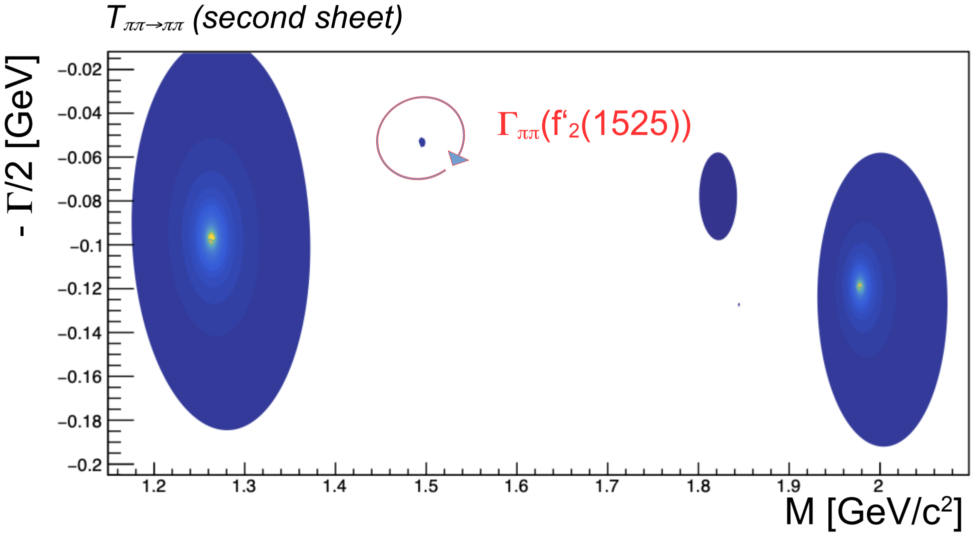


Extraction of Resonance Properties

- K-matrix contains all resonance parameters
- Masses and widths defined by the pole position in the complex energy plane of the T-matrix sheet closest to the physical sheet
- Related partial decay width can be extracted via the residues:

$$Res^{\alpha}_{k\to k} = \frac{1}{2\pi i}\oint_{C_{z\alpha}} \sqrt{\rho_k}\cdot T_{k\to k}(z)\cdot\sqrt{\rho_k}\,dz$$

Example: f_2-wave



More than 50 different resonance properties extracted on the relevant Riemann-sheets for f_0, f_2, a_0, a_2 and ρ resonances

$\gamma\gamma \to K^+K^-, \pi^0\pi^0, \pi^0\eta$ @ BESIII

- Electromagnetic interaction of the production → access to the inner structure
- Gluon poor process with weak coupling of some resonances
- Fixed K-matrix parametrization used
- Good description with the K-matrix parametrization for f_0, f_2, a_0 and a_2

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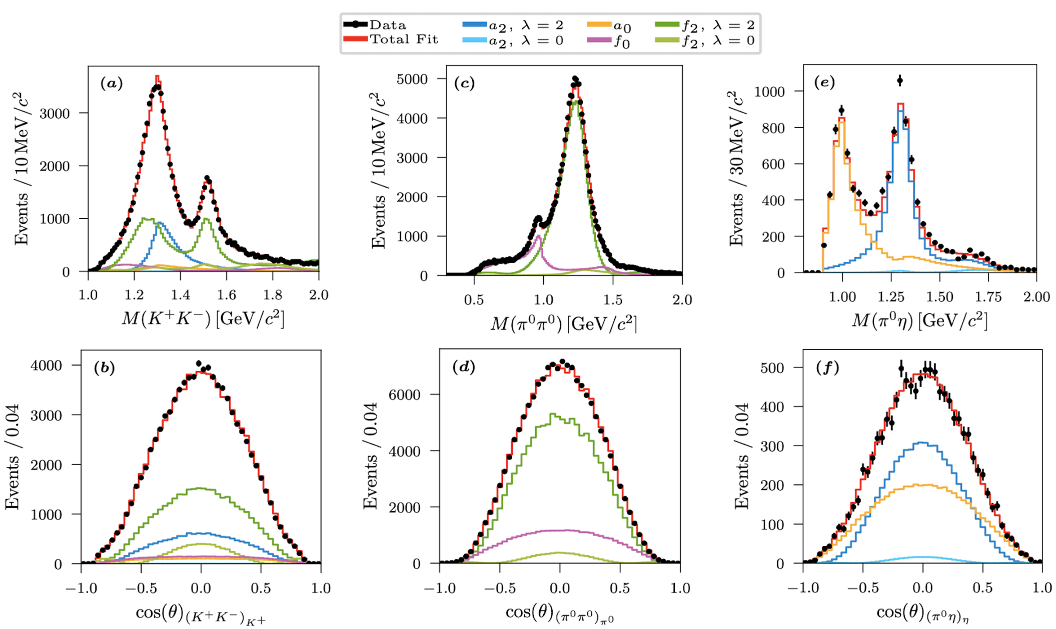
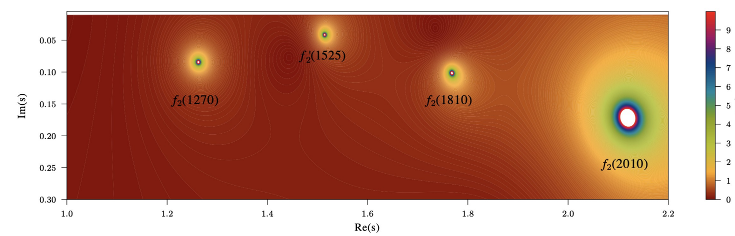
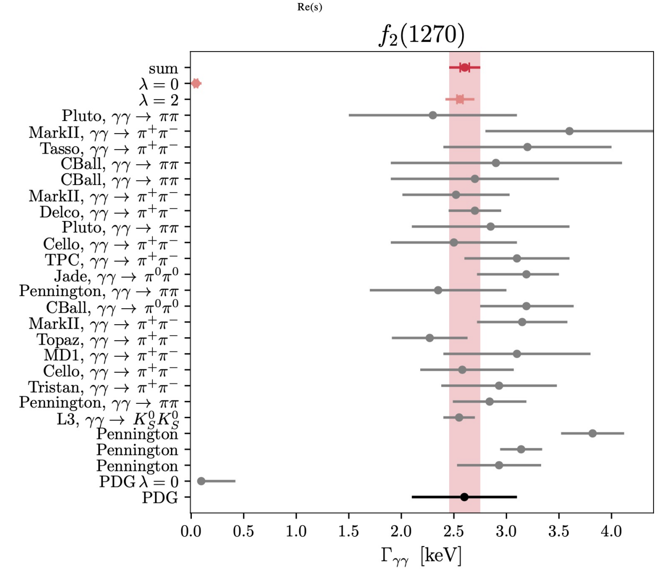


M. Küßner, PhD-Thesis, RUB (2022)

- Extraction of the $\gamma\gamma$-widths via the pole residues of the F-vector on the second Rieman-sheet





Radiative J/ψ Decays @ BESIII

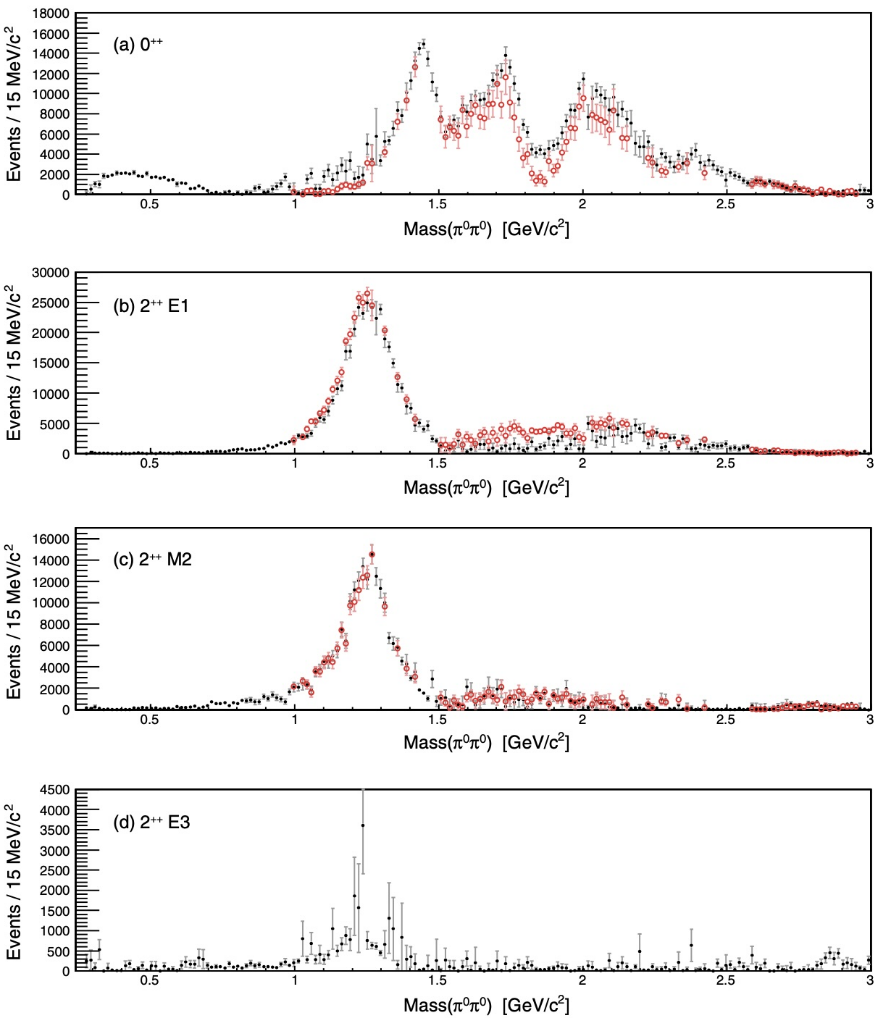
- Gluon rich process
- Electromagnetic part can be calculated by QED
- Multipole amplitudes give access to the transition form factors of the contributing resonances
  - access to the inner structure
  - e.g. for production of conventional f_2 mesons consisting of $q\bar{q}$ pair: E1>M2>E3
- Single channel fits feasible in a model independent way for channels like $J/\psi \to \gamma\pi\pi$ or $J/\psi \to \gamma K\bar{K}$
  - poor contributions of resonances in the $\gamma\pi$ and γK systems
  - outcome can be used for mass dependent couple channel fits

J/ψ–>γπ⁰π⁰ @ BESIII

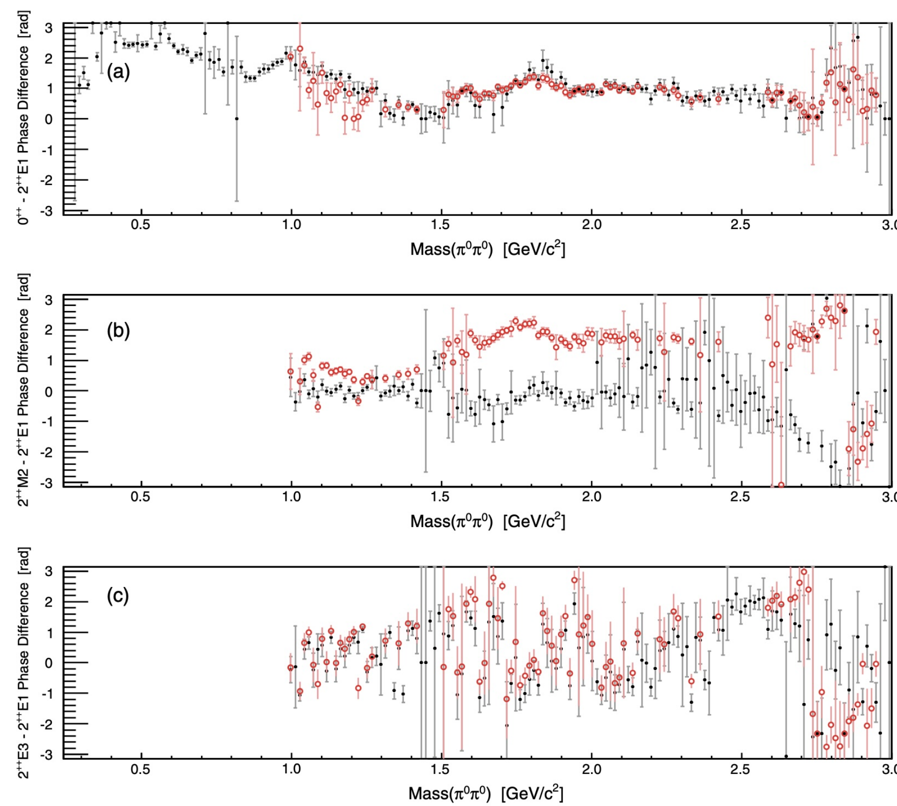
- Independent fits for each mass bin
- Mass-independent fits lead to ambiguities → here 2 solutions (marked in black and red)
- 2^{++} wave: E1 dominates over M2 and E3

BESIII: Phys.Rev.D 92, 052003

Amplitudes



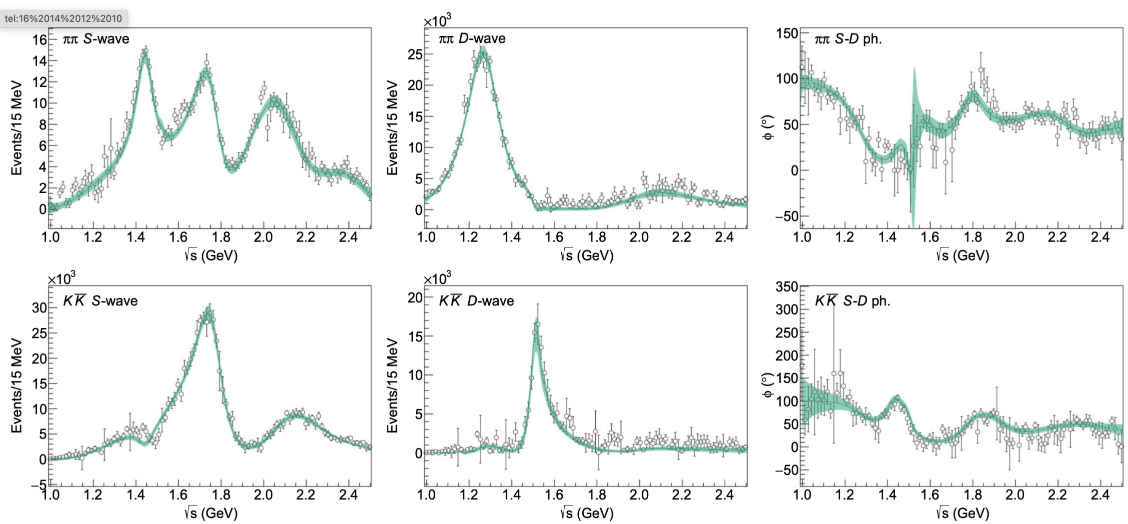
Phase Differences

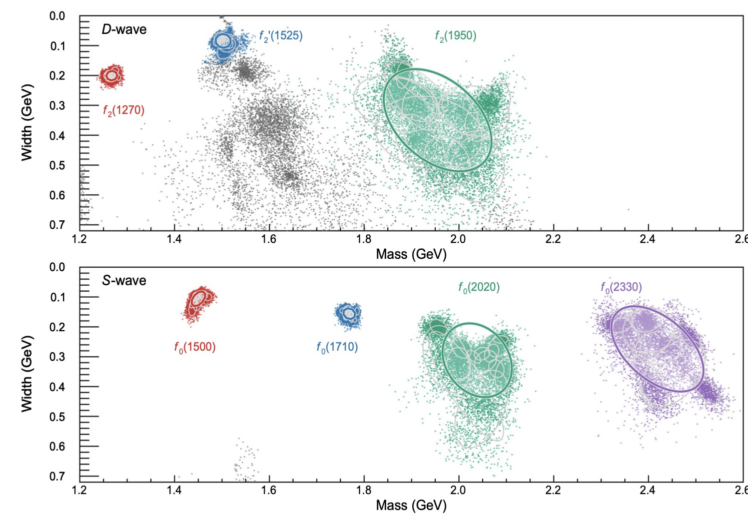


Coupled Cannel Analysis with radiative J/ψ Decays

- Mass-independent fit results for $J/\psi \rightarrow \gamma\pi^0\pi^0$ and $J/\psi \rightarrow \gamma K_s K_s$ used as input for coupled channel analyses
- JPAC
  - mass range: 1 - 2.5 GeV
  - f_0 and f_2 E1-wave are taken into account
  - 2- and 3-channel fit with coupled channel N/D formalism
  - identification of 4 scalar and 3 tensor states

JPAC: Eur.Phys.J.C.82, 80 (2022)





Summary

- Event based maximum likelihood fits of the complete phase space often needed
  - consideration of the complete reaction chain from the initial to the final states
  - structures originated from reflections or interference effects are better under control
- Approaches with an adequate consideration of analyticity and unitarity important for the description of the dynamics
  - Breit-Wigner functions only a good approximation for isolated resonances appearing in a single channel
  - sophisticated formalisms like K-matrix with Chew-Mandelstam functions, N/D etc. are preferable
- Coupled channel analyses with a reasonable choice of channels can guarantee a good approach to unitarity with access to (almost) all K-matrix parameters
- Fit examples
  - one-pole scenario can describe the π_1 peak in $\pi\eta$ at 1.4 GeV and in $\pi\eta'$ at 1.6 GeV
  - coupled channel fits using the outcome of model-independent single channel analyses