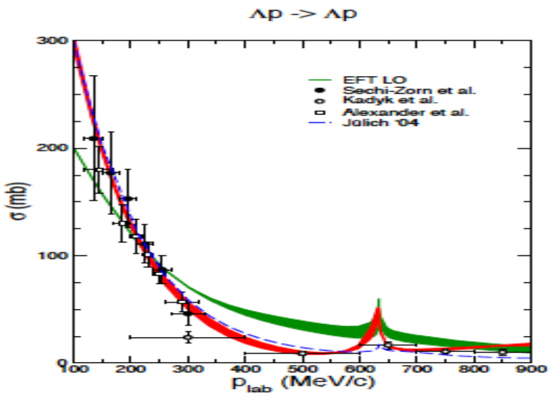

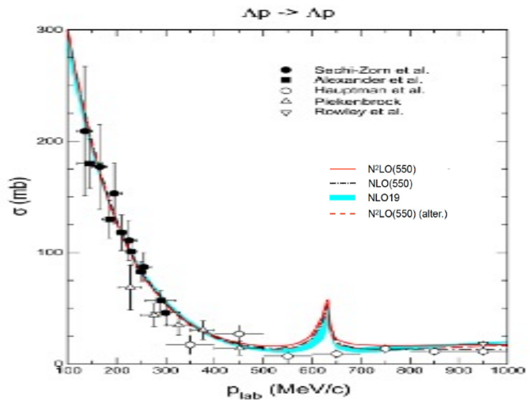

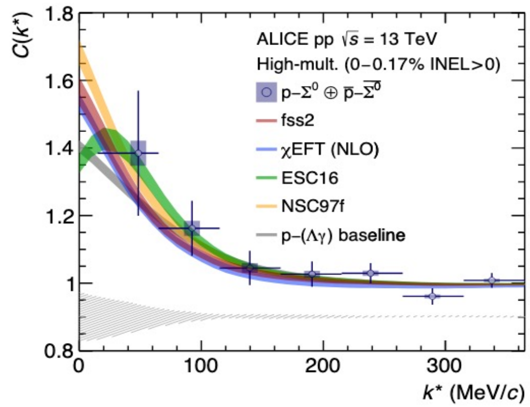

Interactions with Hyperons

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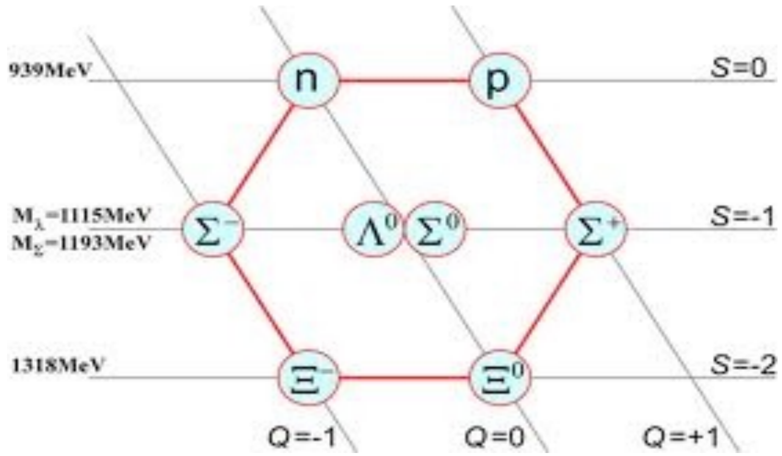
CSIC
Consejo Superior de Investigaciones Científicas
IEEC

Outline

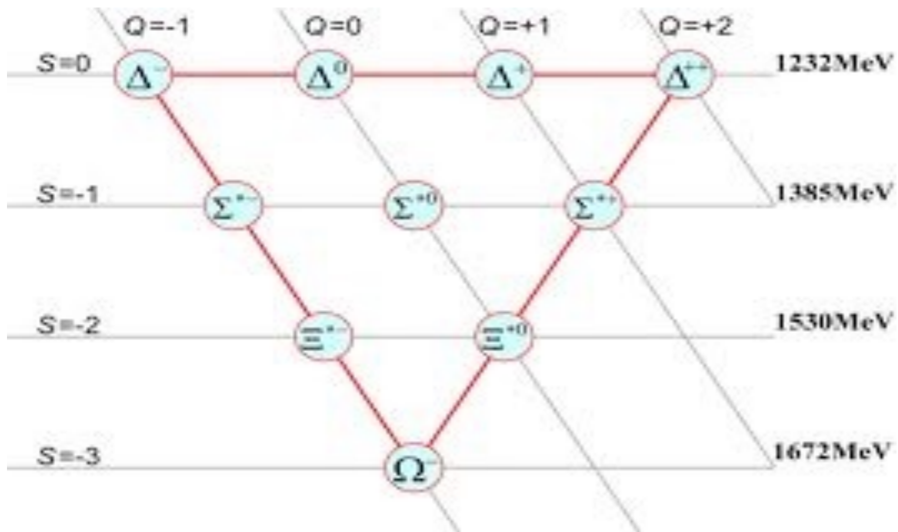
- Hyperons and where to find them
- YN and YY interactions
- Theoretical approaches to YN and YY
- YN (and YY) in meson-exchange models
- YN (and YY) in χEFT
- Bibliography

Hyperons and where to find them

A hyperon is a baryon containing one or more strange quarks



Hyperon	Mass (MeV/c^2)
Λ	1115.57 ± 0.06
Σ^+	1189.37 ± 0.06
Σ^0	1192.55 ± 0.10
Σ^-	1197.50 ± 0.05
Ξ^0	1314.80 ± 0.8
Ξ^-	1321.34 ± 0.14
Ω^-	1672.43 ± 0.14



On Earth: Hypernuclei

A hyperon is a baryon containing one or more strange quarks

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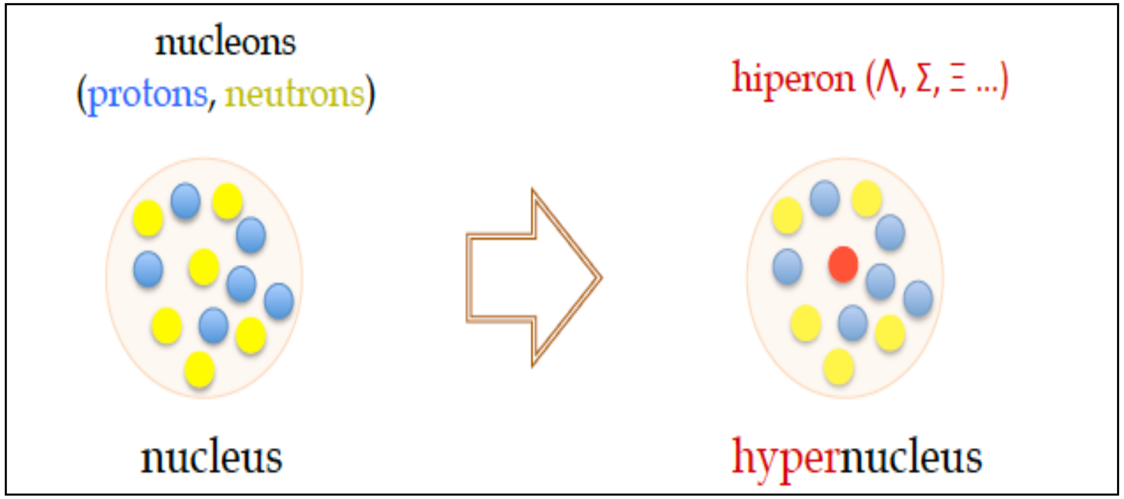
Ξ^0	1314.80 ± 0.8
Ξ^-	1321.34 ± 0.14
Ω^-	1672.43 ± 0.14

The study of hypernucleus allows for

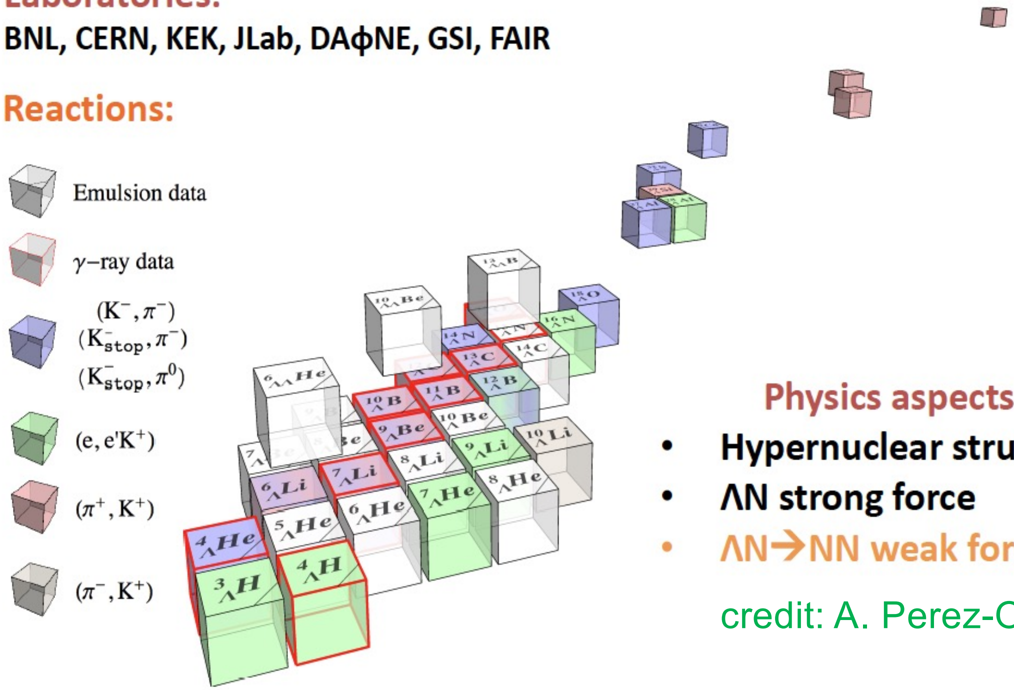
- new spectroscopy
- information on strong and weak interactions between hyperons and nucleons



credit: A. Parreno

Laboratories:
BNL, CERN, KEK, JLab, DAΦNE, GSI, FAIR

Reactions:

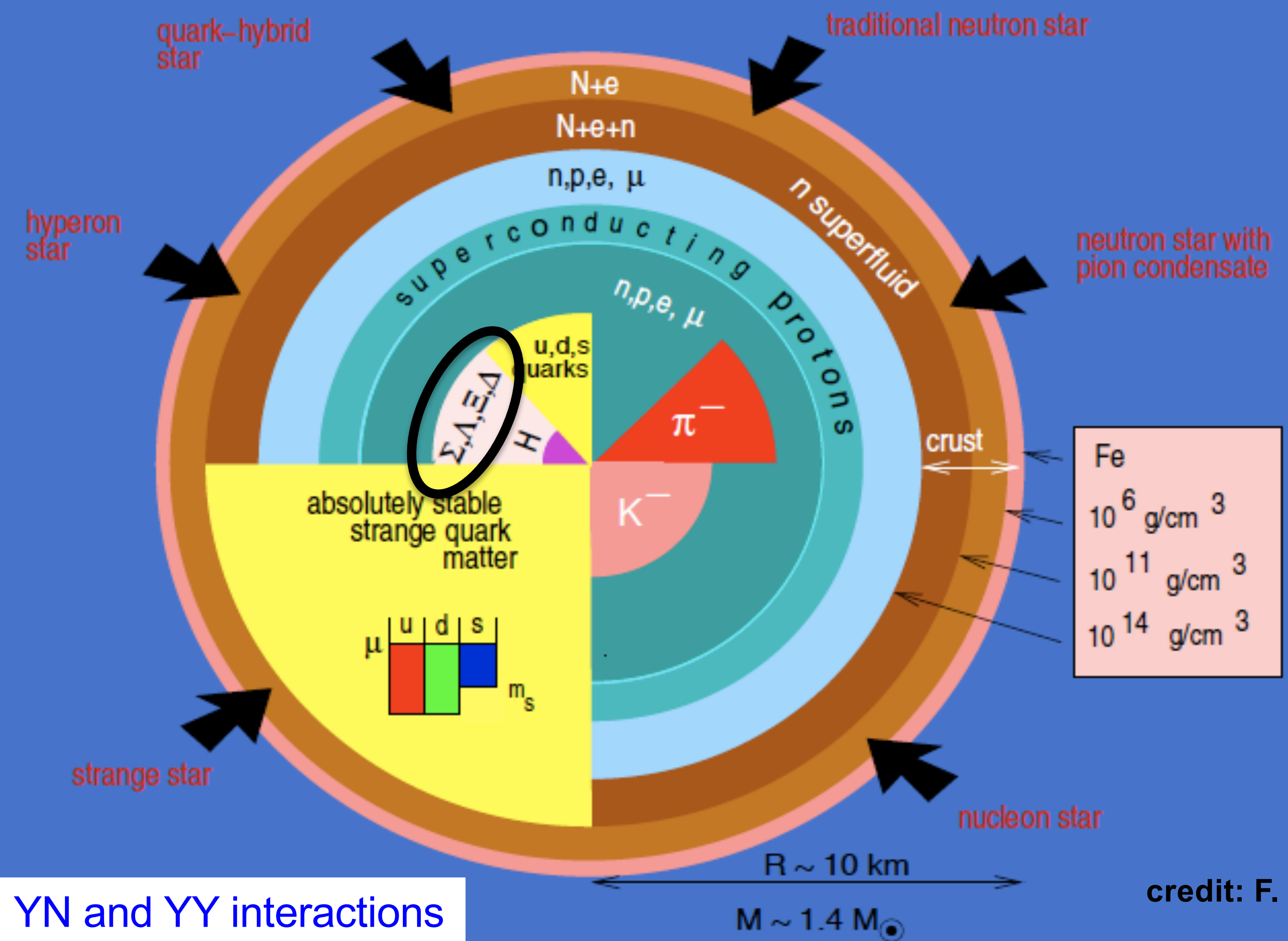


Physics aspects

- **Hypernuclear structure**
- **ΛN strong force**
- **ΛN→NN weak force**

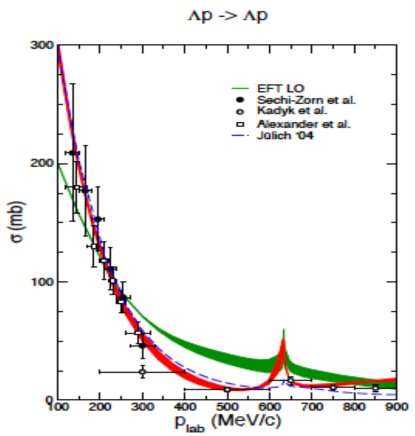
credit: A. Perez-Obiol

In Neutron Stars

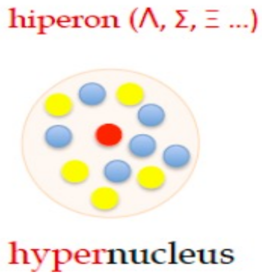


YN and YY interactions

credit: F. Weber



YN and YY interactions



- Study strangeness in nuclear physics
- Provide input for hypernuclear physics and astrophysics

Scarce YN scattering data due to the short life of hyperons and the low-density beam fluxes

ΛN and ΣN: < 50 data points
ΞN very few events

NN: > 5000 data
for E_{lab}<350 MeV

Data from hypernuclei:

- more than 40 Λ-hypernuclei (ΛN attractive)
- few ΛΛ- hypernuclei ($\Lambda\Lambda$ weak attraction)
- few Ξ-hypernuclei (ΞN attractive)
- evidence of 1 Σ-hypernuclei ? (ΣN repulsive)

New data on femtoscopy!

Theoretical approaches to YN and YY

- Meson exchange models (Juelich/Nijmegen models)

To build YN and YY from a NN meson-exchange model imposing $SU(3)_{flavor}$ symmetry

Juelich: Holzenkamp, Holinde, Speth '89; Haidenbauer and Meißner '05
Nijmegen: Maesen, Rijken, de Swart '89; Rijken, Nagels and Yamamoto '10

- Chiral effective field theory approach (Juelich-Bonn-Munich group)

To build YN and YY from a chiral effective Lagrangian similarly to NN interaction

Juelich-Bonn-Munich: Polinder, Haidenbauer and Meißner '06; Haidenbauer, Petschauer, Kaiser, Meißner, Nogga and Weise '13
Kohno '10; Kohno '18

- Quark model potentials

To build YN and YY within constituent quark models

Fujiwara, Suzuki, Nakamoto '07
Garcilazo, Fernandez-Carames and Valcarce '07 '10

- $V_{low\ k}$ approach

To calculate a "universal" effective low-momentum potential for YN and YY using RG techniques

Schaefer, Wagner, Wambach, Kuo and Brown '06

- Lattice calculations (HALQCD/NPLQCD)

To solve YN and YY interactions on the lattice

HALQCD: Ishii, Aoki, Hatsuda '07; Aoki, Hatsuda and Ishii '10; Aoki et al '12
NPLQCD: Beane, Orginos and Savage '11; Beane et al '12

Theoretical approaches to YN and YY

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NPLQCD: Beane, Orginos and Savage '11; Beane et al '12

YN (and YY) in meson-exchange models

Let's start with NN! Yukawa's idea

1930's: NN finite-range nature well established

Yukawa (1935): to construct a force of finite range in analogy to QED

QED	MESON THEORY
A field of particles with	
zero mass (photons)	non-zero mass (mesons)
is assumed and fulfills a field equation (in static approximation)	
POISSON	KLEIN-GORDON
$-\Delta A^0(\vec{r}) = e\delta(\vec{r})$	$(-\Delta + \mu^2)\phi(\vec{r}) = g\delta(\vec{r})$
and the solution is	
$A^0(\vec{r}) = \frac{e}{4\pi}\frac{1}{r}\hat{r}$	$\phi(\vec{r}) = \frac{g}{4\pi}\frac{e^{-\mu r}}{r}\hat{r}$ **finite range!!**
COULOMB POTENTIAL	**YUKAWA POTENTIAL**

QED

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\psi}\not{A}\psi$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

→ INSERT INTO EULER-LAGRANGE EQ.

Euler-Lagrange $\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} \right) - \frac{\partial \mathcal{L}}{\partial A_\nu} = 0$

$$\partial_\mu(\partial^\nu A^\mu - \partial^\mu A^\nu) - (-e\bar{\psi}\gamma^\nu\psi) = 0$$

→ OBTAIN FIELD EQ. FOR PHOTON FIELD

$$\partial_\mu F^{\mu\nu} = e\bar{\psi}\gamma^\nu\psi$$

→ $\nu = 0$ [COULOMB GAUGE]

$$\partial_\mu(\partial^\mu A^0 - \partial^0 A^\mu) = e\bar{\psi}\gamma^0\psi \simeq e\,\delta(\vec{r})$$

→ STATIC

$$-\Delta A^0(\vec{r}) = e\,\delta(\vec{r}) \quad \text{LAPLACE}$$

$$\Longrightarrow \boxed{A^0(\vec{r}) = \frac{e}{4\pi}\frac{1}{r}}$$

MESON-THEORY

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi + \frac{1}{2}(\partial_\mu\varphi\partial^\mu\varphi - \mu^2\varphi^2) + g\bar{\psi}\psi\varphi$$

→ EULER-LAGRANGE TO OBTAIN EQ. FOR MESON FIELD

$$(\partial_\mu\partial^\mu + \mu^2)\varphi(\vec{r}) = g\bar{\psi}\psi \simeq g\,\delta(\vec{r})$$

KLEIN-GORDON EQ.

→ STATIC

$$(-\Delta + \mu^2)\varphi(\vec{r}) = g\,\delta(\vec{r})$$

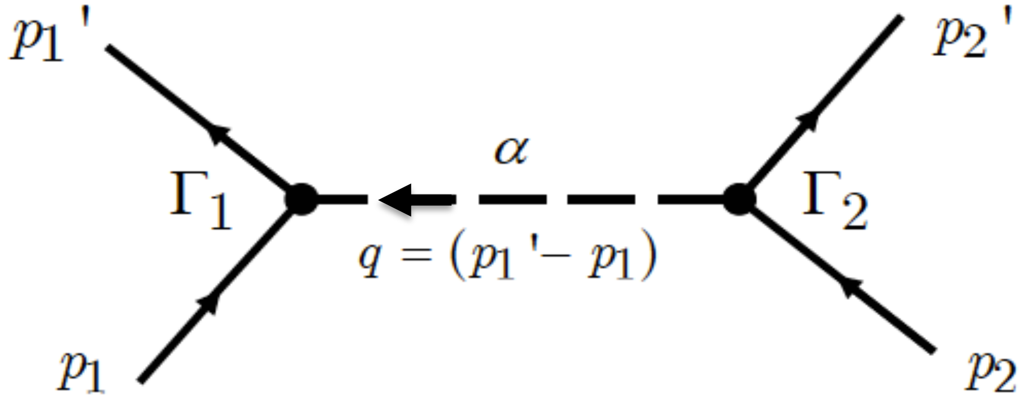
$$\Longrightarrow \boxed{\varphi(\vec{r}) = \frac{g}{4\pi}\frac{e^{-\mu r}}{r}}$$

finite range!

The One Boson Exchange model

Idea: to consider the exchange of bosons among nucleons within quantum field theory in terms of perturbation theory using Feynman diagrams

At lowest order



$$\text{Amplitude: } F_\alpha(p\,', p) = \frac{\bar{u}_1\,'\Gamma_1 u_1 \; P_\alpha \; \bar{u}_2\,'\Gamma_2 u_2}{q^2 - m_\alpha^2}$$

with Dirac spinor $u(p,s) = \sqrt{\frac{E+M}{2M}} \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M}\chi_s \end{pmatrix} \approx \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M}\chi_s \end{pmatrix} \approx \begin{pmatrix} \chi_s \\ 0 \end{pmatrix}$

where $E = \sqrt{\vec{p}^2 + M^2}$ and χ_s is a two-component Pauli spinor.

Citation: S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004) (URL: http://pdg.lbl.gov)

LIGHT UNFLAVORED MESONS
$(S = C = B = 0)$

For $I = 1$ (π, b, ρ, a): $u\bar{d}$, $(u\bar{u} - d\bar{d})/\sqrt{2}$, $d\bar{u}$;
for $I = 0$ (η, η', h, h', ω, ϕ, f, f'): $c_1(u\bar{u} + d\bar{d}) + c_2(s\bar{s})$

$\pi^\pm$ $I^G(J^P) = 1^-(0^-)$

Mass $m = 139.57018 \pm 0.00035$ MeV (S = 1.2)
Mean life $\tau = (2.6033 \pm 0.0005) \times 10^{-8}$ s (S = 1.2)
$c\tau = 7.8045$ m

π^0 $I^G(J^{PC}) = 1^-(0^{-+})$

Mass $m = 134.9766 \pm 0.0006$ MeV (S = 1.1)
$m_{\pi^\pm} - m_{\pi^0} = 4.5936 \pm 0.0005$ MeV
Mean life $\tau = (8.4 \pm 0.6) \times 10^{-17}$ s (S = 3.0)
$c\tau = 25.1$ nm

η $I^G(J^{PC}) = 0^+(0^{-+})$

Mass $m = 547.75 \pm 0.12$ MeV [f] (S = 2.6)
Full width $\Gamma = 1.29 \pm 0.07$ keV [g]

$f_0(600)$ [i] or σ $I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = (400–1200)$ MeV
Full width $\Gamma = (600–1000)$ MeV

$f_0(600)$ DECAY MODES	Fraction (Γ_i/Γ)
$\pi\pi$	dominant

$\rho(770)$ [j] $I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.8 \pm 0.5$ MeV
Full width $\Gamma = 150.3 \pm 1.6$ MeV
$\Gamma_{ee} = 7.02 \pm 0.11$ keV

$\rho(770)$ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
$\pi\pi$	$\sim$ 100 %	

$\omega(782)$ $I^G(J^{PC}) = 0^-(1^{--})$

Mass $m = 782.59 \pm 0.11$ MeV (S = 1.7)
Full width $\Gamma = 8.49 \pm 0.08$ MeV
$\Gamma_{ee} = 0.60 \pm 0.02$ keV

$\omega(782)$ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
$\pi^+\pi^-\pi^0$	(89.1 ± 0.7) %	S=1.1

Exchanged bosons

Citation: S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004) (URL: http://pdg.lbl.gov)

LIGHT UNFLAVORED MESONS
$(S = C = B = 0)$

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for $I = 0$ (η, η', h, h', ω, ϕ, f, f'): $c_1(u\bar{u} + d\bar{d}) + c_2(s\bar{s})$

$\pi^{\pm}$ $\qquad I^G(J^P) = 1^-(0^-)$

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$c\tau = 7.8045$ m

π^0

$c\tau = 25.1$ nm

PSEUDOSCALAR

η $\qquad I^G(J^{PC}) = 0^+(0^{-+})$

Mass $m = 547.75 \pm 0.12$ MeV [f] (S = 2.6)
Full width $\Gamma = 1.29 \pm 0.07$ keV [g]

$f_0(600)$ [i] **or σ** $\qquad I^G(J^{PC}) = 0^+(0^{++})$

Mass
Full w

SCALAR

$f_0(600)$ DECAY MO	
$\pi\pi$	dominant

$\rho(770)$ [j] $\qquad I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.8 \pm 0.5$ MeV
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$\rho(770)$ DECAY MOD	Scale factor/ Confidence level
$\pi\pi$	

VECTOR

$\omega(782)$ $\qquad I^G(J^{PC}) = 0^-(1^{--})$

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$\pi^+\pi^-\pi^0$	(89.1 ± 0.7) %	S=1.1

Exchanged bosons

Lagrangian density

main constraint: Lorentz scalar and Hermitean

$$\mathcal{L}_I = g[\bar{\psi}\tilde{\Gamma}\psi]\phi$$

Vertex (Γ): "i" times the Lagrangian stripped off the fields

Potential: "i" times the amplitude

Example: One-Pion Exchange for NN

$$\mathcal{L}_{\pi NN} = -g_{\pi NN}\ \bar{\psi}\mathrm{i}\gamma_5\vec{\tau}\psi\ \vec{\phi}^{(\pi)}$$

Vertex:

$$\Gamma = g_{\pi NN}\gamma_5\vec{\tau}$$

$$\bar{u}(p_1')\Gamma_1 u(p_1) = -g_{\pi NN}\frac{\vec{\sigma}_1\cdot\vec{q}}{2M}\vec{\tau}_1$$

$$\bar{u}(p_2')\Gamma_1 u(p_2) = g_{\pi NN}\frac{\vec{\sigma}_2\cdot\vec{q}}{2M}\vec{\tau}_2$$

Potential: $(P_\pi = i,\ q^2 \approx -\vec{q}^{\,2})$

$$V_\pi = iF_\pi = -\frac{g_{\pi NN}^2}{(2M)^2}\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{\vec{q}^{\,2}+m_\pi^2}\vec{\tau}_1\cdot\vec{\tau}_2$$

ONE-PION EXCHANGE

$$V_\pi = i\, F_\pi = i\, \bar{u}(p_1', s_1)\, \Gamma_1\, u(p_1, s_1) \frac{i}{(p_1'-p_1)^2 - m_\pi^2} \bar{u}(p_2', s_2)\, \Gamma_2\, u(p_2, s_2)$$

→ vertex: $\Gamma = -i\, i\, g_{\pi NN}\, \gamma_5 \vec{\tau} = g_{\pi NN}\, \gamma_5 \vec{\tau}$

→ full vertex ① $\bar{u}(p_1', s_1)\, \Gamma_1\, u(p_1, s_1)$

$$= g_{\pi NN}\, \bar{u}(p_1', s_1)\, \gamma_5\, u(p_1, s_1)\, \vec{\tau}_1$$

$$= g_{\pi N}\, u^+(p_1', s_1)\, \gamma^0 \gamma_5\, u(p_1, s_1)\, \vec{\tau}_1$$

$$= g_{\pi N} \left(\chi_{s_1}^+ ,\ \frac{\vec{\sigma}_1 \cdot \vec{p}_1'}{2M} \chi_{s_1}^+ \right) \gamma^0 \gamma_5 \begin{pmatrix} \chi_{s_1} \\ \frac{\vec{\sigma}_1 \cdot \vec{p}_1}{2M} \chi_{s_1} \end{pmatrix} \vec{\tau}_1$$

$E \approx M$

$u^+(p_1' s_1)\, u(p_1 s_1)$

$$= g_{\pi NN} \left(\chi_{s_1}^+ ,\ -\frac{\vec{\sigma}_1 \cdot \vec{p}_1'}{2M} \chi_{s_1}^+ \right) \gamma_5 \begin{pmatrix} \chi_{s_1} \\ \frac{\vec{\sigma}_1 \cdot \vec{p}_1}{2M} \chi_{s_1} \end{pmatrix} \vec{\tau}_1$$

$u^+(p_1' s_1)\, \gamma^0$

$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$

$$= g_{\pi NN} \left(-\frac{\vec{\sigma}_1 \cdot \vec{p}_1'}{2M} \chi_{s_1}^+ ,\ \chi_{s_1}^+ \right) \begin{pmatrix} \chi_{s_1} \\ \frac{\vec{\sigma}_1 \cdot \vec{p}_1}{2M} \chi_{s_1} \end{pmatrix} \vec{\tau}_1$$

$u^+(p_1' s_1)\, \gamma^0 \gamma_5$

$\gamma_5 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$

$u^+(p_1' s_1)\, \gamma^0 \gamma_5\, u(p_1 s_1)$

$\chi_{s_1}^+ \chi_{s_1} = 1$

$$= g_{\pi NN} \left(-\frac{\vec{\sigma}_1 \cdot \vec{p}_1'}{2M} + \frac{\vec{\sigma}_1 \cdot \vec{p}_1}{2M} \right) \vec{\tau}_1 = g_{\pi NN} \left(-\frac{\vec{\sigma}_1 \cdot (\vec{p}_1' - \vec{p}_1)}{2M} \right) \vec{\tau}_1$$

$q = \vec{p}_1' - \vec{p}_1$

→ full vertex ② : $\bar{u}(p_2', s_2)\, \Gamma_2\, u(p_2, s_2)$

$$= g_{\pi NN} \frac{-\vec{\sigma}_2 \cdot (\vec{p}_2' - \vec{p}_2)}{2M} \vec{\tau}_2 = g_{\pi NN} \frac{\vec{\sigma}_2 \cdot \vec{q}}{2M} \vec{\tau}_2$$

$\vec{q} = \vec{p}_2 - \vec{p}_2'$

Potential

$$V_\pi = i\, g_{\pi NN}^2 \left(\frac{-\vec{\sigma}_1 \cdot \vec{q}}{2M} \vec{\tau}_1 \right) \frac{i}{q^2 - m_\pi^2} \left(\frac{\vec{\sigma}_2 \cdot \vec{q}}{2M} \vec{\tau}_2 \right)$$

$$= g_{\pi NN}^2 \frac{\vec{\sigma}_1 \cdot \vec{q}}{2M} \frac{1}{q^2 - m_\pi^2} \left(\frac{\vec{\sigma}_2 \cdot \vec{q}}{2M} \right) \vec{\tau}_1 \cdot \vec{\tau}_2$$

$$= -\frac{g_{\pi NN}^2}{(2M)^2} \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^2 + m_\pi^2} \vec{\tau}_1 \cdot \vec{\tau}_2$$

static: $q^2 \simeq -\vec{q}^2$

Using the operator identity

$$(\vec{\sigma_1} \cdot \vec{q})(\vec{\sigma_2} \cdot \vec{q}) = \frac{\vec{q}^{\,2}}{3}[\vec{\sigma_1} \cdot \vec{\sigma_2} + S_{12}(\hat{q})]$$

$$S_{12}(\hat{q}) \equiv 3(\vec{\sigma_1} \cdot \hat{q})(\vec{\sigma_2} \cdot \hat{q}) - \vec{\sigma_1} \cdot \vec{\sigma_2}$$ (tensor operator)

the one-pion exchange potential (OPEP) can be written

$$V_\pi = \frac{g^2_{\pi NN}}{3(2M)^2} \frac{\vec{q}^{\,2}}{\vec{q}^{\,2} + m^2_\pi}[-\vec{\sigma_1} \cdot \vec{\sigma_2} - S_{12}(\hat{q})]\vec{\tau_1} \cdot \vec{\tau_2}$$

Also OPEP from pseudo-vector or gradient coupling to the nucleon (suggested by chiral symmetry)

$$\mathcal{L}_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi}\; \bar{\psi}\gamma^\mu\gamma_5\vec{\tau}\psi \;\cdot \partial_\mu \vec{\phi}^{(\pi)}$$

$$\Gamma_{\pi NN} = (i)^2 \frac{f_{\pi NN}}{m_\pi}\gamma^\mu\gamma_5\vec{\tau}q_\mu$$

(incoming pion)

Other meson exchanges:

$$\mathcal{L}_{\sigma NN} = -g_{\sigma NN}\ \bar{\psi}\psi\ \phi^{(\sigma)}$$

$$\mathcal{L}_{\omega NN}^{(vector)} = -g_{\omega}\ \bar{\psi}\gamma^{\mu}\psi\ \phi_{\mu}^{(\omega)}$$

$$\mathcal{L}_{\rho NN}^{(tensor)} = -\frac{f_{\rho}}{4M}\ \bar{\psi}\sigma^{\mu\nu}\vec{\tau}\psi \cdot (\partial_{\mu}\vec{\phi}_{\nu}^{(\rho)} - \partial_{\nu}\vec{\phi}_{\mu}^{(\rho)})$$

Summary

$\pi(138)$

$$V_\pi = \frac{f_{\pi NN}^2}{3m_\pi^2}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\left[-\vec{\sigma}_1\cdot\vec{\sigma}_2 - S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$$

Long-ranged tensor force

$\sigma(600)$

$$V_\sigma \approx \frac{g_\sigma^2}{\vec{q}^2+m_\sigma^2}\left[-1 - \frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$$

intermediate-ranged, attractive central force plus LS force

$\omega(782)$

$$V_\omega \approx \frac{g_\omega^2}{\vec{q}^2+m_\omega^2}\left[+1 - 3\frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$$

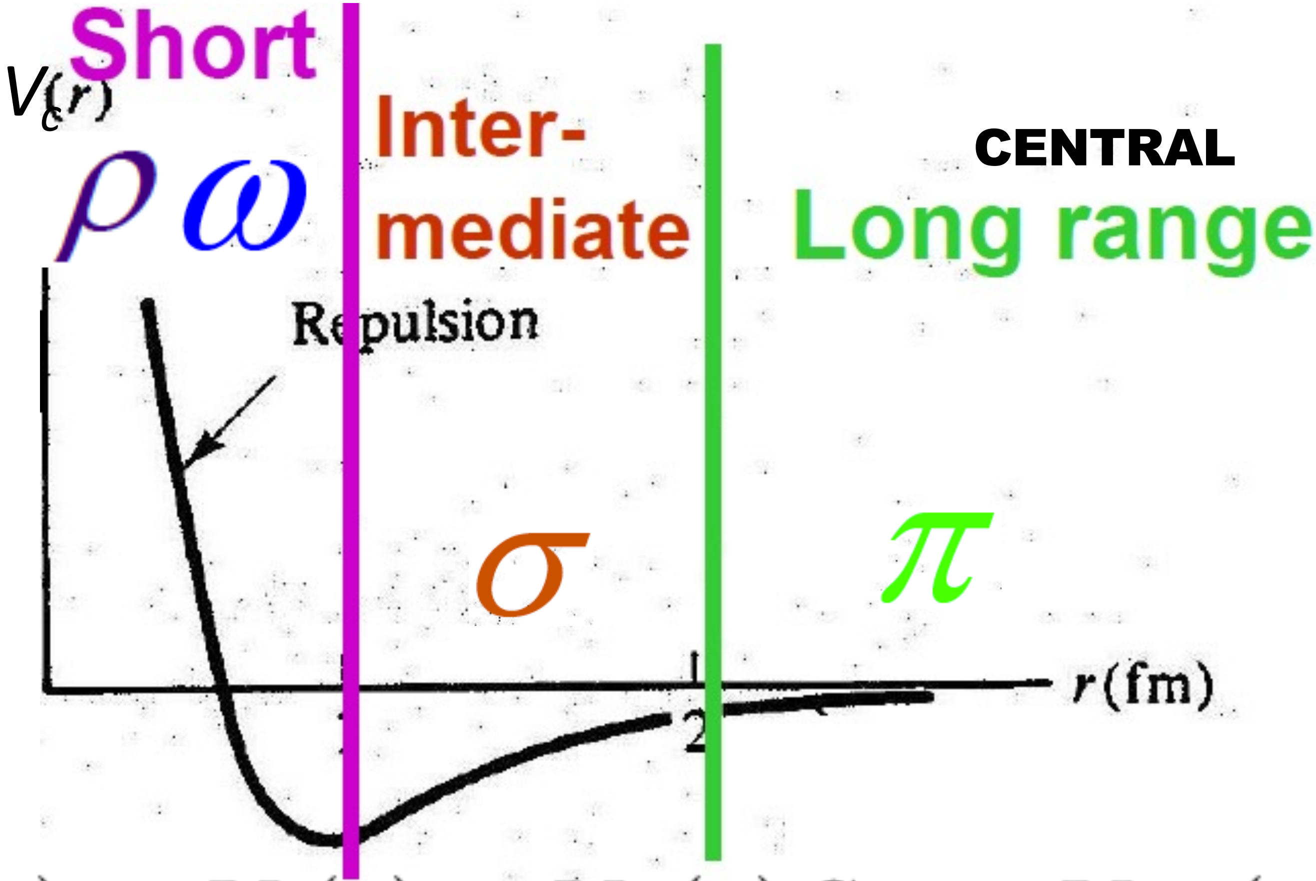
short-ranged, repulsive central force plus strong LS force

$\rho(770)$

$$V\rho = \frac{f_\rho^2}{12M^2}\frac{\vec{q}^2}{\vec{q}^2+m_\rho^2}\left[-2\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$$

short-ranged tensor force, opposite to pion

We can describe NN !!



$$V(r) = V_c(r) + V_T(r) S_{12} + V_{LS}(r) \vec{L}\vec{S}$$

CENTRAL **TENSOR** **SPIN-ORBIT**

ρ π ω σ

One Boson Exchange Potential

$$V_{\rm OBEP} = \sum_{\alpha=\pi,\sigma,\rho,\omega,\eta,a_0,\dots} V_\alpha$$

$\eta(548)$ is a pseudo-scalar meson with $I=0$, therefore, V_η is given by the same expression as V_π, except that V_η carries no $(\vec{\tau}_1 \cdot \vec{\tau}_2)$ factor.

$a_0(980)$ is a scalar meson with $I=1$, therefore, V_{a_0} is given by the same expression as V_σ, except that V_{a_0} carries a $(\vec{\tau}_1 \cdot \vec{\tau}_2)$ factor.

Note: Include FORM FACTORS
to implement the substructure of hadrons

OBE x $\left(\dfrac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 + \vec{k}^2}\right)^{n_\alpha}$

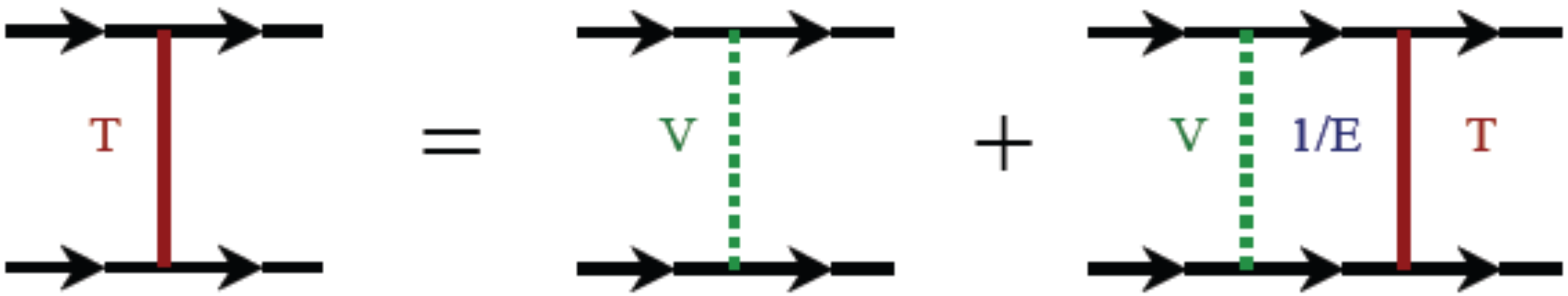
Theory (OBEP) vs Experiment

S-matrix (collision operator)

final state after collision $|f> = S\,|i>$ initial state

S-matrix $S_{if} = \delta_{if} - i(2\pi)^4\delta^4(P_i - P_f)T_{if}$ scattering amplitude

Lippman-Schwinger Equation:



Deuteron and Low-Energy Scattering Parameters as Predicted by the Relativistic OBEP Defined in Table 4.1 (Theory) and from Experiment (Experiment)

	Theory	Experiments[a]	References
Deuteron			
Binding energy-ϵ_d (MeV)	2.2246	2.224575 (9)	LA 82
D-state probability P_D (%)	4.99	—	—
Quadrupole moment Q_d (fm^2)	0.278[b]	0.2860 (15)	RV 75, BC 79
		0.2859 (3)	ER 83, BC 79
Magnetic moment μ_d (μ_N)	0.8514[b]	0.857406 (1)	Lin 65
Asymptotic *S*-state A_S ($fm^{-1/2}$)	0.8860	0.8846 (8)	ER 83
Asymptotic *D/S*-state *D/S*	0.0264	0.0271 (8)	GKT 82
		0.0272 (4)	Bor+ 82
		0.0256 (4)	RK 86
Root-mean-square radius r_d (fm)	1.9688	1.9635 (45)	Bér+ 73
		1.9560 (68)	SSW 81, KMS 84
		1.953 (3)	Kla+ 86
Neuteron–proton low-energy scattering (scattering length *a*, effective range *r*):			
$^1S_0$: a_{np} (fm)	−23.75	−23.748 (10)	Dum+ 83
r_{np} (fm)	2.71	2.75 (5)	Dum+ 83
$^3S_1$: a_t (fm)	5.424	5.419 (7)	Hou 71, Dil 75, KMS 84
$r_t = \rho(0, 0)$ (fm)	1.761	1.754 (8)	Hou 71, Dil 75, KMS 84

[a] The figures in parentheses after the values give the one-standard-deviation uncertainties in the last digits.
[b] The meson exchange current contributions to the moments are not included in the theoretical values.

NN

R. Machleidt, Adv. Nucl. Phys. 19 (1989) 189-736

YN (and YY) meson-exchange models

Built from a NN meson-exchange model imposing $SU(3)_{flavor}$ symmetry

NIJMEGEN

(Nagels, Rijken, de Swart, Timmermans, Maessen..)

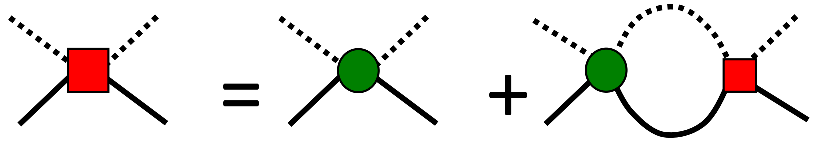
- ✓ Based on Nijmegen NN potential
- ✓ Momentum and Configuration Space
- ✓ Exchange of pseudoscalar, vector and scalar nonets
- ✓ SU(3) symmetry to relate YN to NN vertices
- ✓ Gaussian form factors

JUELICH

(Holzenkamp, Reube, Holinde, Speth, Haidenbauer, Meissner, Melnitchouck..)

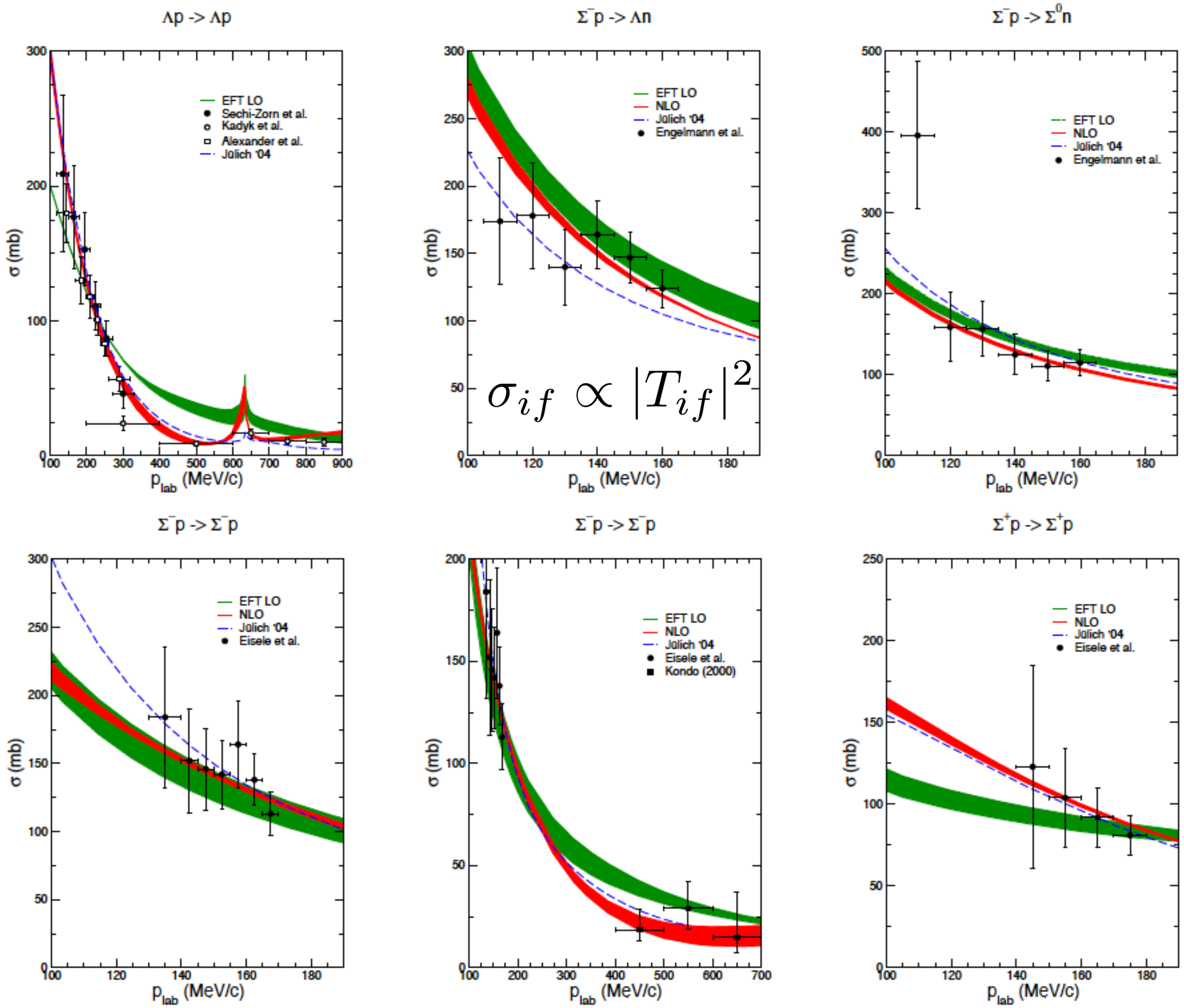
- ✓ Based on Bonn NN potential
- ✓ Momentum Space, Full Energy Dependence & Non-localities
- ✓ Exchange of single mesons and higher order processes
- ✓ SU(6) symmetry to relate YN to NN vertices
- ✓ Dipolar form factors

ΛN and ΣN scattering



LO: H. Polinder, J.H., U. Meißner, NPA 779 (2006) 244
NLO: J.H., N. Kaiser, et al., NPA 915 (2013) 24
Jülich '04: J.H., U.-G. Meißner, PRC 72 (2005) 044005

$$T = V + V \frac{1}{E_0 - H_0 + i\eta} T$$

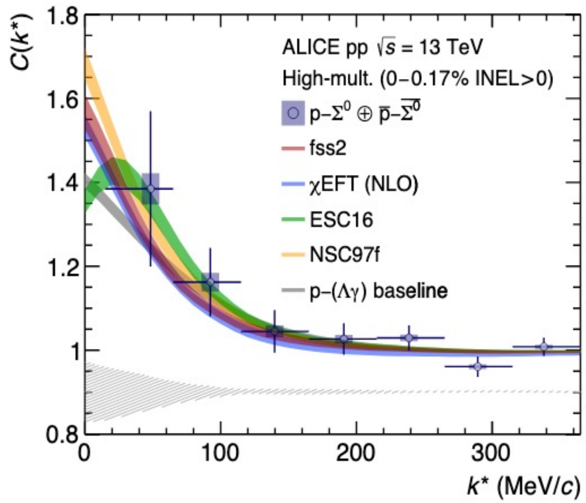


$$\sigma_{if} \propto |T_{if}|^2$$

Results from
femtoscopy for $\Sigma^0 p$

$$C(k^*) = \mathcal{N} \times \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$

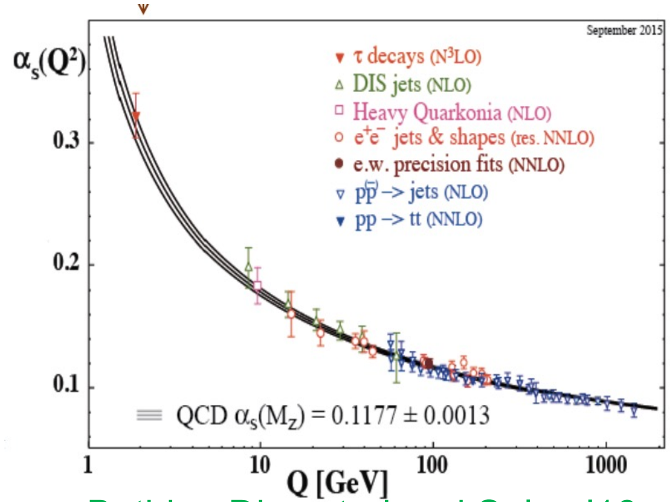
$$k^* = \frac{1}{2} \times |\mathbf{p}_1^* - \mathbf{p}_2^*|$$



Acharya et al. '19

YN (and YY) interactions in χEFT

QCD: the interaction at short distances between quarks is weak (asymptotic freedom). However, the interaction is strong at long distances giving rise to confinement in colorless objects

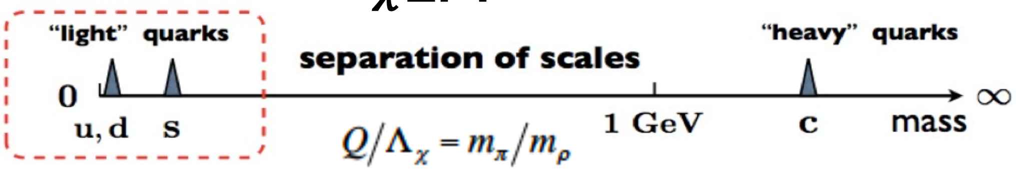


Bethke, Dissertori and Salam'16

Effective Theories are developed to treat the non-perturbative regime of QCD. The main premise is that the dynamics of low energies does not depend on the details of the dynamics at high energies.

Effective Theory is systematic approach to a certain dynamics (known or not) that governs a physical process in a certain regime of energies. It is not a model, since its systematic character means that, in principle, one can make predictions with arbitrary precision.

χEFT



However, in order to be possible, some small parameter must govern the systematic approach (expansion). In most physical processes, that parameter it is constructed through the quotient of two of the physical scales present in that process, that are clearly separated.

credit: A. Parreno

To describe the interaction between hadrons at low energy, we only need mesons and baryons, as relevant degrees of freedom.

The physics that appears in the fundamental Lagrangian density (QCD) is mimicked in the effective Lagrangian density through a set of operators and associated constants.

If we could solve QCD exactly, we could find the value of these constants comparing the effective theory with the complete theory. But since finding the exact solution is not possible, we use experimental data to determine these constants.

The **power of effective theory** lies in the fact that:
→ it contains the *symmetries* of the fundamental theory
→ it provides a *power counting* that allows us to make consistent calculations order by order
→ it allows us, a priori, to *estimate the corrections* introduced at each order
→ it is *systematic*, so independent of models

ALGORITHM

- identify the “soft” and “hard” scales and the appropriate degrees of freedom

-identify the relevant symmetries of low-energy QCD and investigate whether they are broken or not

-write the most general possible Lagrangian that contains all the terms consistent with the symmetries of our problem (and with the symmetry breaking)

-design an organizational scheme to distinguish the more relevant contributions from the non-relevant: the low-momentum expansion

QCD Lagrangian and Chiral Symmetry

$$\mathcal{L}_{QCD} = \sum_{f=\substack{u,d,s\\c,b,t}} \bar{q}_f \left(i \not{D} - m_f\right) q_f - \frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu}_{a}$$

$$q_f = \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix}$$

$$D_\mu \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} = \partial_\mu \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} + ig \sum_{a=1}^{8} \frac{\lambda_a}{2} A^a_\mu \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

assuming $m_u, m_d \approx 0$,

$$\mathcal{L}^0_{QCD} = \sum_{l=u,d} \bar{q}_l i \not{D} q_l - \frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu}_{a}$$

Chirality is the generalization of helicity for particles without mass or highly energetic (ultrarelativistic)

Introducing $P_L = \frac{1-\gamma_5}{2}$ "Left-handed", $P_R = \frac{1+\gamma_5}{2}$ "Right-handed" with

$P_R + P_L = 1$ completeness

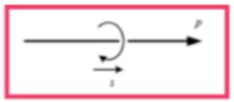
$P_R^2 = P_R \quad P_L^2 = P_L$ idempotent

$P_R P_L = P_L P_R = 0$ orthogonal

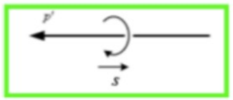
Eigenstates of chirality exist for massless particles

$$q_L = \frac{1-\gamma_5}{2} q \qquad q_R = \frac{1+\gamma_5}{2} q$$

$$(P_L q_L = q_L \,,\; P_R q_R = q_R)$$



$q_R = P_R q,$



$q_L = P_L q.$

We can rewrite the (massless) QCD lagrangian

$$\mathcal{L}^0_{QCD} = \sum_{l=u,d} \left(\bar{q}_{R,l} i \not{D} q_{R,l} + \bar{q}_{L,l} i \not{D} q_{L,l} \right) - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \quad \rightarrow$$

SU(2)$_L$ x SU(2)$_R$ symmetry
or chiral symmetry
(extensible to SU(3)$_L$ x SU(3)$_R$)

Chiral symmetry:
the Lagrangian is invariant under a global phase transformation
(interaction between quarks and gluons is
independent of flavour and retains helicity)

$$q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix} \rightarrow \exp\left(-i \sum_{a=1}^{3} \alpha^L_a \frac{\lambda_a}{2} \right) \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$q_R \equiv \begin{pmatrix} u_R \\ d_R \end{pmatrix} \rightarrow \exp\left(-i \sum_{a=1}^{3} \alpha^R_a \frac{\lambda_a}{2} \right) \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

Pauli matrices

According to Noether's Theorem
(to every continuous symmetry of a physical system belongs to a conserved quantity and viceversa)
there are 3 left-handed and 3 right-handed conserved currents

$$L^\mu_a = \bar{q}_L \gamma^\mu \frac{\lambda_a}{2} q_L \qquad \text{con } \partial_\mu L^\mu_a = 0$$

$$R^\mu_a = \bar{q}_R \gamma^\mu \frac{\lambda_a}{2} q_R \qquad \text{con } \partial_\mu R^\mu_a = 0$$

or 3 vector currents and 3 axial-vectors currents

$$V^\mu_a = R^\mu_a + L^\mu_a = \bar{q} \gamma^\mu \frac{\lambda_a}{2} q \qquad \text{con } \partial_\mu V^\mu_a = 0$$

$$A^\mu_a = R^\mu_a - L^\mu_a = \bar{q} \gamma^\mu \gamma_5 \frac{\lambda_a}{2} q \qquad \text{con } \partial_\mu A^\mu_a = 0$$

ISOSPIN rotation

$$q \equiv \begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \exp\left(-i \sum_{a=1}^{3} \alpha^V_a \frac{\lambda_a}{2} \right) \begin{pmatrix} u \\ d \end{pmatrix}$$

$$q \equiv \begin{pmatrix} u \\ d \end{pmatrix} \rightarrow \exp\left(-i \sum_{a=1}^{3} \gamma_5 \alpha^A_a \frac{\lambda_a}{2} \right) \begin{pmatrix} u \\ d \end{pmatrix}$$

SU(2)$_V$ x SU(2)$_A$

The QCD Lagrangian is invariant under $SU(2)_V$ x $SU(2)_A$ for massless quarks, thus, chiral invariant. However, if the mass of quarks is not negligible, then the mass term breaks chiral symmetry explicitely

$$-\sum_{l=u,d}\bar{q}_l m_l q_l = -\bar{q}Mq = -(\bar{q}_R M q_L + \bar{q}_L M q_R) \quad \text{with} \quad M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

Moreover, the axial-vector symmetry is spontaneously broken *(a spontaneously broken symmetry is a symmetry of the Hamiltonian that is not realized in the ground state)*

Experimental indication: the hadronic spectrum (fundamental state of QCD) does not contain parity doublets (axial-vector current related to the parity of the state)

Consequence of the spontaneously broken axial-vector symmetry: existence of (pseudo-)Goldstone bosons

Goldstone theorem: when a continuous symmetry is spontaneously broken, new scalar particles without mass (or very light, if the symmetry is not exact) appear within the spectrum of possible excitations

$$\pi^+, \pi^0, \pi^-$$

credit: A. Parreno

Effective Theory: Chiral Perturbation Theory

ALGORITHM

- identify the "soft" and "hard" scales and the appropriate degrees of freedom

-identify the relevant symmetries of low-energy QCD and investigate whether they are broken or not

-write the most general possible Lagrangian that contains all the terms consistent with the symmetries of our problem (and with the symmetry breaking)

-design an organizational scheme to distinguish the more relevant contributions from the non-relevant: the low-momentum expansion

- "soft" and "hard" scales (Q,Λ_χ) and degrees of freedom (pions, nucleons..)

$$Q/\Lambda_\chi = m_\pi/m_\rho$$

- identify relevant symmetry: *chiral symmetry*

- write the most general Lagrangian compatible with the chiral symmetry of QCD

$$\mathcal{L}_{eff} = \sum_{i=1}^{\infty} \mathcal{L}_{\pi\pi}^{(2i)} + \sum_{i=1}^{\infty} \mathcal{L}_{\pi N}^{(i)} + \ldots$$

power of momenta

- while designing an organizational scheme

Pich, Rept. Prog. Phys. 58 (1995) 563-610
Koch, Int. J. Mod. Phys. E6 (1997) 203-250

Chiral Effective Theory for BB Interaction

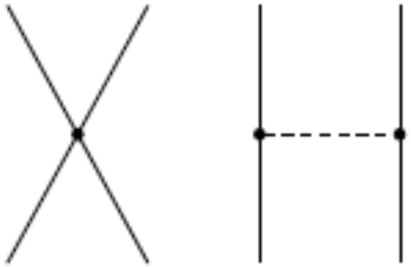
Baryon-Baryon interaction in SU(3) χEFT **a la Weinberg** (1990);

- **power counting** allowing for a systematic improvement by going to higher order
- derivation of **two- and three-baryon forces** in a consistent way

Degrees of freedom: **octet of baryons** (N, Λ ,Σ, Ξ) & **pseudoscalar mesons** (π,K,η)
Diagrams: **pseudoscalar-meson exchanges and contact terms**

credit: Haidenbauer

LO :

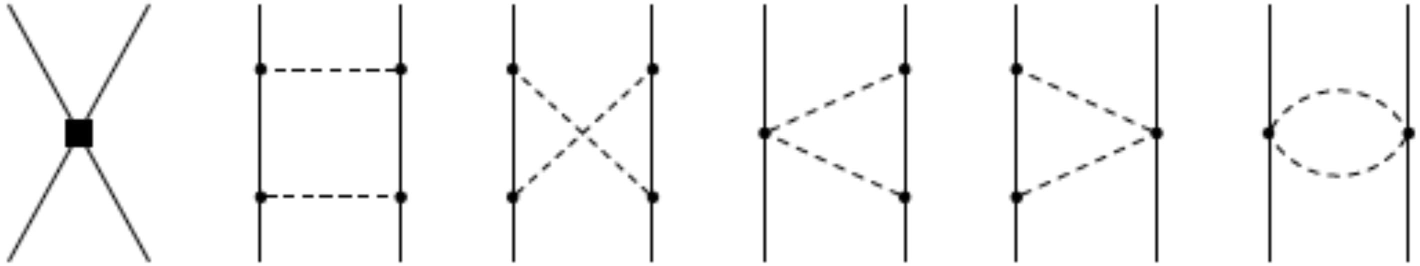


$$\nu = 2 - B + 2L + \sum_i v_i \Delta_i \ ,$$

$$\Delta_i = d_i + \frac{1}{2} b_i - 2 \ ,$$

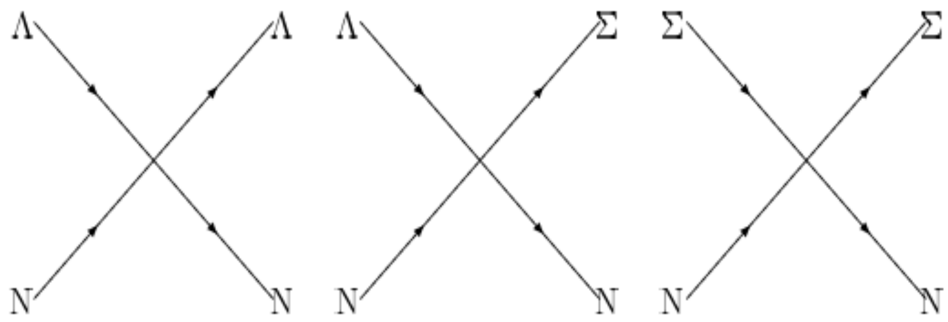
B: number of incoming (outgoing) baryons
L: number of Goldstone boson loops
v_i: number of vertices with dimensión Δ_i
d_i: derivatives
b_i: number of internal baryons at vertex

NLO :



LO: H. Polinder, J.H., U. Meißner, NPA 779 (2006) 244
NLO: J.H., N. Kaiser, U.-G. Meißner, A. Nogga, S. Petschauer, W. Weise, NPA 915 (2013) 24

LO Contact Terms



$$\mathcal{L}^1 = C_i^1 \langle \bar{B}_a \bar{B}_b (\Gamma_i B)_b (\Gamma_i B)_a \rangle,$$

$$\mathcal{L}^2 = C_i^2 \langle \bar{B}_a (\Gamma_i B)_a \bar{B}_b (\Gamma_i B)_b \rangle$$

$$\mathcal{L}^3 = C_i^3 \langle \bar{B}_a (\Gamma_i B)_a \rangle \langle \bar{B}_b (\Gamma_i B)_b \rangle.$$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

with *a* and *b* the Dirac indices of the particles
and Γ_i the five elements of Clifford algebra (1, γ^μ, $\sigma^{\mu\nu}$, $\gamma^\mu\gamma^5$ and γ^5)
The LO contact potential is

$$V_{LO}^{BB} = C_S^{BB} + C_T^{BB} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2,$$

where the coupling constants for the central and spin-spin parts are linear combinations of the six independent low-energy coefficients

Polinder, Haidenbauer and Meissner, Nucl. Phys. A779 (2006) 244-266

LO: One-Pseudoscalar Meson Exchange

$$\mathcal{L} = \left\langle i\bar{B}\gamma^\mu D_\mu B - M_0\bar{B}B + \frac{D}{2}\bar{B}\gamma^\mu\gamma_5\left\{u_\mu, B\right\} + \frac{F}{2}\bar{B}\gamma^\mu\gamma_5\left[u_\mu, B\right]\right\rangle$$

$$D_\mu B = \partial_\mu B + [\Gamma_\mu, B] \ ,$$

$$\Gamma_\mu = \frac{1}{2}\left[u^\dagger\partial_\mu u + u\partial_\mu u^\dagger\right] \ ,$$

$$u^2 = U = \exp(2iP/\sqrt{2}F_\pi)$$

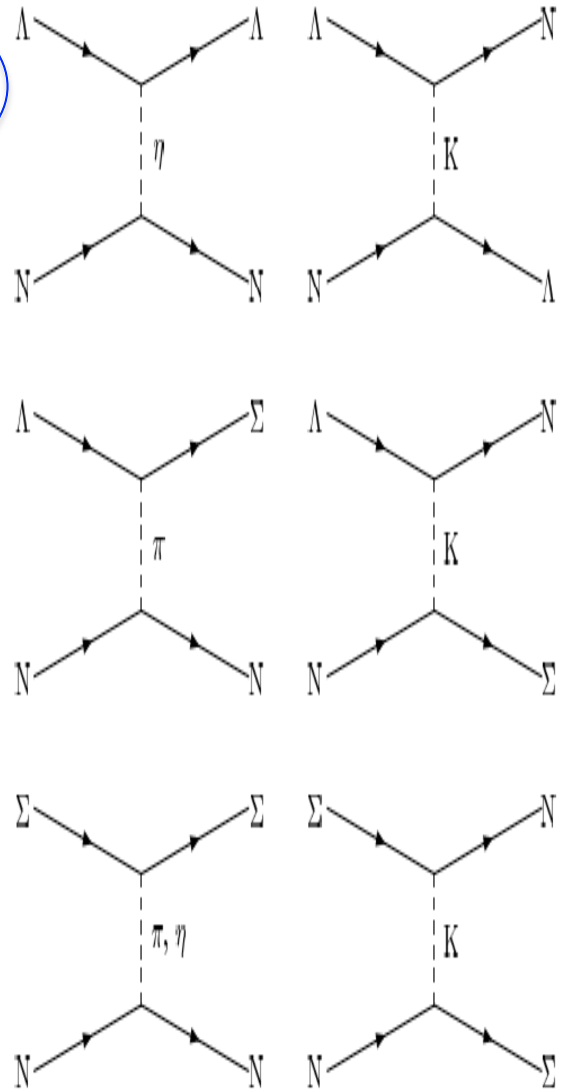
$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ -K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

$$\frac{1}{2}u_\mu = \frac{i}{2}\left(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger\right) = \frac{i}{2}\left\{u^\dagger, \partial_\mu u\right\} = \frac{i}{2}u^\dagger\partial_\mu U u^\dagger$$

one pseudoscalar-meson exchanges

The one-pseudoscalar meson exchange is given by

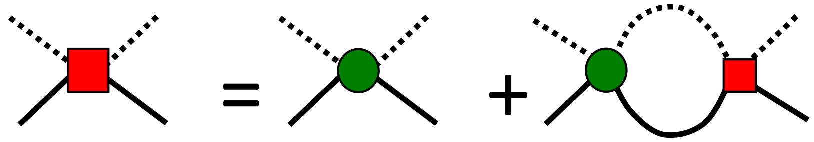
$$V^{\mathrm{BB}}_{\mathrm{OBE}} = -f_{B_1B_2P}f_{B_2B_4P}\frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})}{\boldsymbol{q}^2 + m_{ps}^2}\mathcal{I}_{B_1B_2\to B_3B_4}$$



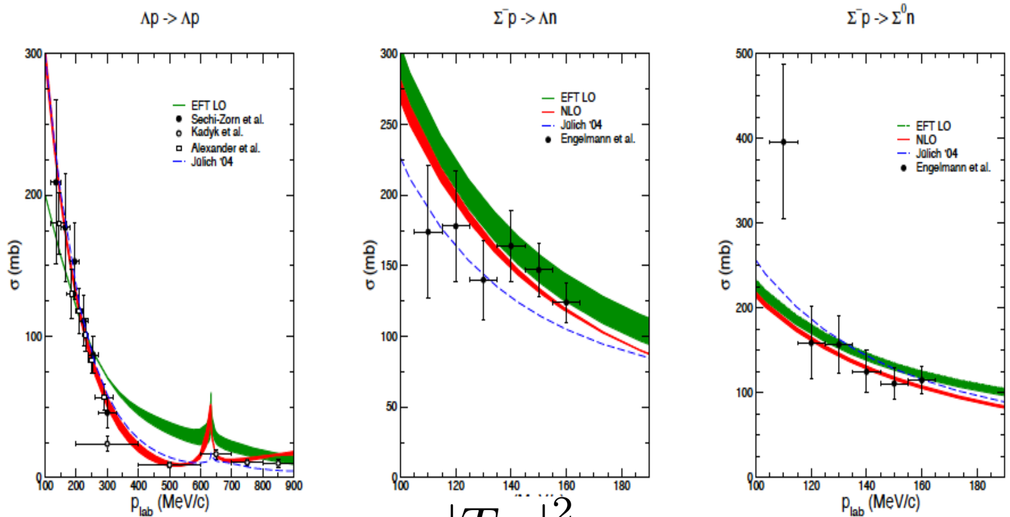
Polinder, Haidenbauer and Meissner, Nucl. Phys. A779 (2006) 244-266

ΛN and ΣN scattering

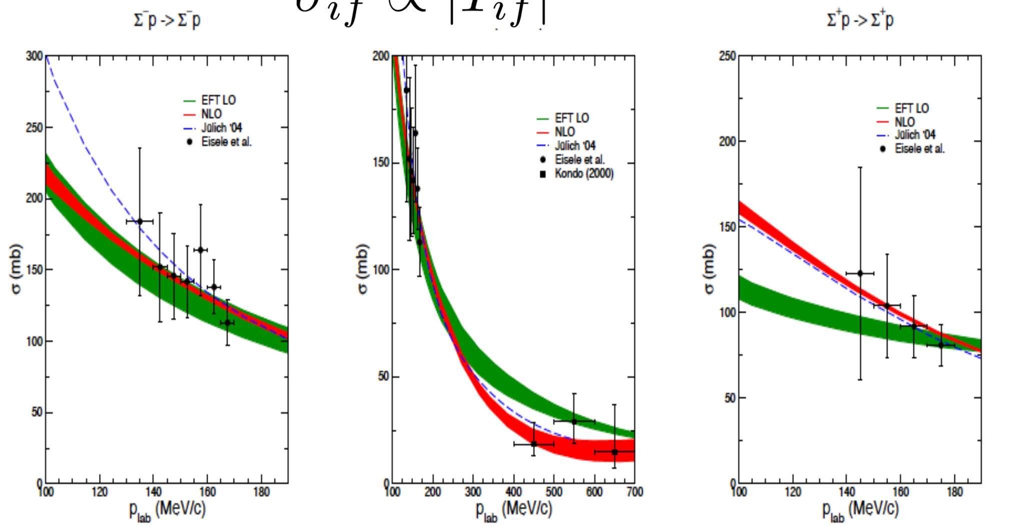
LO: H. Polinder, J.H., U. Meißner, NPA 779 (2006) 244
NLO: J.H., N. Kaiser, et al., NPA 915 (2013) 24
Jülich '04: J.H., U.-G. Meißner, PRC 72 (2005) 044005

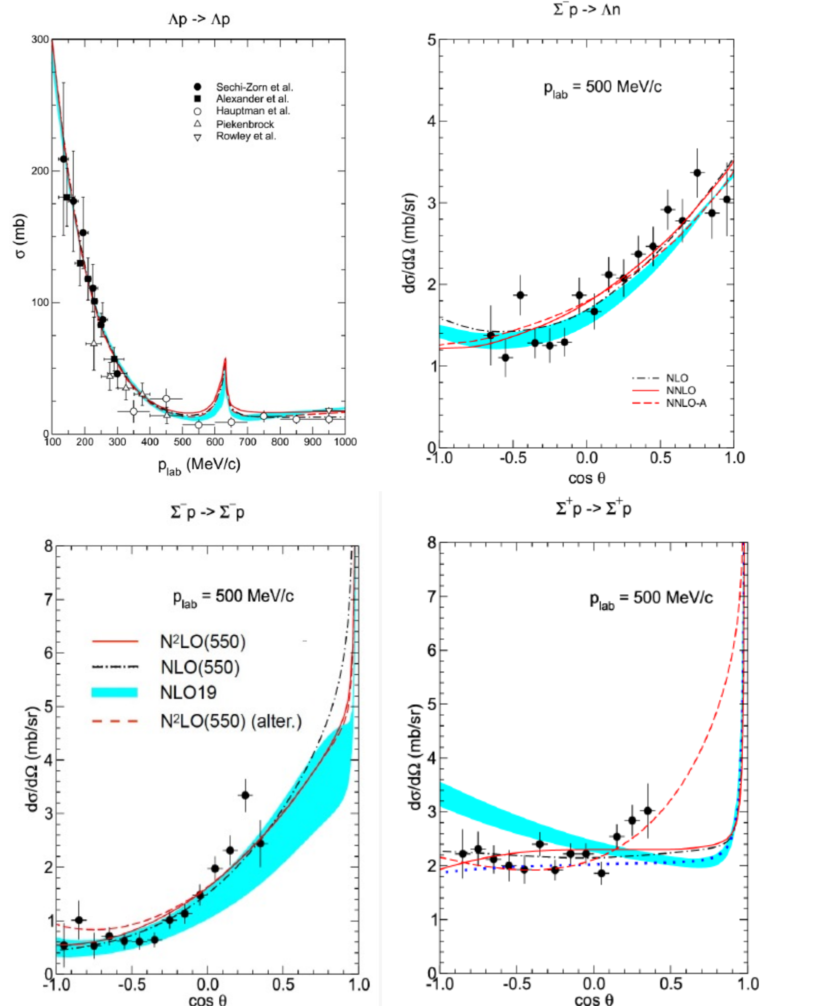


$$T = V + V \frac{1}{E_0 - H_0 + i\eta} T$$



$$\sigma_{if} \propto |T_{if}|^2$$

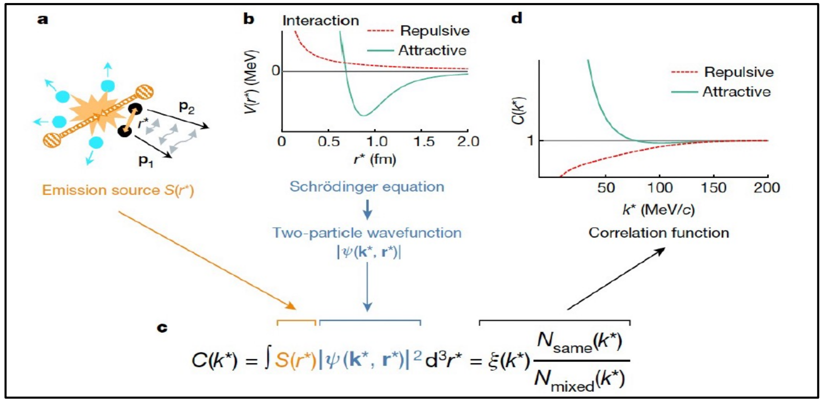




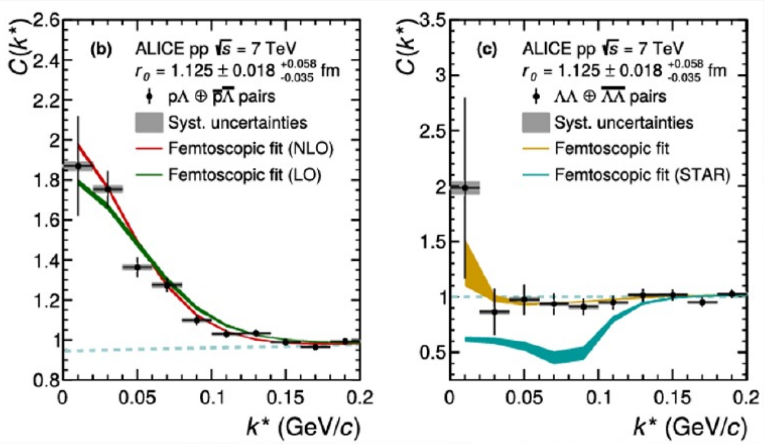
latest data on YN scattering
using new data from J-PARC and CLAS
Haidenbauer, Meißner, Nogga and Le ´23

Femtoscopy (ALICE@LHC)

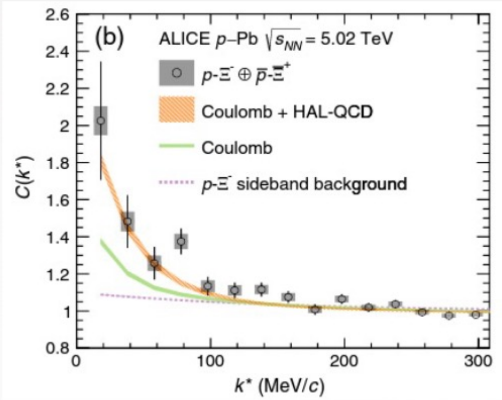
ALICE Collaboration, Nature 588 (2020) 232
Fabbietti, Mantovani-Sarti, Vazquez-Doce ‘21



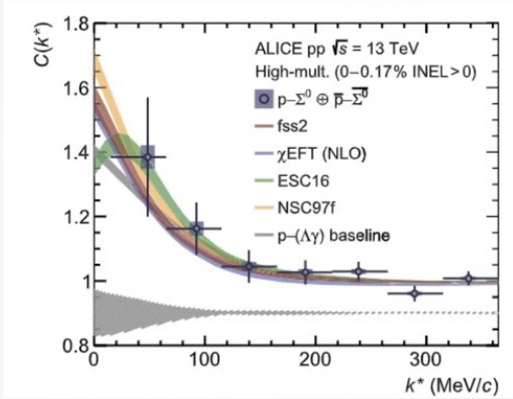
ALICE, PRC (2019)



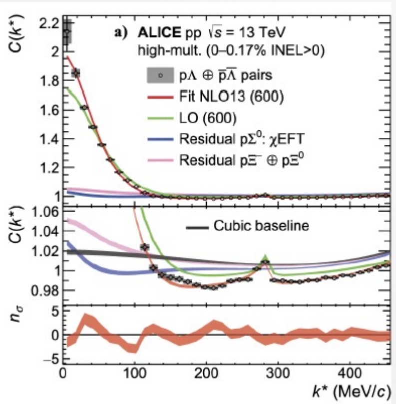
ALICE, PRL (2019)



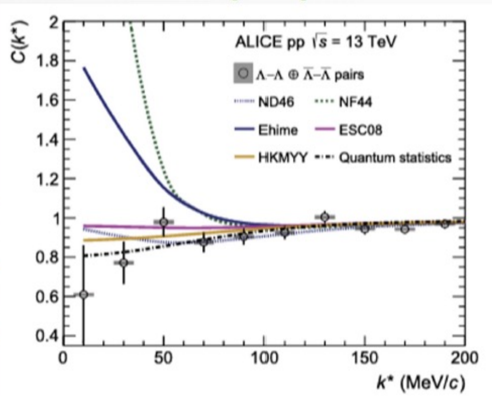
ALICE, PLB (2020)



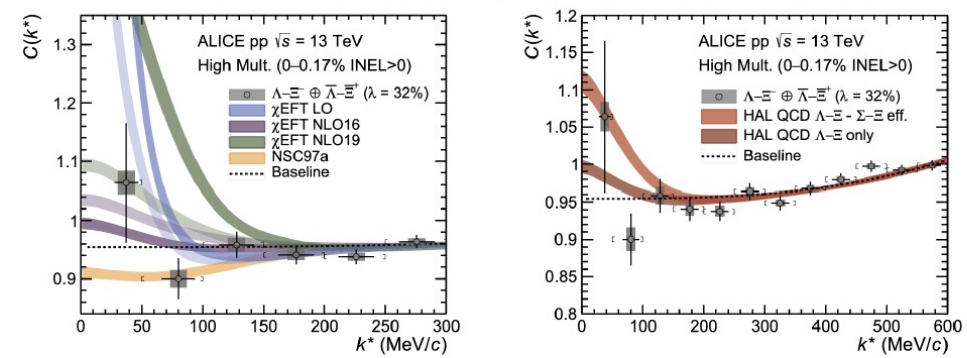
ALICE, PLB (2022)



ALICE, PLB (2019)

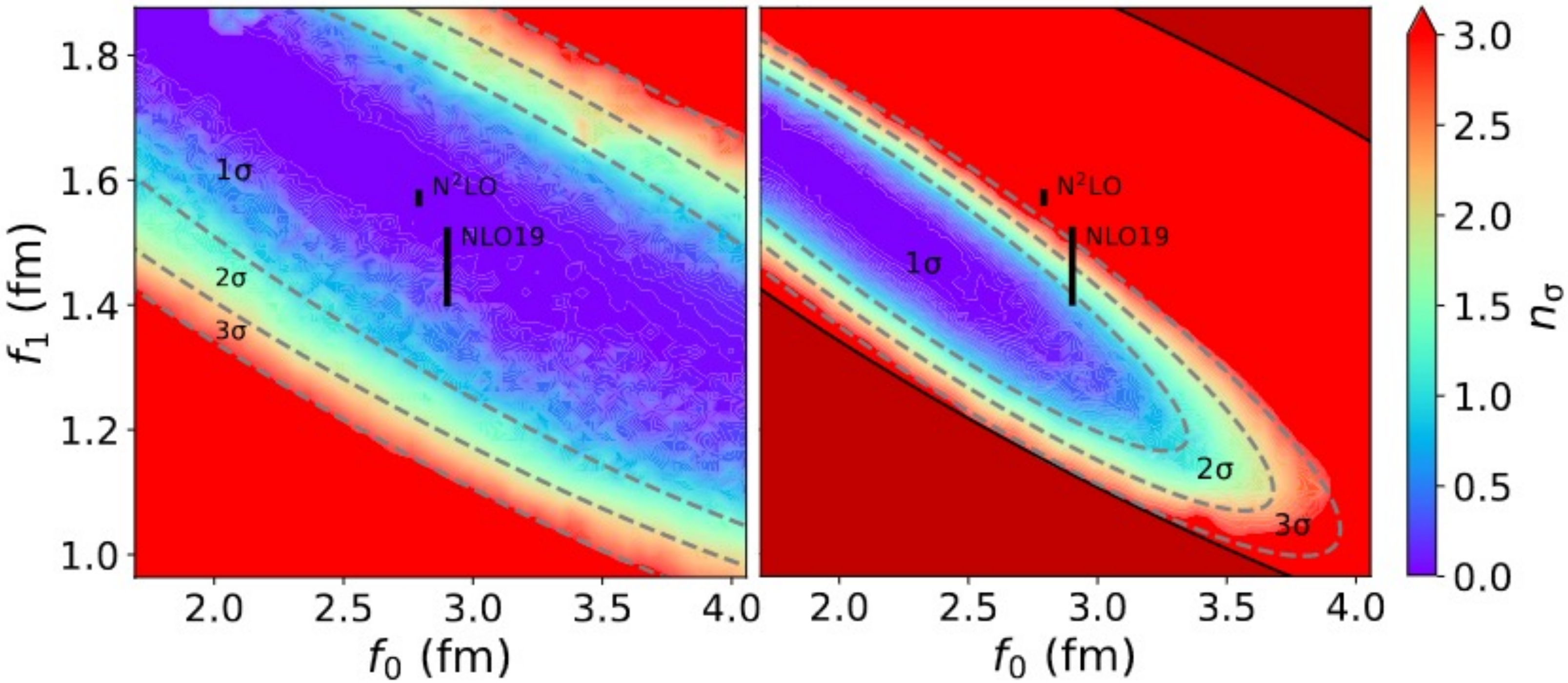


ALICE, PLB (2023)



credit: A. Ramos

First combined analysis of low-energy femtoscopic and scattering data to constrain the s-wave scattering parameters of the Λp interaction



Λp interaction is overall less attractive!

Mihaylov, Haidenbauer and Mantovani-Sarti
PLB 850 (2024) 138550

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H. Polinder, J. Haidenbauer, and U.-G. Meissner, “Hyperon-nucleon interactions - a chiral effective field theory approach”, Nucl. Phys. A779 (2006) 244-266

Other references mentioned in the lecture!