RUB

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Modern Techniques in Hadron Spectroscopy
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Chiral Dynamics of QCD

Concepts

Effective field theory, chiral perturbation theory, renormalization, predictive power, KSW vs Weinberg, power counting…

Methods

Effective Lagrangian, heavy-baryon expansion, perturbative calculation of the amplitude, methods to derive nuclear forces (and currents), …

DFG Deutsche Forschungsgemeinschaft
Bundesministerium für Bildung und Forschung
erc European Research Council

Summary Part I

Underlying theory:

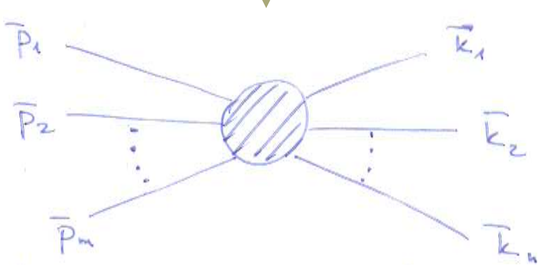
$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2}_{\textit{light}} + \underbrace{\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M^2\Phi^2}_{\textit{heavy}} - \frac{\lambda}{2}\phi^2\Phi$$

For low-energy scattering of ϕ-particles, the heavy field Φ can be integrated out. Alternatively, write down the most general $\mathscr{L}_{\rm eff}$ for ϕ's respecting all ***symmetries*** *of the underlying theory and fix the LECs from data…*

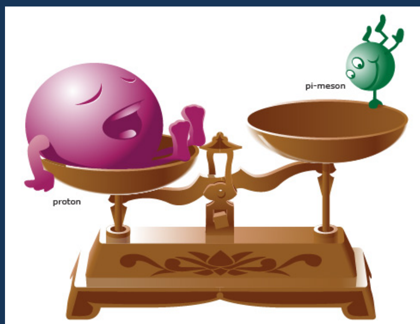
Effective theory:

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{l_1}{4!}\phi^4 - \frac{l_2}{4}(\partial_\mu\phi)(\partial^\mu\phi)\phi^2 + \ldots$$

Scattering amplitude is computed via an expansion in powers of soft momenta. Vertices with more ∂_μ are suppressed when using ***proper renormalization conditions*** *(power counting).*



Part II: Chiral Perturbation Theory



1. Effective Lagrangian for pions
2. From the chiral Lagrangian to S-matrix
3. Inclusion of nucleons: HB ChPT
4. Beyond the HB approach

Selected review articles

Bernard, Kaiser, Meißner, Int. J. Mod. Phys. E4 (1995) 193
Pich, Rep. Prog. Phys. 58 (1995) 563
Bernard, Prog. Part. Nucl. Phys. 60 (2007) 82
Scherer, Prog. Part. Nucl. Phys. 64 (2010) 1

Lecture notes

Scherer, Adv. Nucl. Phys. 27 (2003) 277
Gasser, Lect. Notes Phys. 629 (2004) 1

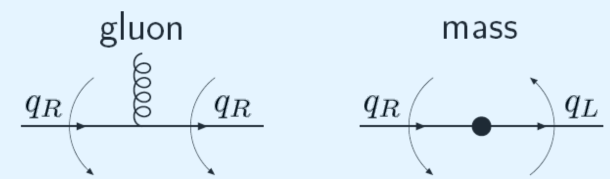
Text book

Scherer, Schindler, *A Primer for Chiral Perturbation Theory*, Lecture Notes in Physics, 2012

Chiral perturbation theory

1. Effective Lagrangian for pions

Chiral symmetry of QCD



$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G_a^{\mu\nu} G_{a,\mu\nu} + \bar{q}(i\gamma^\mu D_\mu - \mathcal{M})q$$

$$= -\frac{1}{4} G_a^{\mu\nu} G_{a,\mu\nu} + \underbrace{\bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R}_{\mathrm{SU(N_f)_L \times SU(N_f)_R}\ \text{invariant}} \underbrace{- q_L \mathcal{M} q_R - q_R \mathcal{M} q_L}_{\text{small}}$$

with $q_L = \frac{1}{2}(1-\gamma_5)q$, $q_R = \frac{1}{2}(1+\gamma_5)q$

$\mathrm{SU(N_f)_L \times SU(N_f)_R}$ invariant

small for $\mathrm{N_f} = 2, (3)$. Indeed: $m_u \sim 3$ MeV, $m_d \sim 5$ MeV ($\overline{\text{MS}}$, $\mu = 2$ GeV)

SSB to $\mathrm{SU(N_f)_V \leq SU(N_f)_L \times SU(N_f)_R}$ $\Rightarrow$ $\mathrm{N_f}^2 - 1$ GBs

Chiral perturbation theory

Ideal world [$m_u = m_d = 0$], zero-energy limit: non-interacting massless GBs (+ strongly interacting massive hadrons)

Real world [$m_u, m_d \ll \Lambda_{QCD}$], low energy: weakly interacting light GBs (pions) (+ strongly interacting massive hadrons)

$\Rightarrow$ expand about the ideal world (ChPT)

Effective Lagrangian for pions

Pions transform linearly under isospin (iso-triplet): $|\pi_1\rangle = \frac{|\pi^+\rangle - |\pi^-\rangle}{\sqrt{2}}\,, \quad |\pi_2\rangle = \frac{|\pi^+\rangle + |\pi^-\rangle}{\sqrt{2}i}\,, \quad |\pi^3\rangle = |\pi^0\rangle$

Pions have to transform nonlinearly under chiral rotations

($SU(2)_L$ x $SU(2)_R$ ~ SO(4) $\Rightarrow$ pion fields as coordinates on a 4-dimensional sphere)

Nonlinear field redefinitions of the kind $\vec{\pi} \to \vec{\pi}' = \vec{\pi}\, F[\vec{\pi}],\ F[0] = 1$ do not change physics

$\Rightarrow$ all nonlinear realizations of χ symmetry are equivalent $\Rightarrow$ use the most convenient one!

Haag '58; Coleman, Callan, Wess, Zumino '69

Example of an explicit construction:

Infinitesimal SO(4) rotation of a 4-vector $(\pi_1,\, \pi_2,\, \pi_3,\, \sigma)$:

$$\begin{pmatrix} \boldsymbol{\pi} \\ \sigma \end{pmatrix} \xrightarrow{SO(4)} \begin{pmatrix} \boldsymbol{\pi}' \\ \sigma' \end{pmatrix} = \left[\mathbf{1}_{4\times 4} + \sum_{i=1}^{3} \theta_i^V V_i + \sum_{i=1}^{3} \theta_i^A A_i \right] \begin{pmatrix} \boldsymbol{\pi} \\ \sigma \end{pmatrix}$$

where:
$$\sum_{i=1}^{3} \theta_i^V V_i = \begin{pmatrix} 0 & -\theta_3^V & \theta_2^V & 0 \\ \theta_3^V & 0 & -\theta_1^V & 0 \\ -\theta_2^V & \theta_1^V & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \sum_{i=1}^{3} \theta_i^A A_i = \begin{pmatrix} 0 & 0 & 0 & \theta_1^A \\ 0 & 0 & 0 & \theta_2^A \\ 0 & 0 & 0 & \theta_3^A \\ -\theta_1^A & -\theta_2^A & -\theta_3^A & 0 \end{pmatrix}$$

Switch to a nonlinear realization: only 3 out of 4 components of the vector $(\boldsymbol{\pi},\, \sigma)$ are independent, i.e. $\boldsymbol{\pi}^2 + \sigma^2 = F^2$

$$\boldsymbol{\pi} \xrightarrow{\theta^V} \boldsymbol{\pi}' = \boldsymbol{\pi} + \boldsymbol{\theta}^V \times \boldsymbol{\pi}\,, \qquad \longleftarrow \textit{linear under } \vec{\theta}^{\,V}$$

$$\boldsymbol{\pi} \xrightarrow{\theta^A} \boldsymbol{\pi}' = \boldsymbol{\pi} + \boldsymbol{\theta}^A \sqrt{F^2 - \boldsymbol{\pi}^2} \qquad \longleftarrow \textit{nonlinear under } \vec{\theta}^{\,A}$$

Effective Lagrangian for pions

Can be rewritten in terms of a 2×2 matrix:

$$U = \frac{1}{F}(\sigma\, \mathbf{1}_{2\times 2} + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}) \xrightarrow{\text{nonlinear realization}} U = \frac{1}{F}\left(\sqrt{F^2 - \boldsymbol{\pi}^2}\, \mathbf{1}_{2\times 2} + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\right) \in SU(2)$$

Chiral rotations: $U \longrightarrow U' = LUR^\dagger$ with $L = \exp[-i(\boldsymbol{\theta}^V - \boldsymbol{\theta}^A)\cdot\boldsymbol{\tau}/2], \quad R = \exp[-i(\boldsymbol{\theta}^V + \boldsymbol{\theta}^A)\cdot\boldsymbol{\tau}/2]$

Derivative expansion for the effective Lagrangian $\mathcal{L}_{\text{eff}} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$

0 derivatives: $UU^\dagger = U^\dagger U = 1$ — irrelevant $\longleftarrow$ only derivative couplings of GBs

2 derivatives: $\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \xrightarrow{g\in G} \mathrm{Tr}(L\partial_\mu U R^\dagger R\, \partial^\mu U^\dagger L^\dagger) = \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$

$$\longrightarrow \quad \mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$$

derivatives act only on the next U

4 derivatives: $[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)]^2, \; \mathrm{Tr}(\partial_\mu U \partial_\nu U^\dagger)\mathrm{Tr}(\partial^\mu U \partial^\nu U^\dagger), \; \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger)$

(terms with $\partial_\mu\partial_\nu U,\ \partial_\mu\partial_\nu\partial_\rho U,\ \partial_\mu\partial_\nu\partial_\rho\partial_\sigma U$ can be eliminated via EOM/partial integration)

…

Chiral symmetry breaking terms

$\delta\mathcal{L}_{\text{QCD}} = -\bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M}^\dagger q_L$ can be made χ-invariant by requiring $\mathcal{M} \to L\mathcal{M}R^\dagger$

$\Rightarrow$ construct all possible χ-invariant terms involving $\mathcal{M}$ and freeze out $\mathcal{M}$ at the end

LO term: $\delta\mathcal{L}_{\text{SB}} = \frac{BF^2}{2}\mathrm{Tr}[\mathcal{M}U^\dagger + U\mathcal{M}^\dagger] = 2BF^2 m_q - Bm_q\vec{\pi}^2 + \ldots \longrightarrow M_\pi^2 = 2m_q B + \mathcal{O}(m_q^2)$

Effective Lagrangian for pions

The leading and subleading effective Lagrangians for pions

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + 2B(\mathcal{M}U + \mathcal{M}U^\dagger)\rangle\,,$$

$$\mathcal{L}_\pi^{(4)} = \frac{l_1}{4}\langle \partial_\mu U \partial^\mu U^\dagger\rangle^2 + \frac{l_2}{4}\langle \partial_\mu U \partial_\nu U^\dagger\rangle\langle \partial^\mu U \partial^\nu U^\dagger\rangle + \frac{l_3}{16}\langle 2B\mathcal{M}(U+U^\dagger)\rangle^2 + \ldots$$
$$- \frac{l_7}{16}\langle 2B\mathcal{M}(U-U^\dagger)\rangle^2$$

Gasser, Leutwyler '84

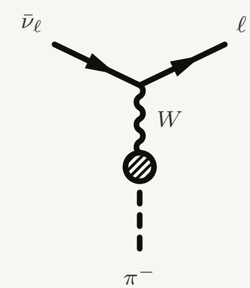
terms involving external fields

Low-energy constants of $\mathcal{L}_\pi^{(2)}$

- F is related to the pion decay constant F_π: $\langle 0|J^i_{A_\mu}(0)|\pi^j(\vec{p})\rangle = i p_\mu F_\pi \delta^{ij}$

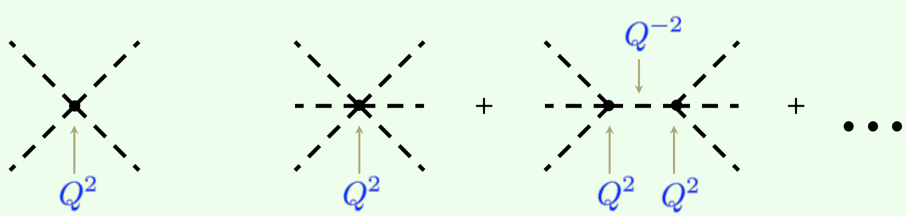
  axial current from $\mathcal{L}_\pi^{(2)}$: $J^i_{A\mu} = i\mathrm{Tr}[\tau^i(U^\dagger\partial_\mu U - U\partial_\mu U^\dagger)] = -F\partial_\mu\pi^i + \ldots$

  $\longrightarrow$ F is F_π in the chiral limit: $F_\pi = F + \mathcal{O}(m_q) \simeq 92.4\ \mathrm{MeV}$
- B is related to the chiral quark condensate



Tree-level multi-pion connected diagrams from $\mathcal{L}_\pi^{(2)}$

$$U(\boldsymbol{\pi}) = \mathbf{1}_{2\times 2} + i\frac{\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{F} - \frac{\boldsymbol{\pi}^2}{2F^2} - i\alpha\frac{\boldsymbol{\pi}^2\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{F^3} + \mathcal{O}(\boldsymbol{\pi}^4) \longrightarrow \mathcal{L}_\pi^{(2)} = \frac{\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}}{2} - \frac{M^2\boldsymbol{\pi}^2}{2} + \frac{(\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\pi})^2}{2F^2} - \frac{M^2\boldsymbol{\pi}^4}{8F^2} + \ldots$$



- all diagrams scale as Q^2
- insertions from $\mathcal{L}_\pi^{(4)}$, $\mathcal{L}_\pi^{(6)}$, ... suppressed by powers of Q^2
- remarkable predictive power!

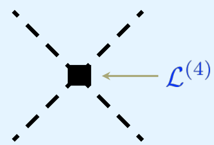
Scattering amplitude

2. From the chiral Lagrangian to S-matrix

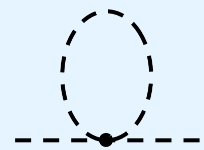
Scattering amplitude is obtained via an expansion in Q/Λ_χ.

Power counting for GBs:

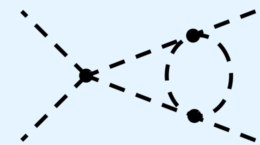
$$D = 2 + 2L + \sum_d N_d(d-2)$$



$D = 2+0+2 = 4$



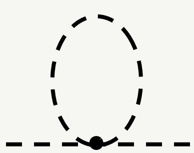
$D = 2+2+0 = 4$



$D = 2+4+0 = 6$

But what is the value of Λ_χ?

- Chiral expansion breaks down for $E \sim M_\rho \longrightarrow \Lambda_\chi \sim M_\rho = 770$ MeV
- An upper bound for Λ_χ from pion loops: $\Lambda_\chi \sim 4\pi F_\pi$ Manohar, Georgi '84



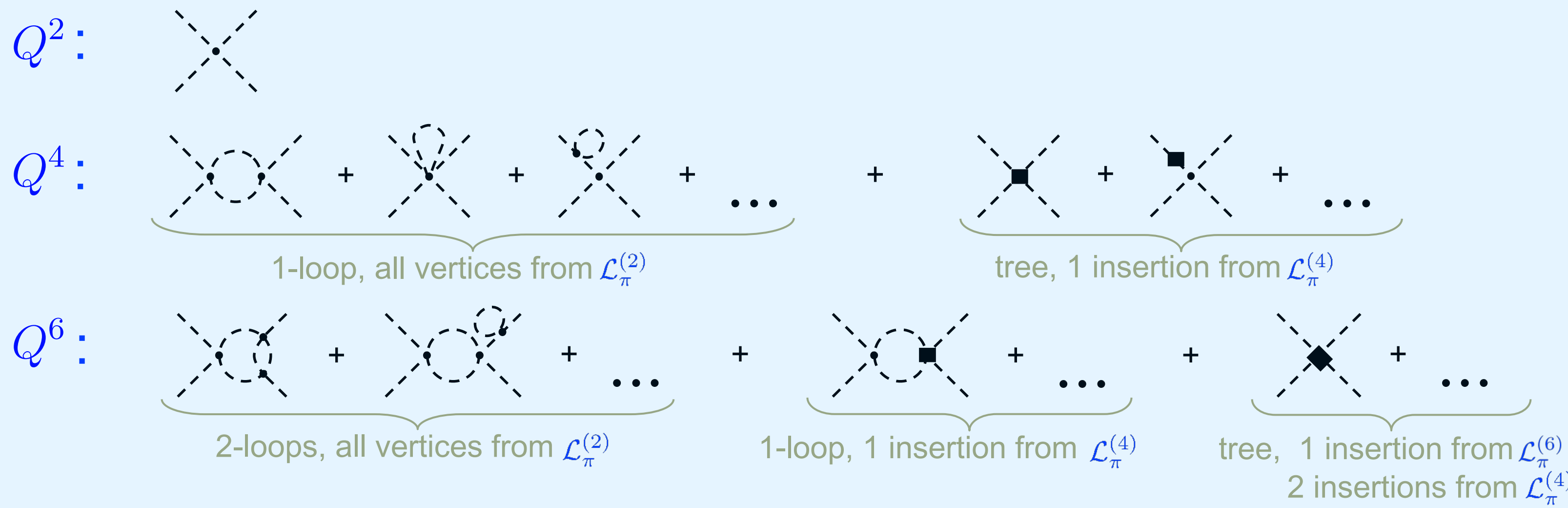
$$\frac{M^2}{F^2}\int \frac{d^4 l}{(2\pi)^4}\frac{i}{l^2 - M^2 + i\epsilon} \xrightarrow{\text{DR}} M^2 \frac{M^2}{(4\pi F)^2}\left[\ln\frac{M^2}{\mu^2} + 2\mu^{d-4}\left(\frac{1}{d-4} + \text{const}\right)\right]$$

angular integration in 4 dimensions

dimensional argument

$$\int \frac{d^d l}{(2\pi)^d} = \int \frac{d\Omega_d}{(2\pi)^d}\int l^{d-1} dl = \frac{1}{2^{d-1}\pi^{d/2}\Gamma(d/2)}\int l^{d-1} dl \xrightarrow{d\to 4} \frac{2}{(4\pi)^2}\int l^3 dl$$

Pion scattering length in ChPT



Predictive power?

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \mathcal{L}_\pi^{(6)} + \ldots$$

# of LECs increasing…

S-wave $\pi\pi$ scattering length

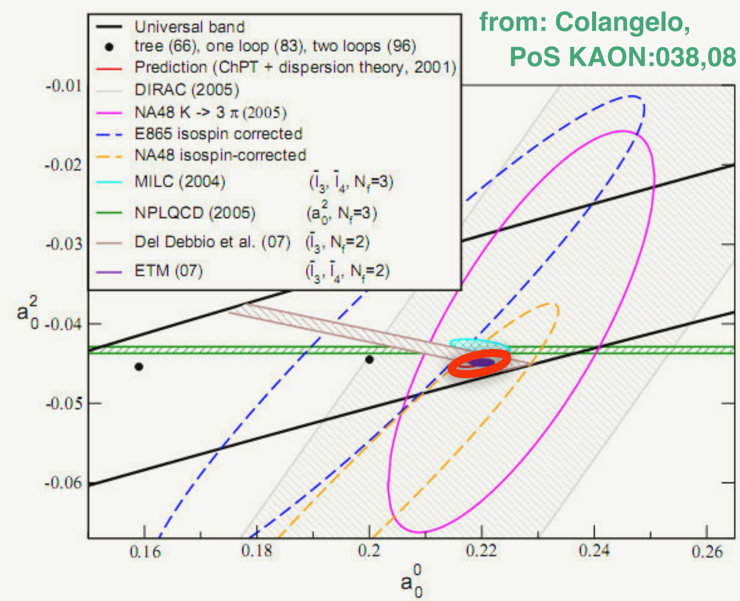
LO: $a_0^0 = 0.16$ [Weinberg '66]

NLO: $a_0^0 = 0.20$ [Gasser, Leutwyler '83]

NNLO: $a_0^0 = 0.217$ [Bijnens et al. '95]

NNLO + disp. relations: [Colangelo et al.]

$$a_0^0 = 0.217 \pm 0.008\,(\text{exp}) \pm 0.006\,(\text{th})$$



from: Colangelo, PoS KAON:038,08

Inclusion of nucleons

3. Inclusion of the nucleons: HB ChPT

In the baryon sector, it is more convenient to work with u defined via $U =: u^2$. Then:

$$u \longrightarrow u' = \sqrt{RUL^\dagger} =: RuK^{-1} \quad \Rightarrow \quad K = (\sqrt{RUL^\dagger})^{-1} R \sqrt{U}$$

Notice: the transformation property of u can also be written as $u \longrightarrow u' = KuL^\dagger$.

The so-called compensator field K is a complicated SU(2)-valued function of θ^L, θ^R, U (and thus of space-time), K = K(L,R,U), except for isospin (i.e., vector) rotations with $\theta^L = \theta^R = \theta^V$:

$$K(V, V, U) = V$$

Then, one **defines** the transformation properties of the nucleon fields via: $N \longrightarrow N' = KN$

The Coleman-Callan-Wess-Zumino (CCWZ) nonlinear realization of the chiral group:

$$\begin{pmatrix} U \\ N \end{pmatrix} \xrightarrow{g} \begin{pmatrix} U' \\ N' \end{pmatrix} = \begin{pmatrix} RUL^\dagger \\ K(L,R,U)N \end{pmatrix}$$

[see, e.g., the book by Scherer and Schindler]

To construct the effective Lagrangian, one uses building blocks which transform covariantly with respect to $SU(2)_R \times SU(2)_L$

$$N \longrightarrow N' = KN, \quad O_i \longrightarrow O_i' = KO_iK^{-1} = KO_iK^\dagger$$

to write terms like: $\bar{N} O_1 \ldots O_n N \ \ \mathrm{Tr}\,(O_{n+1} \ldots O_m) \ \ldots \ \mathrm{Tr}\,(O_{m+1} \ldots O_k)$

Inclusion of nucleons

Covariant derivatives of the nucleon and pion fields:

$$D_\mu N := (\partial_\mu + \Gamma_\mu)\, N, \qquad \Gamma_\mu := \frac{1}{2}\left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right)$$

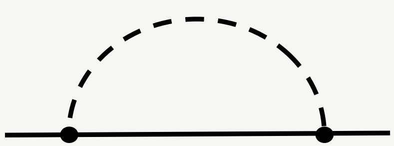
$$u_\mu := i u^\dagger (\partial_\mu U)\, u^\dagger$$

homework problem: verify $D_\mu \to K D_\mu$, $u_\mu \to K u_\mu K^{-1}$

$$\longrightarrow \quad \mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \ldots\,, \qquad \mathcal{L}_{\pi N}^{(1)} = \bar{N}\left(i\gamma^\mu D_\mu - m + \frac{\mathring{g}_A}{2}\gamma^\mu \gamma_5 u_\mu\right) N$$

where $g_A = G_A(0)$ is axial charge of the nucleon

Problem (?): new hard mass scale *m* $\longrightarrow$ power counting ??



$$\longrightarrow \quad \delta m \xrightarrow{\mathcal{M}\to 0} -m\,\frac{3 g_A^2 m^2}{(4\pi F)^2}\left[\log\frac{m}{\mu} + \mu^{d-4}\left(\frac{1}{d-4} + \text{const}\right)\right]$$

$$\frac{m_N}{4\pi F_\pi} \sim 1$$

divergence has to be absorbed by m from the LO Lagrangian…

Inclusion of nucleons

Making power counting manifest: The Heavy-Baryon approach

Jenkins & Manohar '91; Bernard, Keiser, Meißner '92; Mannel, Roberts, Ryzak '92

Write the nucleon momentum as $p^\mu = m v^\mu + l^\mu$ with $v^2 = 1$ and $l_\mu \ll m$

Split the nucleon fields into $N_v = e^{imv\cdot x} P_v^+ N, \quad h_v = e^{imv\cdot x} P_v^- N$ with $P_v^\pm := \frac{1 \pm \not{v}}{2}$

For the free Dirac Lagrangian, one then obtains:

$$\bar{N}(i\not{\partial} - m)N = \ldots = \bar{N}_v iv\cdot\partial N_v - \bar{h}_v(iv\cdot\partial + 2m)h_v + \bar{N}_v i\underbrace{\partial_\perp}_{A_\perp := A - (v\cdot A)v} h_v + \bar{h}_v i\partial_\perp N_v$$

⇒ the small component h_v behaves as a heavy field and can be integrated out:

$$\boxed{\mathcal{L}^{(1)}_{\pi N} = \bar{N}_v \left(iv\cdot D + g_A S\cdot u\right) N_v \; + \; \mathcal{O}(m^{-1})} \quad \text{where} \quad S_\mu \equiv i\gamma_5 \sigma_{\mu\nu} v^\nu$$

HB propagator: $S(p) = \frac{i}{v\cdot p + i\epsilon} \quad \Rightarrow \quad S(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p_0 + i\epsilon} e^{-ip\cdot(x-y)} = \theta(x_0 - y_0)\delta^3(\vec{x} - \vec{y})$

In the HB formulation, the nucleon mass does not appear in the propagator and contributes only through 1/m corrections to vertices ⇒ power counting is manifest!

Connected 1N diagrams scale as Q^D with $D = 1 + 2L + \sum_i V_i \Delta_i, \quad \Delta_i = -2 + \frac{n_i}{2} + d_i$

Or use covariant BChPT with PC enforced through renormalization conditions Becher, Leutwyler; Gegelia et al.

Pion-nucleon scattering

Effective chiral Lagrangian:

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

low-energy constants

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N}\left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\right)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c_i}\bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d_i}\bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \underbrace{\sum_i \boldsymbol{e_i}\bar{N}\hat{O}_i^{(4)}[\pi]N}_{\mathcal{L}_{\pi N}^{(4)}} + \ldots$$

Pion-nucleon scattering amplitude for $\pi^a(q_1) + N(p_1) \to \pi^b(q_2) + N(p_2)$:

$$T_{\pi N}^{ba} = \frac{E+m}{2m}\left(\delta^{ba}\left[g^+(\omega,t) + i\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\, h^+(\omega,t)\right] + i\epsilon^{bac}\tau^c\left[g^-(\omega,t) + i\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\, h^-(\omega,t)\right]\right)$$

calculated within the chiral expansion

Pion-nucleon scattering up to Q⁴ in heavy-baryon ChPT

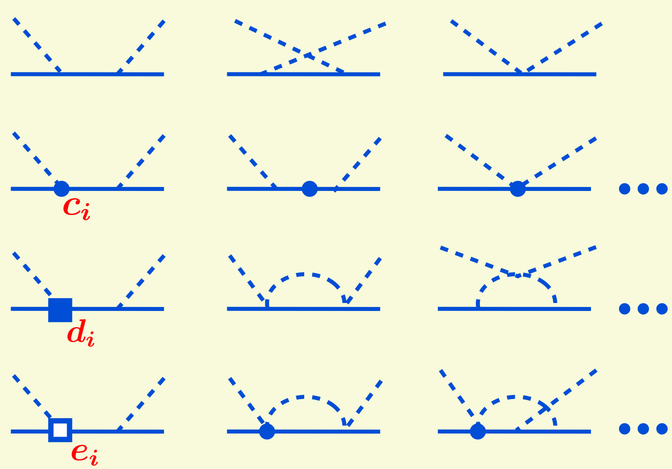
Fettes, Meißner '00; Krebs, Gasparyan, EE '12

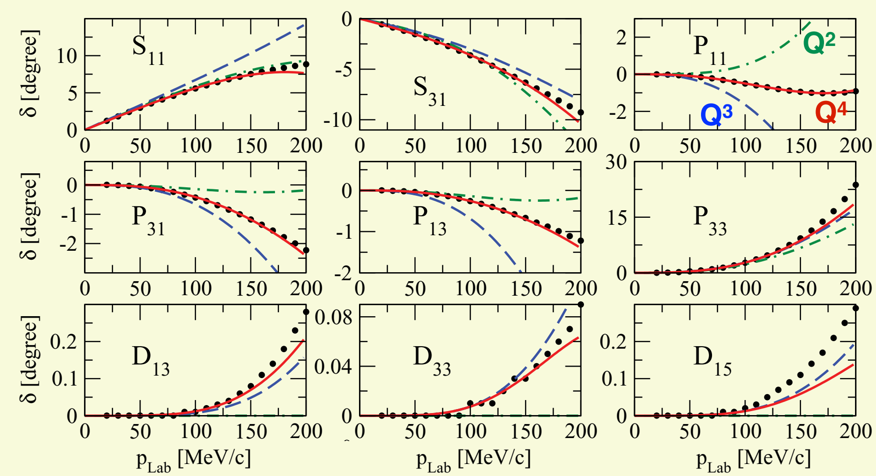
Order Q:

Order Q²:

Order Q³:

Order Q⁴:

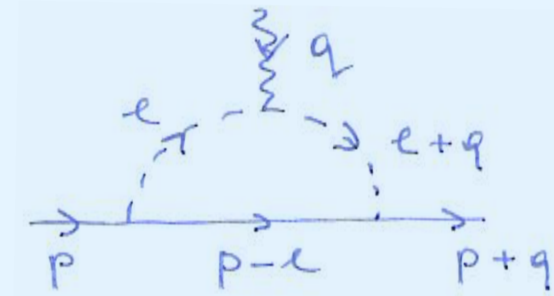




Limitations of the HB approach

4. Beyond the HB approach

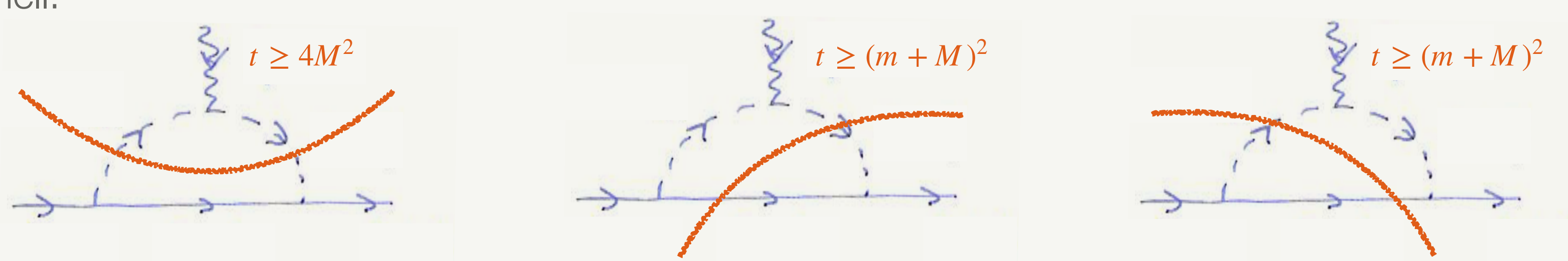
Consider the scalar form factor of the nucleon. Using relativistic nucleon propagators (i.e., no 1/m-expansion) and ignoring the vertex structure, the amplitude is:



$$J = i\int \frac{d^4 l}{(2\pi)^4}\frac{1}{[l^2 - M^2 + i\epsilon][(l+q)^2 - M^2 + i\epsilon][(p-l)^2 - m^2 + i\epsilon]}$$

For on-shell nucleons ($p^2 = (p+q)^2 = m^2$), we have $J = J(t)$ with $t = q^2$.

The amplitude develops an imaginary part when two intermediate particles can become on-shell:



Instead of calculating $J(t)$ (tedious…), one can compute $\mathrm{Im}[J(t)]$ using Cutkosky cutting rules (i.e., replace the cut propagators by the δ-functions enforcing the on-shell relation, see, e.g., Schröder & Peskin)

Limitations of the HB approach

For $t \geq 4M^2$ (1st graph): $\mathrm{Im}[J(t)] = \frac{1}{8\pi}\frac{1}{\sqrt{t(4m^2-t)}}\arctan\left[\frac{\sqrt{(t-4M^2)(4m^2-t)}}{t-2M^2}\right]$

We now perform the $1/m$-expansion of this result for $t = \mathcal{O}(M^2) \equiv \mathcal{O}(Q^2)$:

$$\frac{\sqrt{(t-4M^2)(4m^2-t)}}{t-2M^2} \sim \mathcal{O}\left(\frac{m}{Q}\right) \gg 1 \quad \Rightarrow \quad \arctan(x) \overset{x\gg 1}{\longrightarrow} \underbrace{\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} + \ldots}_{\text{converges for } x > 1}$$

This expansion (+ pre-factor) is the HB expansion: $\mathrm{Im}[J(t)] \overset{\text{HB}}{=} \frac{1}{8\pi}\frac{1}{2m\sqrt{t}}\frac{\pi}{2} + \mathcal{O}(m^0)$

Problem: it breaks down in the vicinity of $t \approx 4M^2$: $x < 1 \Rightarrow 4M^2 \leq t \lesssim 4M^2\left(1 + \frac{M^2}{m^2}\right)$

Solution 1: Infrared Regularization Becher, Leutwyler '99

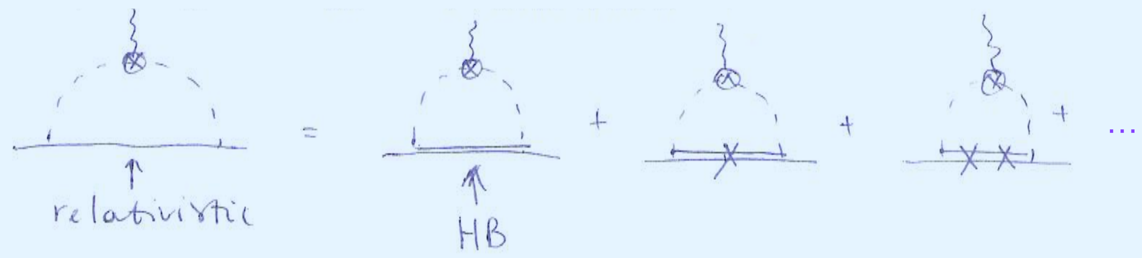
$$i\frac{\not p + m}{p^2 - m^2 + i\epsilon} = i\frac{\not p + m}{2mv\cdot l + l^2 + i\epsilon} = i\frac{\not p + m}{2mv\cdot l + i\epsilon}\frac{1}{1 + \frac{l^2}{2mv\cdot l + i\epsilon}}$$

$$= \frac{\not p + m}{2m}\frac{i}{v\cdot l + i\epsilon}\left[1 + \underbrace{i\frac{l^2}{2m}}_{\text{vertex from } \mathscr{L}_{\pi N}^{(2)}}\frac{i}{v\cdot l + i\epsilon} + \left(i\frac{l^2}{2m}\frac{i}{v\cdot l + i\epsilon}\right)^2 + \ldots\right]$$

use: $p = mv + l$

Limitations of the HB approach

The IR approach: expand the integrand in 1/m, compute the integrals using DR (power counting manifest) and resum all contributions… Schindler et al.'03



At the 1-loop level can also be realized by selecting out the infrared singular parts of the integrals. Becher, Leutwyler '99

Disadvantage: violates the analytic structure of the amplitude (for hard momenta).

Solution 2: The Extended On-Mass-Shell (EOMS) Renormalization scheme

Gegelia, Japaridze '99; Fuchs et al. '03

Main idea: work in the manifestly covariant approach and restore chiral power counting by using appropriate renormalization conditions (i.e., perform additional finite subtractions of PC violating terms involving positive powers of the nucleon mass)

This is nowadays the standard approach for baryon ChPT…

Summary Part II

Part II: Chiral Perturbation Theory

— ChPT = low-energy EFT of QCD

— For GB and 1N processes, the amplitude is calculable perturbatively via an expansion in powers of $|\vec{p}| \sim M_\pi \equiv Q$ (thanks to the spontaneously broken chiral symmetry)

— In the 1N sector, additional efforts are needed to maintain the power counting: HB ChPT or covariant BChPT (using IR or EOMS)

Notice: Interactions between two nucleons are **not** suppressed at low-energy and must be re-summed non-perturbatively

$\Rightarrow$ π-less EFT $(p \ll M_\pi)$ or χEFT $(p \sim M_\pi)$

For an introduction see EE, *Nuclear Forces from Chiral Effective Field Theory: A Primer*, e-Print: 1001.3229