Hadron physics with functional methods

Reviews: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]
Eichmann, CF, Heupel, Santowsky, Wallbott, FBS 61 (2020) [2008.10240]

Topic

Blue

Important !

… please pay attention…

Topic

Red

Derivation/Detour

… you may take a nap
if you are not interested…

Why hadron physics ??

- Bridge to particle physics (standard model and beyond)
  - Confinement
  - Dynamical mass generation
  - Properties of baryons, mesons, exotics, …
- Bridge from fundamental physics to effective nuclear forces

Overview

1. Bound states in the quark model
   - Construction
   - Assets, shortcomings and how to do better
2. Properties of QCD and correlation functions
   - Dynamical chiral symmetry breaking
   - Correlation functions and Dyson-Schwinger equations (DSEs)
   - The quark DSE
3. Mesons
   - The pion Bethe-Salpeter equation (BSE): Goldstone bosons, GMOR and all that…
   - Spectra: light and charm
4. Exotic mesons
   - Confinement and glueballs
   - Four-quark states
5. Baryons
   - Spectra: light and strange
6. Form factors
   - Meson form factors
   - Baryon form factors

Nonpert. QCD: Complementary approaches

Quarks and gluons

- Lattice simulations
  - Ab initio
  - Gauge invariant

Hadrons

- Effective theories (χPT, ...)
- Models
  - physical dof

- Functional approaches (DSE, FRG):
  - Space-time continuum
  - Chiral symmetry: light quarks and mesons
  - Multi-scale problems feasible
  - Chemical potential: no sign problem
  - Access to structural information

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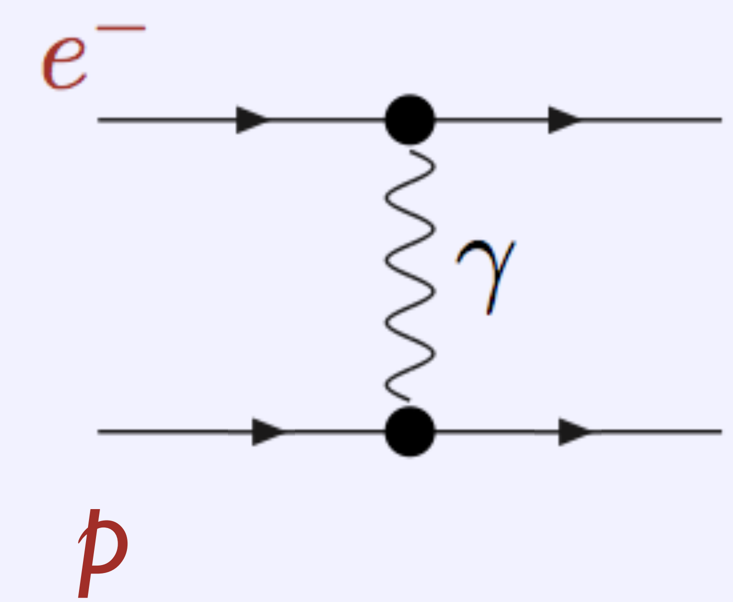
Phenomenological tool: Quark-model

Bound states in QED and QCD

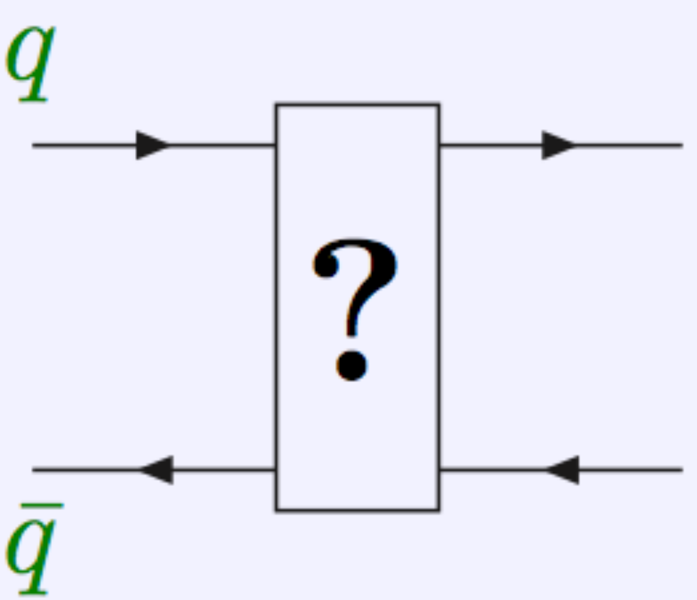
Proton-electron-system:

- $F(r) \sim \frac{1}{r^2}$
- Hydrogen can be ionised…



Quark-Antiquark-System

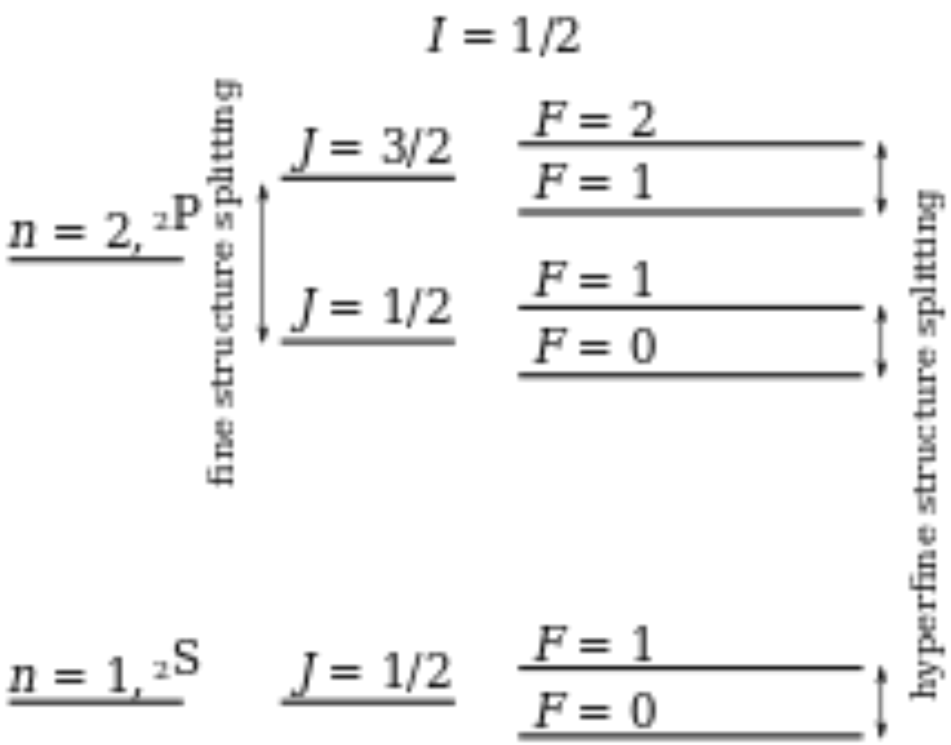
- $F(r) \sim const.$
- Confinement



Similarities: bound states of two spin 1/2 particles…

QED: electron-proton interaction

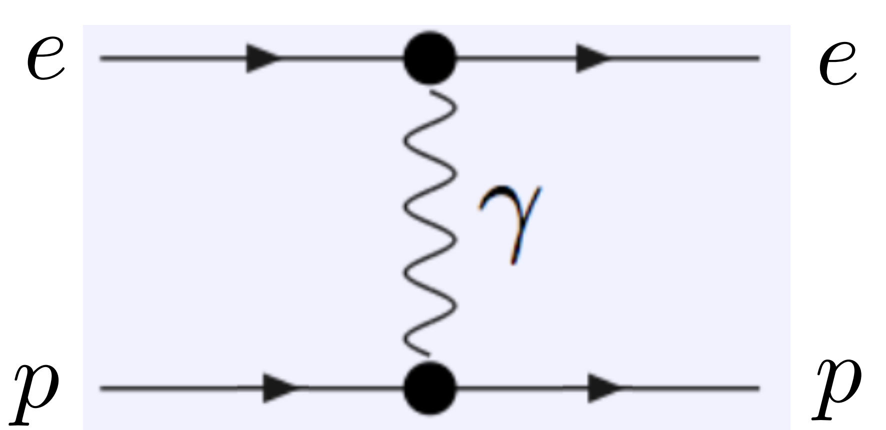
- hydrogen:



Coulomb potential
spin-orbit coupling (LS): fine splitting
spin-spin coupling (SS): hyperfine splitting

Calculation e.g. via Schrödinger equation and perturbation theory

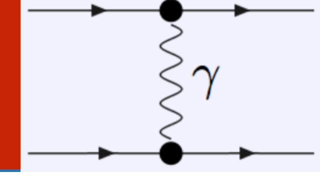
- field theory:



$$
\begin{aligned}
V_{Fermi-Breit} = &-\frac{\alpha}{r} + \frac{\alpha\pi}{2}\delta(\mathbf{r})\left[\frac{1}{m_e^2}+\frac{1}{m_p^2}\right] + \frac{8\pi\alpha}{3m_e m_p}\mathbf{S}_e\mathbf{S}_p\delta(\mathbf{r}) \\
&+ \frac{\alpha}{m_e m_p r^3}\left[3(\mathbf{S}_e\mathbf{r})(\mathbf{S}_p\mathbf{r}) - \mathbf{S}_e\mathbf{S}_p\right] \\
&+ \frac{\alpha}{r^3}\left[\frac{\mathbf{S}_e\mathbf{L}_e}{2m_e^2} - \frac{\mathbf{S}_p\mathbf{L}_p}{2m_p^2} + \frac{\mathbf{S}_p\mathbf{L}_e - \mathbf{S}_e\mathbf{L}_p}{2m_p m_e}\right] \\
&+ \frac{\alpha}{2m_e m_p r}\left(\mathbf{p}_e\mathbf{p}_p + \frac{(\mathbf{r}\mathbf{p}_p)(\mathbf{r}\mathbf{p}_e)}{r^2}\right)
\end{aligned}
$$

Donoghue, Golowich, Holstein, Dynamics of the Standard Model , Cambridge University Presse, Chapter V

Derivation: Fermi-Breit force



We start with the formula for the scattering amplitude in momentum space

$$M = e^2 \bar{u}_e(\mathbf{p}'_e)\gamma_\mu u_e(\mathbf{p}_e)\frac{1}{\mathbf{q}^2}\bar{u}_p(\mathbf{p}'_p)\gamma_\mu u_p(\mathbf{p}_p) \tag{1}$$

with generic spinors in non-relativistic approximation

$$u(\mathbf{p}) = \sqrt{\frac{E+m}{2E}}\begin{pmatrix} \chi \\ \frac{\boldsymbol{\sigma}\mathbf{p}}{E+m}\chi \end{pmatrix} \quad \rightarrow \quad u(\mathbf{p}) = \left(1 - \frac{\mathbf{p}^2}{8m^2}\right)\begin{pmatrix} \chi \\ \frac{\boldsymbol{\sigma}\mathbf{p}}{2m}\chi \end{pmatrix} \tag{2}$$

Using this and $\sigma_i\sigma_j = \delta_{ij} + i\epsilon_{ijk}\sigma_k$ we obtain

$$\begin{aligned} M = &-\frac{e^2}{\mathbf{q}^2}\left(1 - \frac{\mathbf{p}_e^2 + \mathbf{p}_e'^2}{8m_e^2}\right)\left(1 - \frac{\mathbf{p}_p^2 + \mathbf{p}_p'^2}{8m_p^2}\right) \times \\ &\left[\chi_p^\dagger\left(1 + \frac{\mathbf{p}_p\mathbf{p}'_p + i\boldsymbol{\sigma}(\mathbf{p}'_p \times \mathbf{p}_p)}{4m_p^2}\right)\chi_p\chi_e^\dagger\left(1 + \frac{\mathbf{p}_e\mathbf{p}'_e + i\boldsymbol{\sigma}(\mathbf{p}'_e \times \mathbf{p}_e)}{4m_e^2}\right)\chi_e \right. \\ &\left. - \chi_p^\dagger\frac{\mathbf{p}_p + \mathbf{p}'_p - i\boldsymbol{\sigma}\mathbf{q}}{2m_p}\chi_p\chi_e^\dagger\frac{\mathbf{p}_e + \mathbf{p}'_e - i\boldsymbol{\sigma}\mathbf{q}}{2m_e}\chi_e\right] \end{aligned} \tag{3}$$

Donoghue, Golowich, Holstein, Dynamics of the Standard Model , Cambridge University Presse, Chapter V

Derivation: Fermi-Breit force

Reminding ourselves that scattering amplitude and potential are connected via a Fourier-transformation

$$V(\mathbf{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\mathbf{r}} M \tag{1}$$

we obtain the familiar Coulomb potential from the leading term

$$V_{Coulomb}(r) = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\mathbf{r}} \frac{-e^2}{\mathbf{q}} = -\frac{\alpha}{r} \tag{2}$$

and with $\mathbf{S} = \boldsymbol{\sigma}/2$ as well as $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ we obtain the **LS**-coupling term

$$V_{LS} = -\int \frac{d^3q}{(2\pi)^3} \frac{e^2}{\mathbf{q}^2} \frac{i\boldsymbol{\sigma}(\mathbf{p}'_e \times \mathbf{p}_e)}{4m_e^2} e^{-i\mathbf{q}\mathbf{r}} \tag{3}$$

$$= \frac{e^2}{em_e^2} \frac{\boldsymbol{\sigma}(\mathbf{r} \times \mathbf{p}_e)}{4\pi r^3} = \frac{\alpha}{2m_e^2 r^3} \mathbf{LS} \tag{4}$$

Donoghue, Golowich, Holstein, Dynamics of the Standard Model , Cambridge University Presse, Chapter V

Derivation: Fermi-Breit force

The other terms of order p^2/m_e^2 combine to the Darwin-term

$$V_D = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\mathbf{r}} \frac{\mathbf{p}_e^2 + \mathbf{p}_e'^2 - 2\mathbf{p}_e\mathbf{p}_e'}{8m_e^2} = \frac{\alpha\pi}{2m_e^2}\delta(\mathbf{r})\,, \tag{1}$$

whereas terms of order p/m_e can be interpreted as spin-spin interactions between proton and electron

$$V_{SS} = V_{hyp} + V_{tensor} \tag{2}$$

$$V_{hyp} = \frac{8\pi\alpha}{3m_e m_p}\mathbf{S}_e\mathbf{S}_p\delta(\mathbf{r}) \tag{3}$$

$$V_{tensor} = \frac{\alpha}{m_e m_p r^3}\left(3(\mathbf{S}_e\mathbf{r})(\mathbf{S}_p\mathbf{r}) - \mathbf{S}_e\mathbf{S}_p\right)\,. \tag{4}$$

This is what makes the hyperfine structure of the hydrogen atom !
Finally the remaining term denotes the **LL** interaction and can be written as

$$V_{LL} = \frac{\alpha}{2m_e m_p r}\left(\mathbf{p}_e\mathbf{p}_p + \frac{(\mathbf{r}\mathbf{p}_p)(\mathbf{r}\mathbf{p}_e)}{r^2}\right) \tag{5}$$

Donoghue, Golowich, Holstein, Dynamics of the Standard Model , Cambridge University Presse, Chapter V

(Non-relativistic) Quark model

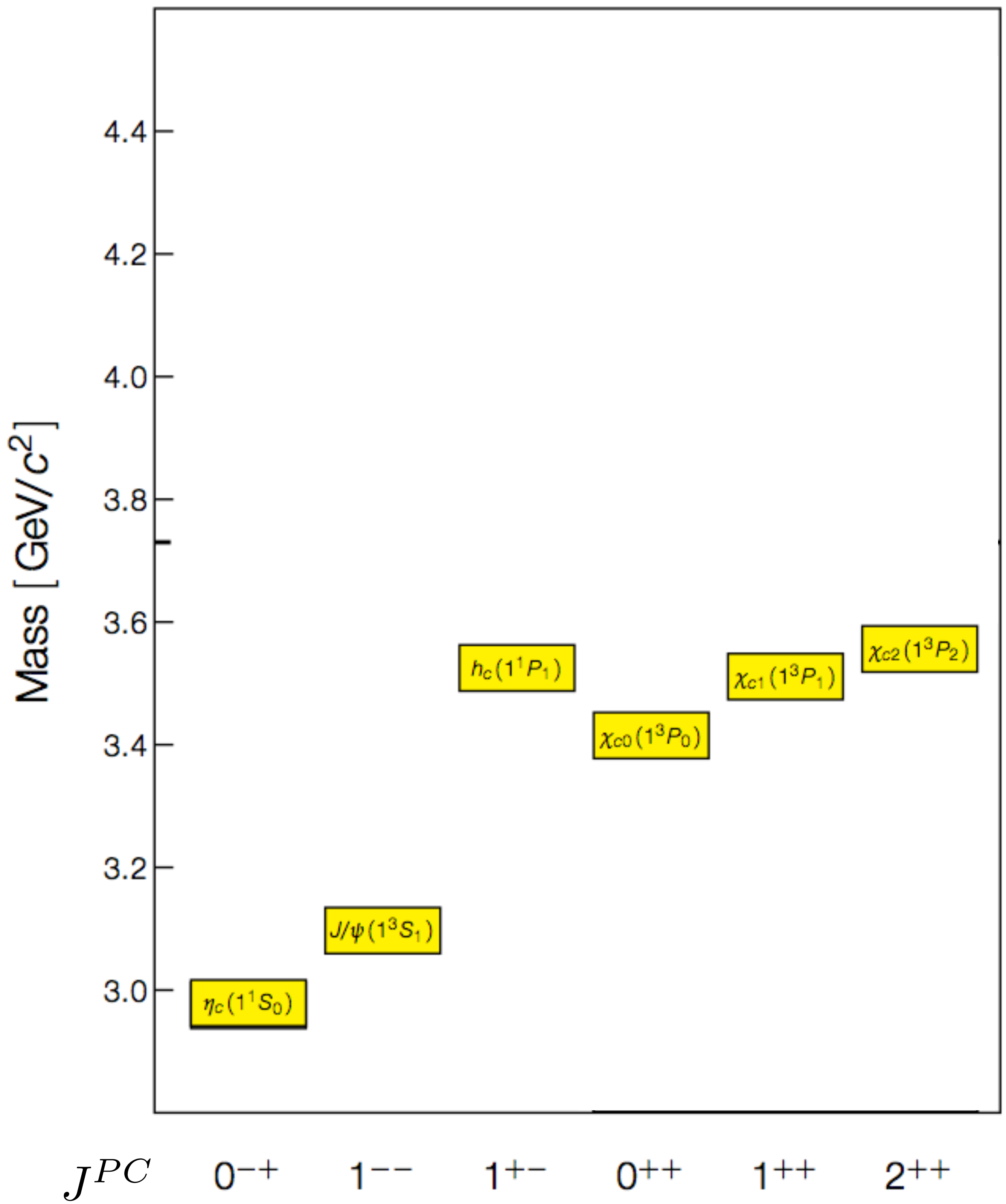
Basic ideas:

- Consider heavy quarks (charm, bottom): non-relativistic
- Bound states of two spin 1/2 particles:
  similar forces than QED ?
- Quarks are pointlike (=contituents) with mass m
- simplest assumption: interaction dominated by one-gluon exchange (vector-vector type of interaction) —> Fermi-Breit
- replace α_{QED} with α_S and Coulomb- by Cornell-potential

$$V_{Coulomb} = -\frac{\alpha}{r} \qquad \rightarrow \qquad V_{Cornell} = br - \frac{\alpha_S}{r}$$

- introduce parameters to play with strength of different contributions

Spectrum of ground state charmonia



Wolfgang Gradl, BESIII, St Goar 2015

Do we understand level ordering ?

Quantum numbers (quark model)

Coupling a quark and an antiquark:

$$S: 1/2 \otimes 1/2 \rightarrow 0 \oplus 1$$

$$P: (-1)^{L+1}$$

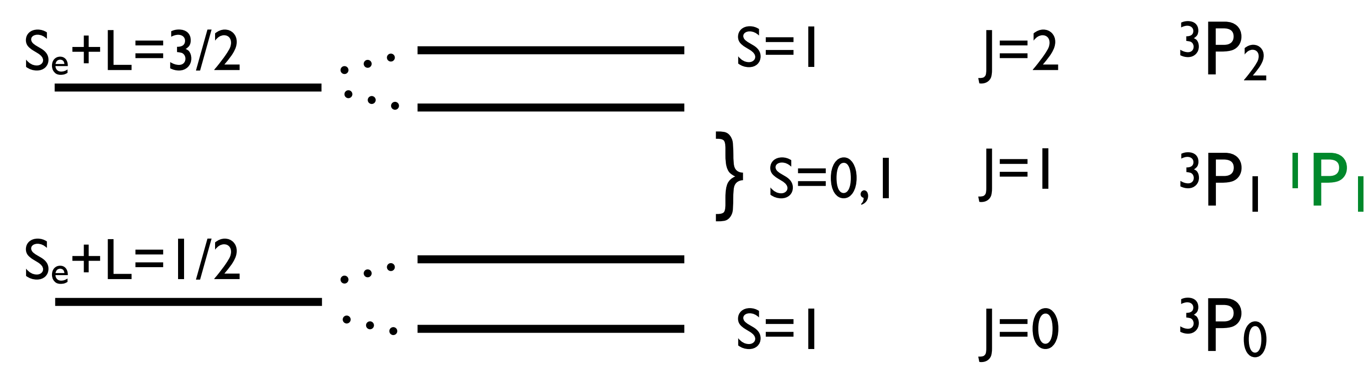
$$C: (-1)^{L+S}$$

S	L	J^{PC}	
0	0	0^{-+}	
1	0	1^{--}	
0	1	1^{+-}	$^1P_1$
1	1	0^{++}	$^3P_0$
		1^{++}	$^3P_1$
		2^{++}	$^3P_2$

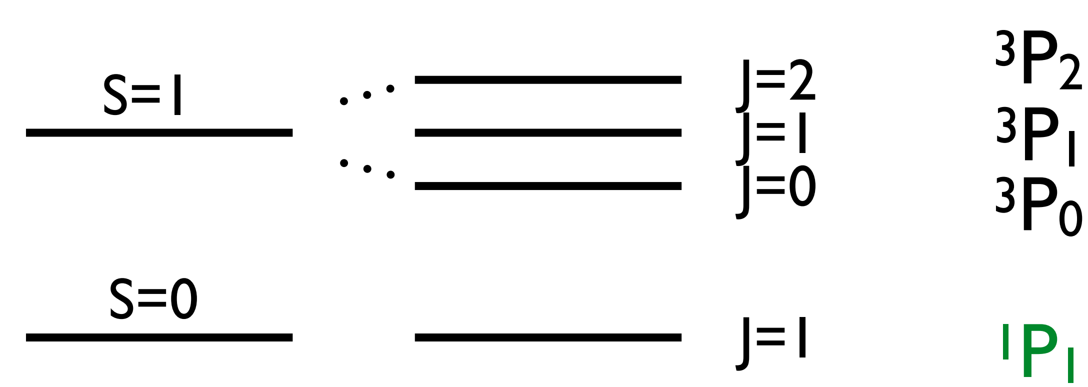
J^{PC} or $^{2S+1}L_J$

Spectrum for L=1 states

dominant LS-coupling:

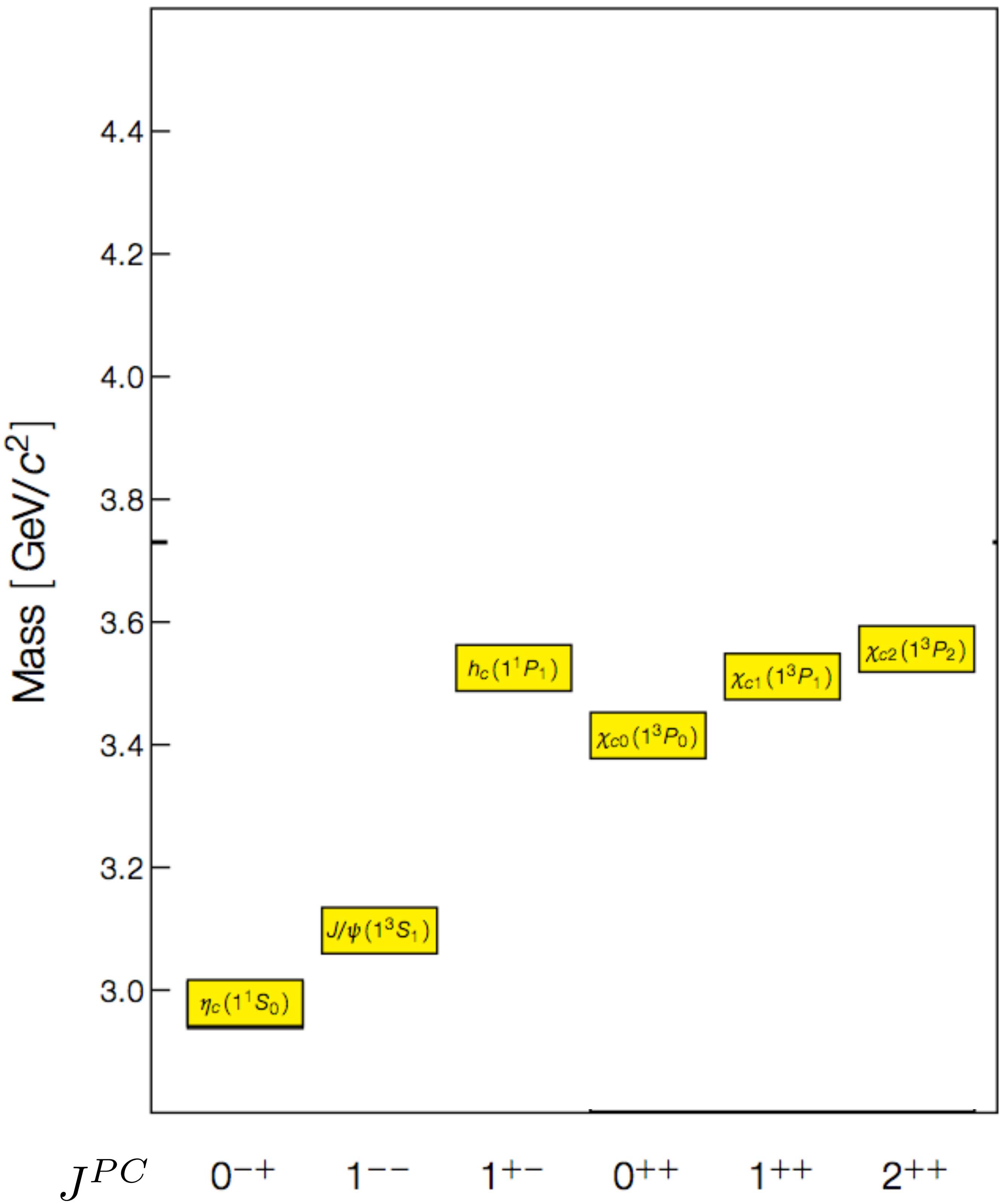


dominant SS-coupling:

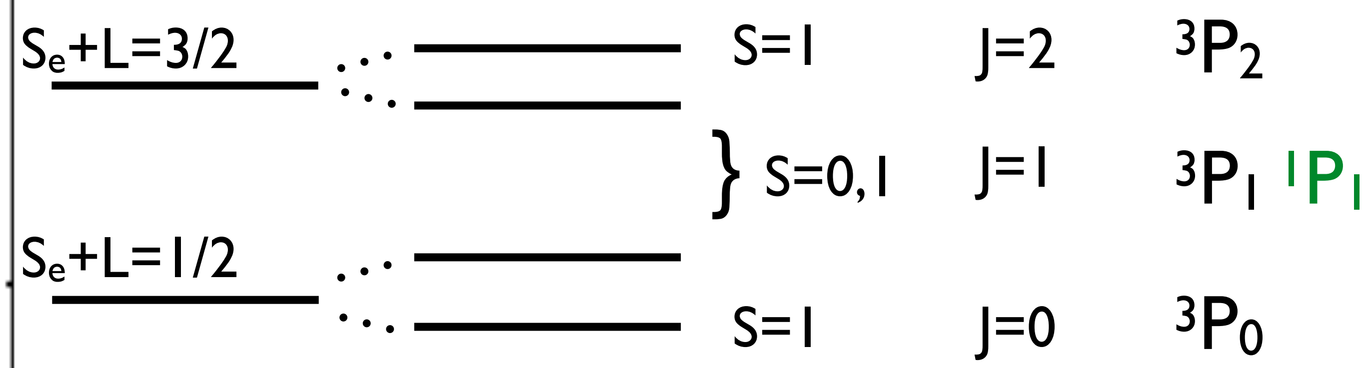


Note: 'exotic' quantum numbers such as $0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$ etc. not possible !

Spectrum of states in the charmonia region



dominant LS-coupling:



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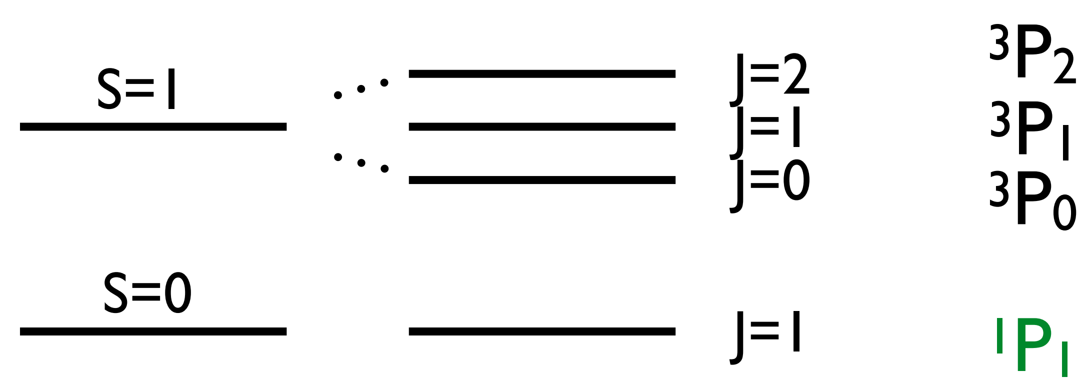

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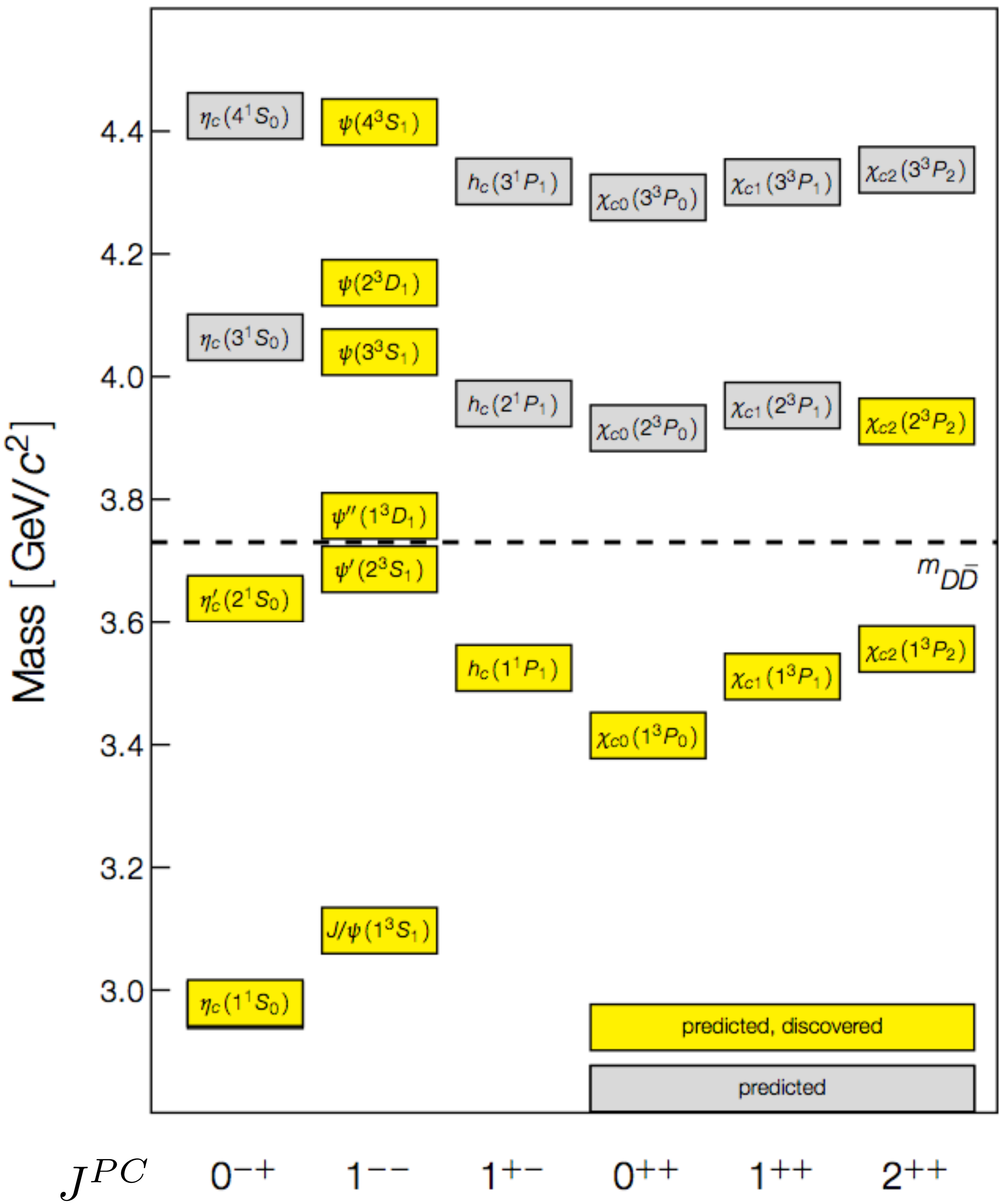

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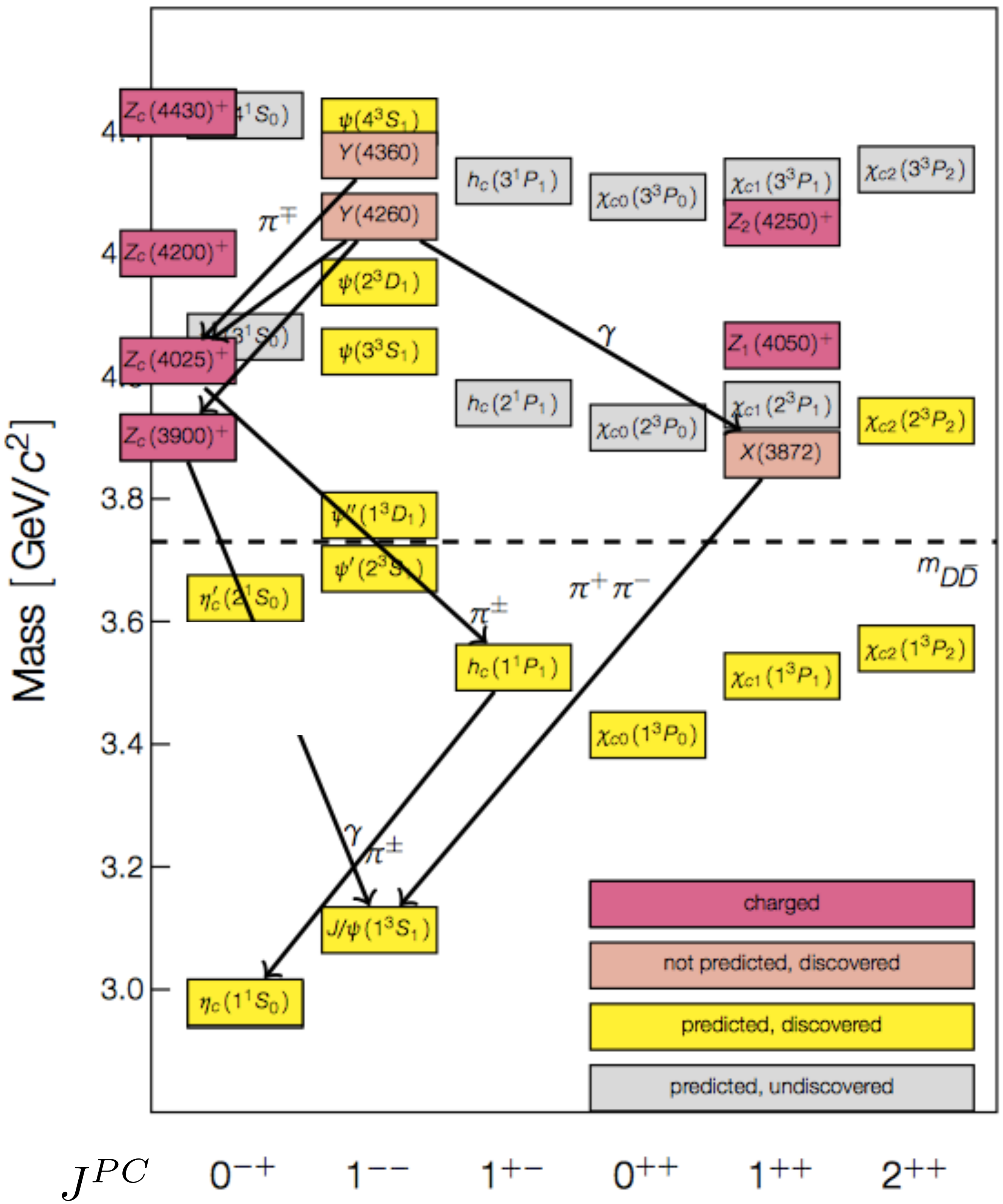
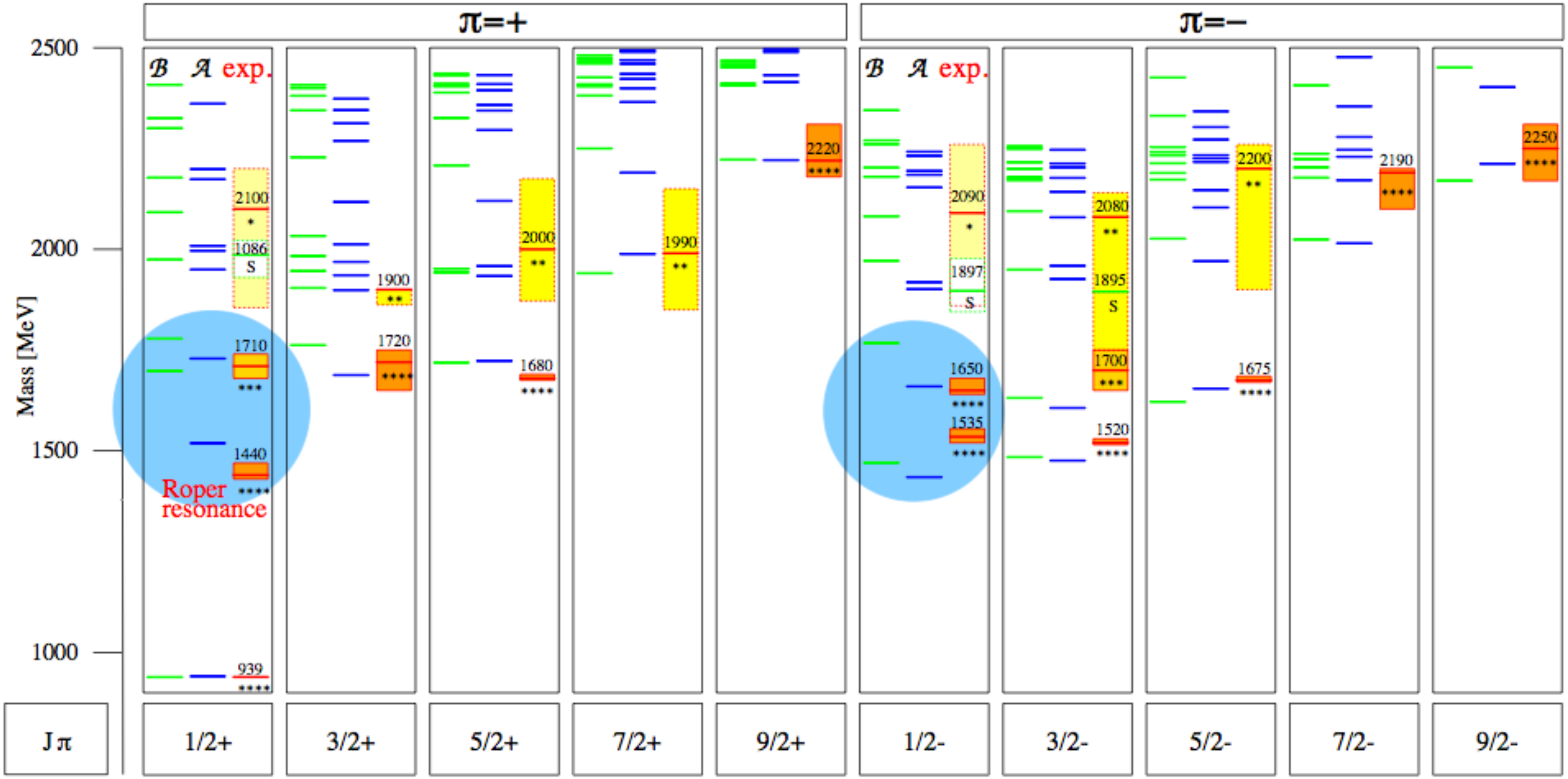


- many new states, not predicted by quark model
- some of these are charged... : candidates for tetraquarks
- but also: hybrids ? glueballs ?

Experiments: Belle (II), BaBAR, BES III, LHCb, GlueX/JLAB, PANDA/FAIR

Baryons: quark model



Loring, Metsch, Petry, EPJA 10 (2001) 395

- ‘missing resonances’ ?! diquarks ??
- level ordering between channels with opposite parity ??

Shortcomings of quark model

- Concept of constituent quarks ? see later…
- Use of potentials justified for light quarks ? No !
- Use of potentials justified even for bottom/charm ? NRQCD
- Relation of (phenomenological) potential to QCD ? unclear…
- Different parameters for different problems (mesons-baryons)
- Exotic states ? (tetraquarks, hybrids…) not well developed
- Many unsolved problems: Roper …

> Still: quark model provides base line calculation
> which allows us to formulate many useful questions !

S. Capstick and W. Roberts,
Quark models of baryon masses and decays,
Prog. Part. Nucl. Phys. 45 (2000) S241

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   - Dynamical chiral symmetry breaking
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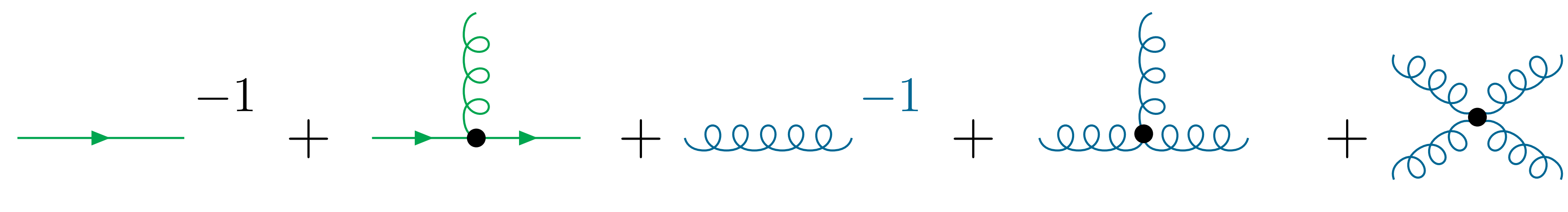
The QCD generating functional

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \; \exp\left\{ -\int d^4x \Big(\overline{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \text{gauge fixing} \Big) \right\}$$

$S_{QCD} = \int d^4x \;\Big($



$\Big)$

- Euclidean space
- $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$
- $D_\mu = \partial_\mu + i g t^a A^a_\mu$
- Landau gauge: $\partial_\mu A^a_\mu = 0$

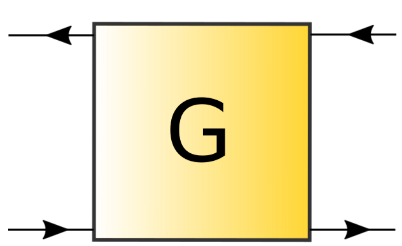
QCD correlation functions

$$\mathcal{Z} = \int \mathcal{D}[A,\Psi,\bar{\Psi}] \quad e^{-(\bar{\Psi}\mathbf{D}\Psi + S_{YM})}$$

$$\rightarrow \langle \mathcal{O} \rangle = \int \mathcal{D}[A,\Psi,\bar{\Psi}]\ \mathcal{O}\ e^{-(\bar{\Psi}\mathbf{D}\Psi + S_{YM})}$$

Examples:

$$\langle \Psi\bar{\Psi} \rangle = \int \mathcal{D}[A,\Psi,\bar{\Psi}]\ \Psi\bar{\Psi}\ e^{-(\bar{\Psi}\mathbf{D}\Psi + S_{YM})}$$

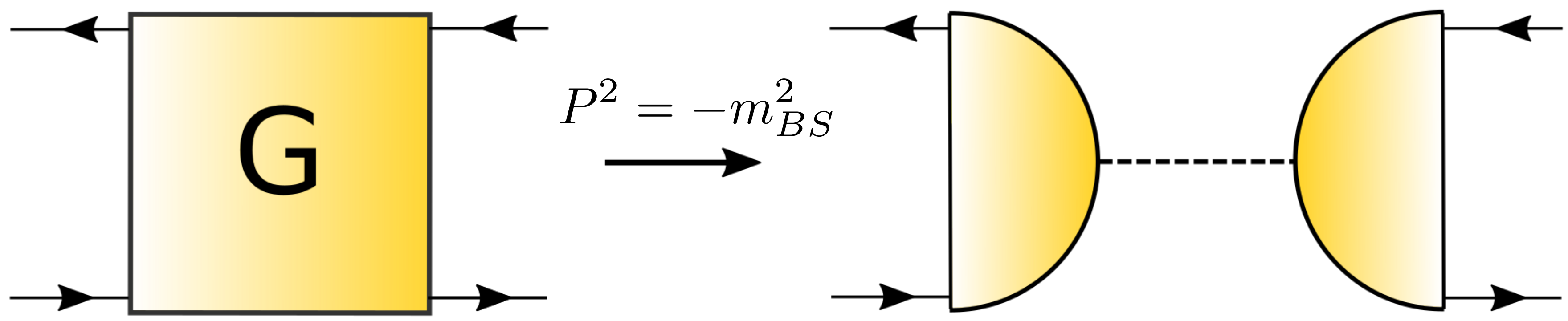


$$\langle \Psi\bar{\Psi}\Psi\bar{\Psi} \rangle = \int \mathcal{D}[A,\Psi,\bar{\Psi}]\ \Psi\bar{\Psi}\Psi\bar{\Psi}\ e^{-(\bar{\Psi}\mathbf{D}\Psi + S_{YM})}$$

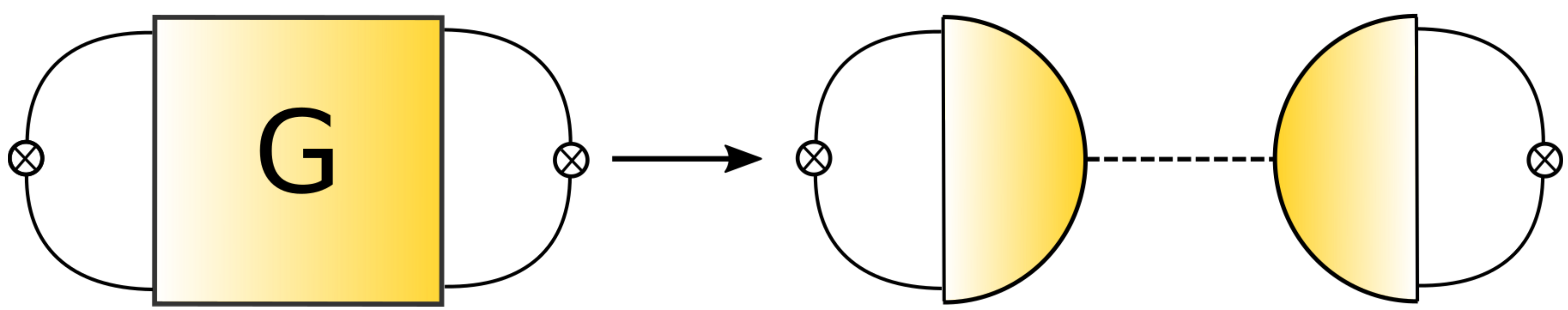
- can be gauge invariant or gauge dependent

Extracting spectra from QCD-correlators

functional:



Lattice:



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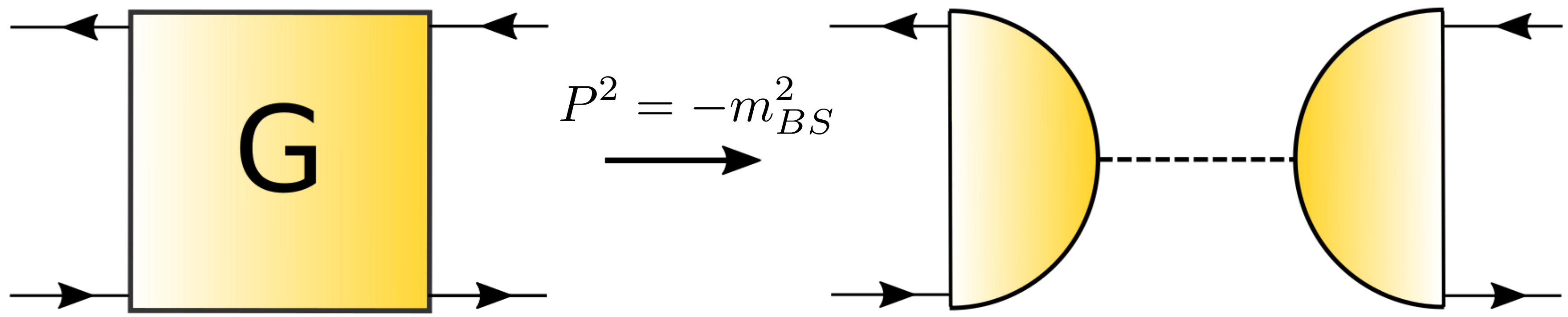

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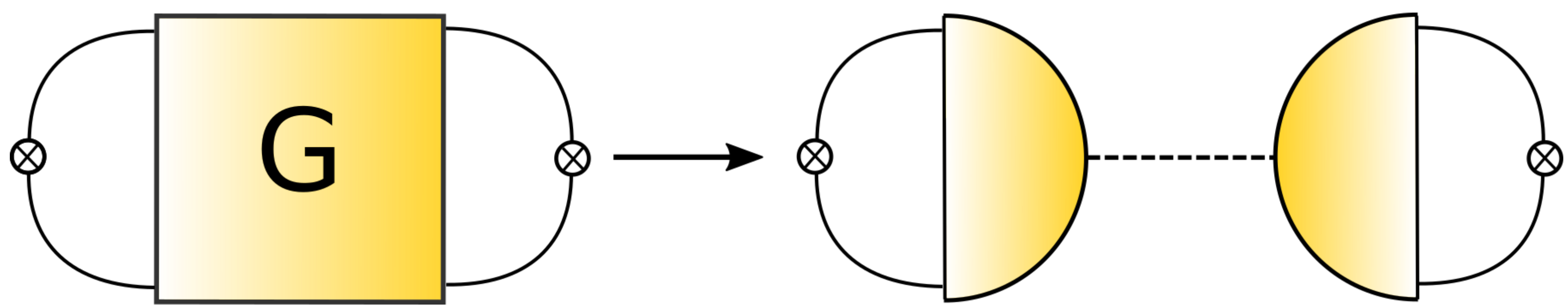
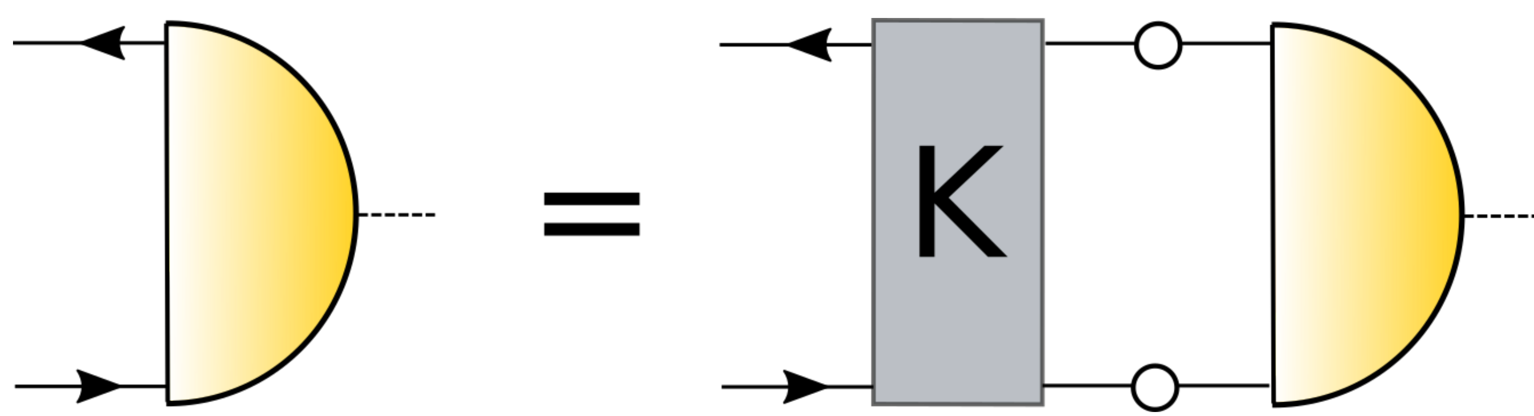
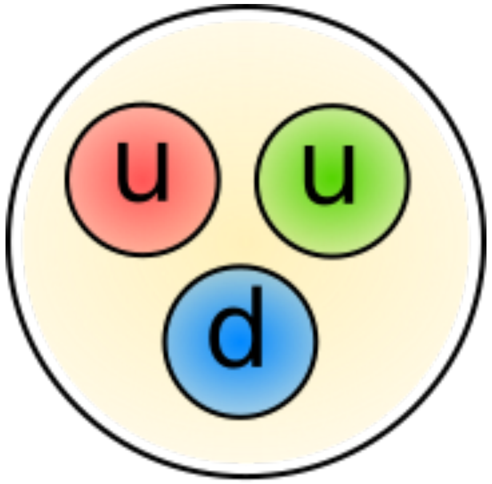


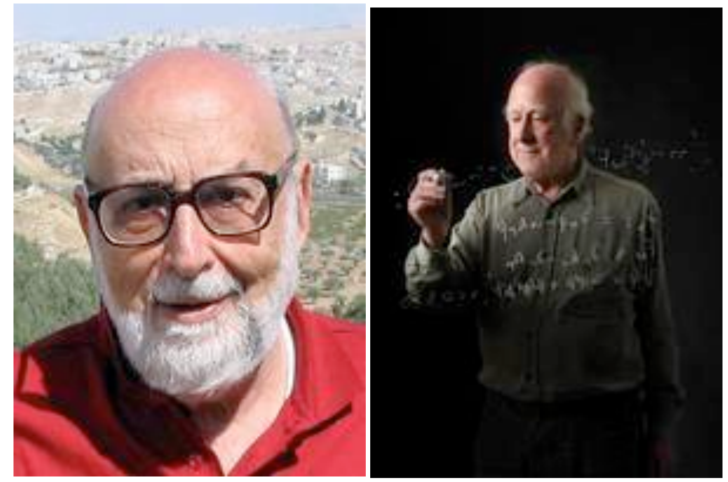
exact BSE:



Properties of QCD: Dynamical mass generation



m_{proton} = 938 MeV



Francois Englert, Peter Higgs
Nobel prize 2013

Dynamical quark masses via weak force

quarks	u	d	s	c	b	t
M_{weak} [MeV]	3	5	80	1200	4500	176000

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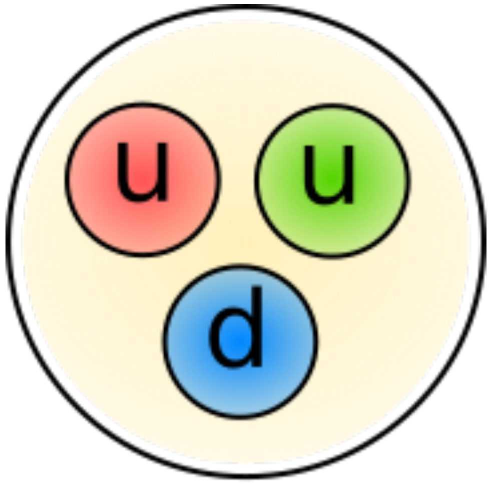

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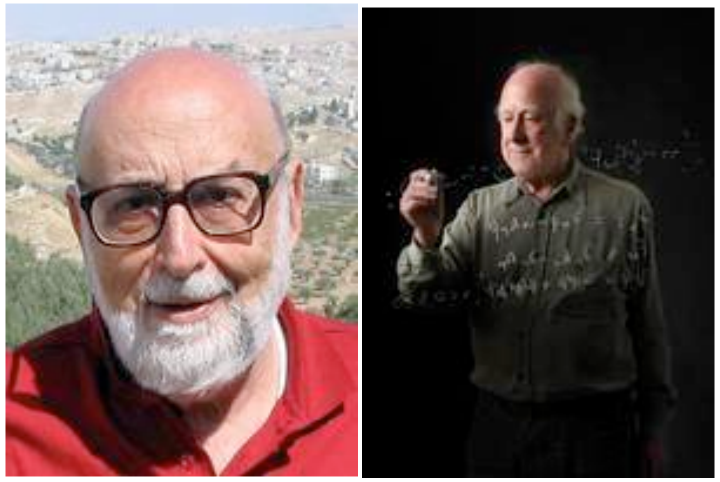


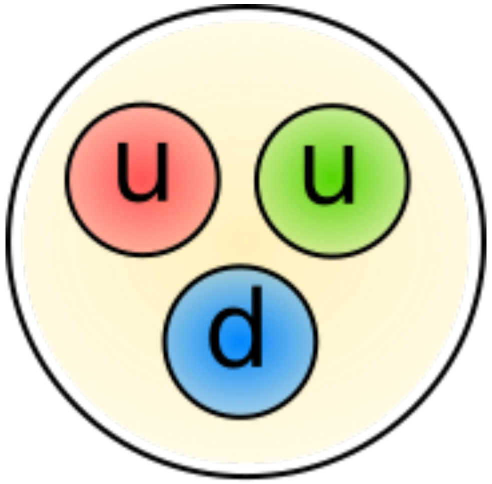
Francois Englert, Peter Higgs
Nobel prize 2013

Yoichiro Nambu,
Nobel prize 2008

Dynamical quark masses via weak force and strong force:

quarks	u	d	s	c	b	t
M_{weak} [MeV]	3	5	80	1200	4500	176000
M_{strong} [MeV]	350	350	350	350	350	350

Properties of QCD: Dynamical mass generation



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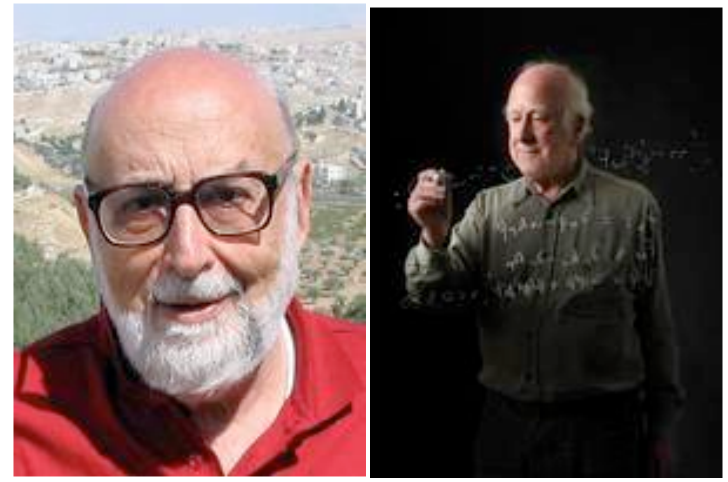

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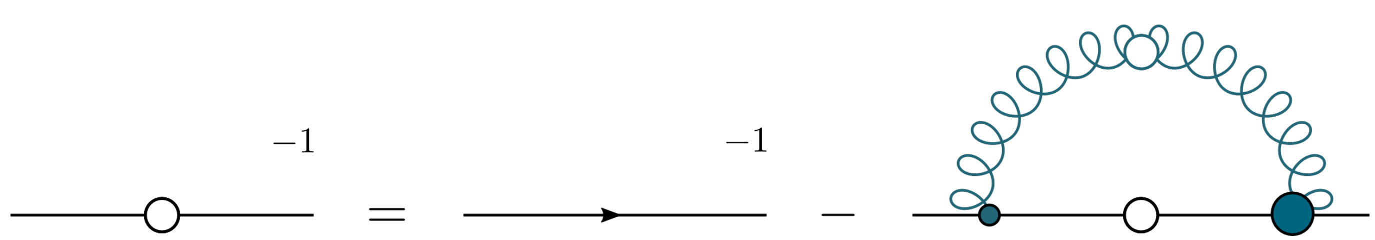
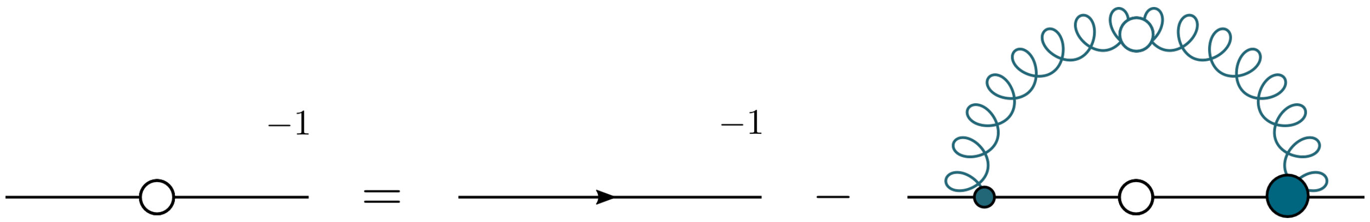
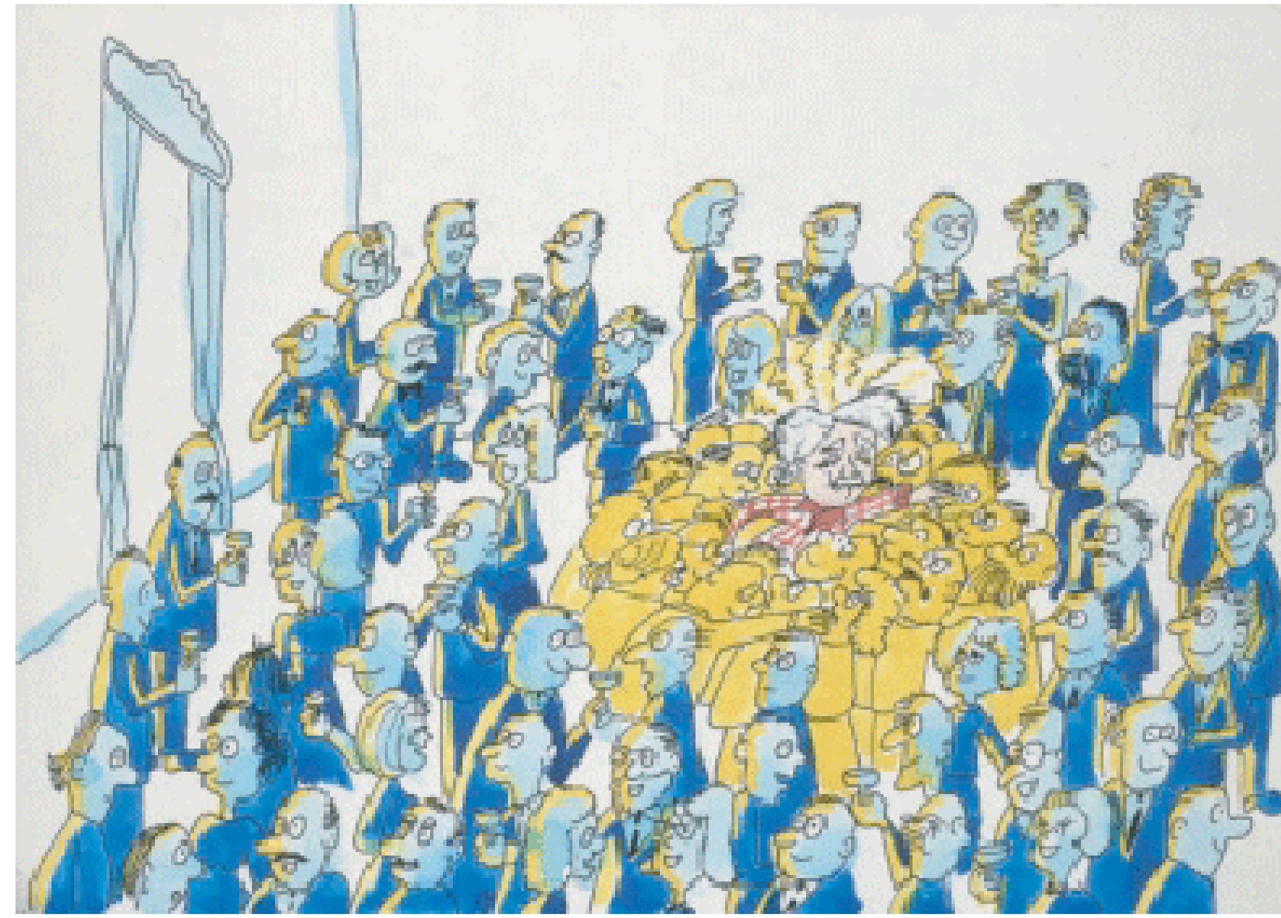
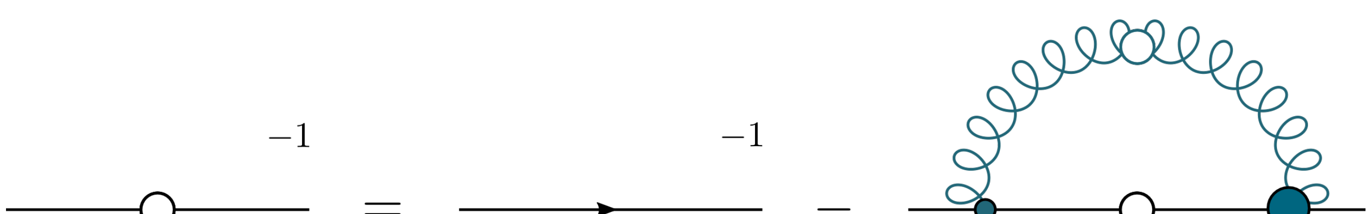


Dynamische Massenerzeugung

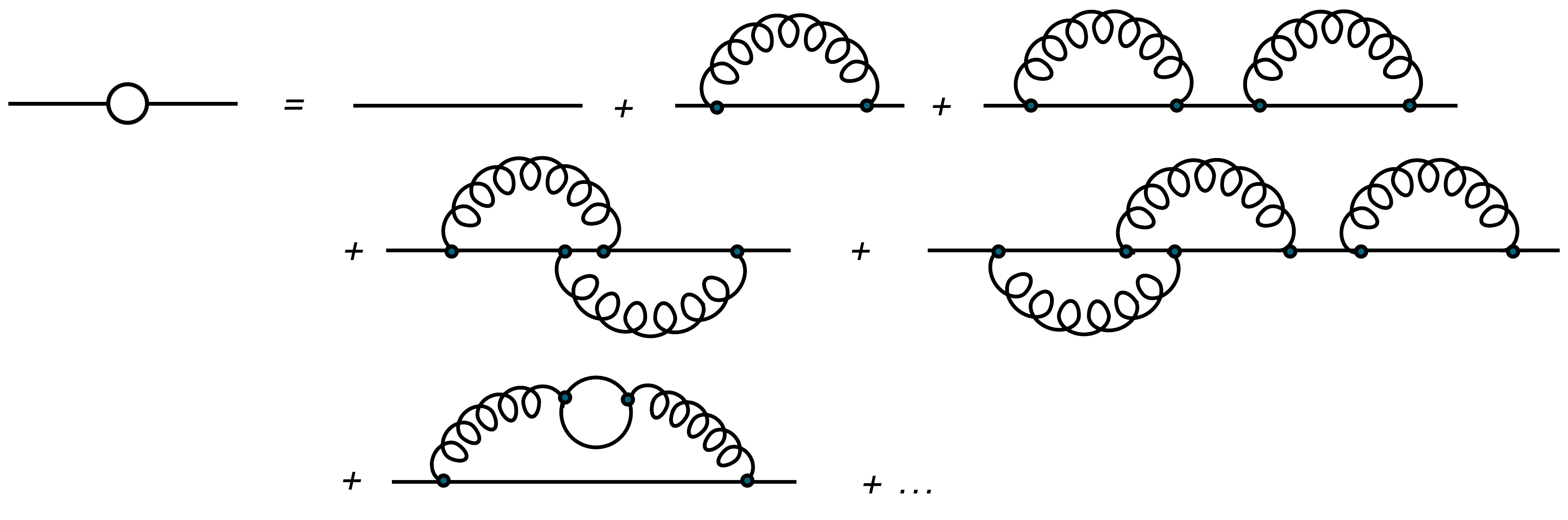


Dynamische Massenerzeugung



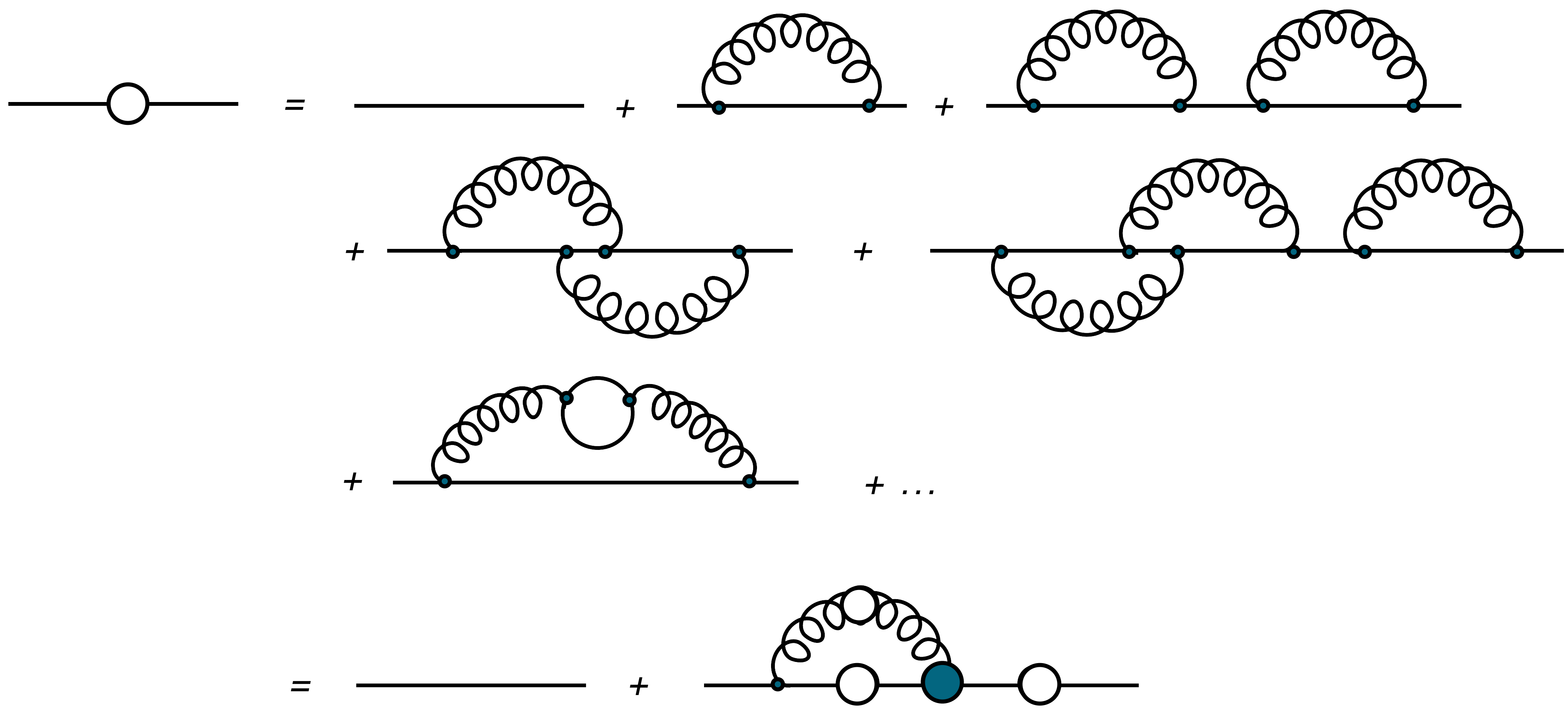


Graphical: start with perturbation theory and resum



Derivation of DSEs I

Graphical: start with perturbation theory and resum



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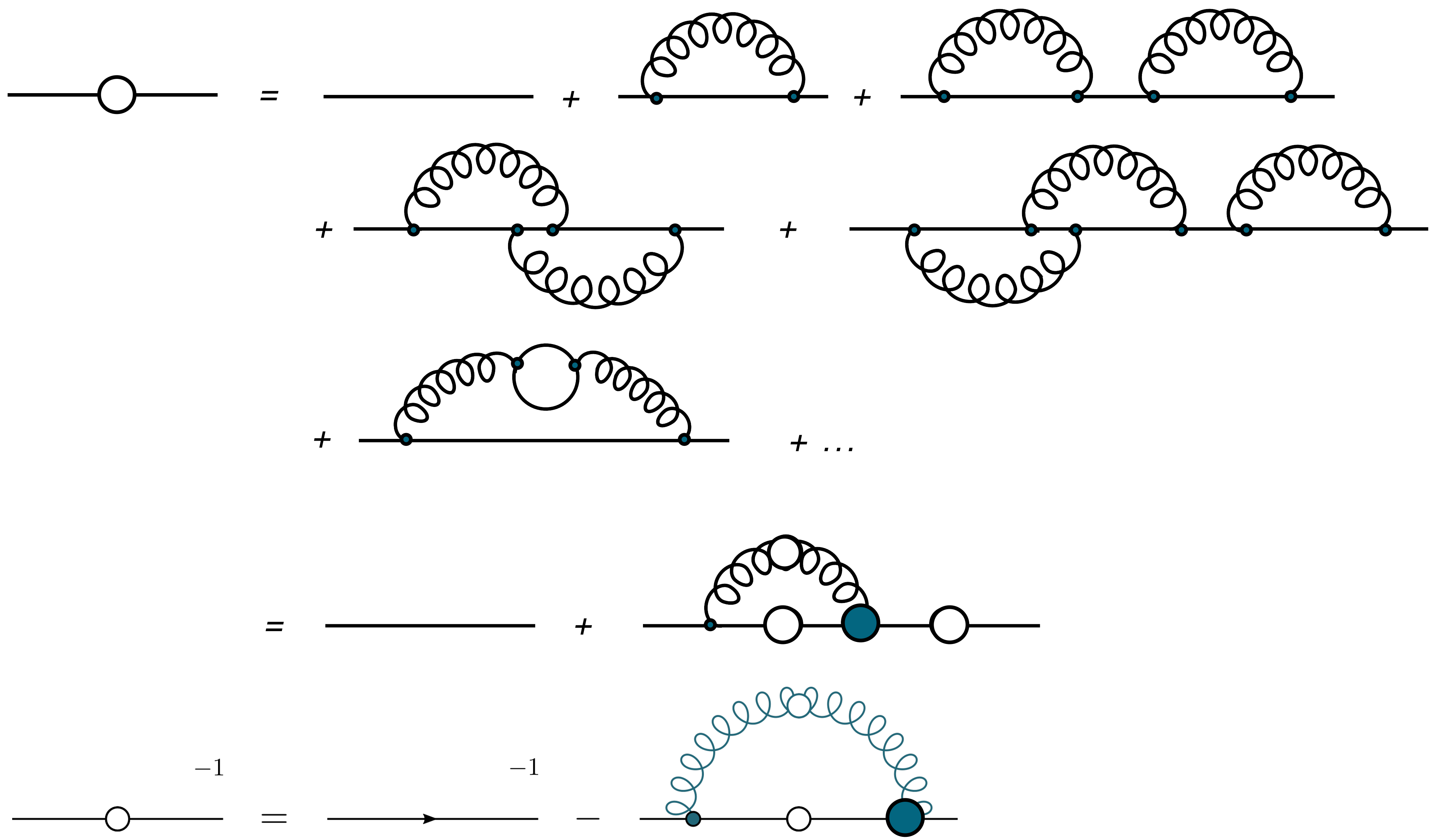


$$S^{-1}(p) = i\not{p}\, A(p^2) + B(p^2) \qquad S_0^{-1}(p) = i\not{p} + m$$

$$S^{-1}(p) = [i\not{p} + M(p^2)]/Z_f(p^2)$$

Derivation of DSEs II

Start from generating functional:

$$\mathcal{Z}[j] = \int \mathcal{D}[\Phi] \ \exp\{-S(\Phi) + j\Phi\} \qquad \text{with} \quad j\Phi = \int d^4x j(x)\Phi(x)$$

The integral of a total derivative vanishes:

$$0 = \frac{\delta}{\delta\Phi(y)}\mathcal{Z}[j] = \int \mathcal{D}[\Phi] \ \left(-\frac{\delta S(\Phi)}{\delta\Phi(y)} + j(y)\right) \ \exp\{-S(\Phi) + j\Phi\}$$

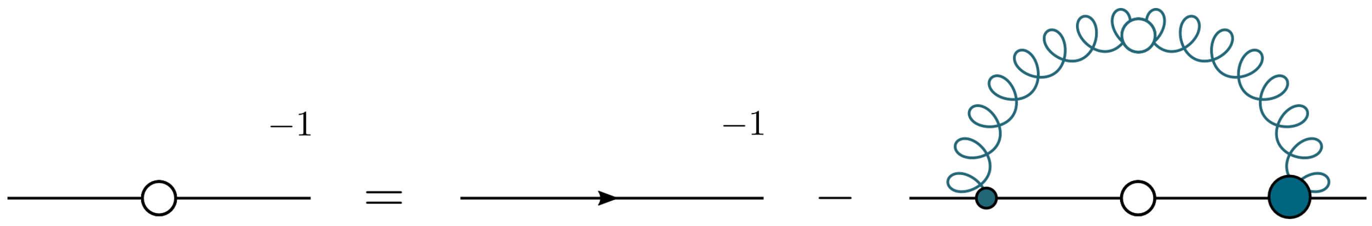
$$= \left\langle -\frac{\delta S(\Phi)}{\delta\Phi(y)} + j(y) \right\rangle$$

After a further derivative we set j=0 and obtain the DSE for the propagator:

$$0 = \frac{\delta^2}{\delta j(z)\delta\Phi(y)}\mathcal{Z}[j] = \left\langle -\frac{\delta S(\Phi)}{\delta\Phi(y)}\Phi(z) \right\rangle + \delta(y-z)$$

For the DSE of the quark propagator we obtain:

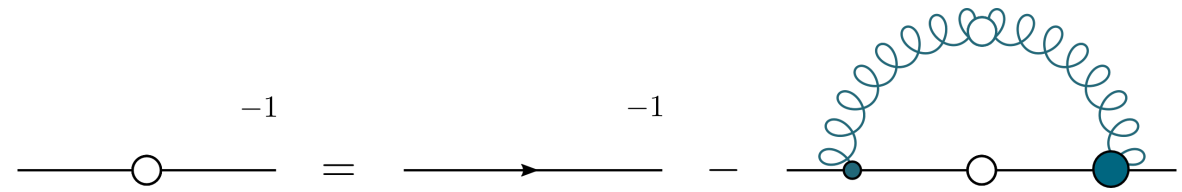
$$S^{-1}(p) = S_0^{-1}(p) + g^2 \int \frac{d^4q}{(2\pi)^4}\, t^a \gamma_\mu\, S(q)\, D^{ab}_{\mu\nu}(q-p)\, \Gamma^b_\nu(q,p)$$



- Tower of DSEs for Euclidean n-point functions
- Similar tower from functional renormalization group (FRG): different structure but similar content !

FRG: H. Gies,``Introduction to the functional RG and applications to gauge theories,'' hep-ph/0611146.
J.M.Pawlowski,``Aspects of the functional renormalisation group,'' Annals Phys. 322 (2007) 2831 [hep-th/0512261].

Simple example:



Take bare gluon propagator: $D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{1}{k^2}$

and bare quark-gluon-vertex: $\Gamma_\mu(p,q) = i\gamma_\mu$

$$S^{-1}(p) = S_0^{-1}(p) + g^2 C_F \int \frac{d^4q}{(2\pi)^4}\, \gamma_\mu D_{\mu\nu}(k)\, S(q)\, \gamma_\nu$$

with $C_F = \frac{N_c^2-1}{2N_c} \to \frac{4}{3}$

$$S^{-1}(p) = i\not{p}A(p^2) + B(p^2)$$

$$S_0^{-1}(p) = i\not{p} + m \qquad \longrightarrow \text{project onto Dirac structures}$$

$$B(p^2) = m + g^2\,\frac{4}{3}\int \frac{d^4q}{(2\pi)^4}\,\frac{1}{k^2}\,\frac{3B(q^2)}{q^2A^2(q^2)+B^2(q^2)}$$

$$A(p^2) = 1 + g^2\,\frac{4}{3}\int \frac{d^4q}{(2\pi)^4}\,\frac{1}{k^2}\,\frac{A(q^2)}{q^2A^2(q^2)+B^2(q^2)}\left[-\frac{k^2}{p^2} + \frac{p^2+q^2}{2p^2} + \frac{(p^2-q^2)^2}{2p^2k^2}\right]$$

Dynamical chiral symmetry breaking II

In our simple example $A \approx 1$, then:

$$B(p^2) = m + g^2 \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} \frac{1}{k^2} \frac{3B(q^2)}{q^2 + B^2(q^2)}$$

Transform $\int d^4q$ in hyperspherical coordinates and perform angular integrals analytically ($\alpha = g^2/4\pi$):

$$B(p^2) = m + \alpha \int_0^{p^2} dq^2 \, \frac{q^2}{p^2} \frac{B(q^2)}{q^2 + B^2(q^2)} + \alpha \int_{p^2}^{\Lambda^2} dq^2 \, \frac{B(q^2)}{q^2 + B^2(q^2)}$$

This equation for the quark mass function $M(p^2) = B(p^2)/A(p^2)$ has a typical structure. ⟶

Dynamical chiral symmetry breaking III

Consider chiral limit $m=0$:

$$B(p) = \alpha \int_0^{p^2} dq^2 \, \frac{q^2}{p^2} \, \frac{B(q)}{q^2+B^2(q)} + \alpha \int_{p^2}^{\Lambda^2} dq^2 \, \frac{B(q)}{q^2+B^2(q)}$$

Dynamical chiral symmetry breaking III

Consider chiral limit $m=0$:

$$B(p) = \alpha \int_0^{p^2} dq^2 \, \frac{q^2}{p^2} \, \frac{B(q)}{q^2+B^2(q)} + \alpha \int_{p^2}^{\Lambda^2} dq^2 \, \frac{B(q)}{q^2+B^2(q)}$$

Three solutions:

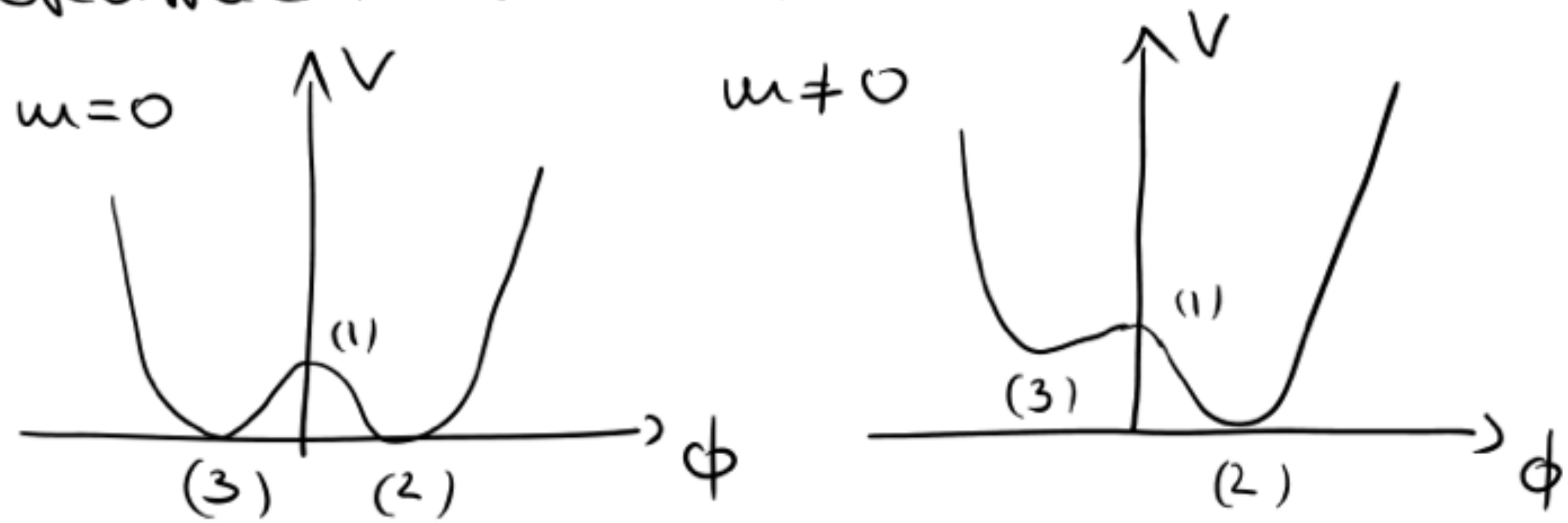
(1) $B(p) \equiv 0$ $\longrightarrow$ chiral symmetric: Wigner-Weyl

(2,3) $\pm B(p) \neq 0$ $\longrightarrow$ chiral symmetry broken: Nambu-Goldstone

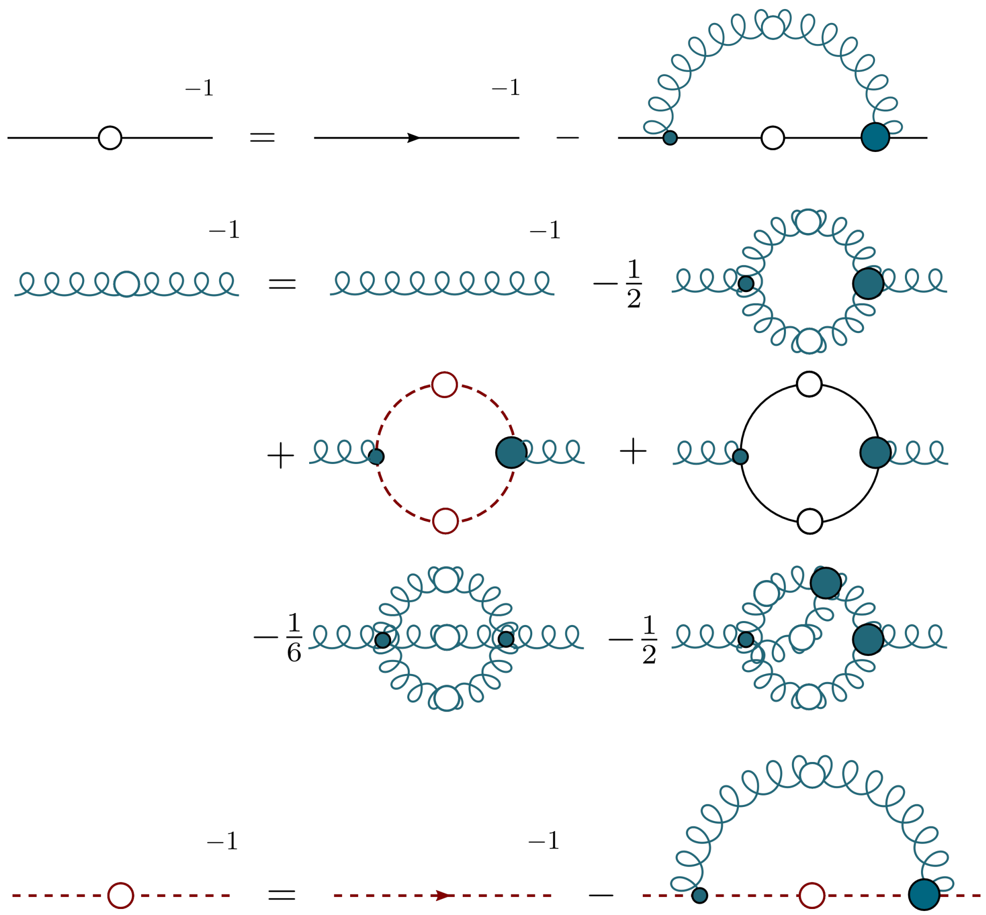
cp. to effective potential in scalar models:

(1) metastable

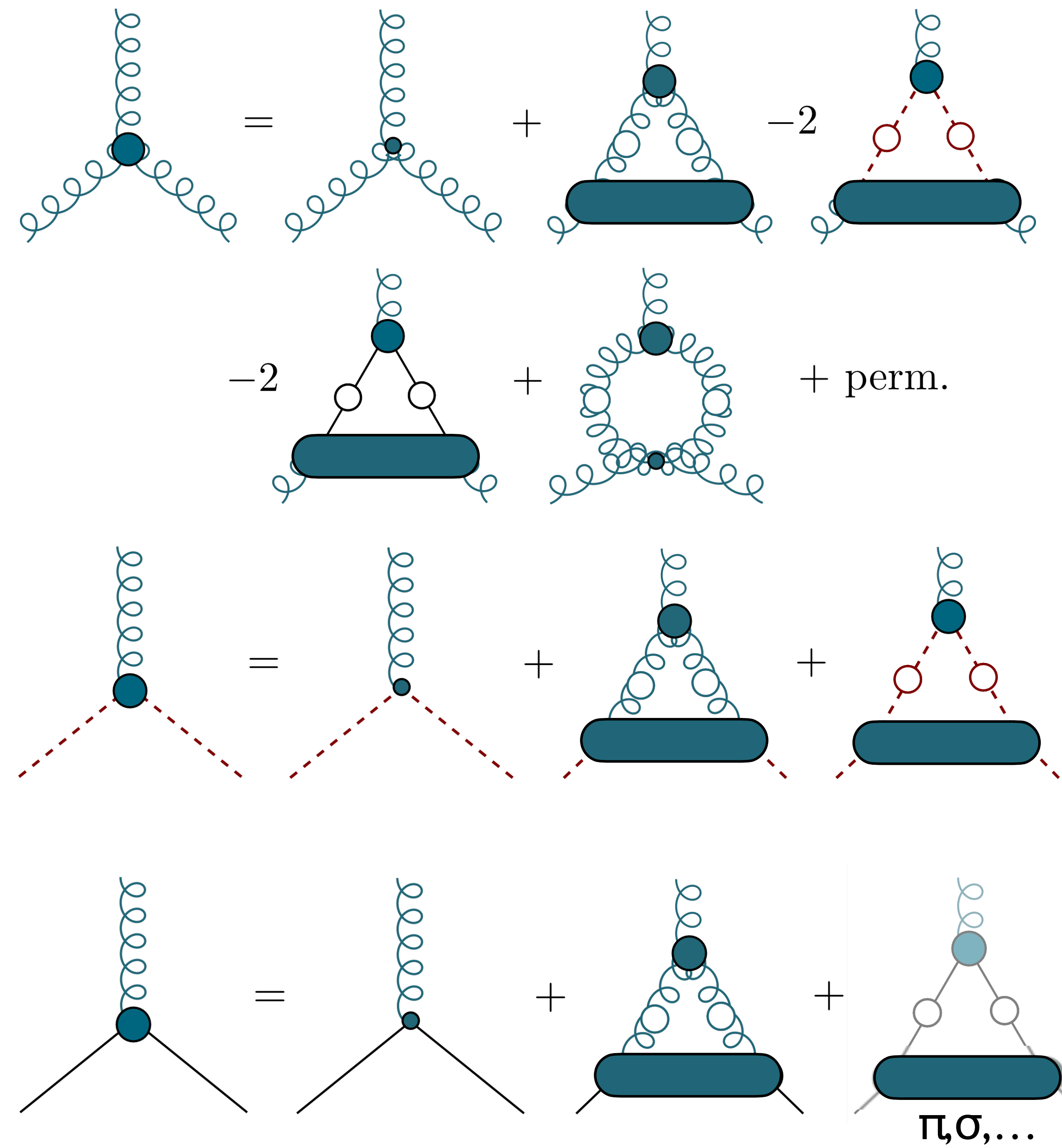
(2,3) stable



propagators



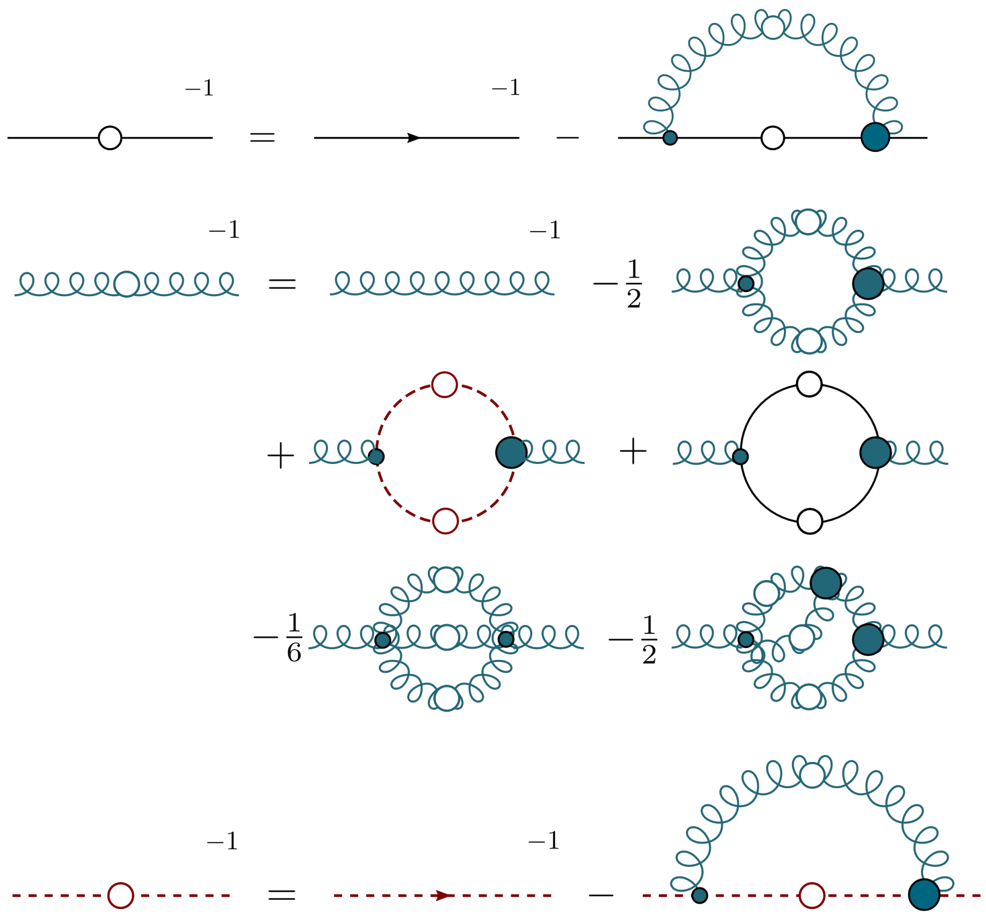
vertices



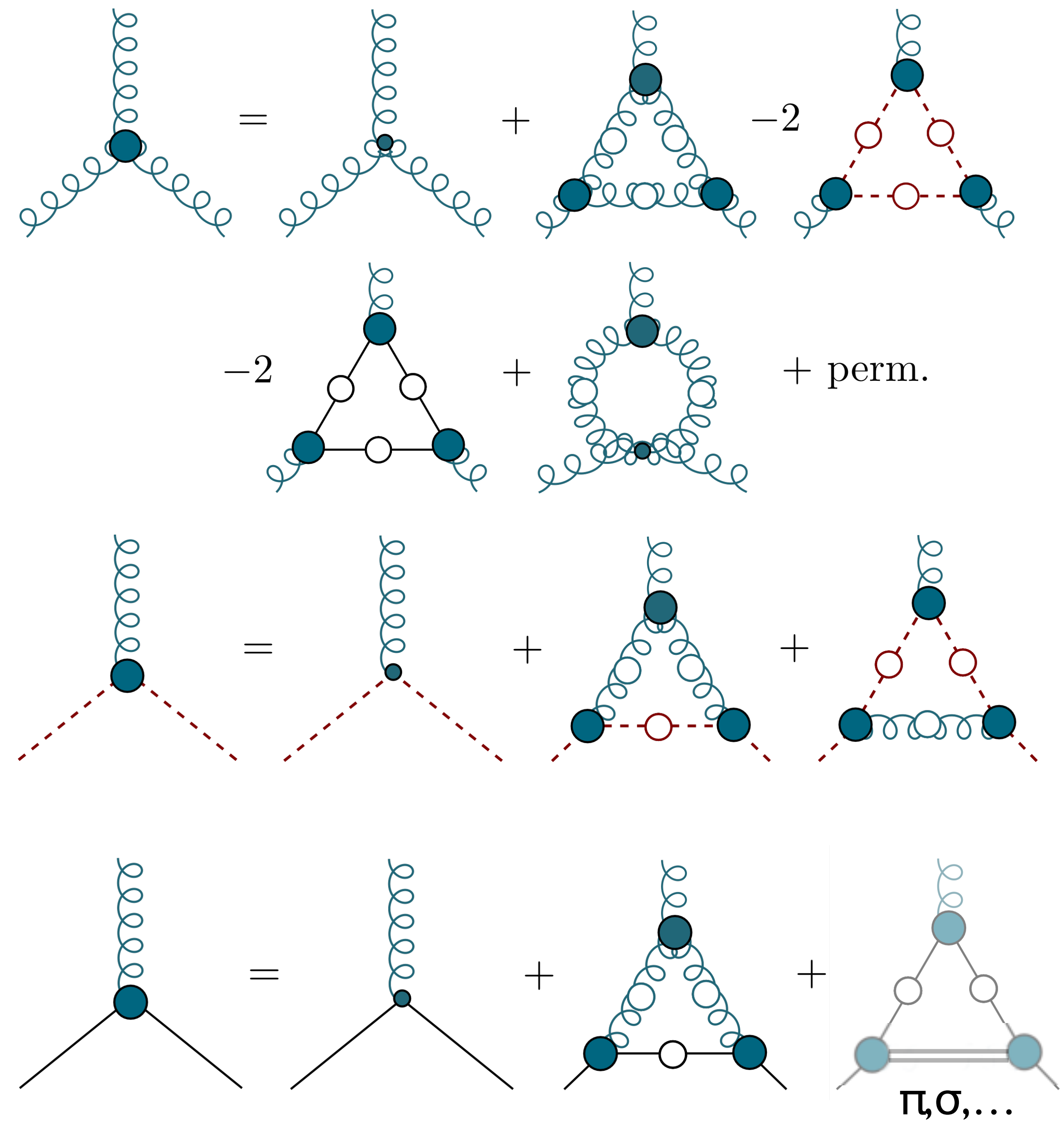
CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

propagators



vertices



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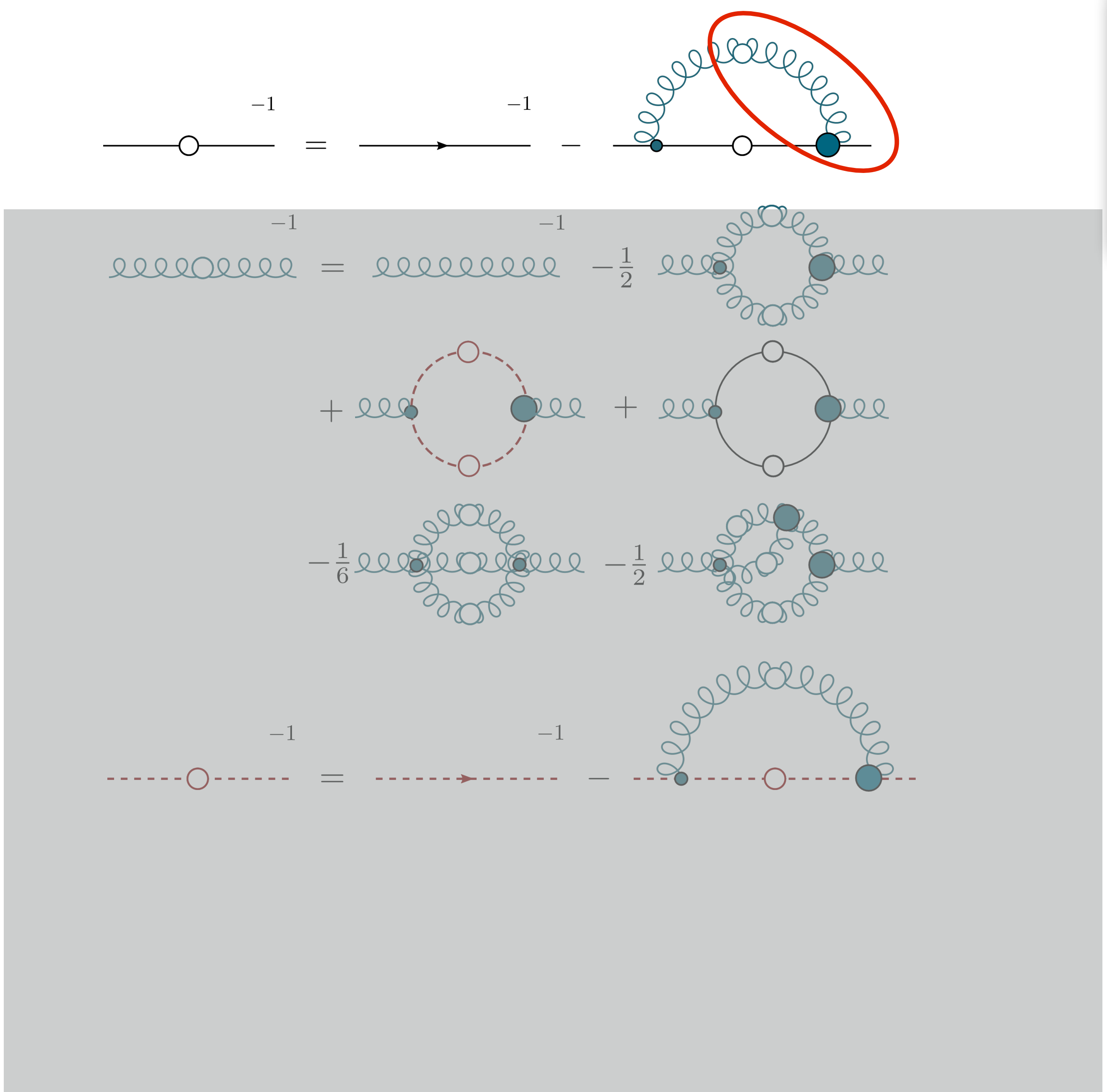
propagators



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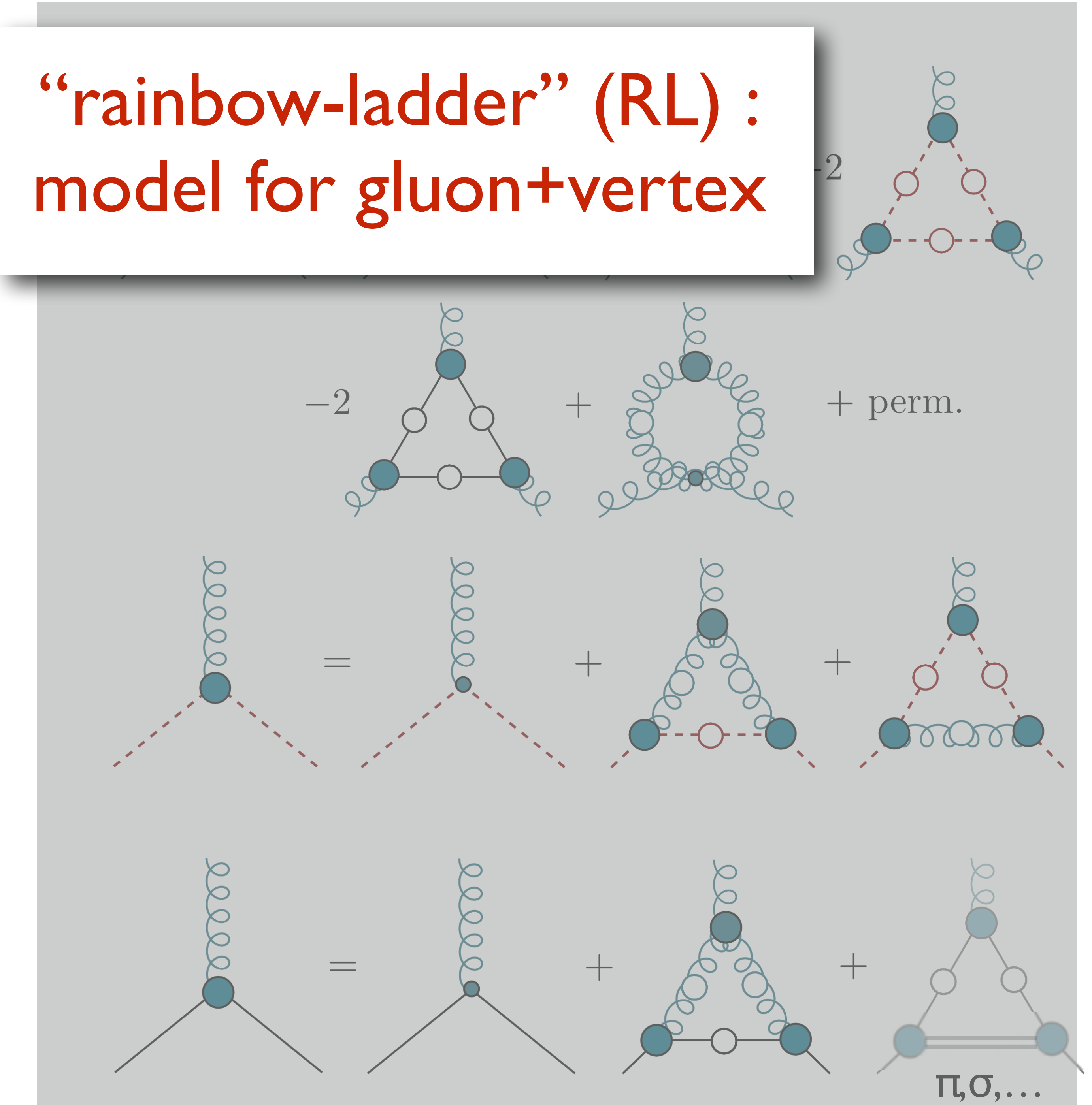

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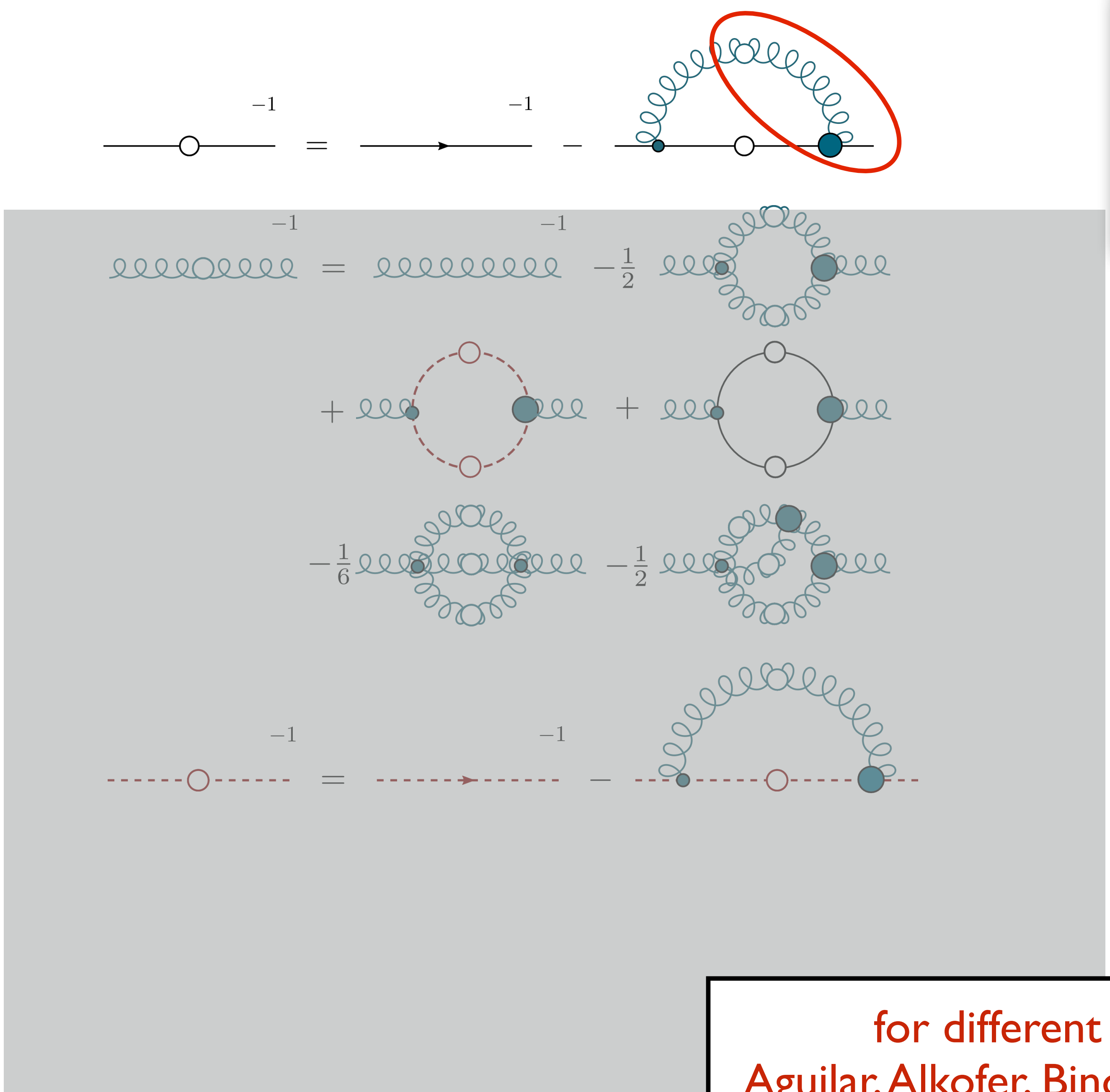
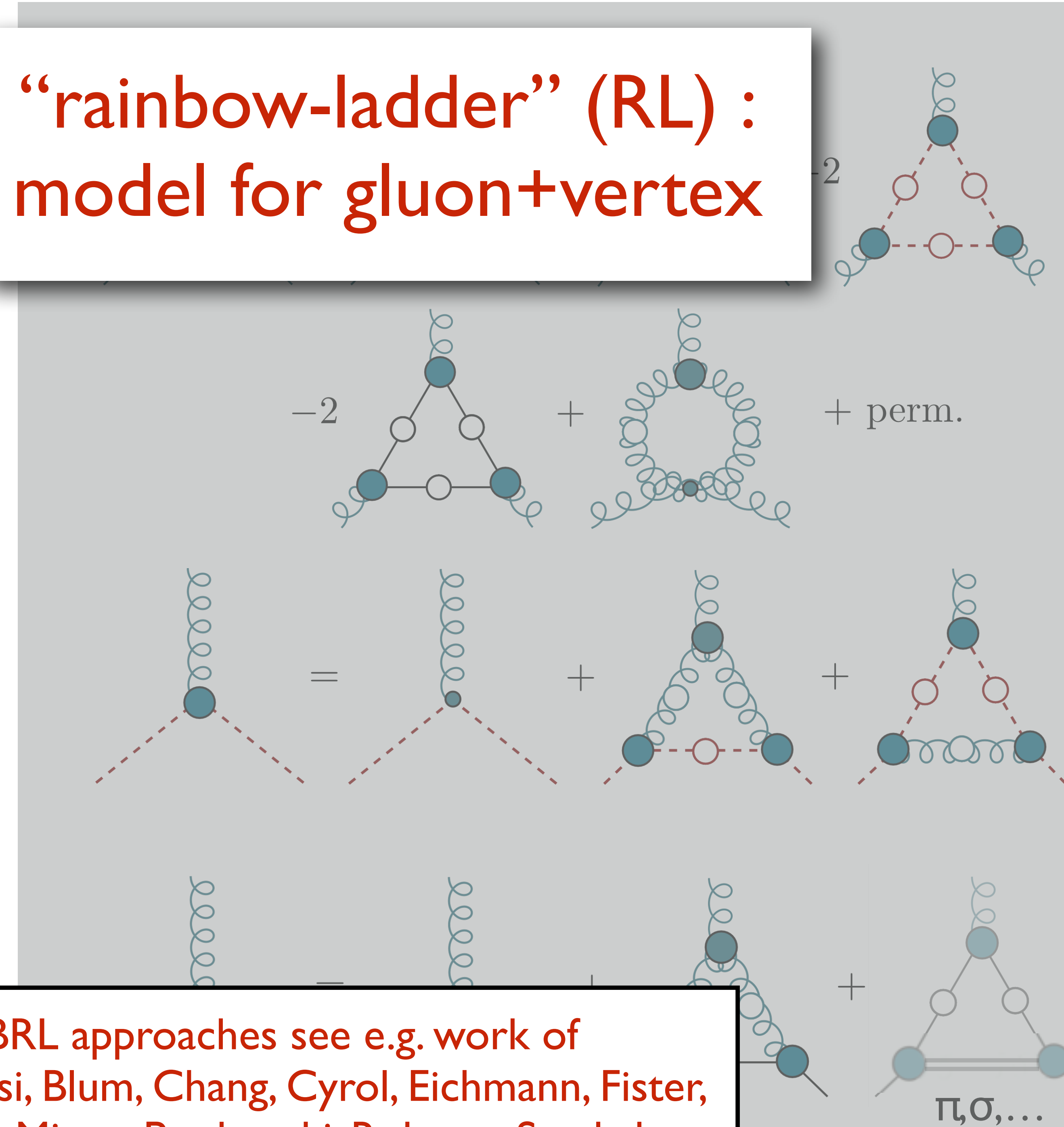


vertices



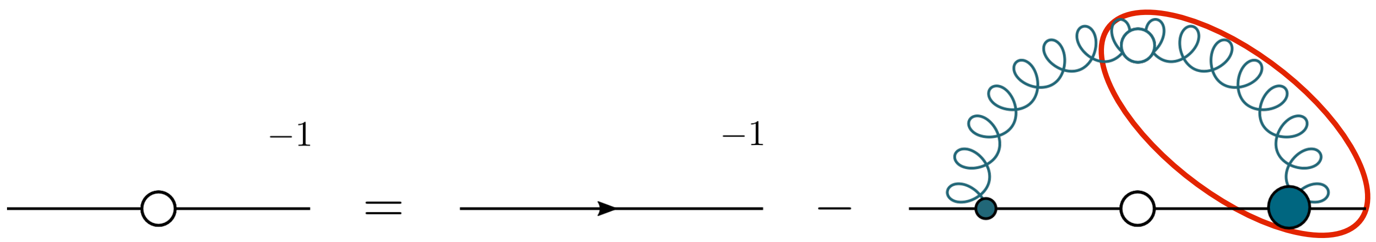
"rainbow-ladder" (RL) :
model for gluon+vertex

for different BRL approaches see e.g. work of
Aguilar, Alkofer, Binosi, Blum, Chang, Cyrol, Eichmann, Fister,
Gao, Huber, Maas, Mitter, Pawlowski, Roberts, Smekal,
Strodthoff, Vujinovic, Watson, Williams…

Review: Eichmann, Sanchis-Alepuz, Williams, Alko…

…67 (2003) 094020
…upel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

Rainbow-ladder model for quark-gluon interaction



Combine gluon with quark-gluon vertex:

$$\Gamma^\mu(p,k) = \sum_{i=1,12} \tau_i(p,k) T_i^\mu$$

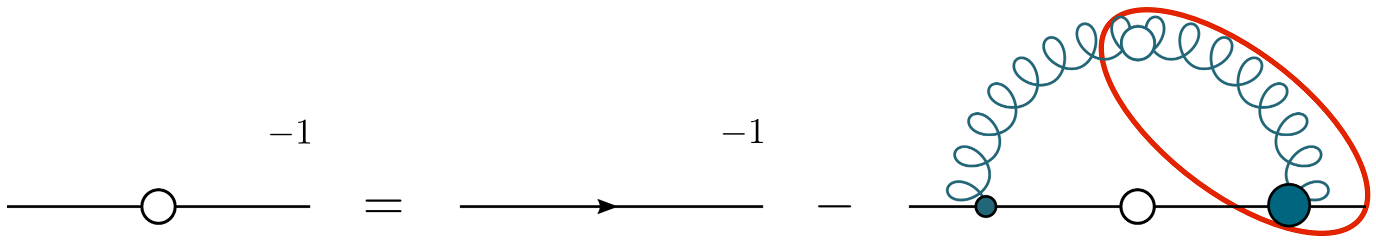
$$\sim \gamma^\mu \tau(k^2)$$

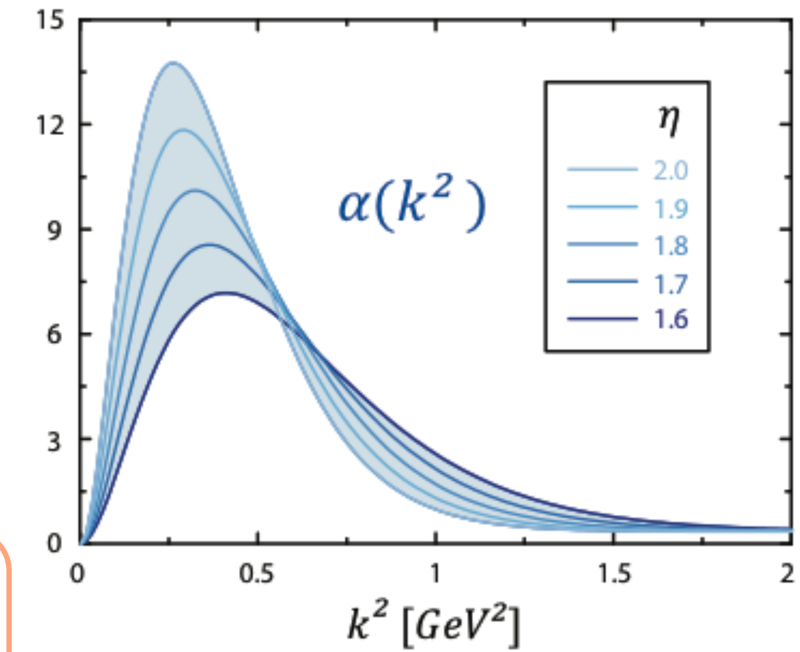
"approximation" !

$$D^{\mu\nu}(k) = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) \frac{Z(k^2)}{k^2}$$

$$\frac{g^2}{4\pi} \tau(k^2) Z(k^2) \sim \alpha(k^2)$$

Rainbow-ladder model for quark-gluon interaction





Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

Combine gluon with quark-gluon vertex:

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

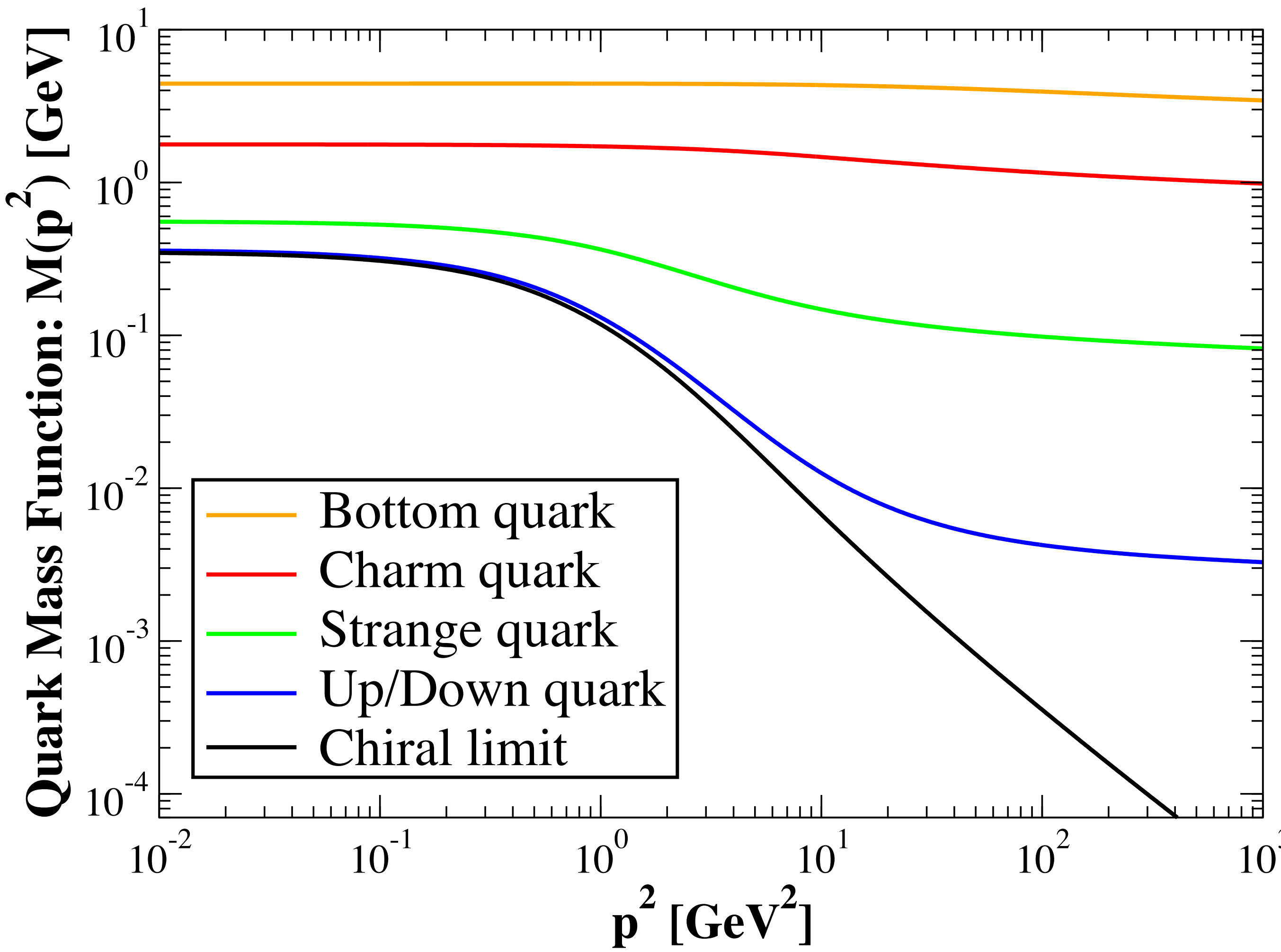
- scale Λ from f_π, masses $m_u=m_d$, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : results almost independent
- qualitatively similar to explicit calc.

Williams, EPJA 51 (2015) 5, 57.
Sanchis-Alepuz, Williams, PLB 749 (2015) 592;
Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein

Quark mass: flavor dependence

Typical solution:

$$S(p) = \frac{-i\not{p}\, A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}$$
$$= Z_f(p^2)\, \frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

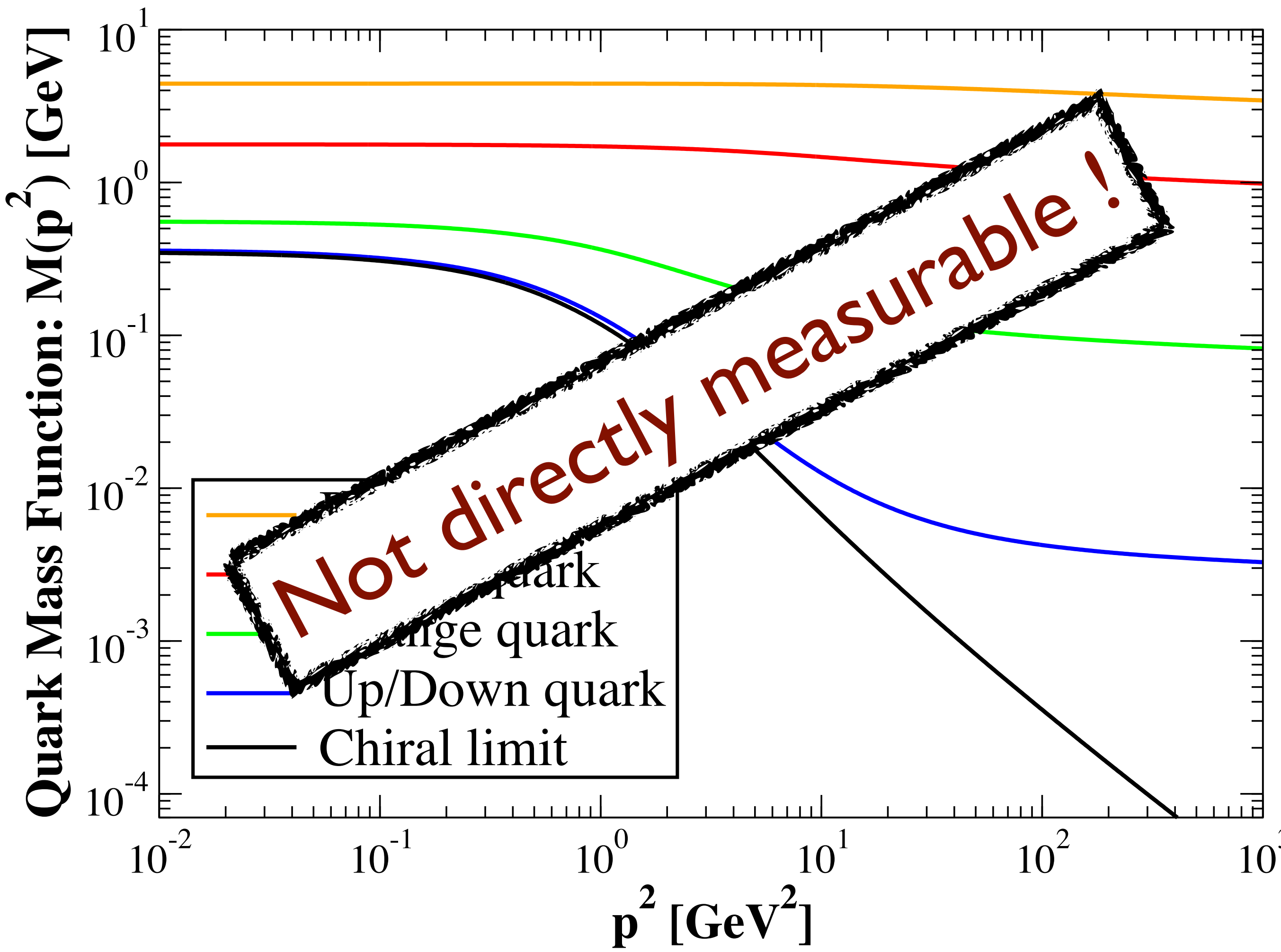


- $M(p^2)$: momentum dependent!
- Dynamical mass: $M_{strong} \approx 350$ MeV
- Flavour dependence because of m_{weak}
- Chiral condensate: $-\langle\bar\Psi\Psi\rangle \approx (250\,\mathrm{MeV})^3$

$$-\langle\bar\Psi\Psi\rangle = Z_2 Z_m N_c \int_p Tr S(p)$$

Quark mass: flavor dependence

Typical solution:



$$S(p) = \frac{-i\not{p}\, A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}$$

$$= Z_f(p^2)\, \frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

- M(p²): momentum dependent!
- Dynamical mass: $M_{strong} \approx 350$ MeV
- Flavour dependence because of m_{weak}
- Chiral condensate: - $\langle \bar{\Psi}\Psi \rangle \approx (250\,\mathrm{MeV})^3$

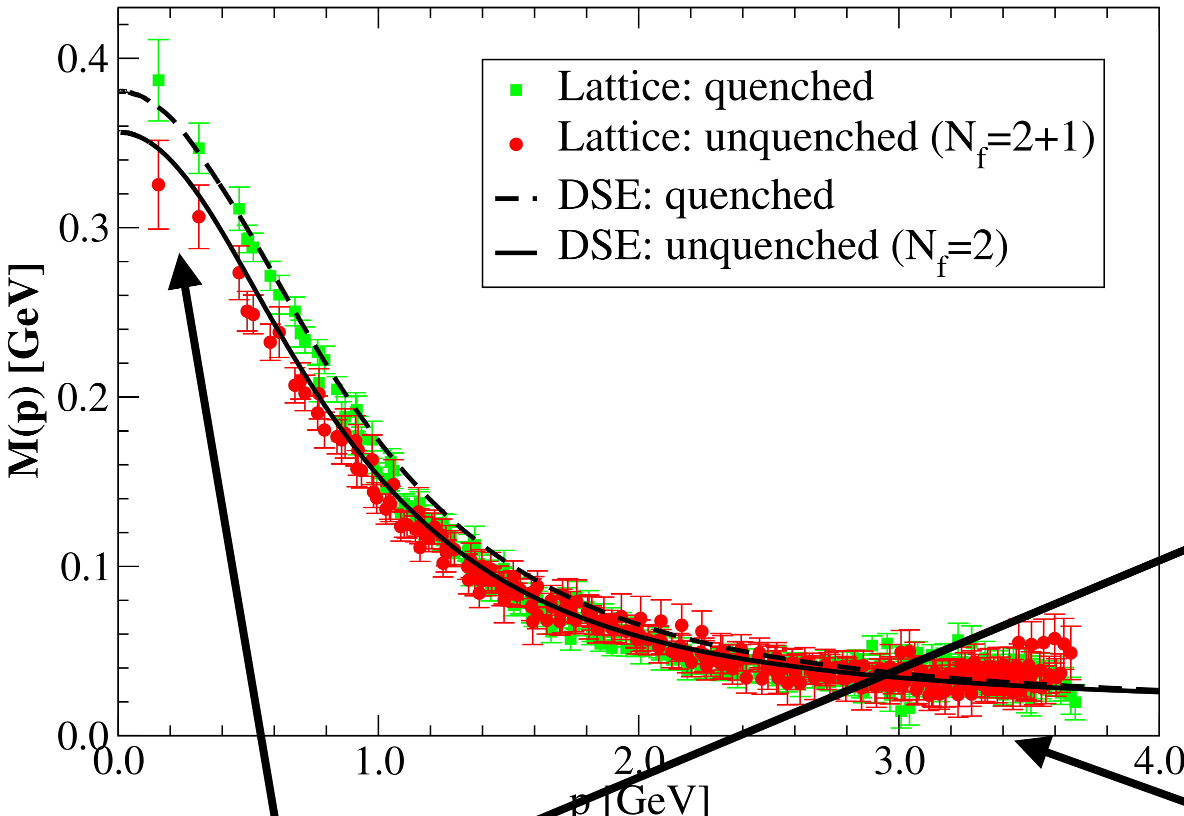
$$-\langle \bar{\Psi}\Psi \rangle = Z_2 Z_m N_c \int_p Tr S(p)$$

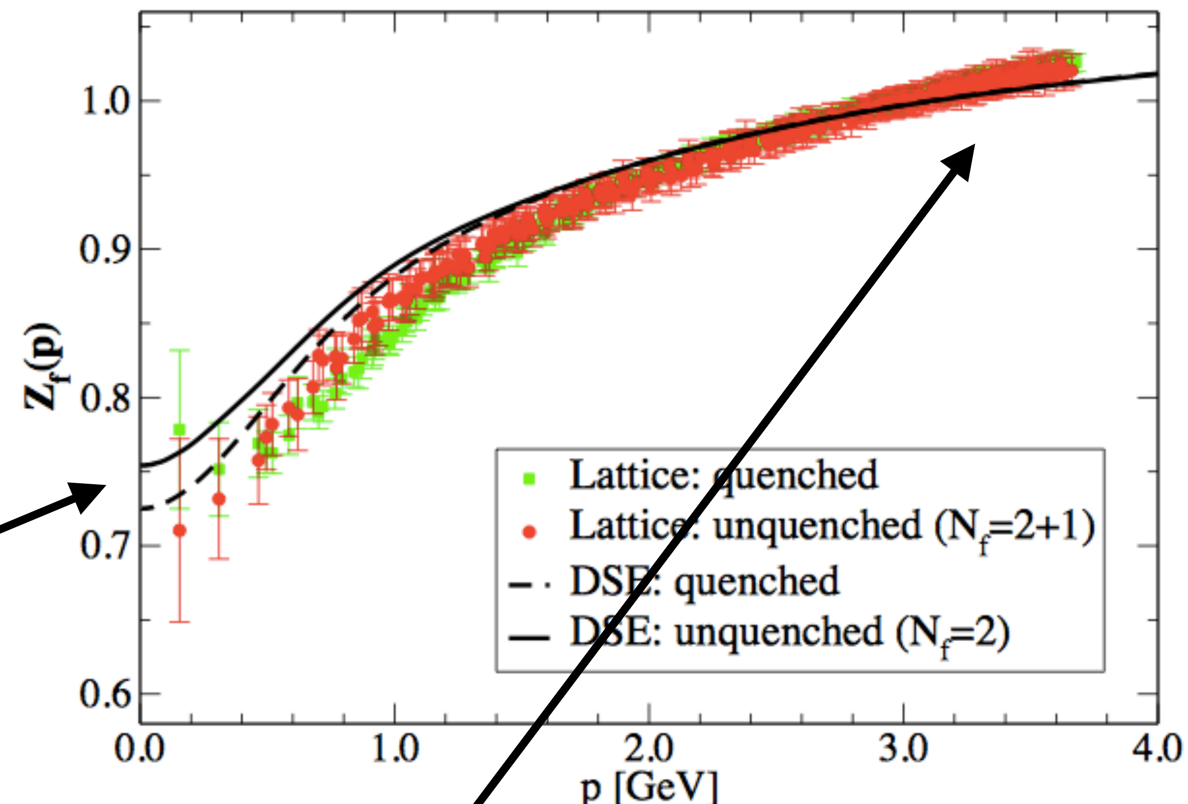
Quark dressing - comparison with lattice

$$S(p) = Z_f(p^2)\frac{-i\not p + M(p^2)}{p^2 + M^2(p^2)}$$

Beyond rainbow-ladder:

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507





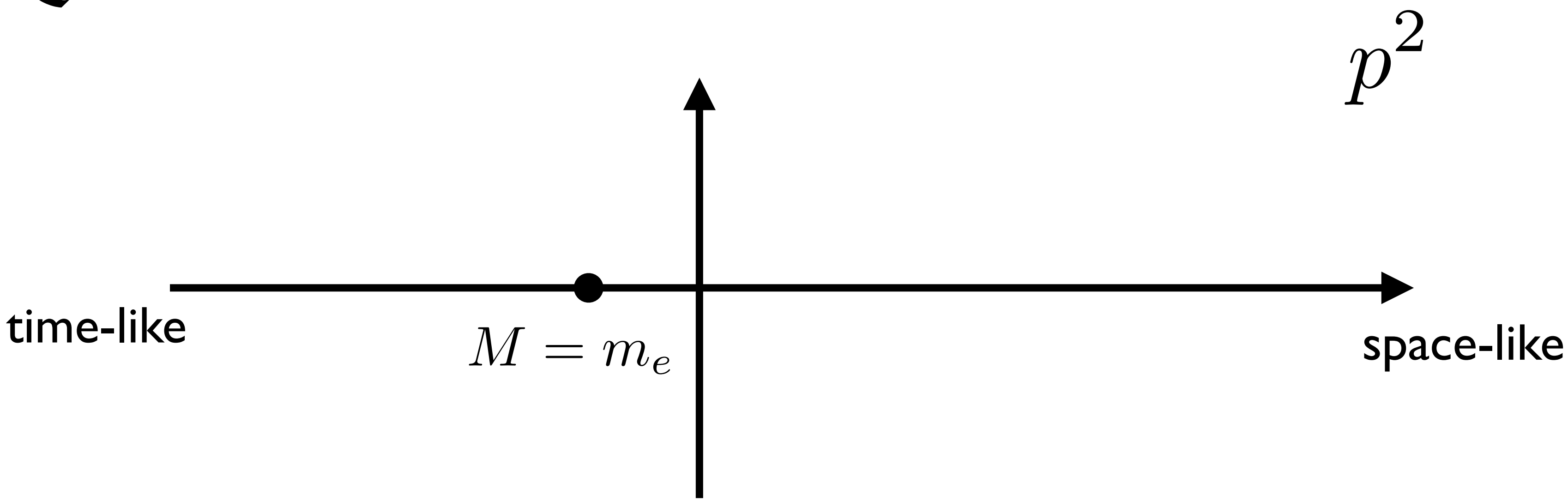
‘constituent quark’:
large mass; very composite

‘current quark’:
- small mass; non-composite

What about time-like momenta?

$$S(p) = Z_f(p^2)\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

QED: electron



Alkofer, Detmold, CF, Maris, PRD 70 (2004) 014014
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

What about time-like momenta?

$$S(p) = Z_f(p^2)\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

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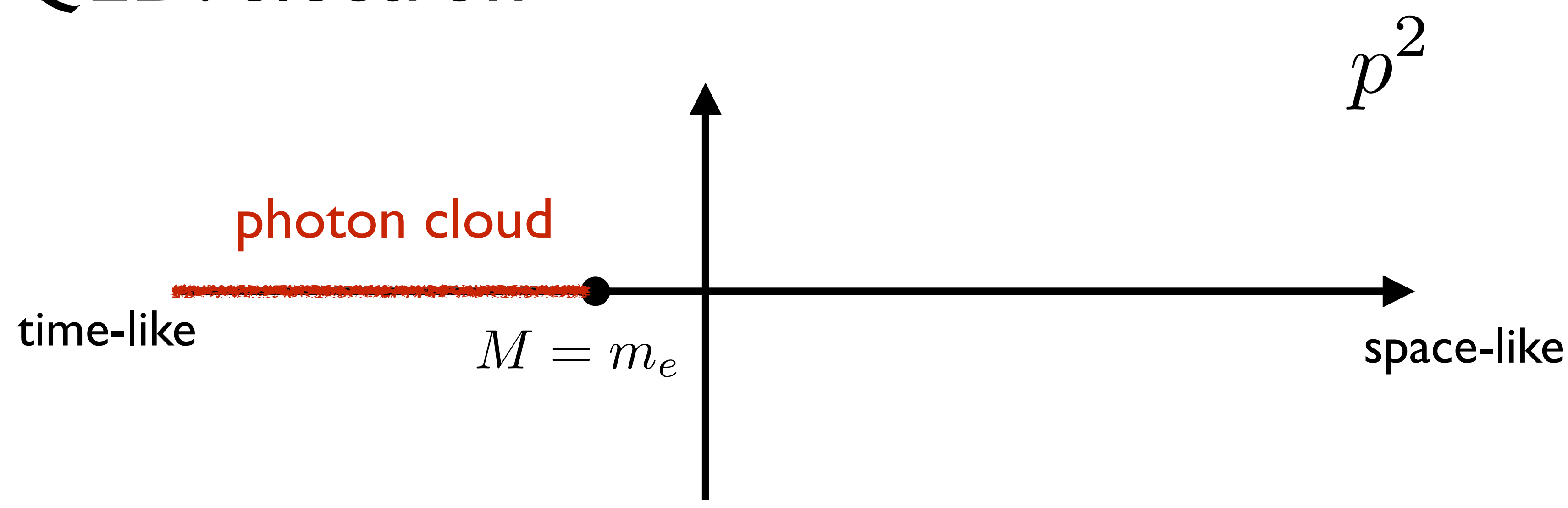

physical particle:
electron+photon cloud

Alkofer, Detmold, CF, Maris, PRD 70 (2004) 014014
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

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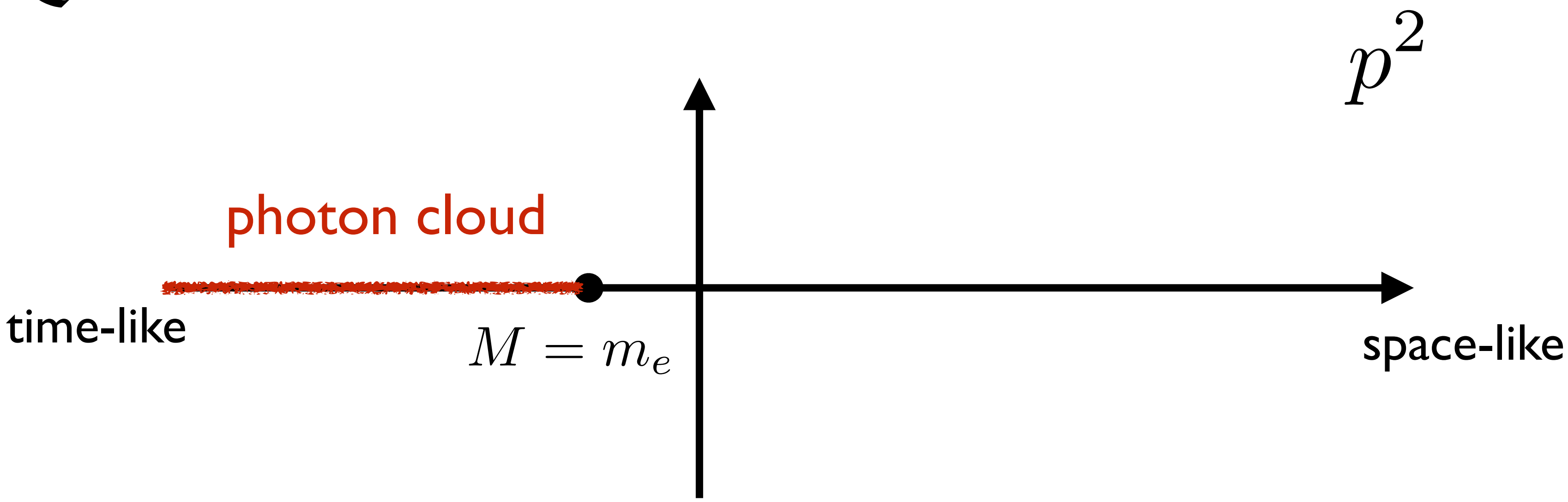
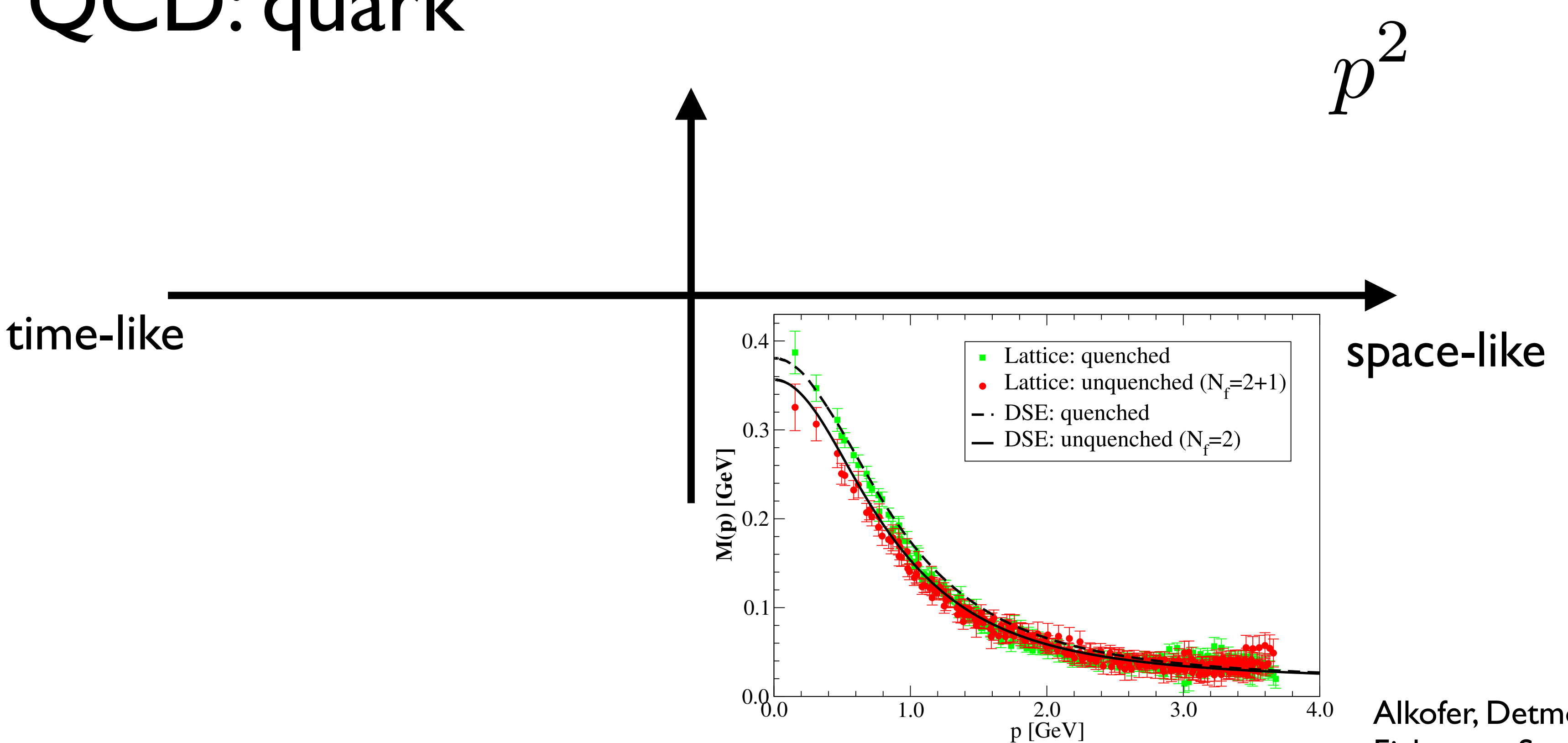
QED: electron



physical particle:
electron+photon cloud

QCD: quark



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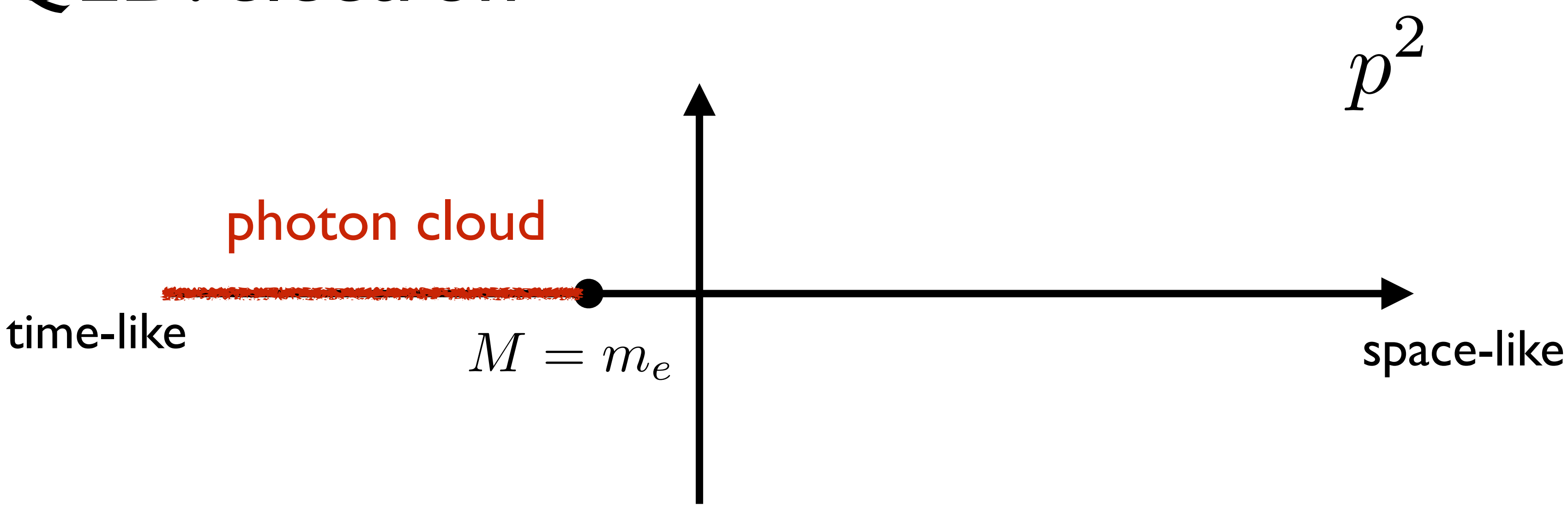
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

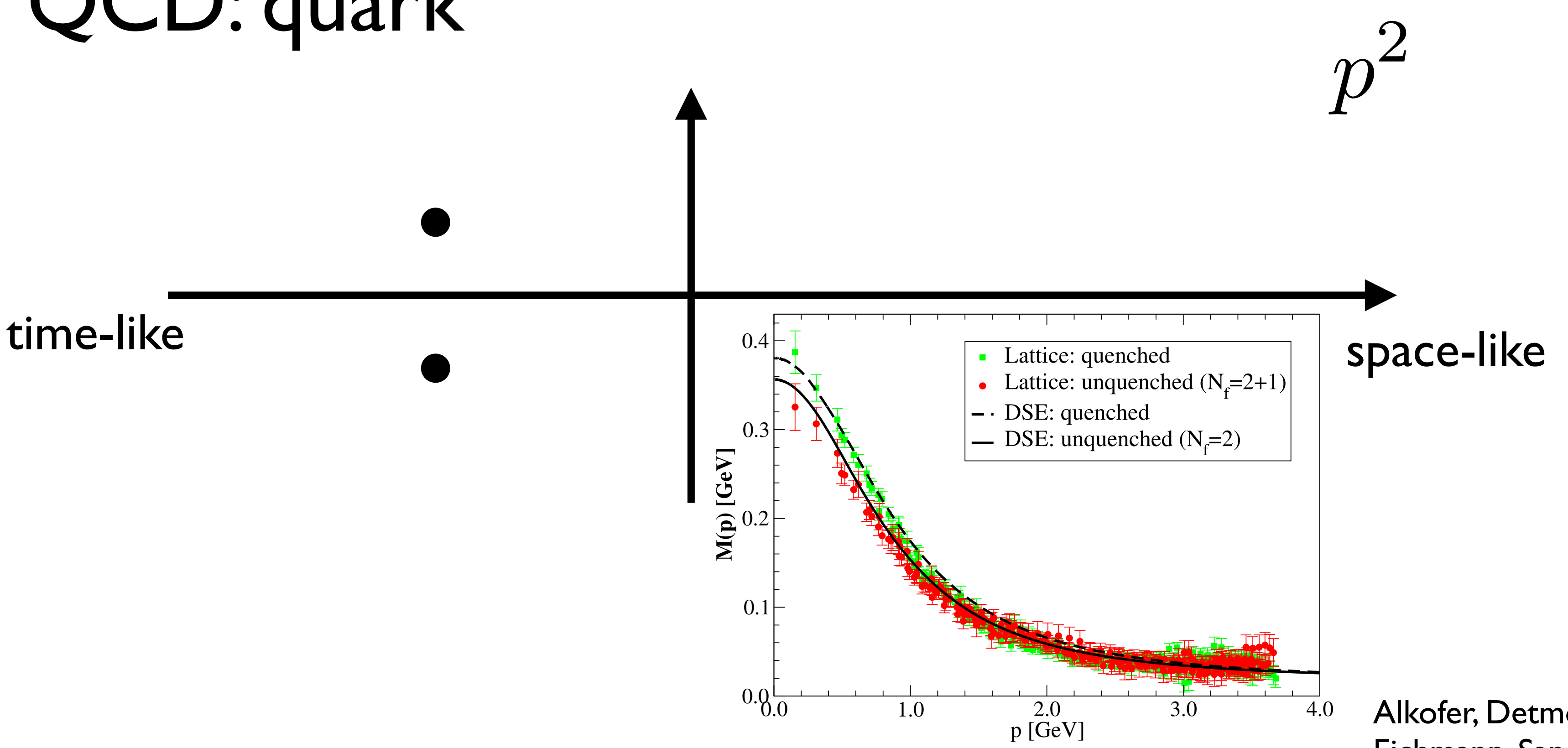
$$S(p) = Z_f(p^2)\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

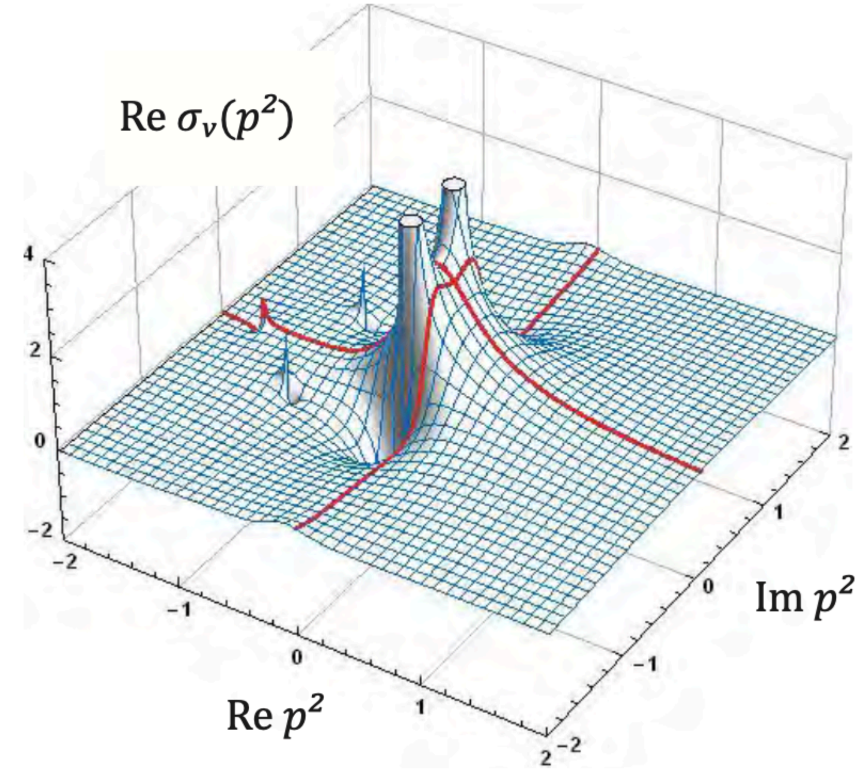
QED: electron



physical particle:
electron+photon cloud

QCD: quark





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Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

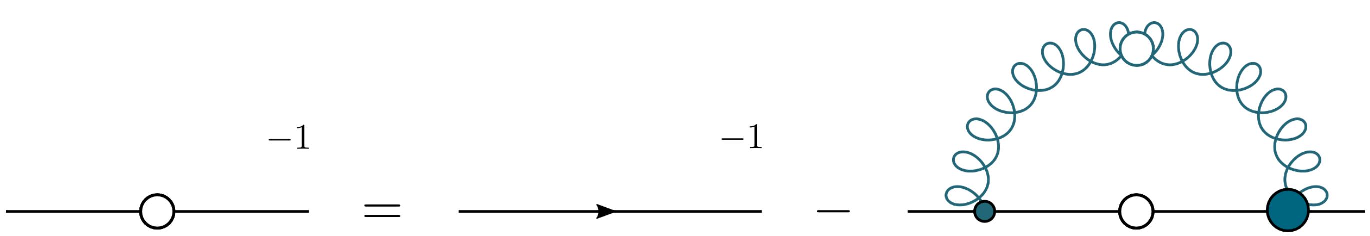
Hadron physics with functional methods

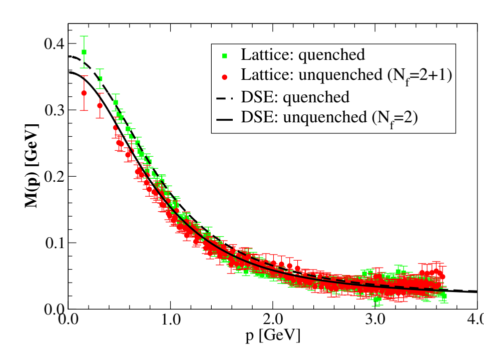
Lecture 2

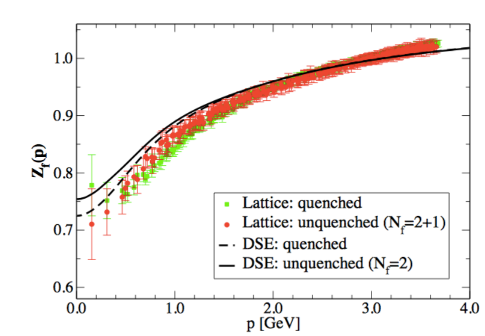
Bits and pieces to remember from Lecture 1

- non-relativistic quark model
  - source for classification of 'exotic quantum numbers'
    —> absent in quark-model but possible in relativistic theory
  - works with non-relativistic structure for forces (+rel. corr.)
    —> cp with exp. spectrum: LS dominates over SS

- functional methods: DSEs and BSEs (and FRGs)
  - derived exactly from QCD path integral
  - quark-DSE displays mechanism for dynamical mass generation
    —> already visible at simplest possible approximation
    —> not present in perturbation theory
    —> important part of dynamical chiral symmetry breaking

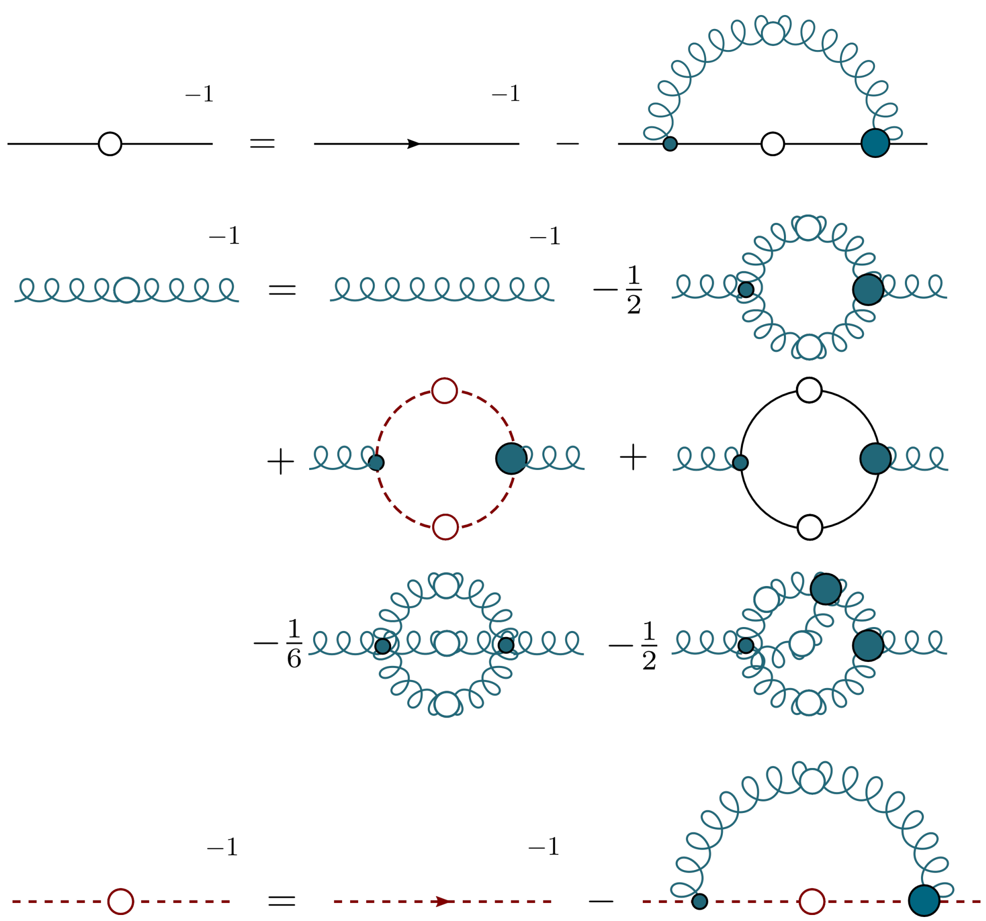




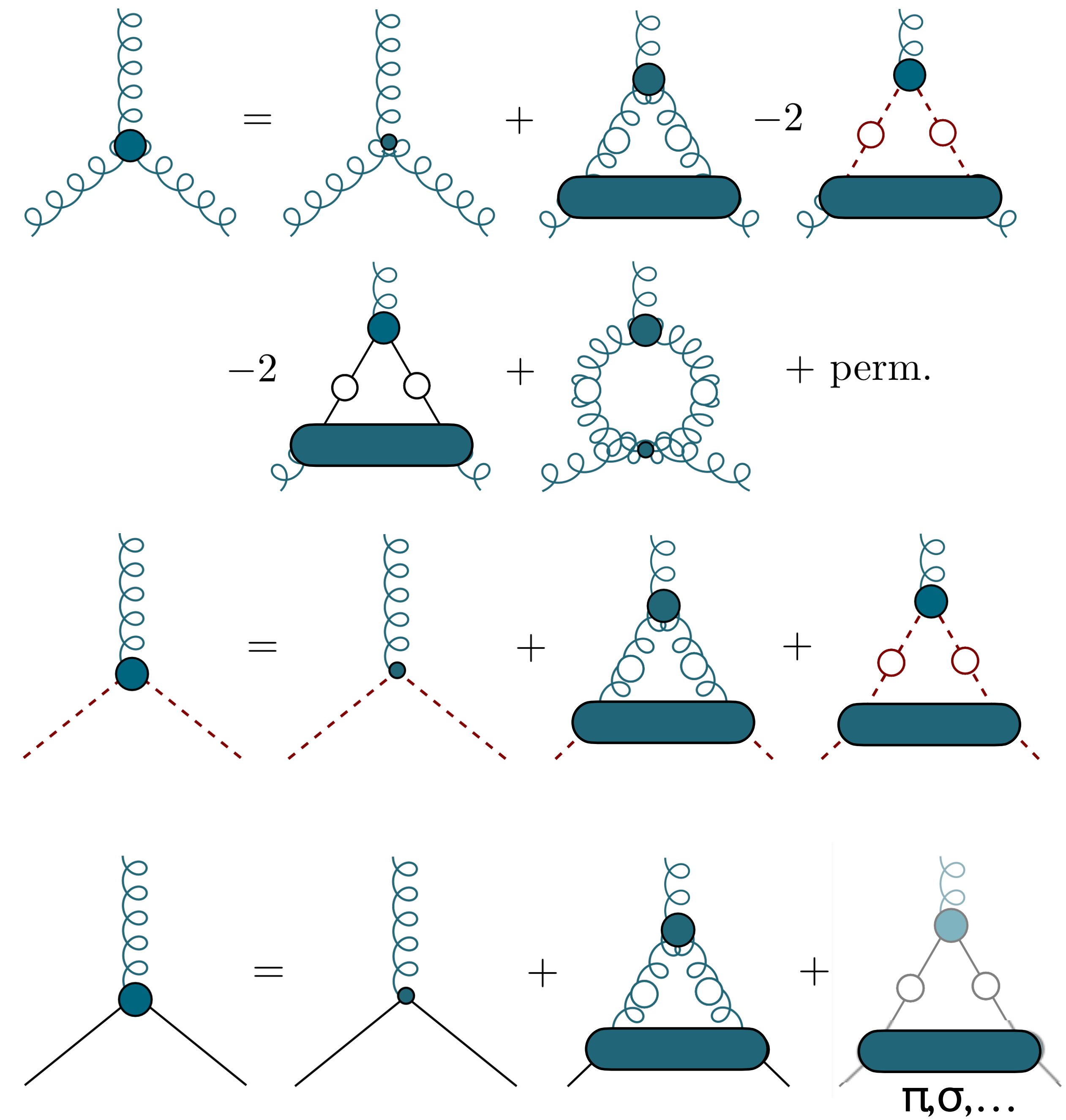


QCD with DSE

propagators



vertices



CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

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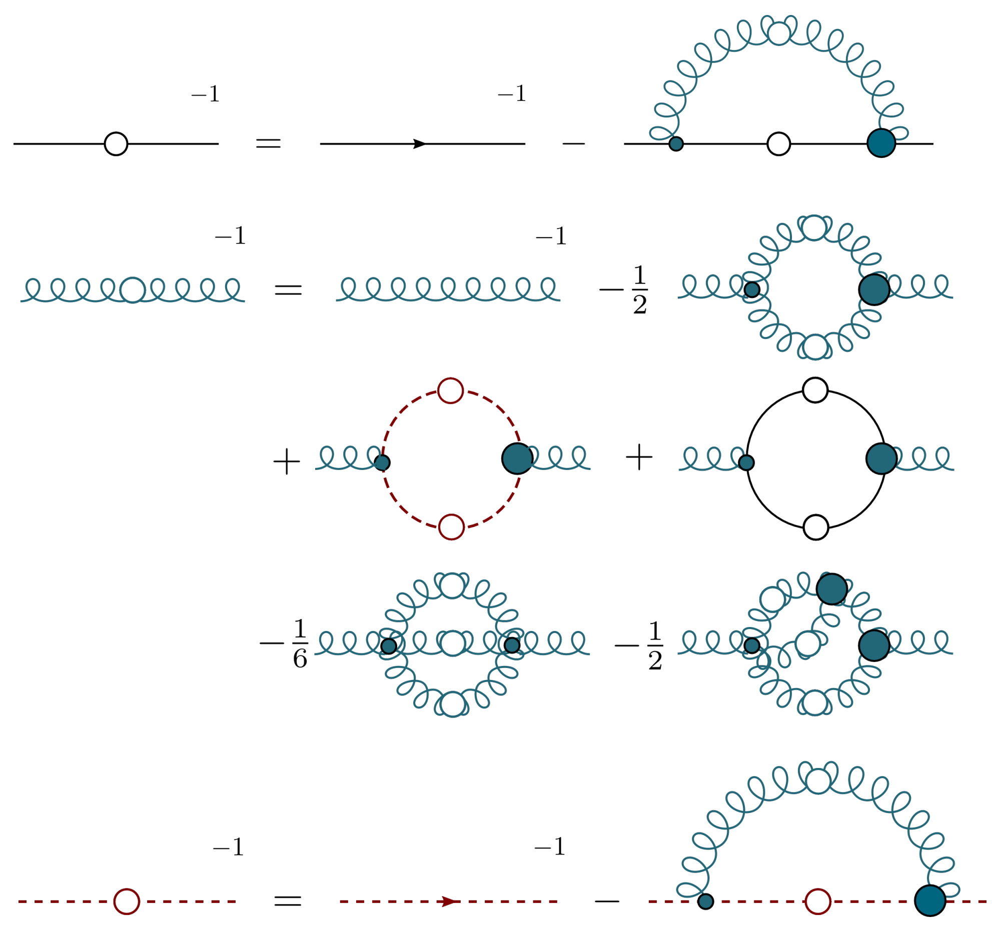
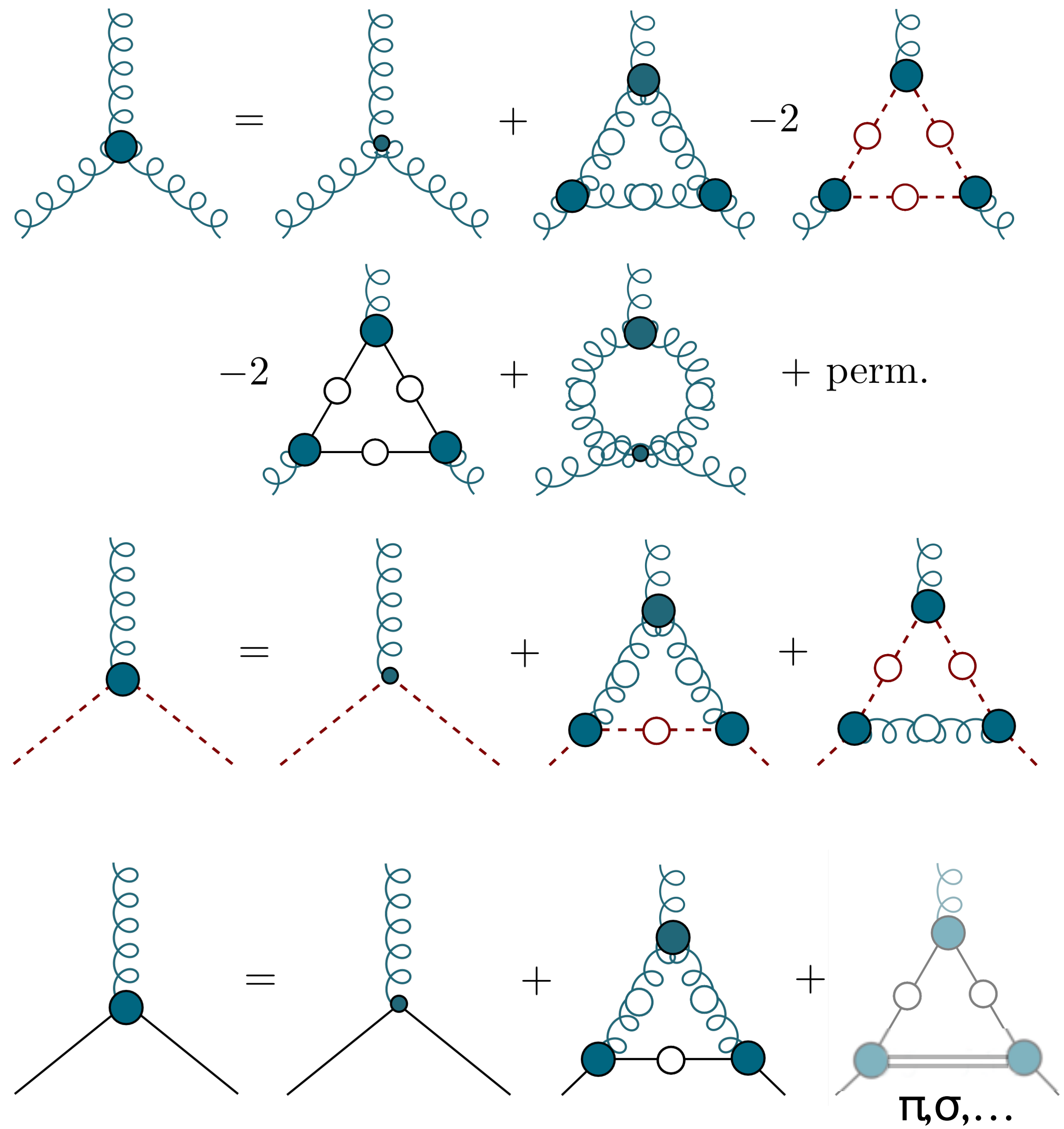
propagators



vertices



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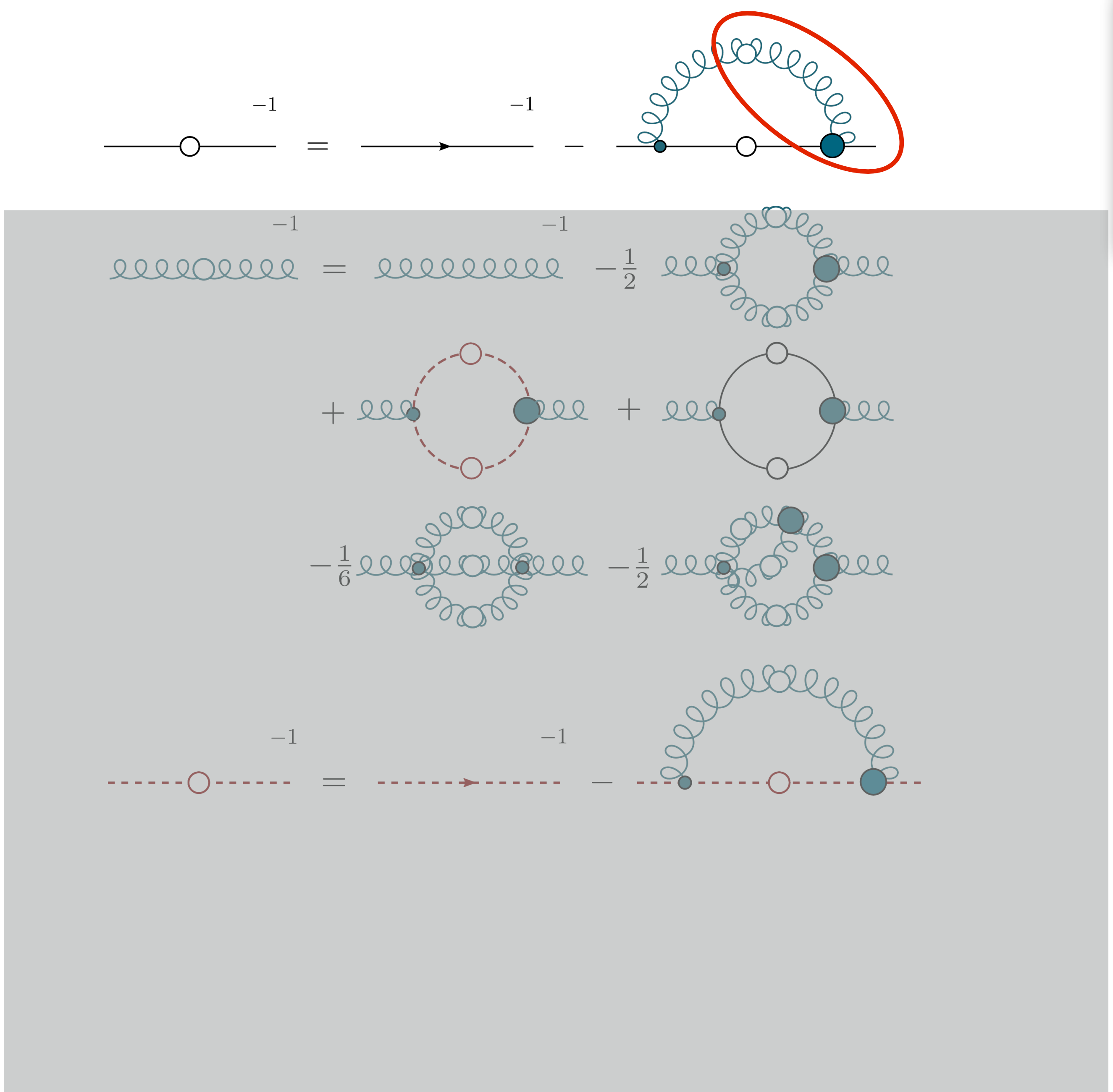
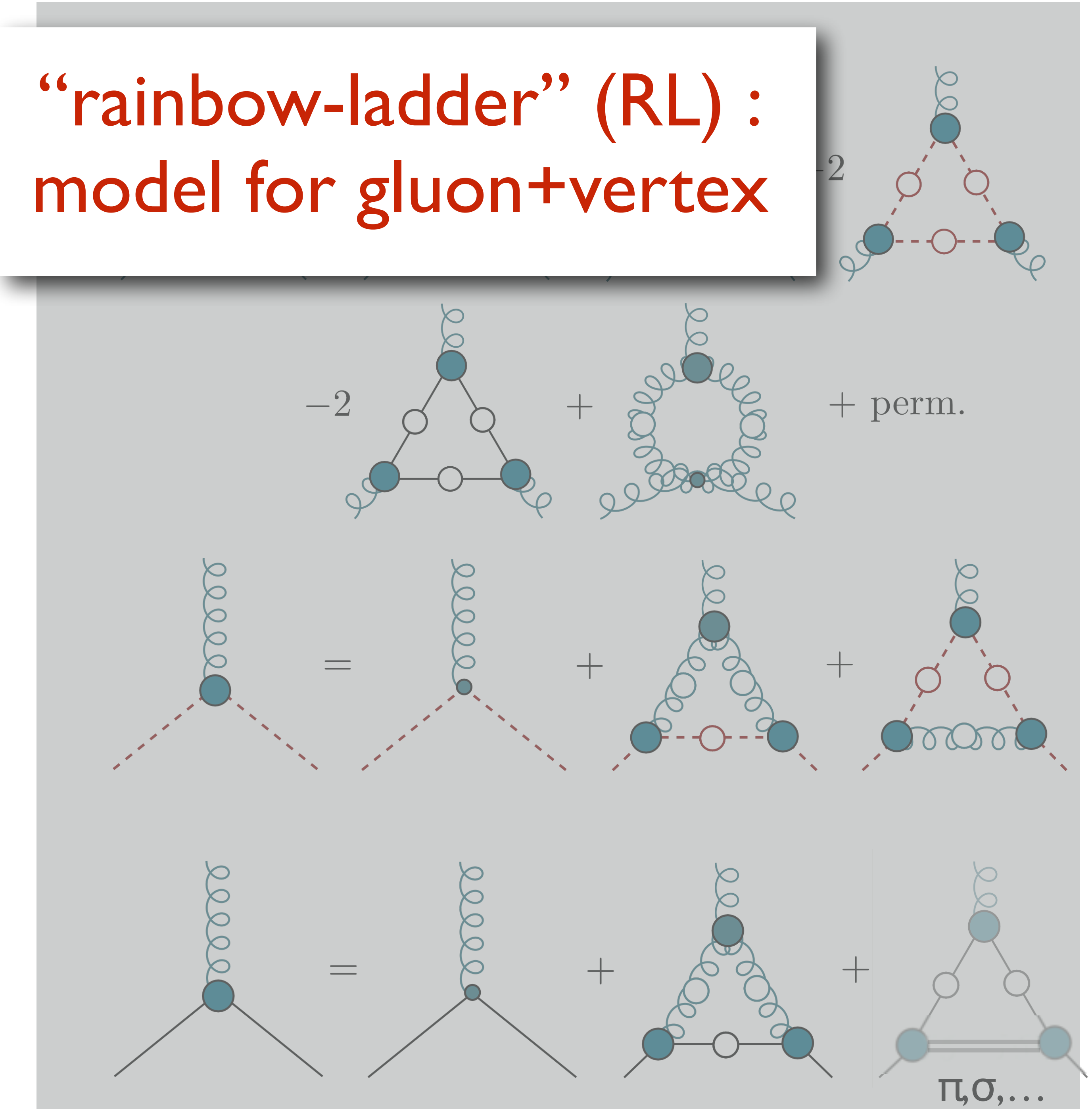
QCD with DSE

propagators



vertices



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QCD with DSE

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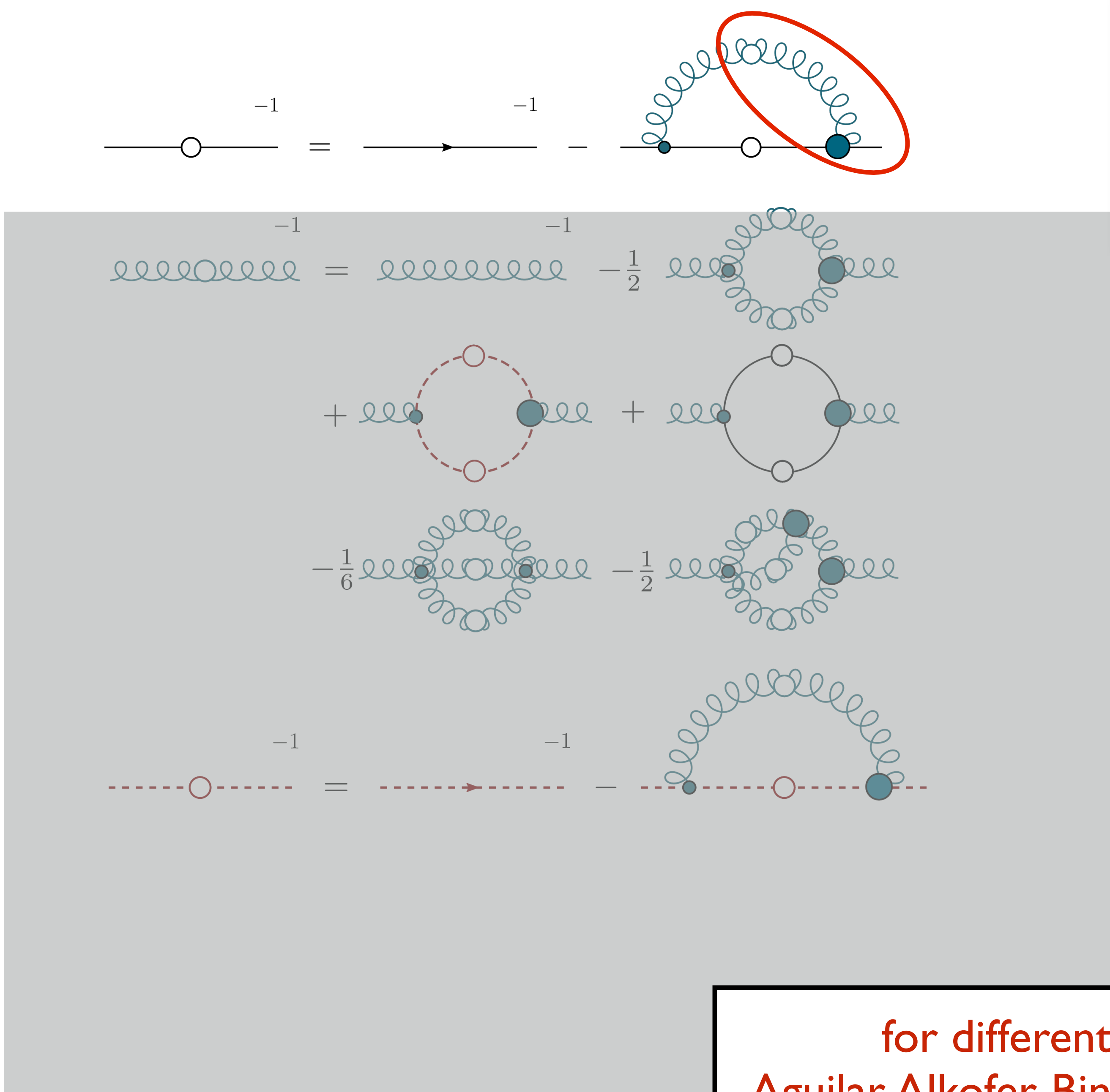
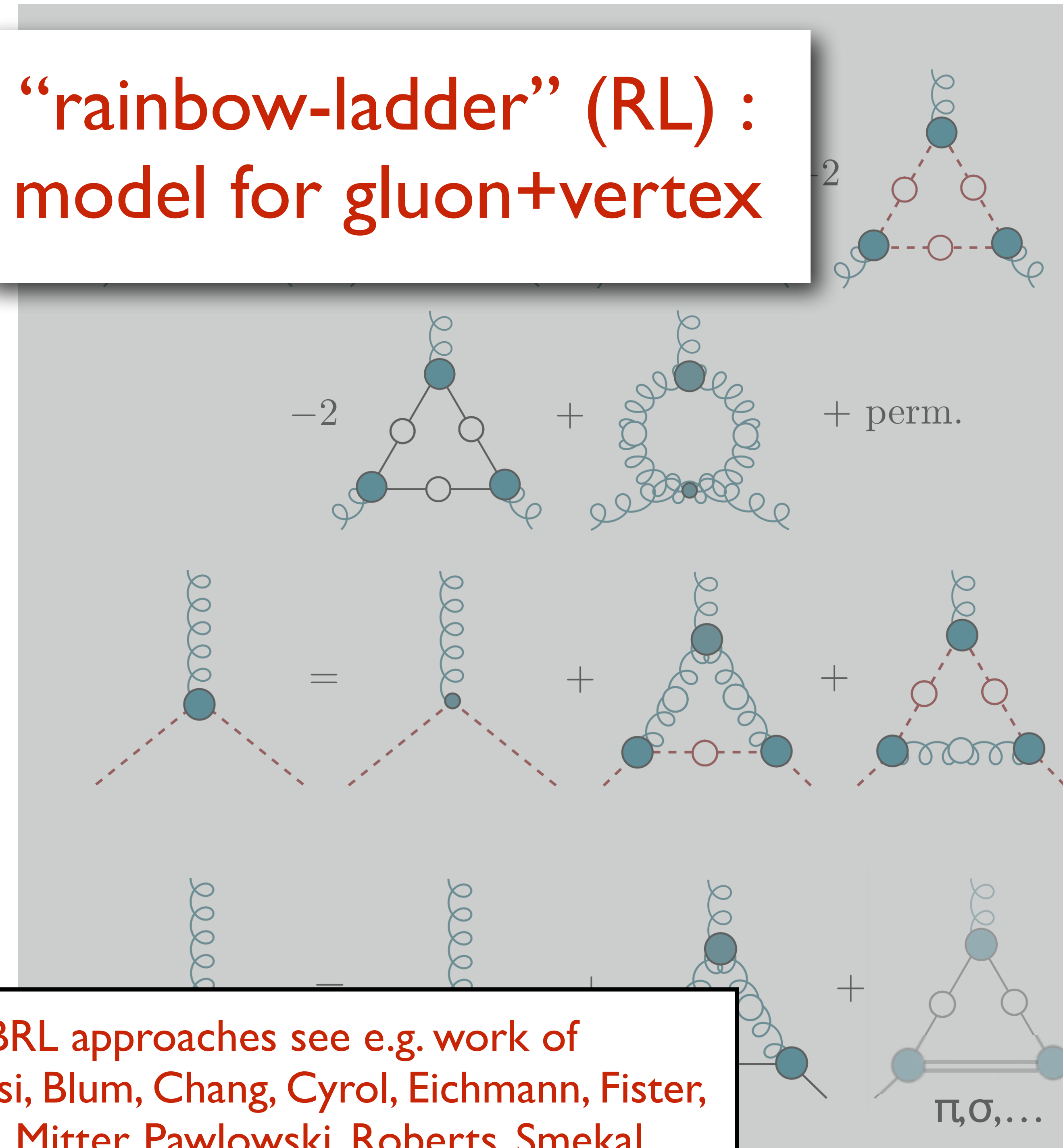


vertices



"rainbow-ladder" (RL) :
model for gluon+vertex

for different BRL approaches see e.g. work of
Aguilar, Alkofer, Binosi, Blum, Chang, Cyrol, Eichmann, Fister,
Gao, Huber, Maas, Mitter, Pawlowski, Roberts, Smekal,
Strodthoff, Vujinovic, Watson, Williams…

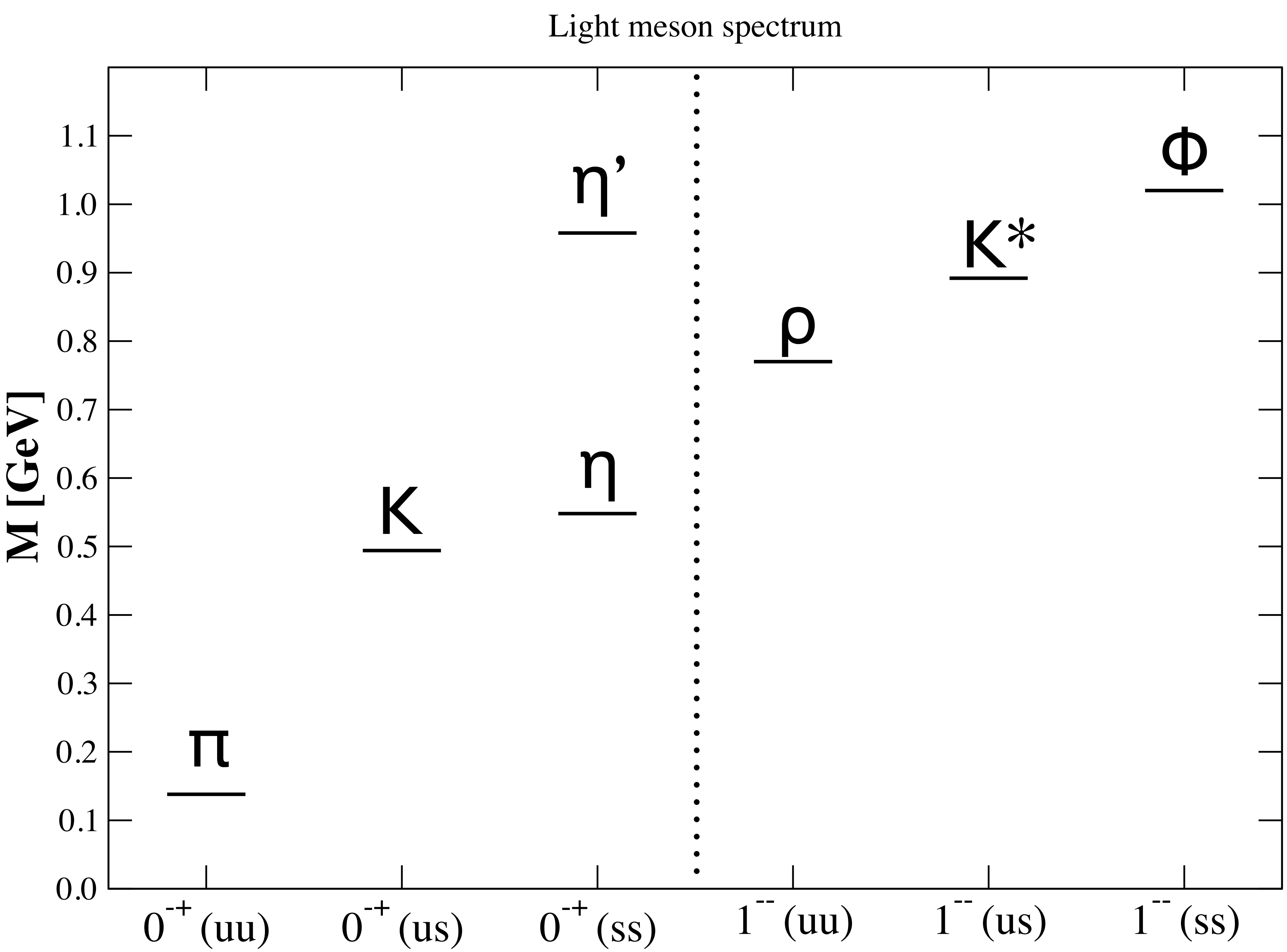
Review: Eichmann, Sanchis-Alepuz, Williams, Alk…

…67 (2003) 094020
…upel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

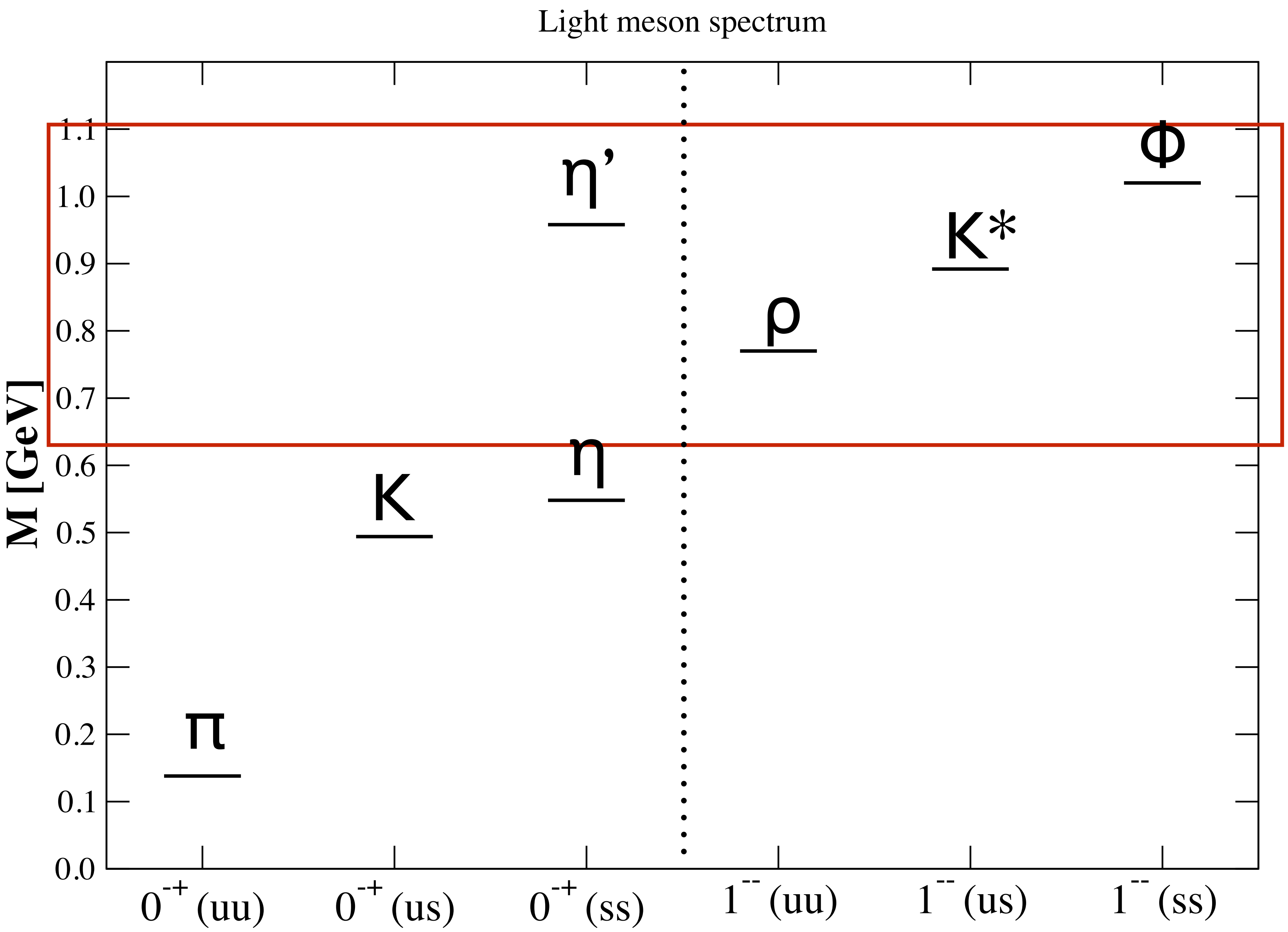
Overview

1. Bound states in the quark model
   - Construction
   - Assets, shortcomings and how to do better
2. Properties of QCD and correlation functions
   - Dynamical chiral symmetry breaking
   - Correlation functions and Dyson-Schwinger equations (DSEs)
   - The quark DSE
3. Mesons
   - The pion Bethe-Salpeter equation (BSE): Goldstone bosons, GMOR and all that…
   - Spectra: light and charm
4. Exotic mesons
   - Confinement and glueballs
   - Four-quark states
5. Baryons
   - Spectra: light and strange
6. Form factors
   - Meson form factors
   - Baryon form factors

Experimental light meson spectrum

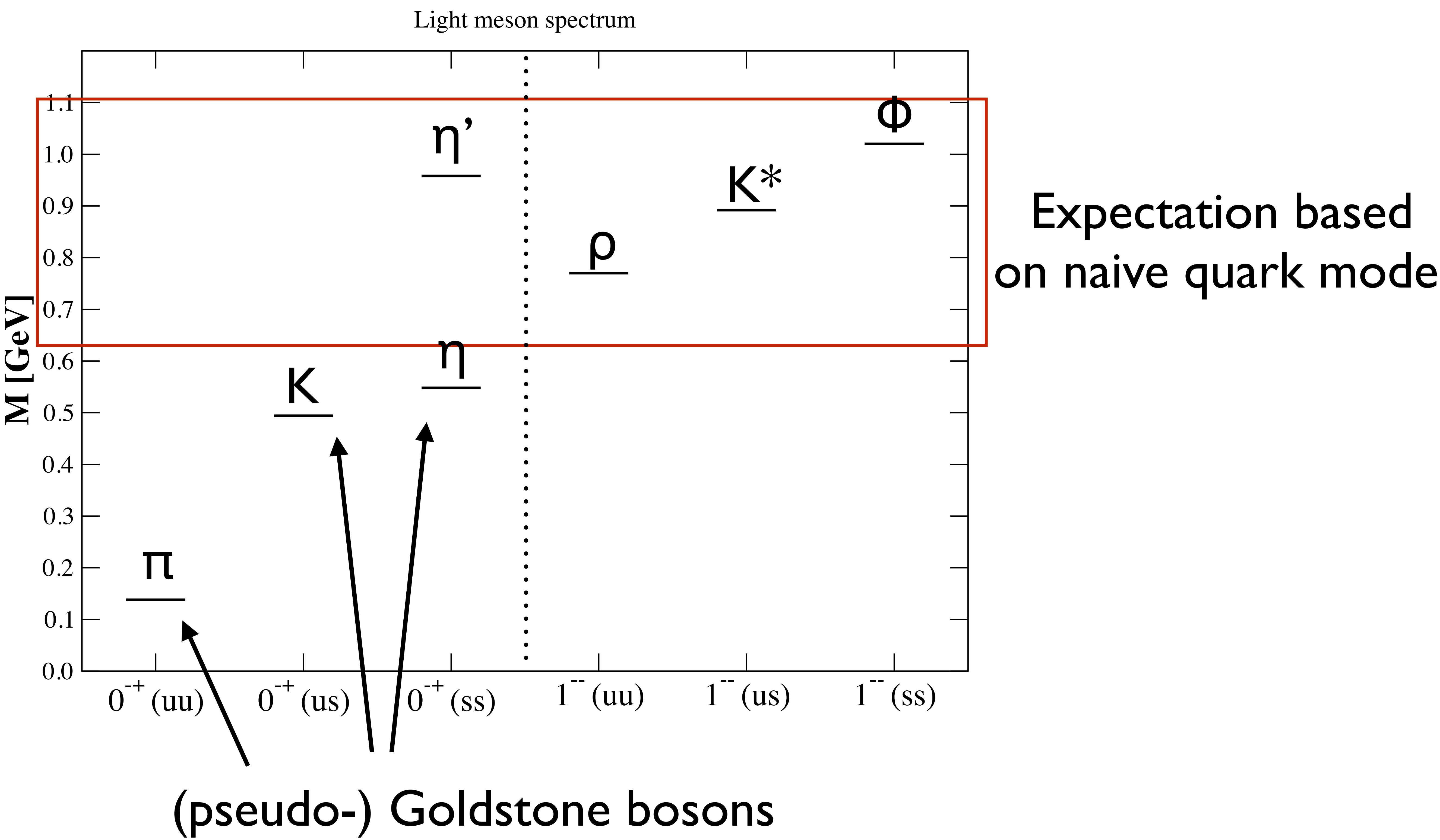


Experimental light meson spectrum



Expectation based on naive quark model

Experimental light meson spectrum



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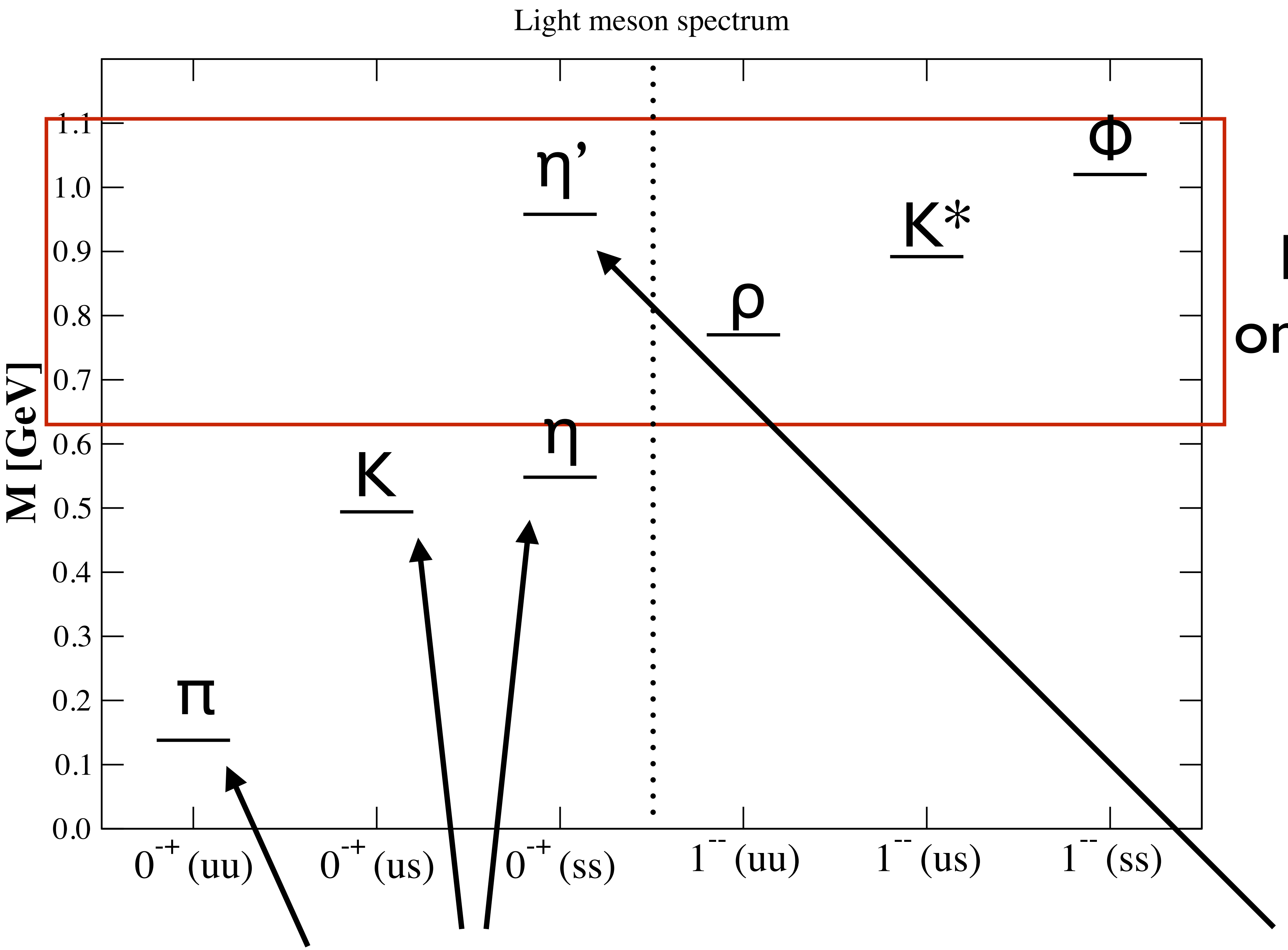


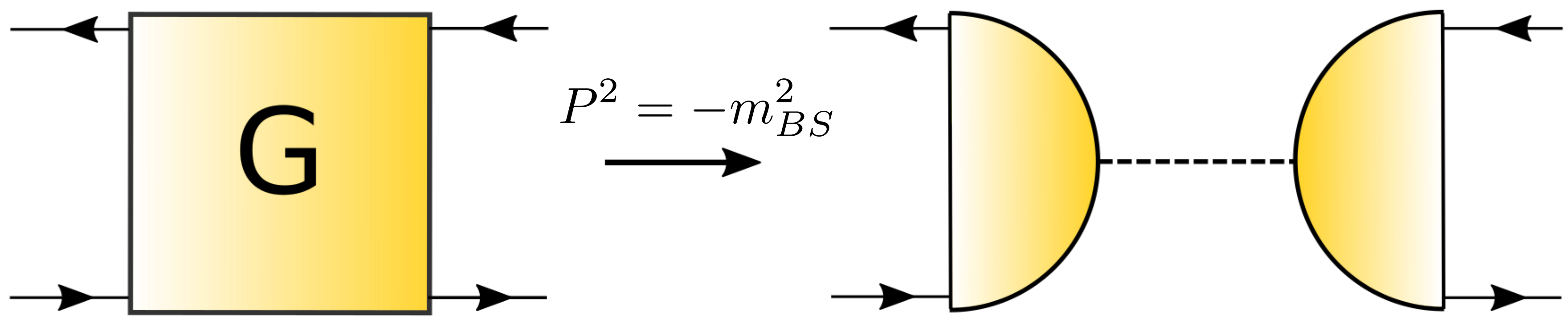
Expectation based on naive quark model

(pseudo-) Goldstone bosons

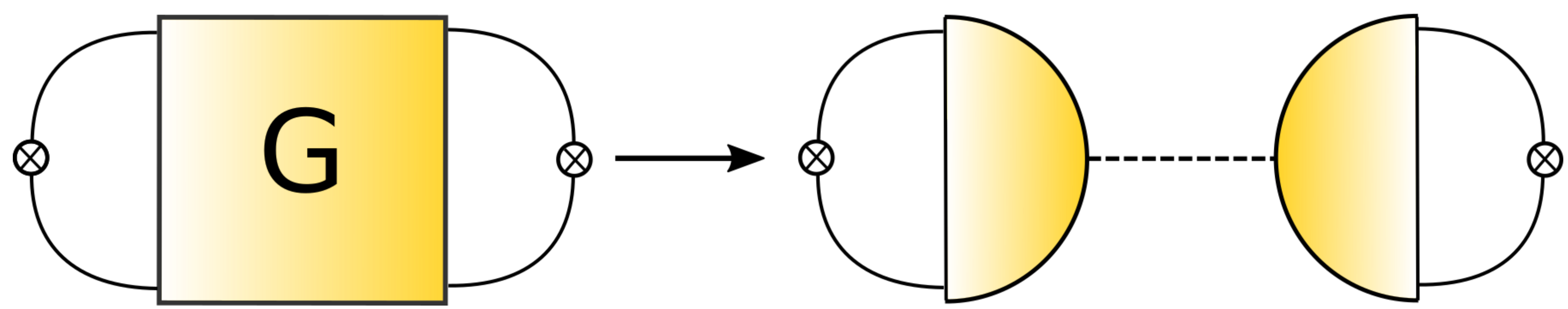
QCD-anomaly

Extracting spectra from QCD-correlators

functional:



Lattice:



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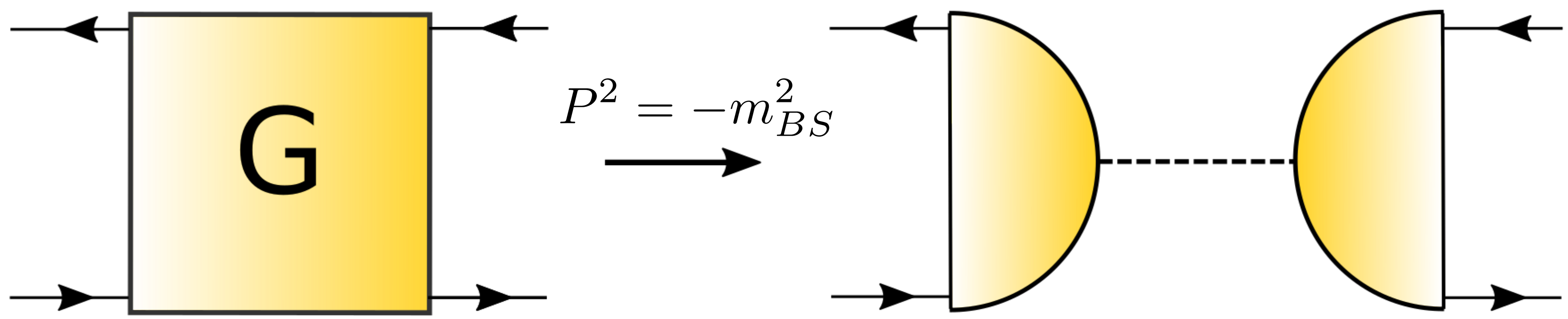

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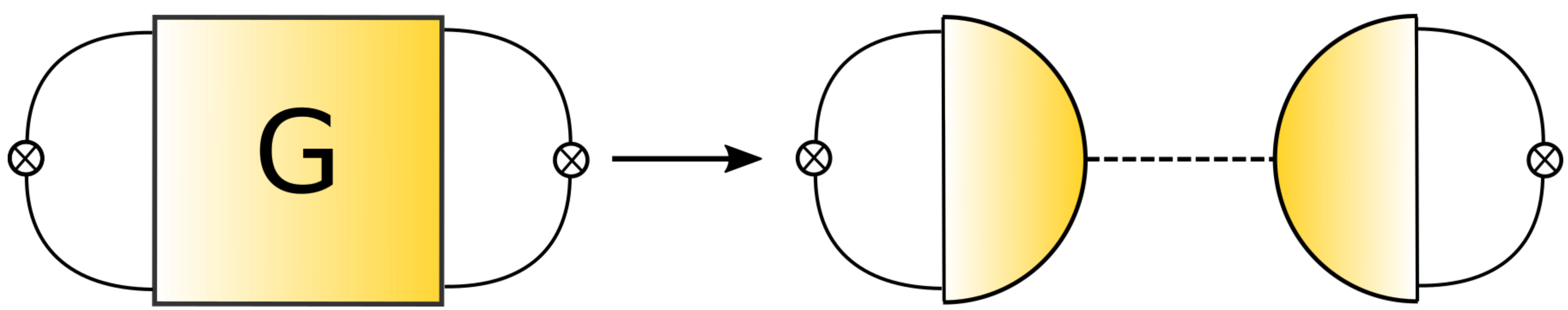
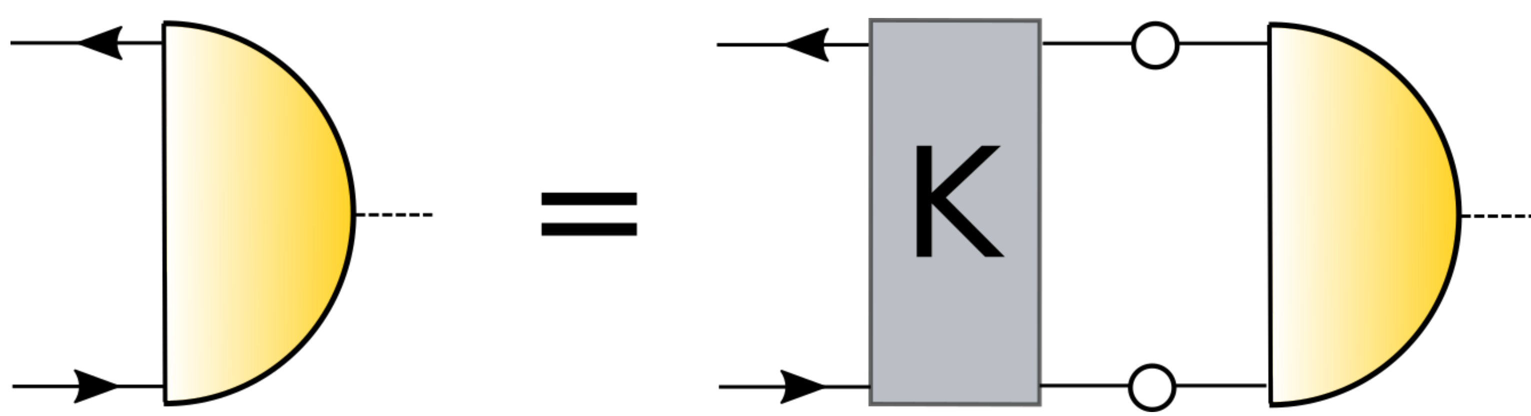
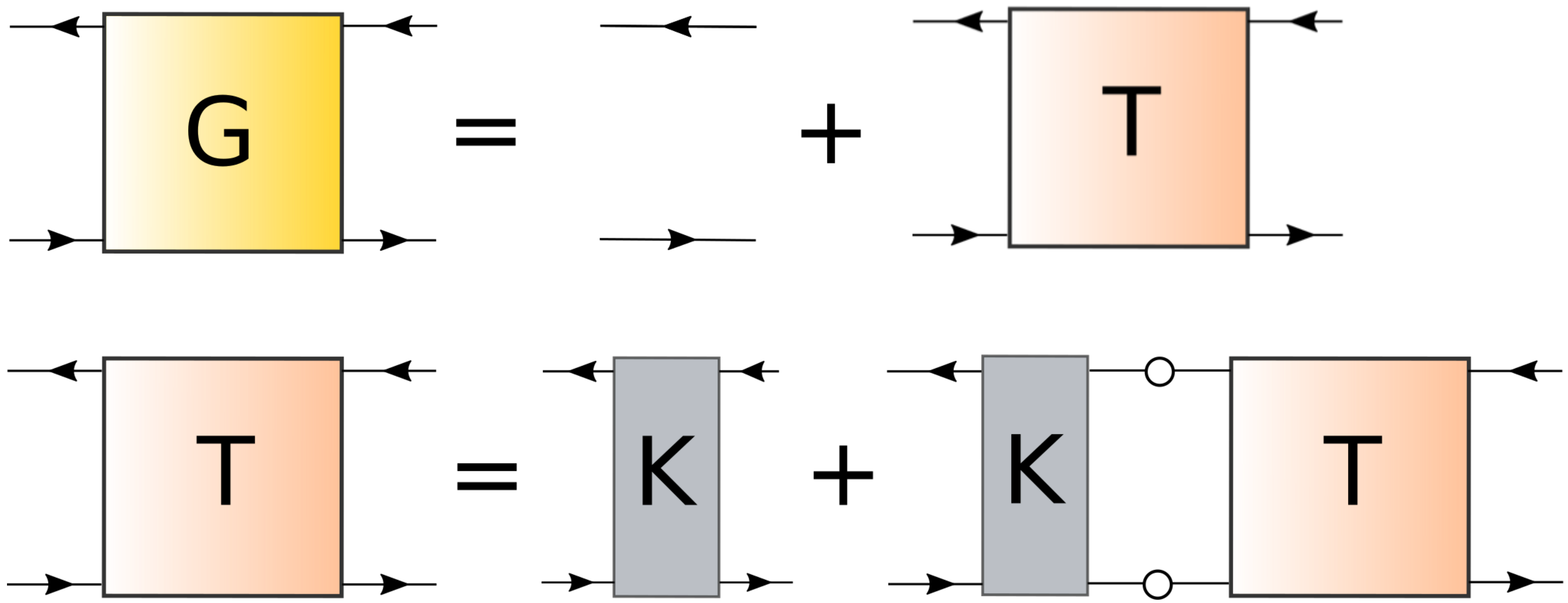
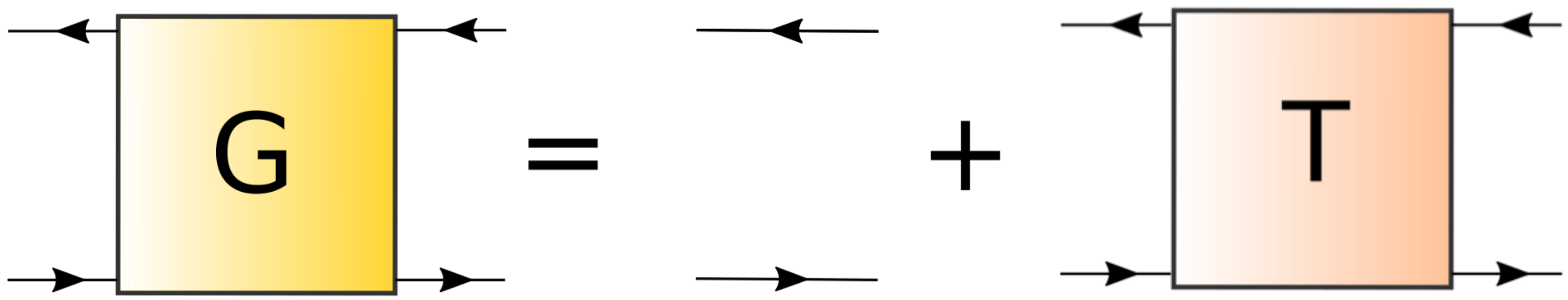


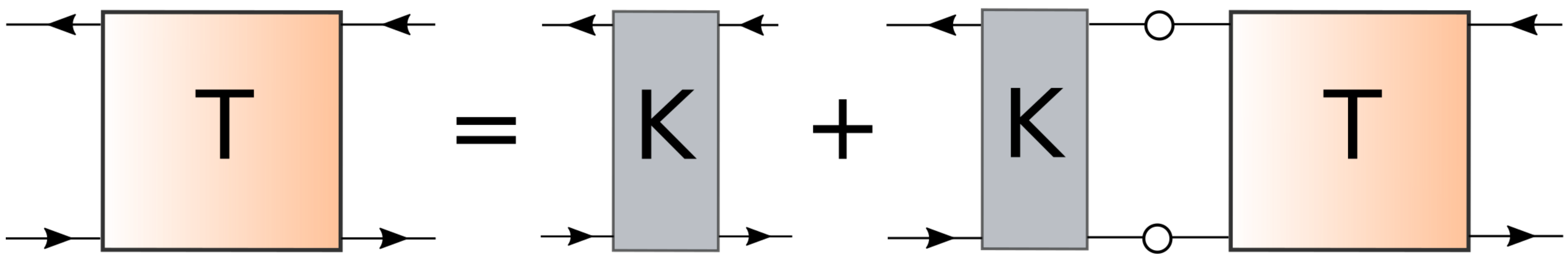
exact BSE:



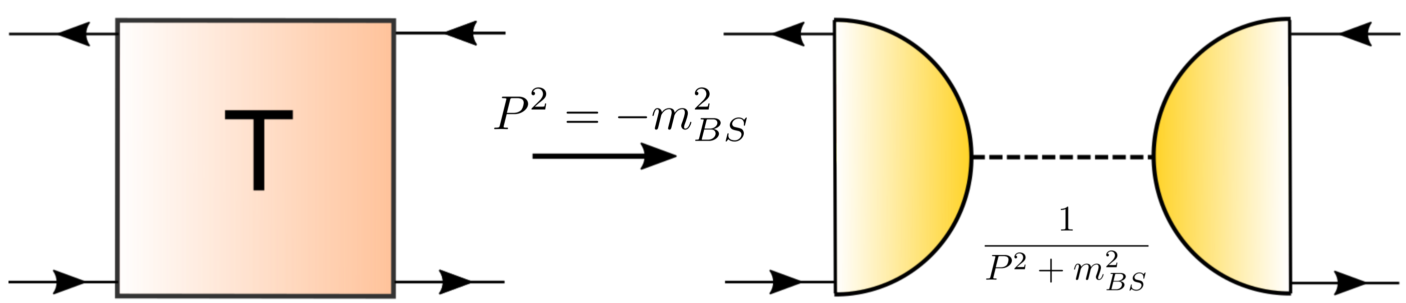
Bound states and Bethe-Salpeter equations



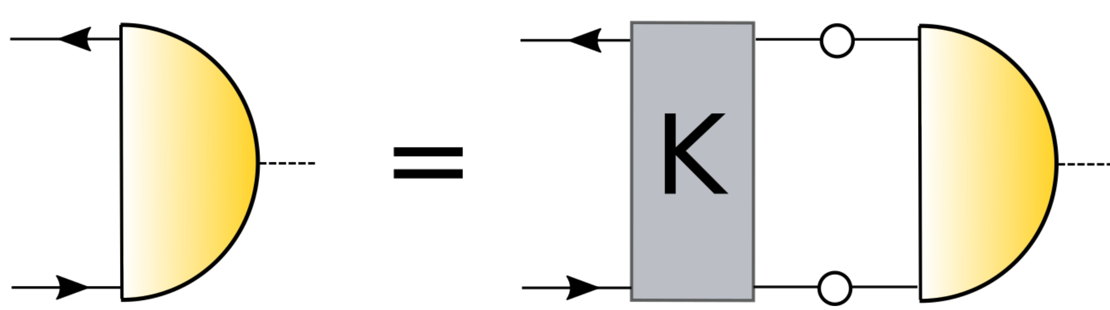




Bound states appear as poles in T:



BSE:



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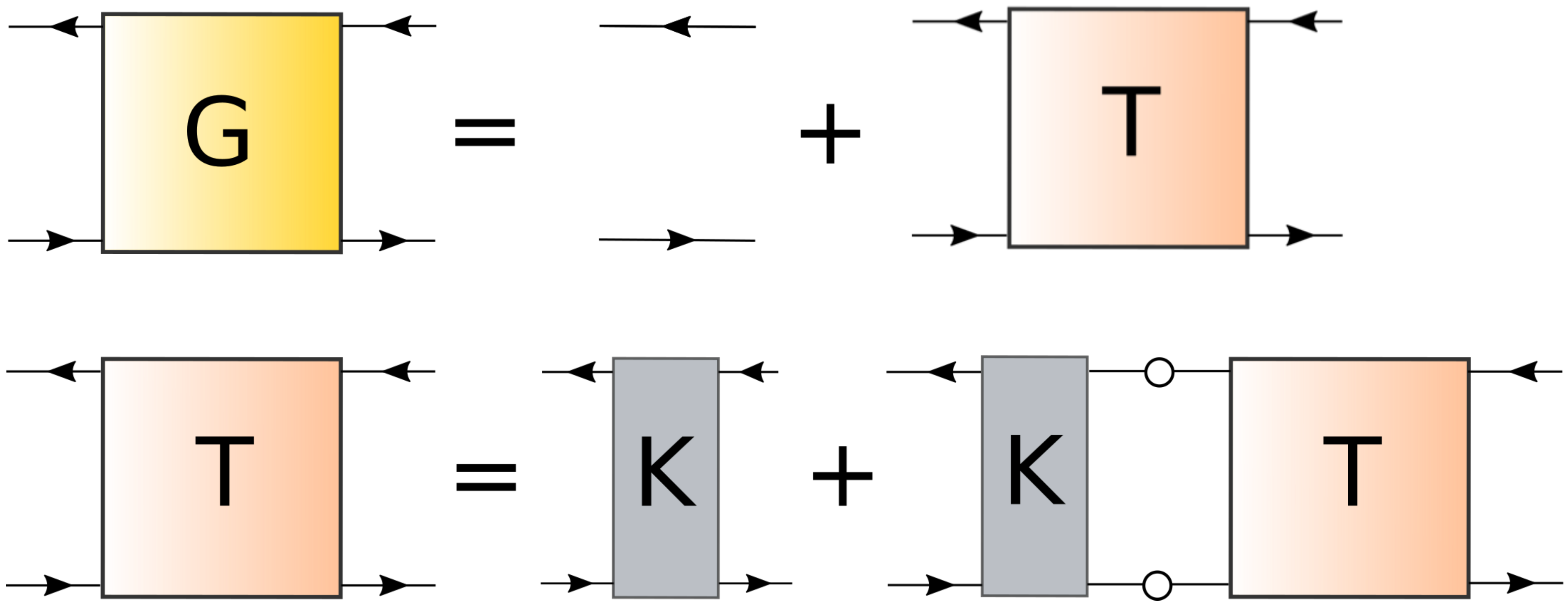

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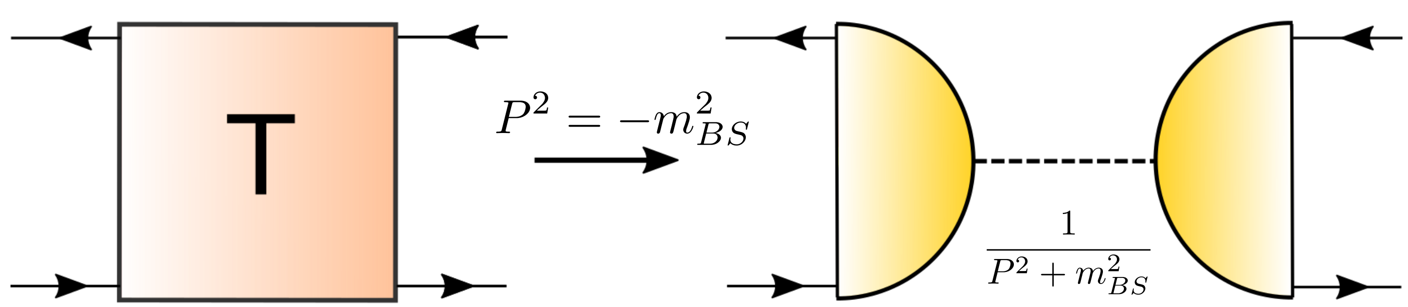
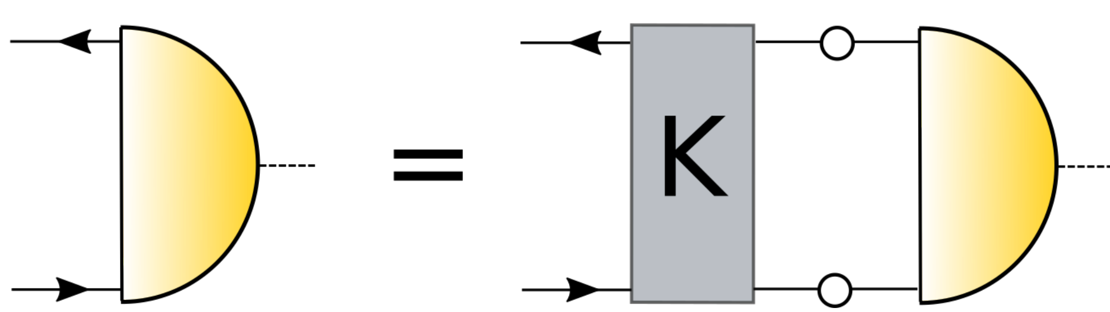


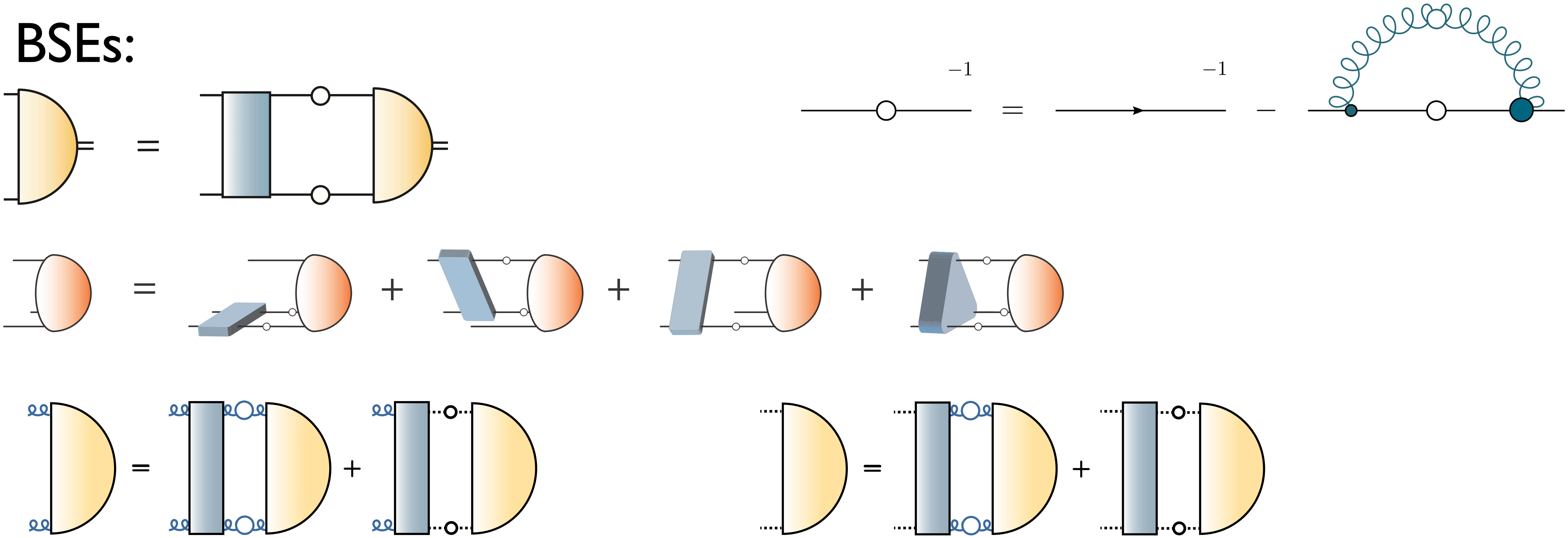
BS-wave functions = residue of bound state pole

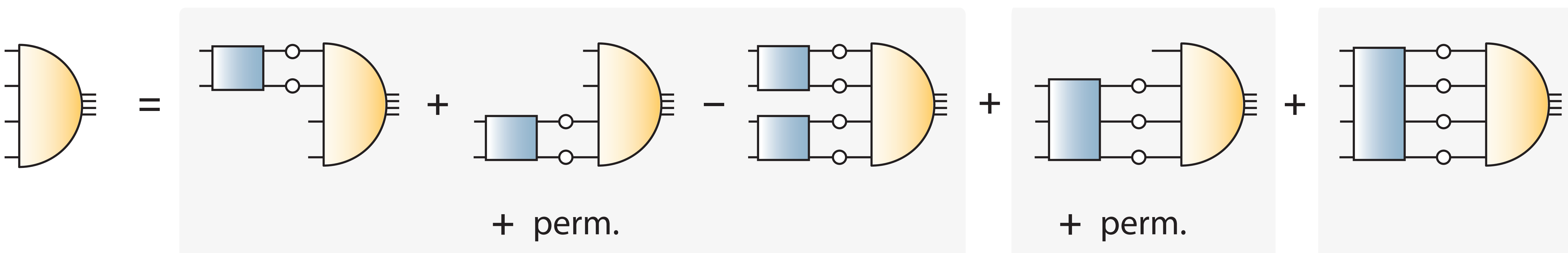
BSE:



Bound states and Bethe-Salpeter equations

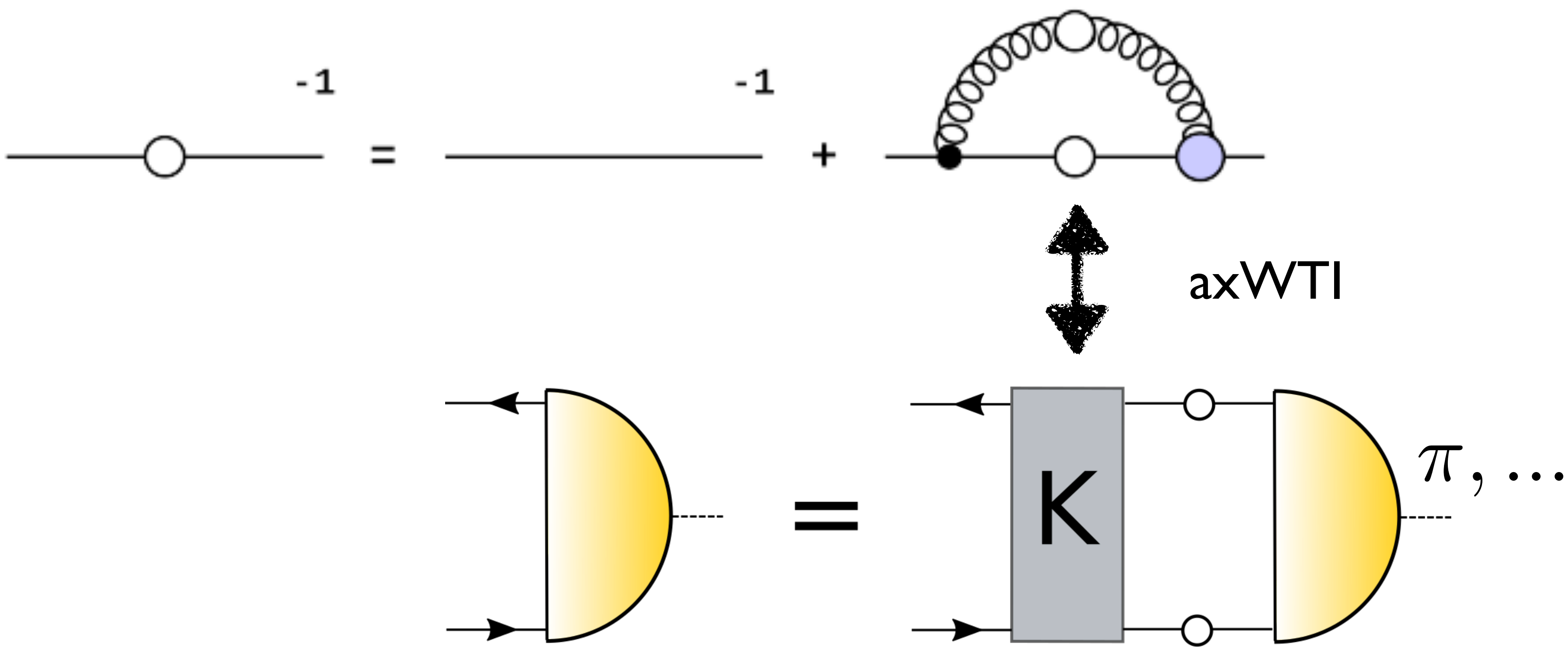
BSEs:





Eigenvalue equations: masses and wave functions

DSEs and Bethe-Salpeter equation



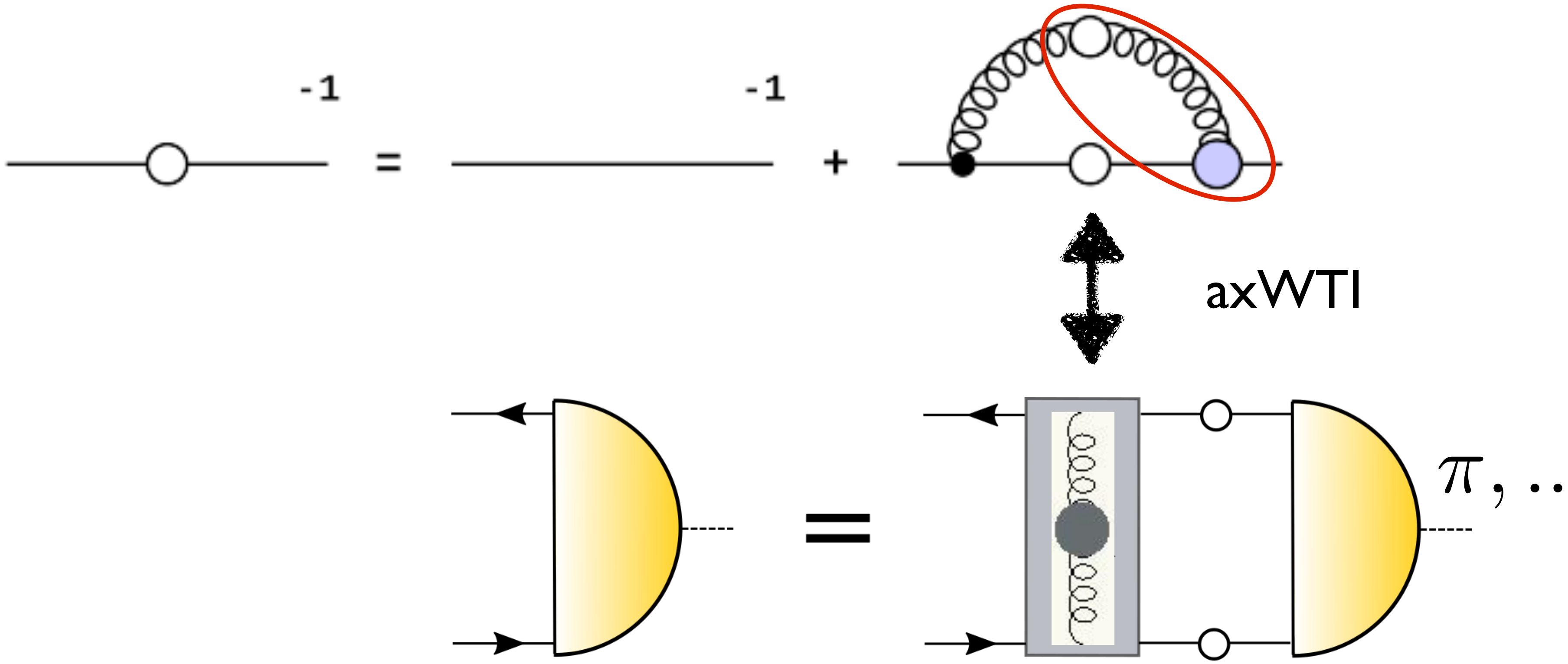
Kernel K uniquely related to quark-DSE via axialvector Ward-Takahashi-Identity (axWTI):

$$-i \int (K\gamma_5 S_- + K S_+ \gamma_5) = \int \gamma_\mu S_+ D_{\mu\nu} \Gamma_\nu \gamma_5 + \int \gamma_5 \gamma_\mu S_- D_{\mu\nu} \Gamma_\nu$$

→Pion is bound state **and** Goldstone boson

Maris, Roberts, Tandy, PLB 420 (1998) 267

DSEs and Bethe-Salpeter equation



RL: QED-structure of binding force

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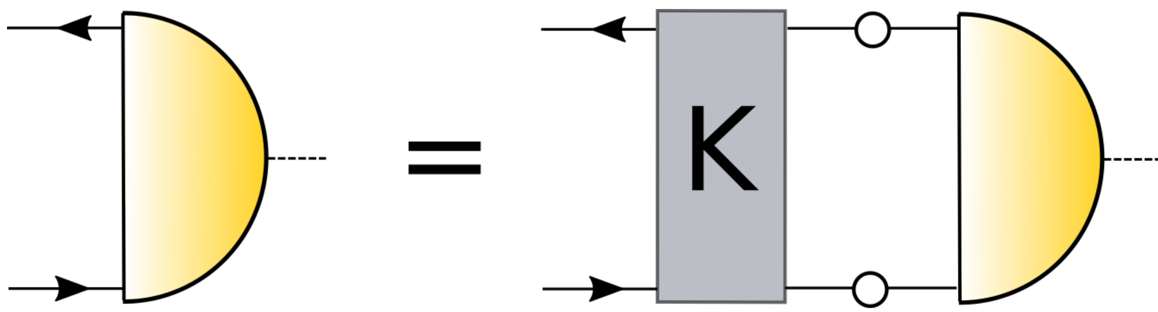
Maris, Roberts, Tandy, PLB 420 (1998) 267

Pion-BSE



$$P^2 = -m_{BS}^2$$

$$\mathbf{1} \times EV = operator \times EV$$

- Structure: eigenvalue equation
- Eigenvector is 'Bethe-Salpeter wave function':

$$[\Gamma_\pi(P,p)]^e_{\alpha,\beta,A,B,a,b} = \{\gamma_5\,[F_1(P,p) + F_2(P,p)\,i\not{P} + F_3(P,p)\,pP\,i\not{p} + F_4(P,p)\,[\not{p},\not{P}]]\}_{\alpha,\beta} \times \frac{\delta_{AB}}{\sqrt{3}} \times r^e_{ab}$$

Llewelyn-Smith 1965

(pseudo-) scalar: 4 Dirac tensor structures
(axial-)vector: 8

Bethe-Salpeter wave function

$$[\Gamma_\pi(P,p)]^e_{\alpha,\beta,A,B,a,b} = \{\gamma_5\,[F_1(P,p) + F_2(P,p)\, i\not{P} + F_3(P,p)\, pP\, i\not{p} + F_4(P,p)\,[\not{p}\,,\not{P}]]\}_{\alpha,\beta} \times \frac{\delta_{AB}}{\sqrt{3}} \times r^e_{ab}$$

- why four tensor structures ?

  quark legs ⟶ Dirac-structure

  pseudoscalar ⟶ no Lorenz-index, overall γ_5

  two independent momenta $P_\mu, p_\mu\, \gamma_\mu$

- comparison with quark model:

  same flavor and color part of wave function

  relativistic: spin and spatial wave function combined !!

Quantum numbers: non-relativistic vs relativistic

non-relativistic $q\bar{q}$

S	L	J^{PC}
0	0	0^{-+}
1	0	1^{--}
0	1	1^{+-}

$$P : (-1)^{L+1}$$

relativistic $q\bar{q}$

$$\Gamma_\pi(P,p) = \gamma_5 \big(F_1(P,p) + F_2(P,p)\, \not{P} + F_3(P,p)\, \not{p} + F_4(P,p)\, [\not{p}\,, \not{P}] \big)$$

s-wave (F_1, F_2)

p-wave (F_3, F_4)

Pauli-Lubanski-vector ($[\not{p}\,, \not{P}]$)

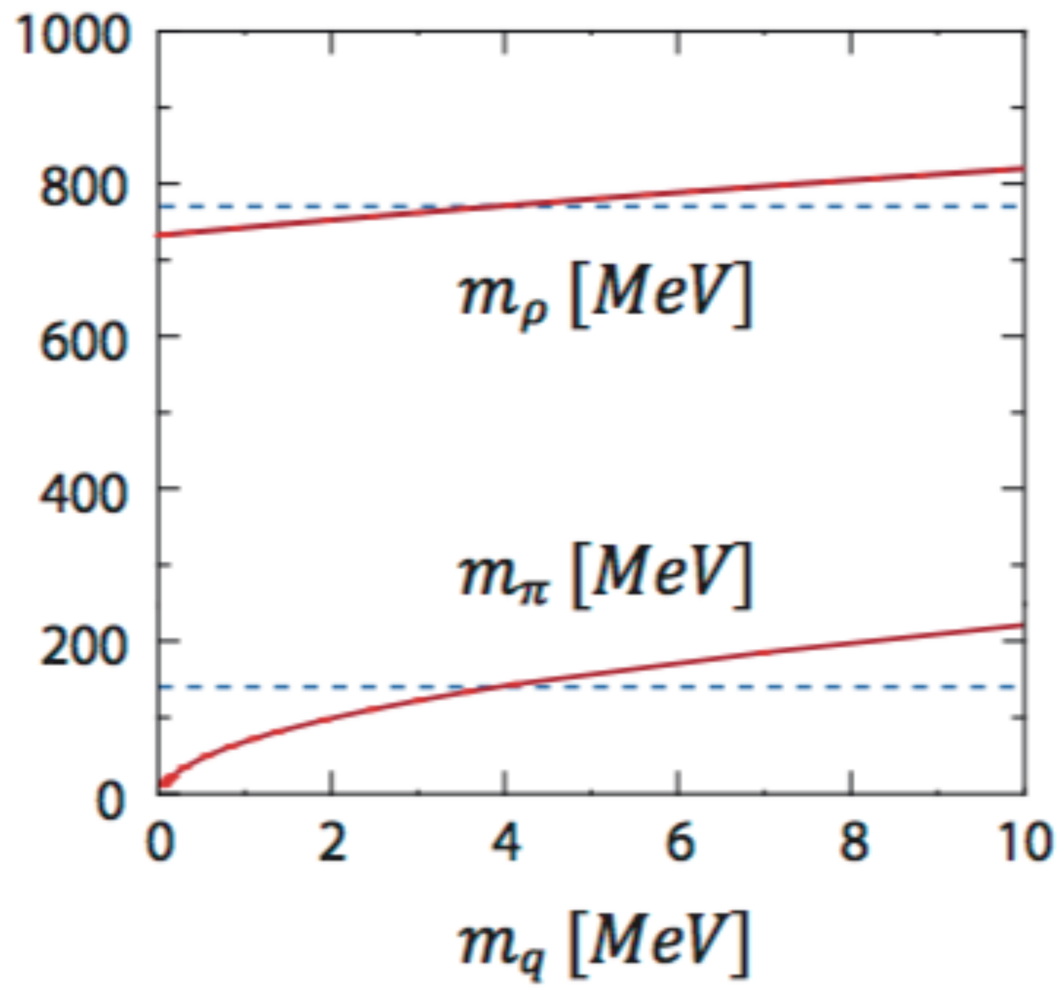
Llewelyn-Smith 1965

~~$P : (-1)^{L+1}$~~

- mesons: 'exotic' quantum numbers possible:

$$0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$$

Pions as Goldstone bosons



- Gell-Mann-Oakes-Renner: $f_\pi^2 m_\pi^2 = -2\, m \langle \bar{\Psi}\Psi \rangle$
- Pion BS-amplitude: $f_\pi \Gamma_\pi(P^2 = 0, p) = B(p^2)\gamma_5$

Pion decay constant does not vanish in chiral limit !

Excited states: no GB, decay constant must vanish in chiral limit!

Hoell, Krassnigg, Roberts, PRC 70 (2004)

Noether Theorem:

Consider field Ψ with $\mathcal{L}(\Psi, \partial\Psi)$ and unitary transformation with generators λ^a:

$$\Psi \to \exp(-i\,\Theta_a \lambda^a)\Psi \approx \Psi + i\Theta_a \lambda^a \Psi$$

then we find a conserved current with

$$\partial_\mu J^a_\mu(\mathbf{x}) = 0 \qquad \text{with} \quad J^a_\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi)} \lambda^a \Psi$$

For conserved currents, the related charge

$$Q^a = \int d^3x J^a_0(\mathbf{x})$$

is conserved if

$$\frac{\partial Q^a}{\partial t} = \int d^3x \frac{\partial J^a_0}{\partial t} = -\int d^3x \nabla \mathbf{J}^a = 0$$

If $\mathbf{J}^a$ does not vanish at infinity we say that the corresponding symmetry is broken and one can show that there are associated massless bosons, the **Goldstone bosons**.

Chiral symmetry II - QCD with N_f=3

With $\Psi^T = (u, d, s)$ the QCD flavour symmetry is given by

$$U_V(3) \times U_A(3) = U_V(1) \times SU_V(3) \times U_A(1) \times SU_A(3)$$

transform	$\mathcal{L}$ inv. iff	current $J_\mu^{(a)}$	charge Q
$U_V(1)$ $e^{i\Theta}$	for all M	$J_\mu = \bar{\Psi}\gamma_\mu\Psi$ $\partial_\mu J_\mu = 0$	baryon number
$SU_V(3)$ $e^{i\Theta^a\lambda^a}$	$m_u = m_d = m_s$	$J_\mu^a = \bar{\Psi}\gamma_\mu\lambda^a\Psi$ $\partial_\mu J_\mu^a = i\bar{\Psi}[\lambda^a, M]\Psi$	isospin hypercharge
$U_A(1)$ $e^{i\Theta\gamma_5}$	$M = 0$	$J_\mu^5 = \bar{\Psi}\gamma_\mu\gamma_5\Psi$ $\partial_\mu J_\mu^5 = 2i\bar{\Psi}m\gamma_5\Psi - g^2/(16\pi^2)\epsilon_{\alpha\beta\gamma\delta}F^c_{\alpha\beta}F^c_{\gamma\delta}$	broken, no GB (QCD anomaly)
$SU_A(3)$ $e^{i\Theta^a\lambda^a\gamma_5}$	$M = 0$	$J_\mu^{5,a} = \bar{\Psi}\gamma_\mu\gamma_5\lambda^a\Psi$ $\partial_\mu J_\mu^{5,a} = i\bar{\Psi}\{\lambda^a, M\}\Psi - \delta_{a3}e^2/(32\pi^2)\epsilon_{\alpha\beta\gamma\delta}F^{QED}_{\alpha\beta}F^{QED}_{\gamma\delta}$	broken, GB (QED-anomaly)

$M = diag(m_u, m_d, m_s)$

Proof of Goldstone theorem

We start by parametrising the matrix elements between the vacuum and bound states λ of the axial and pseudoscalar current ($P^2 = -m_\lambda^2$ fixed):

$$\langle 0|\, j_5^\mu(x)\,|\lambda\rangle = -iP^\mu f_\lambda\, e^{-ix\cdot P}, \qquad \langle 0|\, j_5(x)\,|\lambda\rangle = -ir_\lambda\, e^{-ix\cdot P}. \tag{1}$$

The first quantity encodes the transition from a pseudoscalar meson to an axialvector current and thereby defines its electroweak decay constant f_λ. The pseudoscalar analogue r_λ is not associated with a measurable quantity.
Using now the PCAC-relation (see above)

$$-i\partial_\mu\, j_{5,a}^\mu = Z_4\, i\bar{\psi}\,\{\mathsf{m},\mathsf{t}_a\}\,\gamma_5\,\psi \xrightarrow{\mathsf{m}=m_q} 2m_q\, j_{5,a}\,, \tag{2}$$

where $Z_4 = Z_2\, Z_m$ and $j_{5,a}(z) = Z_4\,\bar{\psi}(z)\, i\gamma_5\,\mathsf{t}_a\,\psi(z)$ is the pseudoscalar density, we arrive at

$$\boxed{f_\lambda\, m_\lambda^2 = 2m_q\, r_\lambda\,,} \tag{3}$$

which is valid for all flavour non-singlet pseudoscalar mesons (in the singlet case there would be an additional term from the axial anomaly).

We proceed with the axial vector Ward takahashi identity (axWTI)

$$Q^\mu\Gamma_5^\mu(k,Q) + 2m\Gamma_5(k,q) = S^{-1}(k_+)i\gamma_5 + i\gamma_5 S^{-1}(k_-) \tag{4}$$

with momenta $k_\pm = k \pm Q/2$, the incoming total momentum Q and the average quark momentum k. A derivation of the vector identity can be found e.g. in Peskin and Schroeder, *Introduction to Quantum Field Theory*, chapter 7.4., from which the axWTI follows by analogy.

Proof of Goldstone theorem

The pseudoscalar and axialvector vertices each contain pole contributions from bound states (similar to the rho-meson contribution to the vector vertex):

$$\Gamma_5^\mu = Q^\mu \sum_\lambda \frac{2if_\lambda}{Q^2+m_\lambda^2}\,\mathbf{\Gamma}_\lambda + \widetilde{\Gamma}_5^\mu\,, \qquad \Gamma_5 = \sum_\lambda \frac{2ir_\lambda}{Q^2+m_\lambda^2}\,\mathbf{\Gamma}_\lambda + \widetilde{\Gamma}_5\,. \qquad (5)$$

see later !

Here the quantities with tilde are regular objects and Γ_λ are the Bethe-Salpeter amplitudes of the respective bound states.

Plugging now Eq.(5) into Eq.(4) and using (3) we arrive at

$$Q^\mu\,\Gamma_5^\mu + 2m_q\,\Gamma_5 = \sum_\lambda 2if_\lambda\,\mathbf{\Gamma}_\lambda + Q^\mu\,\widetilde{\Gamma}_5^\mu + 2m_q\,\widetilde{\Gamma}_5 = S^{-1}(k_+)\,i\gamma_5 + i\gamma_5\,S^{-1}(k_-)\,. \qquad (6)$$

Observe that all hadronic poles contained in the vertices have disappeared, which is consistent because the right-hand side of the axial WTI does not exhibit any such poles. In the chiral limit $m_q \to 0$ and for $Q^\mu \to 0$ this becomes

$$\sum_\lambda f_\lambda\,\mathbf{\Gamma}_\lambda(k,0) = B(k^2)\gamma_5\,. \qquad (7)$$

The sum goes over all pseudoscalar 0^{-+} mesons with identical flavour quantum numbers, i.e., ground states and radial excitations.

In the chiral limit, $B(k^2)$ is only nonzero if chiral symmetry is spontaneously broken. Then there is at least one mode with $f_\lambda \neq 0$. From (3) we must have $m_\lambda \to 0$ in that case, i.e. a massless Goldstone boson.

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$$S_0^{-1} = i\not{p} + m$$
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Proof of GMOR

For excited states with $m_\lambda \neq 0$ the decay constants have to vanish in the chiral limit because of Eq. (3). Therefore the sum in Eq. (7) breaks down and we arrive at

$$f_\pi \, \mathbf{\Gamma}_\pi(k,0) = B(k^2)\gamma_5 \, . \tag{8}$$

Now we multiply on both sides with $S(k)\gamma_5 S(k)$, take the trace and integrate over momentum k. We then find

$$f_\pi \int_k tr\{S(k)\gamma_5 S(k)\Gamma_\pi(k,0)\} = \int_k B \; tr\left\{\frac{(i\not{k}\,A+B)\gamma_5(i\not{k}\,A+B)\gamma_5}{p^2A^2+B^2}\right\} \tag{9}$$

$$f_\pi r_\pi = -\langle\bar{\Psi}\Psi\rangle \tag{10}$$

and substituting this back into Eq. (3) we arrive at the Gell-Mann-Oakes-Renner relation

$$\boxed{f_\pi^2 m_\pi^2 = -2m_q\langle\bar{\Psi}\Psi\rangle} \tag{11}$$

see Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602], sections 3.4 and 4.2

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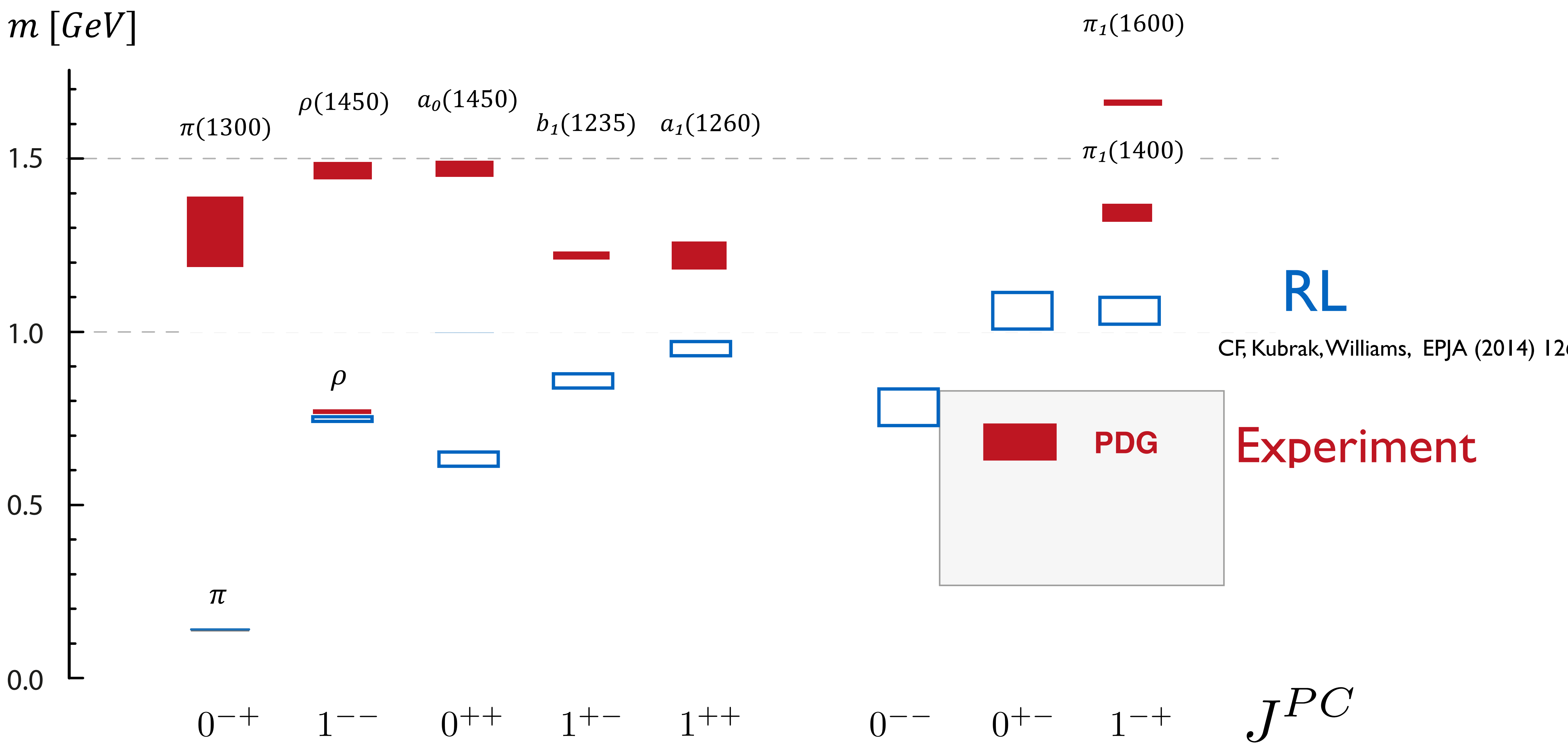
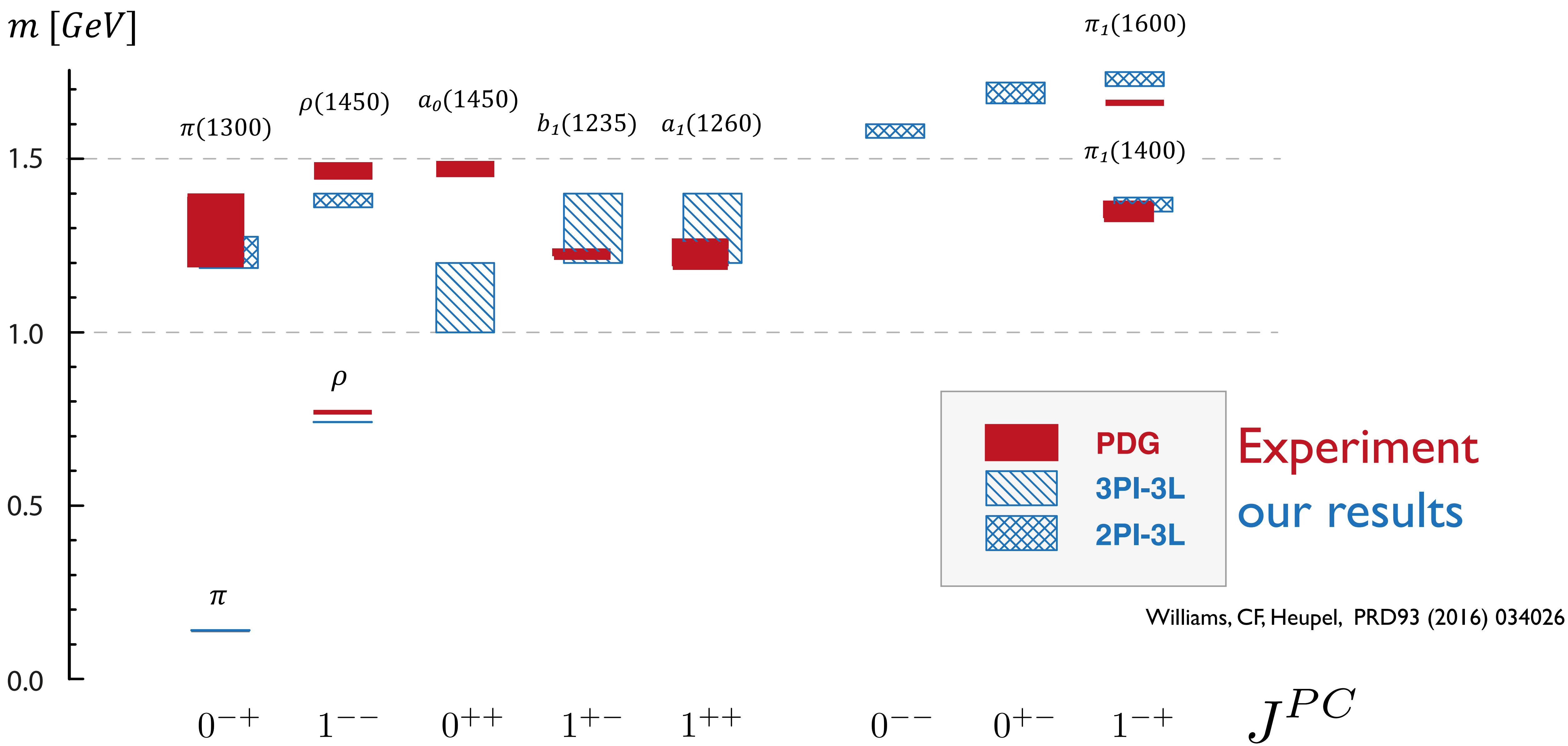
Rainbow-ladder: light meson spectrum



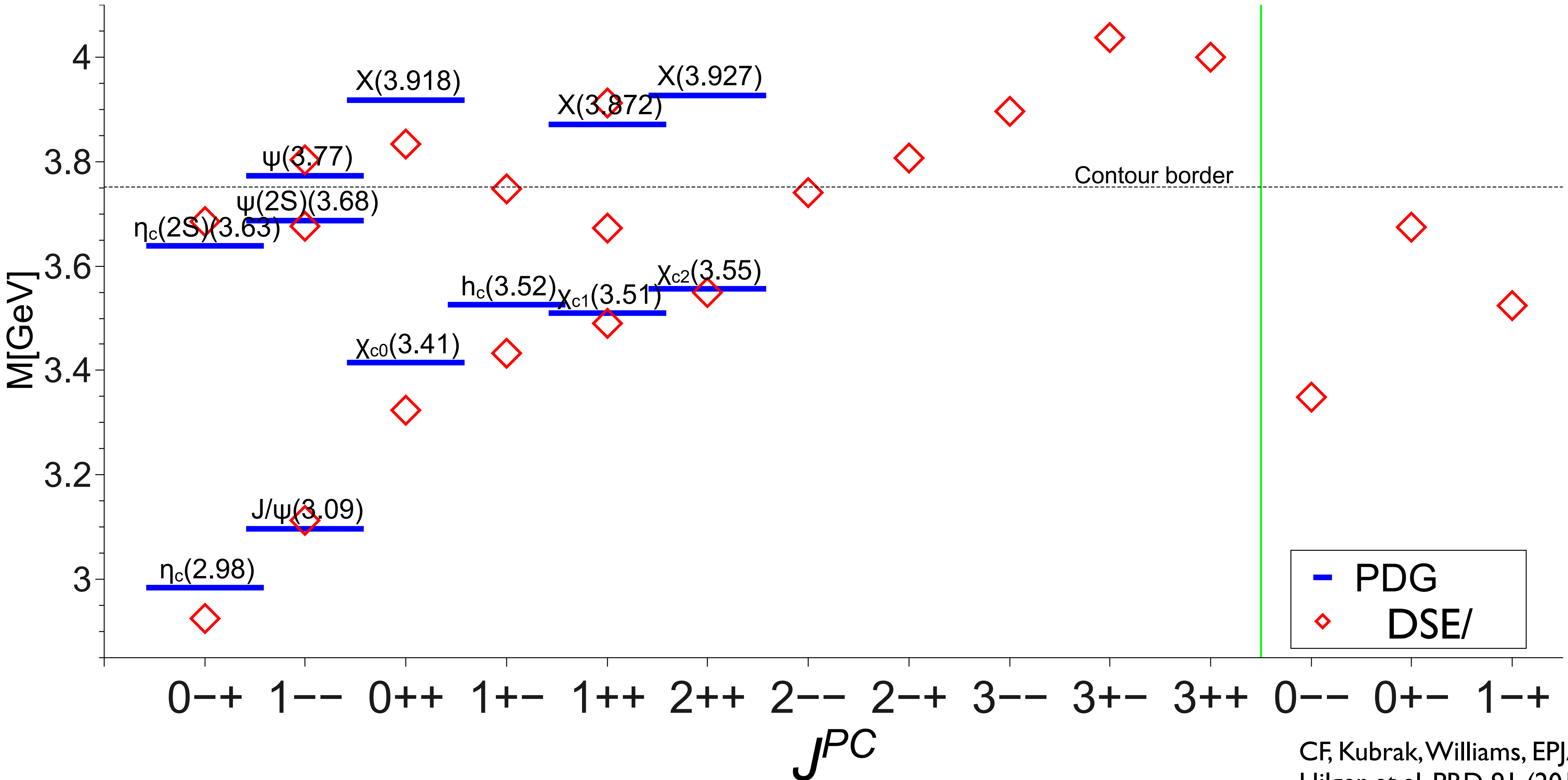
- good channels (ground state): 0^{-+}, 1^{--}
- acceptable channels (ground state) : 2^{++}, 3^{--},...
- clear deficiencies in other channels and excited states

Rainbow-ladder: light meson spectrum



- good agreement with experiment in most channels
- special channels:
  pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons
  scalar 0^{++} : complicated channel…

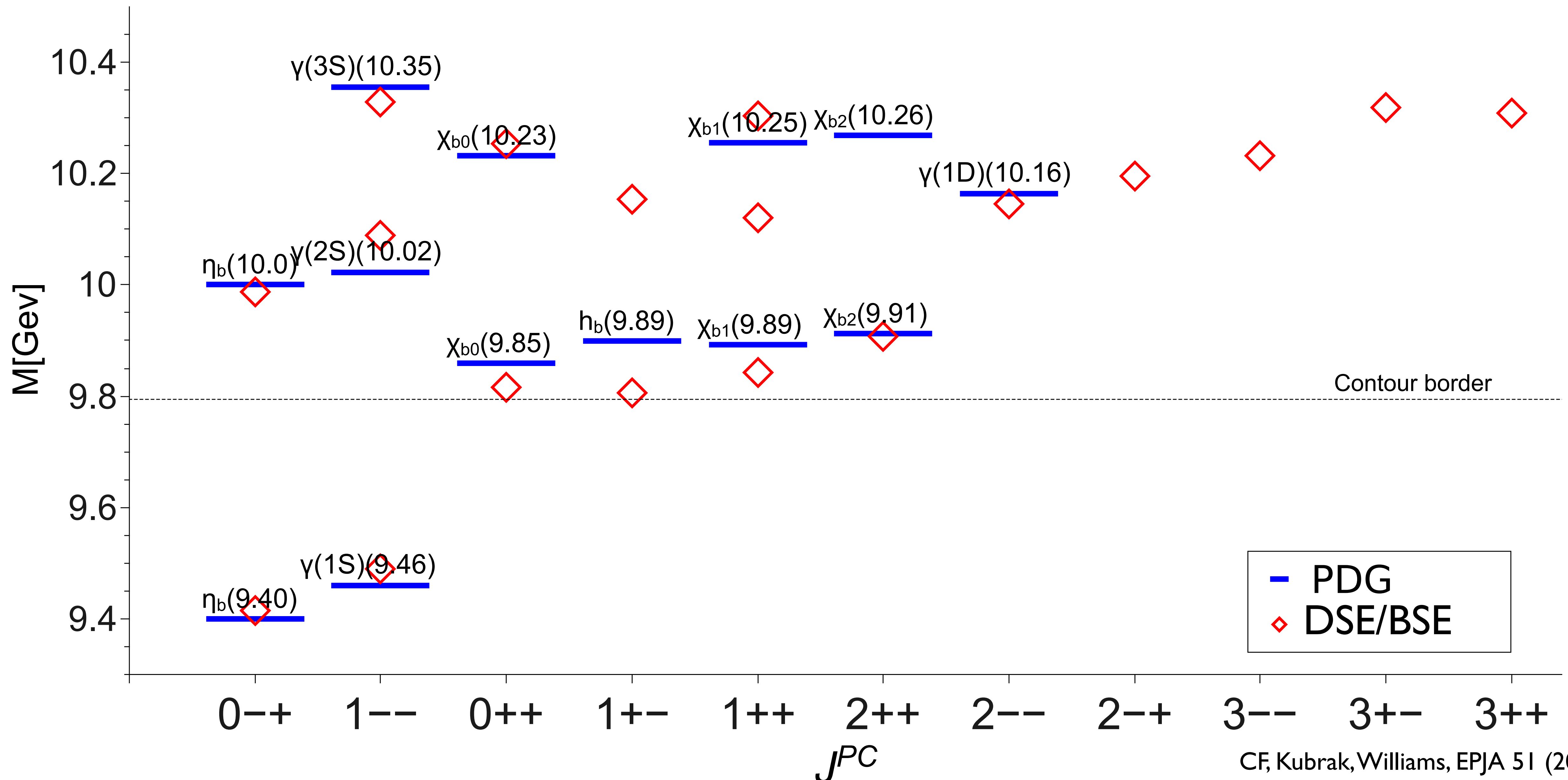
Rainbow-ladder: heavy meson spectrum



CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

- good channels: $1^{--}, 2^{++}, 3^{--}$,…: prediction for tensor state
- acceptable channels : $0^{-+}, 1^{++}$,...
- deficiencies in other channels: ’imbalance’ of spin-structure

Rainbow-ladder: heavy meson spectrum



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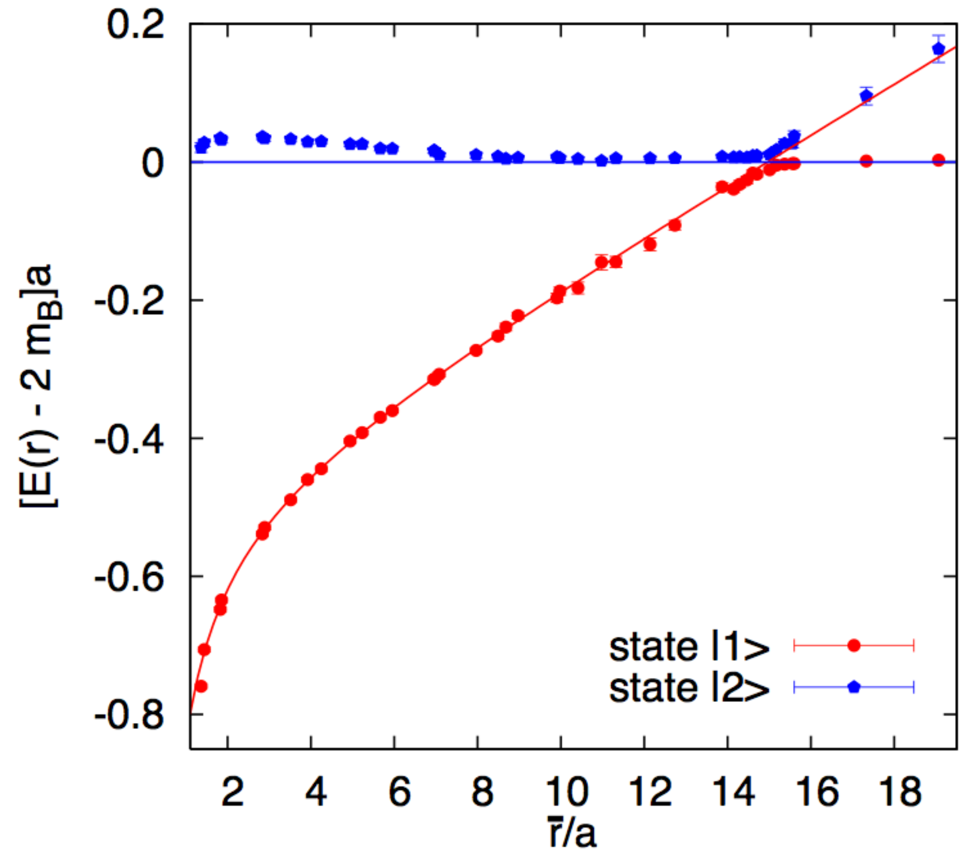
Overview

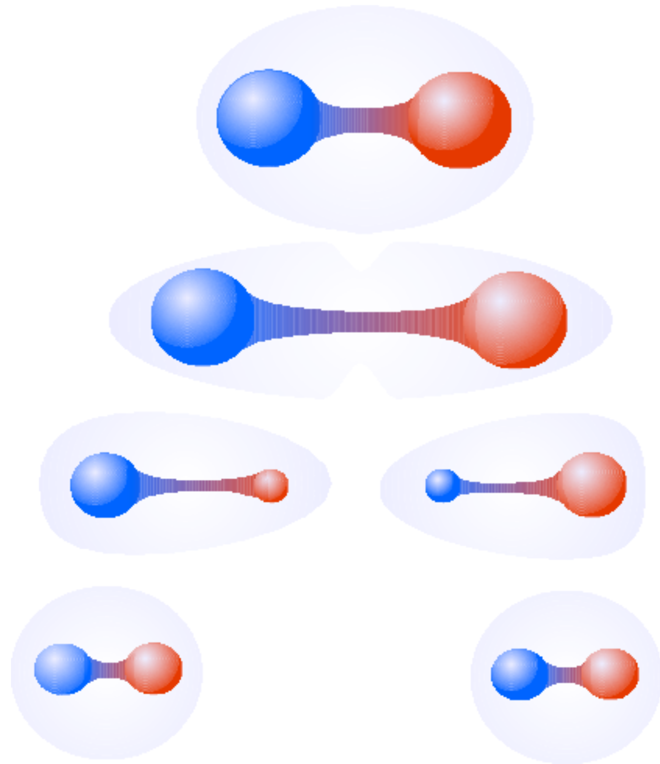
1. Bound states in the quark model
   - Construction
   - Assets, shortcomings and how to do better
2. Properties of QCD and correlation functions
   - Dynamical chiral symmetry breaking
   - Correlation functions and Dyson-Schwinger equations (DSEs)
   - The quark DSE
3. Mesons
   - The pion Bethe-Salpeter equation (BSE): Goldstone bosons, GMOR and all that…
   - Spectra: light and charm
4. Exotic mesons
   - Confinement and glueballs
   - Four-quark states
5. Baryons
   - Spectra: light and strange
6. Form factors
   - Meson form factors
   - Baryon form factors

Confinement: linearly rising potential

QCD:



Bali et al. [SESAM Collaboration], PRD 71 (2005) 114513



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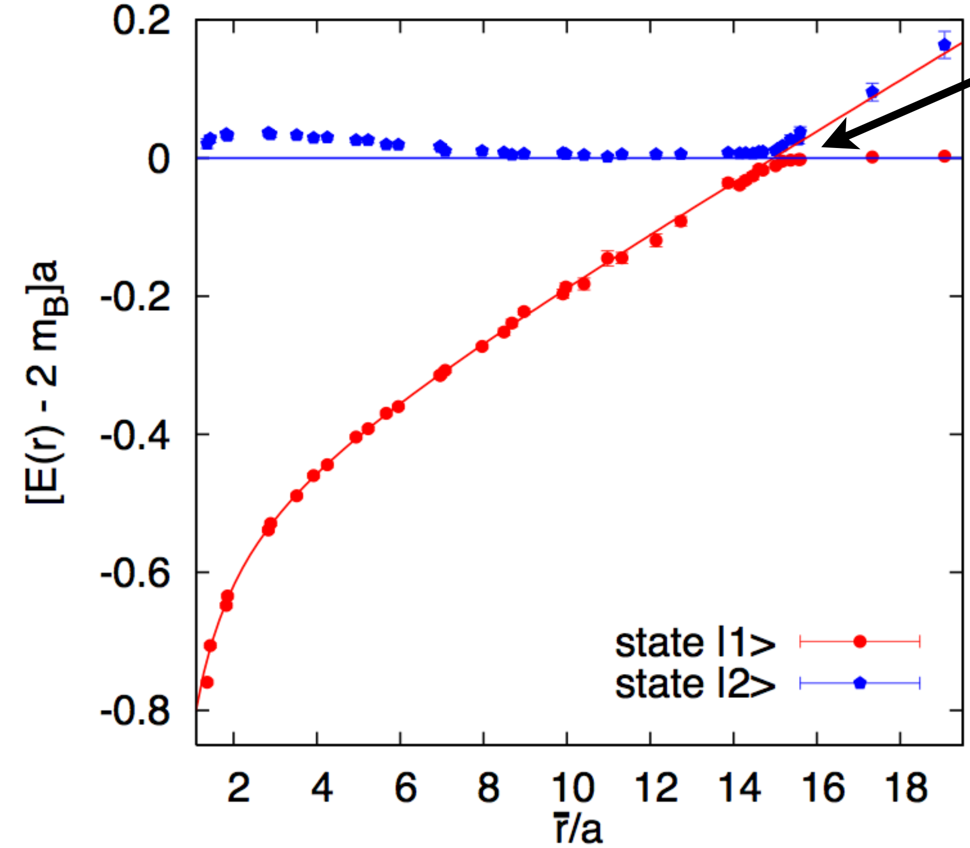
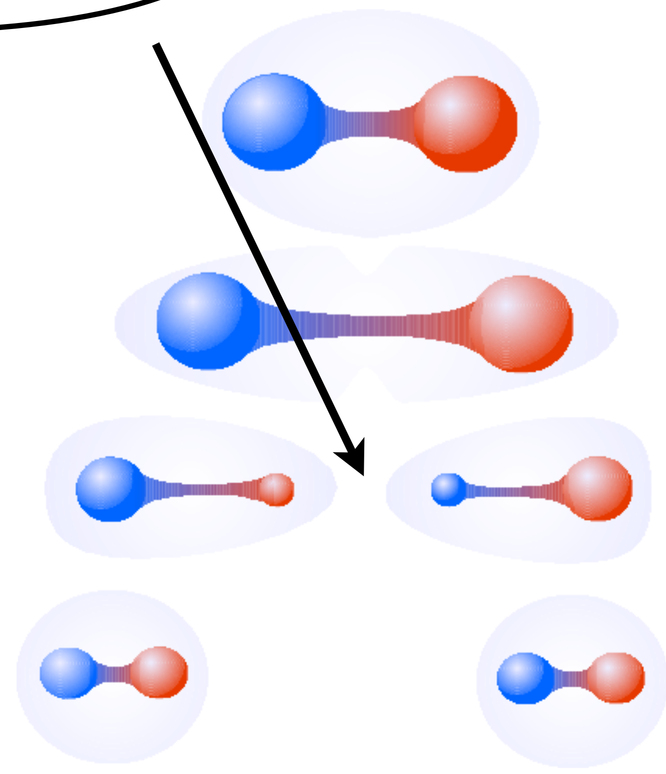


Bali et al. [SESAM Collaboration], PRD 71 (2005) 114513

string breaking



Confinement: linearly rising potential

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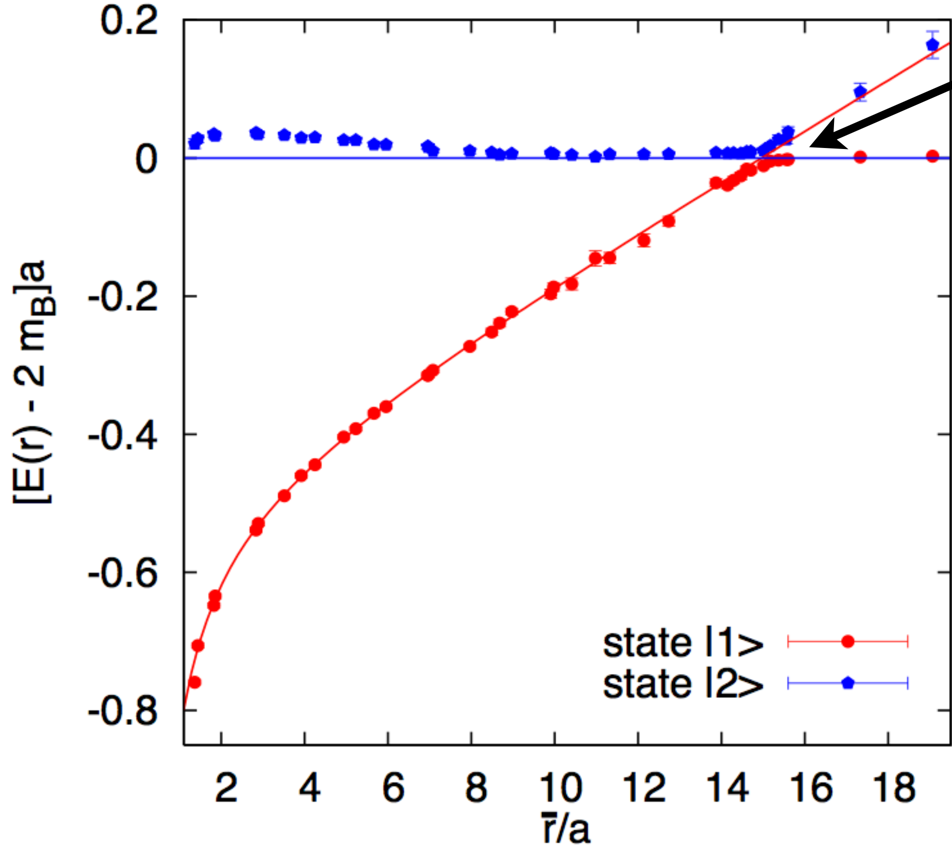
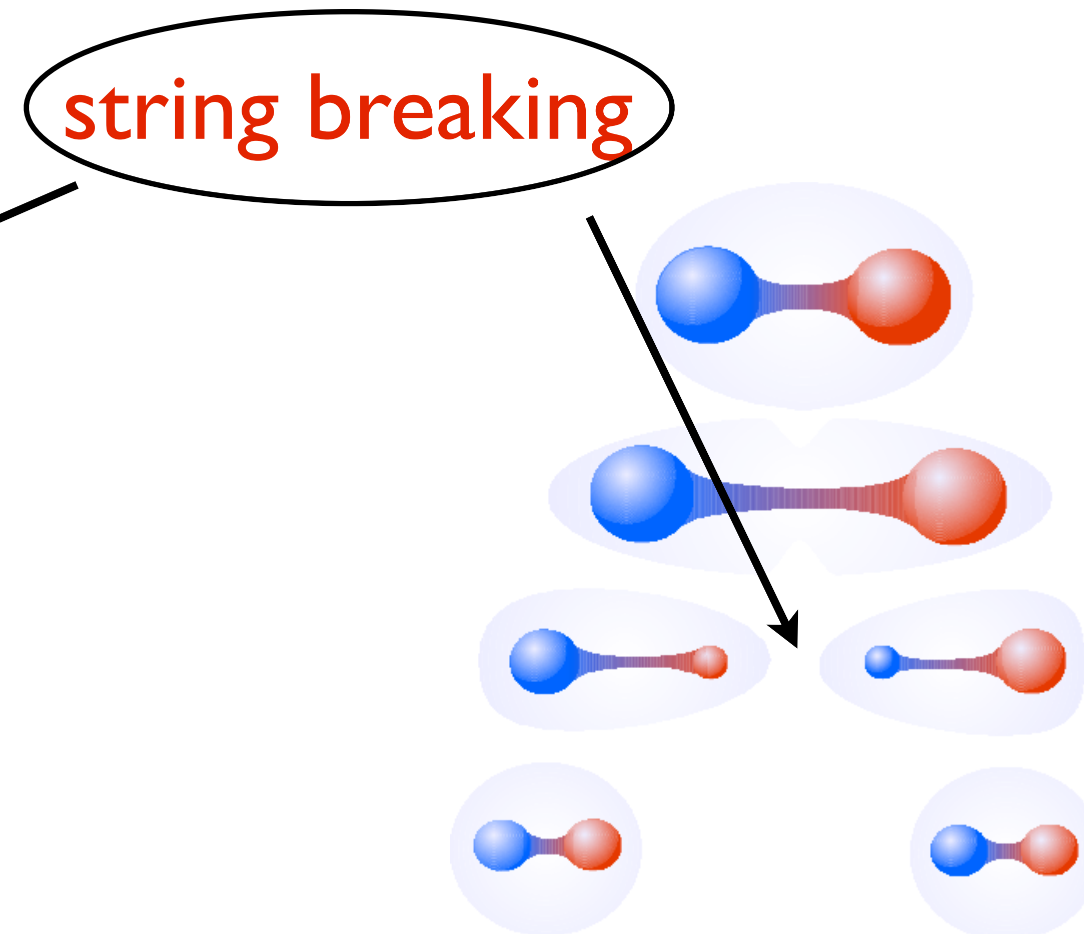


Bali et al. [SESAM Collaboration], PRD 71 (2005) 114513



- String breaking by dynamical charges in fundamental representation of SU(N_c)
- Bound states do not see string breaking scale

Confinement: linearly rising potential

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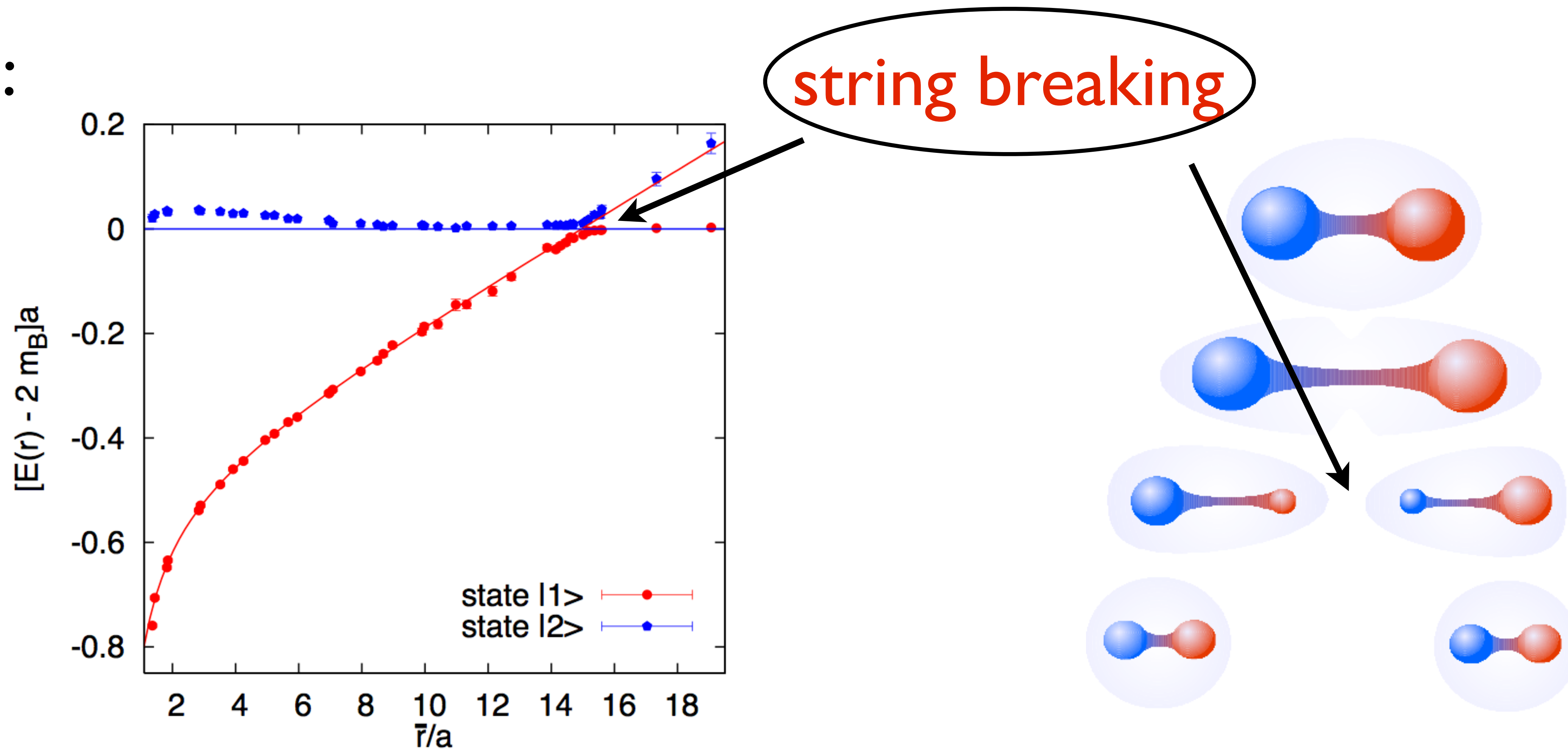
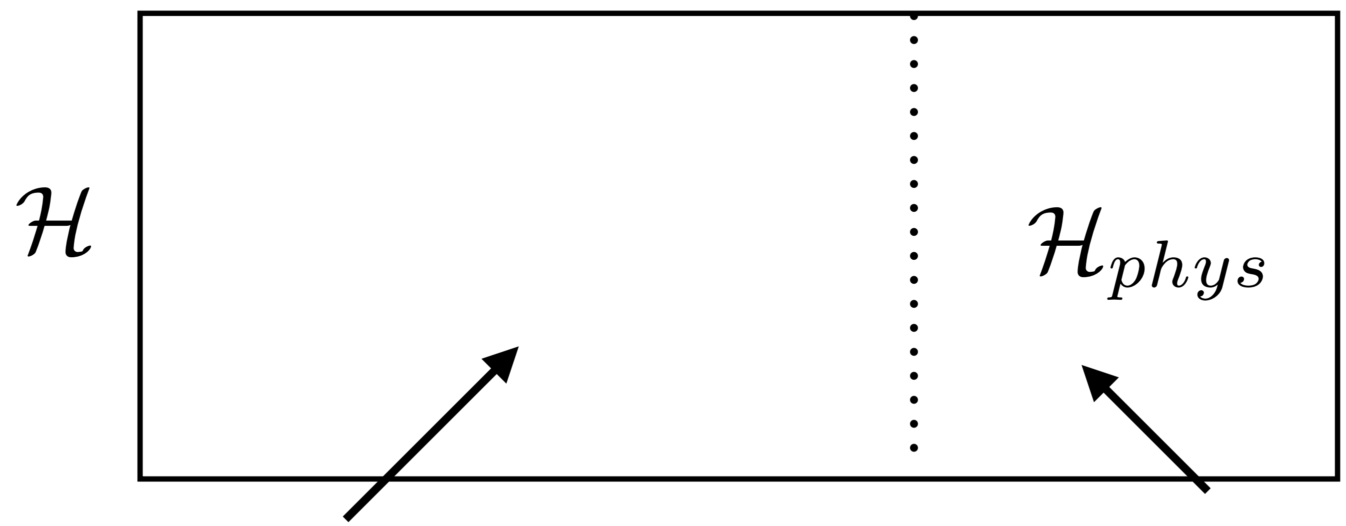


Bali et al. [SESAM Collaboration], PRD 71 (2005) 114513

- String breaking by dynamical charges in fundamental representation of $SU(N_c)$
- Bound states do not see string breaking scale

provides some justification for quark model potential



indefinite metric

positive definite metric, physical subspace of QCD

- If we know that a particle lives not in $\mathcal{H}_{phys}$, it is confined.

> Axiomatic QFT (Osterwalder-Schrader):
>
> physical particle $\longrightarrow$ $D(t, \mathbf{p}) \geq 0$

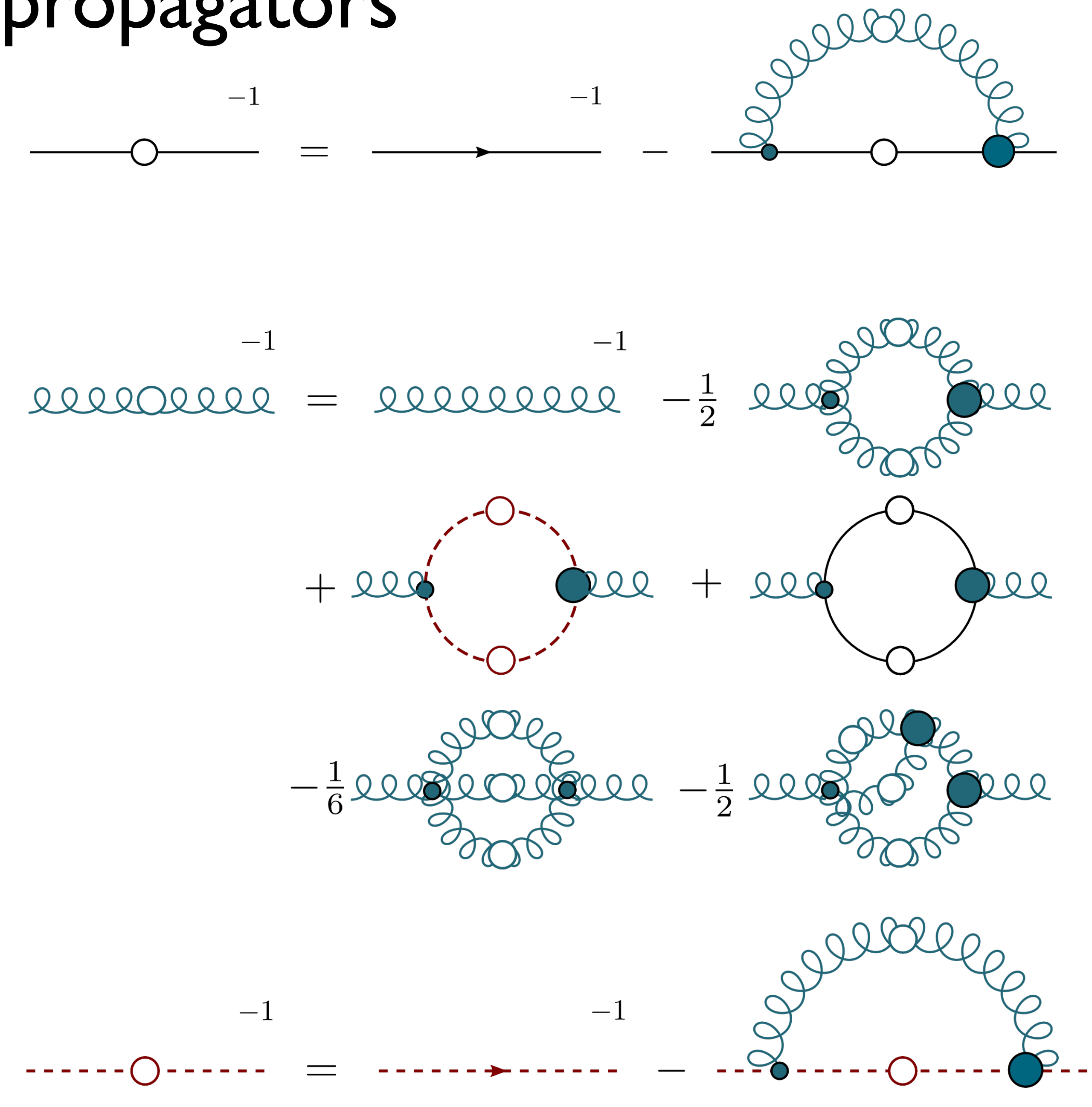
- $\mathcal{H}_{phys}$ needs to have a mass gap!
- related: Kugo-Ojima confinement, IR-behaviour of ghost and glue…

summary: CF J. Phys. G 32 (2006) R253 [hep-ph/0605173]
more details: CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408

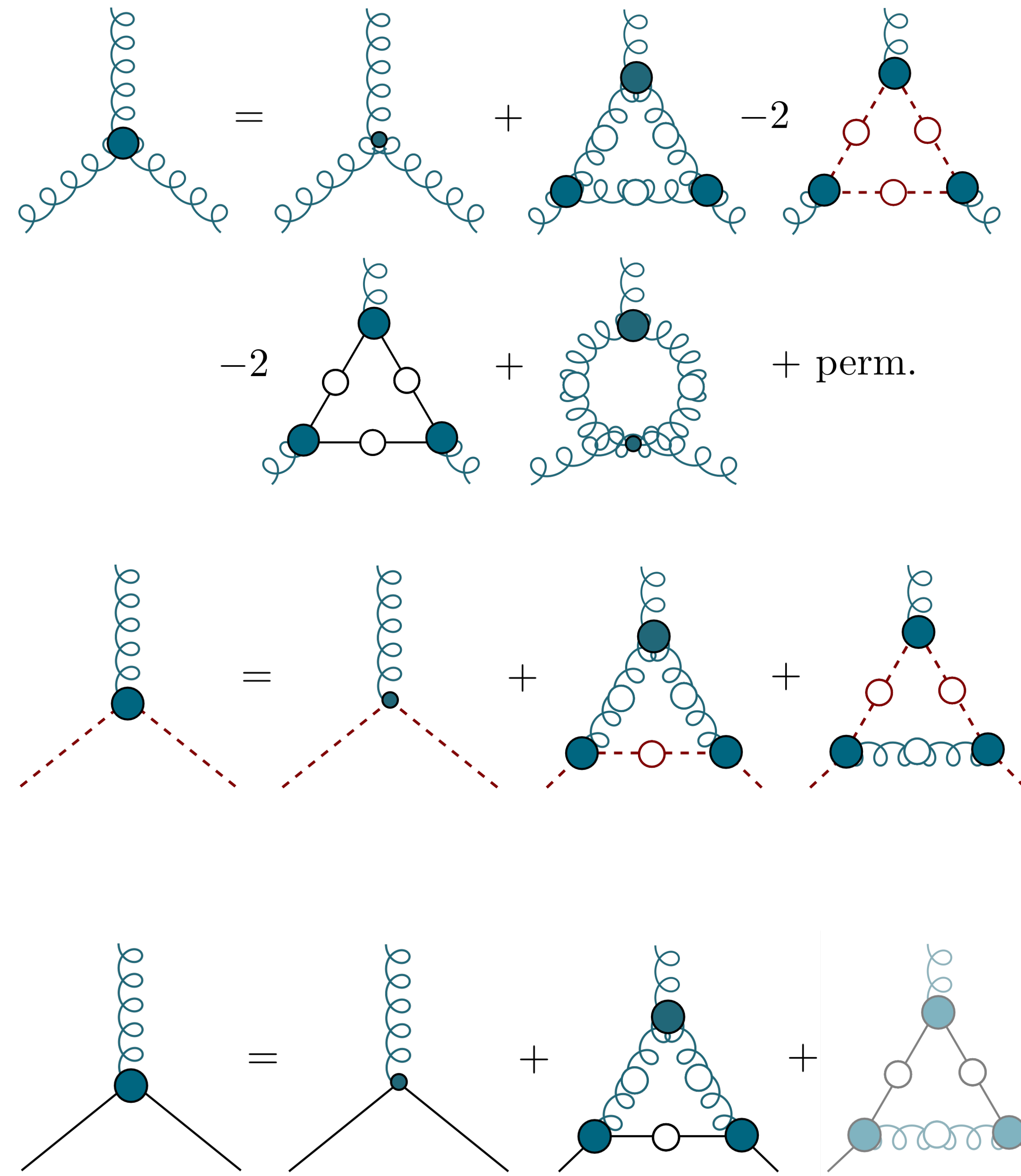
Dyson-Schwinger equations -

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \, (i \right.\right.$$

propagators



vertices



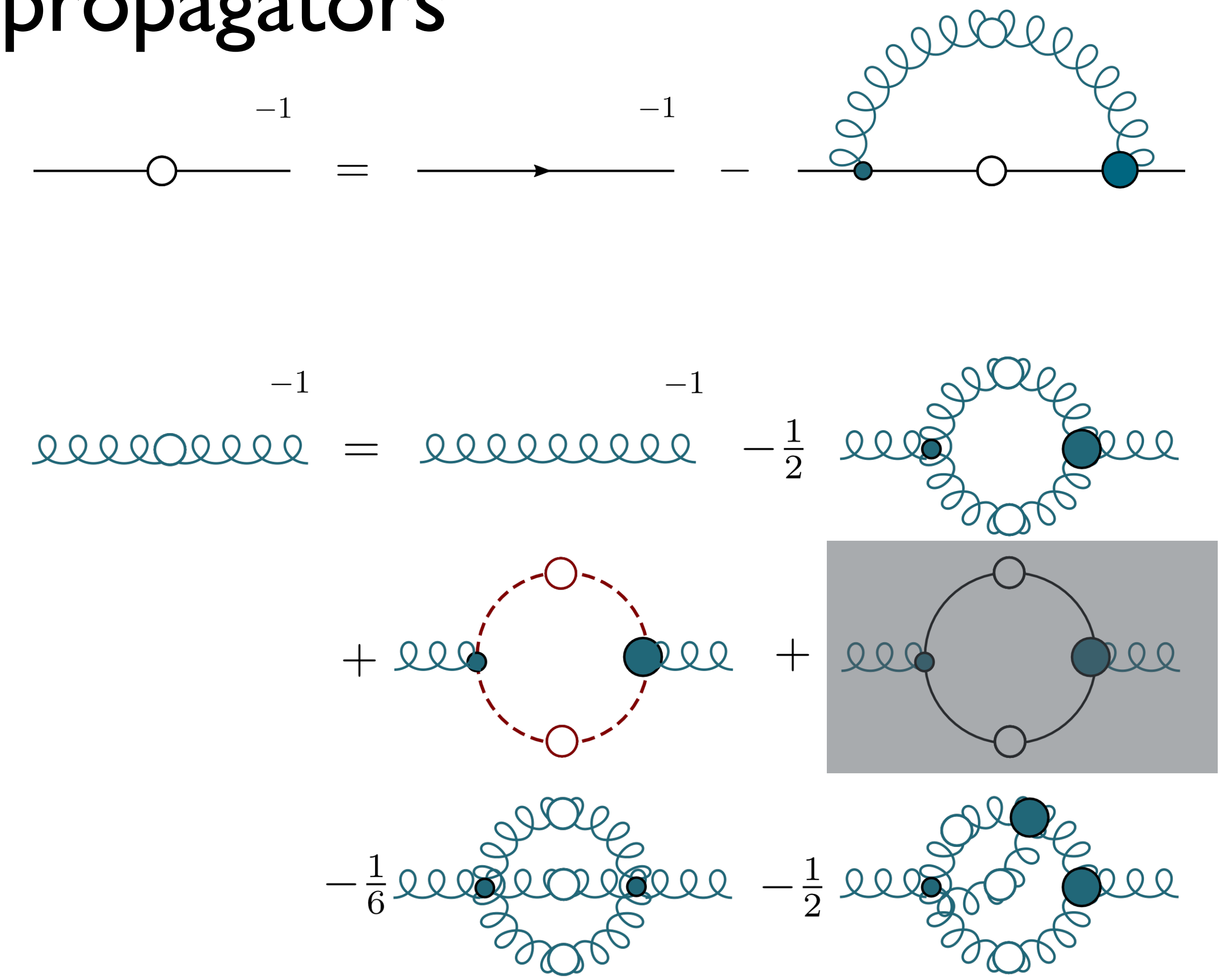
CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

Dyson-Schwinger equations -

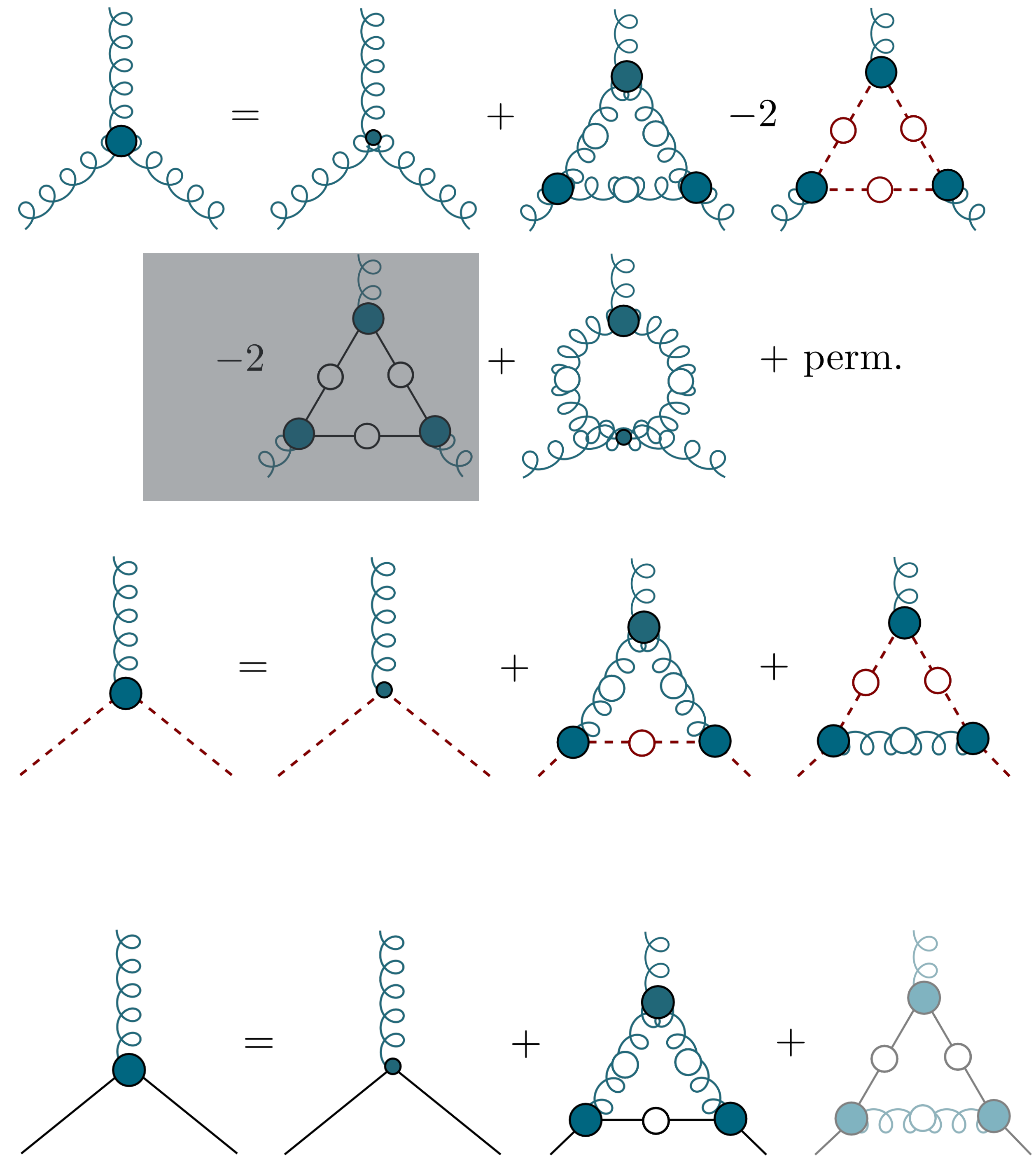
"quenched"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\Big\{ - \int d^4x \Big(\bar{\Psi} \, (i$$

propagators



vertices



CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
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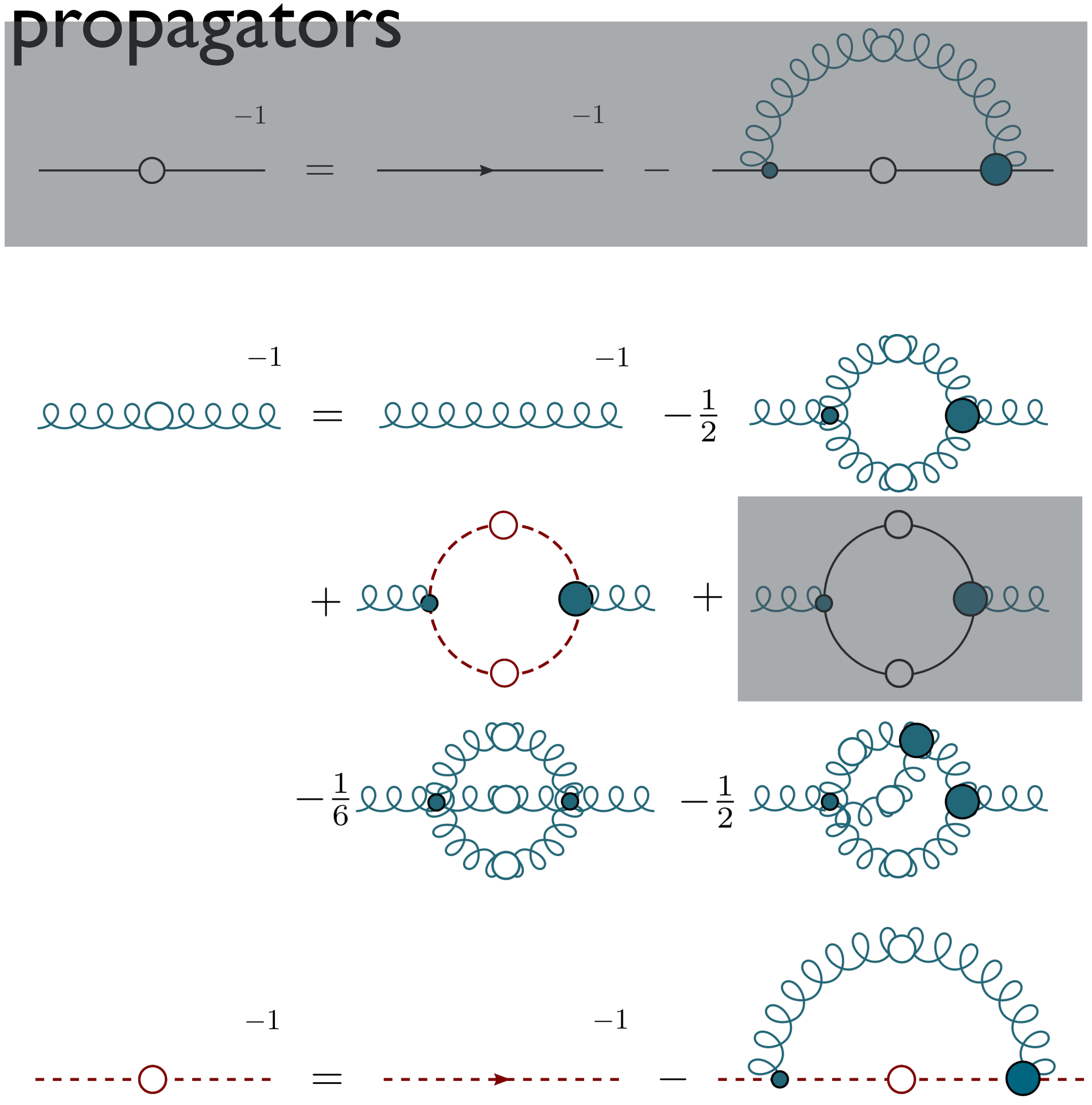
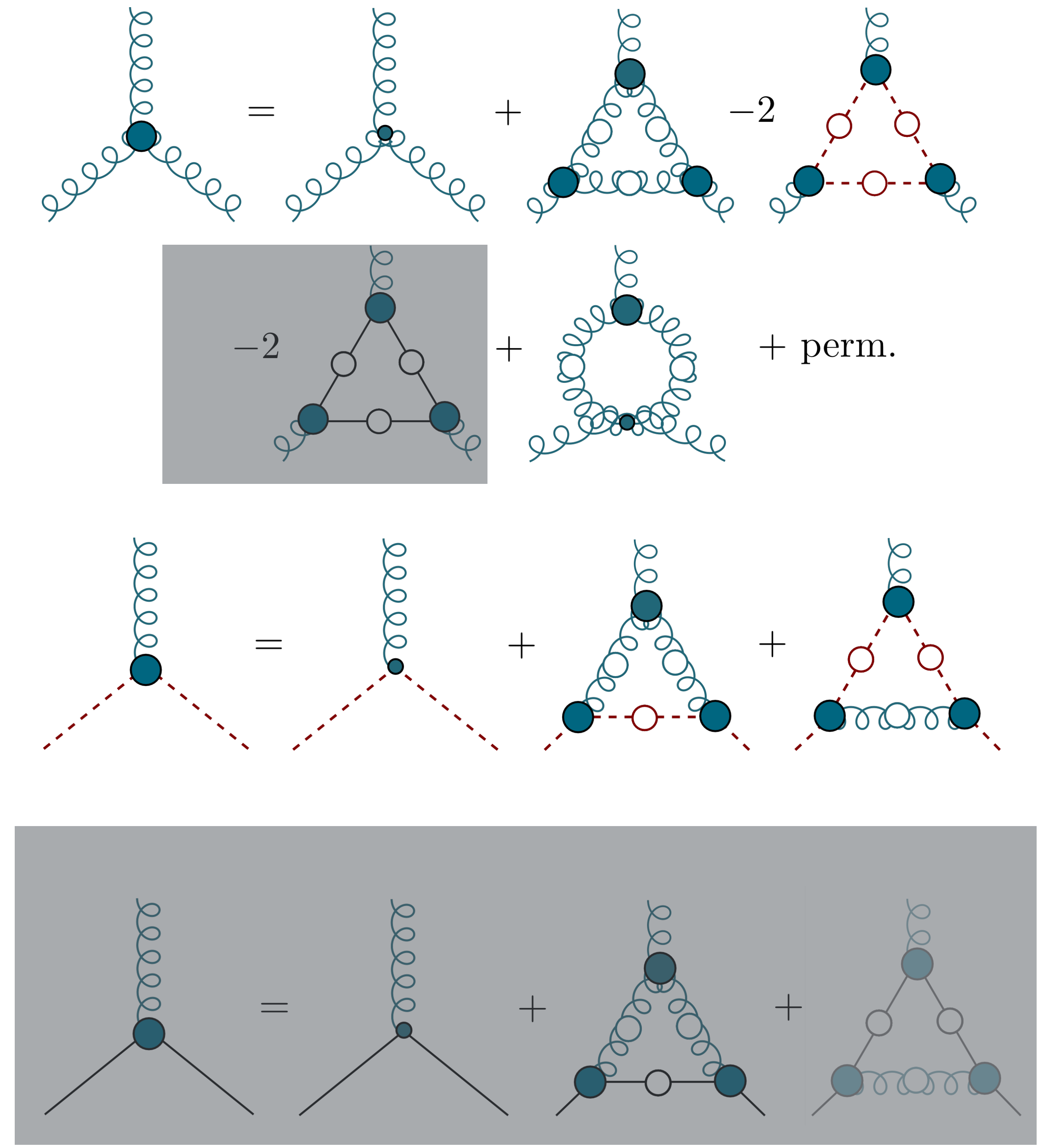
pure YM-Theory

propagators

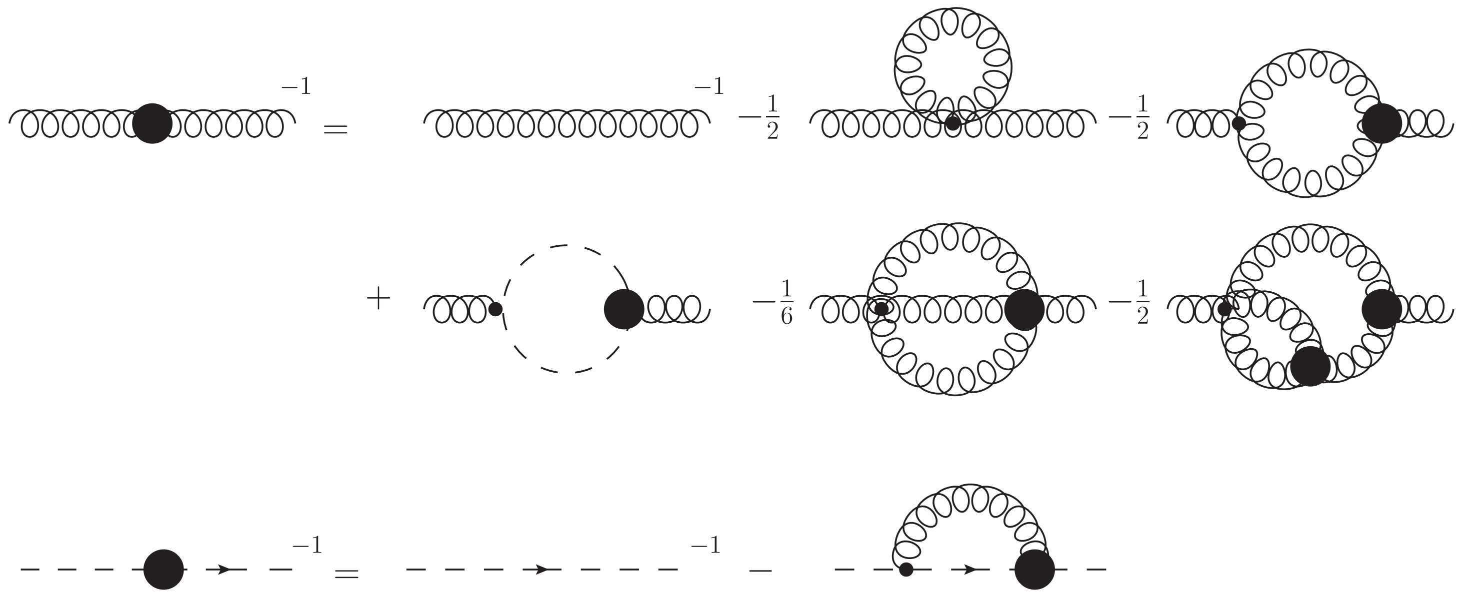


vertices



CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, PRD 101 (2020) 114009

Landau gauge gluon propagator



$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$

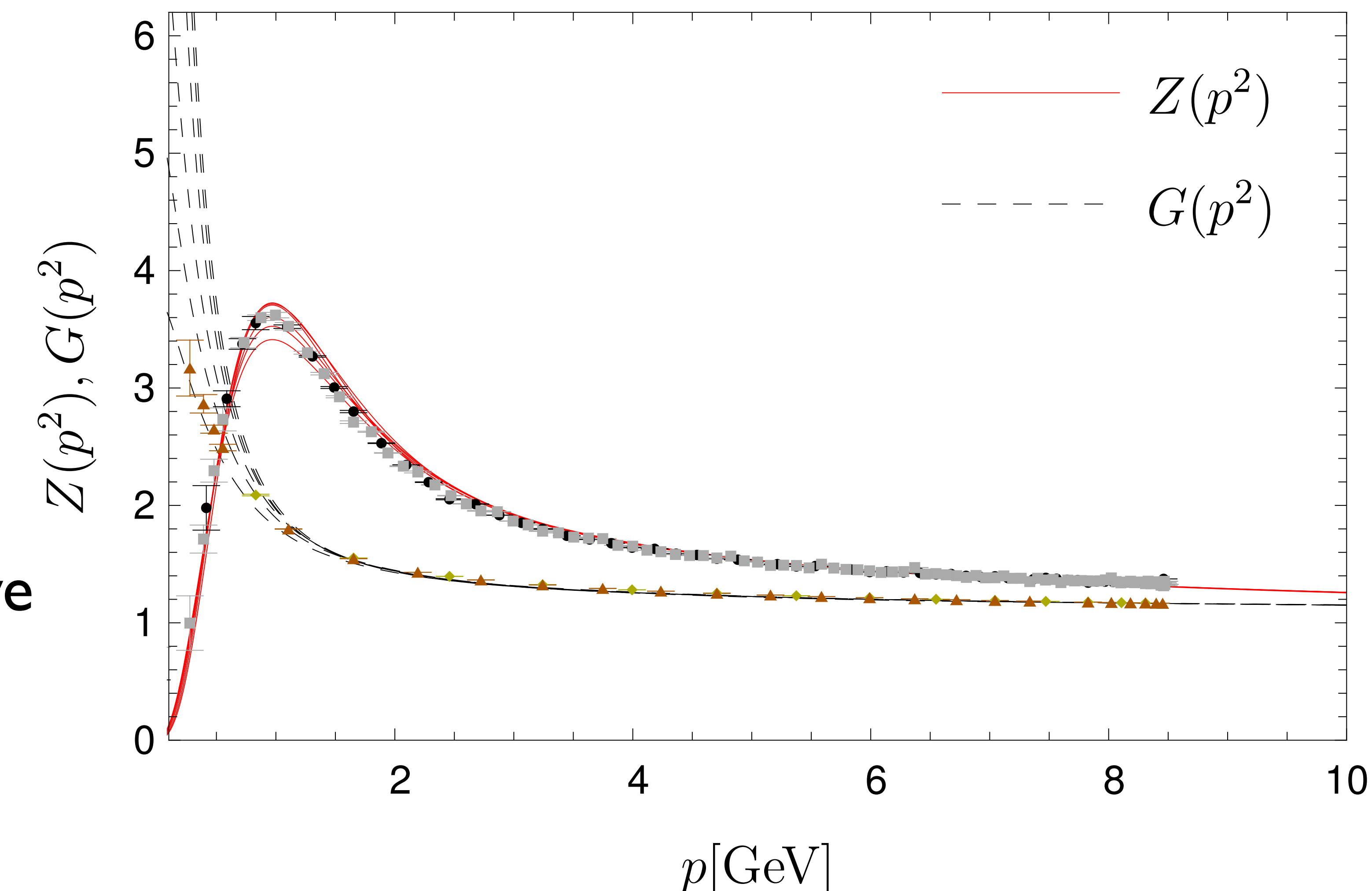
$$D_G(p) = \frac{-G(p^2)}{p^2}$$

- spacelike momenta:
  good agreement with lattice
- fully dressed gluon appears massive

Cornwall PRD 26 (1982);
Cucchieri, Mendes PoS Lat2007 297
Aguilar, Binosi, Papavassiliou, PRD 78, 025010 (2008);
Boucaud et al. JHEP 0806 (2008) 099;
CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408

- time-like momenta: work in progress

CF, Huber, PRD 102 (2020) 094005, arXiv:2007.11505



DSE: Huber, PRD 101 (2020) 114009, arXiv:2003.13703
Lattice: Sternbeck, Müller-Preussker, PLB 726 (2013)

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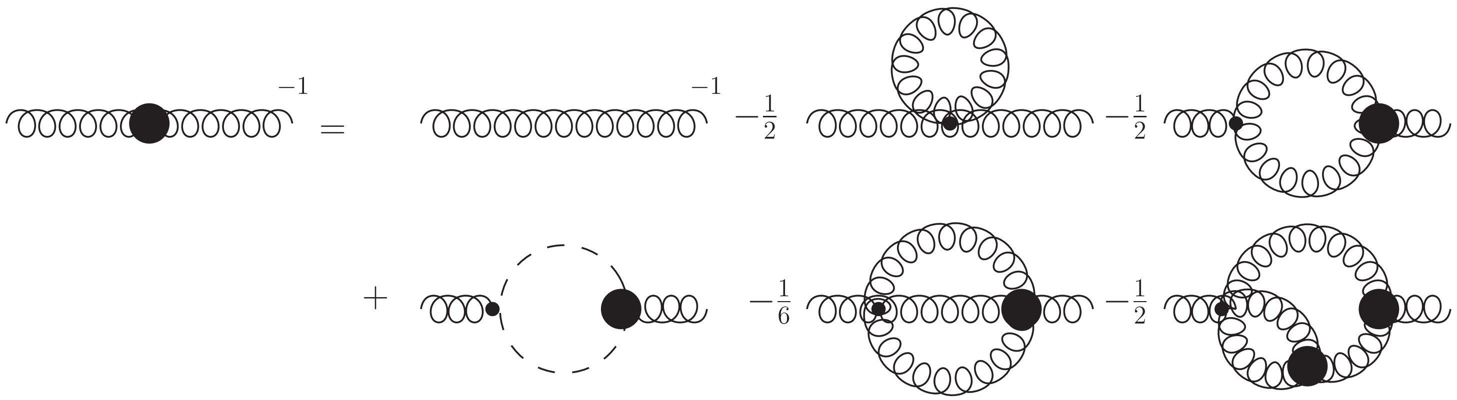

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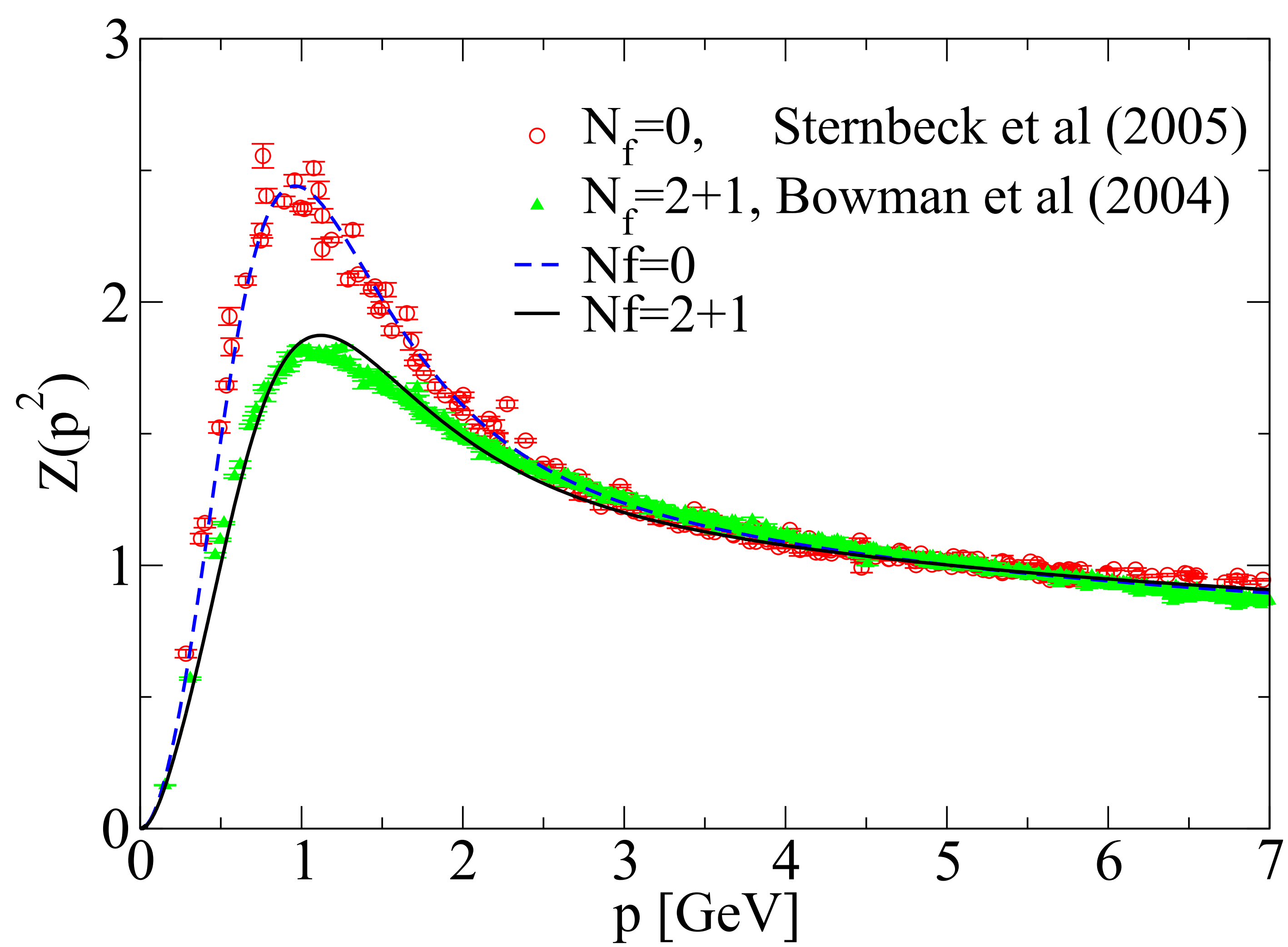

Hopfer, CF and Alkofer, JHEP 1411 (2014) 035

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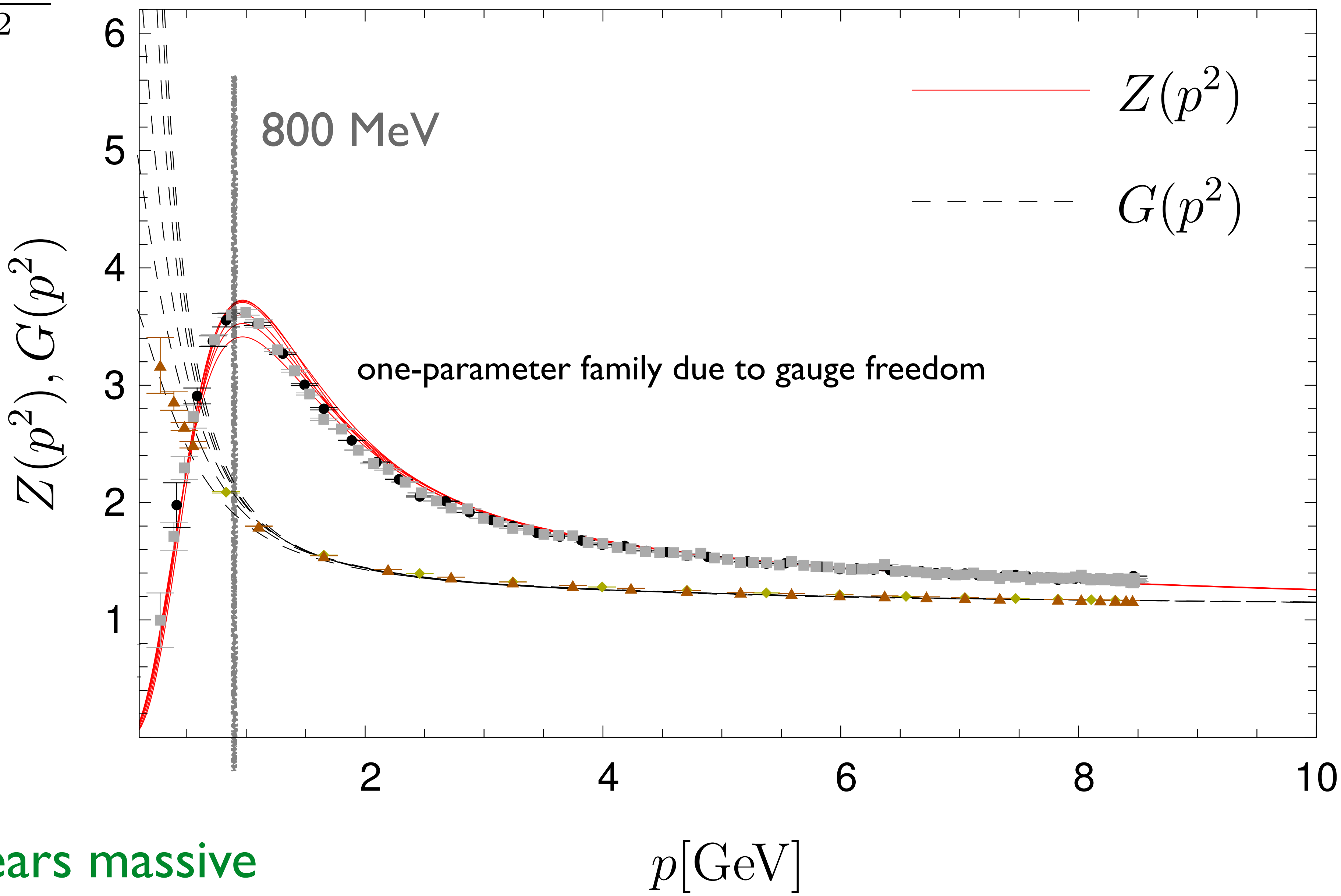

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Boucaud et al. JHEP 0806 (2008) 099;
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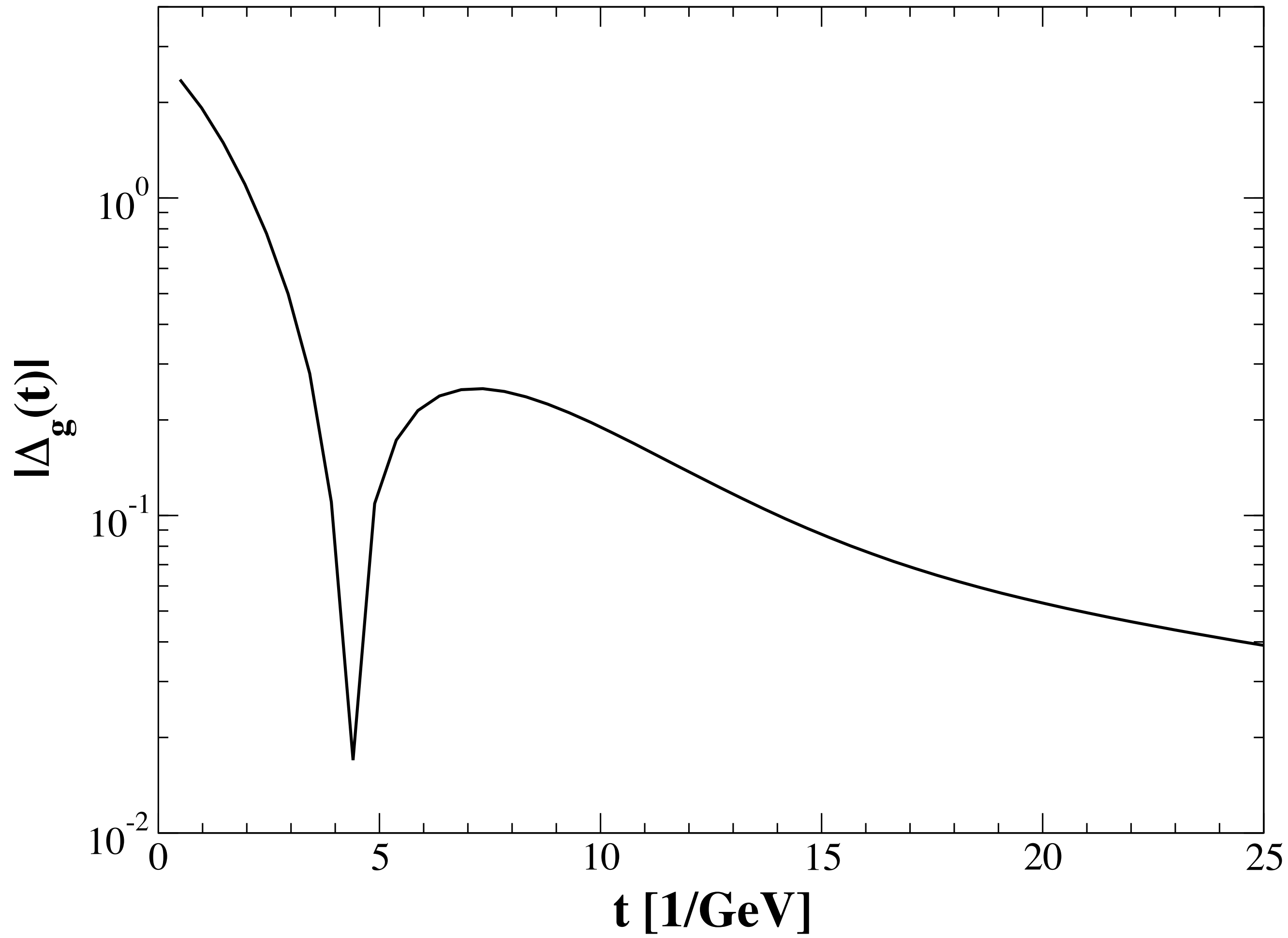
Positivity violations

Schwinger function: $$\Delta_g(t) = \int \frac{dp_0}{2\pi} \, e^{itp_0} \, \left(\frac{Z(p^2)}{p^2} \right) \Bigg|_{\vec{p}=0}$$



Alkofer, Detmold, CF, Maris, PRD 70 (2004) 014014

- Violation of positivity: color screening

Gluons cannot exist as asymptotic states

Positivity violations

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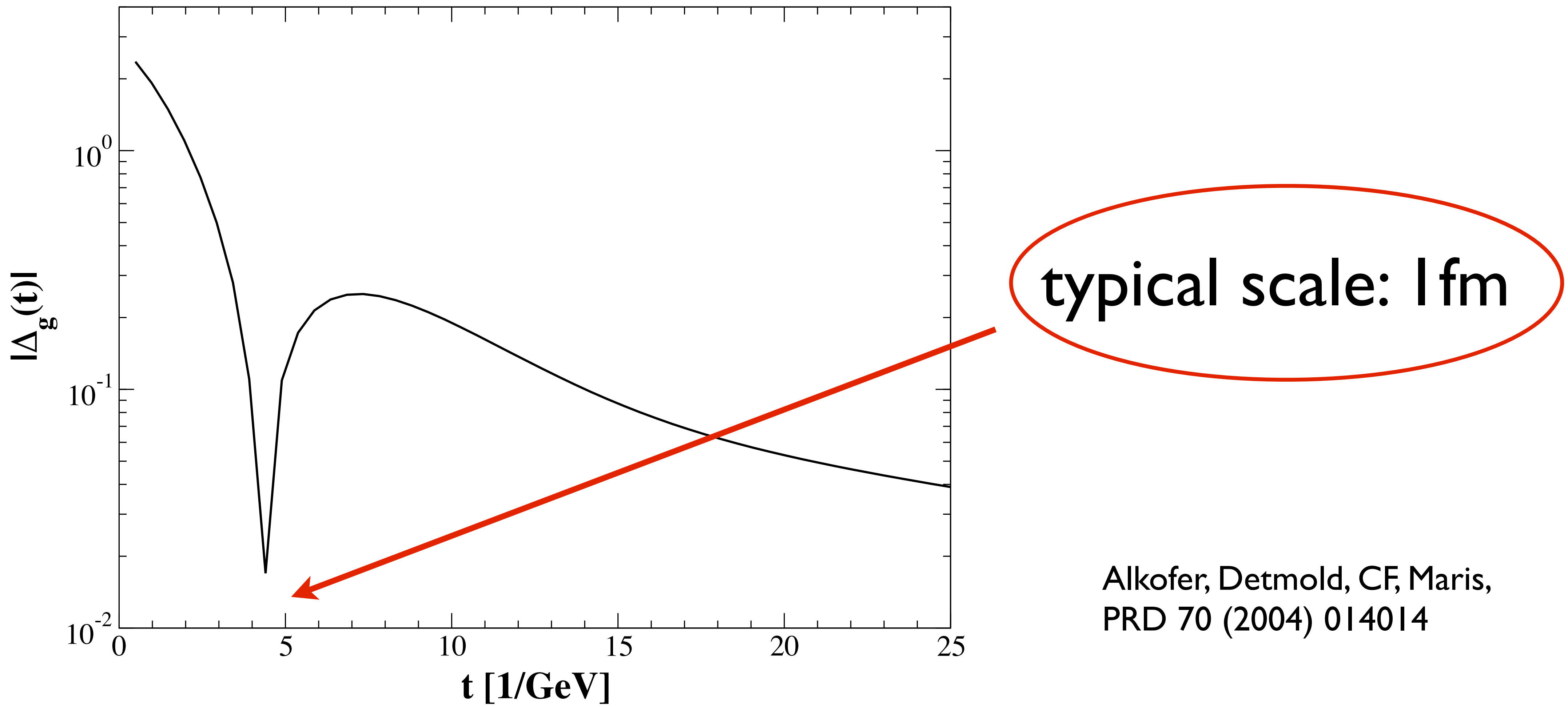


Alkofer, Detmold, CF, Maris, PRD 70 (2004) 014014

- Violation of positivity: color screening

Gluons cannot exist as asymptotic states

Glueballs from DSE/BSEs

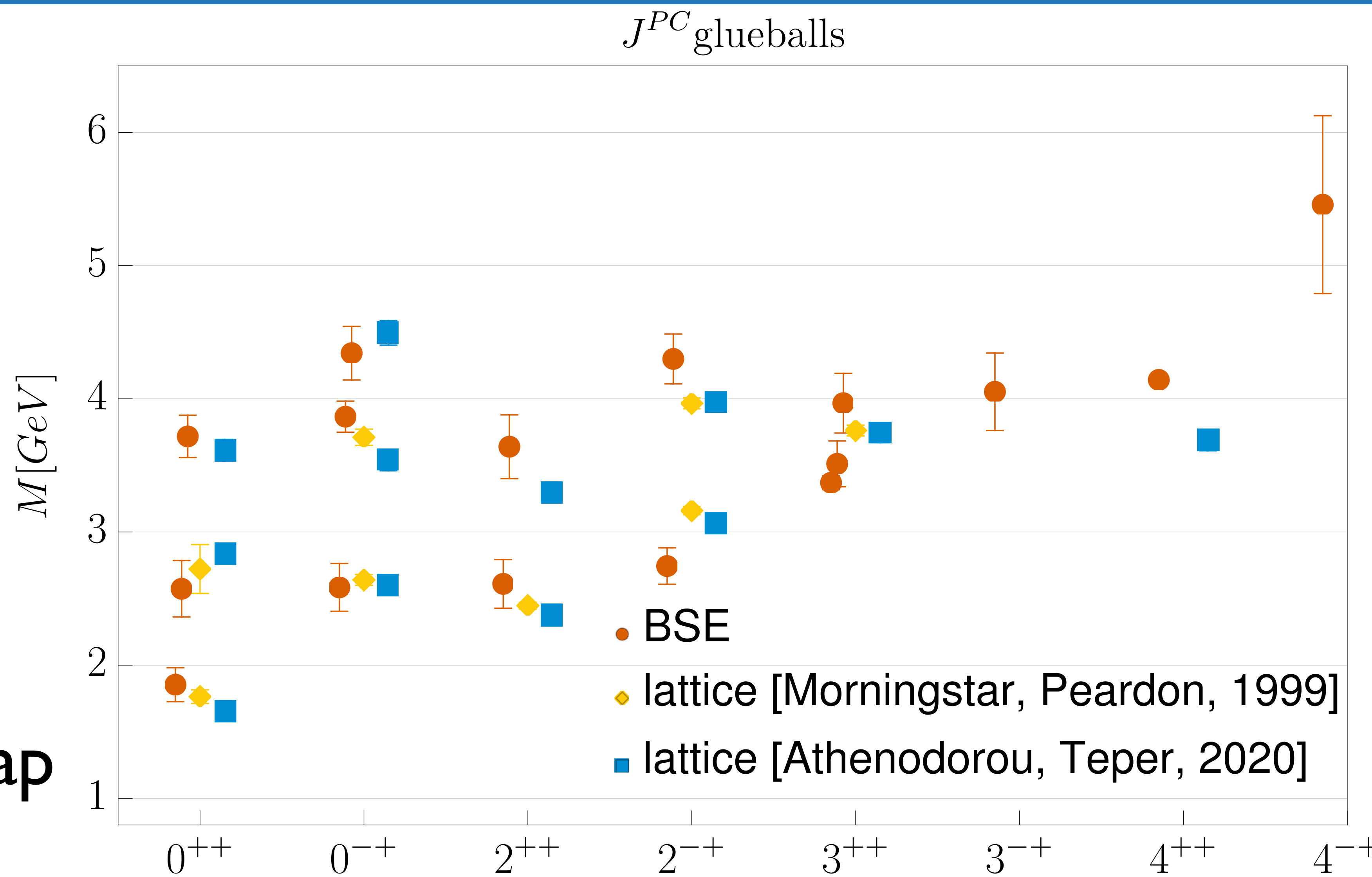
- Mixing of two-gluon amplitudes with ghost-antighost
- exploratory: simple models

Meyers, Swanson, PRD 87 (2013) 3, 036009
Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001
Souza et al. , EPJA 56 (2020) no.1, 25

- new: high quality input from 3PI truncation

Huber, PRD 101 (2020) 114009

Glueballs: results



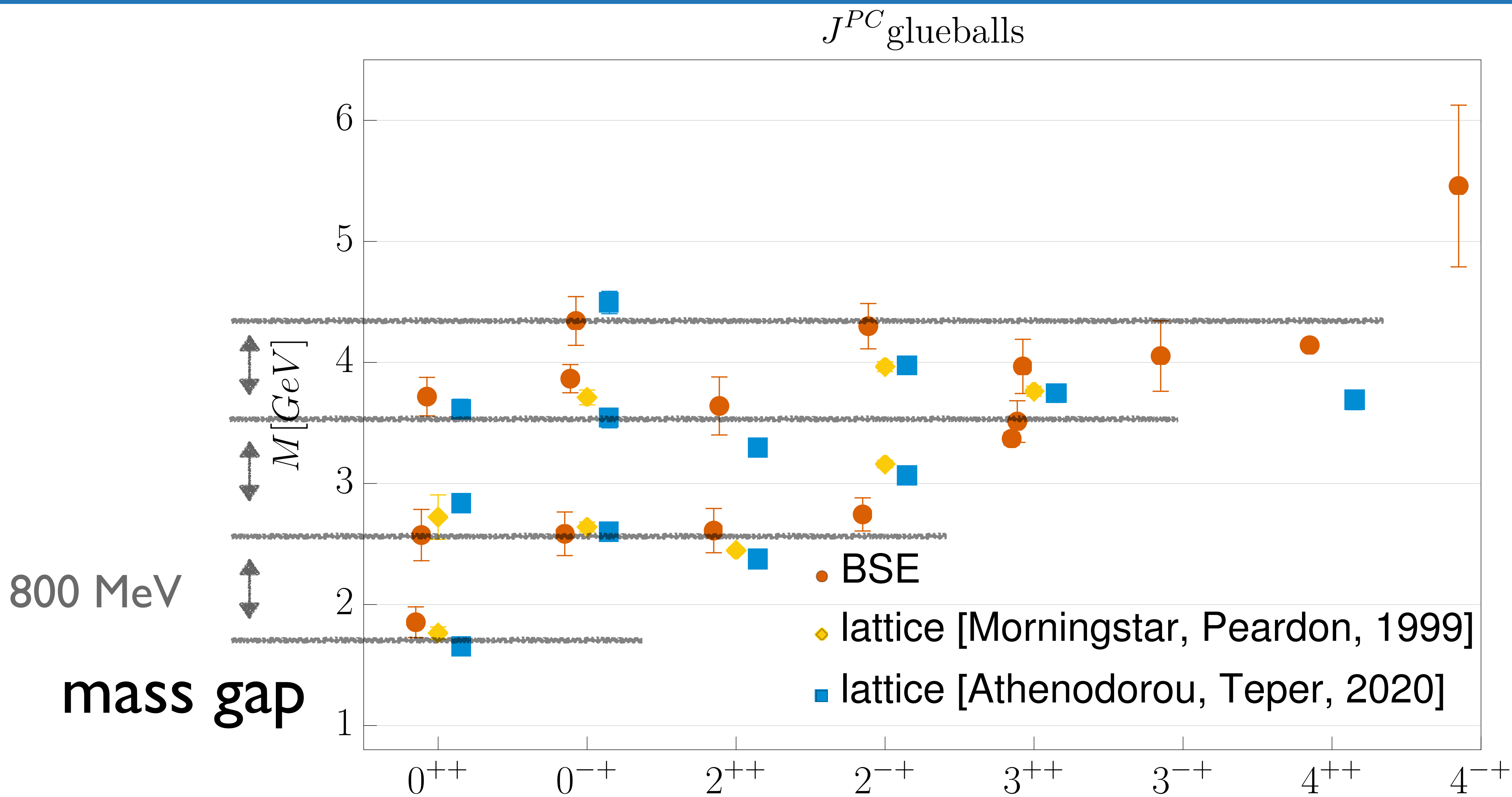
mass gap

- confirmation of results from lattice YM-theory
- predictions for some channels

CF, Huber, Sanchis-Alepuz, EPJC 80 (2020) [arXiv:2004.00415]
Huber, CF, Sanchis-Alepuz, EPJC 81 (2021) [arXiv:2110.09180]

To do:
chart the mixing of glueballs with conventional meson states…

Glueballs: results



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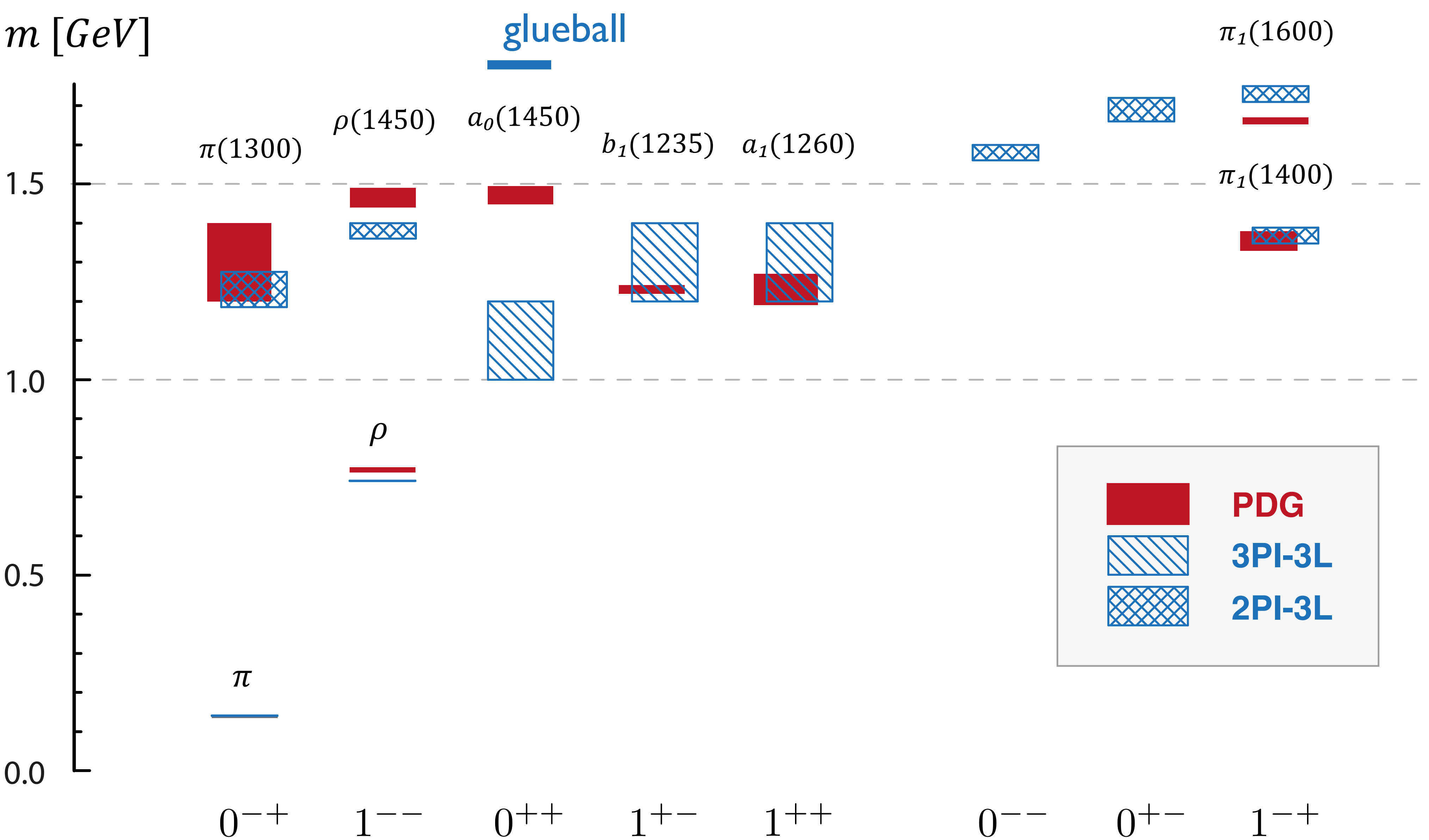
To do:
chart the mixing of glueballs with conventional meson states…

Overview

1. Bound states in the quark model
   - Construction
   - Assets, shortcomings and how to do better
2. Properties of QCD and correlation functions
   - Dynamical chiral symmetry breaking
   - Correlation functions and Dyson-Schwinger equations (DSEs)
   - The quark DSE
3. Mesons
   - The pion Bethe-Salpeter equation (BSE): Goldstone bosons, GMOR and all that…
   - Spectra: light and charm
4. Exotic mesons
   - Confinement and glueballs
   - Four-quark states
5. Baryons
   - Spectra: light and strange
6. Form factors
   - Meson form factors
   - Baryon form factors

Light (conventional) meson spectrum



Williams, CF, Heupel, PRD93 (2016) 034026

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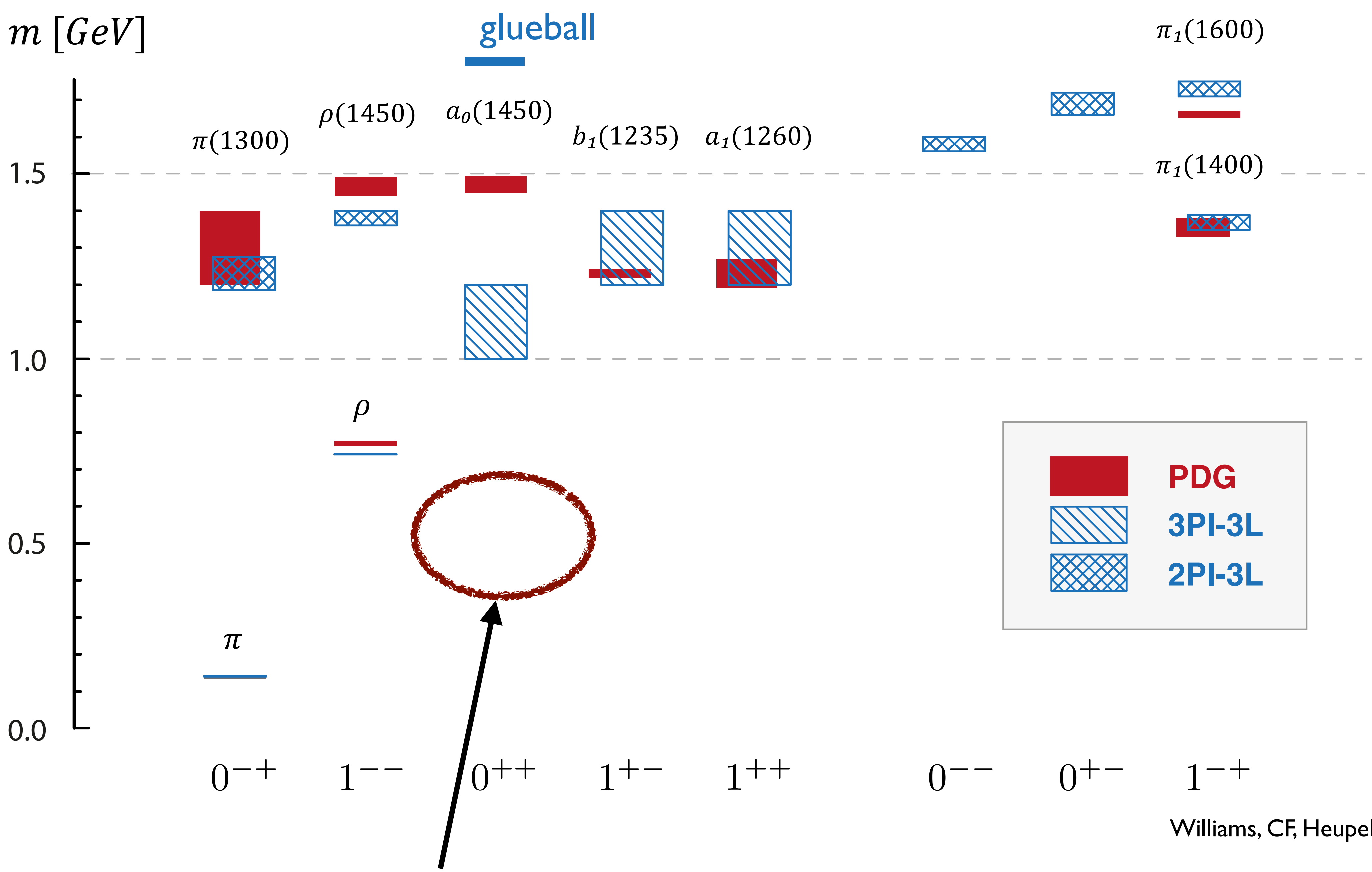
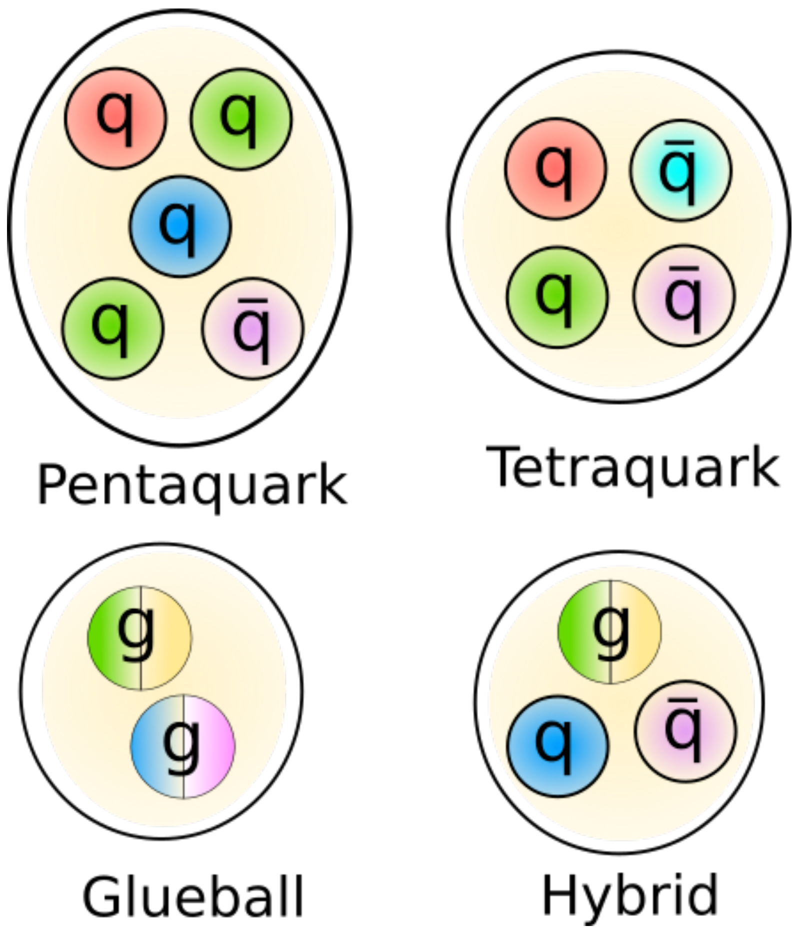


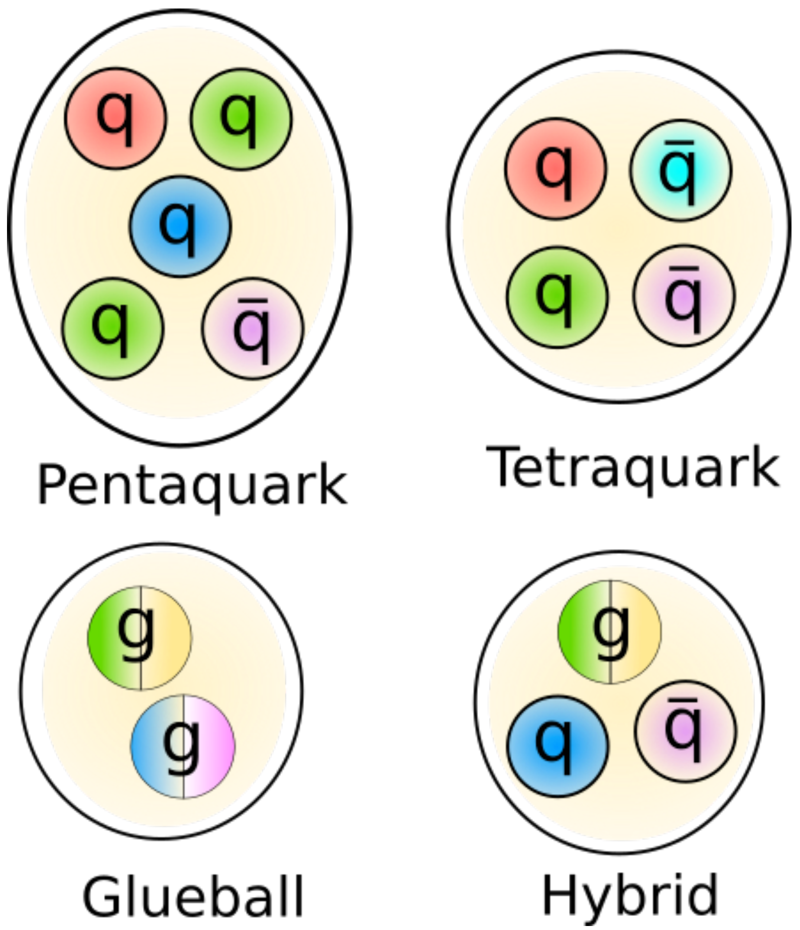
Williams, CF, Heupel, PRD93 (2016) 034026

what about the $f_0(500)$?

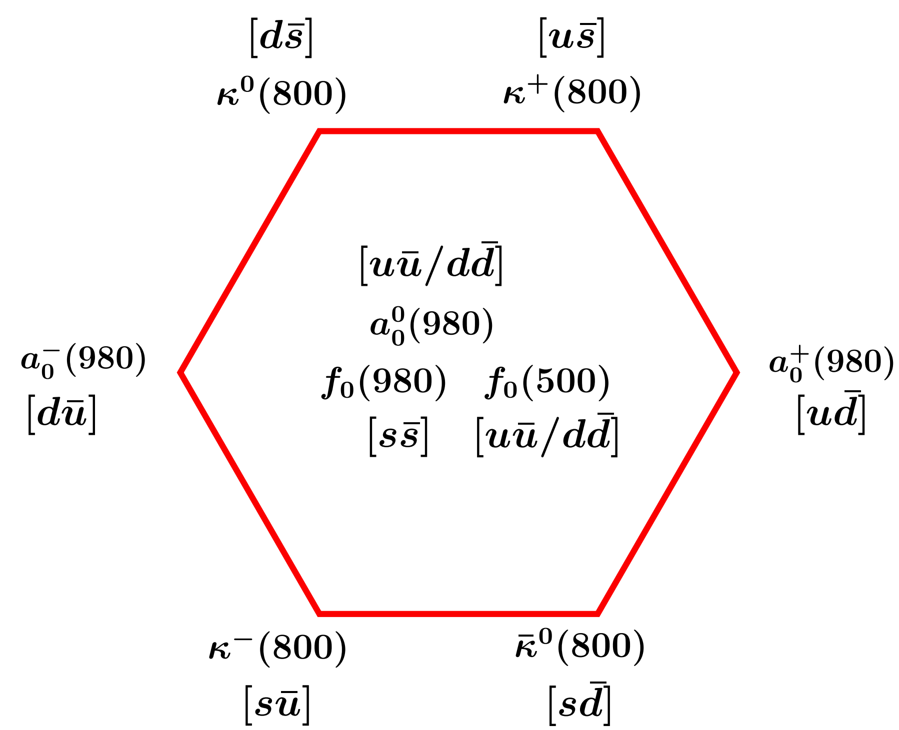
Four-quark states in the light meson sector



K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014) (URL: http://pdg.lbl.gov)



Light meson sector: scalars!



$f_0(980)$ [j] $I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = 990 \pm 20$ MeV
Full width $\Gamma = 40$ to 100 MeV

$f_0(980)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\pi\pi$	dominant	476
$K\overline{K}$	seen	36
$\gamma\gamma$	seen	495

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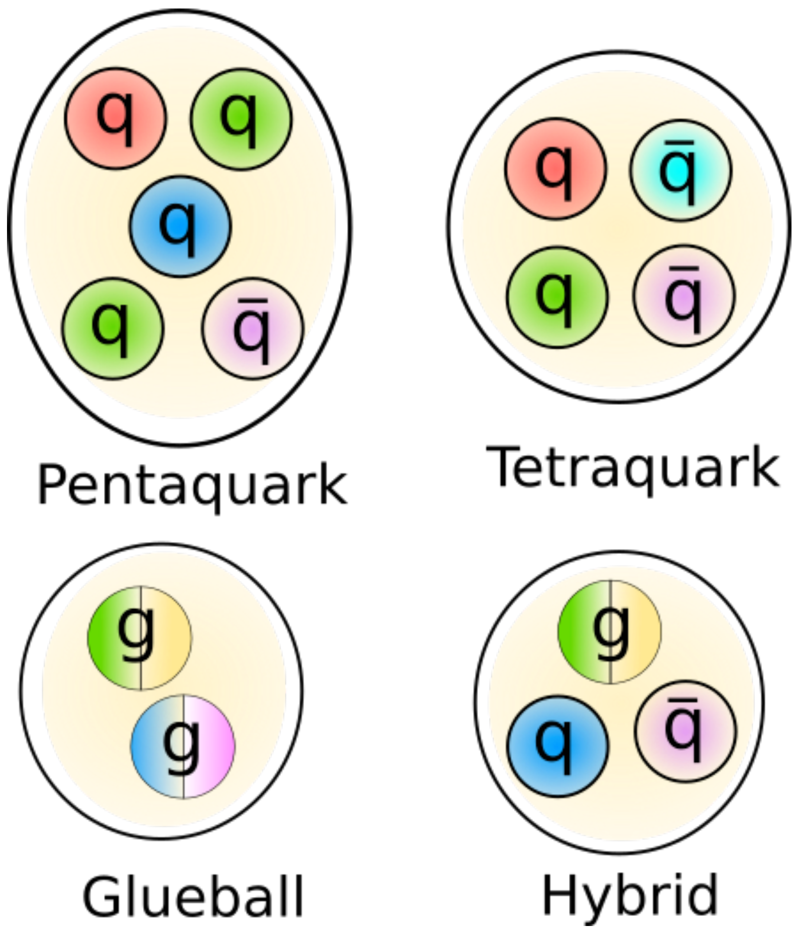

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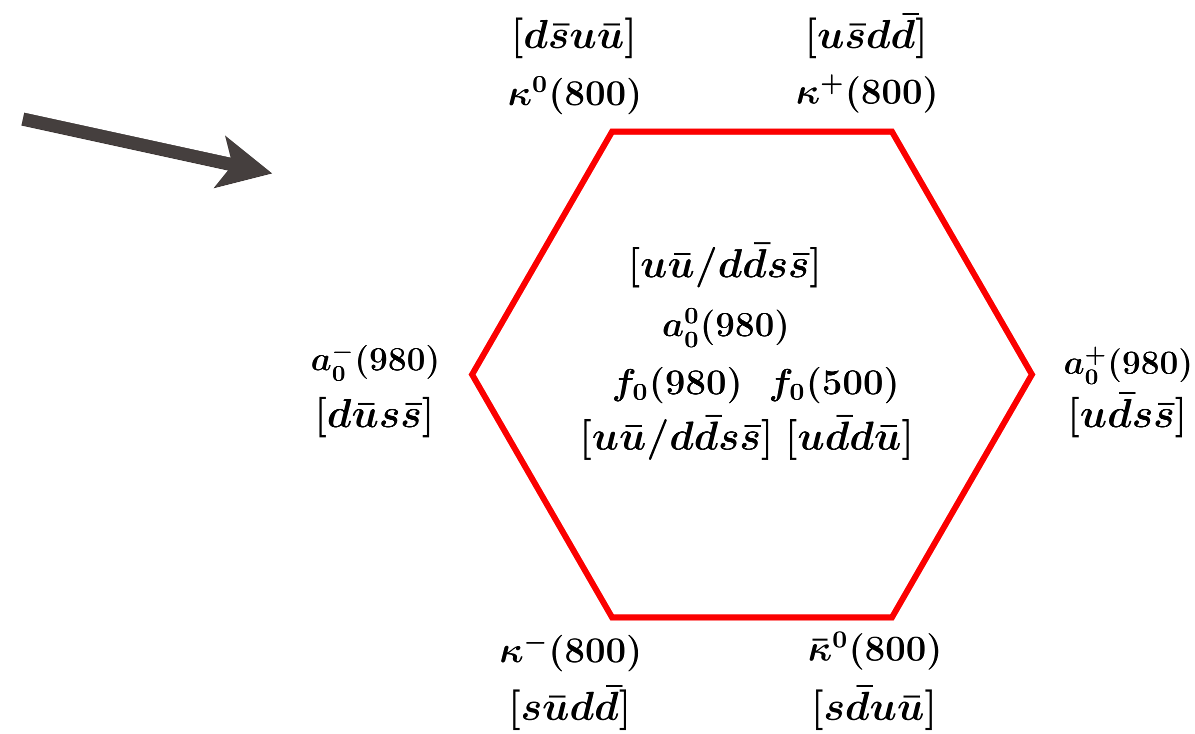

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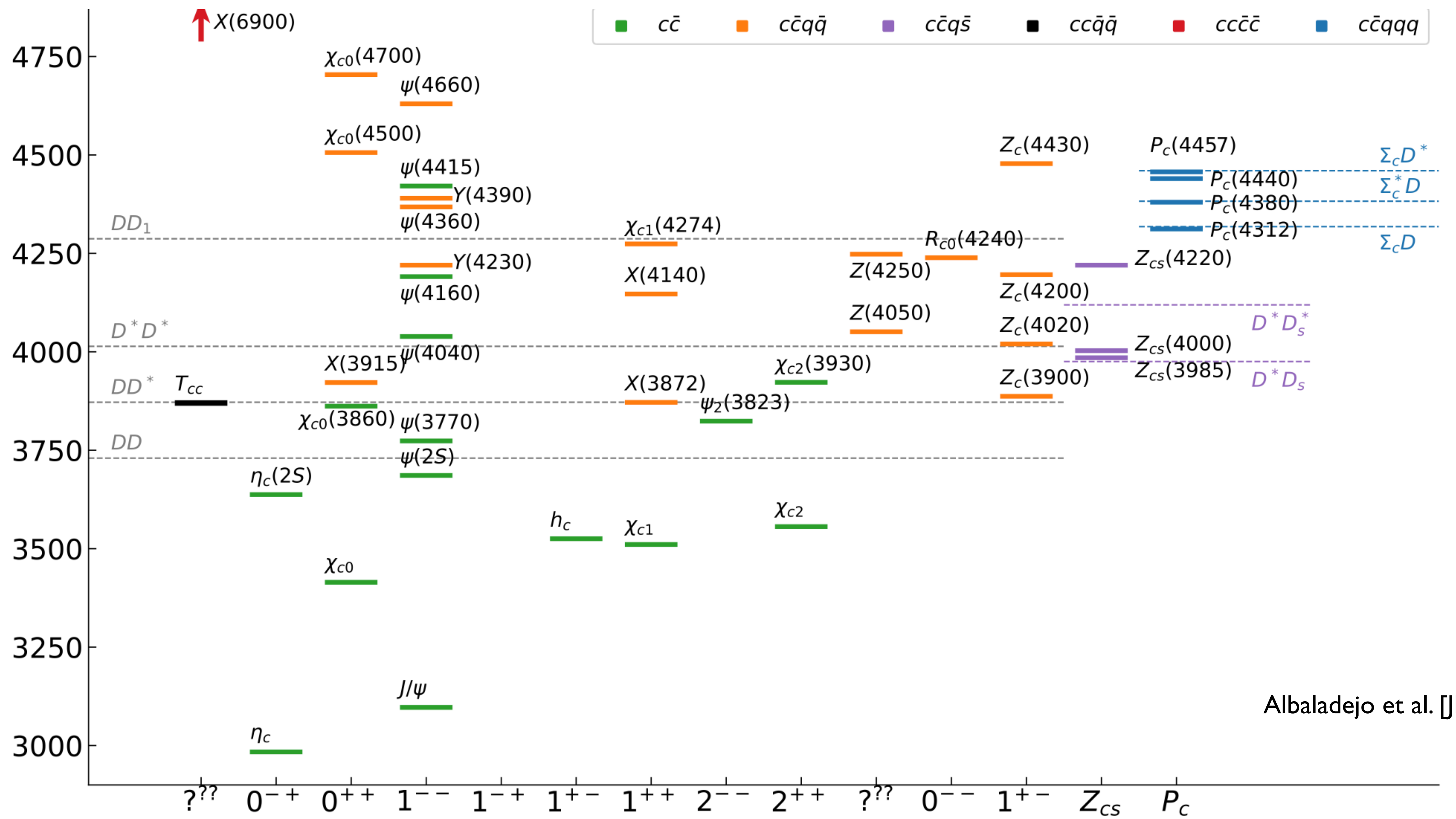
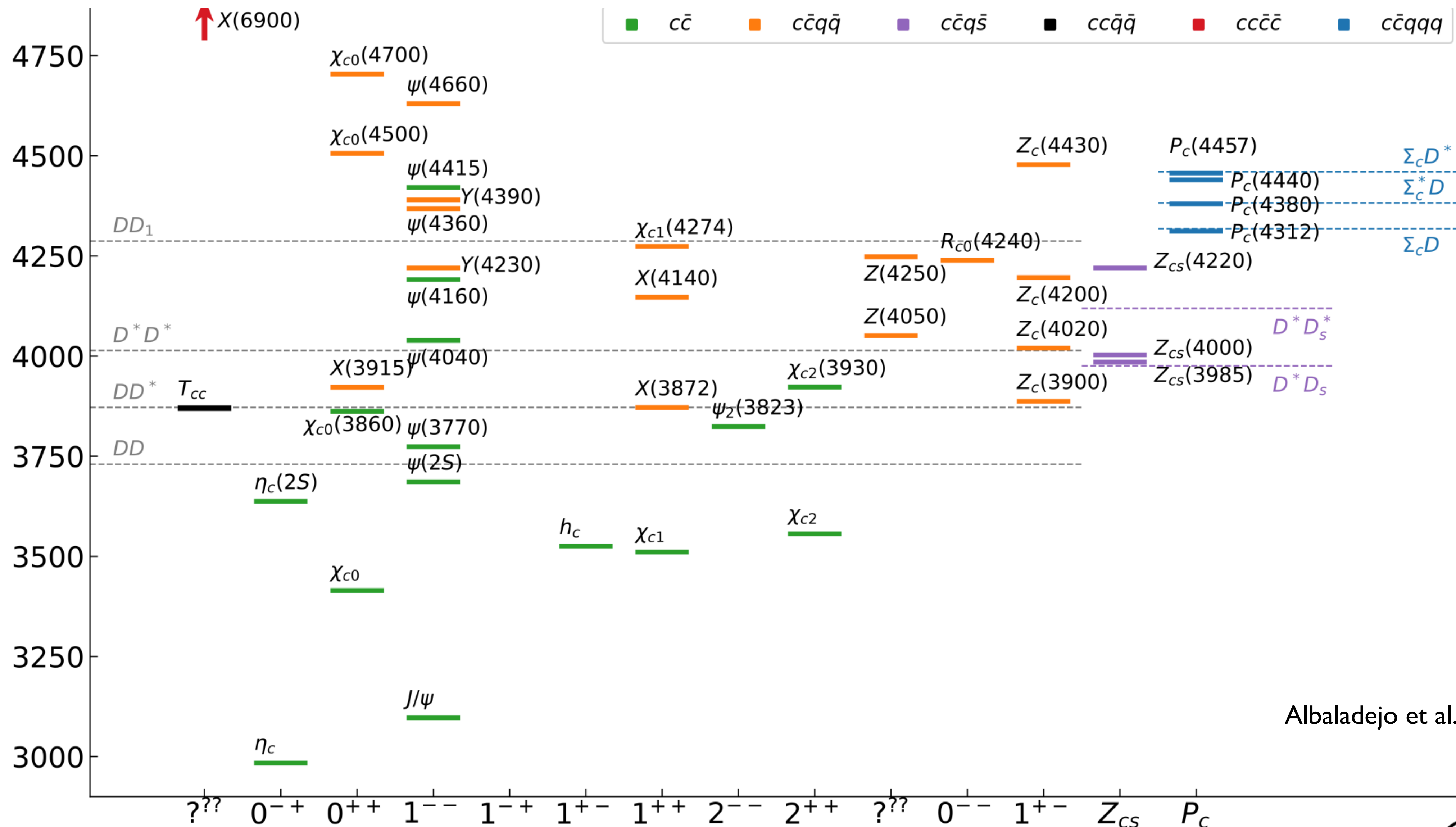
Exotic hadrons at Belle, BABAR, BES, LHCb,…



Albaladejo et al. [JPAC], PPNP 127 (2022), 103981

Four-quark states:

Exotic hadrons at Belle, BABAR, BES, LHCb,…



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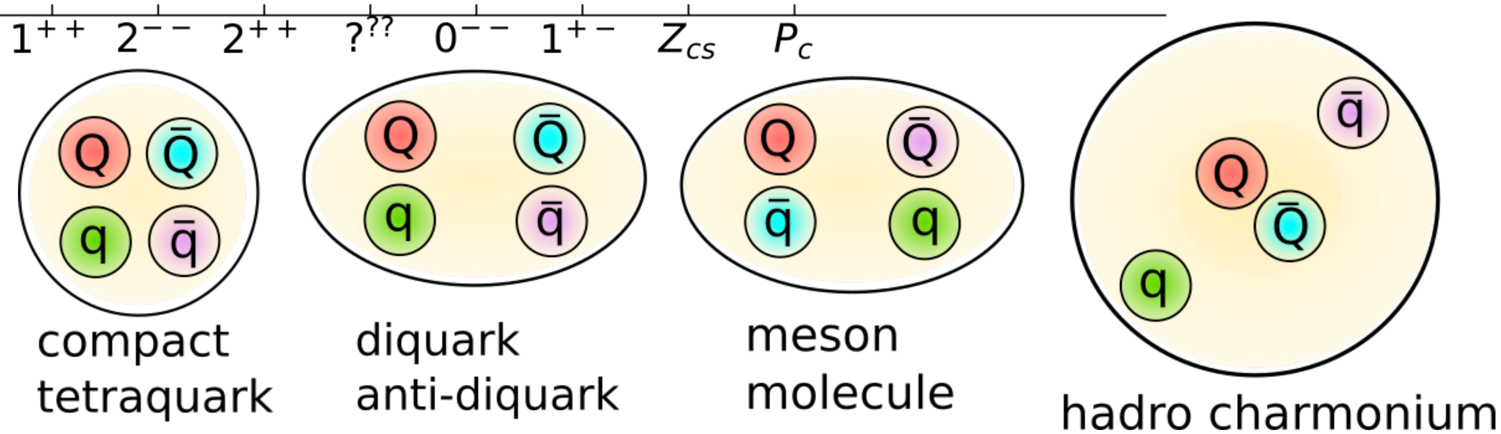
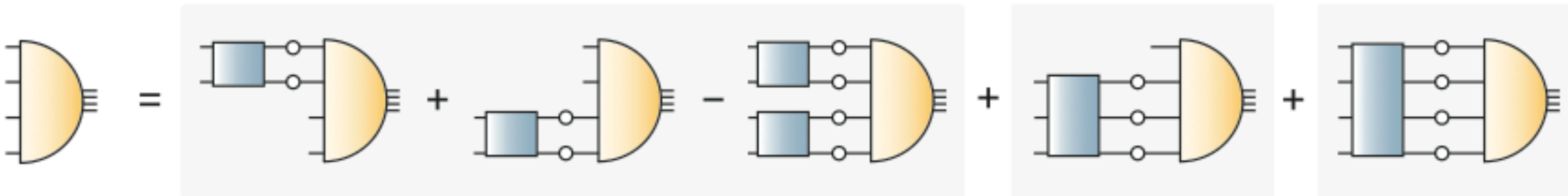


Related to details of underlying QCD forces

Tetraquarks from the four-body interaction

Exact equation:



Two-body interactions

Three- and four-body interactions

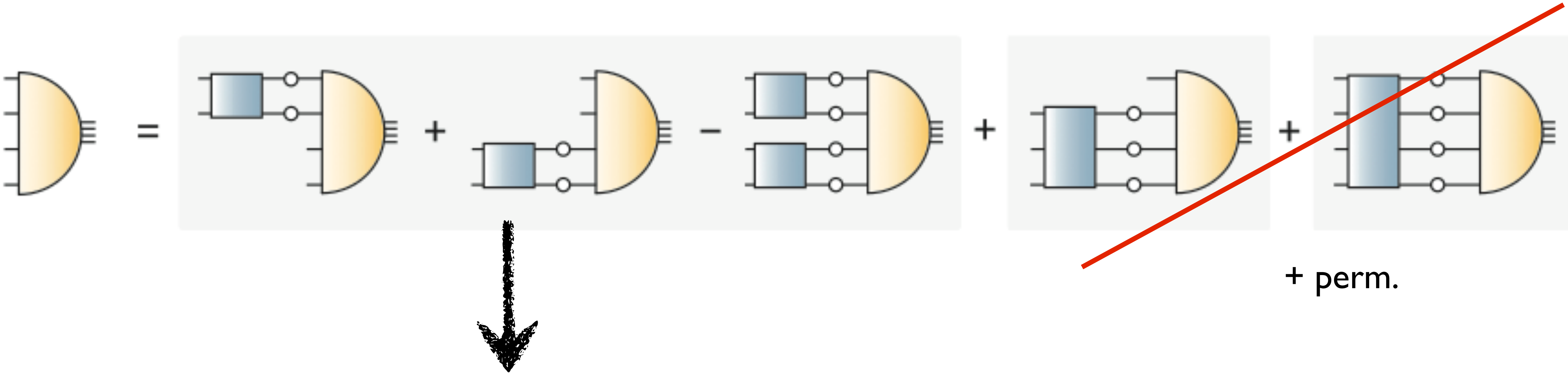
Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

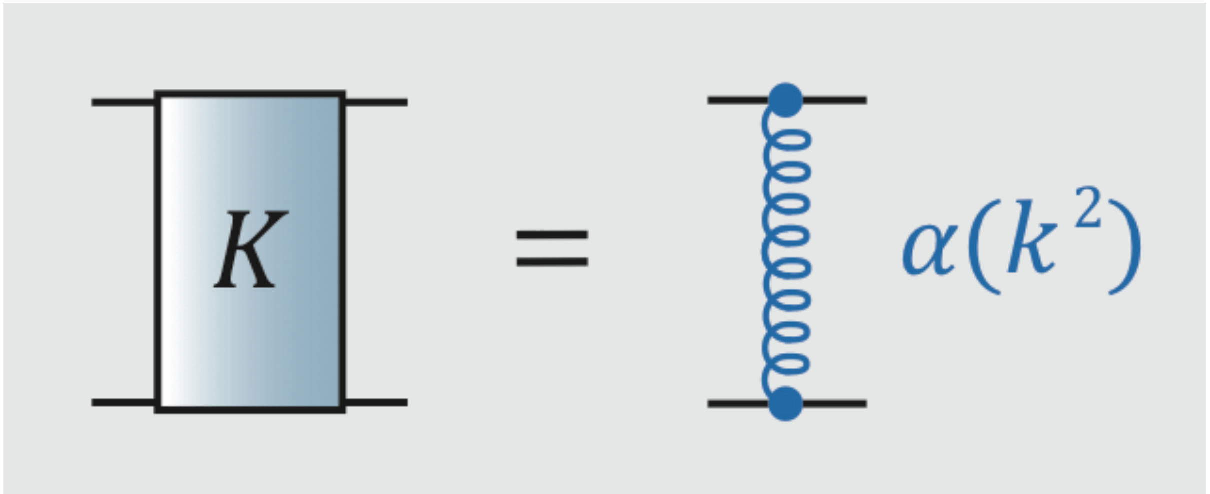
Heupel, Eichman, CF, PLB 718 (2012) 545-549

Eichman, CF, Heupel, PLB 753 (2016) 282-287

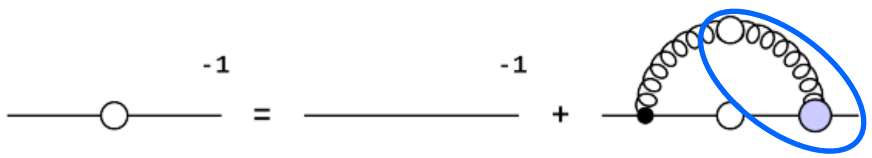
- Basic idea:
  solve four-body equation without any assumption on internal clustering
- Key elements: quark propagator and interaction kernels

Solving the four-body equation



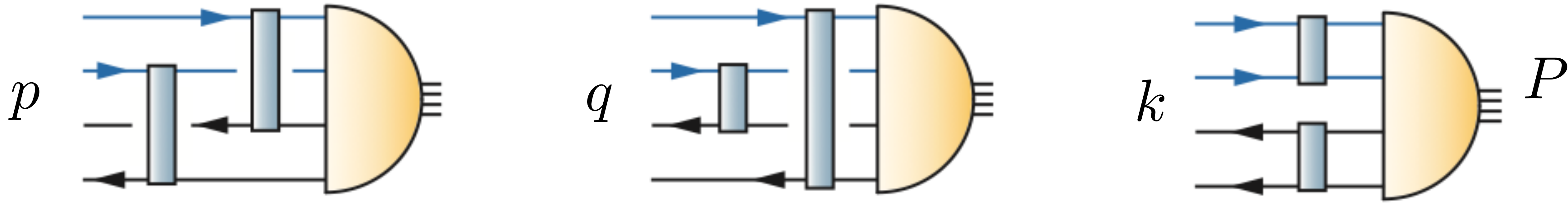


- Input: Non-perturbative quark, quark-gluon interaction



$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

Scalar tetraquark:



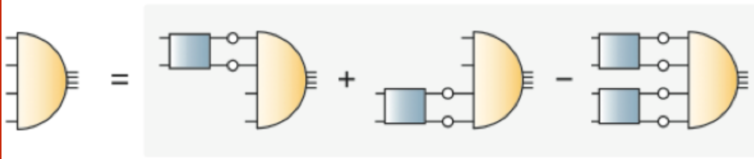
$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\ldots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

9 Lorentz scalars (built from P,p,q,k)

256 tensor structures (scalar tetra)

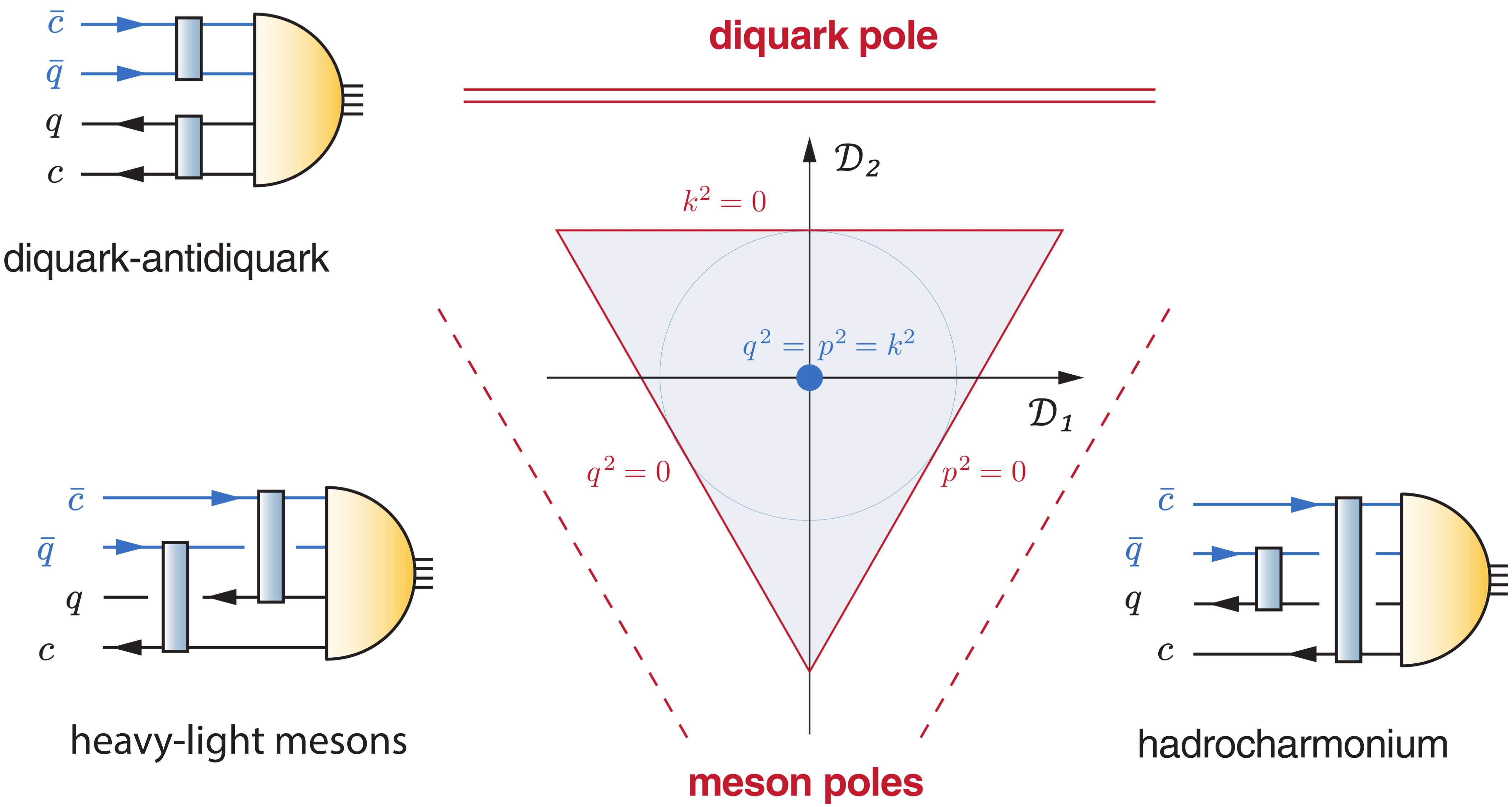
$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

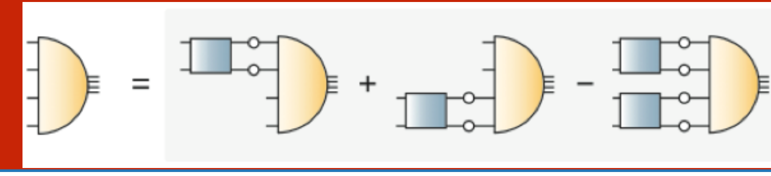
- good approximation: keep s-waves only; 16 tensor structures



- **Singlet:** $S_0 = (p^2 + q^2 + k^2)/4$ $\quad p, q, k$: relative momenta
- **Doublet:** $\mathcal{D}_1 \sim p^2 + q^2 - 2k^2$

  $\mathcal{D}_2 \sim q^2 - p^2$





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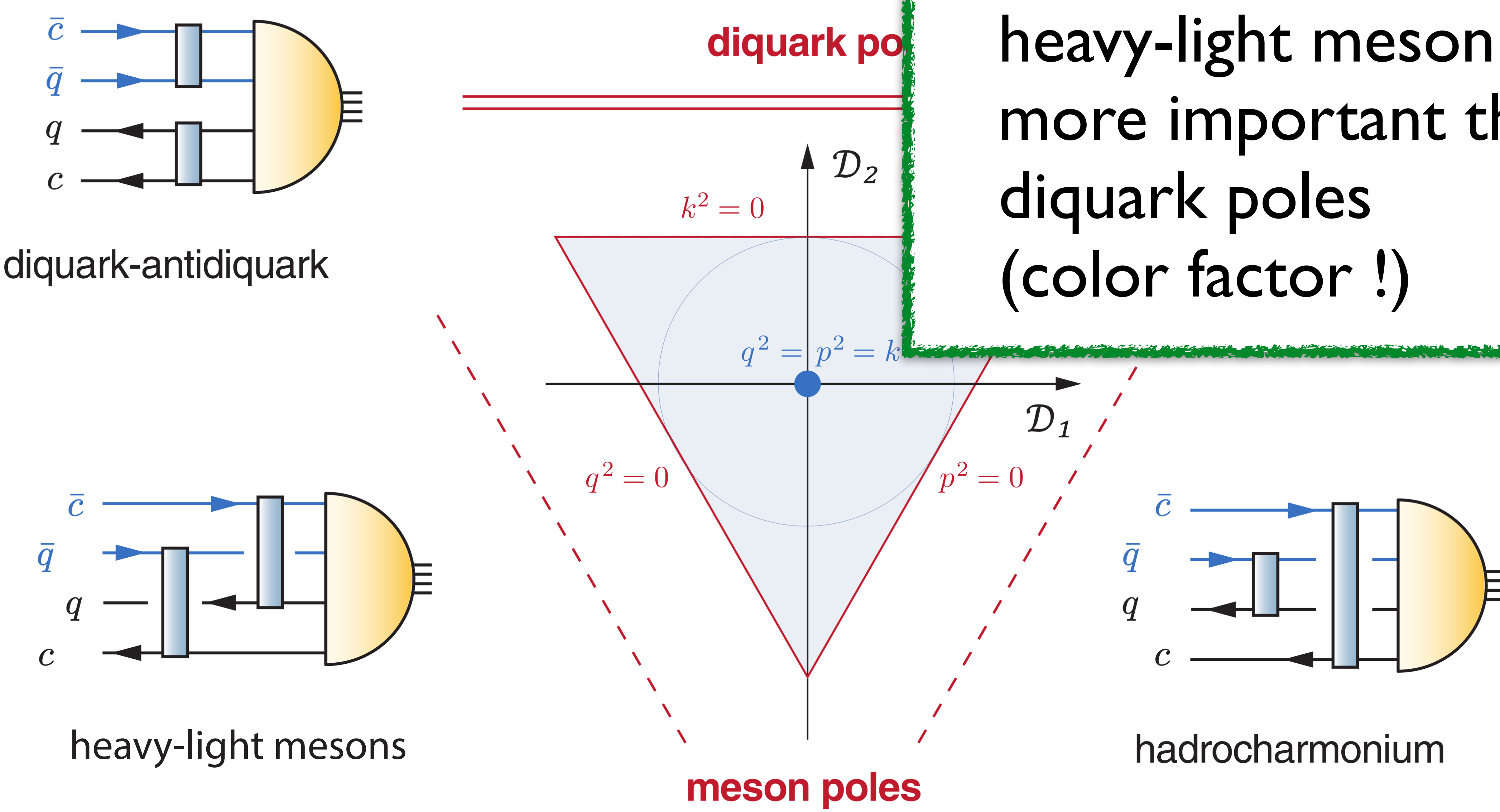
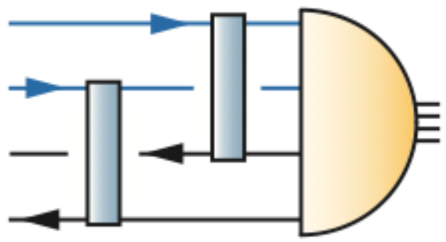


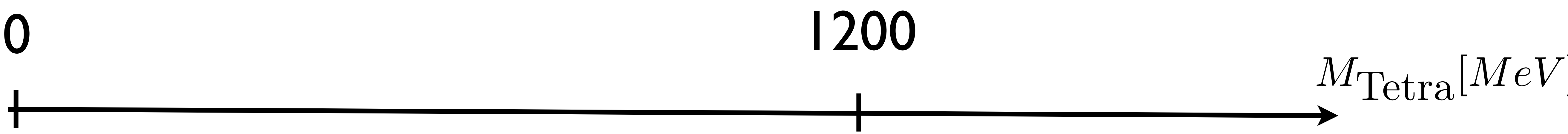
- model independent: heavy-light meson poles more important than diquark poles (color factor !)

Bound state vs resonance: scalar four-quark states



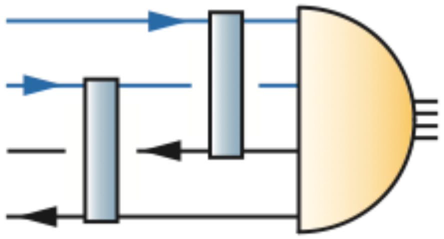
$$\Gamma(S_0, \cancel{s, a, \ldots})$$

without π-clustering



Bound state of
four massive quarks

Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313



$\Gamma(S_0, s, a, \ldots)$

without π-clustering

0 300-400 1200 $M_{\text{Tetra}} [MeV]$

with π-clustering

Two-pion resonance

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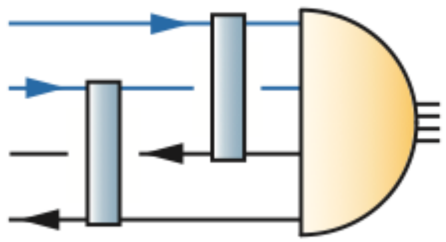
Bound state vs resonance: scalar four-quark states



$\Gamma(S_0, s, a, \ldots)$

without π-clustering

0 — 300-400 — 1200 — $M_{\text{Tetra}}[MeV]$

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⟶ identify with $f_0(500)$ ('σ-meson')

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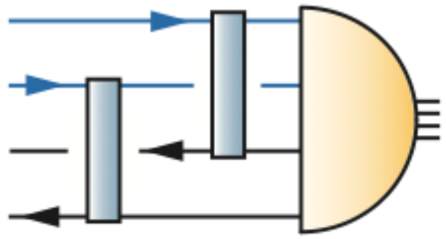

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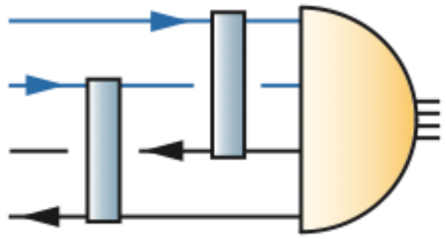

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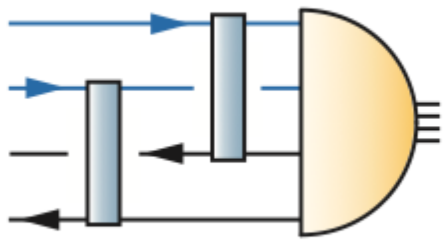
Meson-meson components dominate over diquarks !

Bound state vs resonance: scalar four-quark states



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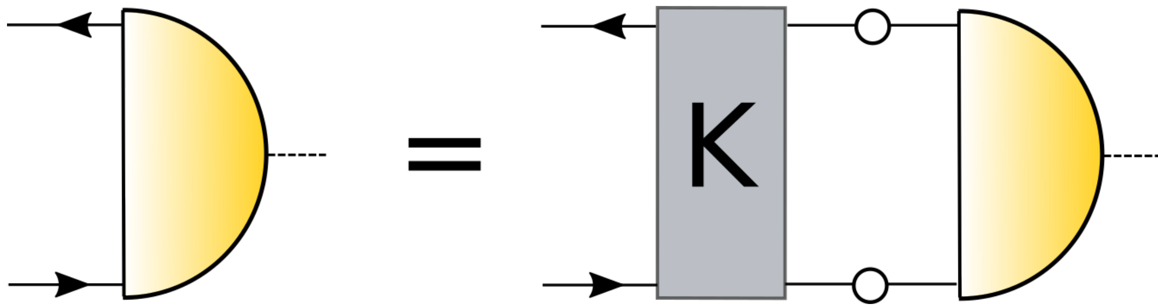
Eichmann, CF, Heupel, PLB 753 (2016) 282-287
Santowsky, CF, PRD 105 (2022) 4,313

Meson-meson components dominate over diquarks !

Mixing with $q\bar{q}$: small effect

Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014
Santowsky, CF, PRD 105 (2022) 4,313

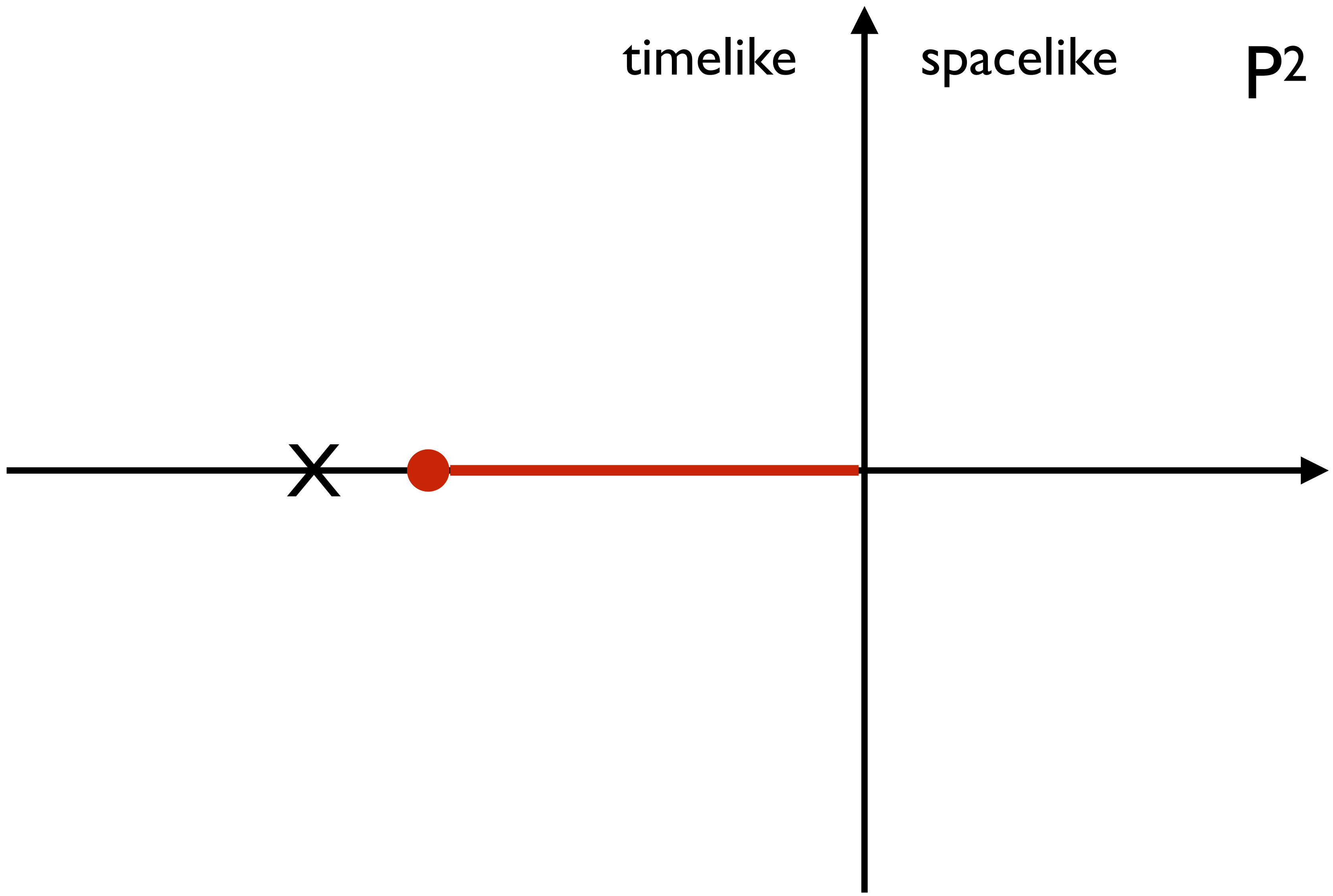
The complex P^2-plane



$\lambda(P^2) \circ$ BSA = kernel $\circ$ BSA

$\lambda(P^2) \overset{!}{=} 1$

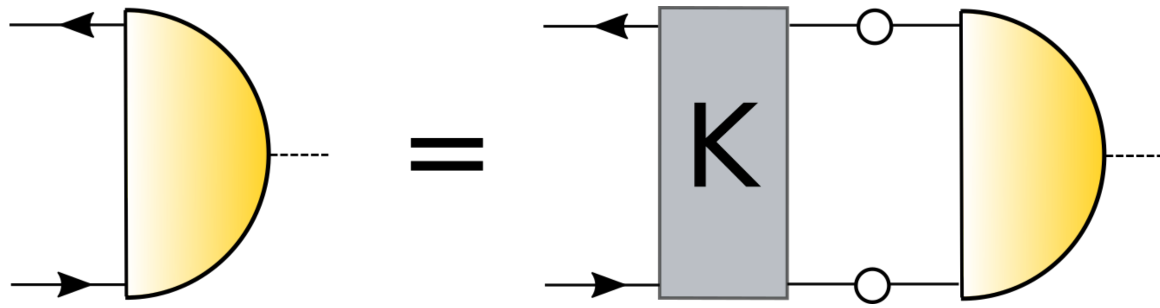
generic situation



Williams, PLB 798 (2019) 134943, [arXiv:1804.11161]

Santowsky, Eichmann, CF, Wallbott and Williams, PRD 102 (2020) no.5, 056014, arXiv:2007.06495.

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755



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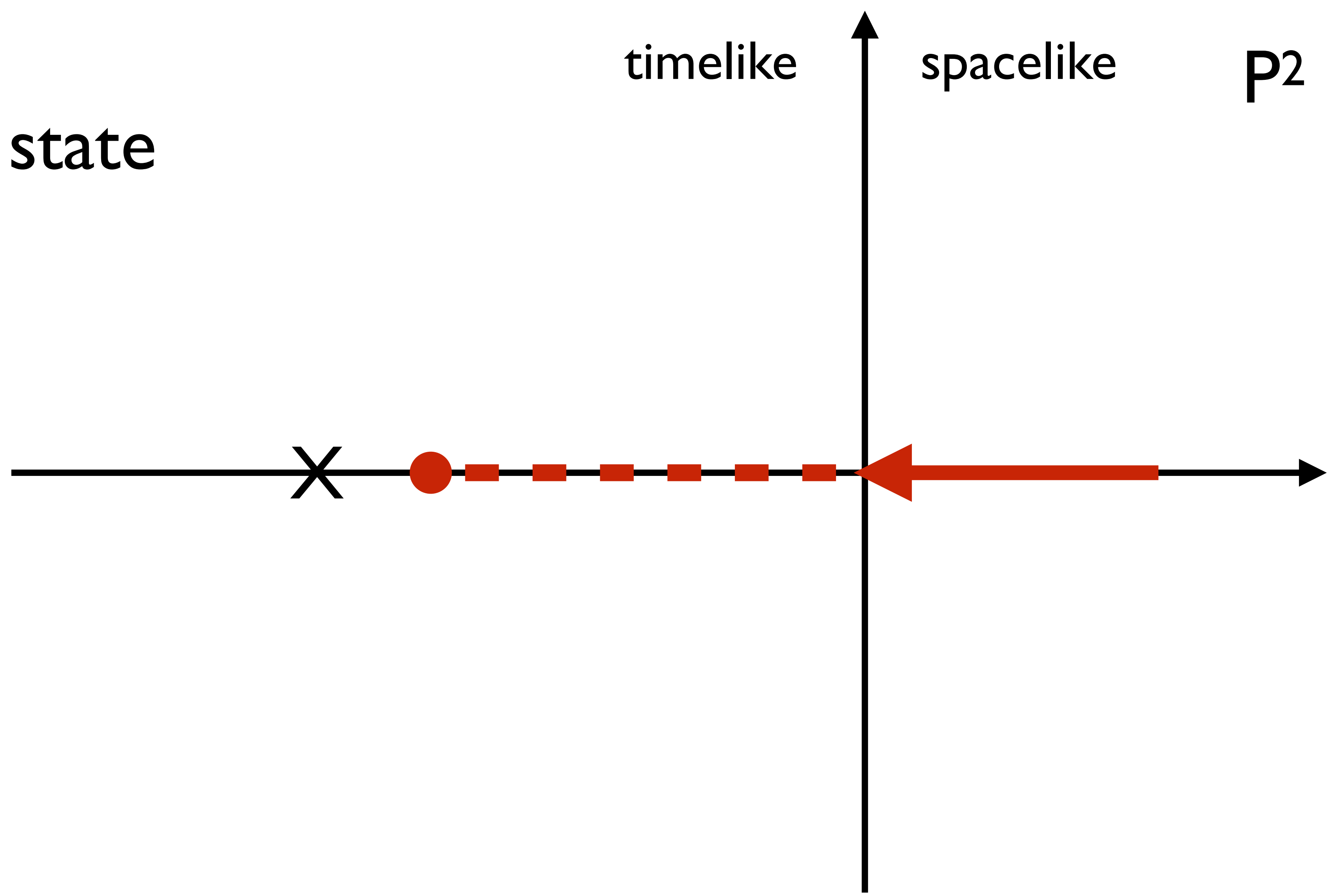
extrapolation to bound state



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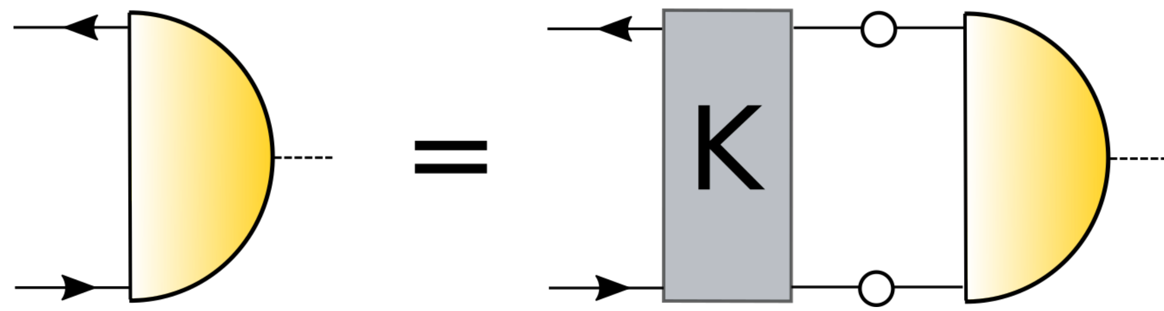


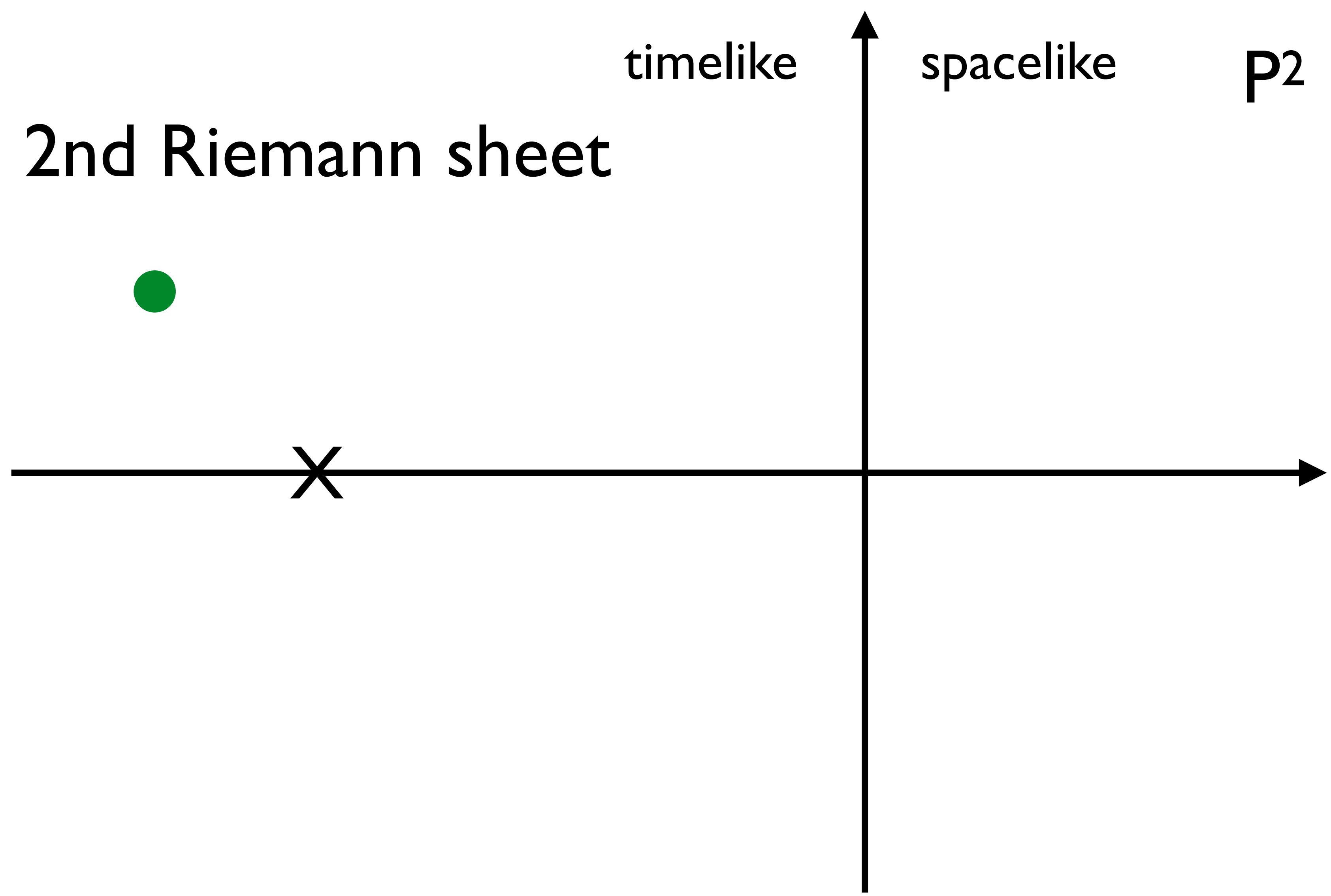
$\lambda(P^2) \stackrel{!}{=} 1$

$\lambda(P^2) \circ \text{BSA} = \text{kernel} \circ \text{BSA}$

extrapolation to pole in 2nd Riemann sheet

$\rho \to \pi\pi$

$\sigma \to \pi\pi$



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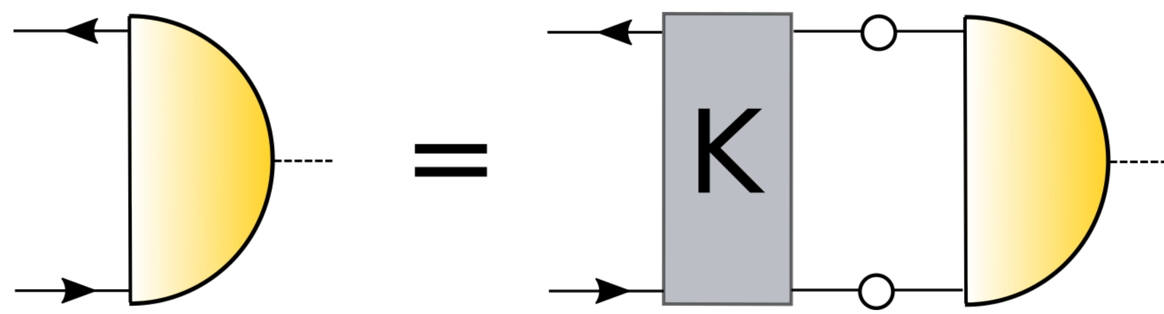

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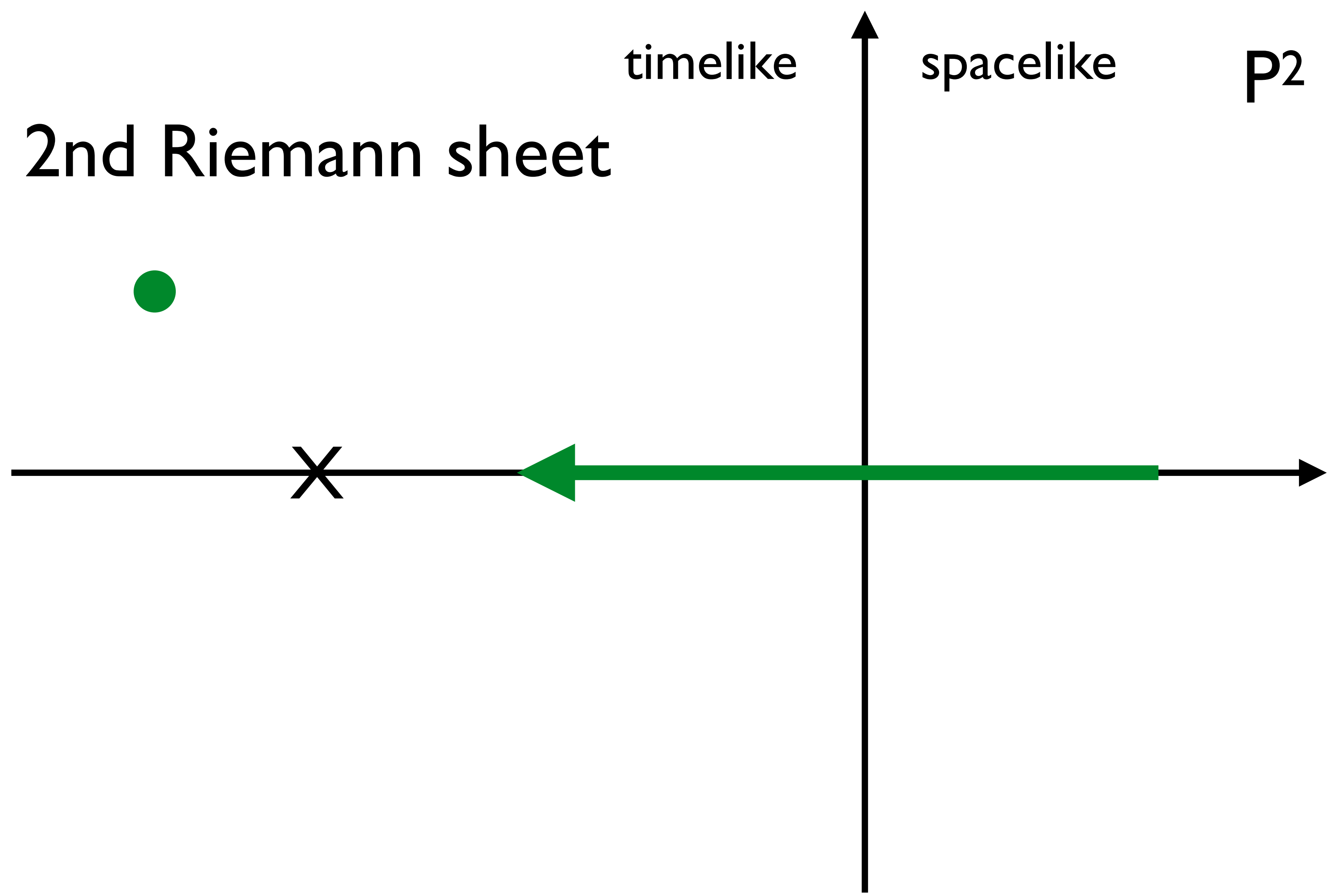

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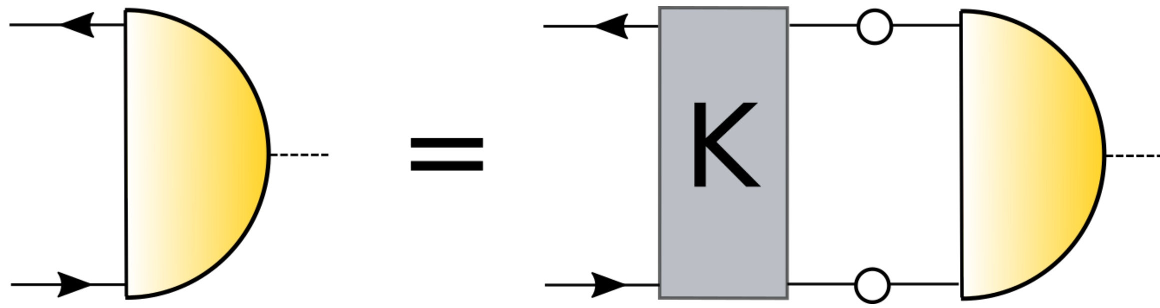

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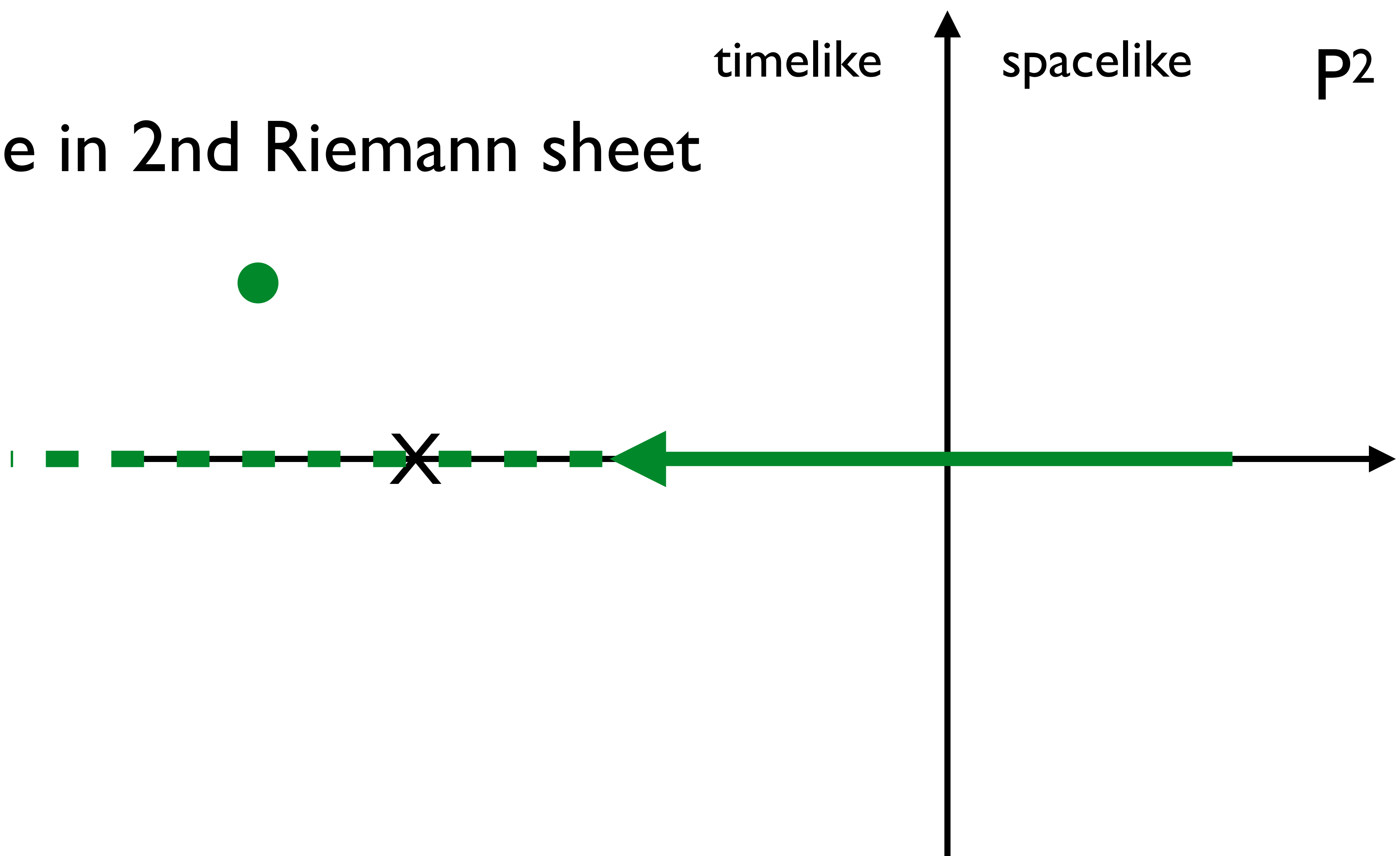

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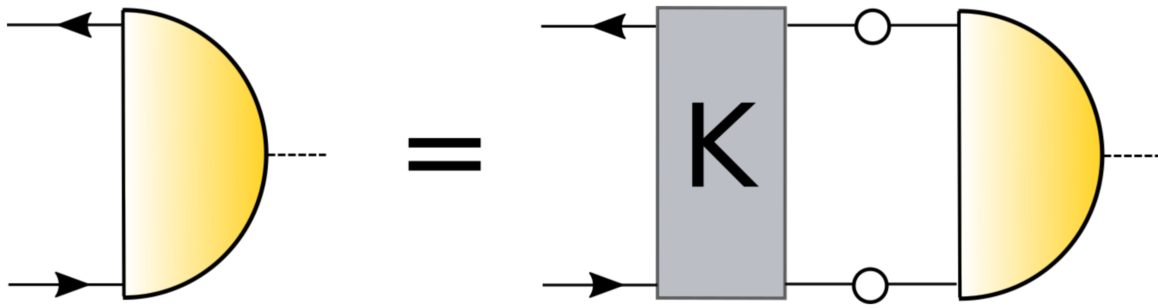

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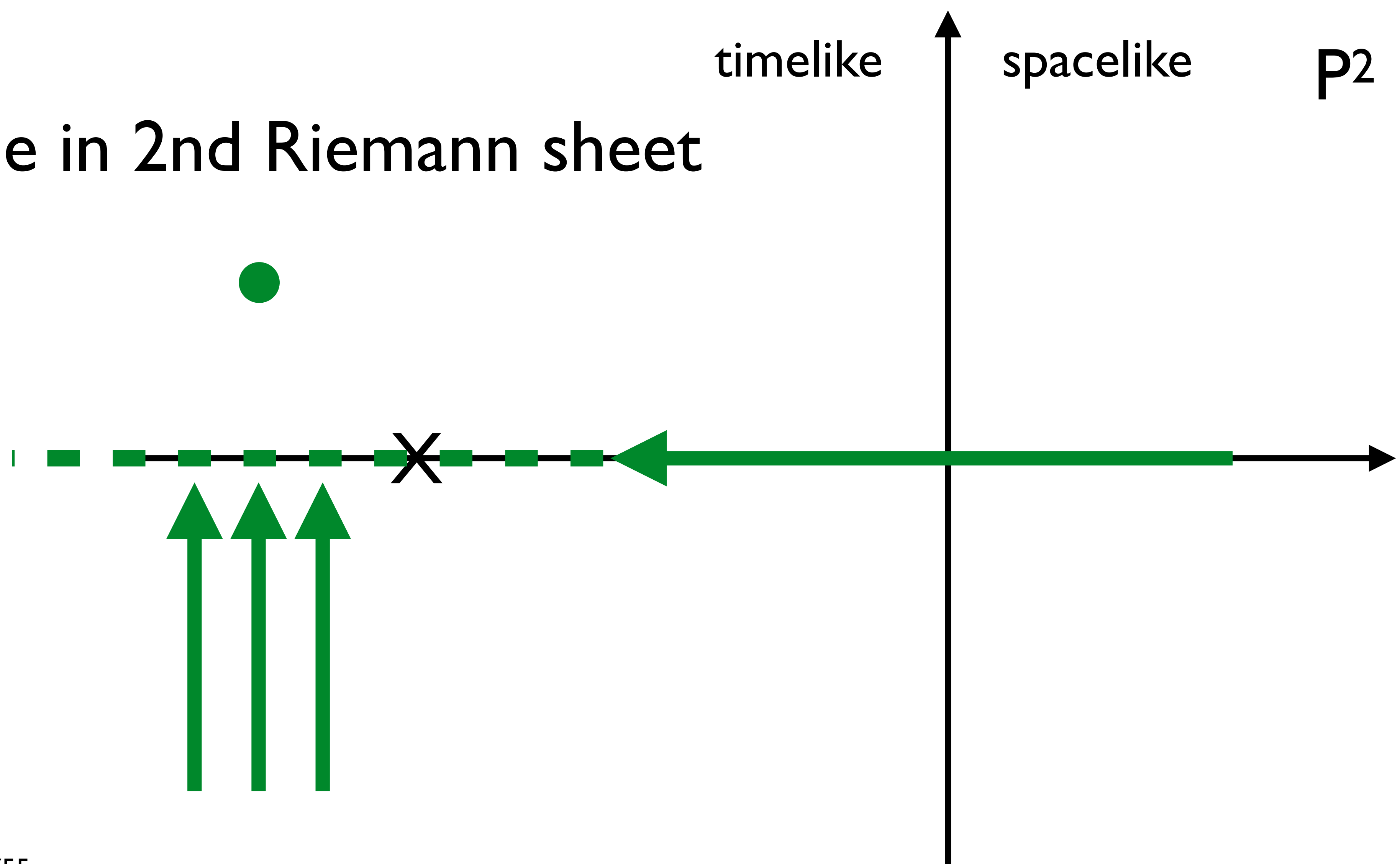

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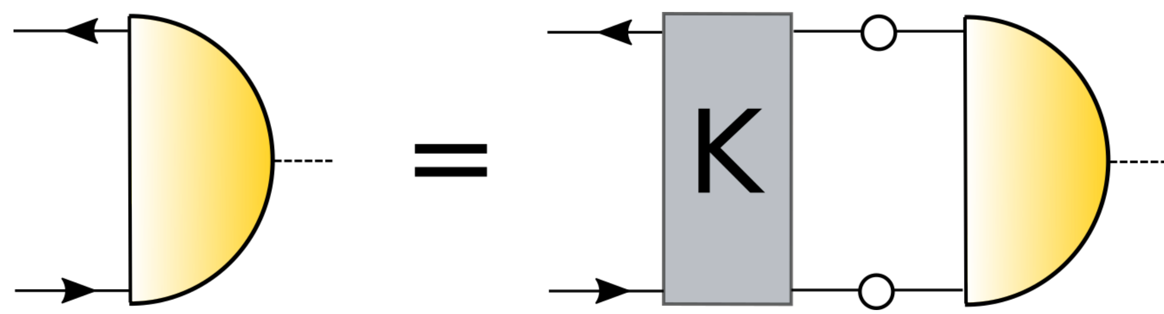

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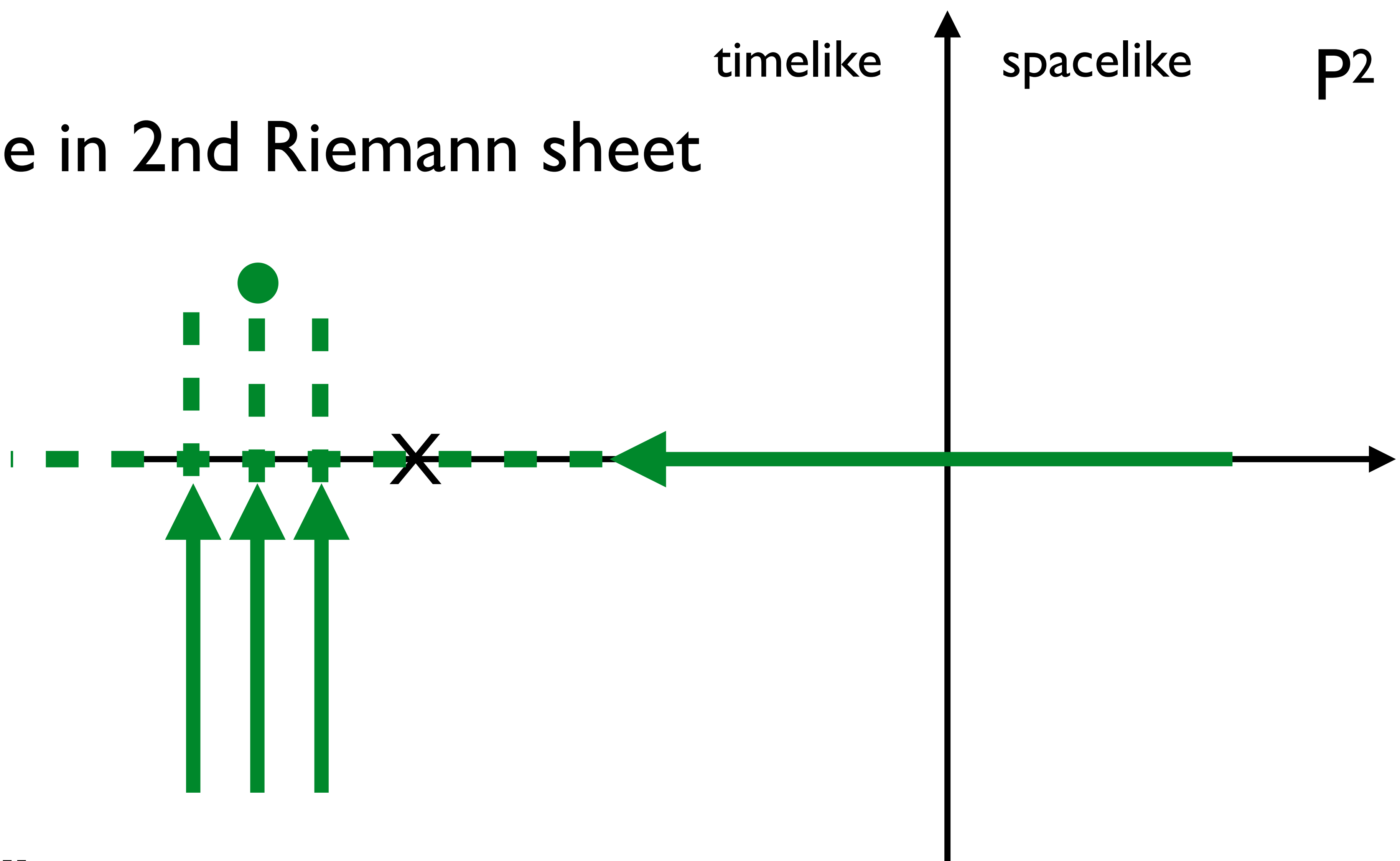

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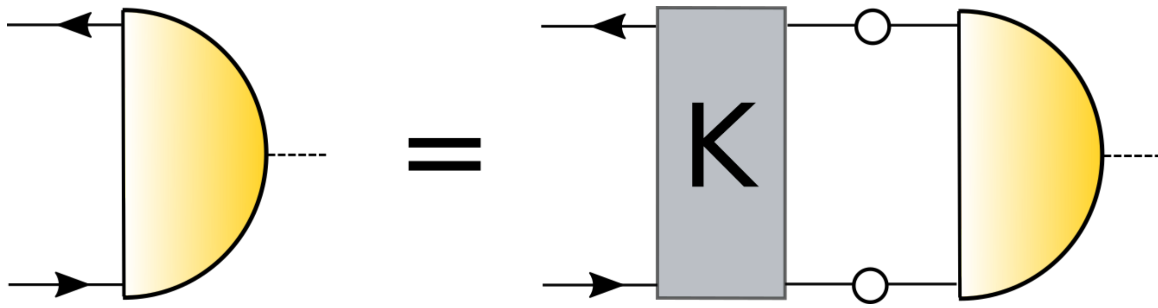

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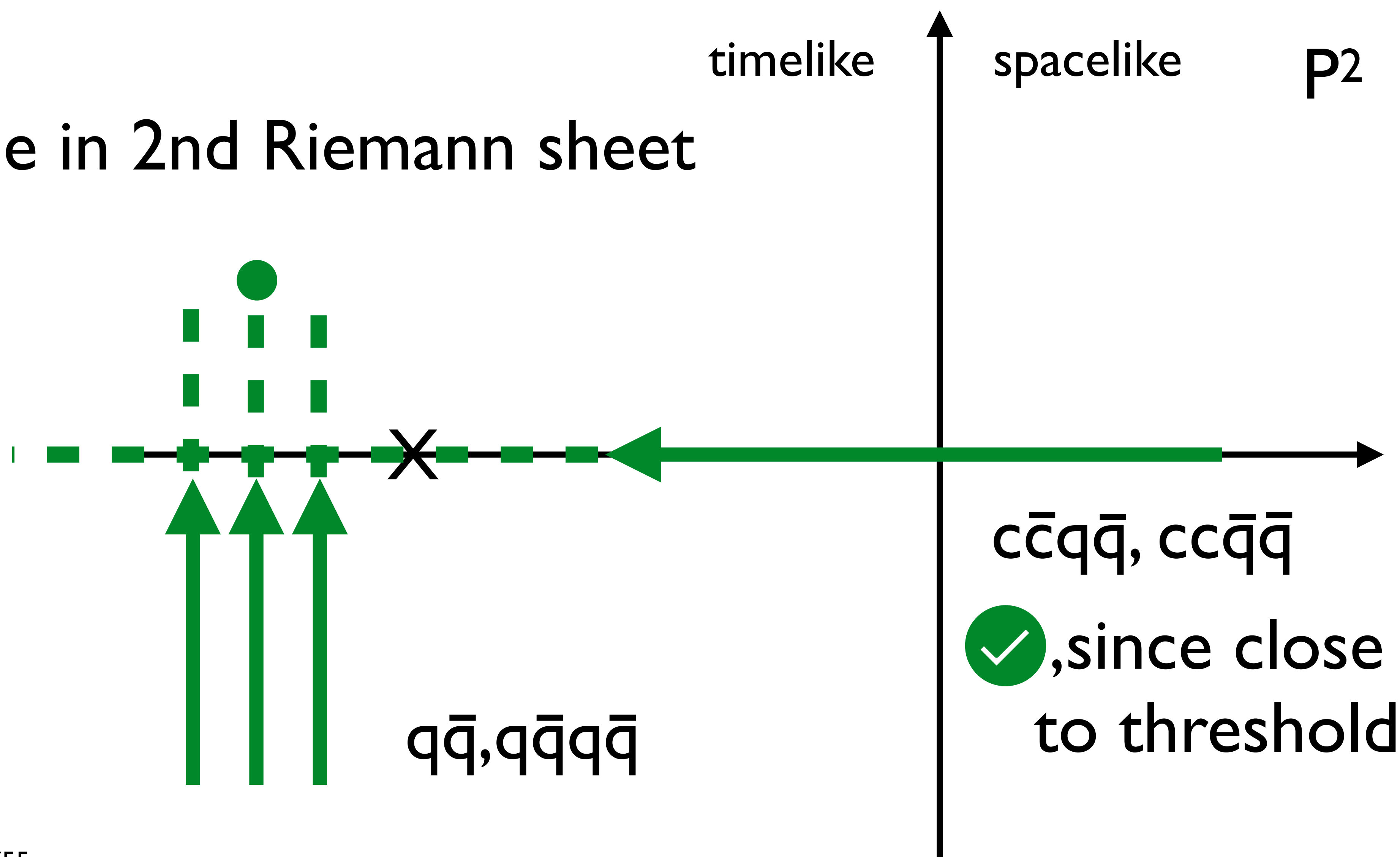

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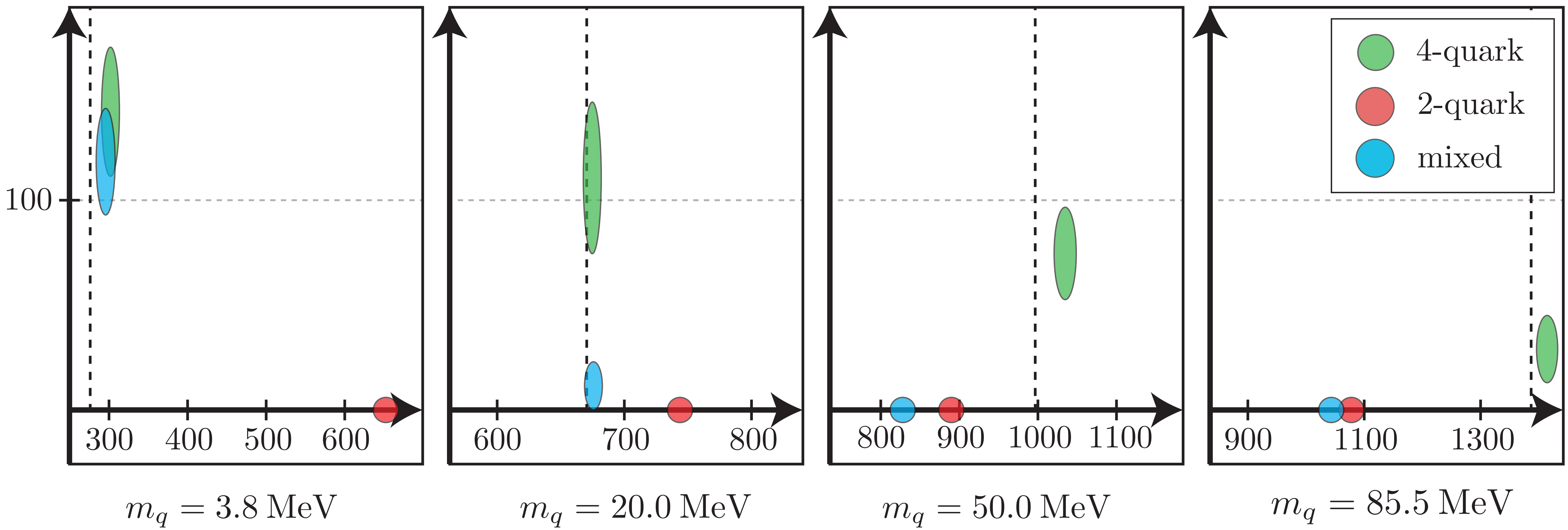
Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

Mass evolution of four-quark state

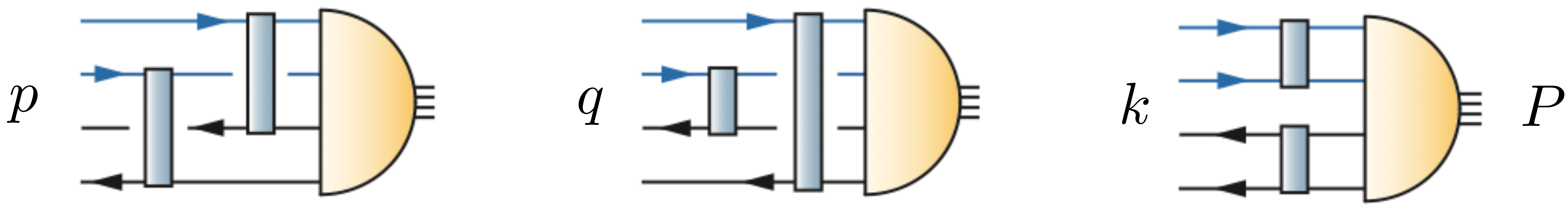


- mixed state becomes qq-dominated for large m_q
- dynamical decision !

Santowsky, CF, PRD 105 (2022) 4,313; arXiv:2109.00755

Structure of the amplitude: open heavy flavour

Scalar tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

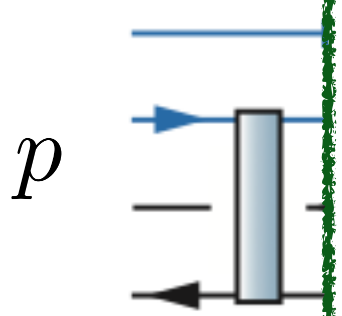
9 Lorentz scalars (built from P,p,q,k)

256 tensor structures (scalar)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or

$1 \otimes 1, 8 \otimes 8$

Scalar te



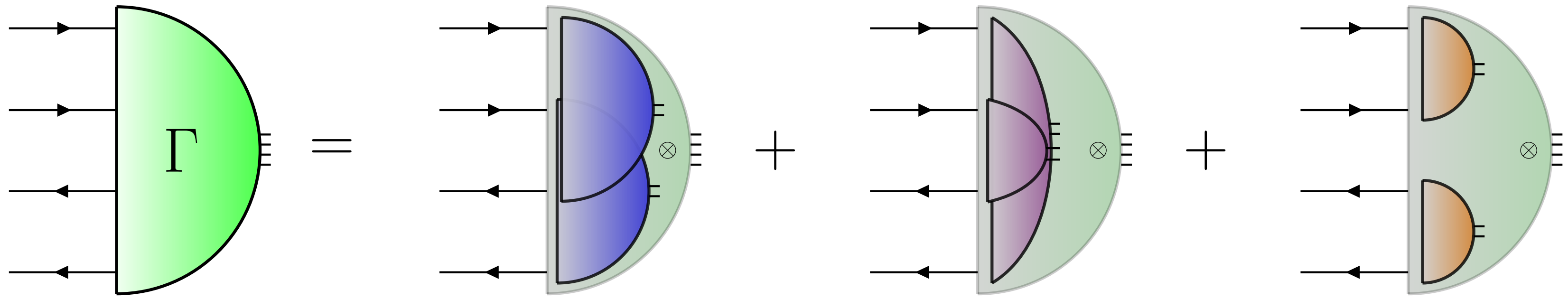
$I(J^P)$	Physical components					
	$\mathbf{1} \otimes \mathbf{1}$		$\bar{\mathbf{3}} \otimes \mathbf{3}$	$\mathbf{8} \otimes \mathbf{8}$		$\bar{\mathbf{6}} \otimes \mathbf{6}$
	f_0	f_1	f_2	f_3	f_4	f_5
$0(1^+)\ bb\bar{n}\bar{n}$	BB^*	B^*B^*	$A_{bb}S$	BB^*	B^*B^*	$S_{bb}A$
$bc\bar{n}\bar{n}$	BD^*	B^*D	$A_{bc}S$	BD^*	B^*D	$S_{bc}A$
$cc\bar{n}\bar{n}$	DD^*	D^*D^*	$A_{cc}S$	DD^*	D^*D^*	$S_{cc}A$
$bb\bar{s}\bar{s}$	$B_sB_s^*$	–	$A_{bb}A_{ss}$	$B_sB_s^*$	–	–
$bc\bar{s}\bar{s}$	$B_sD_s^*$	$B_s^*D_s$	$S_{bc}A_{ss}$	$B_sD_s^*$	$B_s^*D_s$	$A_{bc}S_{ss}$
$cc\bar{s}\bar{s}$	$D_sD_s^*$	–	$A_{cc}A_{ss}$	$D_sD_s^*$	–	–
$1(1^+)\ bb\bar{n}\bar{n}$	BB^*	–	$A_{bb}A$	BB^*	–	–
$bc\bar{n}\bar{n}$	BD^*	B^*D	$S_{bc}A$	BD^*	B^*D	$A_{bc}S$
$cc\bar{n}\bar{n}$	DD^*	–	$A_{cc}A$	DD^*	–	–

$\Gamma(P, p, q$ … $flavor$

r

Junnarkar, Mathur, Padmanath, PRD99, 034507 (2019)

Identifying leading structures...

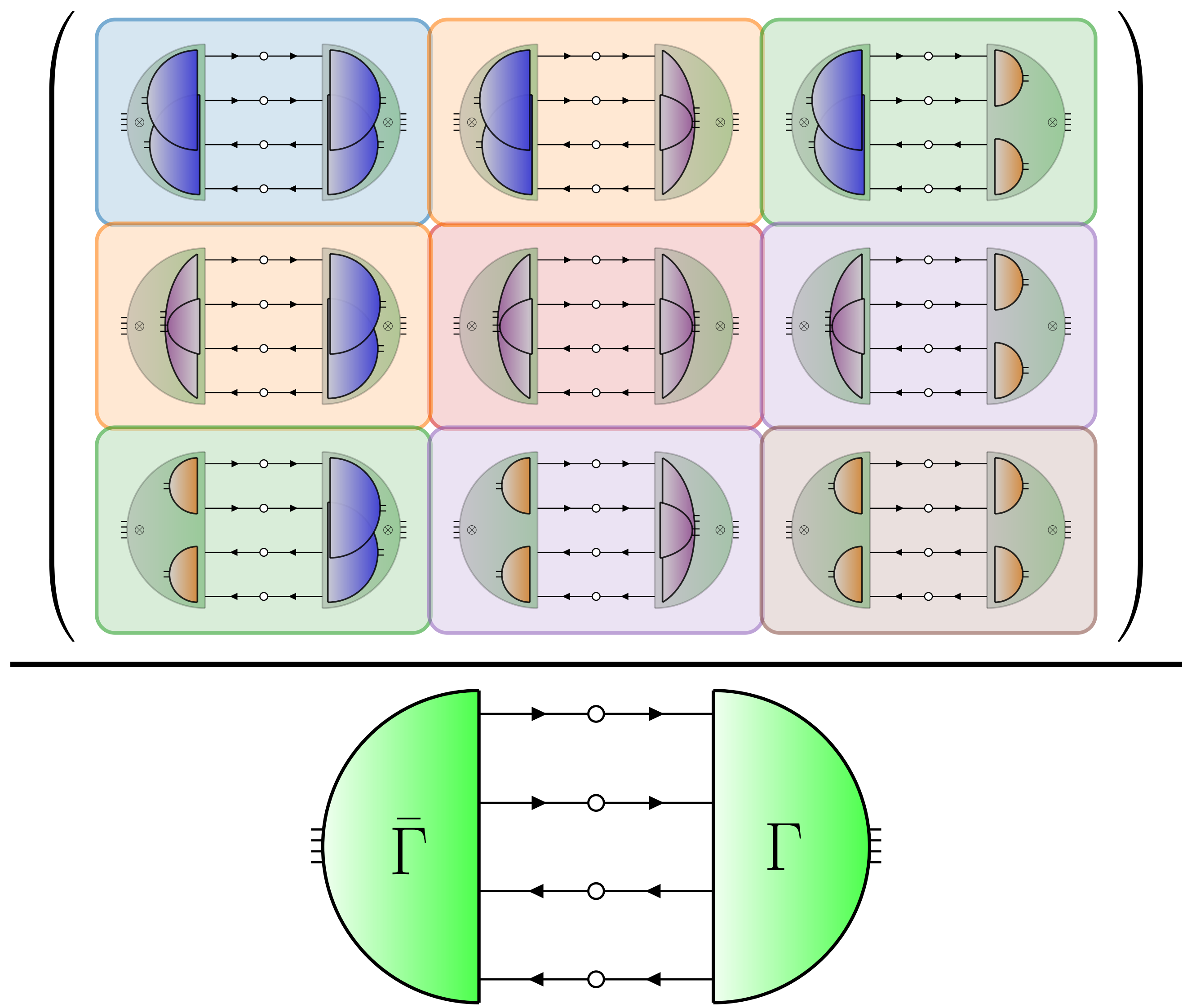


meson-meson f_0, f_3

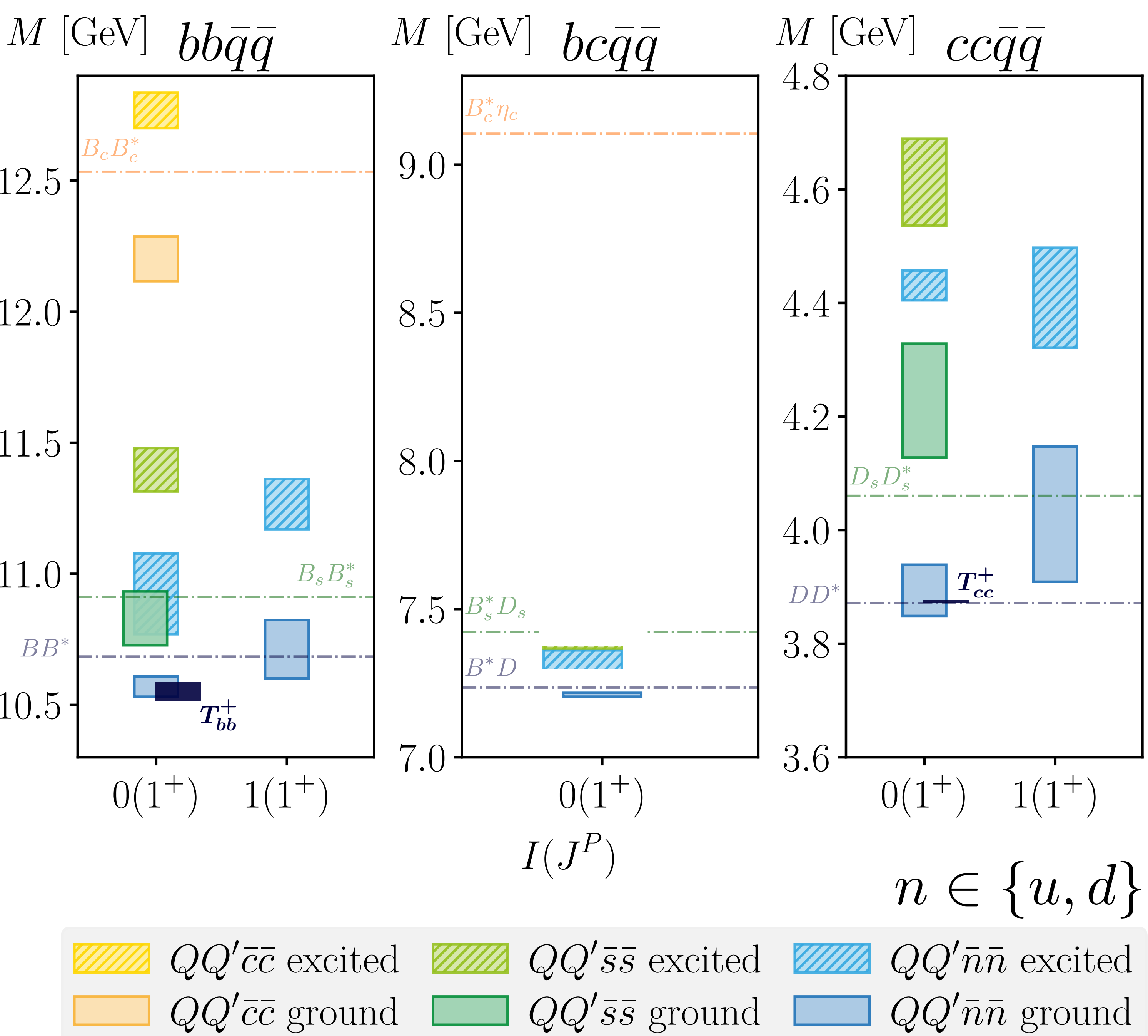
meson-meson f_1, f_4

diquark-antidiquark f_2, f_5

- norm contributions



Spectrum of open heavy-flavour states (prelim. !)



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	$bb\bar{s}\bar{s}$	$B_sB_s^*$	–	$A_{bb}A_{ss}$	$B_sB_s^*$	–	–
	$bc\bar{s}\bar{s}$	$B_sD_s^*$	$B_s^*D_s$	$S_{bc}A_{ss}$	$B_sD_s^*$	$B_s^*D_s$	$A_{bc}S_{ss}$
	$cc\bar{s}\bar{s}$	$D_sD_s^*$	–	$A_{cc}A_{ss}$	$D_sD_s^*$	–	–
$1(1^+)$	$bb\bar{n}\bar{n}$	BB^*	–	$A_{bb}A$	BB^*	–	–
	$bc\bar{n}\bar{n}$	BD^*	B^*D	$S_{bc}A$	BD^*	B^*D	$A_{bc}S$
	$cc\bar{n}\bar{n}$	DD^*	–	$A_{cc}A$	DD^*	–	–

attractive | repulsive

Hoffer, Eichmann, CF, in preparation

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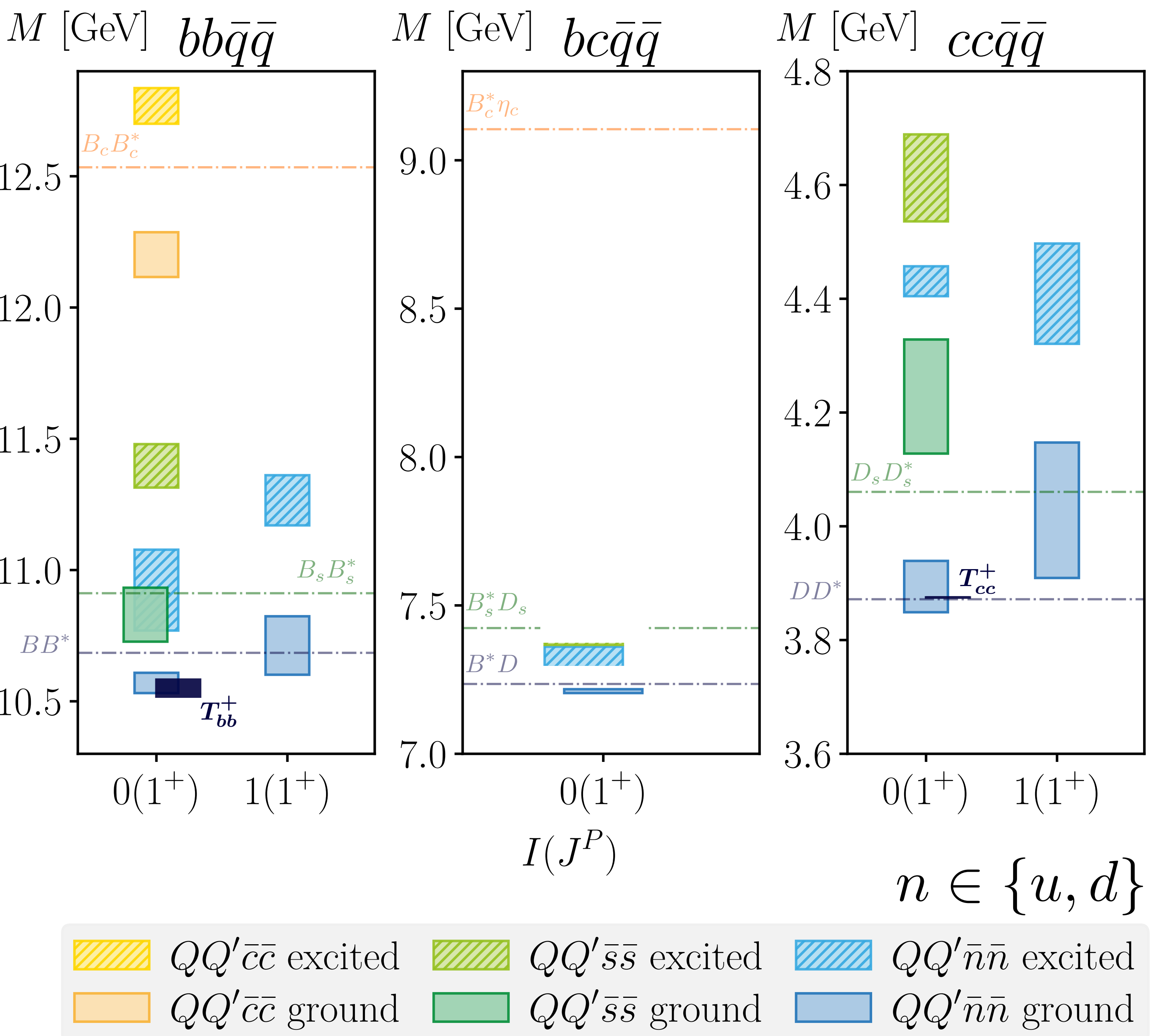

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	$bc\bar{s}\bar{s}$	$B_sD_s^*$	$B_s^*D_s$	$S_{bc}A_{ss}$	$B_sD_s^*$	$B_s^*D_s$	$A_{bc}S_{ss}$
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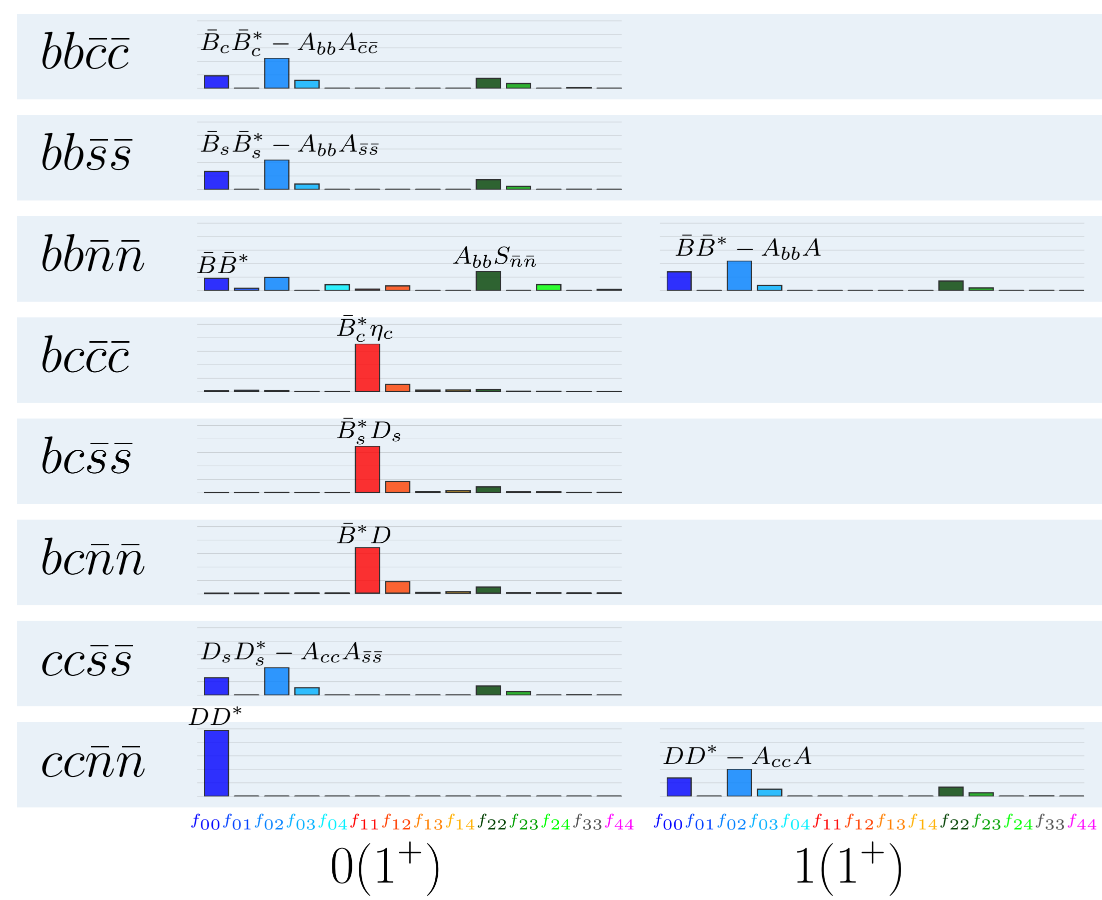
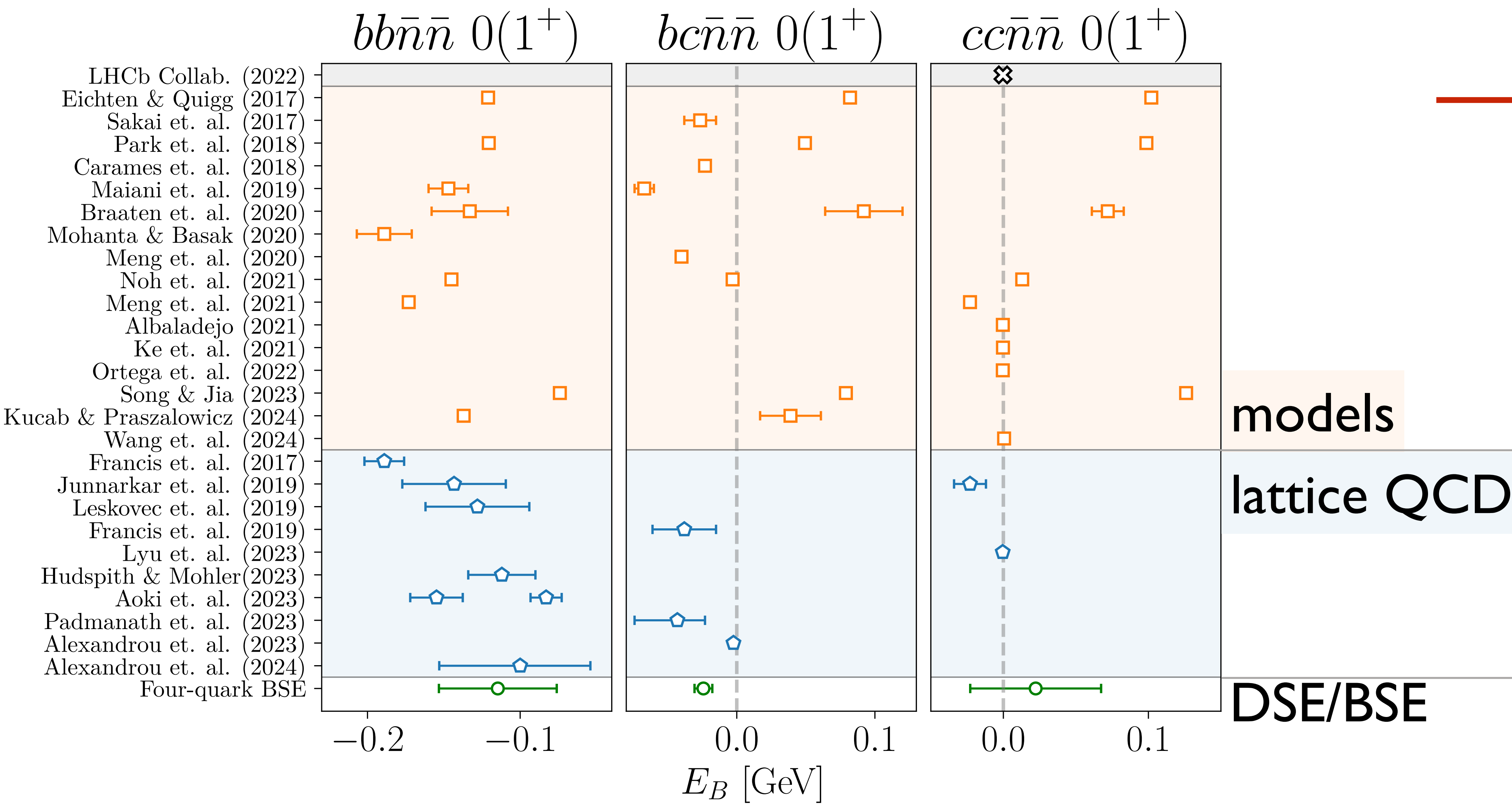
	$cc\bar{n}\bar{n}$	DD^*	–	$A_{cc}A$	DD^*	–	–
		attractive			repulsive		



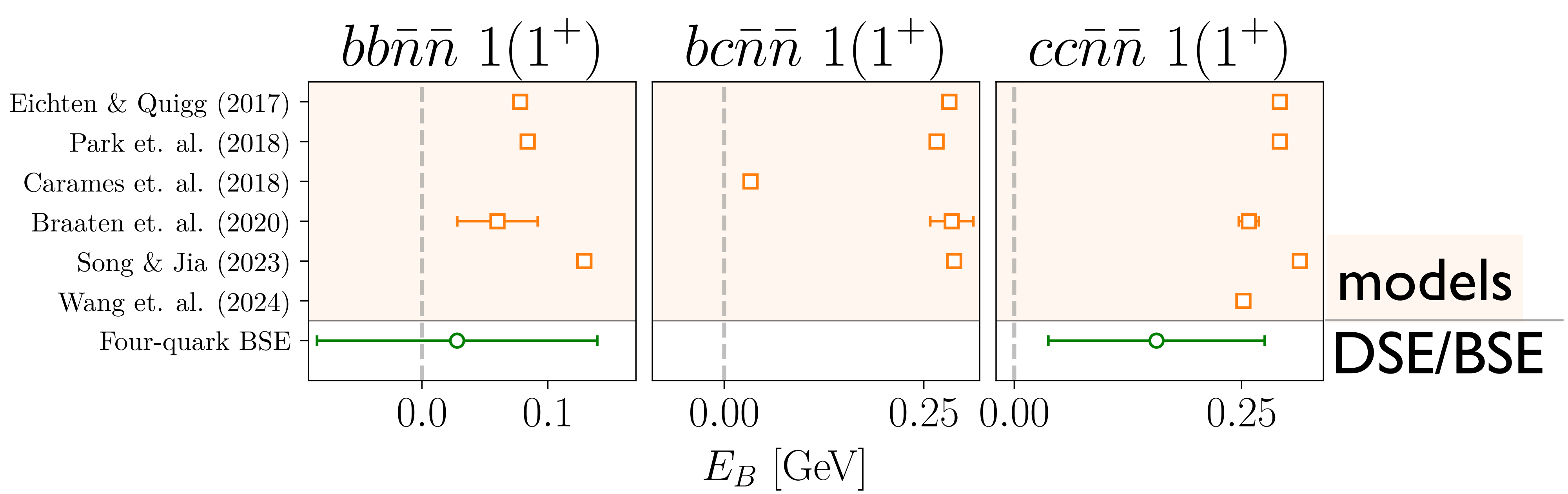
- decided dynamically !
- flavour and spin dependent…

Hoffer, Eichmann, CF, in preparation

Comparison with other approaches



→ seminar by S. Prelovsek

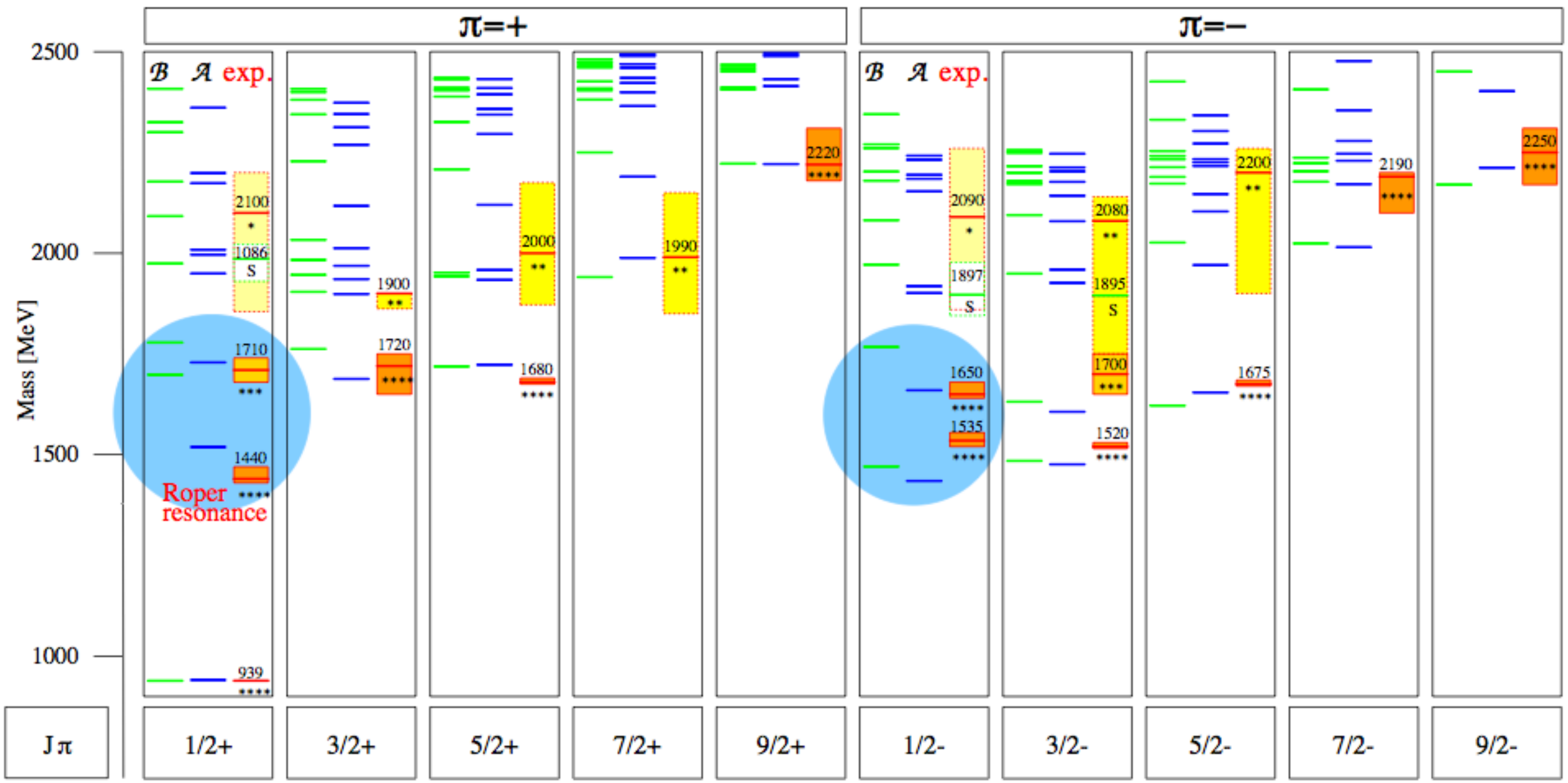


Hoffer, Eichmann, CF, in preparation

Overview

1. Bound states in the quark model
   - Construction
   - Assets, shortcomings and how to do better
2. Properties of QCD and correlation functions
   - Dynamical chiral symmetry breaking
   - Correlation functions and Dyson-Schwinger equations (DSEs)
   - The quark DSE
3. Mesons
   - The pion Bethe-Salpeter equation (BSE): Goldstone bosons, GMOR and all that…
   - Spectra: light and charm
4. Exotic mesons
   - Confinement and glueballs
   - Four-quark states
5. Baryons
   - Spectra: light and strange
6. Form factors
   - Meson form factors
   - Baryon form factors

Light baryon spectrum - quark model



Loring, Metsch, Petry, EPJA 10 (2001) 395

- ‘missing resonances’ - three-body vs. quark-diquark
- level ordering:

$$\boxed{N\frac{1}{2}^{\pm}} \text{ vs. } \quad \Lambda\frac{1}{2}^{\pm}$$

Explaining the Roper (before 2016)

- Quark model: p(2S), but generically too large mass

  e.g. Loring, Metsch, Petry, EPJA 10 (2001) 395 and many others…

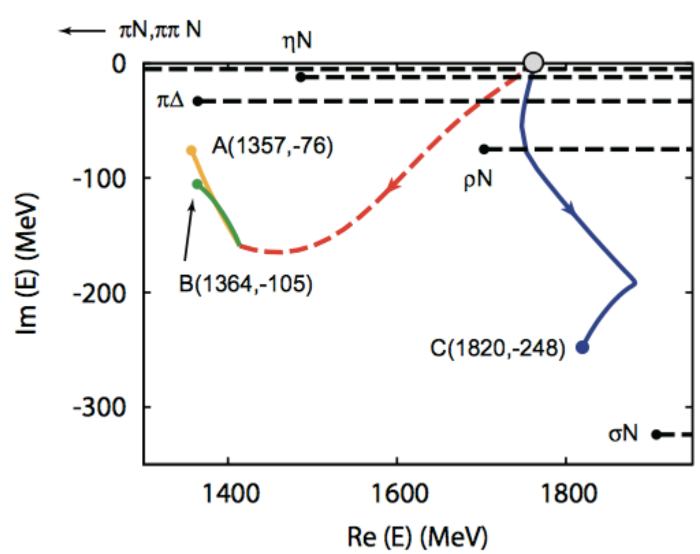
- Hybrid ? Evidence from lattice to the contrary

  Dudek, Edwards, PRD 85 (2012) 054016

- Dynamically generated by coupled channels (no 'bare' state)

  Krehl, Hanhart, Krewald and Speth, PRC C 62 (2000) 025207
  Doring, Hanhart, Huang, Krewald and Meissner, NPA 829 (2009) 170

- Dynamically modified by coupled channels

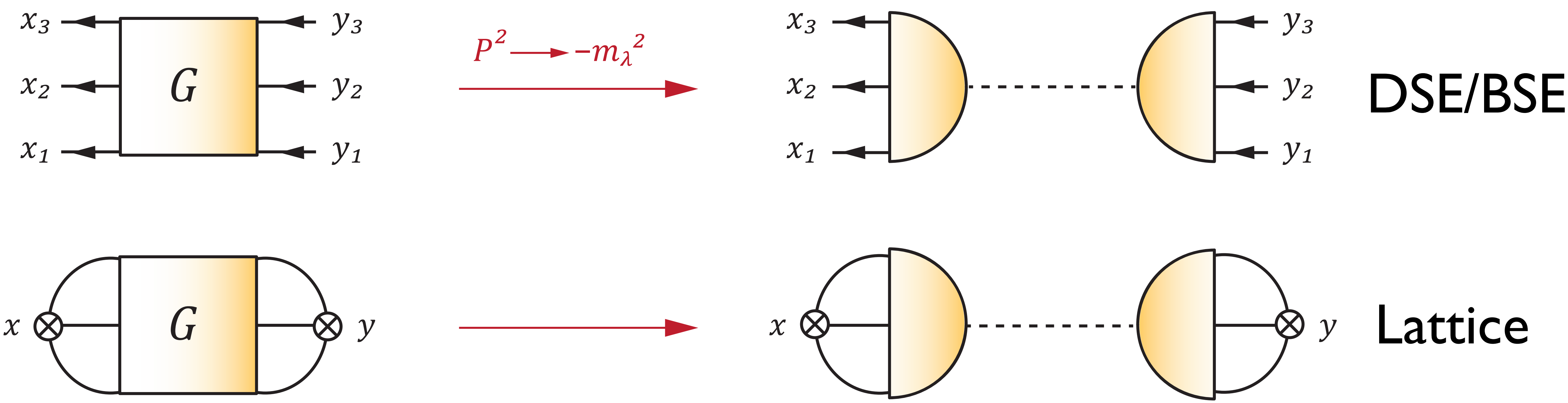


Suzuki, Julia-Diaz, Kamano, Lee, Matsuyama and Sato, PRL 104 (2010) 042302

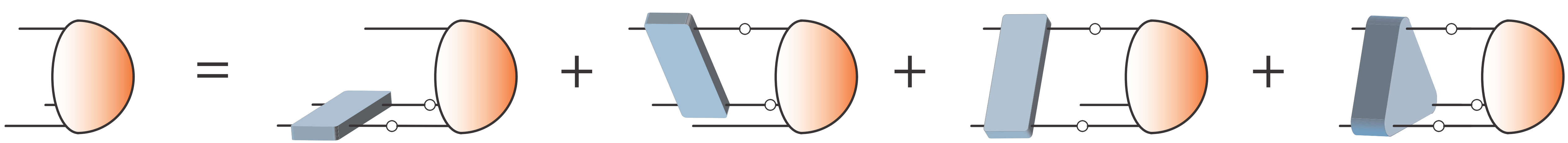
- 'bare' state via DSE/Faddeev (NJL, QCD inspired model)

  Wilson, Cloet, Chang and Roberts, PRC 85 (2012) 025205,
  Segovia, El-Bennich, Rojas, Cloet, Roberts, Xu and Zong, PRL 115 (2015) 17

Extracting spectra from correlators



BSE for baryons (derived from equation of motion for G)



- exact equation for baryon 'wave function'

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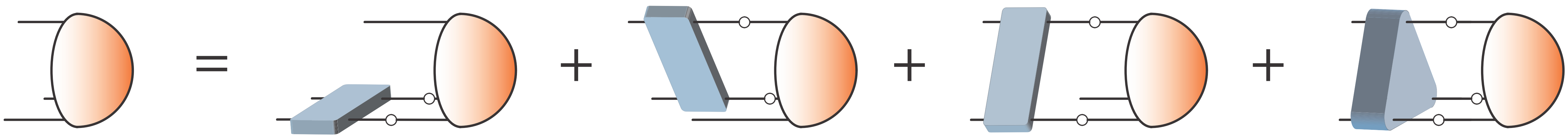
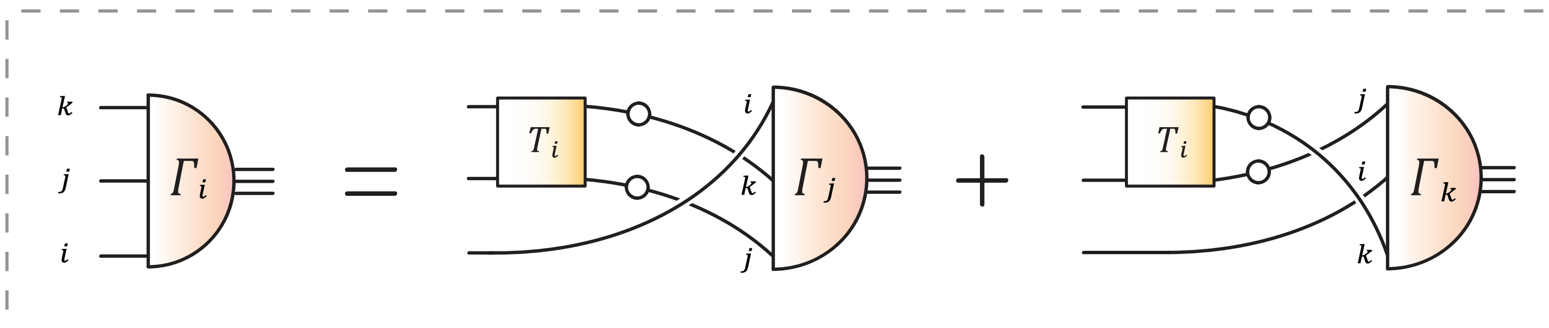
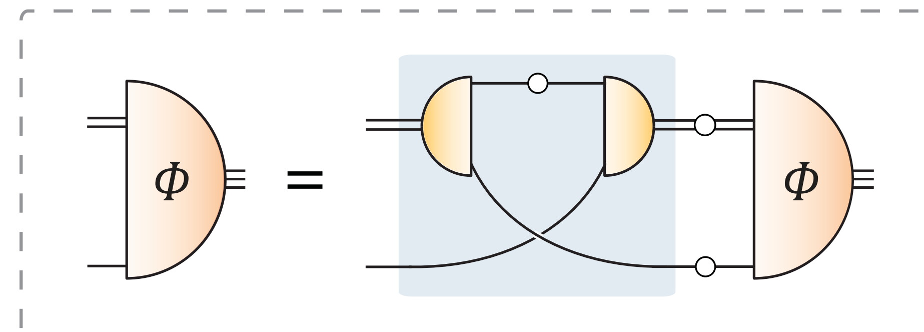


Faddeev equation (no three-body forces)

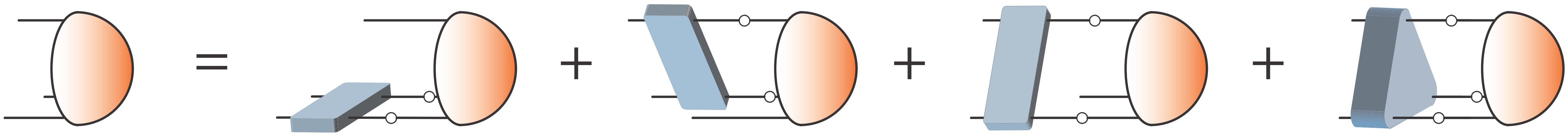


Diquark-quark



- Input in both cases: quark propagator and interaction

BSE for baryons (derived from equation of motion for G)



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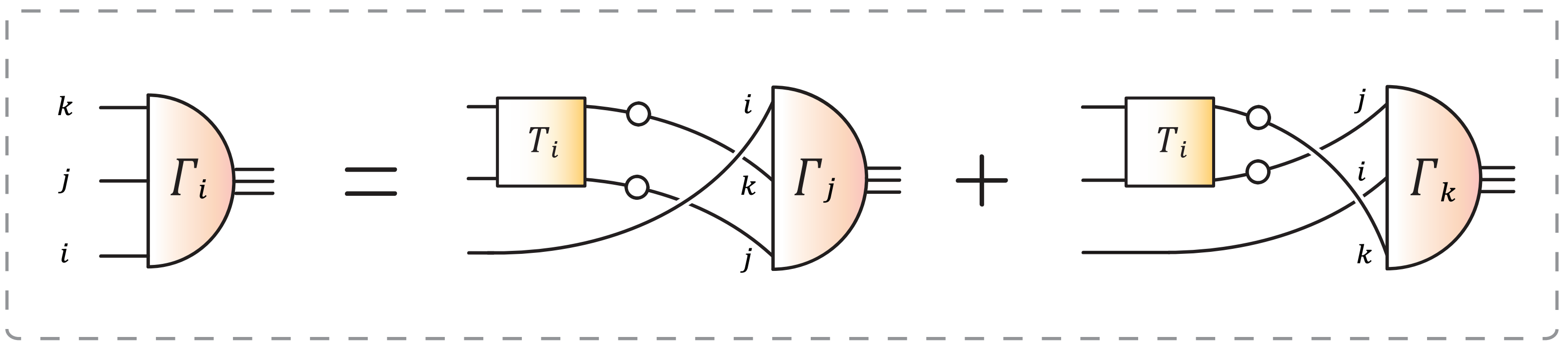
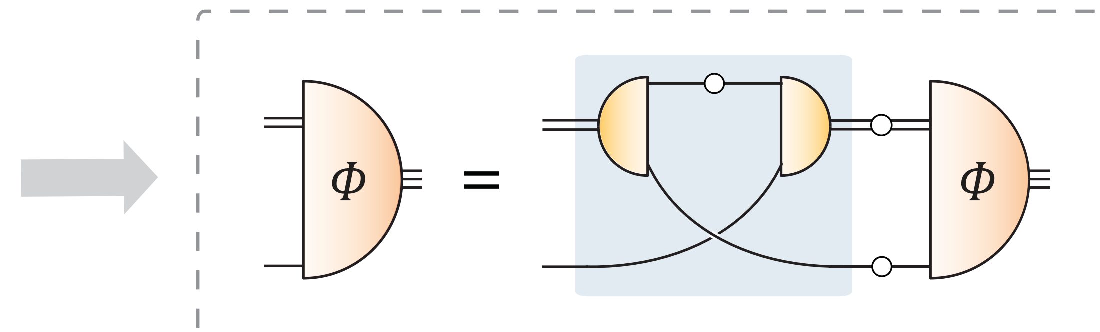
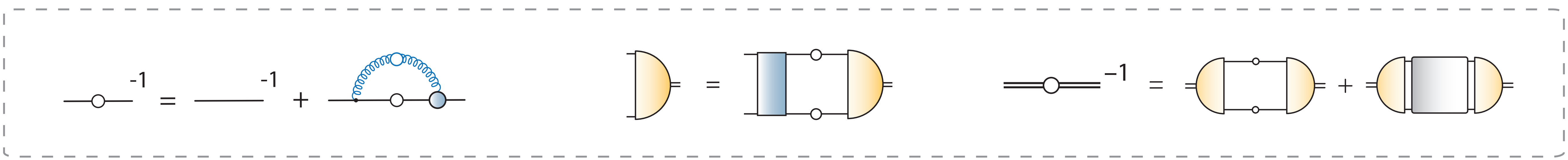


Diquark-quark

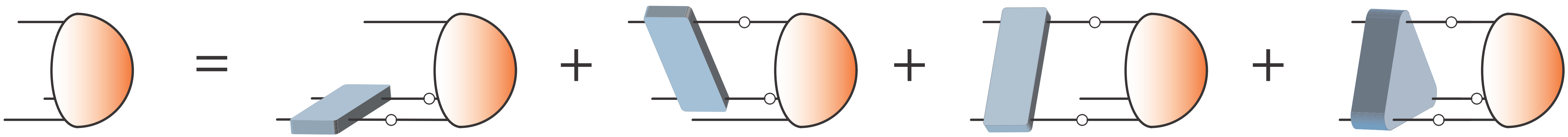




- Input in both cases: quark propagator and interaction

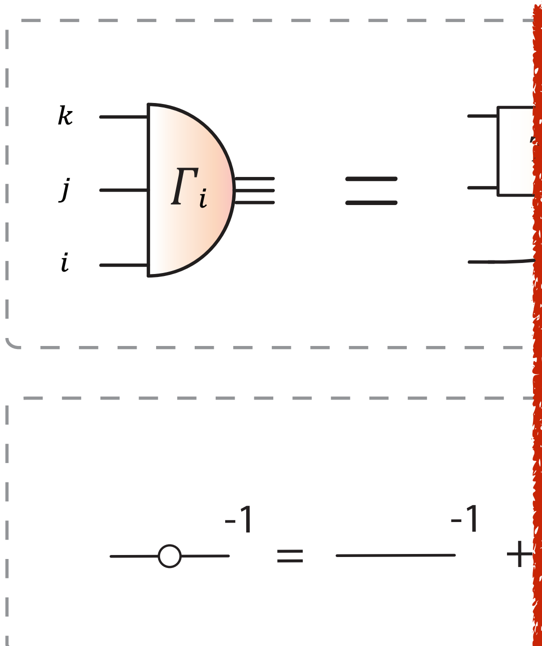
Diquark-Quark approximation

BSE for baryons (derived from equation of motion for G)



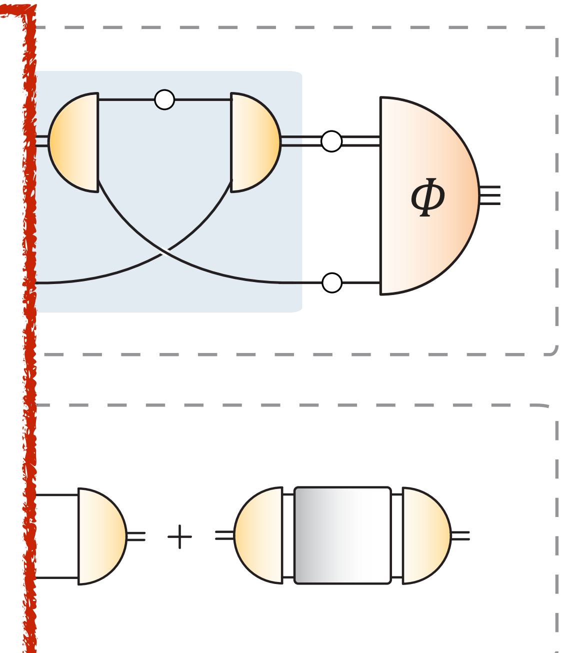
Faddeev equation (no three-body forces)

Diquark-quark



The good, the bad…

scalar	("good")	~ 800 MeV
axialvector	("bad")	~ 1000 MeV



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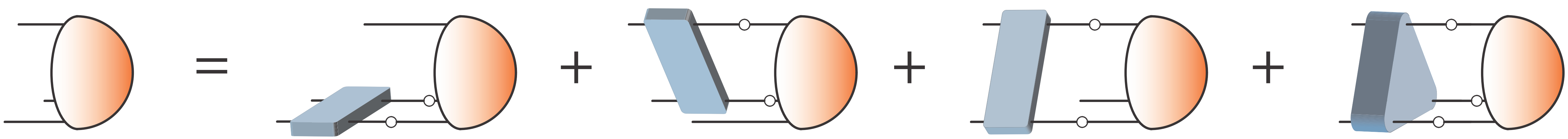
BSE for baryons (derived from equation of motion for G)



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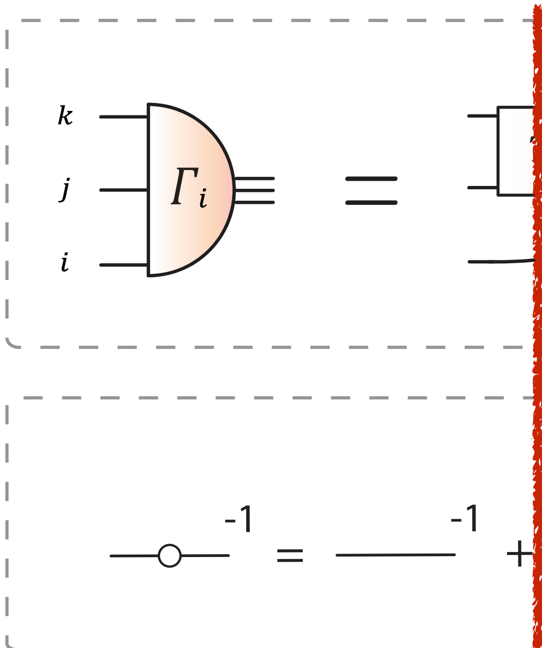
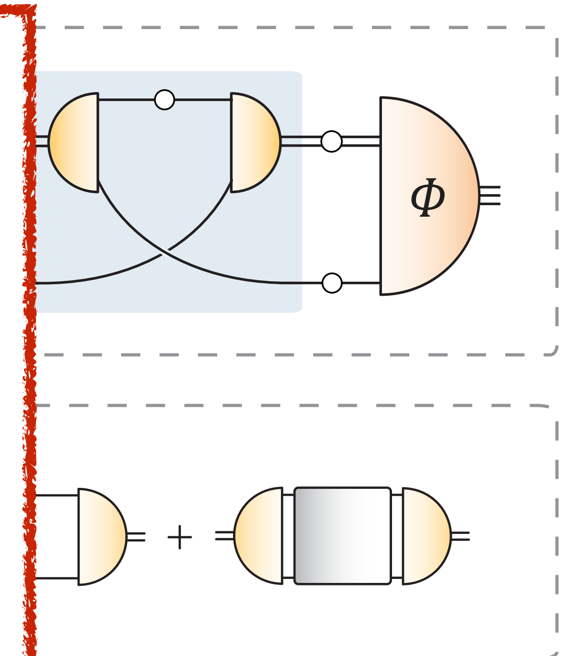


Diquark-quark

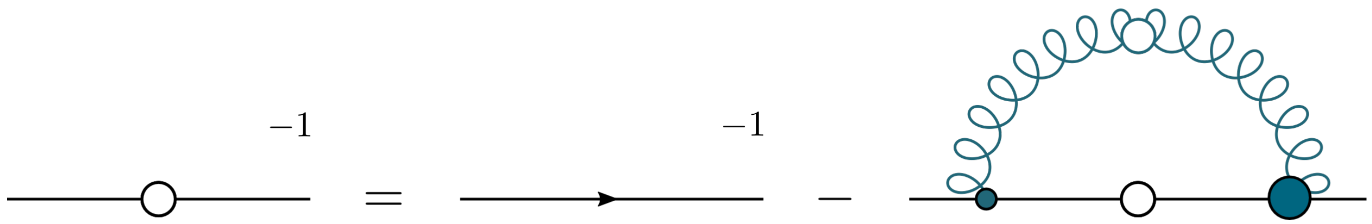


The good, the bad… and the ugly…

scalar	("good")	~ 800 MeV
axialvector	("bad")	~ 1000 MeV
pseudoscalar	("ugly")	~ 1200 MeV
vector		~ 1300 MeV

- Input in both cases: quark propagator and interaction

The DSE for the quark propagator

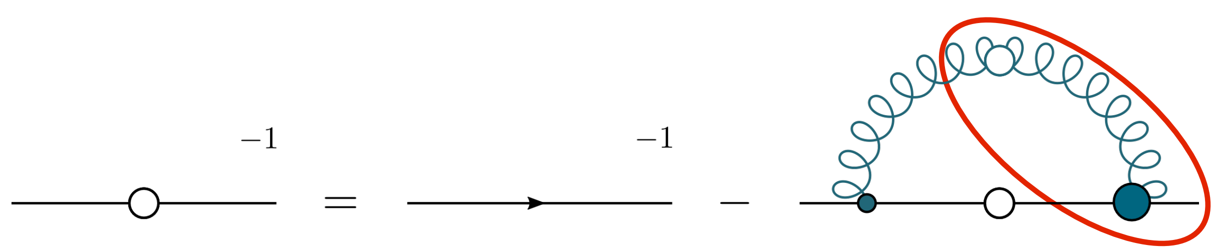


Approximations:

I) NJL/contact model:

II) Quark-diquark model: Ansatz for quark prop
(and diquark wave function)

III) Rainbow-ladder:



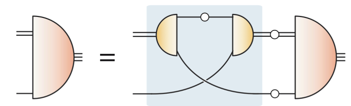
IV) Beyond rainbow-ladder:

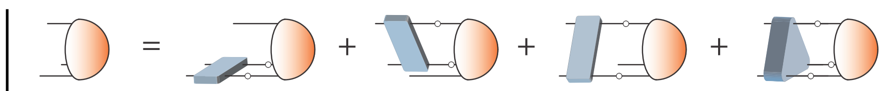
- solve DSEs for quarks, gluons and quark-gluon vertex

CF and Alkofer, PRD 67 (2003) 094020
Williams, EPJA 51 (2015) 5, 57.
Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD 93 (2016) 034026, and refs. therein

DSE/BSE/Faddeev landscape (2015)

level of complexity →

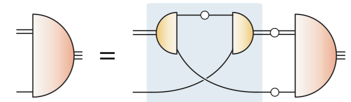


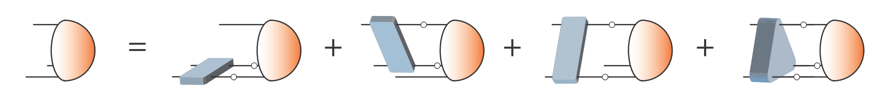


			I) NJL/contact interaction	II) Quark-diquark model	III) DSE (RL)		IV) DSE (bRL)
up/down	$P=\pm$	N,Δ masses	✓	✓	✓	✓	✓
		N,Δ em. FFs	✓	✓	✓	✓	
		$N\to\Delta\gamma$	✓	✓	✓		
	$P=+$	N^*,Δ^* masses	✓	✓			
		$\gamma N\to N^*/\Delta^*$	✓	✓			
	$P=-$	N^*,Δ^* masses		✓			
		$\gamma N\to N^*/\Delta^*$					
strange		ground states		✓			
		excited states					
		em. FF					
		TFFs					
c/b		ground states					
		excited states					
			Cloet, Thomas, Roberts, Segovia, Chen, et al.	Oettel, Alkofer, Bloch, Roberts, Segovia, Chen, et al.	Eichmann, Alkofer, Krassnigg, Nicmorus, Sanchis-Alepuz, CF	Eichmann, Alkofer, Sanchis-Alepuz, CF, Qin, Roberts	Sanchis-Alepuz, Williams, CF

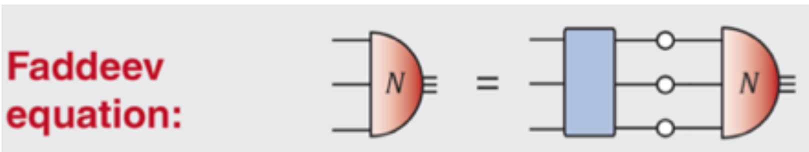
DSE/BSE/Faddeev landscape

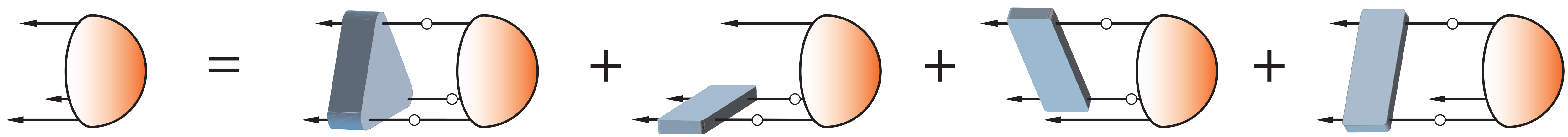
level of complexity →





			I) NJL/contact interaction	II) Quark-diquark model	III) DSE (RL)		IV) DSE (bRL)
up/down	$P=\pm$	N,Δ masses	✓	✓	✓	✓	✓
		N,Δ em. FFs	✓	✓	✓	✓	
		$N\to\Delta\gamma$	✓	✓	✓	✓	
	$P=+$	N^*,Δ^*masses	✓	✓	✓	✓	
		$\gamma N\to N^*/\Delta^*$	✓	✓			
	$P=-$	N^*,Δ^*masses	✓	✓	✓	✓	
		$\gamma N\to N^*/\Delta^*$					
strange		ground states	✓	✓	✓	✓	
		excited states	✓	✓	✓	✓	
		em. FF			✓	✓	
		TFFs				✓	
c/b		ground states	✓	✓		✓	
		excited states		✓		✓	
			Cloet, Thomas, Roberts, Segovia, Chen, et al.	Oettel, Alkofer, Bloch, Roberts, Segovia, Chen, et al.	Eichmann, Alkofer, Krassnigg, Nicmorus, Sanchis-Alepuz, CF	Eichmann, Alkofer, Sanchis-Alepuz, CF, Qin, Roberts	Sanchis-Alepuz, Williams, CF





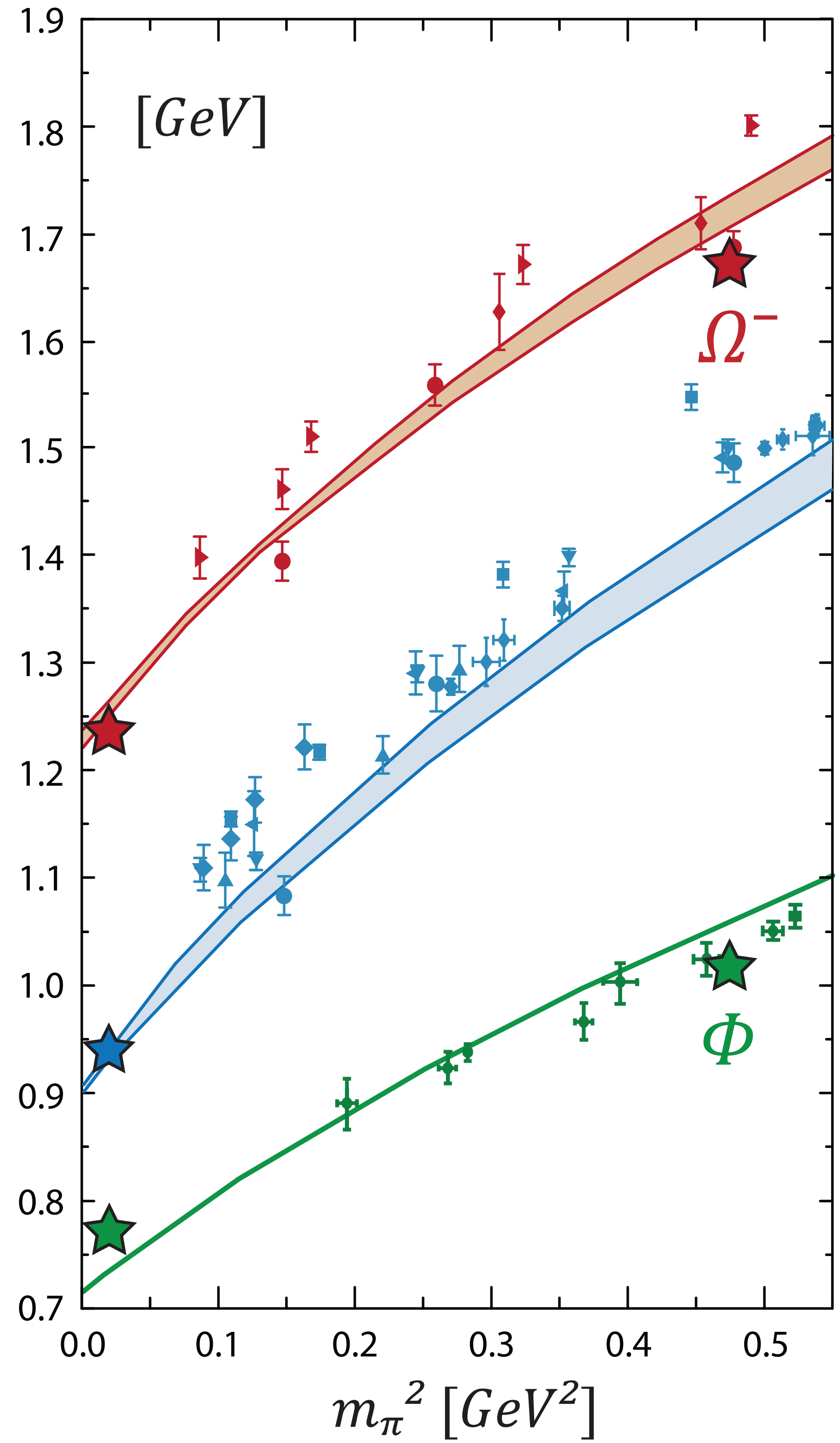
- relativistic bound state:
  - 64 tensor structures for nucleon: s, p, d - wave
  - 128 tensor structures for Delta: s, p, d, f - wave

$$D_i\,\gamma_5\,C \otimes D_j\,\Lambda_+(P)\,, \qquad D_i = \{\mathbb{1},\ \not p,\ \not q,\ \not P,\ [\not p, \not P],\ [\not q, \not P],\ [\not p, \not q],\ [\not p, \not q, \not P]\}\,,$$

$$\gamma_5\,D_i\,\gamma_5\,C \otimes \gamma_5\,D_j\,\Lambda_+(P)\,, \qquad \Lambda_\pm(P) = \tfrac{1}{2}\,(\mathbb{1} \pm \hat{\not P})\,,$$

Baryon masses

- first covariant three-body calculations !
- grosso modo: consistent description of mesons and baryons
- wave functions contain sizable p-wave contributions



Delta:
Sanchis-Alepuz et al., PRD 84 (2011)

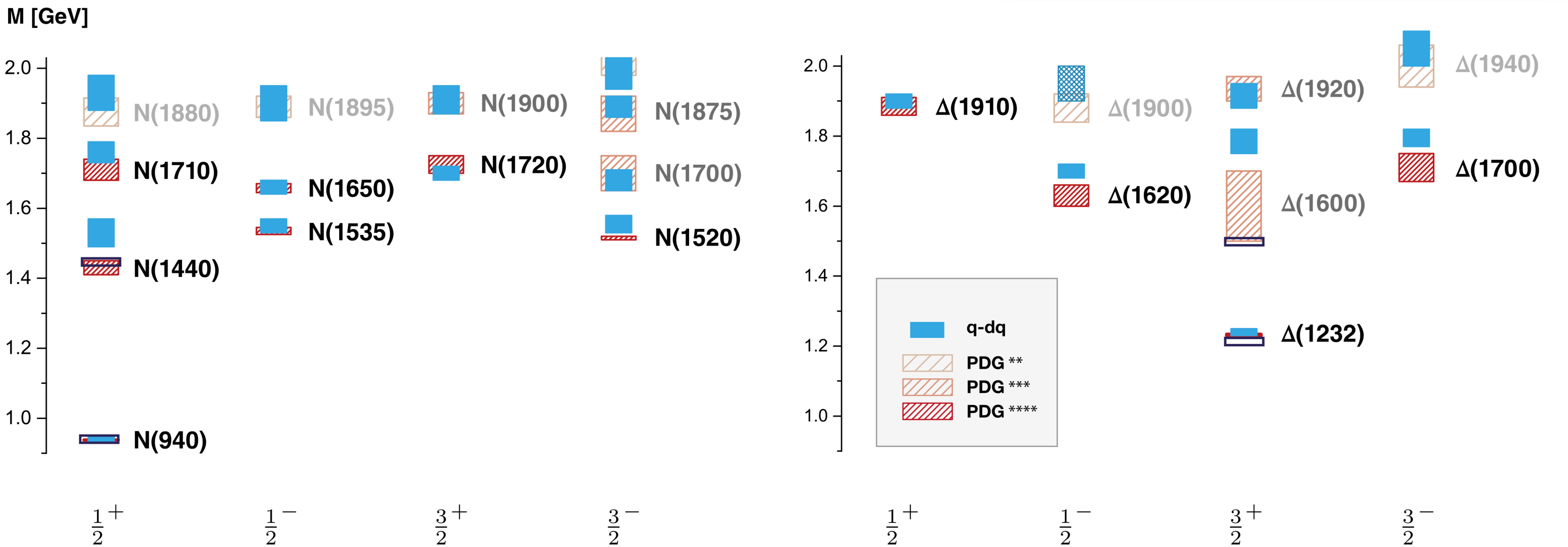
Nucleon:
Eichmann et al., PRL 104 (2010), PRD 84 (2011)

ρ–meson:
Maris & Tandy, PRC 60 (1999)

Eichmann, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)
Eichmann, PRD 84 (2011)
Sanchis-Alepuz ,Eichmann, Villalba-Chavez, Alkofer, PRD (2012)

Light baryon spectrum:

- 3 parameters + $m_{u,d,s}$
  (all fixed in meson sector)



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)

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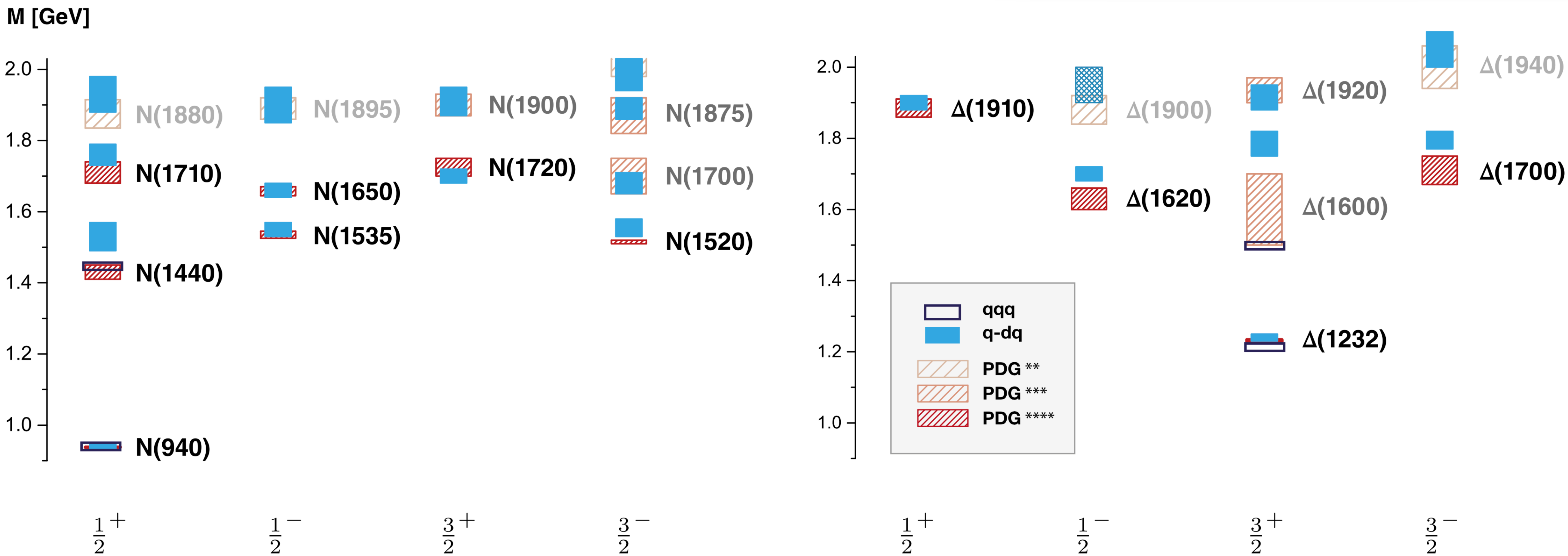

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)
- three-body agrees with diquark-quark where applicable

Relativistic baryons

$J^P = \left(\frac{1}{2}\right)^+$

non-relativistic

three quarks with spin 1/2: $S = 1/2$ or $S = 3/2$

parity $P = (-1)^L$: $L = 0$ or $L = 2$

relativistic

64 components in wave function: 8 s-wave (L=0)
36 p-wave (L=1)
20 d-wave (L=2)

$P = (-1)^L$

%	N	$N^*(1440)$	Δ	$\Delta^*(1600)$
s wave	66	15	56	10
p wave	33	61	40	33
d wave	1	24	3	41
f wave	–	–	< 0.5	16

Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

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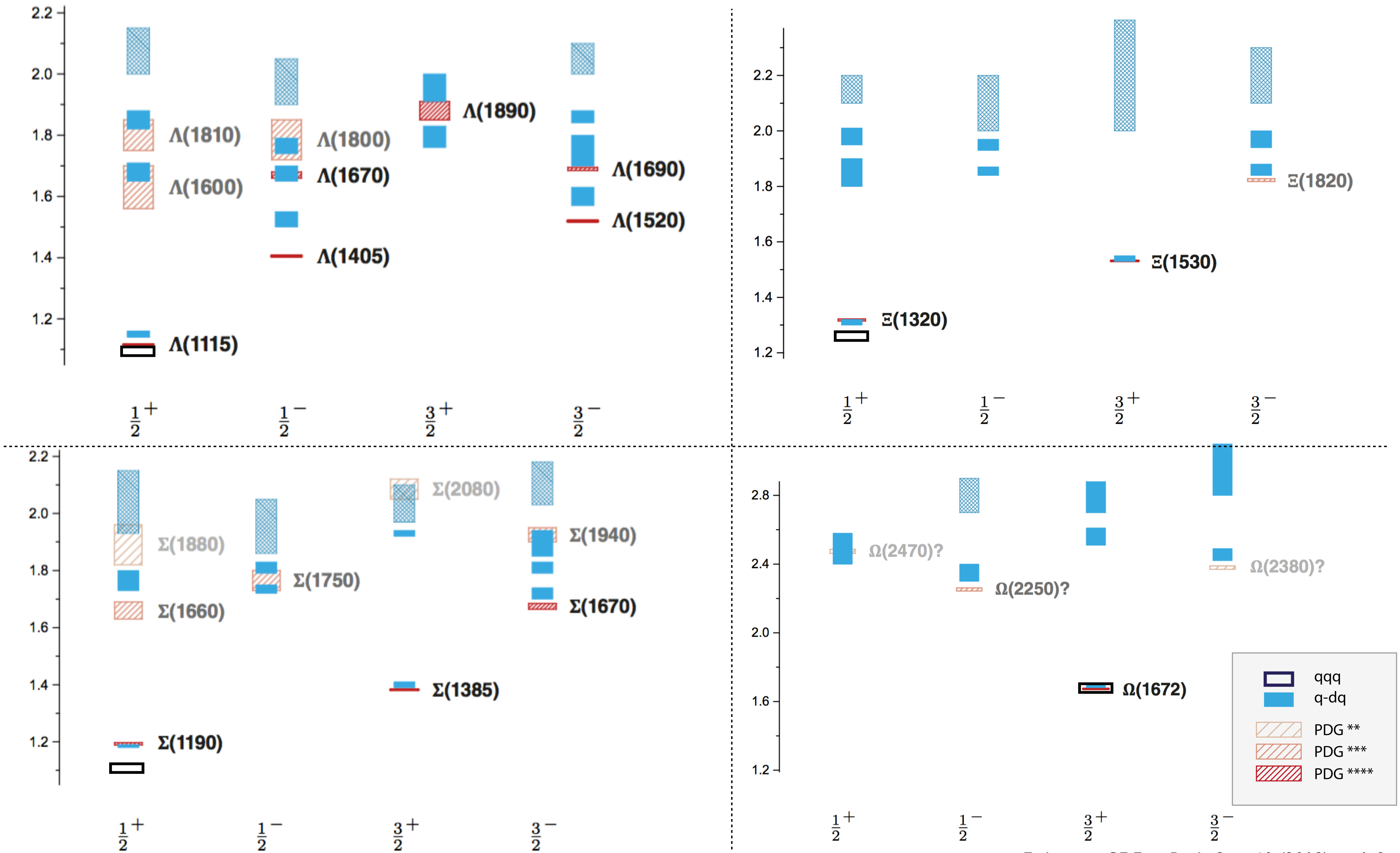
Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

Strange baryon spectrum: DSE-RL (preliminary !)



Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
CF, Eichmann PoS Hadron 2017 (2018) 007
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

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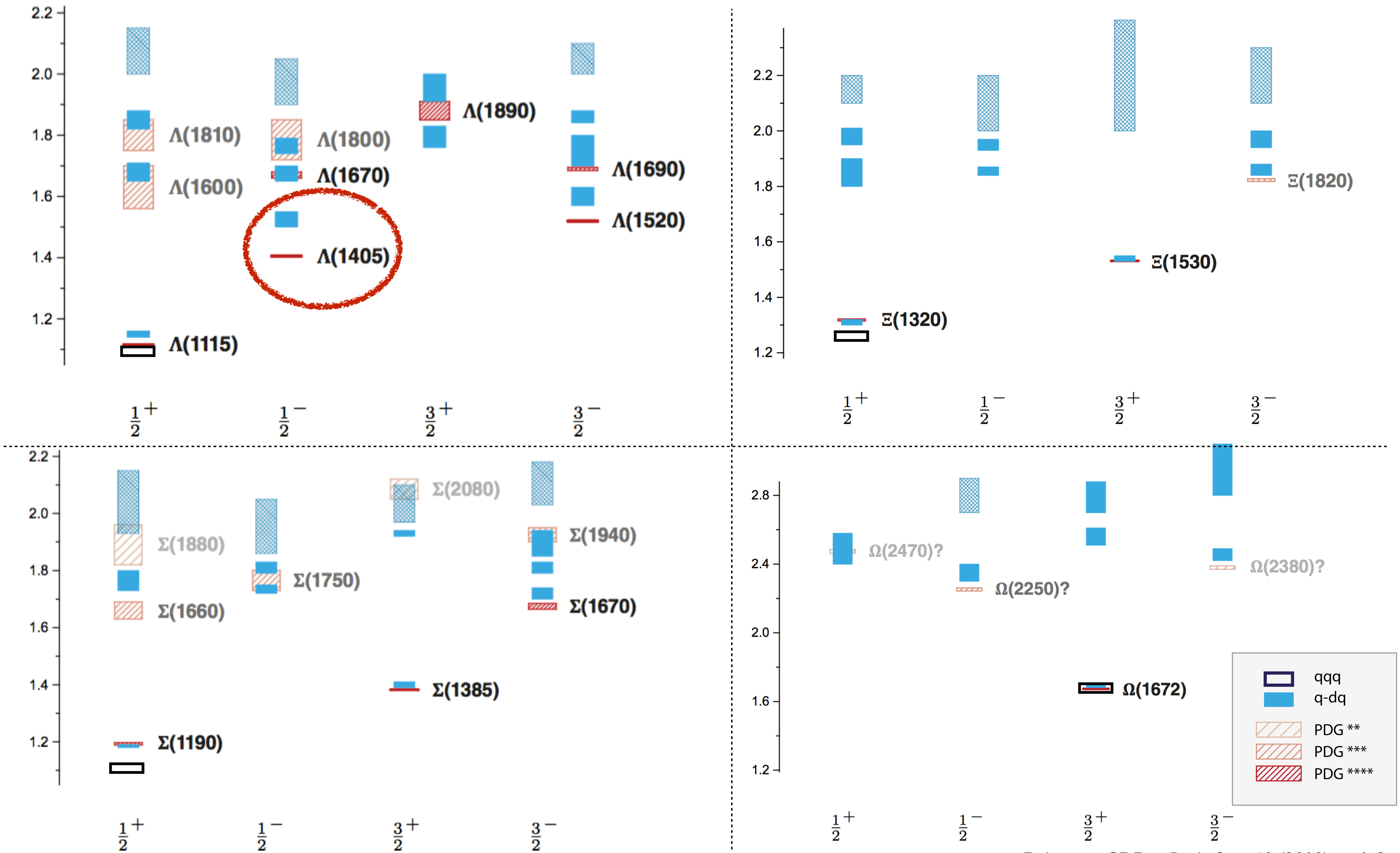

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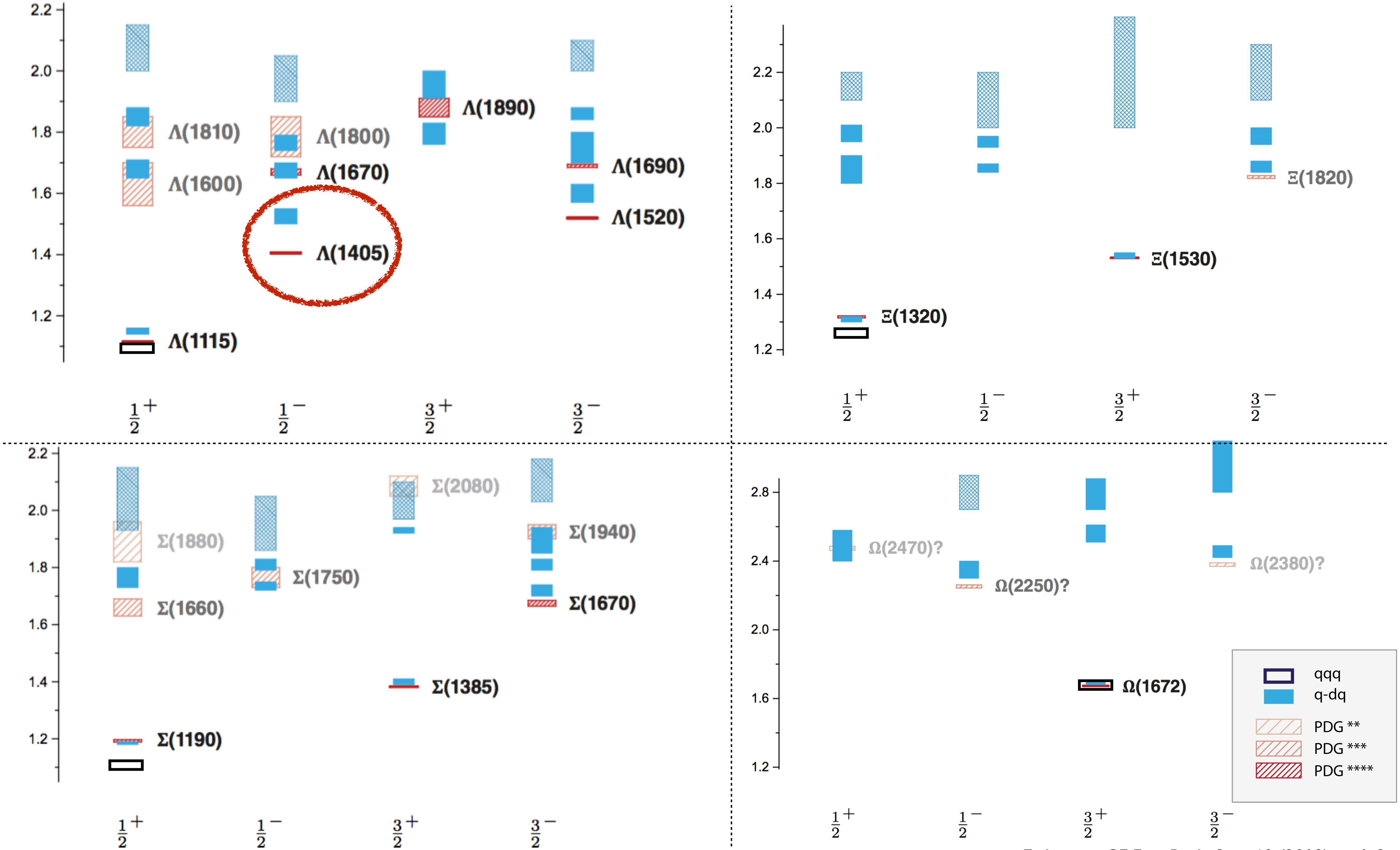

New states: Bonn-Gatchina (talk of M. Matveev at N*2019)

Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
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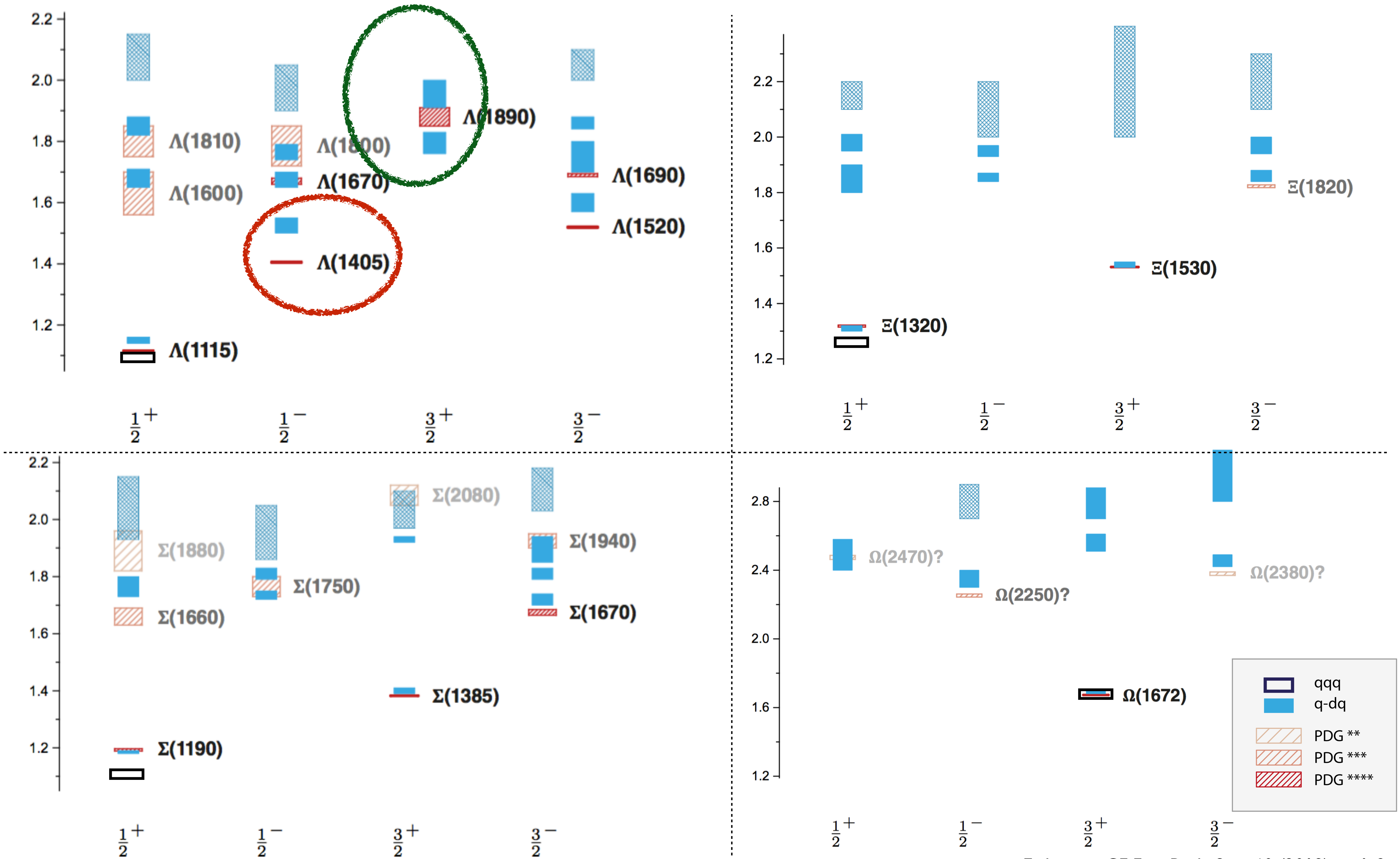

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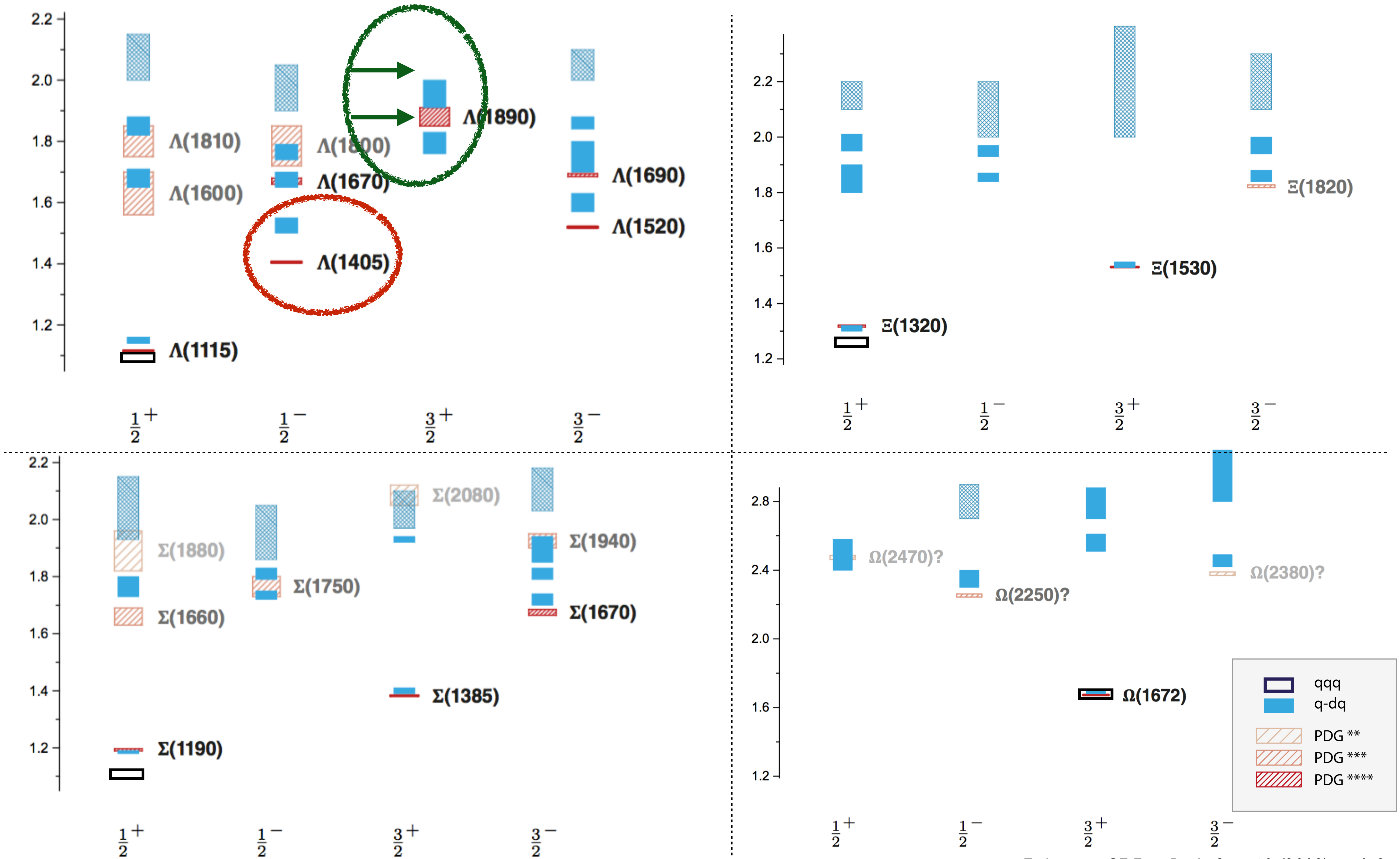

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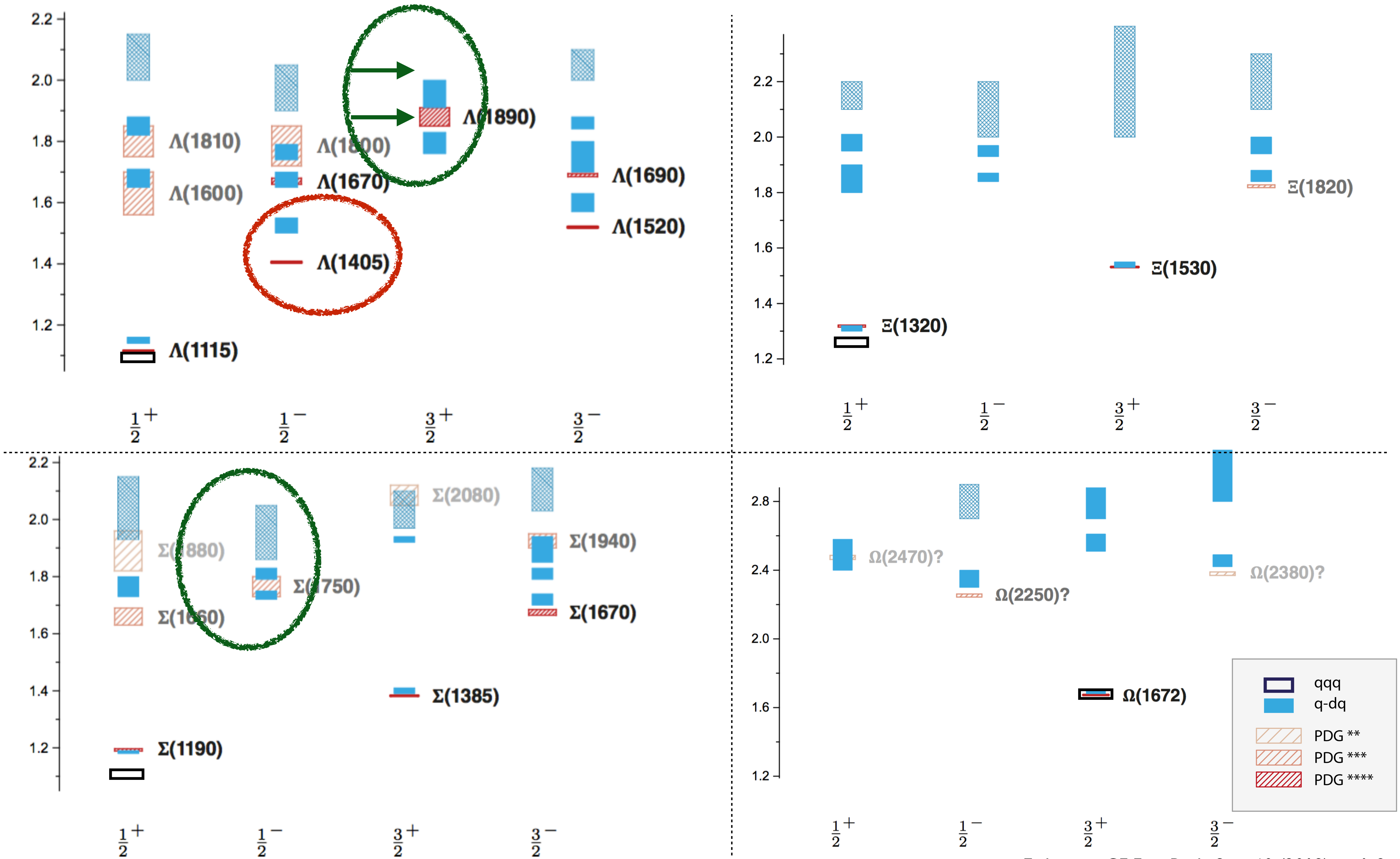

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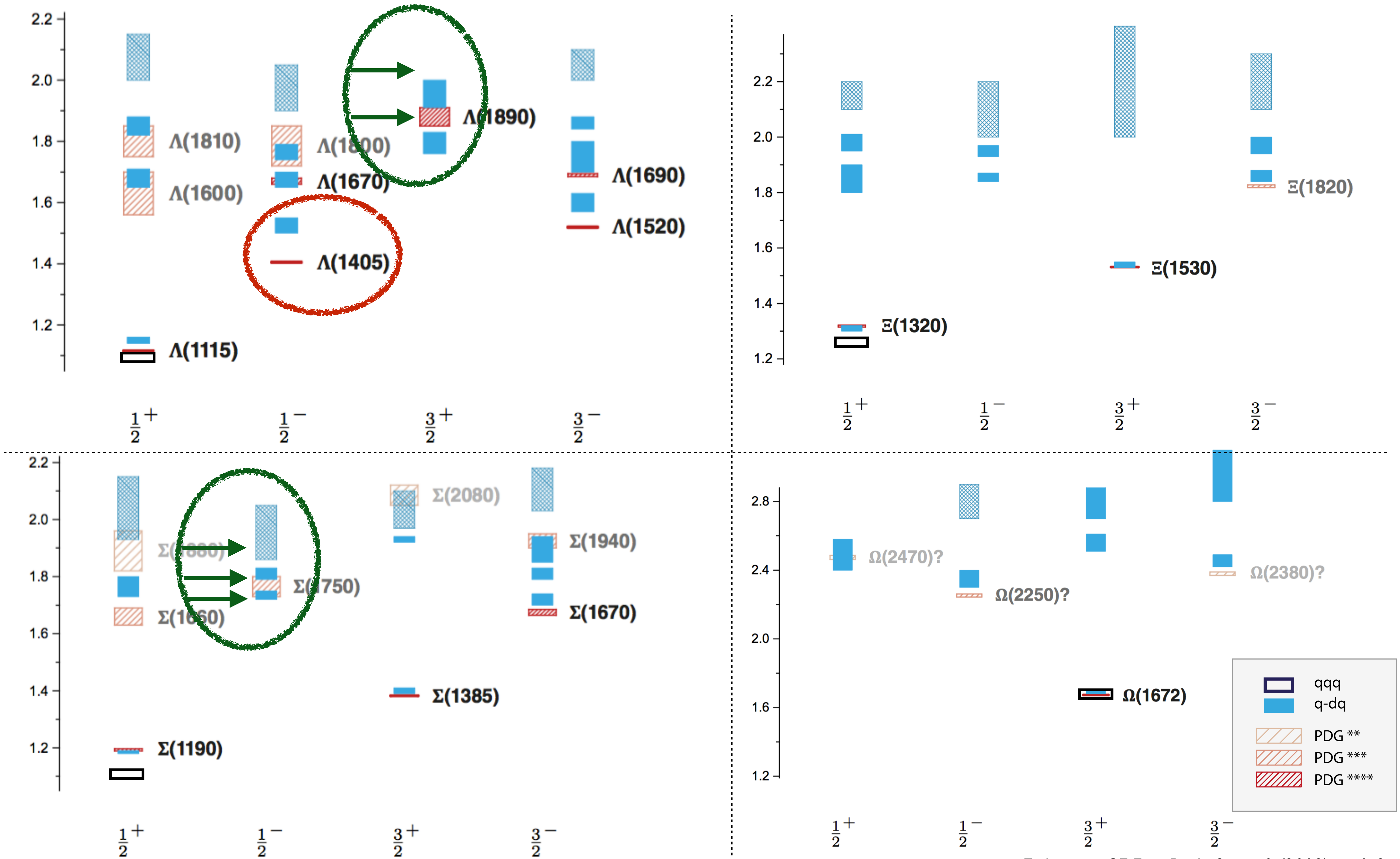

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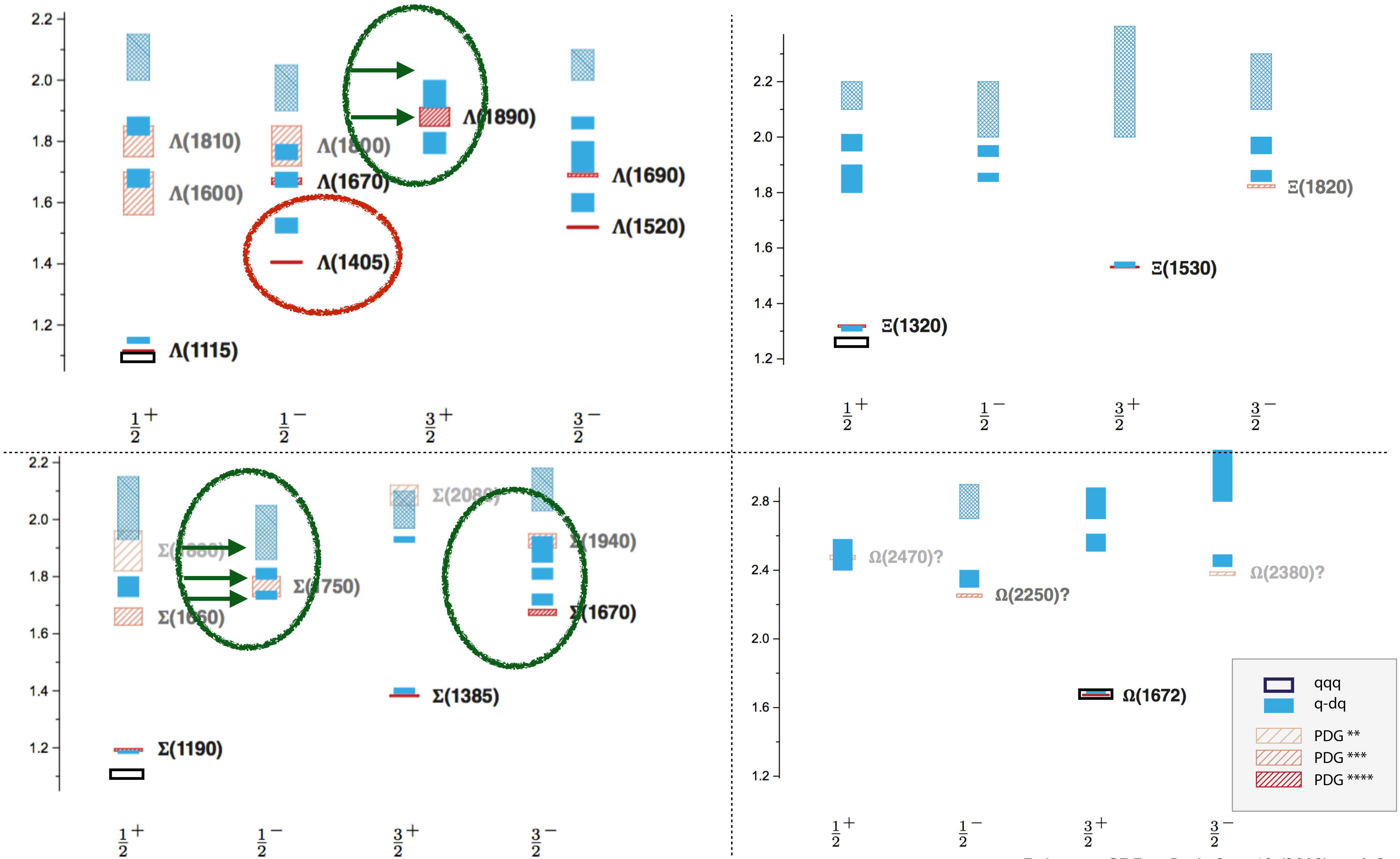

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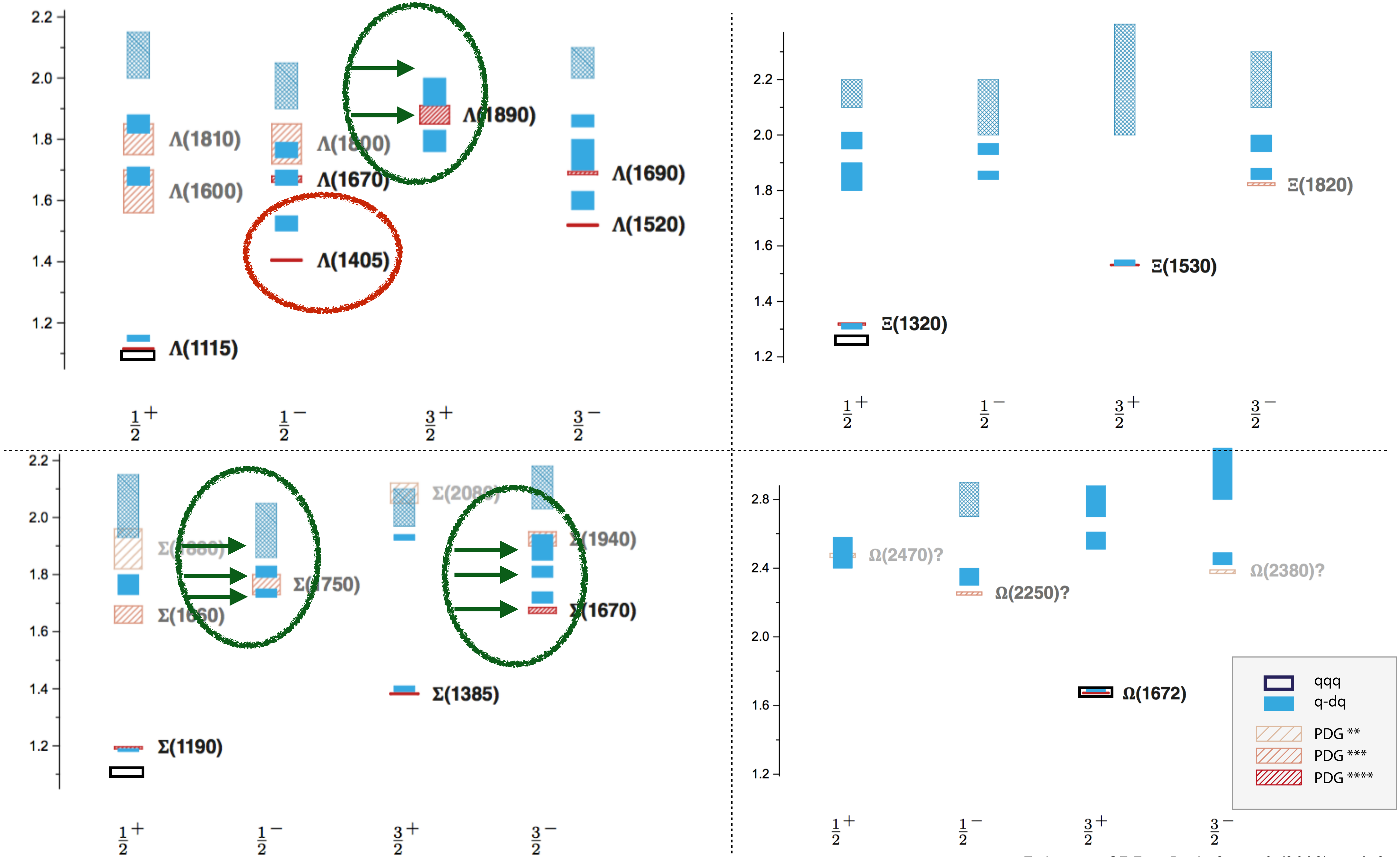

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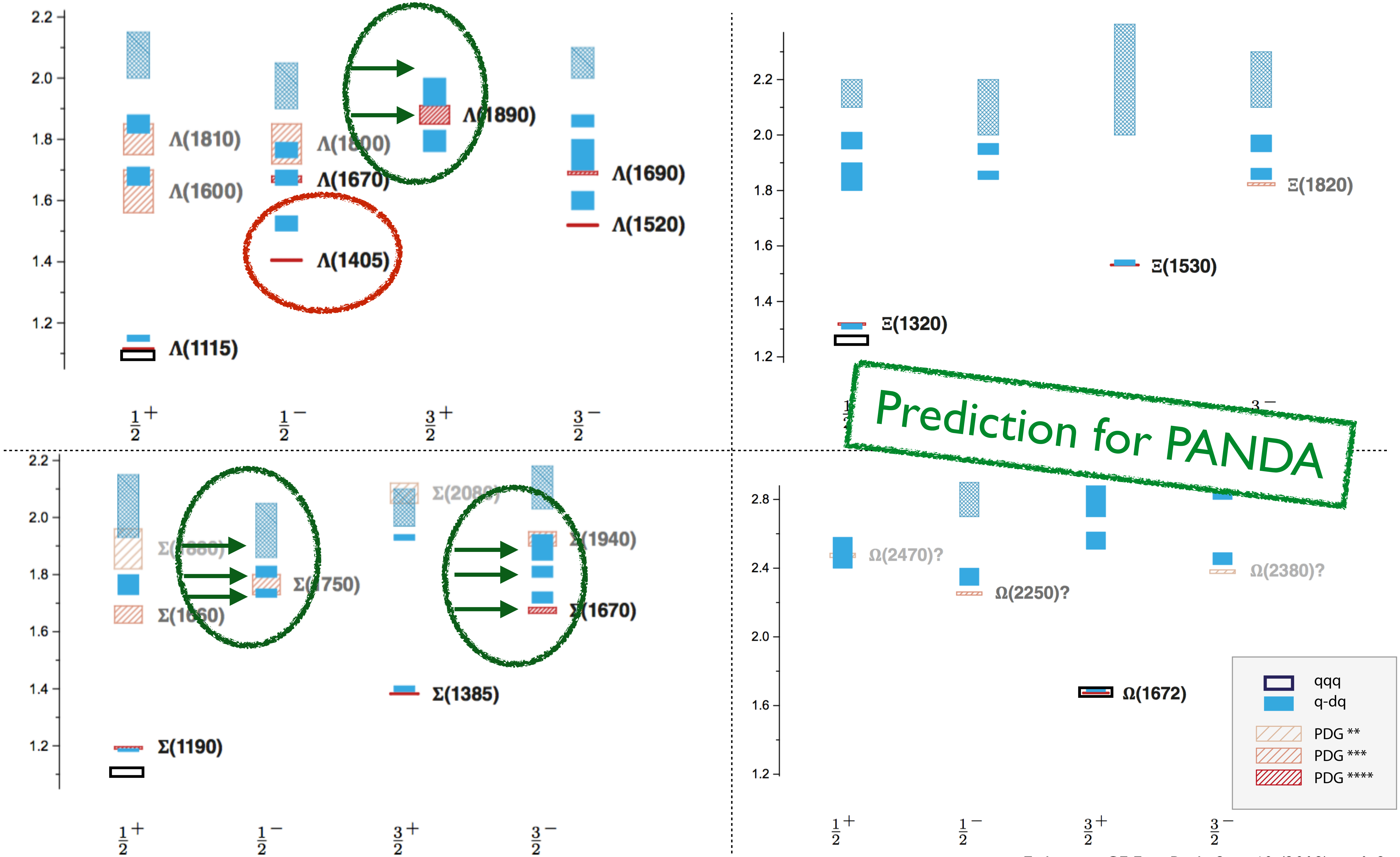

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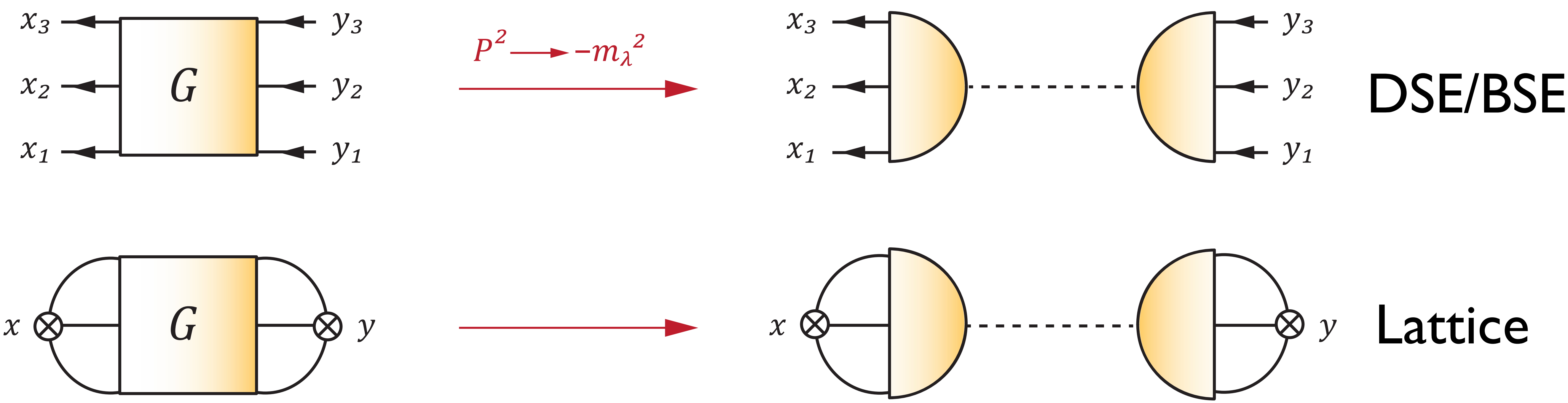
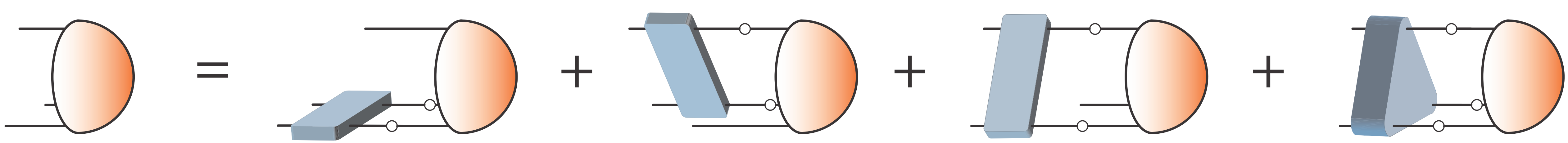
CF, Eichmann PoS Hadron 2017 (2018) 007
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

Overview

1. Bound states in the quark model
   - Construction
   - Assets, shortcomings and how to do better
2. Properties of QCD and correlation functions
   - Dynamical chiral symmetry breaking
   - Correlation functions and Dyson-Schwinger equations (DSEs)
   - The quark DSE
3. Mesons
   - The pion Bethe-Salpeter equation (BSE): Goldstone bosons, GMOR and all that…
   - Spectra: light and charm
4. Exotic mesons
   - Confinement and glueballs
   - Four-quark states
5. Baryons
   - Spectra: light and strange
6. Form factors
   - Meson form factors
   - Baryon form factors

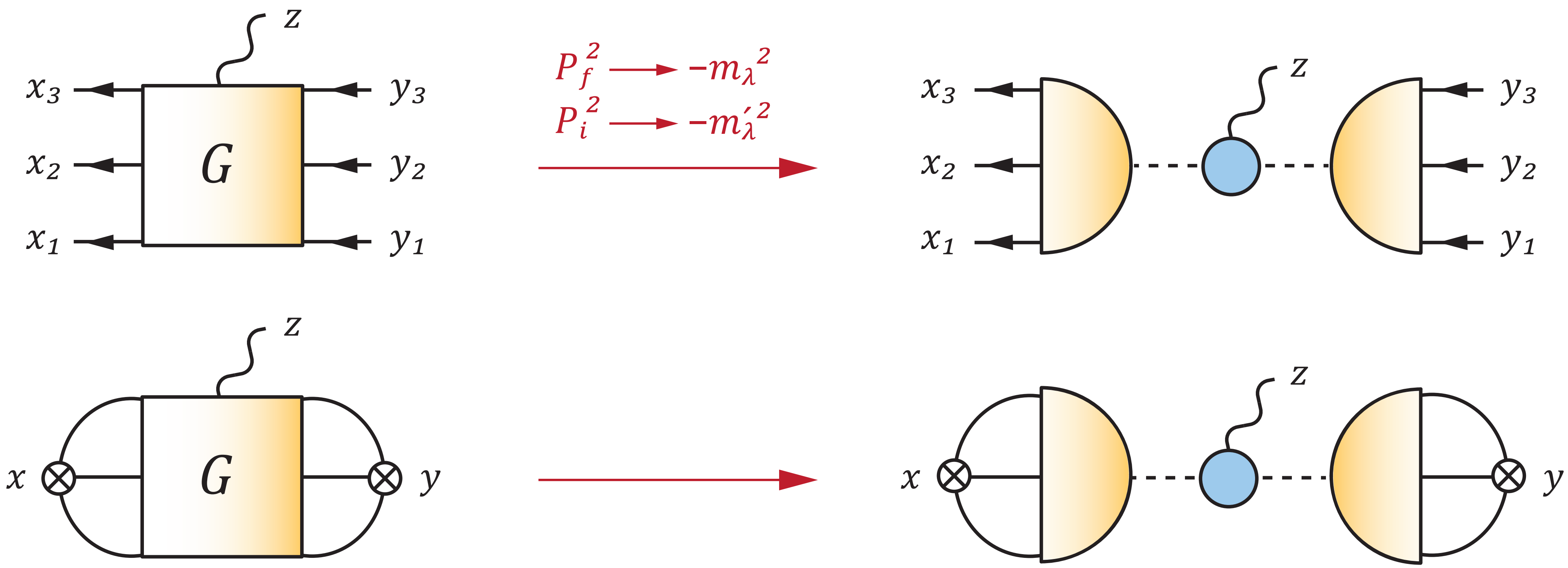


BSE for baryons (derived from equation of motion for G)



- exact equation for baryon 'wave function'

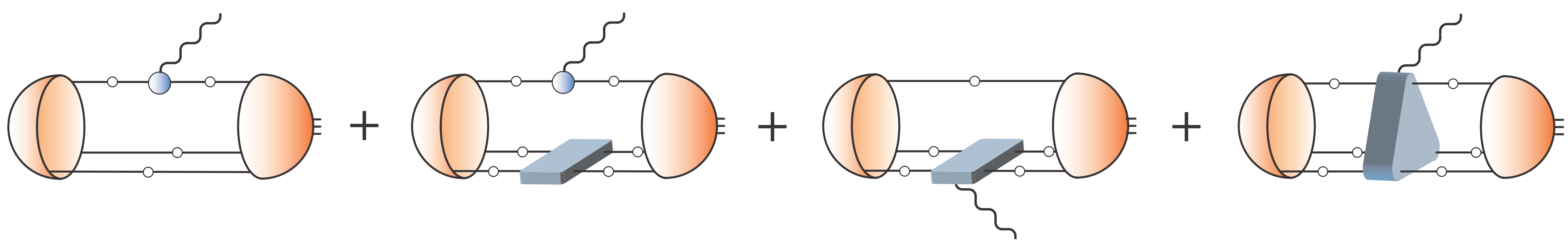
Extracting form factors from correlators



DSE/BSE

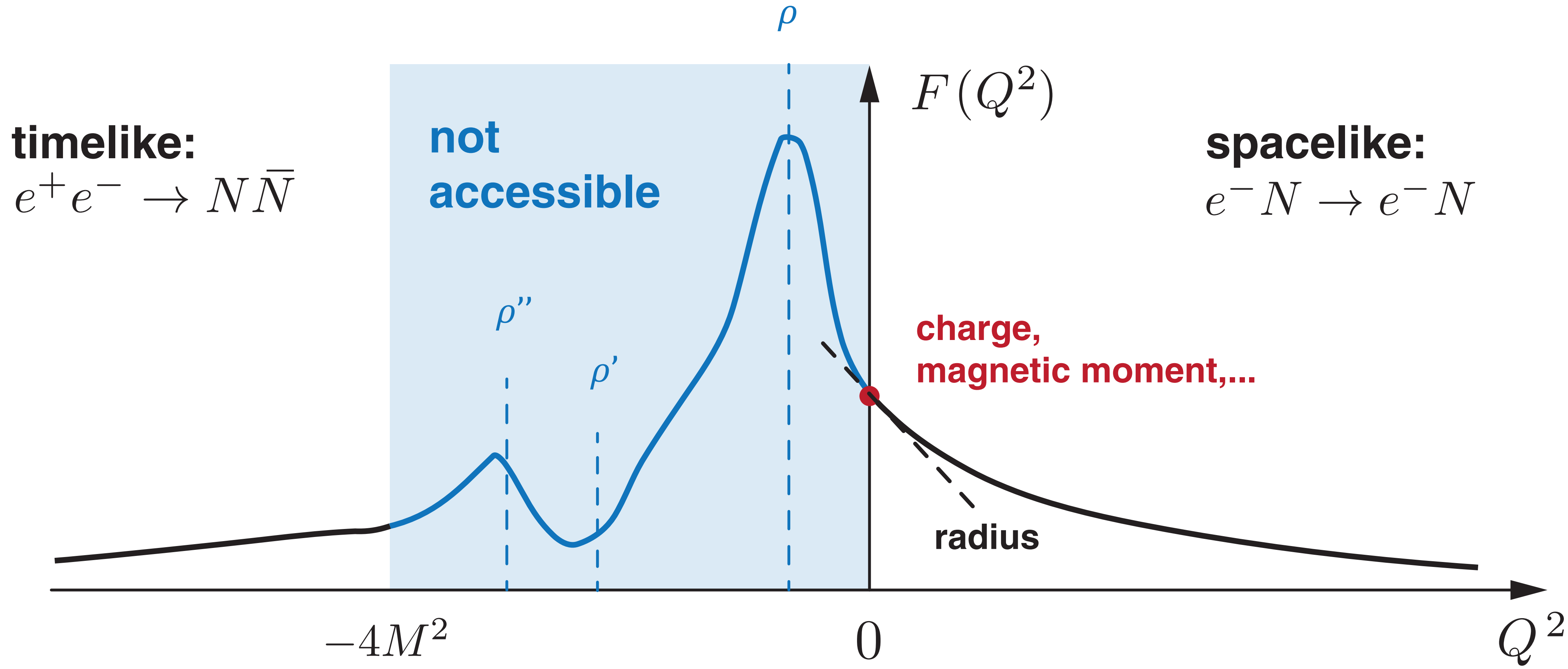
Lattice

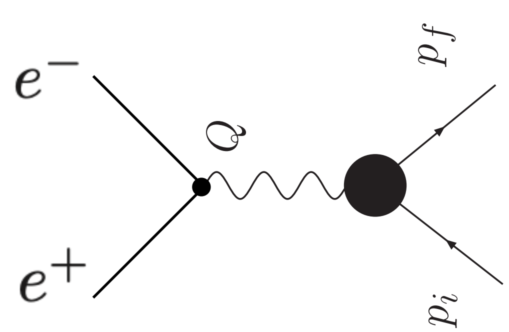
Form factor from BSEs (derived from equation of motion for G and 'gauging')

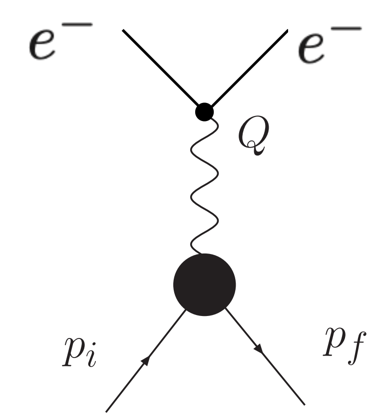
$J^\mu =$ 

- exact equation for baryon form factors

Physics from form factors I







$$Q = (0, 0, 0, 2\sqrt{q^2 + M^2})$$
$$p_i = (0, 0, q, -\sqrt{q^2 + M^2})$$
$$p_f = (0, 0, q, \sqrt{q^2 + M^2})$$

$$Q = (0, 0, q, 0)$$
$$p_i = (0, 0, -q/2, \sqrt{q^2 + M^2})$$
$$p_f = (0, 0, q/2, \sqrt{q^2 + M^2})$$

Physics from form factors II

- Example: pion electromagnetic form factor

$$\mathcal{J}^\mu(p_i,p_f) = (p_i+p_f)^\mu F(Q^2)$$

charge radius

with $$F(Q^2) = F(0) - \frac{r^2}{6} Q^2 + \cdots$$

electric charge

- Example: nucleon electromagnetic form factor

$$\mathcal{J}^\mu(p_i,p_f) = i\bar{u}(p_f)\left(F_1(Q^2)\gamma^\mu + \frac{iF_2(Q^2)}{4M}[\gamma^\mu, \not{Q}]\right)u(p_i)$$

with $$F_1(Q^2) = F_1(0) - \frac{{r_1}^2}{6} Q^2 + \cdots$$ electric charge

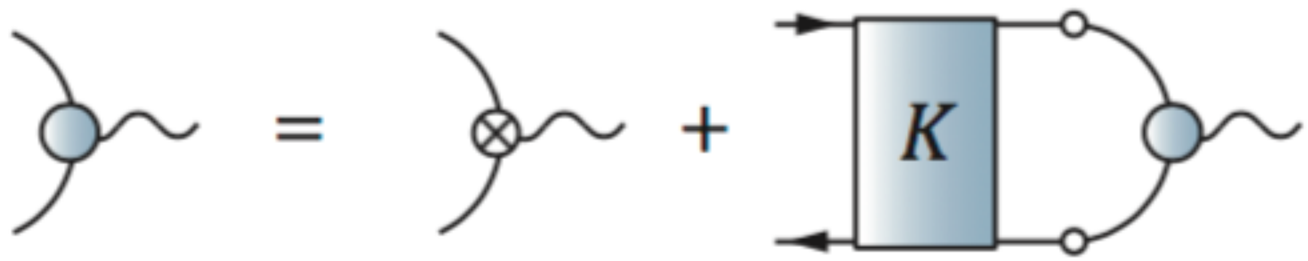
$$F_2(Q^2) = F_2(0)\left[1 - \frac{{r_2}^2}{6} Q^2 + \cdots\right]$$

charge radii

anomalous magnetic moment

Currents coupling to quarks

Exact equation for any vertex in QCD coupling a colorless current to a quark-antiquark pair:

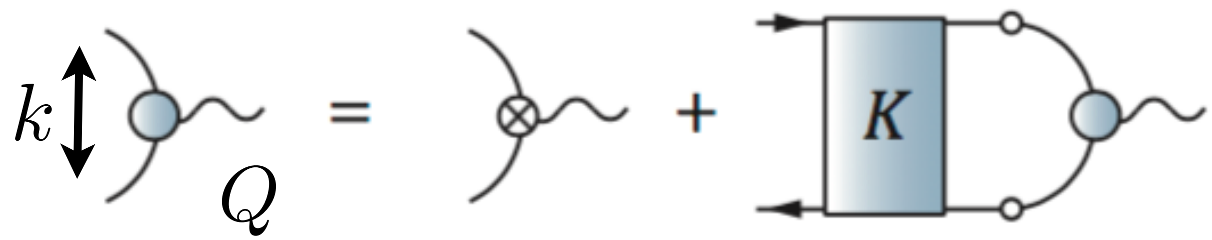


Q

- 'inhomogenuous' Bethe-Salpeter equation
- contains meson poles for on-shell total momenta $Q^2 = -m_{BS}^2$
- physics content determined by quantum numbers

e.g. vector-quark-antiquark vertex contains vector meson poles

Quark-photon vertex and dynamical vector mesons



Basis:

$$\{\gamma^\mu, Q^\mu, k^\mu\} \otimes \{\mathbb{1}, \not{Q}, \not{k}, \not{Q}\not{k}\}$$

⟶ 12 elements

$$\Gamma^\mu(k,Q) = \Gamma^\mu_{\mathrm{BC}}(k,Q) + \Gamma^\mu_{\mathrm{T}}(k,Q) = \sum_{i=1,4} \lambda_i L_i^\mu + \sum_{i=1,8} \tau_i T_i^\mu$$

gauge part 'Ball-Chiu'

transverse part → vector-mesons

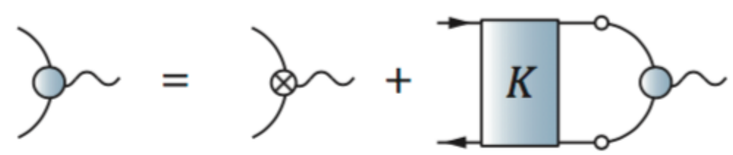
Ball and Chiu, PRD 22 (1980) 2542.

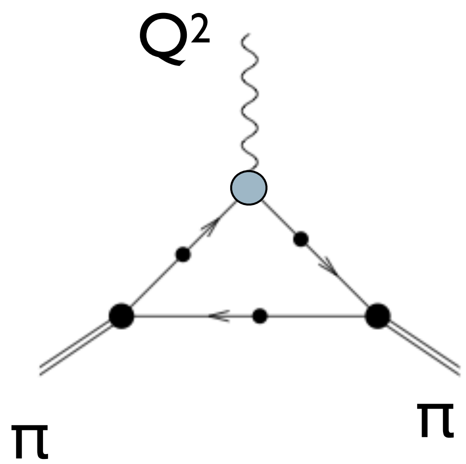
WTI: $Q^\mu \Gamma^\mu(k,Q) = S^{-1}(k+Q/2) - S^{-1}(k-Q/2)$ ✓

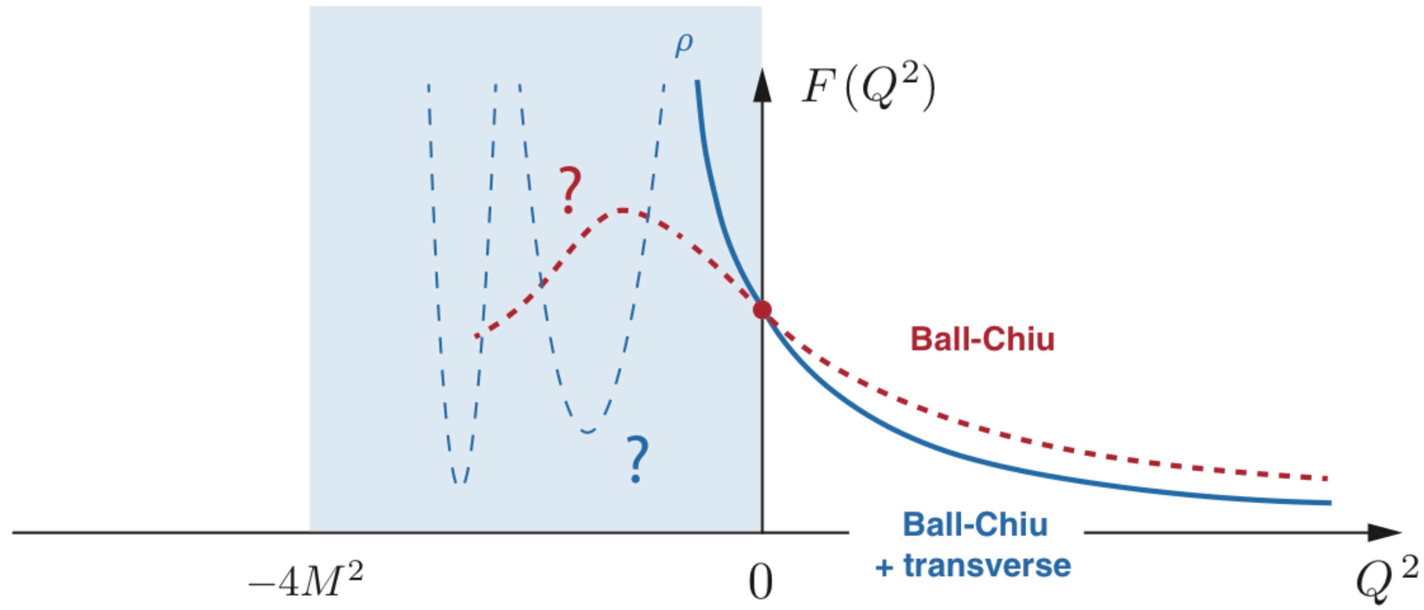
Vector mesons: dynamically generated ✓

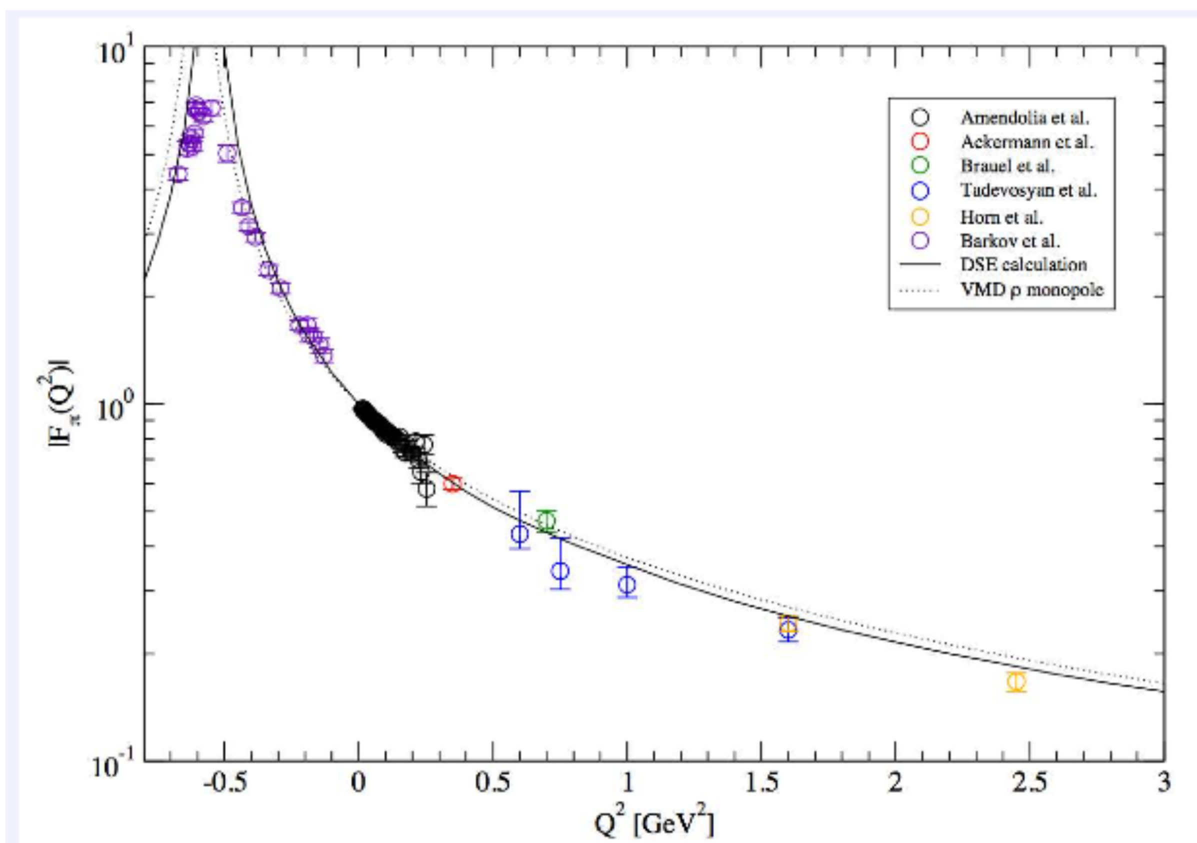
Quark-photon vertex and pion form factors

Pion form factor:







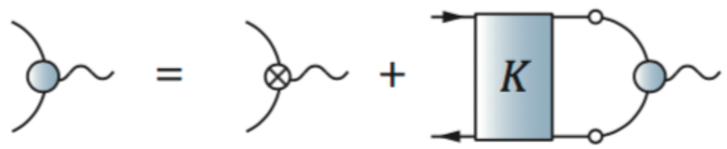


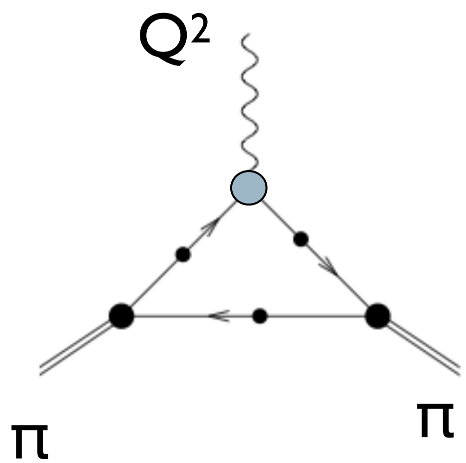
Krassnigg, Schladming 2011; Maris, Tandy NPPS 161, 2006

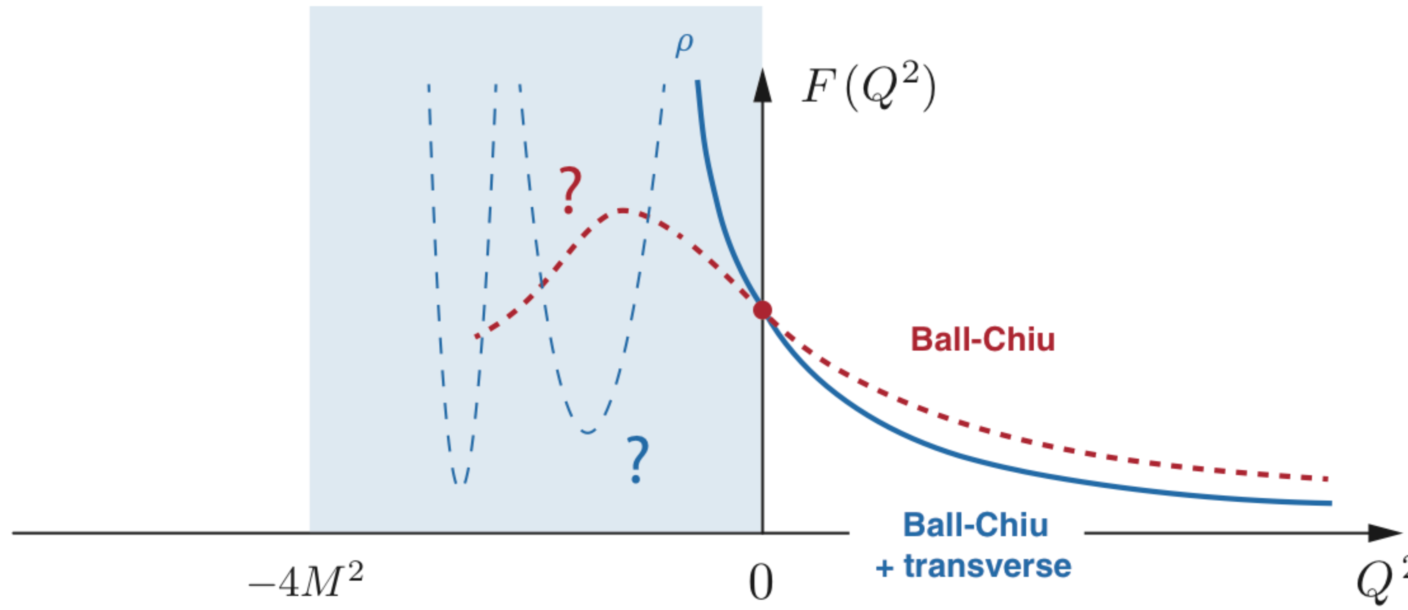
cf. proof of Goldstone theorem

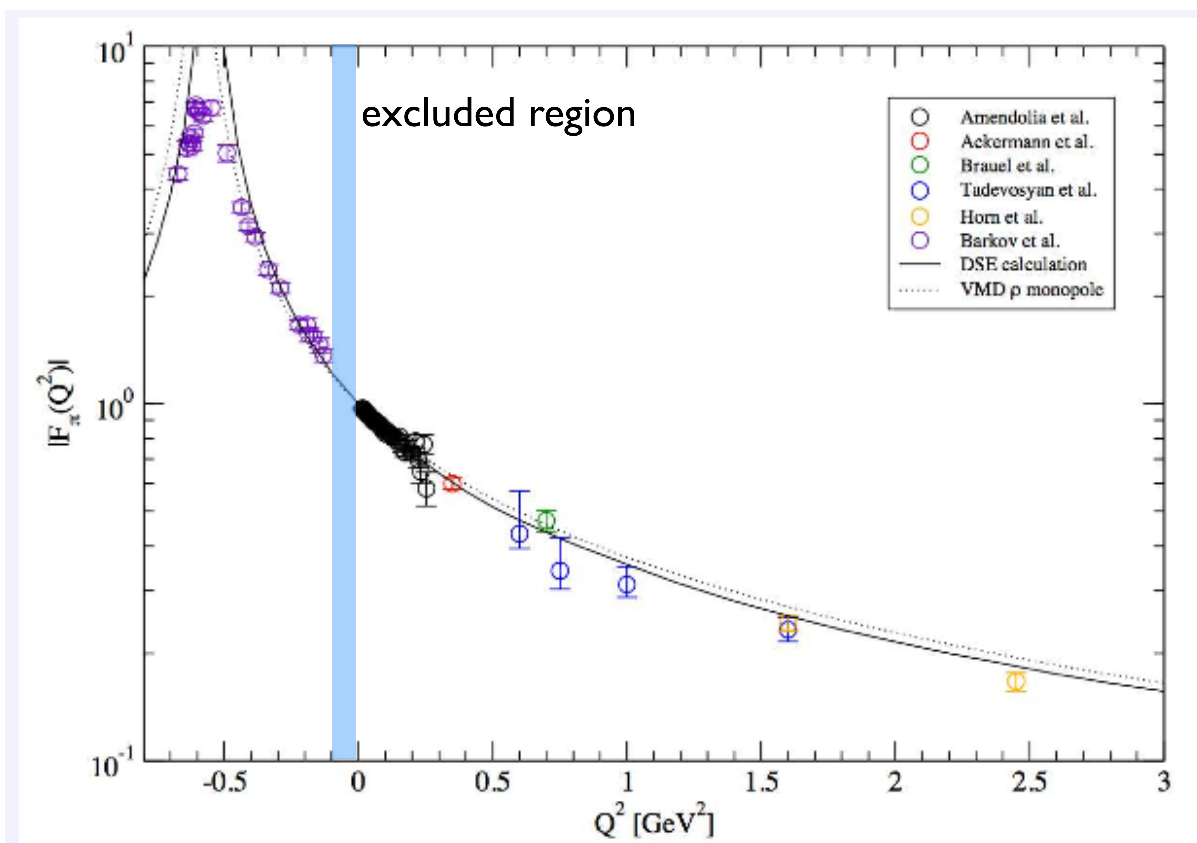
Vector meson poles dynamically generated!

Pion form factor:









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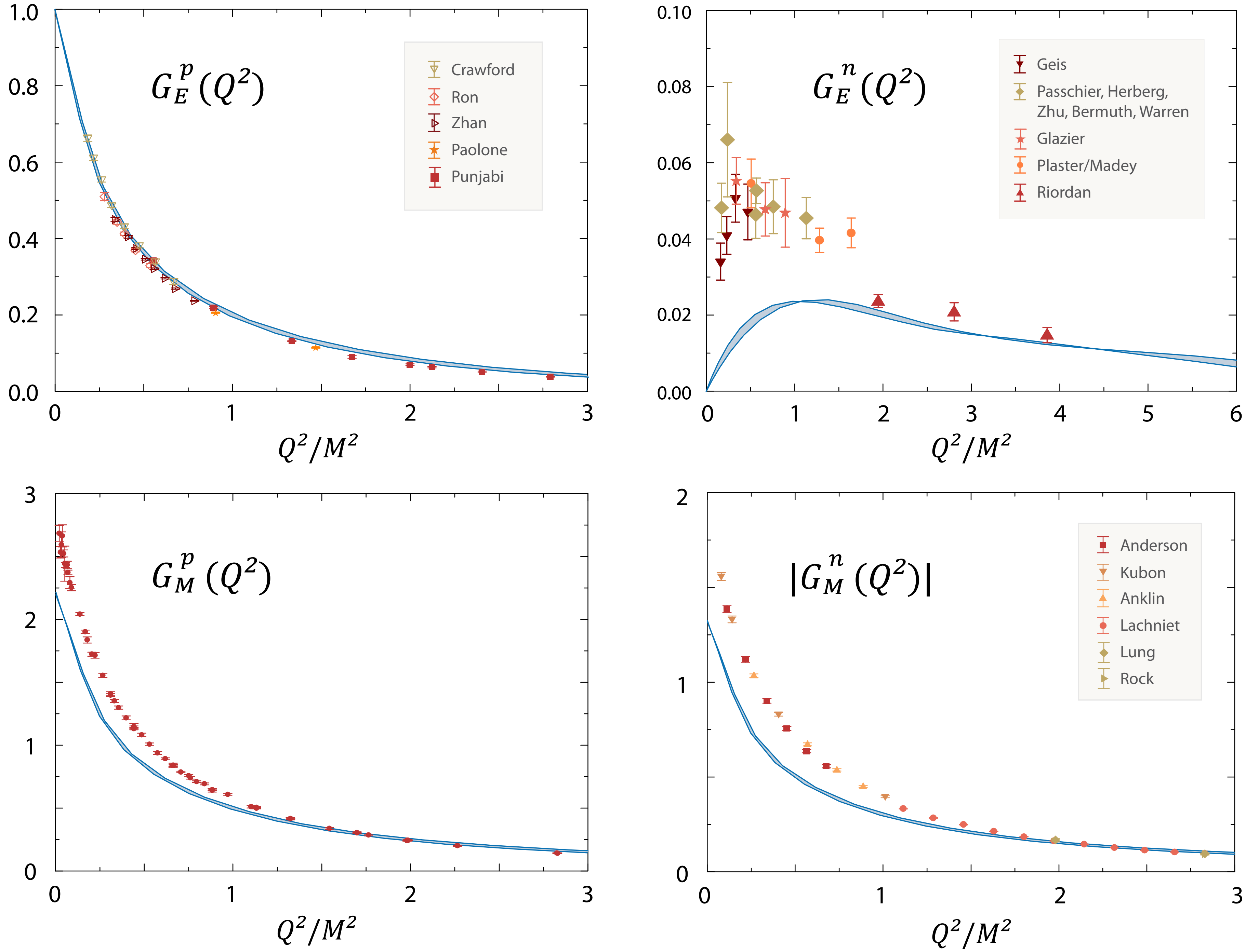
Vector meson poles dynamically generated!

Nucleon form factors and magnetic moments



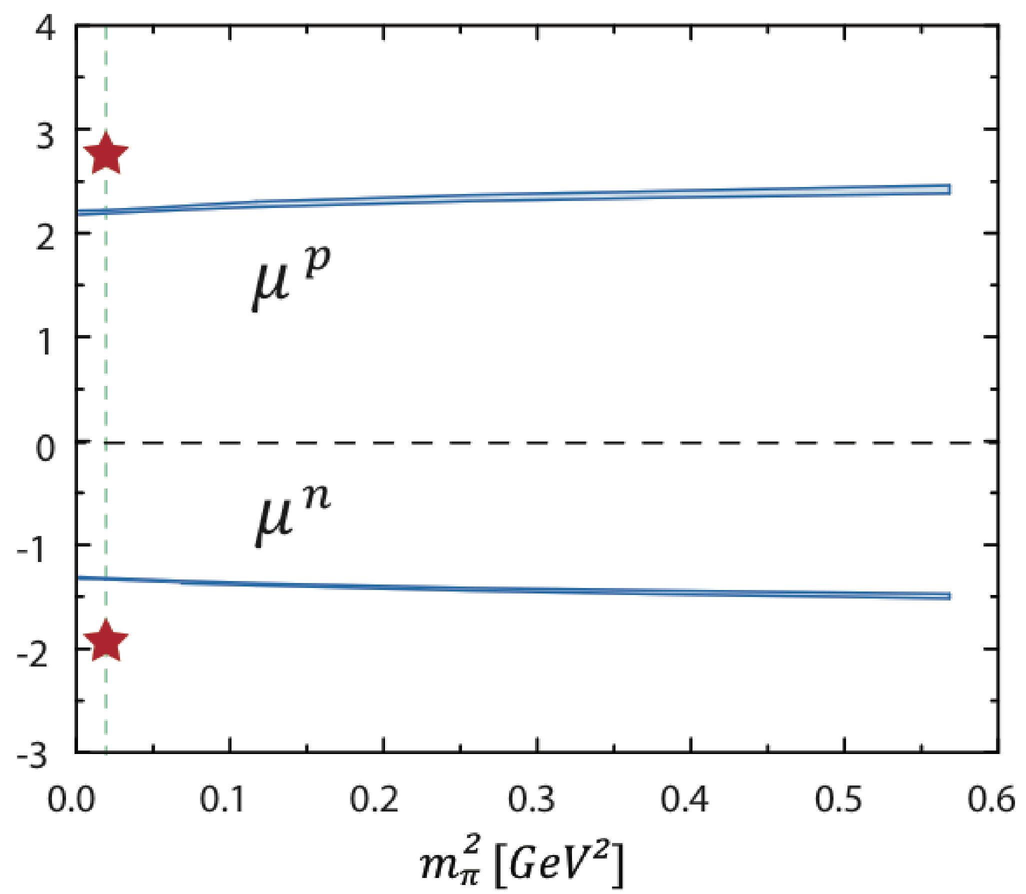
- missing pion cloud effects
- similar for axial form factors

Eichmann, PRD 84 (2011)

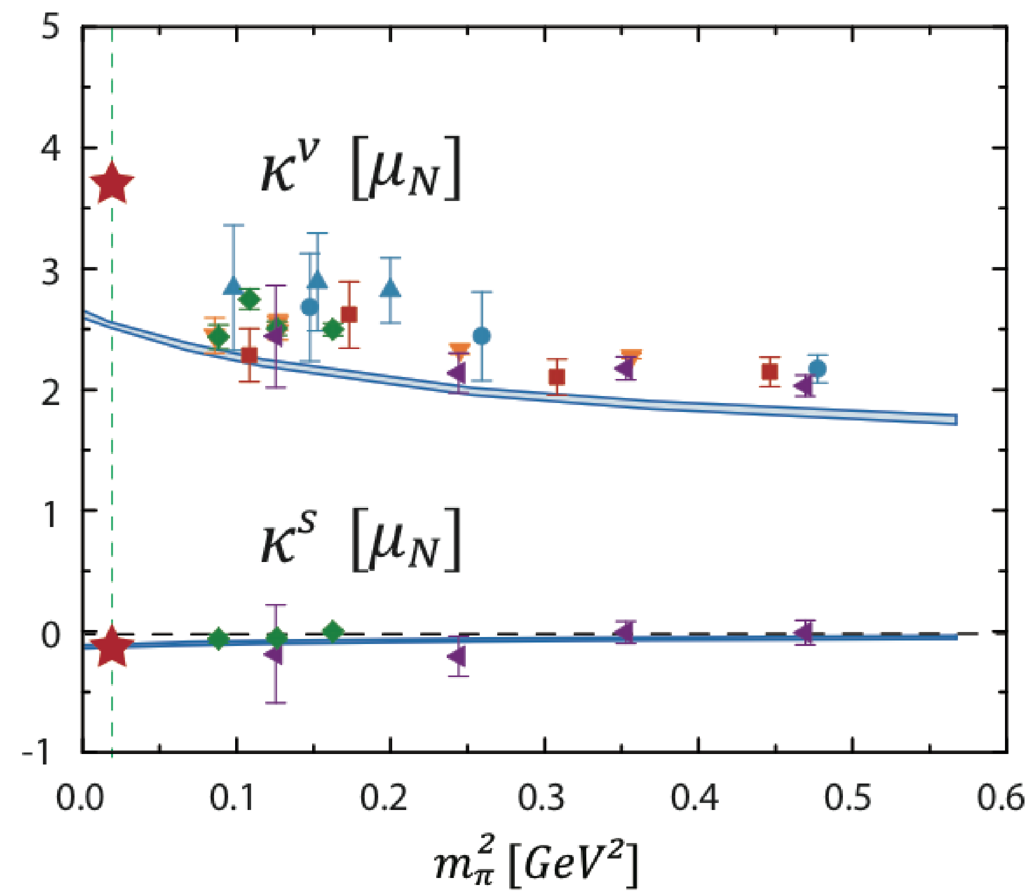
Eichmann and CF, EPJ A48 (2012) 9

Magnetic moments

Magnetic moments (p, n):



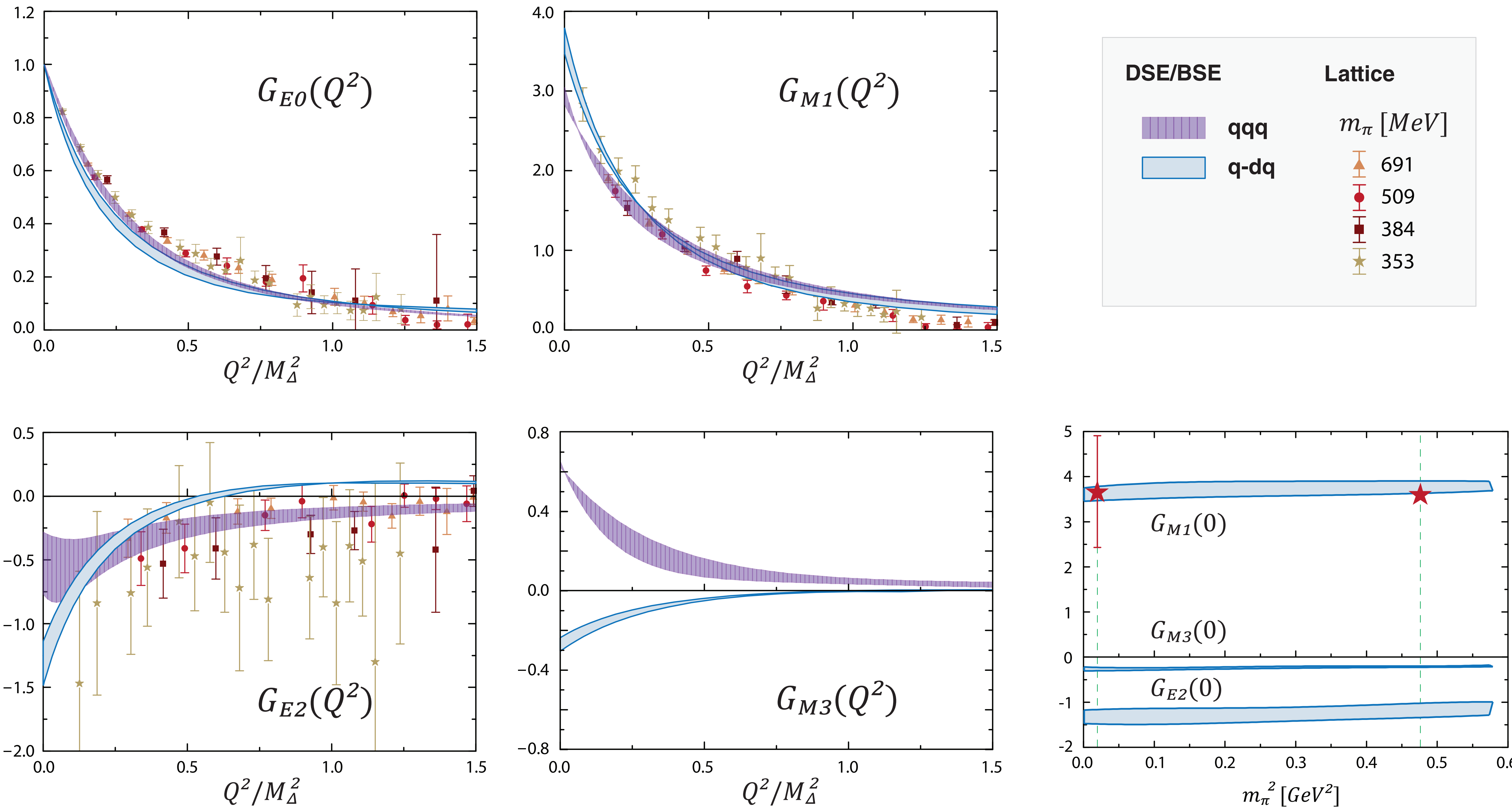
Isovector (p-n), isoscalar (p+n):



- missing pion cloud effects in isovector moment κ^v
- no pion cloud effects in isoscalar moment κ^s

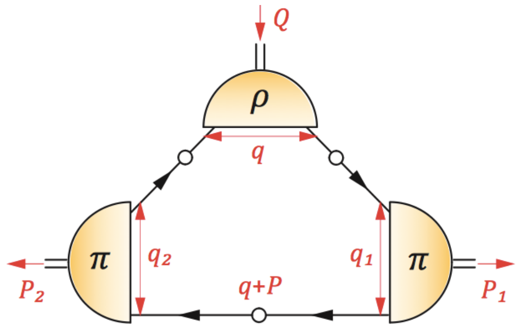
Eichmann, PRD 84 (2011)

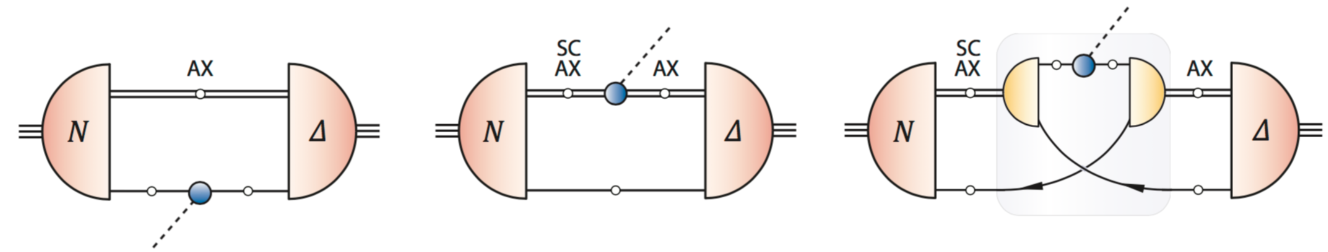
Δ-form factors

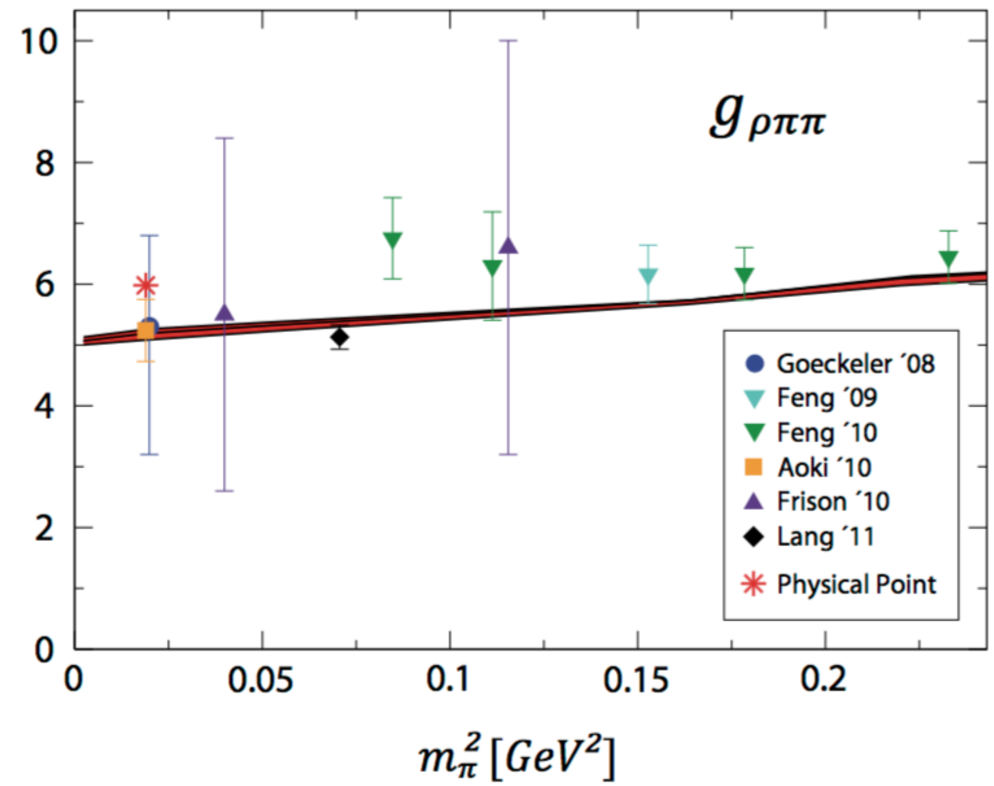


- may serve to distinguish between qqq and q-dq !

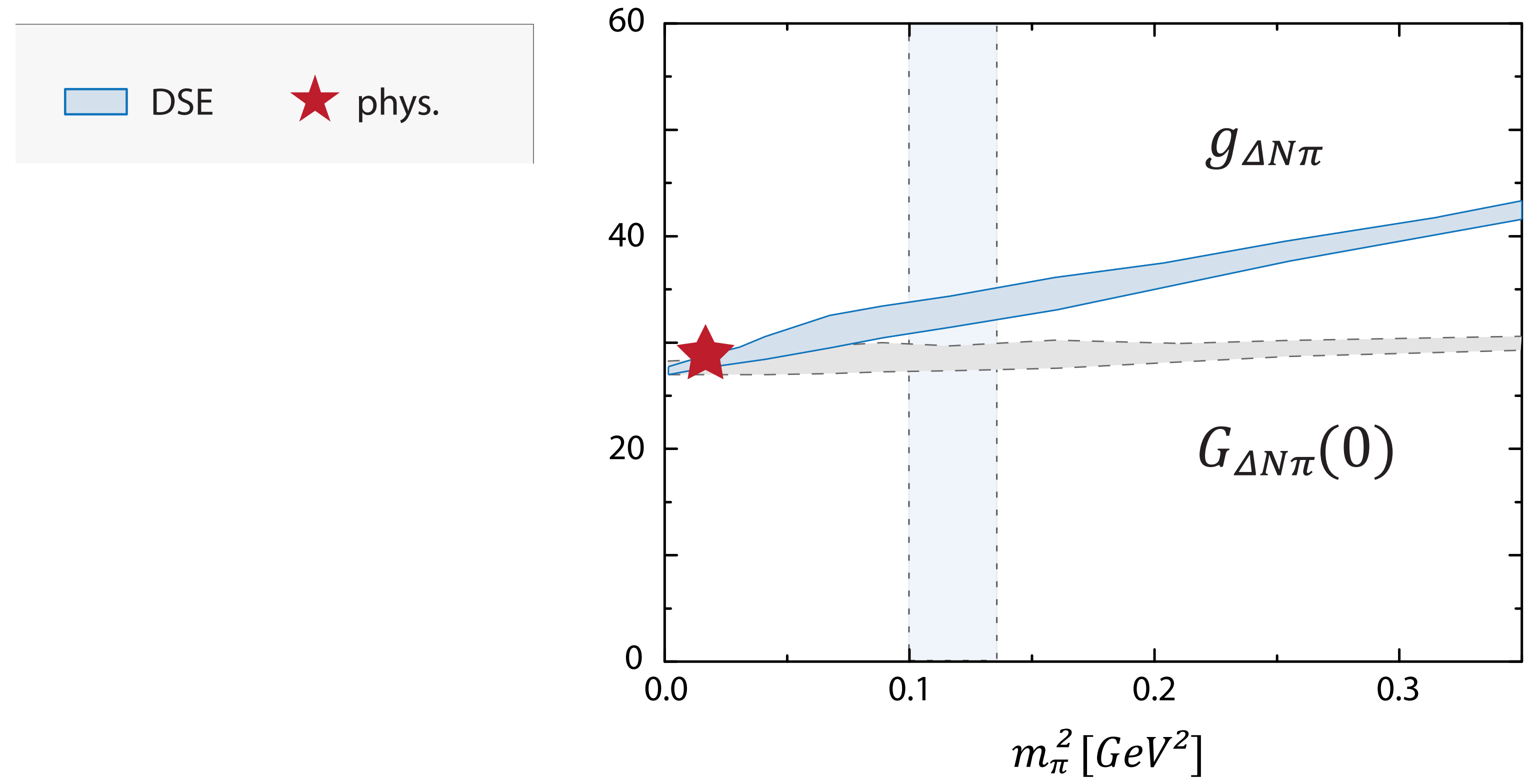
Sanchis-Alepuz, Williams, Alkofer, PRD87 (2013)
Nicmorus, Eichmann, Alkofer, PRD82 (2010)







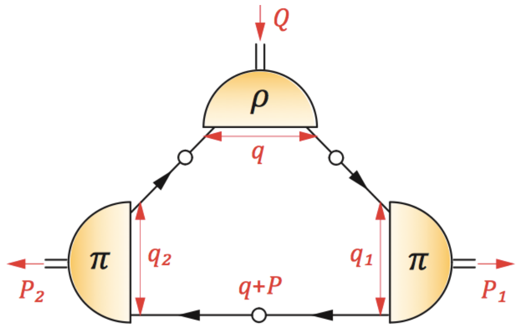
$$g_{\Delta N\pi} = G_{\Delta N\pi}(Q^2 = -m_\pi^2)$$

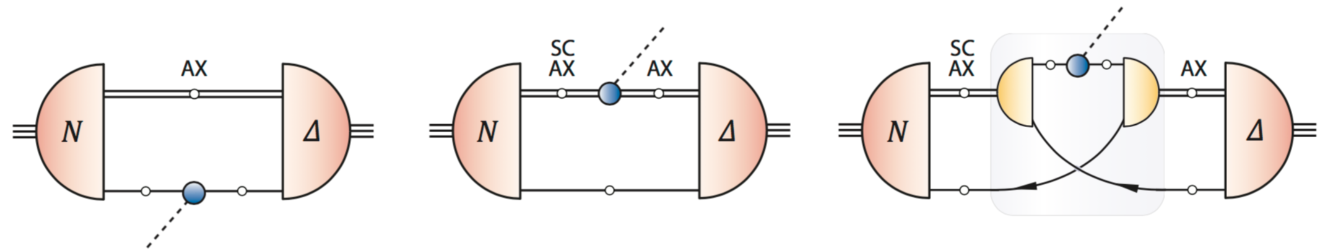


Mader, Eichmann, Blank, Krassnigg PRD84 (2011)

- Decay constants approx. correct in rainbow-ladder
  (although bound states have no width)
- Good agreement with lattice and experiment

Decays: ρππ and ΔNπ





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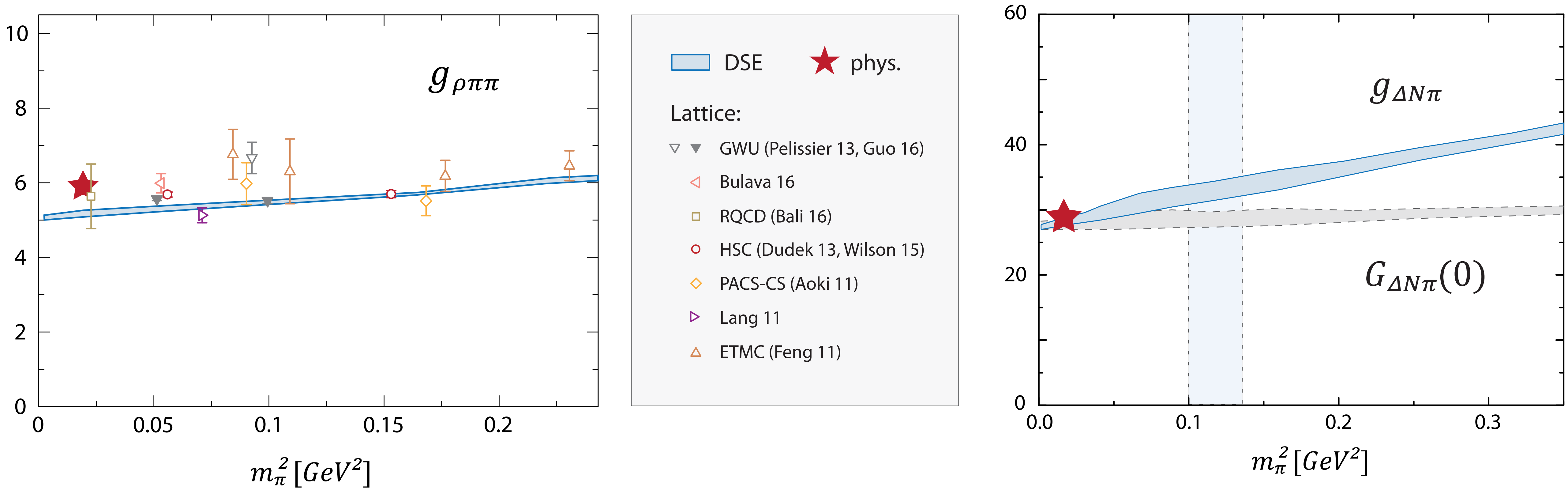

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Summary: Hadron physics with functional methods

Main goals:

- one framework for all areas of hadron physics: mesons, baryons, 'exotic states', form factors, hadronic contributions to standard model
- access to DXSB, confinement,…

Main challenge:

- systematic control over error budget: intrinsic + cp to other methods like lattice QCD

Main results:

- NOT high precision physics
- BUT competitive contributions in many areas of hadron physics