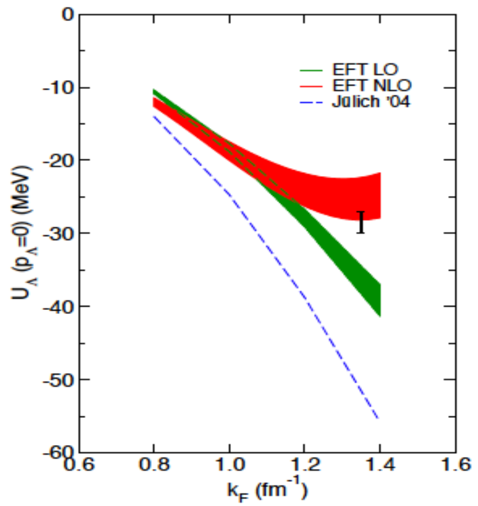

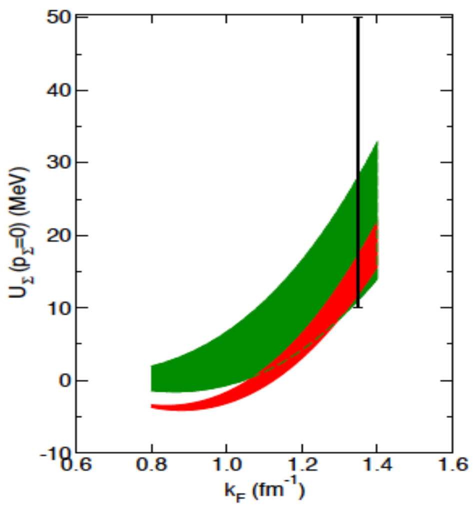

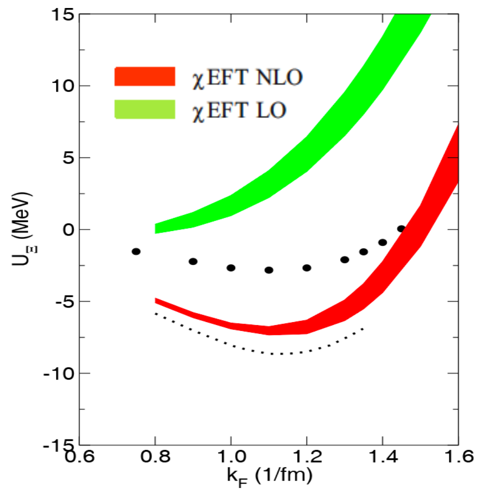

Hyperons in Matter

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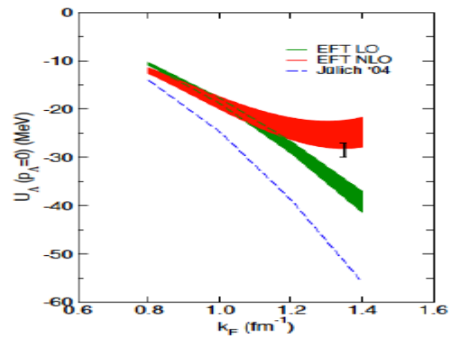
Outline

- Why hyperons in matter?
- Brueckner-Hartree-Fock approach
  - Many-body problem
  - Goldstone Theorem
  - Brueckner-Goldstone Theory: The Bethe-Goldstone Equation
- Lowest order Brueckner Theory
- Hyperons in matter
- Bibliography

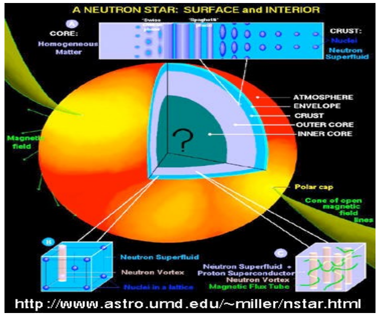
Why hyperons in matter?

To understand..

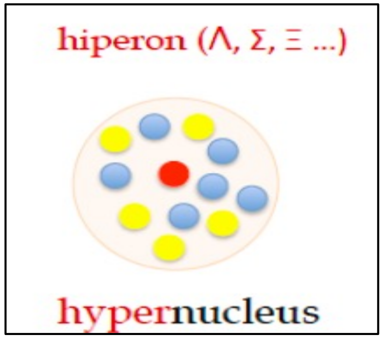


- hadron-hadron interaction in matter

- optical potential of hadrons in matter



- equation of state for hadronic matter: neutron star matter



- nuclear and hypernuclear structure: hypernuclei

- ….

Brueckner-Hartree-Fock approach

Let's start with NN: Many-body problem

• NUCLEAR MATTER is hypothetical system with the same number of protons and neutrons, which fill out the whole space with a uniform density. It is a dilute system: the range of the repulsive "core" (a ~ 0.4 - 0.5 fm) is much less than the distance among nucleons (r_0 ~ 1-2 fm), → $a/r_0 \sim 1/3$

$$\rho = \frac{A}{V} = \frac{A}{\frac{4\pi}{3} r_0^3 A} = 0.17 \text{ fm}^{-3}$$

$$\rho = \frac{A}{V} = 4 \int \frac{d^3k}{(2\pi)^3} \theta(k_F - k) = \frac{2}{3\pi^2} k_F^3$$ (k_F=1.36 fm^{-1})

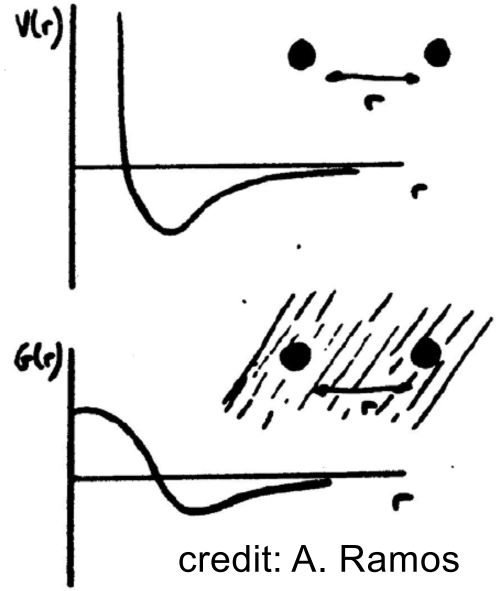
• One of the main difficulties in nuclear many-body systems (finite nuclei and nuclear matter) arises from the fact that baryon-baryon interaction V is repulsive at short distances and, hence, any expansion in terms of V becomes meaningless. A description in terms of a model of independent particles seems "a priori" not reasonable!

• However, the shell model reproduces a great number of nuclear properties under the assumption that each nucleon moves in a single orbital and its dynamics is independent of other surrounding nucleons.

• The success in describing the properties of nucleons with a model of independent particles even if NN is repulsive at short distances induces to think that the NN interaction is shielded when nucleons are submerged in nuclear matter.

• The Brueckner-Goldstone Theory provides the scheme to obtain the effective NN interaction starting from the bare potential V by resuming ladder diagrams. The effective interaction is called G-matrix (or Brueckner reaction matrix) and it is deduced from the Bethe-Goldstone Equation

B. D. Day, Reviews of Modern Physics, Vol. 39, 719 (1967); Vol. 50, 495 (1978)



credit: A. Ramos

DIAGRAM FOR INTERACTIONS

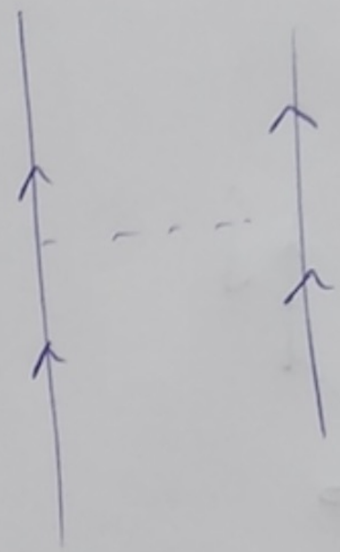


DIAGRAM FOR SEL-ENERGY

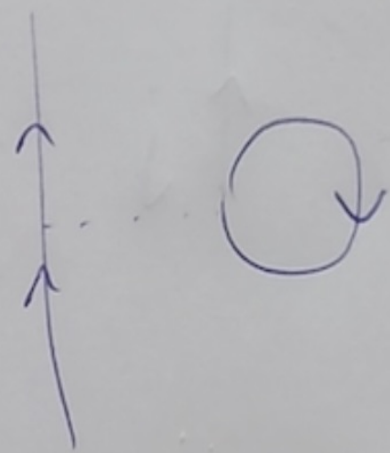
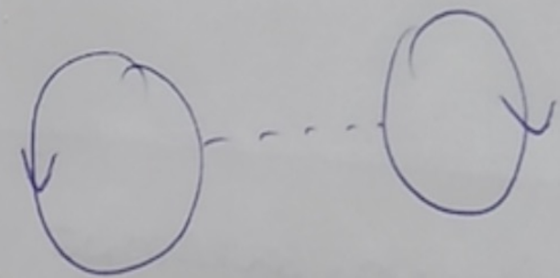


DIAGRAM FOR ENERGY OF THE SYSTEM



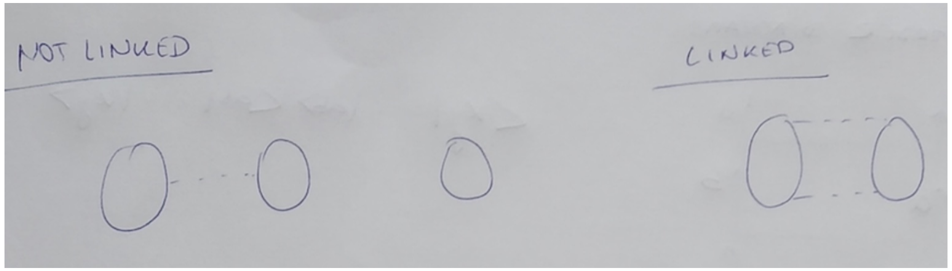
Goldstone Theorem

The starting point of the Brueckner-Goldstone Theory is the expression of the energy of the fundamental state of an interacting many-body system deduced by Goldstone, Goldstone Theorem:

$$E = E_0 + \langle \Phi_0 | H_1 \sum_{n=0}^{\infty} \left[\frac{1 - |\Phi_0\rangle\langle\Phi_0|}{E_0 - H_0} H_1 \right]^n |\Phi_0\rangle_l$$

where H_0 is the free Hamiltonian (or one-body interactions at most)
H_1 is the two-body interaction Hamiltonian
Φ_0 is the fundamental state without correlations
E_0 is the energy of the fundamental state without correlations
$1 - |\Phi_0\rangle\langle\Phi_0|$ is the Pauli operator

and "l" indicates that ***only linked diagrams*** contribute to the Goldstone expansion, i.e., those diagrams which cannot be separated in two pieces by a vertical cut which would not cross any line



Example: Second-order expansion

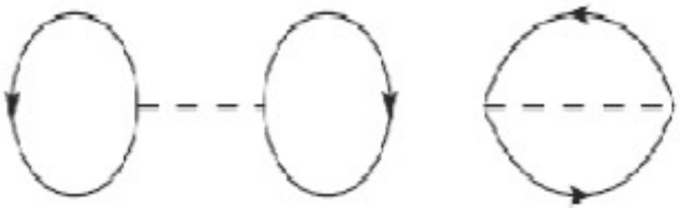
$$E - E_0 = \langle \Phi_0 \mid \hat{H}_1 \mid \Phi_0 \rangle + \sum_{m \neq 0} \frac{\langle \Phi_0 \mid \hat{H}_1 \mid \Phi_m \rangle \langle \Phi_m \mid \hat{H}_1 \mid \Phi_0 \rangle}{E_0 - E_m}$$

due to translational invariance,
single-particle states are plane waves

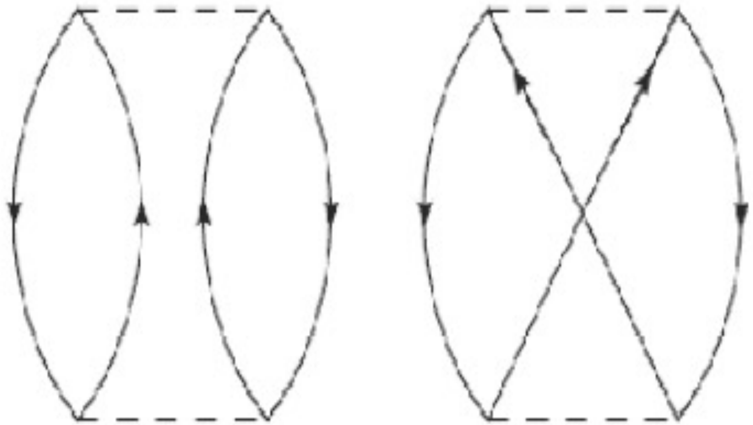
$$|k\rangle = \frac{1}{\sqrt{V}} \mathrm{e}^{i\vec{k}\vec{r}}$$

$$\frac{1}{4} \sum_{k,,j<k_F} \sum_{a,b>k_F} \frac{\langle kj \mid \hat{H}_1 \mid ab - ba \rangle \langle ab - ba \mid \hat{H}_1 \mid kj \rangle}{\varepsilon(k) + \varepsilon(j) - \varepsilon(a) - \varepsilon(b)}$$

$$\langle \Phi_0 \mid \hat{H}_1 \mid \Phi_0 \rangle = \frac{1}{2} \sum_{k,j<k_F} \langle kj \mid \hat{H}_1 \mid kj - jk \rangle$$

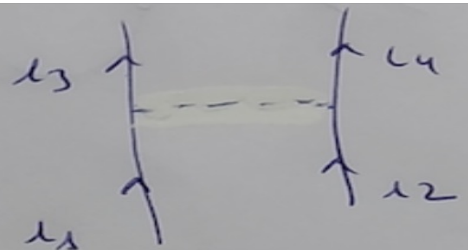


first order

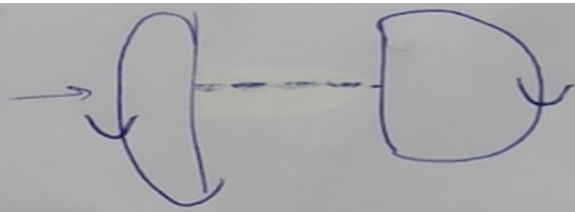


second order

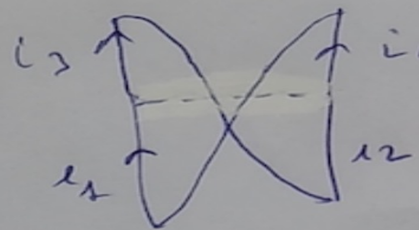
HARTREE
(FIRST ORDER)



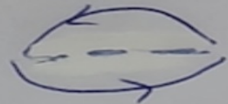
$i_1 = i_3$
$i_2 = i_4$



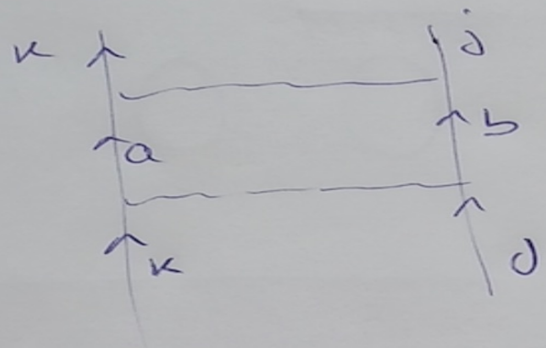
FOCK DIAGRAM
(FIRST ORDER)



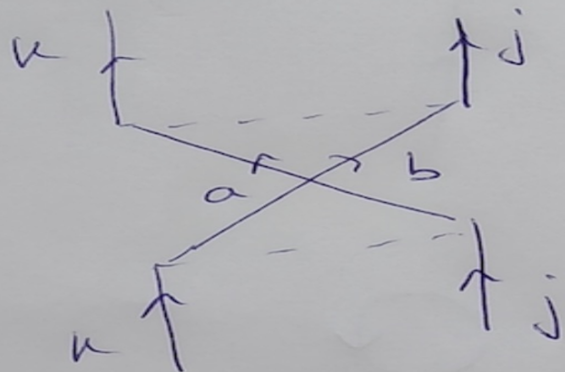
$i_3 = i_2$
$i_1 = i_4$



SECOND ORDER DIAGRAMS

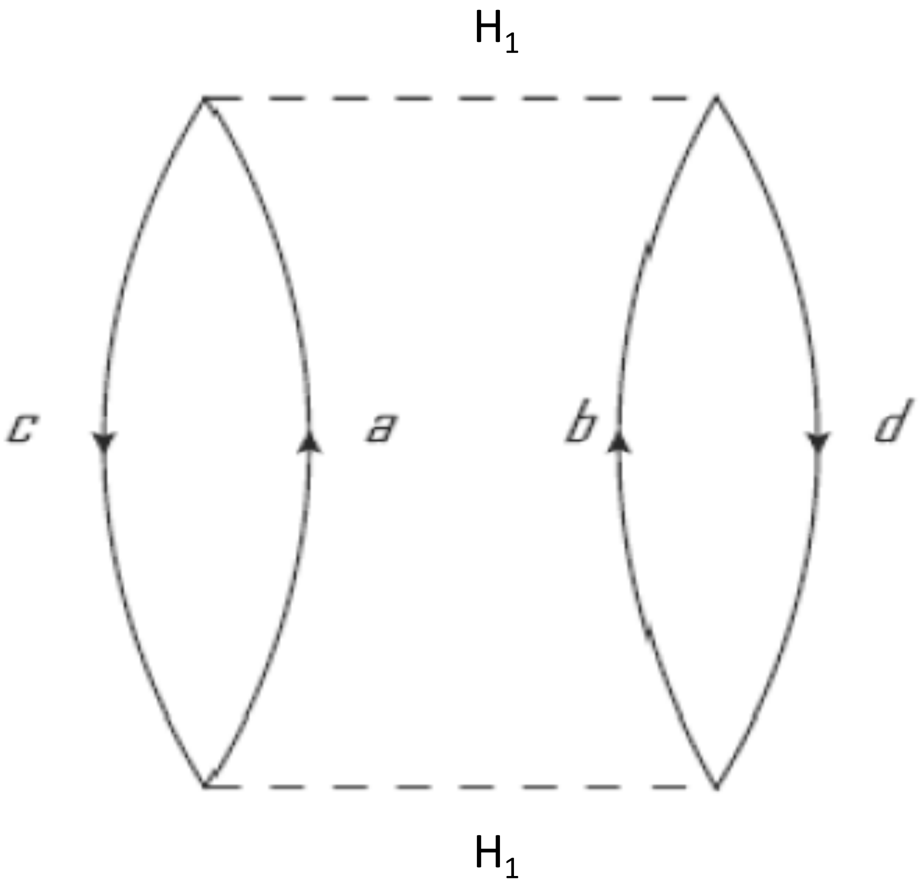


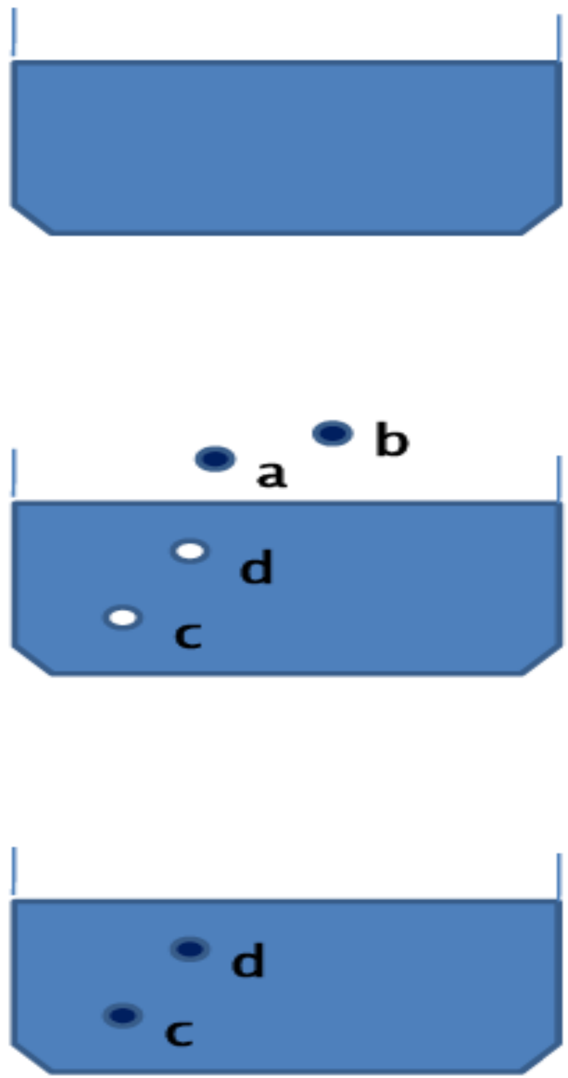
$\langle kj | \ldots | ab \rangle \langle ab | \ldots | kj \rangle$



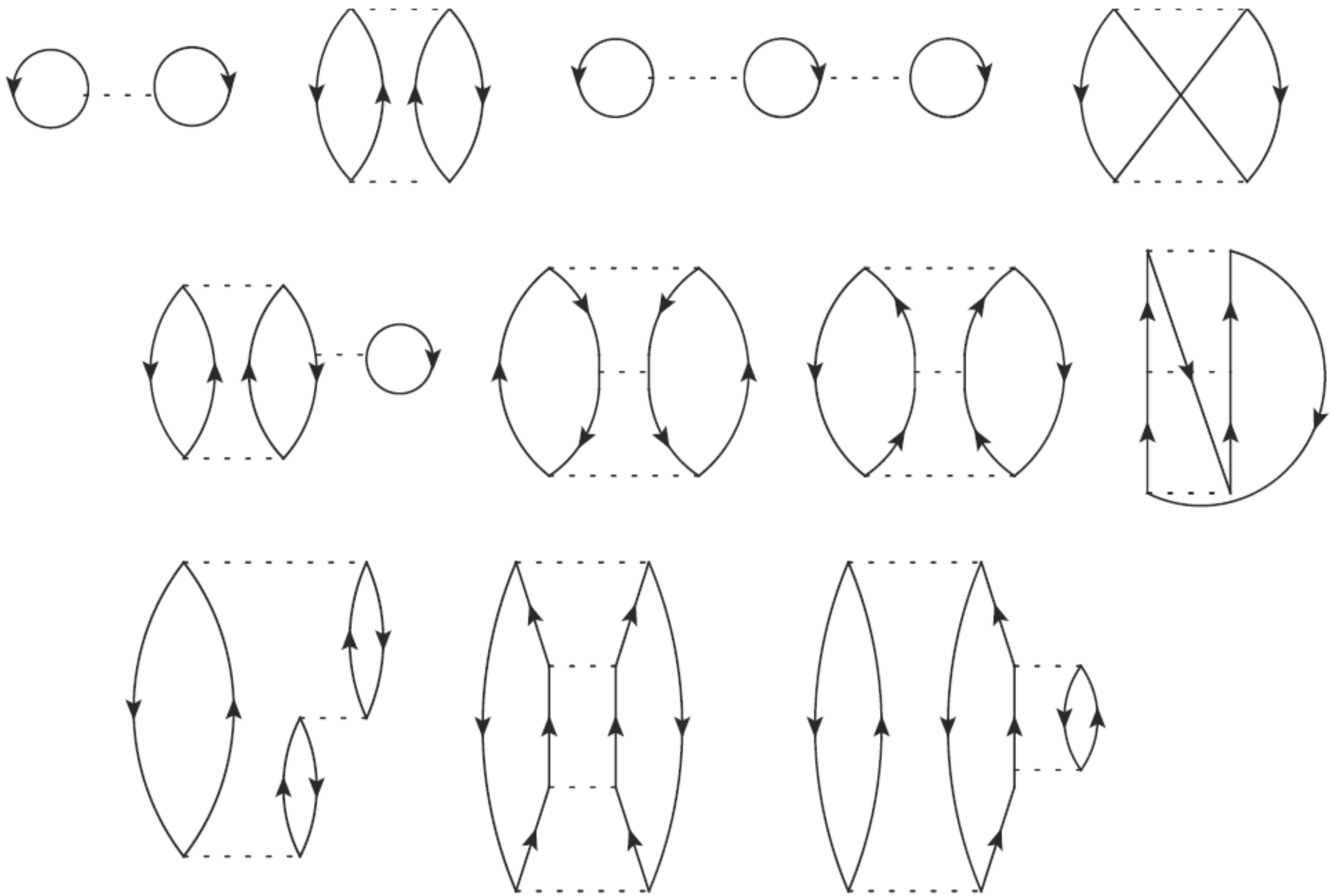
$\langle kj | \ldots | ba \rangle \langle ab | \ldots | kj \rangle$

How to understand those diagrams??





The expansion consists of all possible different linked diagrams:



However, the expansion order by order in V cannot converge because the short-range repulsion in the NN interaction makes all matrix elements very large. Which is the solution???

Brueckner-Goldstone Theory: The Bethe-Goldstone Equation

The solution is given within the Brueckner-Goldstone Theory which resums partially the most relevant terms up to infinity

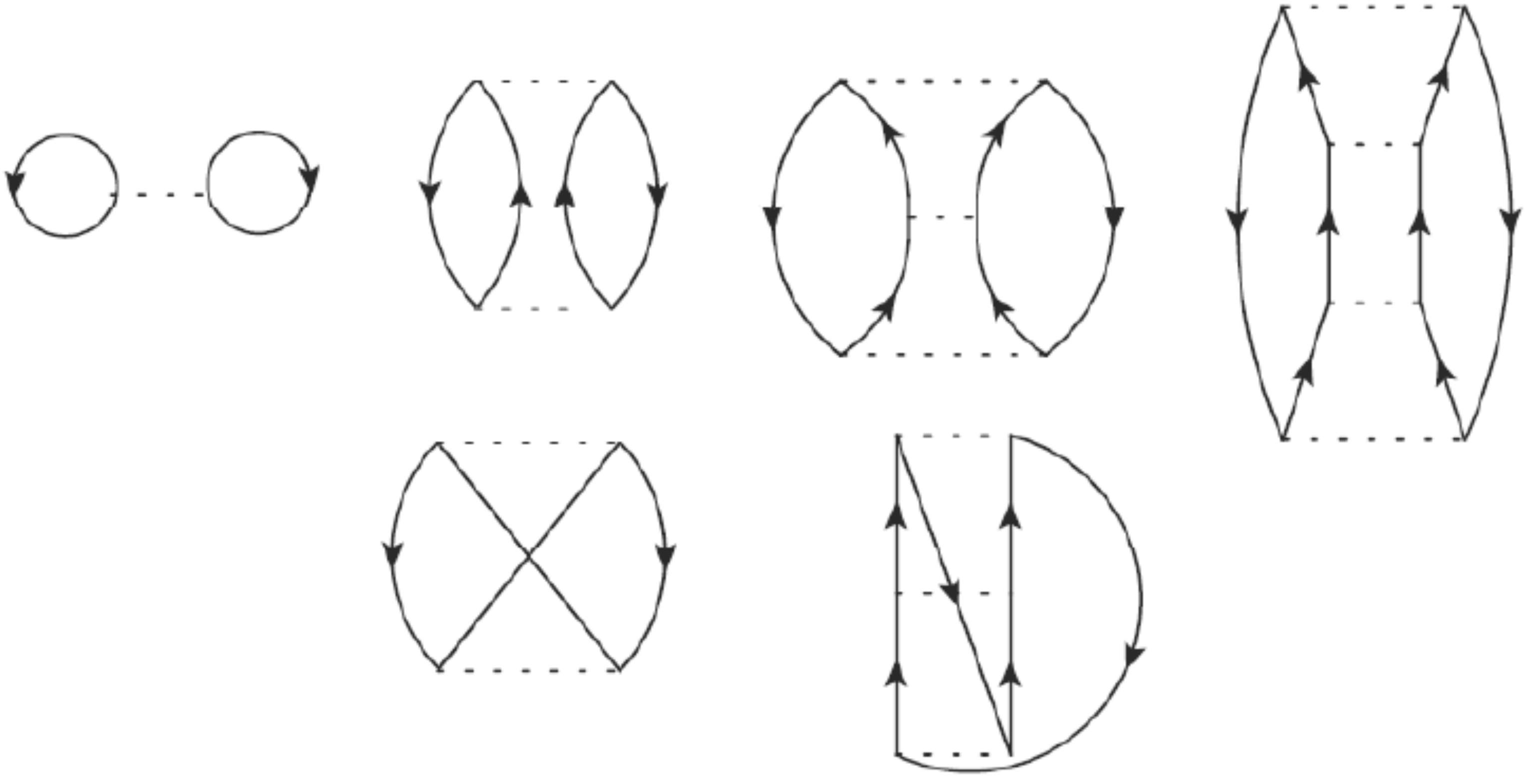
How to select them??

Nuclear matter is a dilute system (a/r_0 <<1, $a \bullet k_F$ <<1).

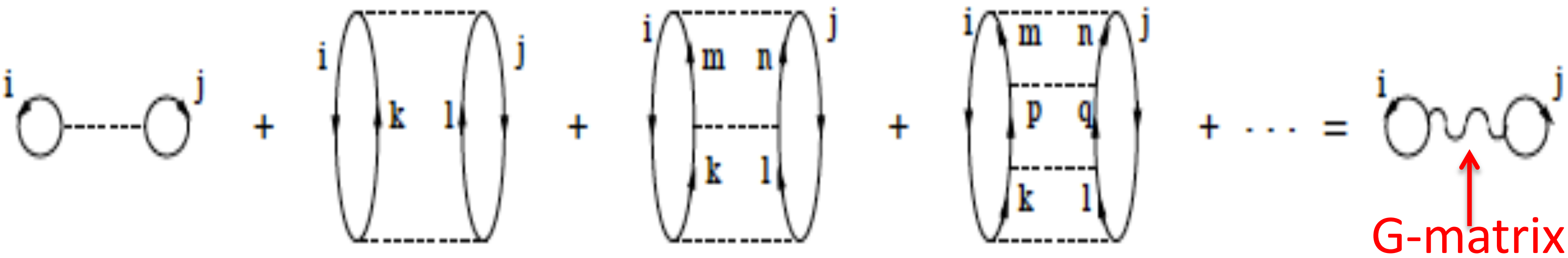
For each diagram, each "hole" line implies an integral over momentum up to k_F while each "particle" line implies integrating above k_F. For low density systems (k_F small), the contribution of "hole" line integrals is smaller than the contribution of "particle" line integrals: the more "hole" lines there are, the more subdominant the diagram is for $a \bullet k_F$ <<1

Diagrams are ordered according to the number of holes and not by number of interactions. The dominant diagrams in the perturbative expansion for the energy (and other observables) are those with less "hole" lines. Those form the ladder expansion.

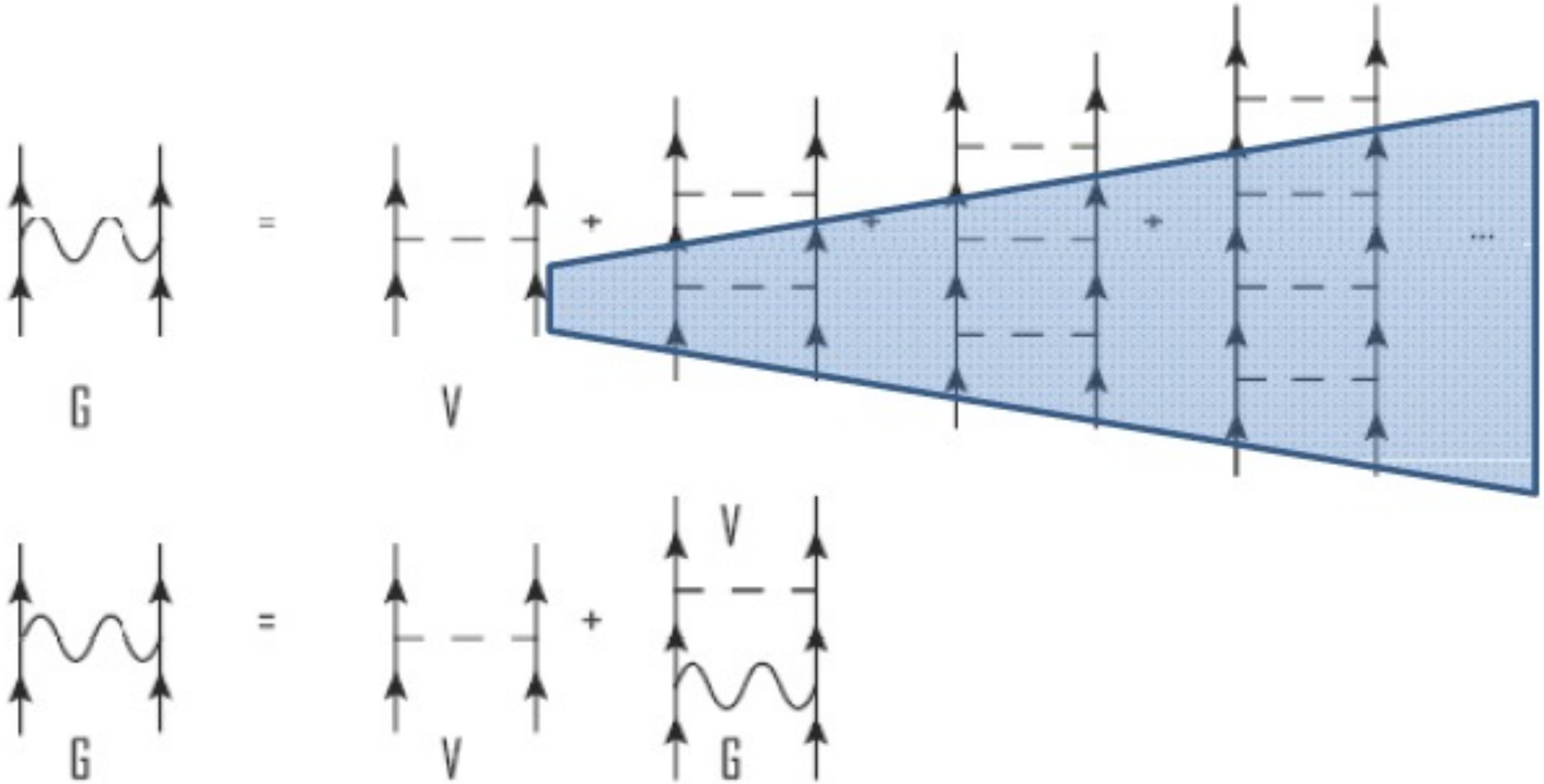
Some ladder diagrams with two-hole lines:



which form part of the ladder series for the effective interaction or G-matrix



The G-matrix (or Brueckner reaction matrix)
is obtained by solving the Bethe-Goldstone Equation



$$G = V + V\frac{Q_{\text{pauli}}}{E_0 - H_0}G$$

Bethe-Goldstone Equation

Formally one has $$G = V + V\frac{Q_{\text{pauli}}}{E_0 - H_0}G$$

• Q_{Pauli}: it only allows two particles above k_F in the intermediate states. This operator is known as Pauli operator

$$Q_{Pauli}\,|p_1\, p_2> = \begin{cases} |p_1\, p_2> & \text{if } |p_1|\ \&\ |p_2| > k_F \\ 0 & \text{otherwise} \end{cases}$$

• The energy denominator, with E_0 being the energy of the fundamental state without correlations and H_0 is the free Hamiltonian acting on the intermediate state (in this case, 2 particle-2 hole)

E_0-H_0 $|p_1p_2\, h_1h_2>= E_0-[E_0+\varepsilon_{p1}+\varepsilon_{p2}-\varepsilon_{h1}-\varepsilon_{h2})\ |p_1p_2\, h_1h_2>=$
$\varepsilon_{h1}+\varepsilon_{h2}-\varepsilon_{p1}-\varepsilon_{p2}\,|p_1p_2\, h_1h_2>=\omega-\varepsilon_{p1}-\varepsilon_{p2}\,|p_1p_2\, h_1h_2>$, ω: starting energy

Then, the Bethe-Goldstone Equation reads

$$G(\omega) = V + V\sum_{p_1,p_2>k_F}\frac{|p_1p_2><p_1p_2|}{\omega-\epsilon_{p_1}-\epsilon_{p_2}}G(\omega)$$

Brueckner found that the G-matrix could be interpreted as describing the collision of two particles in the presence of a medium. There is, therefore, a parallelism with the Lippman-Schwinger equation for the scattering of two free particles satisfied by the T-matrix:

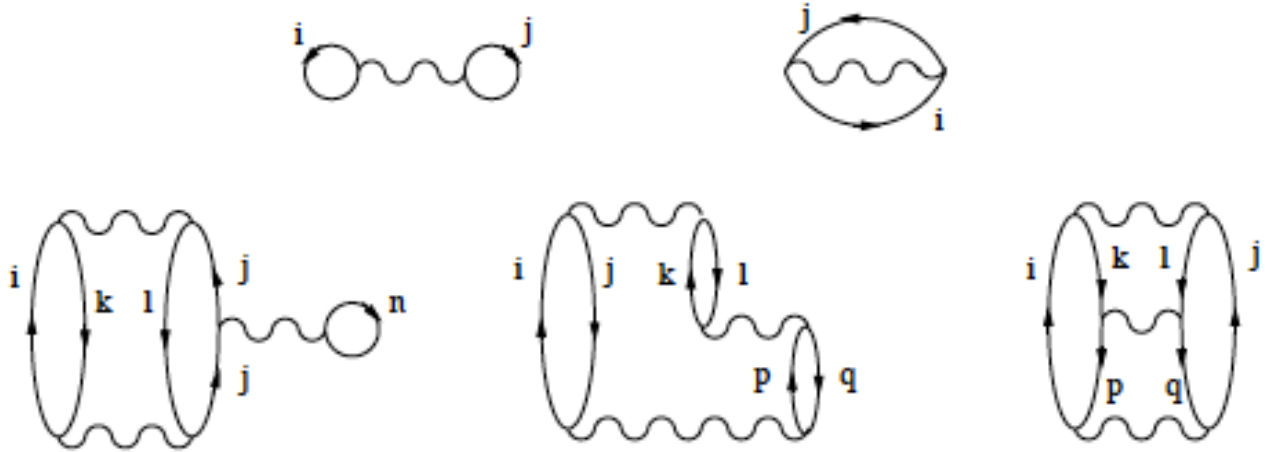
$$T = V + V \frac{1}{E_0 - H_0 + i\eta} T$$

The most important difference lies in the Pauli operator, which indicates that two nucleons interact in the presence of other nucleons and, therefore, there are occupied states not accessible after the interaction.

$$G = V + V \frac{Q_{\mathrm{pauli}}}{E_0 - H_0} G$$

The summation of ladder diagrams has led to a single diagram where the interaction V has been replaced by the reaction matrix G. Now the matrix elements are well-behaved even with short-range repulsion. This suggests to convert each V interaction into a G-matrix line by summing the proper sequences of ladder diagrams avoiding double counting. In this way, one obtains the so-called Brueckner-Goldstone Expansion, in which every term is finite and well behaved.

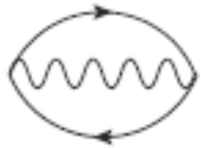
Some of the diagrams appearing in the Brueckner-Goldstone Expansion



How to organize this expansion? The solution was given by Brueckner, Goldstone and Gammel with the Hole-Line Expansion: the diagrams with h (hole) lines being more important than those with h+1 lines (not a theorem!)

Lowest order Brueckner Theory

The leading term of the hole-line expansion for the energy

E = Eo +  + 

$$E/V = \sum_{m<k_F} \langle m \mid T \mid m \rangle + \frac{1}{2} \sum_{m,j<k_F} \langle mj \mid G[\varepsilon(m)+\varepsilon(j)] \mid mj \rangle - \langle jm \mid G[\varepsilon(m)+\varepsilon(j)] \mid mj \rangle$$

$$\langle mn \mid G \mid ij \rangle = \langle mn \mid V \mid ij \rangle + \sum_{p,l>k_F} \langle mn \mid V \mid pl \rangle \frac{1}{\varepsilon(i)+\varepsilon(j)-\varepsilon(p)-\varepsilon(l)} \langle pl \mid G \mid ij \rangle$$

This approximation is also usually called Brueckner-Hartree-Fock (BHF) approximation due to its analogy with Hartree-Fock approximation. The only difference is that the interaction V has been replaced by the Brueckner reaction matrix or G-matrix

Self-consistent calculation of E:

1. Starting from a single-particle spectrum ε(m), one solves the Bethe-Goldstone Equation for G(ω) for a range of ω[=ε(i)+ε(j)]

$$\langle mn \mid G \mid ij \rangle = \langle mn \mid V \mid ij \rangle + \sum_{p,l>k_F} \langle mn \mid V \mid pl \rangle \frac{1}{\varepsilon(i)+\varepsilon(j)-\varepsilon(p)-\varepsilon(l)} \langle pl \mid G \mid ij \rangle$$

1. Calculate single-particle potential U(m)

$$U(m) = \sum_{j<k_F} < mj - jm | G[\epsilon(m)+\epsilon(j)] | mj >$$

2. Adding the kinetic energy to obtain a new single-particle spectrum ε(m)and compare it with the starting one, until both coincide

$$\epsilon(m) =< m|T|m > + \frac{1}{2} U(m)$$

1. Once self-consistency is reached, calculate E (energy density) summing over all single-particle energies up to the Fermi level

$$E/V = \sum_{m<k_F} \left[< m|T|m > + \frac{1}{2} U(m) \right]$$

Let's introduce hyperons: BHF approach with hyperons

- Bethe-Goldstone Equation

credit: I. Vidana

$$G(\omega)_{B_1B_2;B_3B_4} = V_{B_1B_2;B_3B_4} + \sum_{B_5B_6} V_{B_1B_2;B_5B_6} \frac{Q_{B_5B_6}}{\omega - E_{B_5} - E_{B_6} + i\eta} G(\omega)_{B_5B_6;B_3B_4}$$

Pauli blocking

Particle dressing

- Single-particle energy & single-particle potential

$$E_{B_i}(k) = M_{B_i}c^2 + \frac{\hbar^2 k^2}{2M_{B_i}^2} + \mathrm{Re}\left[U_{B_i}(k)\right]$$

$$U_{B_i}(k) = \sum_{B_j} \sum_{k \le k_{F_{B_j}}} \left\langle \vec{k}_i \vec{k}_j \right| G\left(\omega = E_{B_i} + E_{B_j}\right) \left| \vec{k}_i \vec{k}_j \right\rangle$$

- Energy density

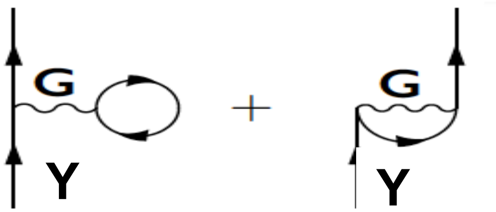
$$\varepsilon = 2\sum_{B_i} \int_0^{k_{F_{B_i}}} \frac{d^3k}{(2\pi)^3} \left[M_{B_i}c^2 + \frac{\hbar^2 k^2}{2M_{B_i}} + \frac{1}{2}\mathrm{Re}[U_{B_i}^N] + \frac{1}{2}\mathrm{Re}[U_{B_i}^Y] \right]$$

Self-consistency in coupled channels!!!

	S = 0	S = -1	S = -2	S = -3	S = -4
I = 0	$(NN \rightarrow NN)$		$\begin{pmatrix} \Lambda\Lambda \rightarrow \Lambda\Lambda & \Lambda\Lambda \rightarrow \Xi N & \Lambda\Lambda \rightarrow \Sigma\Sigma \\ \Xi N \rightarrow \Lambda\Lambda & \Xi N \rightarrow \Xi N & \Xi N \rightarrow \Sigma\Sigma \\ \Sigma\Sigma \rightarrow \Lambda\Lambda & \Sigma\Sigma \rightarrow \Xi N & \Sigma\Sigma \rightarrow \Sigma\Sigma \end{pmatrix}$		$(\Xi\Xi \rightarrow \Xi\Xi)$
I = 1/2		$\begin{pmatrix} \Lambda N \rightarrow \Lambda N & \Lambda N \rightarrow \Sigma N \\ \Sigma N \rightarrow \Lambda N & \Sigma N \rightarrow \Sigma N \end{pmatrix}$		$\begin{pmatrix} \Lambda\Xi \rightarrow \Lambda\Xi & \Lambda\Xi \rightarrow \Sigma\Xi \\ \Sigma\Xi \rightarrow \Lambda\Xi & \Sigma\Xi \rightarrow \Sigma\Xi \end{pmatrix}$	
I = 1	$(NN \rightarrow NN)$		$\begin{pmatrix} \Xi N \rightarrow \Xi N & \Xi N \rightarrow \Lambda\Sigma & \Xi N \rightarrow \Sigma\Sigma \\ \Lambda\Sigma \rightarrow \Xi N & \Lambda\Sigma \rightarrow \Lambda\Sigma & \Lambda\Sigma \rightarrow \Sigma\Sigma \\ \Sigma\Sigma \rightarrow \Xi N & \Sigma\Sigma \rightarrow \Lambda\Sigma & \Sigma\Sigma \rightarrow \Sigma\Sigma \end{pmatrix}$		$(\Xi\Xi \rightarrow \Xi\Xi)$
I = 3/2		$(\Sigma N \rightarrow \Sigma N)$		$(\Sigma\Xi \rightarrow \Sigma\Xi)$	
I = 2			$(\Sigma\Sigma \rightarrow \Sigma\Sigma)$		

credit: I. Vidana

Hyperons in matter



$$G = V + V\frac{Q_{\text{pauli}}}{E_0 - H_0}G$$

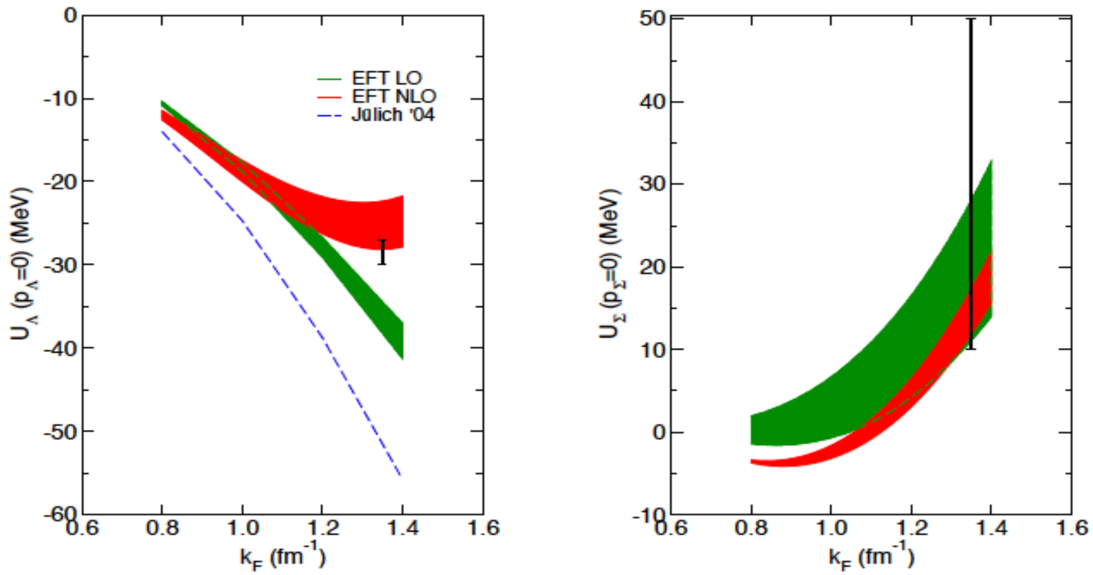
Y in matter within χEFT:

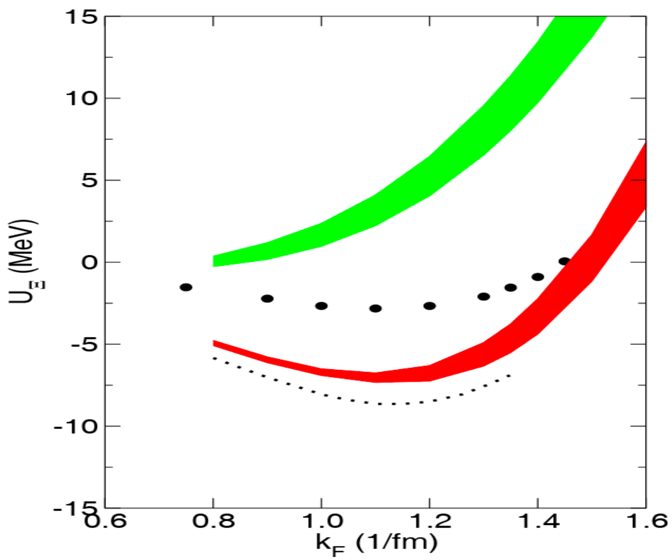
Λ binding in nuclear matter
~-30 to -40 MeV
(exp ≈ -30 MeV)

Repulsion for Σ in matter
(exp repulsive)

Moderately attractive Ξ binding
~ -3 to -5 MeV
(exp ≈ -14, -24 MeV)

Haidenbauer and Meißner , NPA 936 (2015) 29

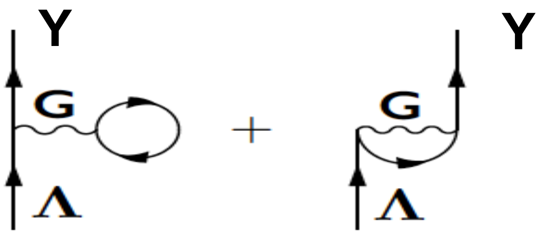




Haidenbauer and Meißner EPJA 55 (2019) 23

Hyperons in matter

Recent results on Λ in dense matter



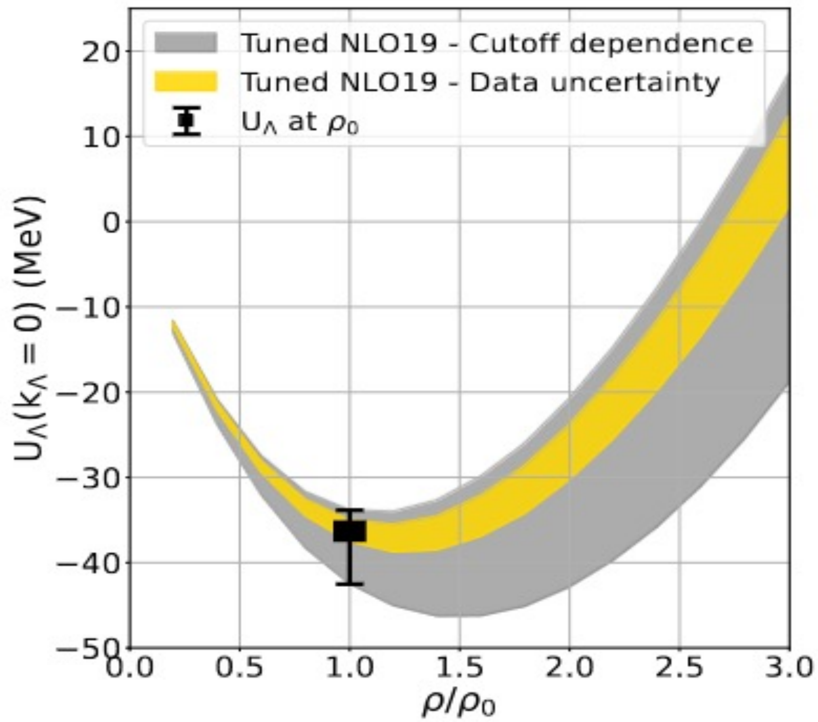
$$G = V + V \frac{Q_{\mathrm{pauli}}}{E_0 - H_0} G$$

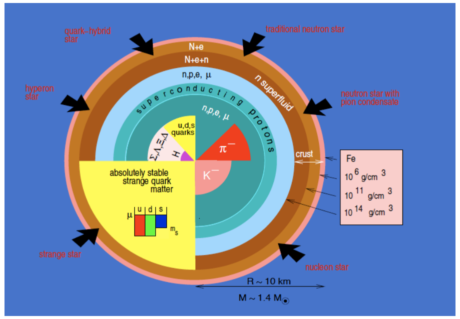
with new parametrization from combined analysis of scattering data and correlation functions

need of additional repulsion acting on Λ, such as, **3-body forces?**

important for neutron stars!!

Mihaylov, Haidenbauer and Mantovani-Sarti
PLB 850 (2024) 138550

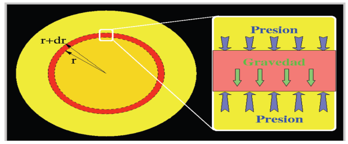




The Hyperon Puzzle in Neutron Stars

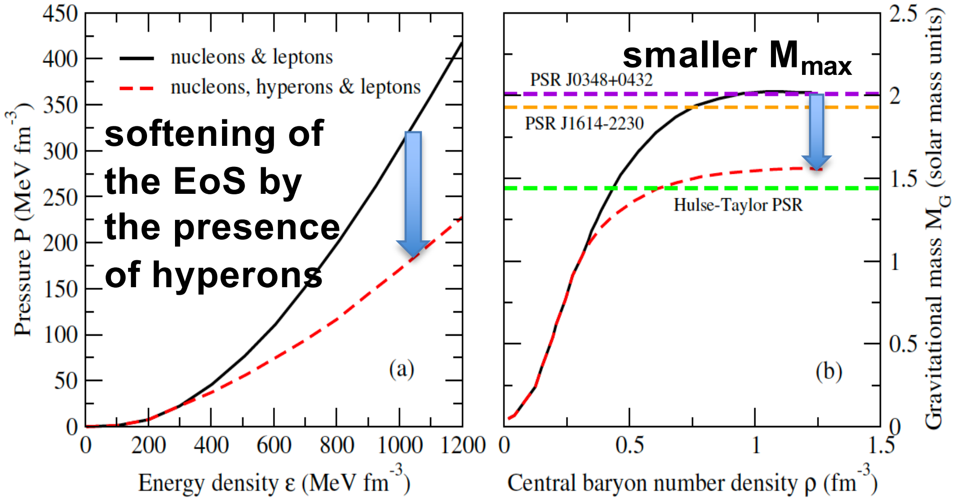
The **presence of hyperons in neutron stars is energetically probable** as density increases. However, it induces a strong softening of the EoS that **leads to maximum neutron star masses < $2M_\odot$**

credit: D. Page



Solution?

- stiffer YN and YY interactions
- **hyperonic 3-body forces**
- push of Y onset by Δ-isobars or meson condensates
- quark matter below Y onset
- dark matter, modified gravity theories…



credit: I. Vidana

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