Modern Techniques in Hadron Spectroscopy

July, 2024

QCD Phenomenology -- Models

Eric Swanson

quark confinement -- definitions?

(i) the absence of free quarks in Nature

[but quarks could combine with a fundamental coloured scalar]

(ii) observable particles are colour singlets

[but this confuses confinement and screening (Higgs phase)]

(iii) quarks interact with a long range linear interaction

[obfuscated by string breaking]

(iv) the work required to separate quarks grows linearly as one takes the quark mass to infinity

[ok, but removes quarks from the definition!]

skip>>

more on confinement

$$H = \frac{1}{2}\int d\mathbf{x}\,(E^2 + B^2) - \frac{1}{2}\int d\mathbf{x}d\mathbf{y}\, f^{abc} E^b(\mathbf{x}) A^c(\mathbf{x}) \langle \mathbf{x}a| \frac{g}{\nabla\cdot D}\nabla^2 \frac{g}{\nabla\cdot D} |\mathbf{y}d\rangle f^{def} E^e(\mathbf{y}) A^f(\mathbf{y})$$

$$K(\mathbf{x}-\mathbf{y};\mathbf{A})$$

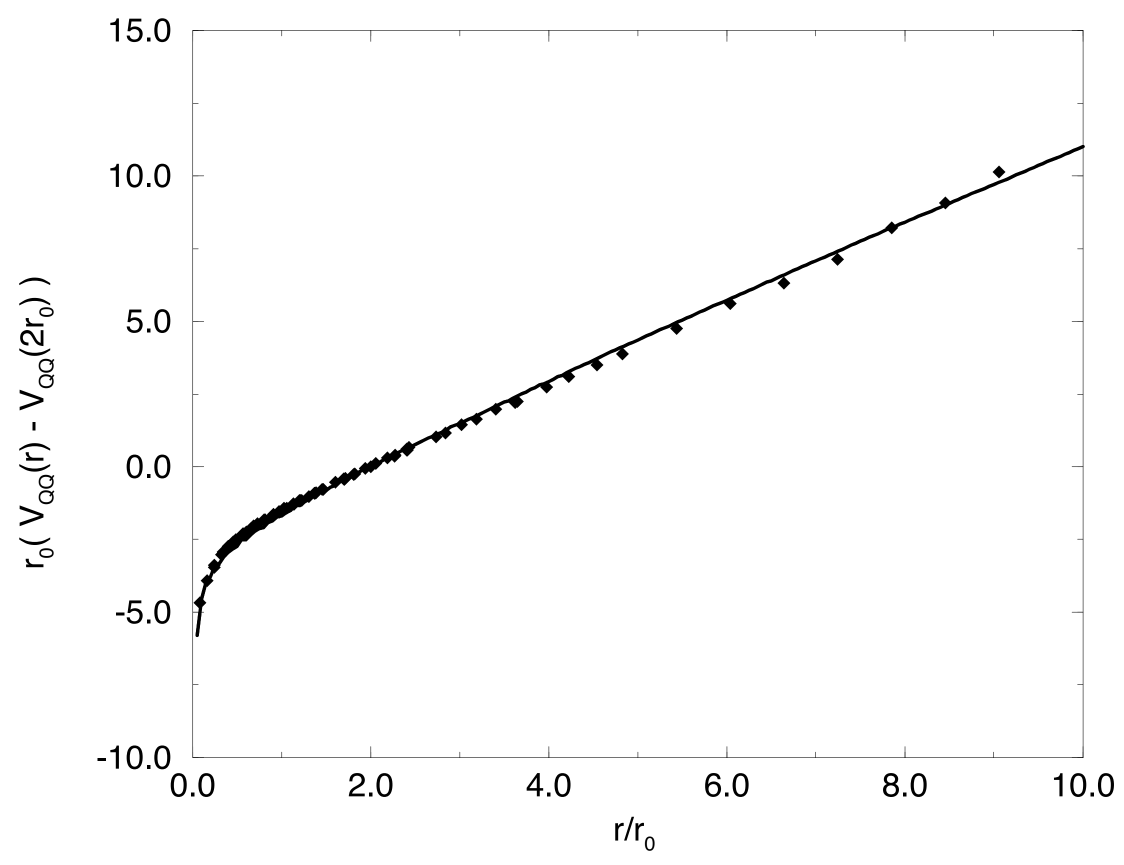
an instantaneous potential that depends on the gauge potential

K generates the beta function

K is renormalization group invariant

K is an upper limit to the Wilson loop potential

K is infrared enhanced at the Gribov boundary



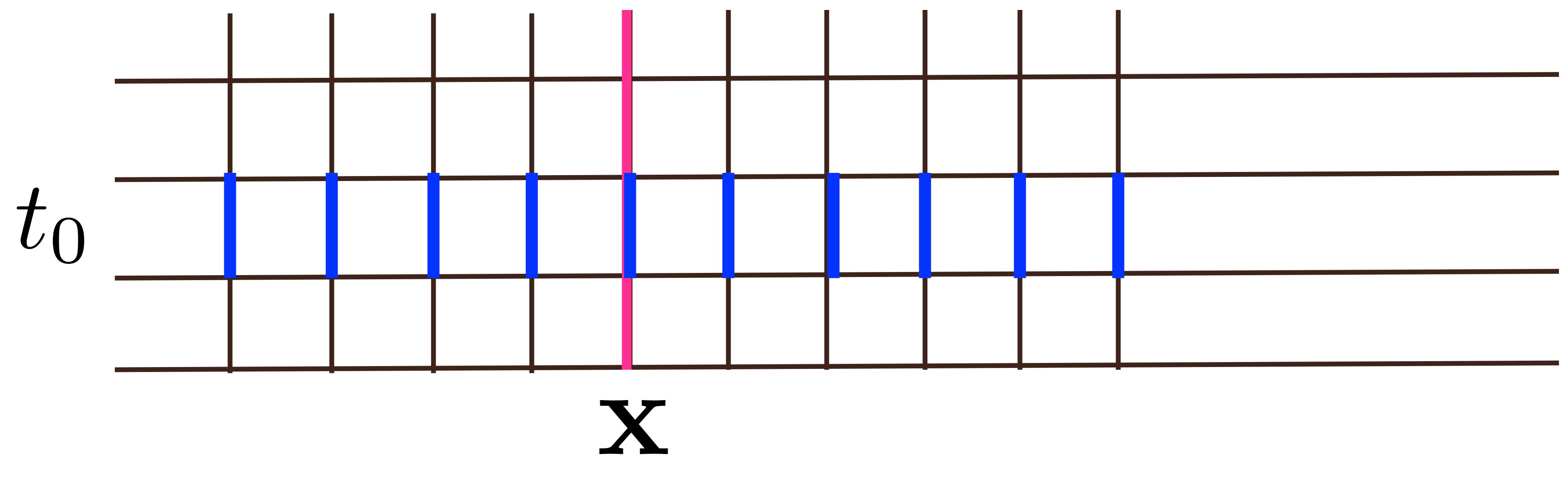
Szczepaniak & Swanson

more on confinement

$$U_t(t_0, \mathbf{x}) \to z U_t(t_0, \mathbf{x}) \quad \forall \mathbf{x}$$

$z \in Z_N$

a global symmetry of QCD



$$P(\mathbf{x}) \to z P(\mathbf{x})$$

more on confinement

either $\langle P(\mathbf{x})\rangle = 0$ symmetric Z_N phase
or $\langle P(\mathbf{x})\rangle \neq 0$ broken Z_N phase

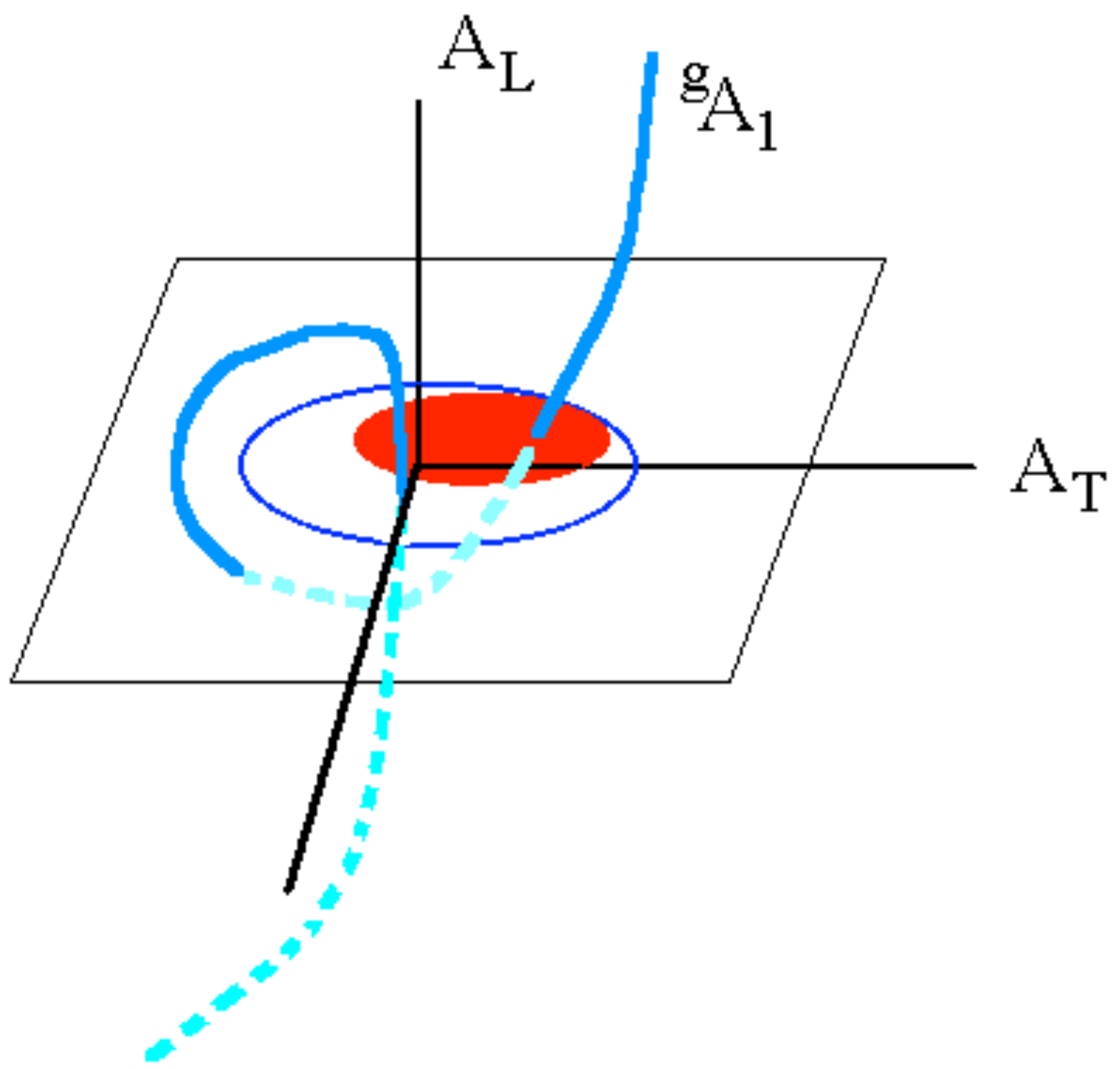
$$\langle P(\mathbf{x})\rangle = \mathrm{e}^{-F_q T}$$

confinement iff QCD is in the symmetric Z_N phase

more on confinement

Coulomb gauge and the Gribov problem

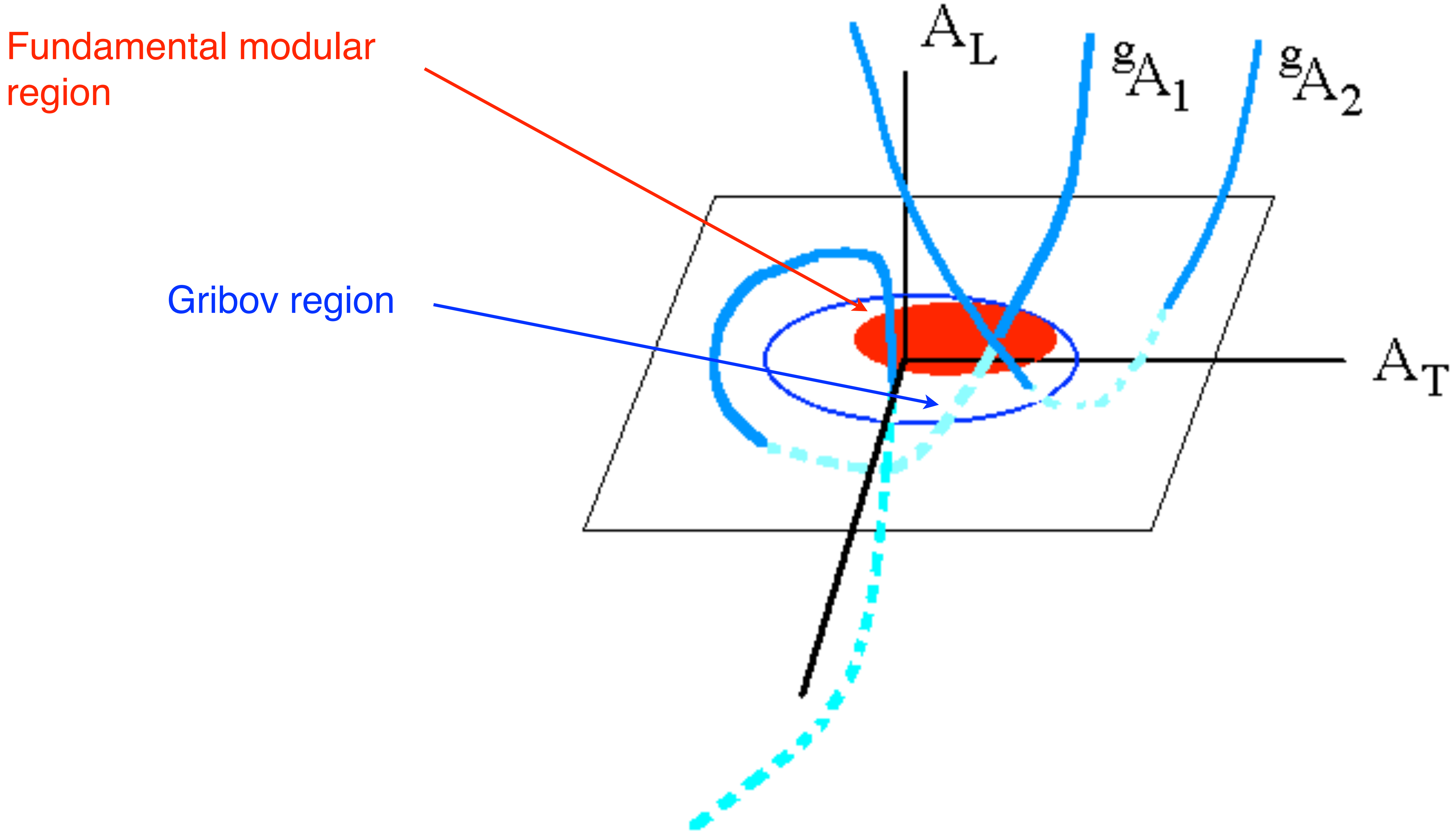
$\nabla \cdot \vec{A}^a = 0 \qquad \det(\nabla \cdot D) = 0$



more on confinement

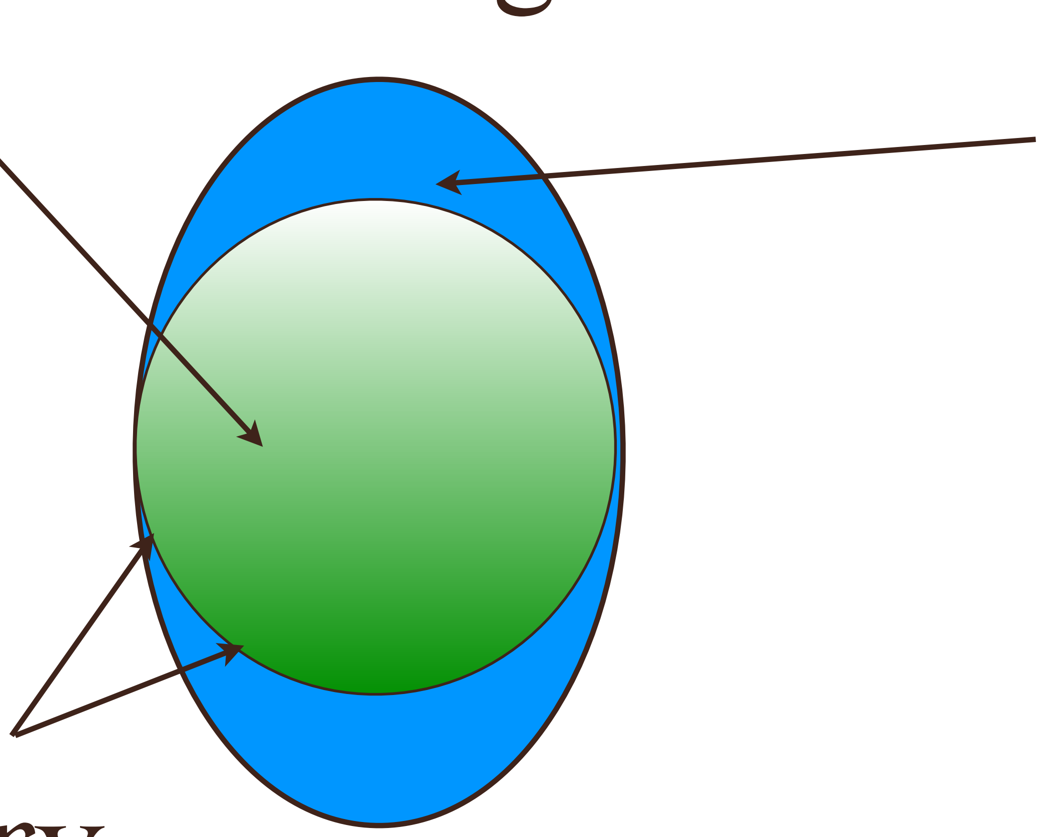
Coulomb gauge and the Gribov problem

$$\nabla \cdot \vec{A}^a = 0 \qquad \det(\nabla \cdot D) = 0$$



more on confinement

Coulomb gauge and the Gribov problem



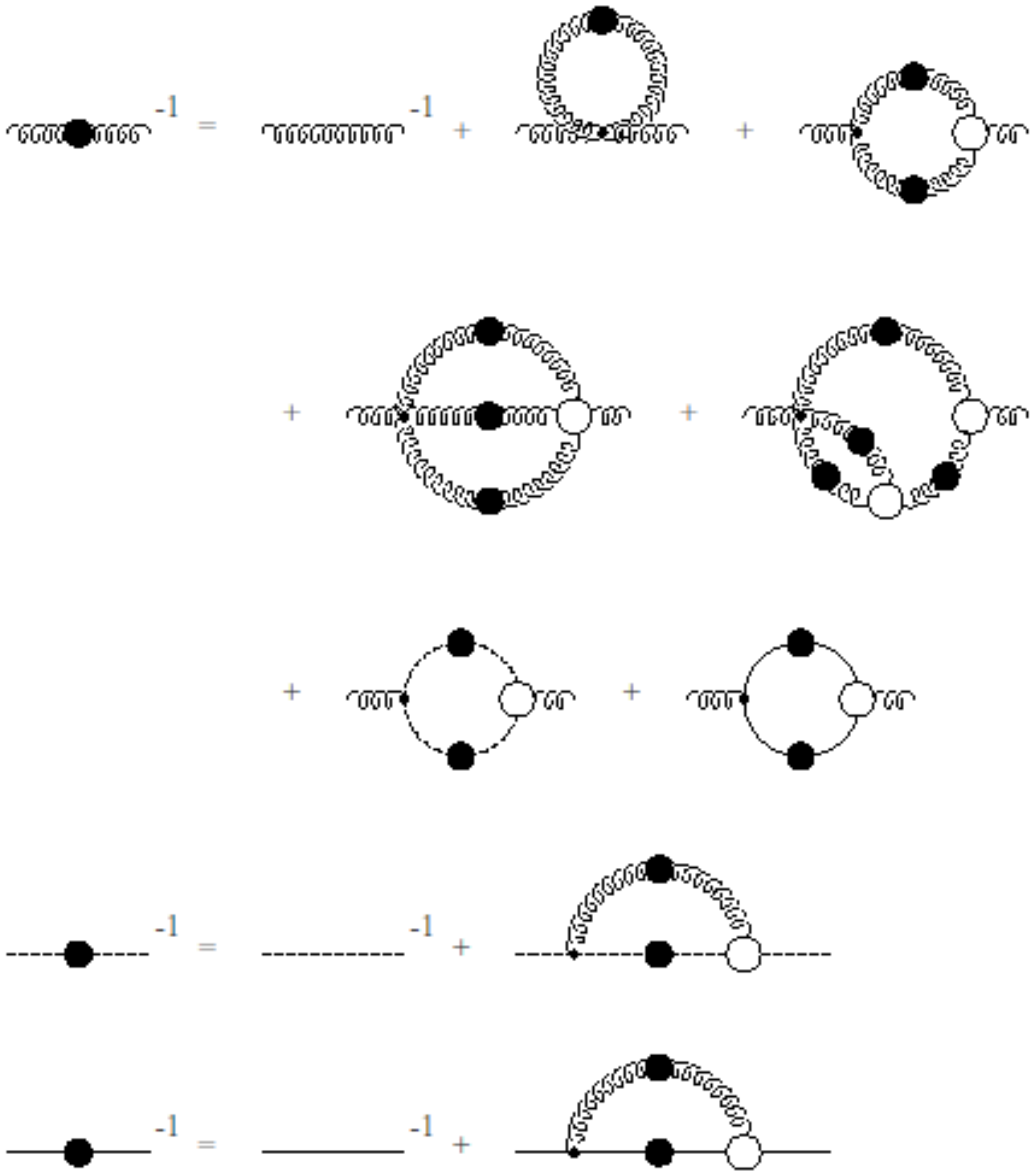
$\det(\nabla \cdot D) = 0$

FMR is convex
GR contains the FMR
FMR contains A=0

physics lies at the intersection of the FMR and GR

Zwanziger; van Baal

more on confinement



$$D(p) = -\frac{1}{p^2}\frac{1}{1+u(p)}$$

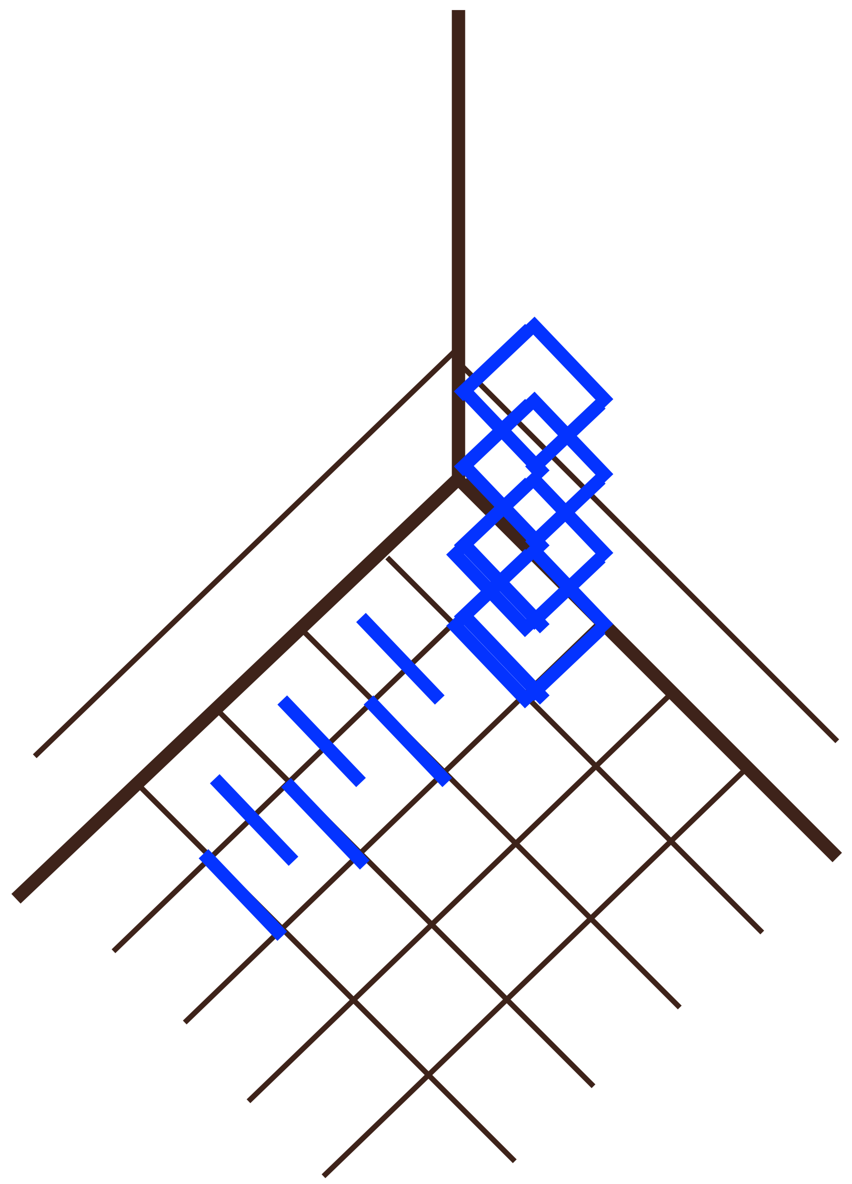
$$u(p) \to -1 \qquad p \to 0$$

Kugo-Ojima confinement criterion

Alkofer, von Smekal, Fischer

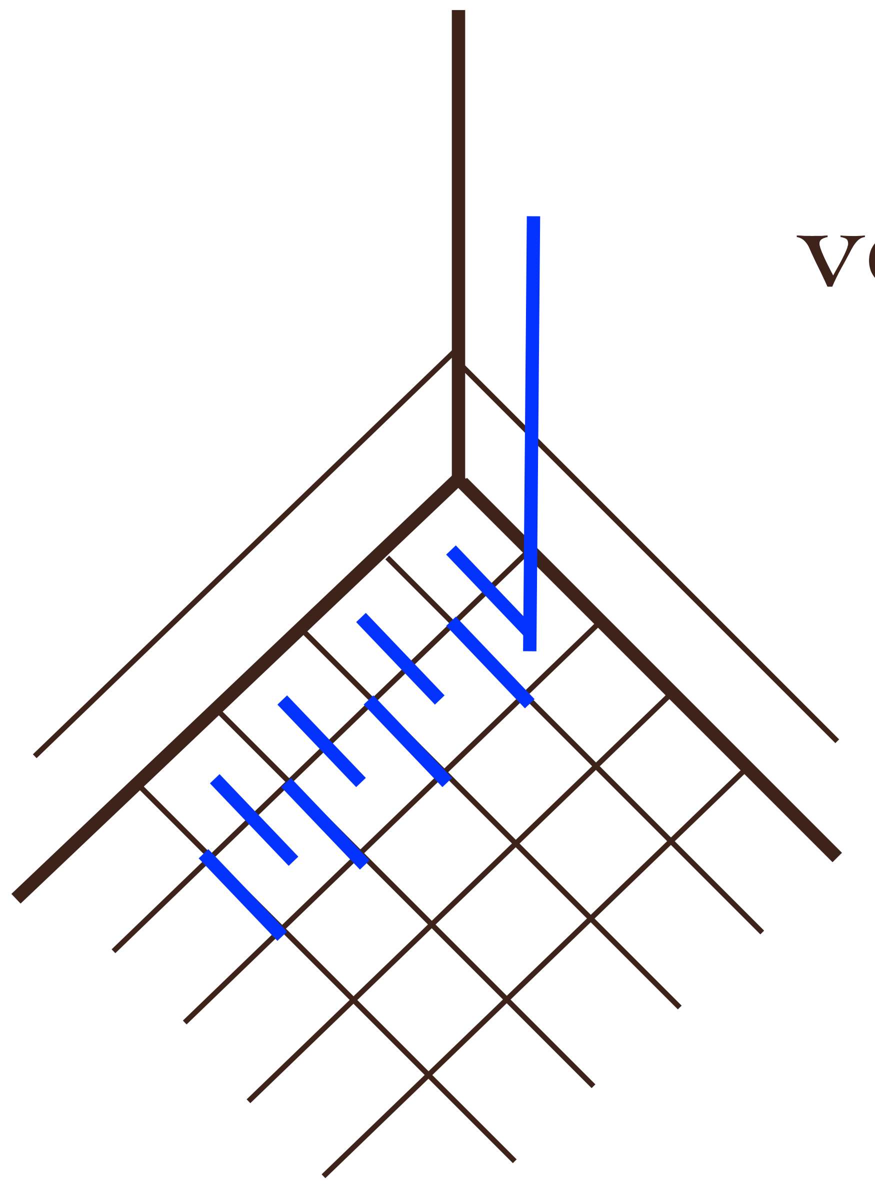
Vortex driven confinement

make a singular gauge transformation



Vortex driven confinement

make a singular gauge transformation (= z phase business)



vortex (locates infinite field strength
caused by the sgt)

Vortex driven confinement

vortex = localized field configuration that 'percolates' the lattice, this can provide confinement since it gives an 'area law'

$$\langle \mathrm{tr} U_{LR}(C) \rangle = \langle \mathrm{tr} \prod_{i=1}^{A/A_0} Z(i) \rangle$$

$$\langle Z(1) Z(2) \rangle \sim \langle Z(1) \rangle \langle Z(2) \rangle$$

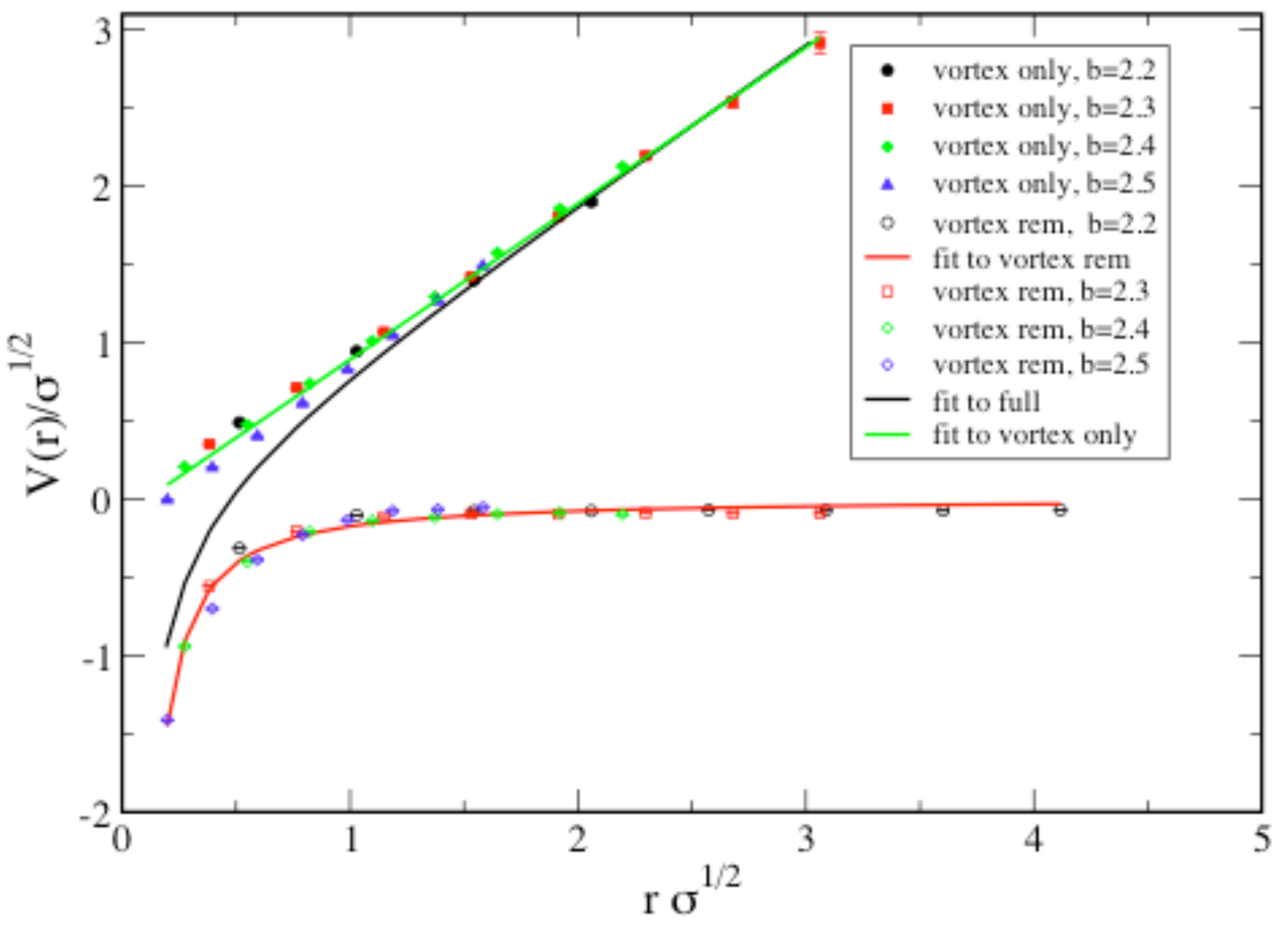
$$= \mathrm{tr} \prod_{i=1}^{A/A_0} \langle Z(i) \rangle$$

$$= \mathrm{e}^{-\sigma A}$$

$$\sigma = -\frac{\log \langle Z(1) \rangle}{A_0}$$

more on confinement

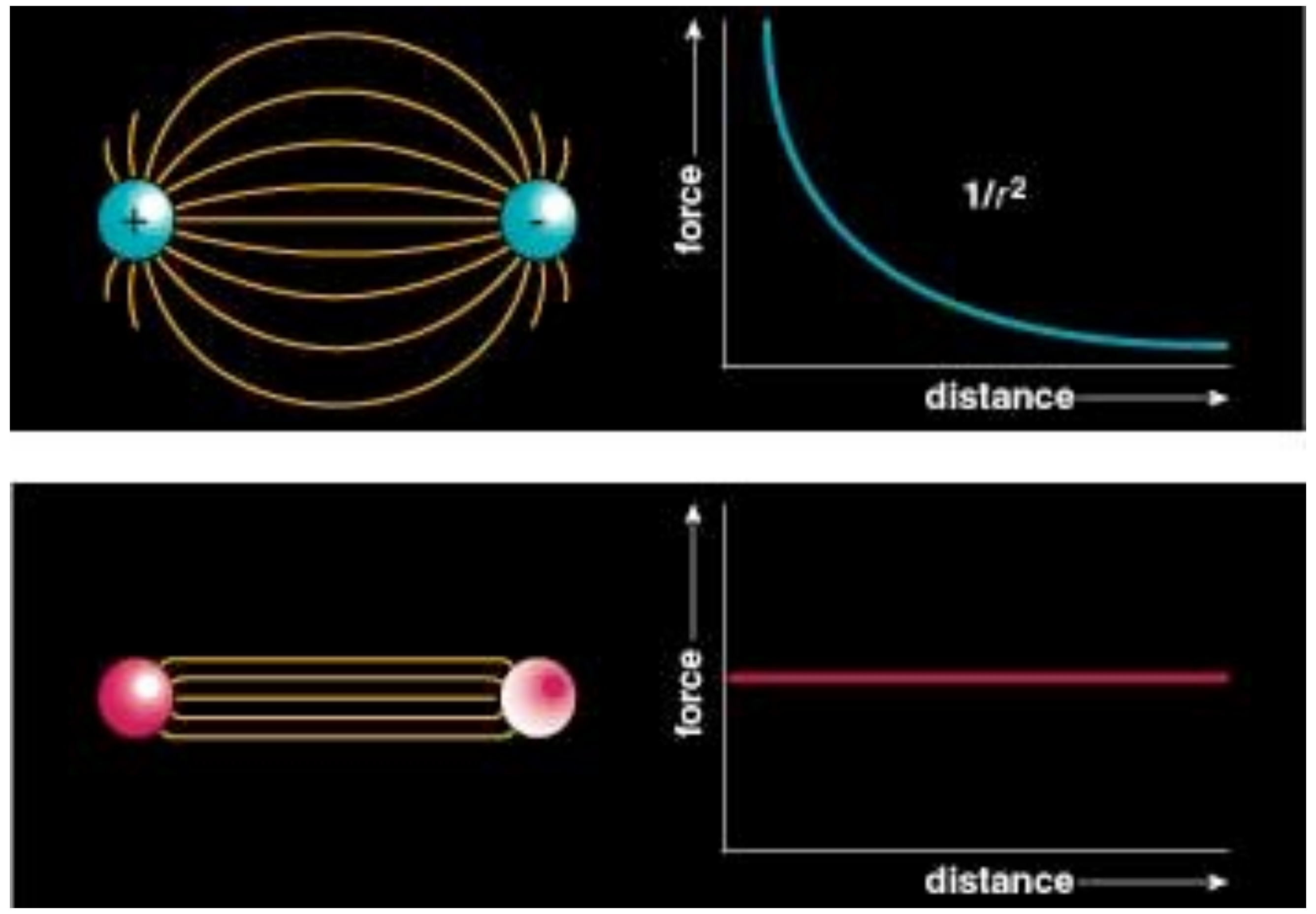
vortices!

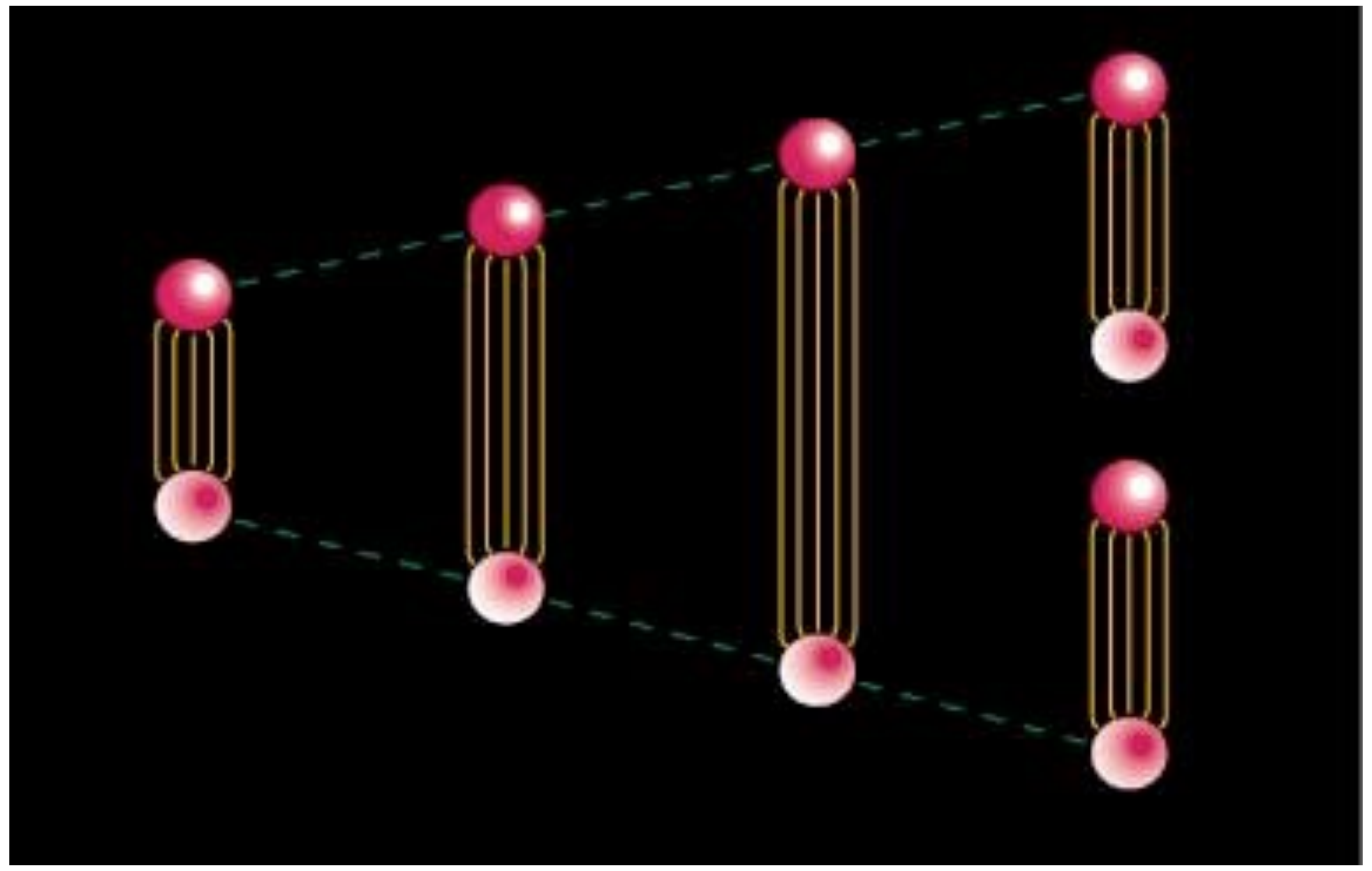


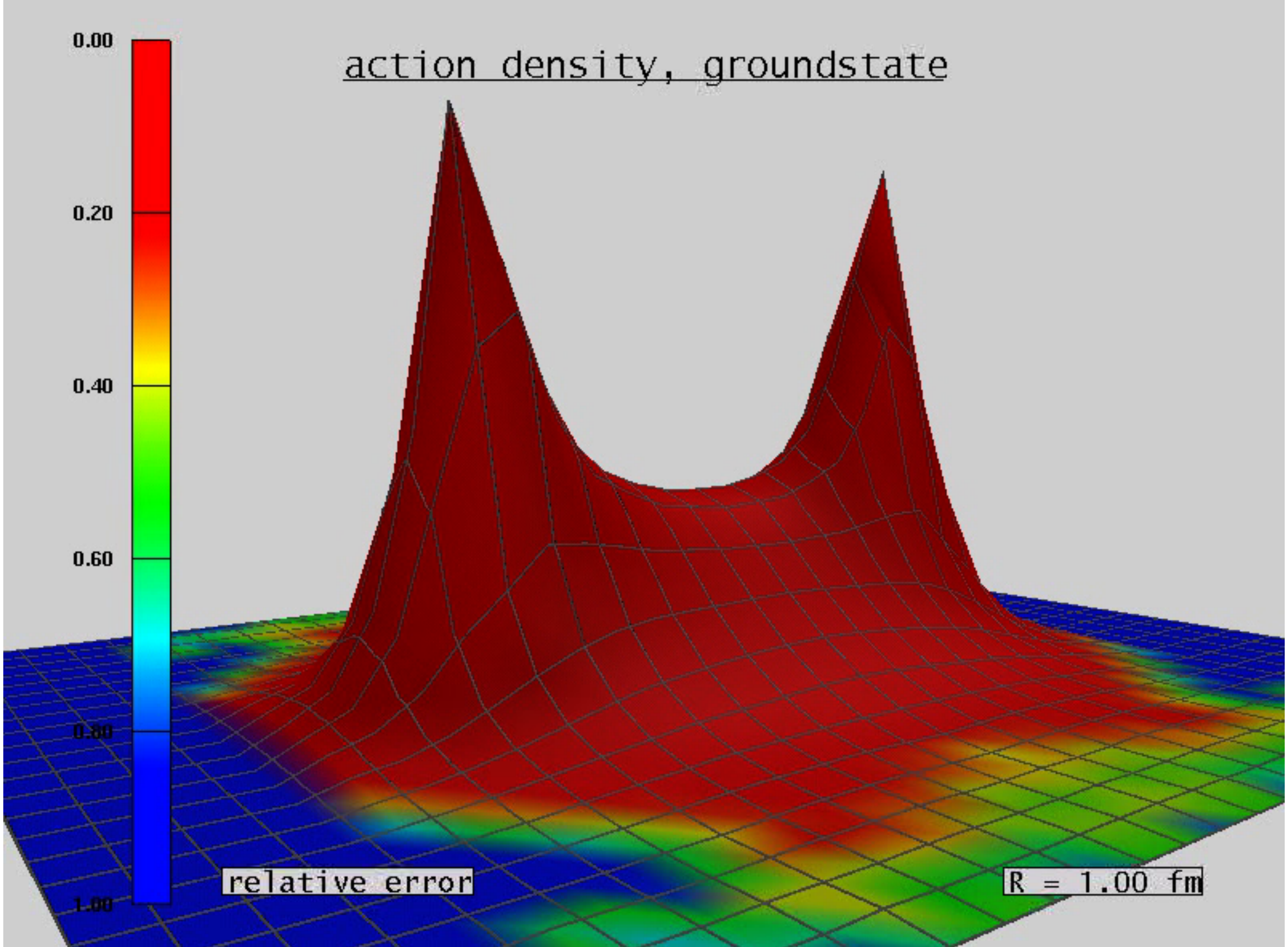
Greensite; Weise; de Forcrand; Reinhardt; Langfeld;...

Lattice Gauge Theory

confinement cartoons

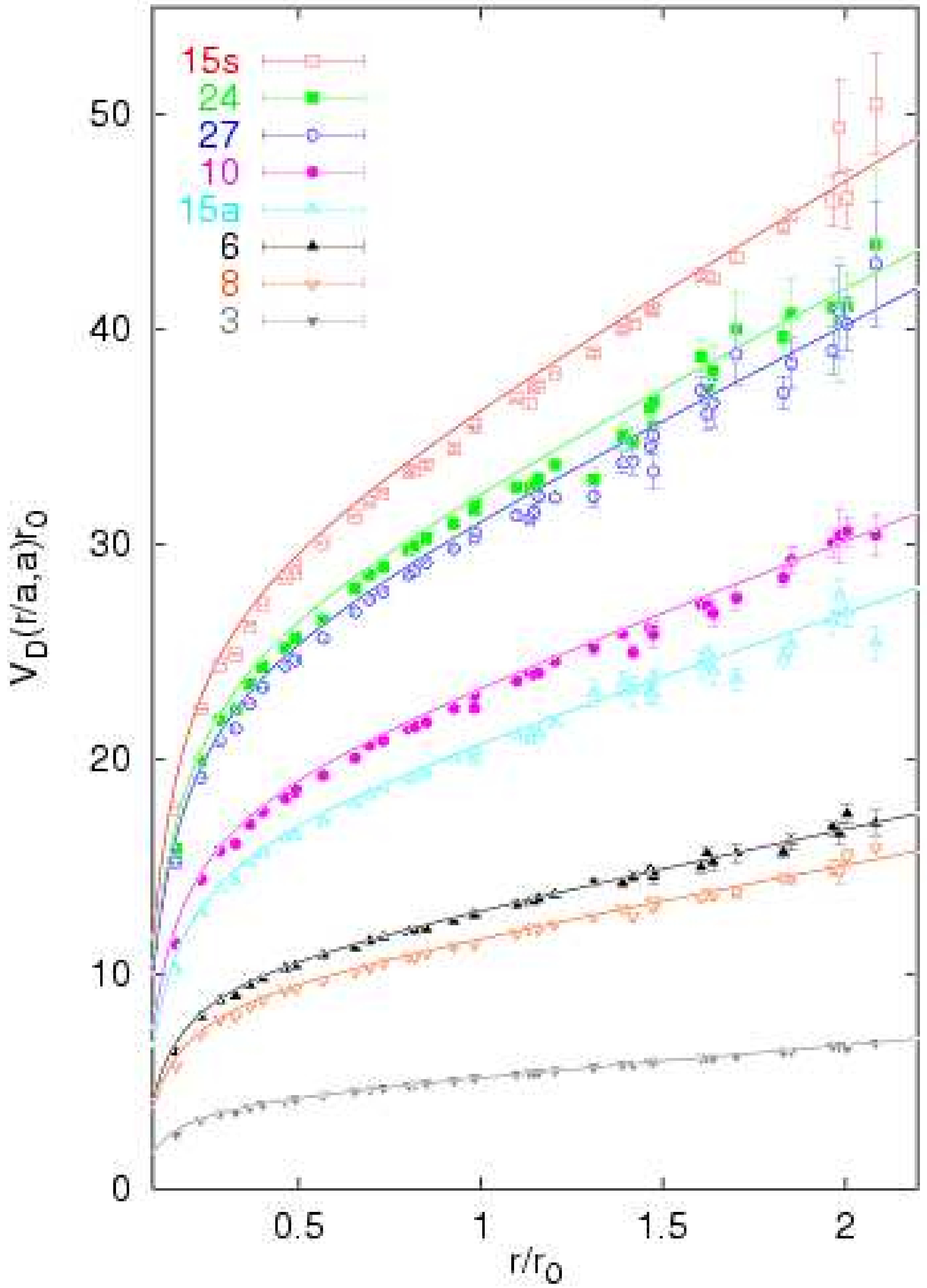





action density, groundstate
0.00
0.20
0.40
0.60
0.80
1.00
relative error
R = 1.00 fm

Lattice Gauge Theory

$Q\bar{Q}|_R$ potential

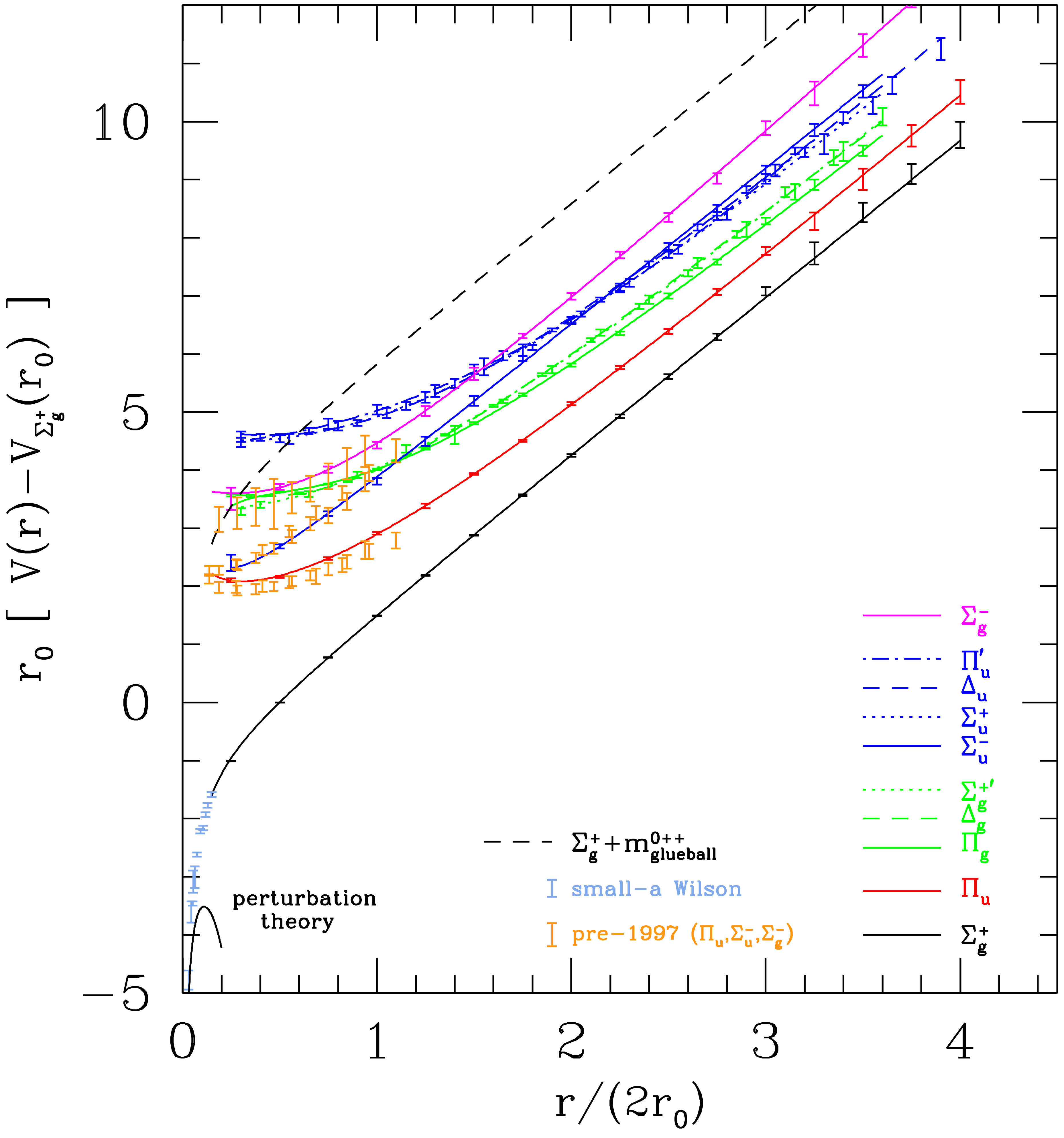


`Casimir scaling'

c= 18/3
c= 16/3
c= 10/3
c= 3
c= 4/3

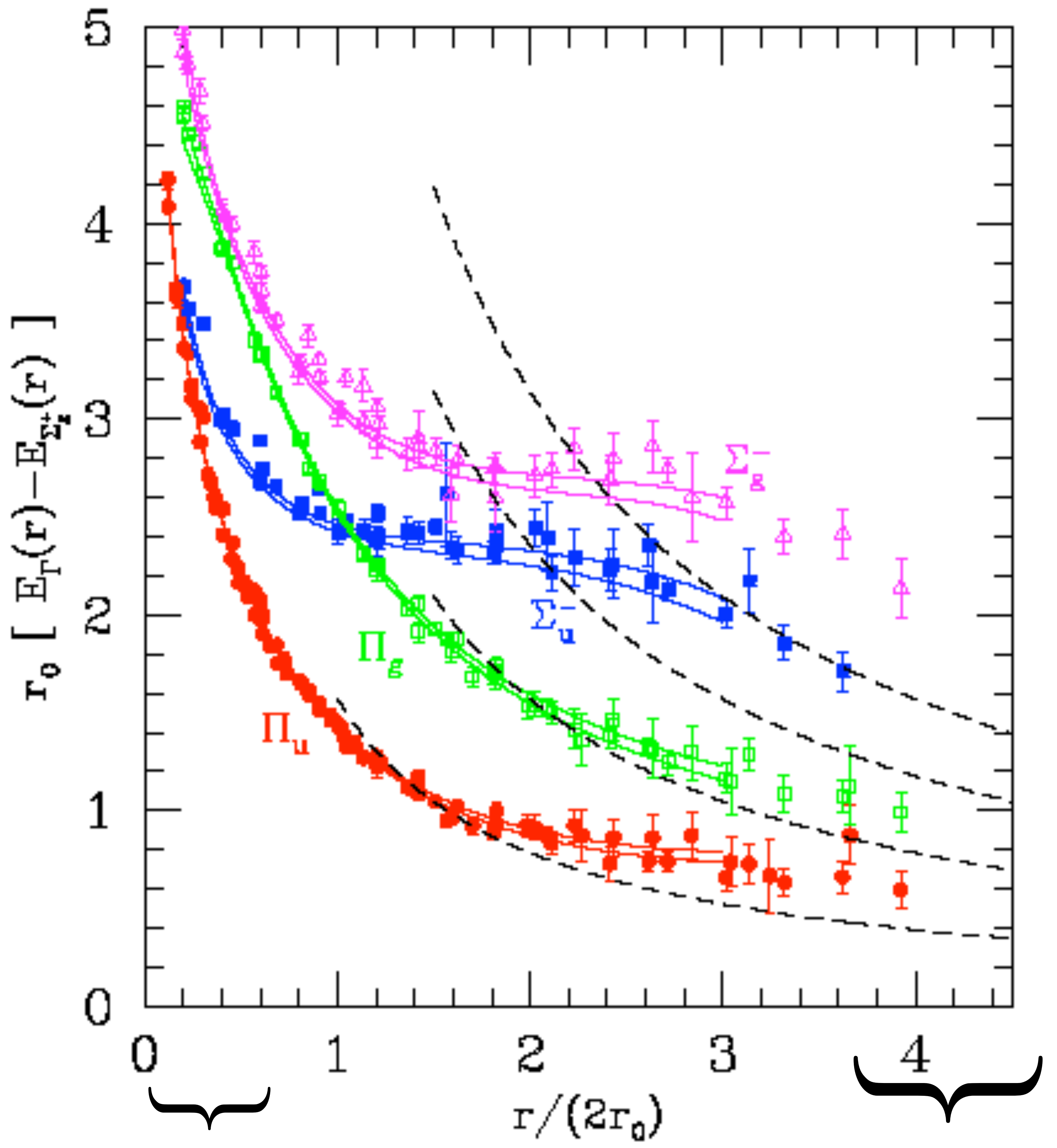
Lattice Gauge Theory

$Q\bar{Q}|_{\Lambda\eta\xi}$ potential



Lattice Gauge Theory

$Q\bar{Q}\,|_{\Lambda\eta\xi}$ potential

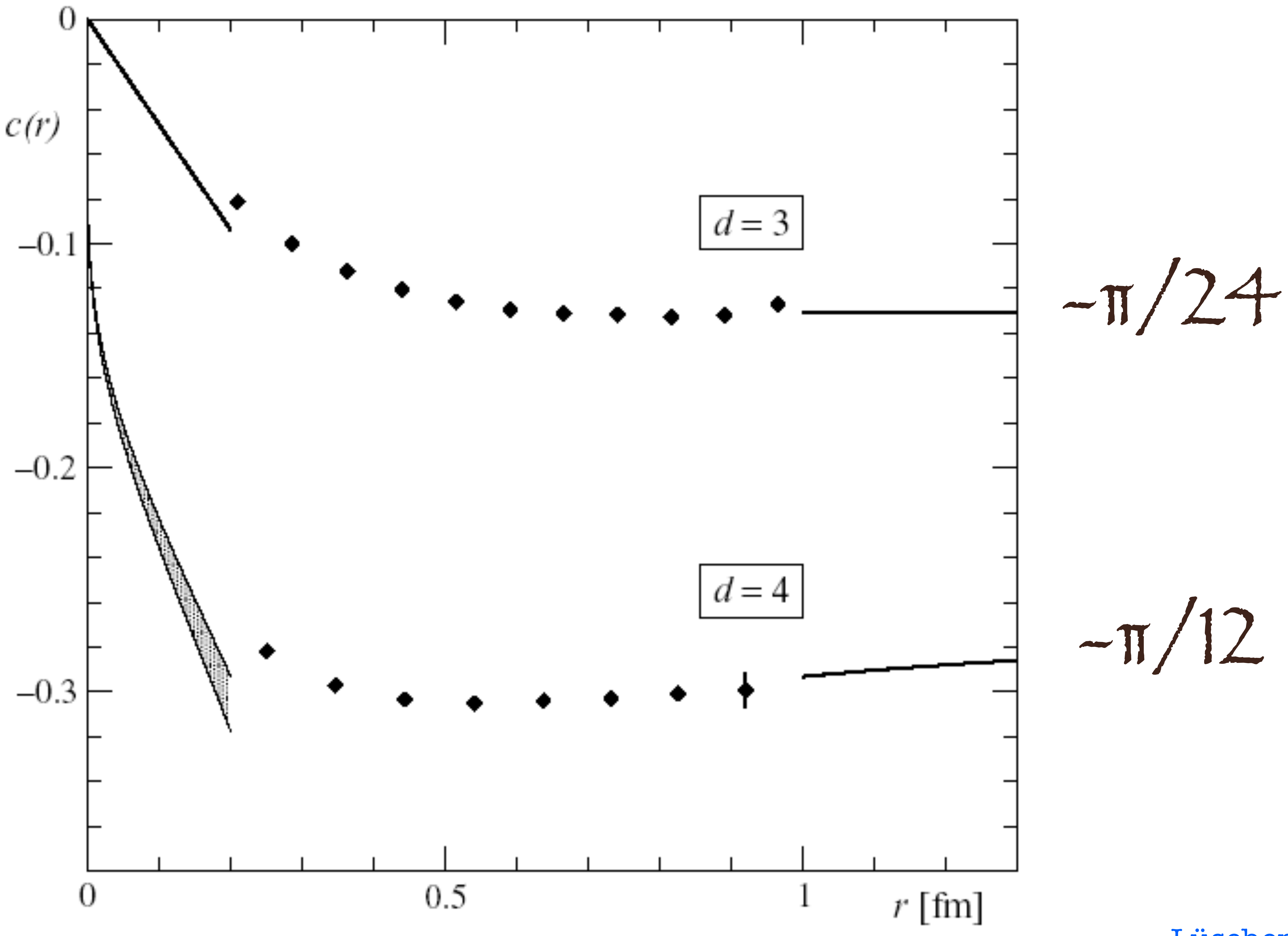


short range intermediate long range

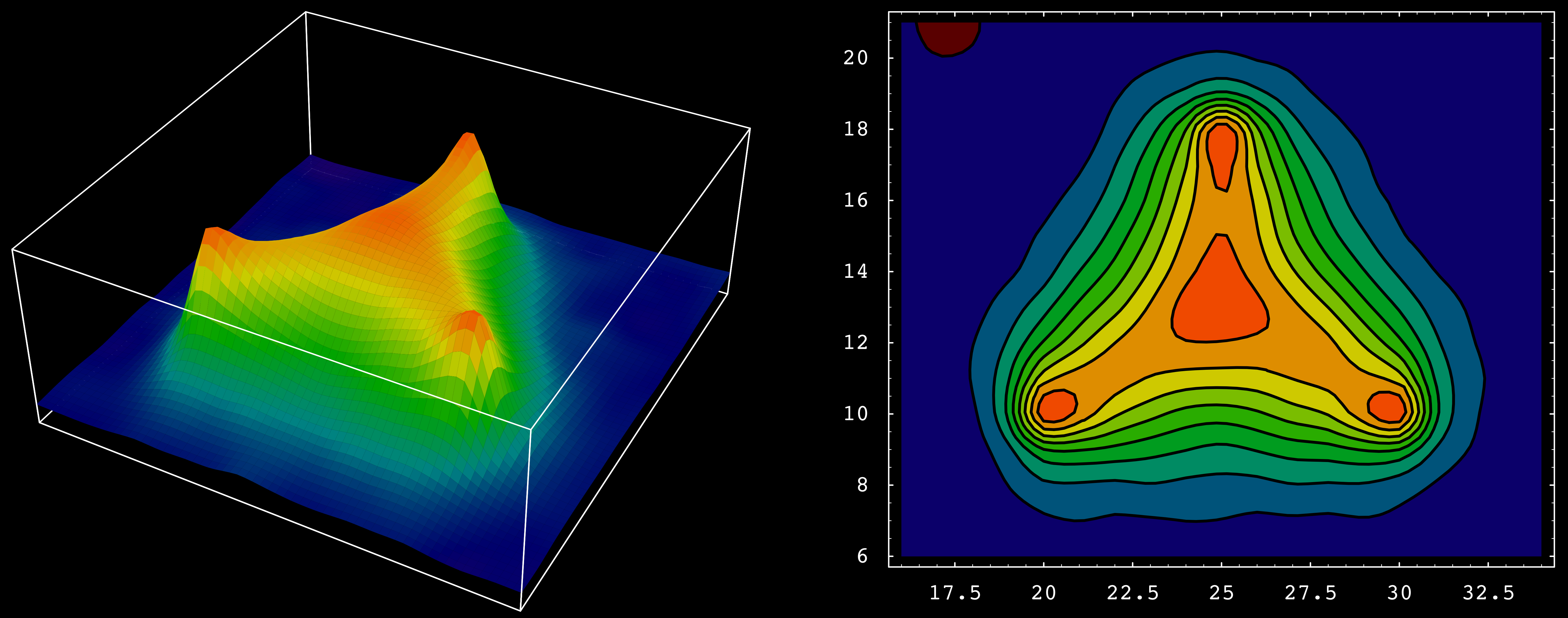
Juge, Kuti, & Morningstar

Lattice Gauge Theory

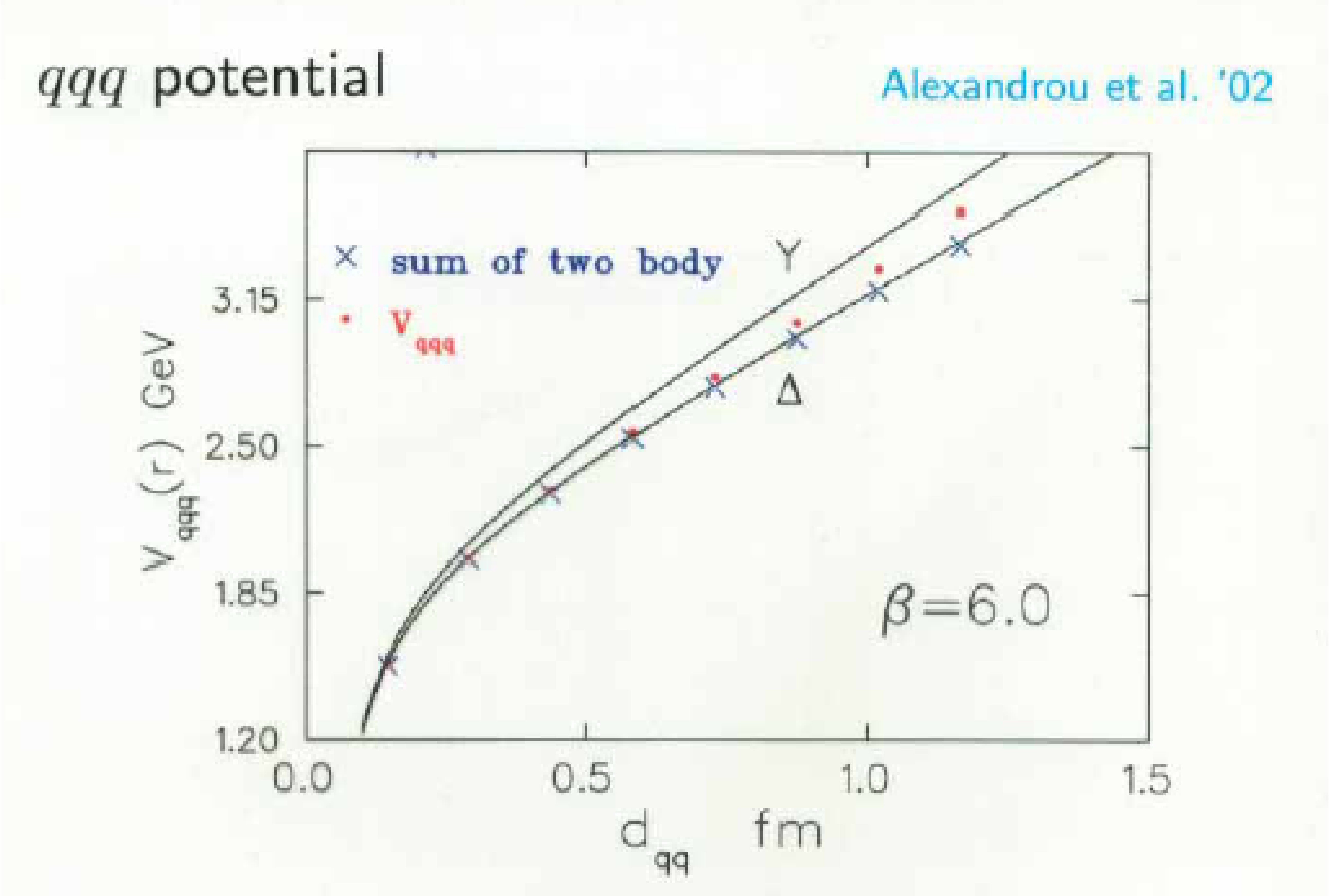
$V = br + c/r$



Lüscher & Weisz

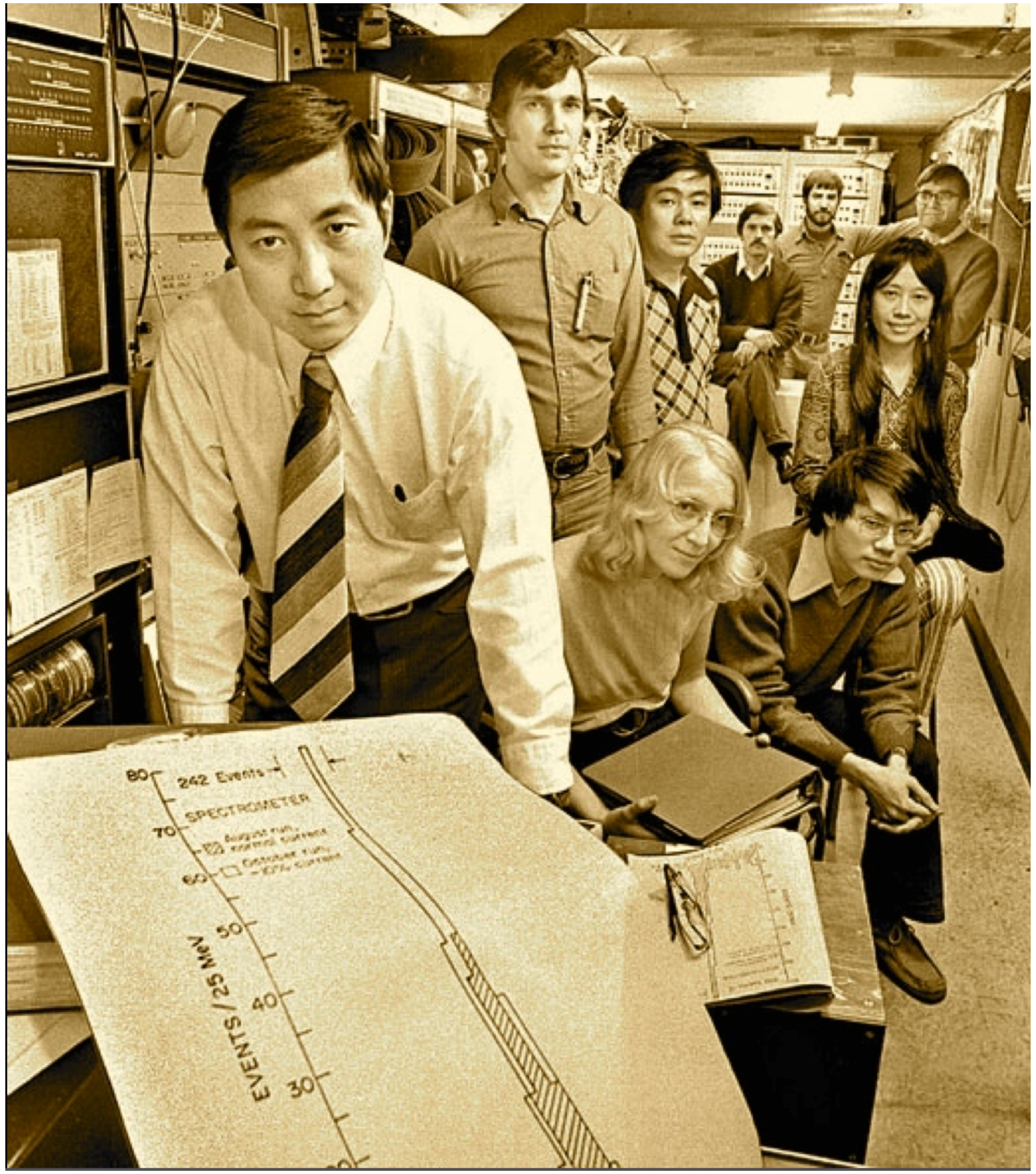

20
18
16
14
12
10
8
6
17.5 20 22.5 25 27.5 30 32.5

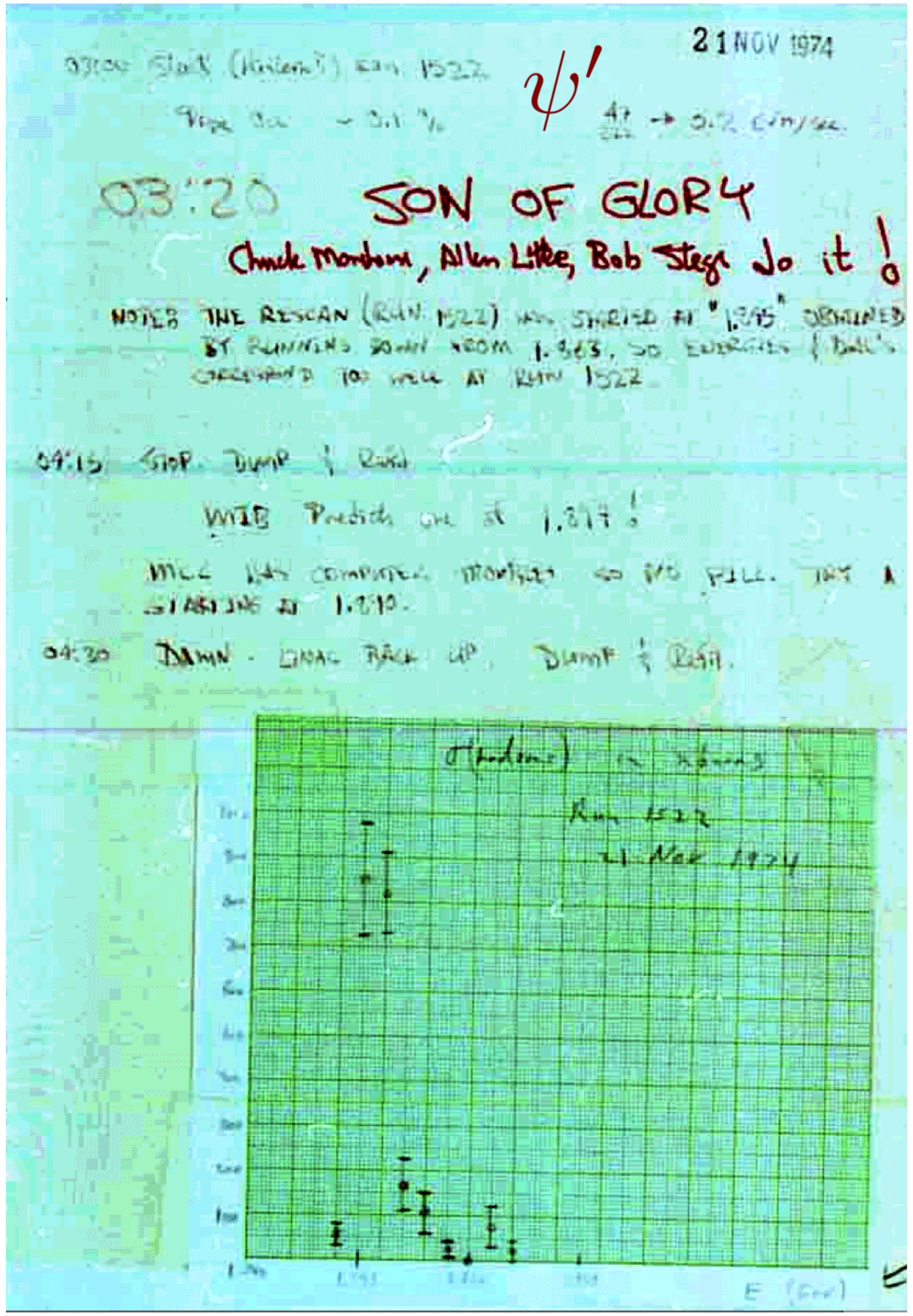
Lattice Gauge Theory



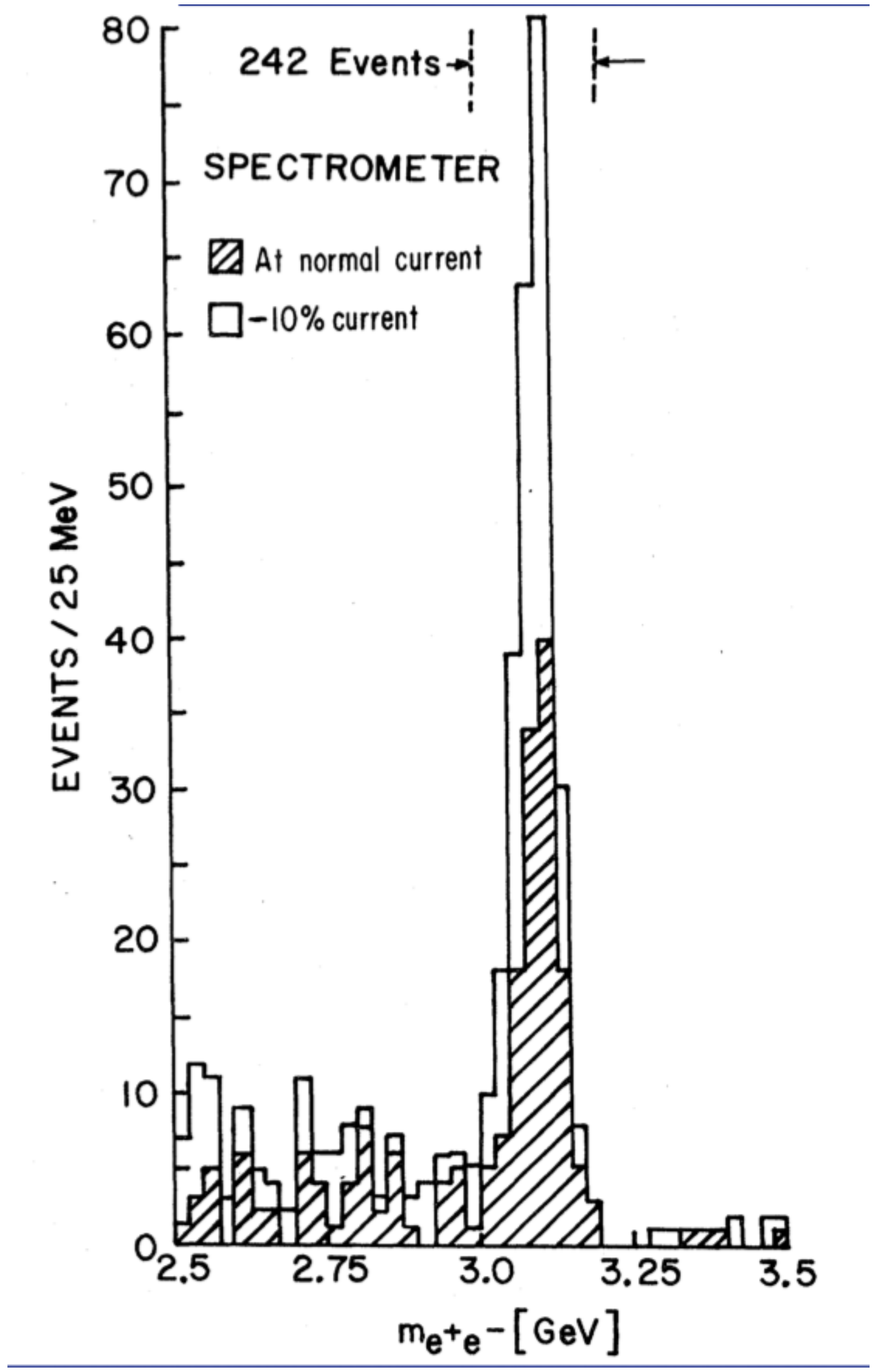
calculating

can we predict these things??





A pole can appear when the components of a complicated system resonate.



Challenges:

QCD is:

- many body
- relativistic
- strong coupling (contrast to QED)
- quantum
- nonlinear

What tools do we have?

1. perturbation theory

[this works well(ish) in the perturbative/high energy transfer regime]

[why the "ish"? Because the theory is asymptotic]

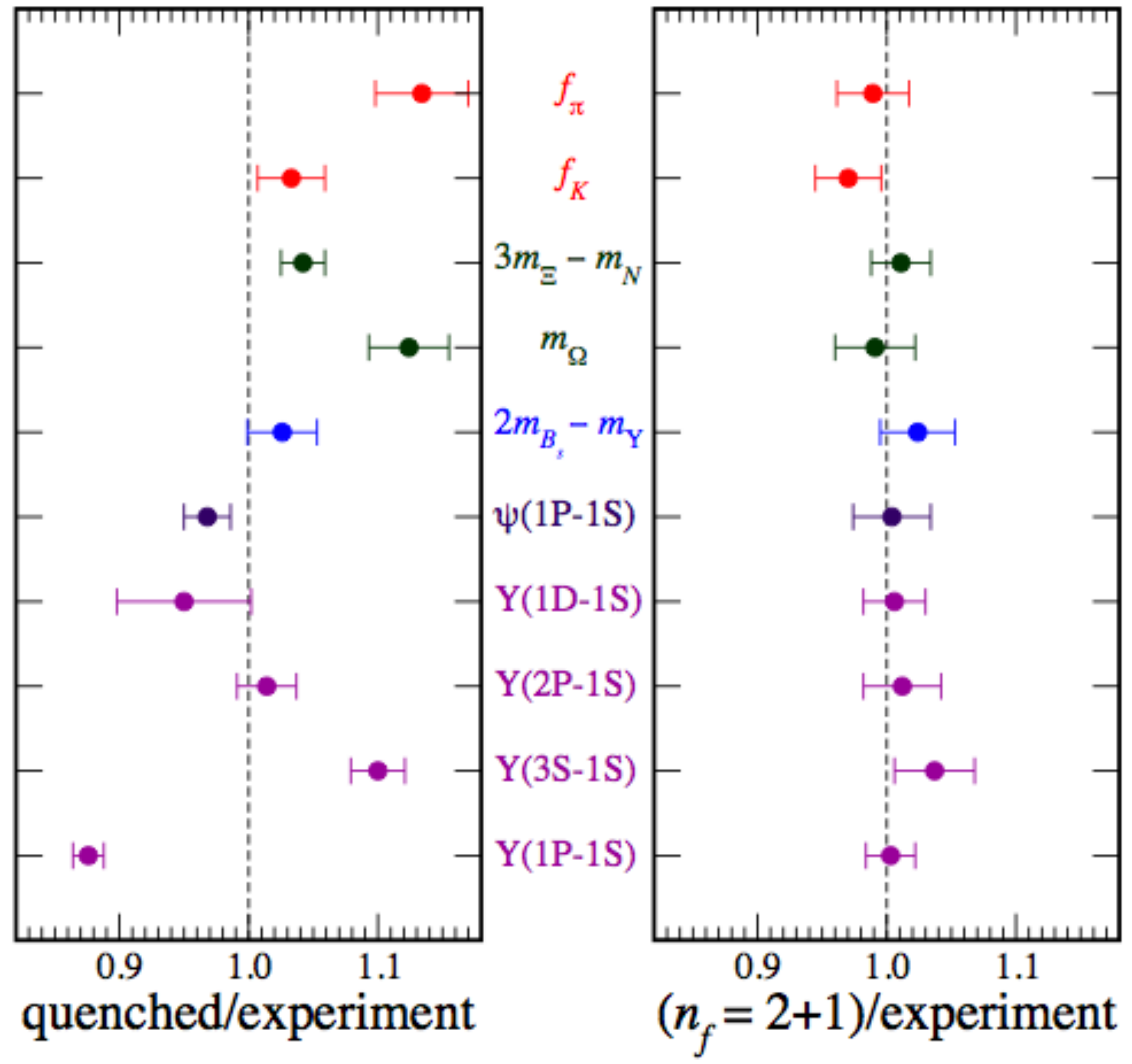
2. nonperturbative methods

[Ex: Schwinger-Dyson equations]

[uncontrolled truncations must be made]

3. lattice field theory

[fermion sign problem. finite machines, Minkowski space]



4. effective field theory

[Ex: chiral perturbation theory, NEFT, HQEFT]]

[limited region of validity, unclear scale separation]

5. models

[Ex: constituent quark models, bag models, string models, color glass,....]

[they're all models]

modelling

having the theory is nice, but not necessarily the end of the story

cf, the Theory of Everything for Life

$$H = -\sum_i \frac{\nabla_i^2}{2m_i} + \sum_{i<j} \frac{q_i q_j}{r_{ij}^2}$$

[plus some nuclear stuff]

need to identify the appropriate degrees of freedom

bag model:

fermions: current quarks
bosons: bag pressure

perturbation theory:

fermions: current quarks
bosons: current gluons

flux tube model:

fermions: constituent quarks
bosons: flux tubes

physical pictures/degrees of freedom can change depending on

scale

quark mass, glue

observables

ρ decay vs. ρ scattering

gauge

confinement in Coulomb gauge vs. Weyl gauge spontaneous chiral symmetry breaking implies both the existence of Goldstone bosons and constituent quarks

current quarks evolve into constituent quarks at scales < Λ_{QCD}

it is the structure of the vacuum that gives chiral symmetry breaking and confinement

it is desirable to incorporate the physics of the vacuum and chiral symmetry breaking into the model from the beginning

effective degrees of freedom should be derived from QCD to the extent possible

only in this way can we recover perturbative QCD in the high energy regime

constituent quark models

Historically, seek to replicate the successes of atomic and nuclear physics

Richard Dalitz
(1925-2006)

Gabriel Karl
(1937-2020)

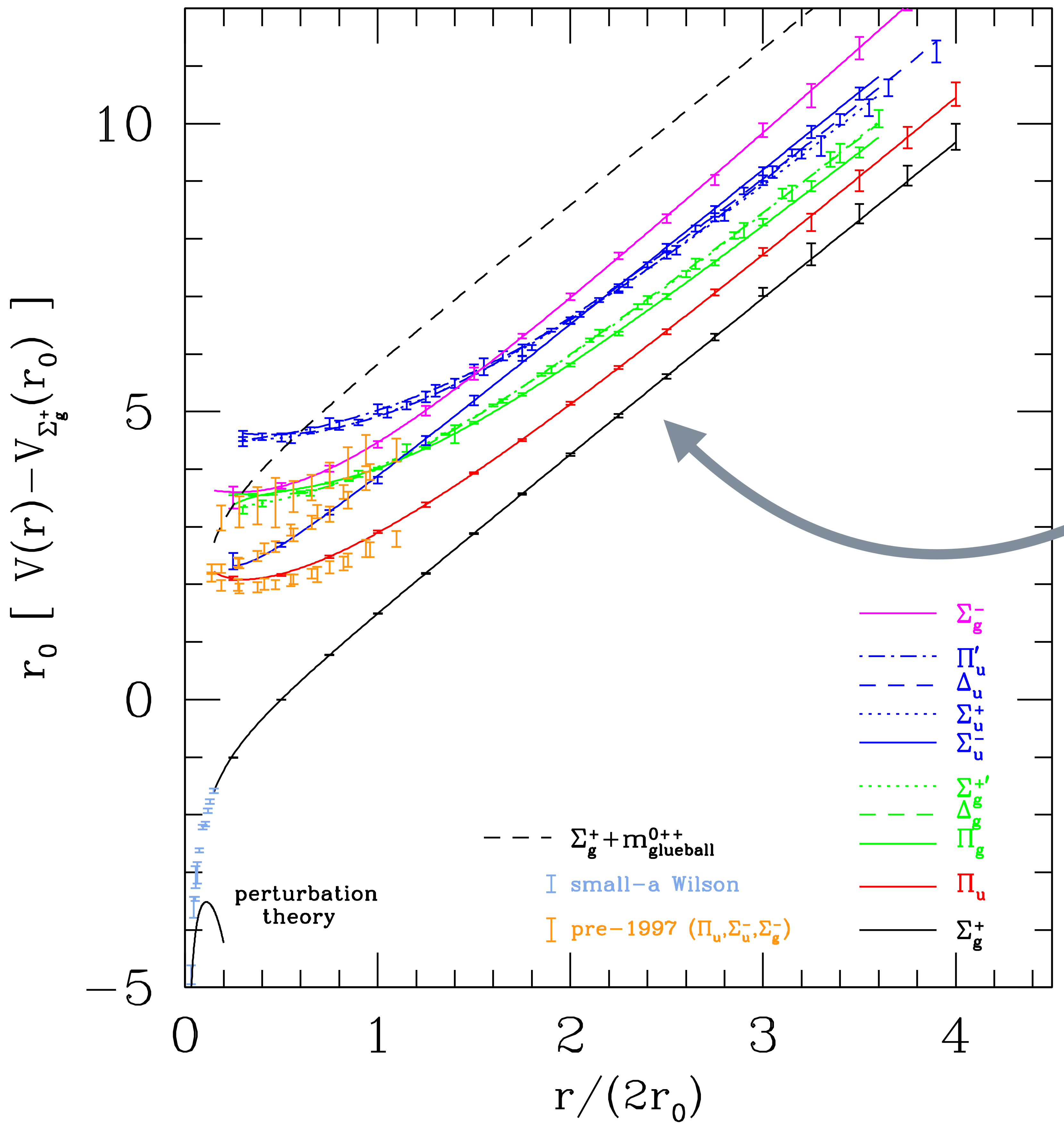
Giacomo Morpugo
(1927-)

Constituent Quarks

- pre-QCD quarks: m~ 5 GeV
- Copley, Karl, & Obryk (1969): m ~ 330 MeV
- QCD (1973): m(2 GeV) ~ 4 MeV
- but recall that quarks are not observable ⇒ different kinds of quark masses exist: current/constituent
- DeRujula, Georgi, Glashow (1975): apply perturbative QCD to splittings in the spectrum

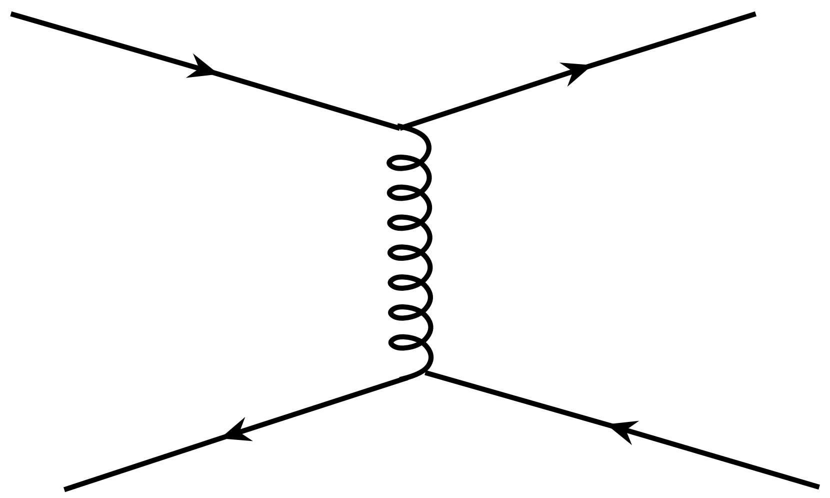
kinetic energy: $-\frac{\nabla^2}{2m}$ $\sqrt{-\nabla^2 + m^2}$ $-i\alpha \cdot \nabla + \beta m$

potential energy:



Cornell potential

potential energy $O(1/m^2)$:



$$
\begin{aligned}
U &= (V_C + V_{so} + V_{hyp})\, \frac{\vec{\lambda}_1}{2} \cdot \frac{-\vec{\lambda}_2{}^*}{2} \\
V_C &= \frac{\alpha}{r} - \frac{\alpha\pi}{2}\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)\delta(\vec{r}) \\
V_{hyp} &= \frac{\alpha}{4m_1m_2}\left(\frac{\vec{\sigma}_1\cdot\vec{\sigma}_2}{r^3} - \frac{3(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{r^5} - \frac{8\pi}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\,\delta(\vec{r})\right) \\
V_{so} &= -\frac{\alpha}{2m_1m_2r}\left(\vec{p}_1\cdot\vec{p}_2 + \frac{\vec{r}\cdot(\vec{r}\cdot\vec{p}_1)\vec{p}_2}{r^2}\right) - \frac{\alpha}{4r^3}\left(\frac{\vec{r}\times\vec{p}_1\cdot\vec{\sigma}_1}{m_1^2} - \frac{\vec{r}\times\vec{p}_2\cdot\vec{\sigma}_2}{m_2^2}\right) \\
&\quad - \frac{\alpha}{2m_1m_2r^3}\left(\vec{r}\times\vec{p}_1\cdot\vec{\sigma}_2 - \vec{r}\times\vec{p}_2\cdot\vec{\sigma}_1\right)
\end{aligned}
$$

potential energy $O(1/m^2)$:

Eichten & Feinberg

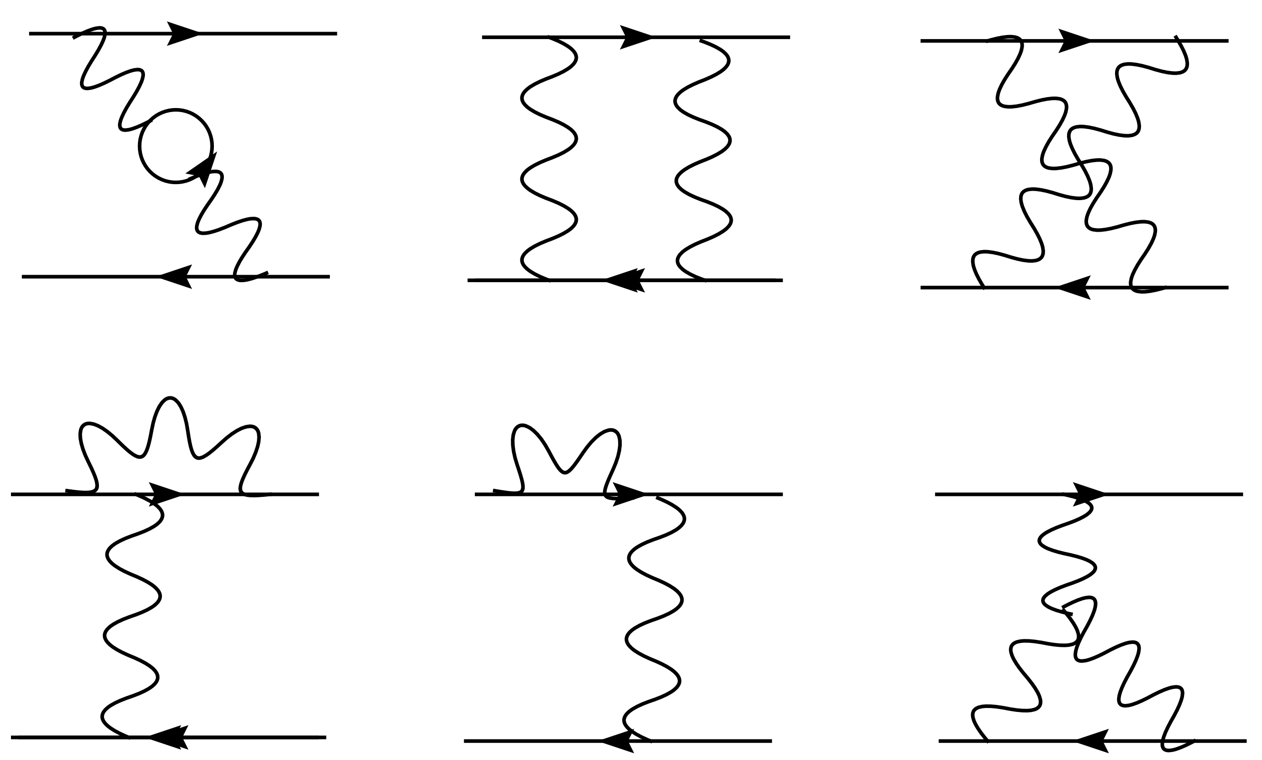
$$V_{conf} = -\frac{3}{4}\frac{\alpha_s}{r} + br$$

$$\begin{aligned} V_{SD}(r) \quad = \quad & \left(\frac{\sigma_q}{4m_q^2} + \frac{\sigma_{\bar{q}}}{4m_{\bar{q}}^2}\right) \cdot \mathbf{L} \left(\frac{1}{r}\frac{dV_{conf}}{dr} + \frac{2}{r}\frac{dV_1}{dr}\right) + \left(\frac{\sigma_{\bar{q}} + \sigma_q}{2m_q m_{\bar{q}}}\right) \cdot \mathbf{L} \left(\frac{1}{r}\frac{dV_2}{dr}\right) \\ & + \frac{1}{12 m_q m_{\bar{q}}}\left(3\sigma_q \cdot \hat{\mathbf{r}}\, \sigma_{\bar{q}} \cdot \hat{\mathbf{r}} - \sigma_q \cdot \sigma_{\bar{q}}\right) V_3 + \frac{1}{12 m_q m_{\bar{q}}} \sigma_q \cdot \sigma_{\bar{q}} V_4 \\ & + \frac{1}{2}\left[\left(\frac{\sigma_q}{m_q^2} - \frac{\sigma_{\bar{q}}}{m_{\bar{q}}^2}\right) \cdot \mathbf{L} + \left(\frac{\sigma_q - \sigma_{\bar{q}}}{m_q m_{\bar{q}}}\right) \cdot \mathbf{L}\right] V_5. \qquad (1) \end{aligned}$$

<<general form of the correction

potential energy $O(1/m^2)$:

Gupta & Radford, PRD33, 777 (86)



$$
\begin{aligned}
V_1(m_q, m_{\bar{q}}, r) &= -br - C_F \frac{1}{2r}\frac{\alpha_s^2}{\pi}\left(C_F - C_A\left(\ln\left[(m_q m_{\bar{q}})^{1/2} r\right] + \gamma_E\right)\right) \\
V_2(m_q, m_{\bar{q}}, r) &= -\frac{1}{r} C_F \alpha_s \left[1 + \frac{\alpha_s}{\pi}\left[\frac{b_0}{2}[\ln(\mu r) + \gamma_E] + \frac{5}{12} b_0 - \frac{2}{3} C_A + \frac{1}{2}\left(C_F - C_A\left(\ln\left[(m_q m_{\bar{q}})^{1/2} r\right] + \gamma_E\right)\right)\right]\right] \\
V_3(m_q, m_{\bar{q}}, r) &= \frac{3}{r^3} C_F \alpha_s \left[1 + \frac{\alpha_s}{\pi}\left[\frac{b_0}{2}[\ln(\mu r) + \gamma_E - \frac{4}{3}] + \frac{5}{12} b_0 - \frac{2}{3} C_A + \right.\right. \\
&\quad \left.\left. + \frac{1}{2}\left(C_A + 2C_F - 2C_A\left(\ln\left[(m_q m_{\bar{q}})^{1/2} r\right] + \gamma_E - \frac{4}{3}\right)\right)\right]\right] \\
V_4(m_q, m_{\bar{q}}, r) &= \frac{32\alpha_s \sigma^3 e^{-\sigma^2 r^2}}{3\sqrt{\pi}} \\
V_5(m_q, m_{\bar{q}}, r) &= \frac{1}{4r^3} C_F C_A \frac{\alpha_s^2}{\pi} \ln\frac{m_{\bar{q}}}{m_q} \qquad (1)
\end{aligned}
$$

potential energy $O(1/m^2)$:

relativistic models

$$V = \frac{1}{2}\int d^3x\, d^3y\, \bar{\psi}\Gamma\psi(y) K(x-y) \bar{\psi}\Gamma\psi(x)$$

$\Gamma = \mathbb{1}$ scalar

$\Gamma = \gamma_\mu$ vector

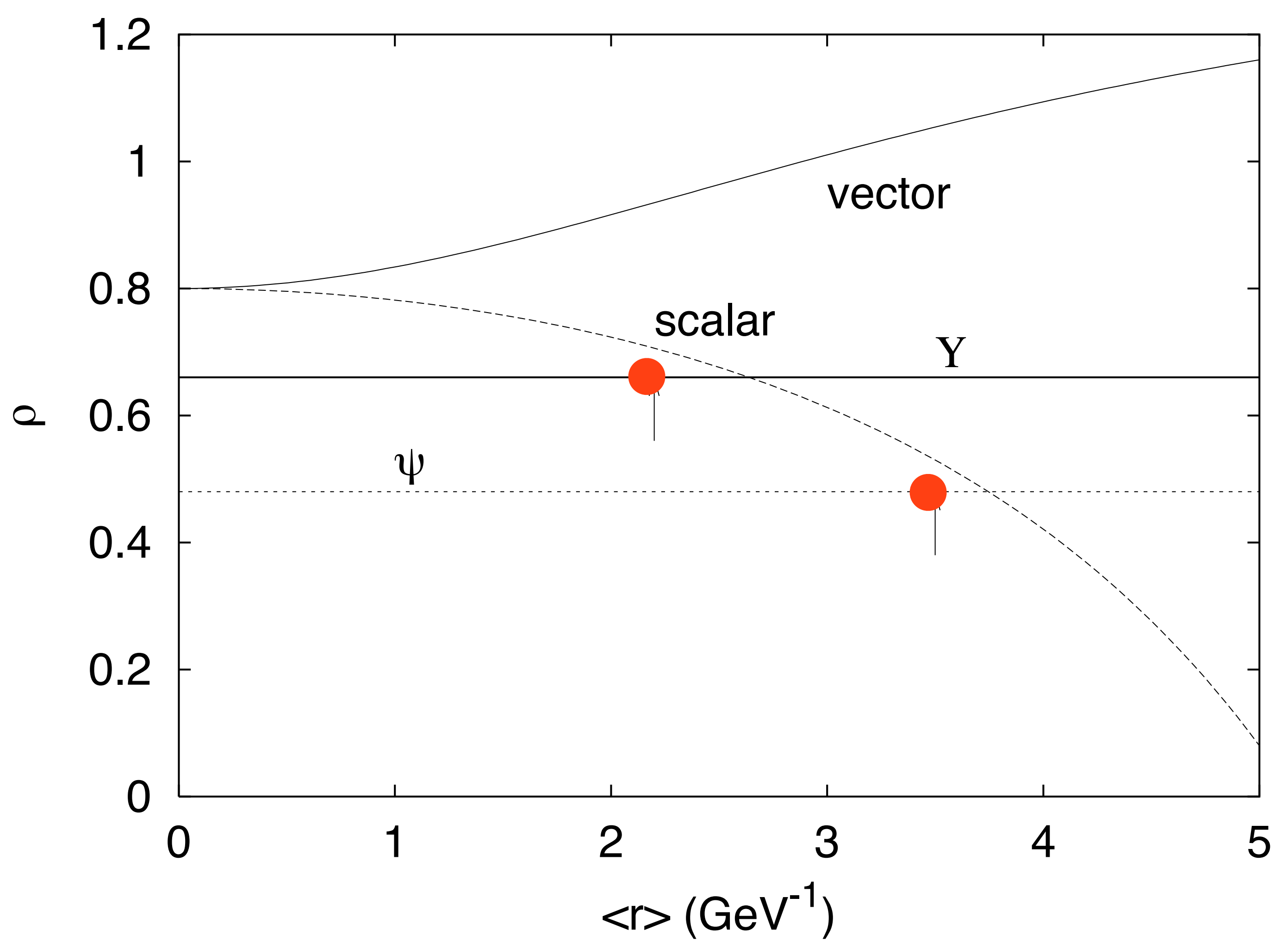
$\Gamma = \gamma_5$ pseudoscalar

⚠ **NB: there is no reason for QCD to take on this simple form!**

$$V_{conf} \to \epsilon + V_{SD} + \ldots$$

Γ	ϵ_Γ	V_1	V_2	V_3	V_4
scalar	S	$-S$	0	0	0
vector	V	0	V	$V'/r - V''$	$2\nabla^2 V$
pseudoscalar	0	0	0	$P'' - P'/r$	$\nabla^2 P$

Dieter Gromes

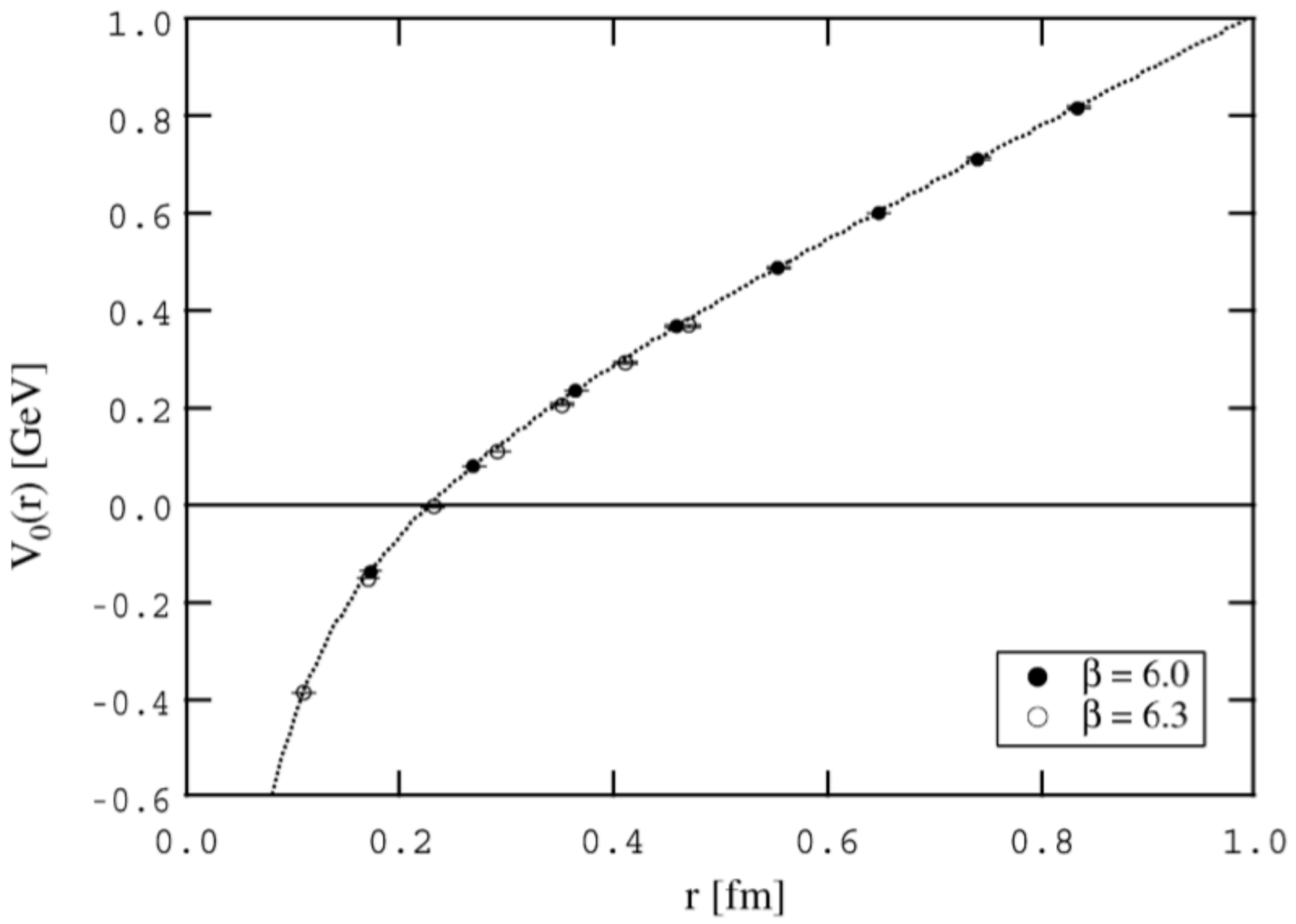


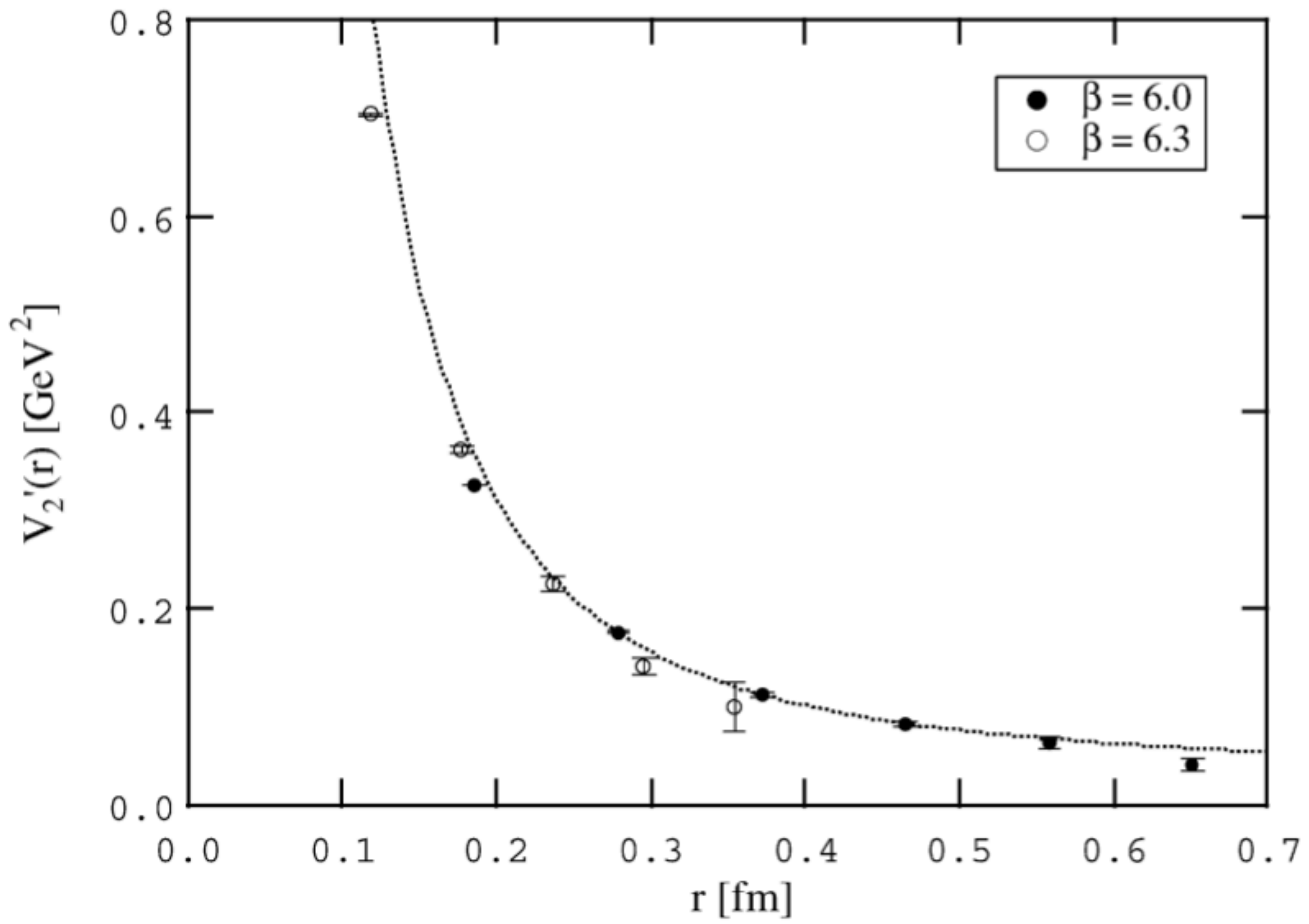
Howard Schnitzer

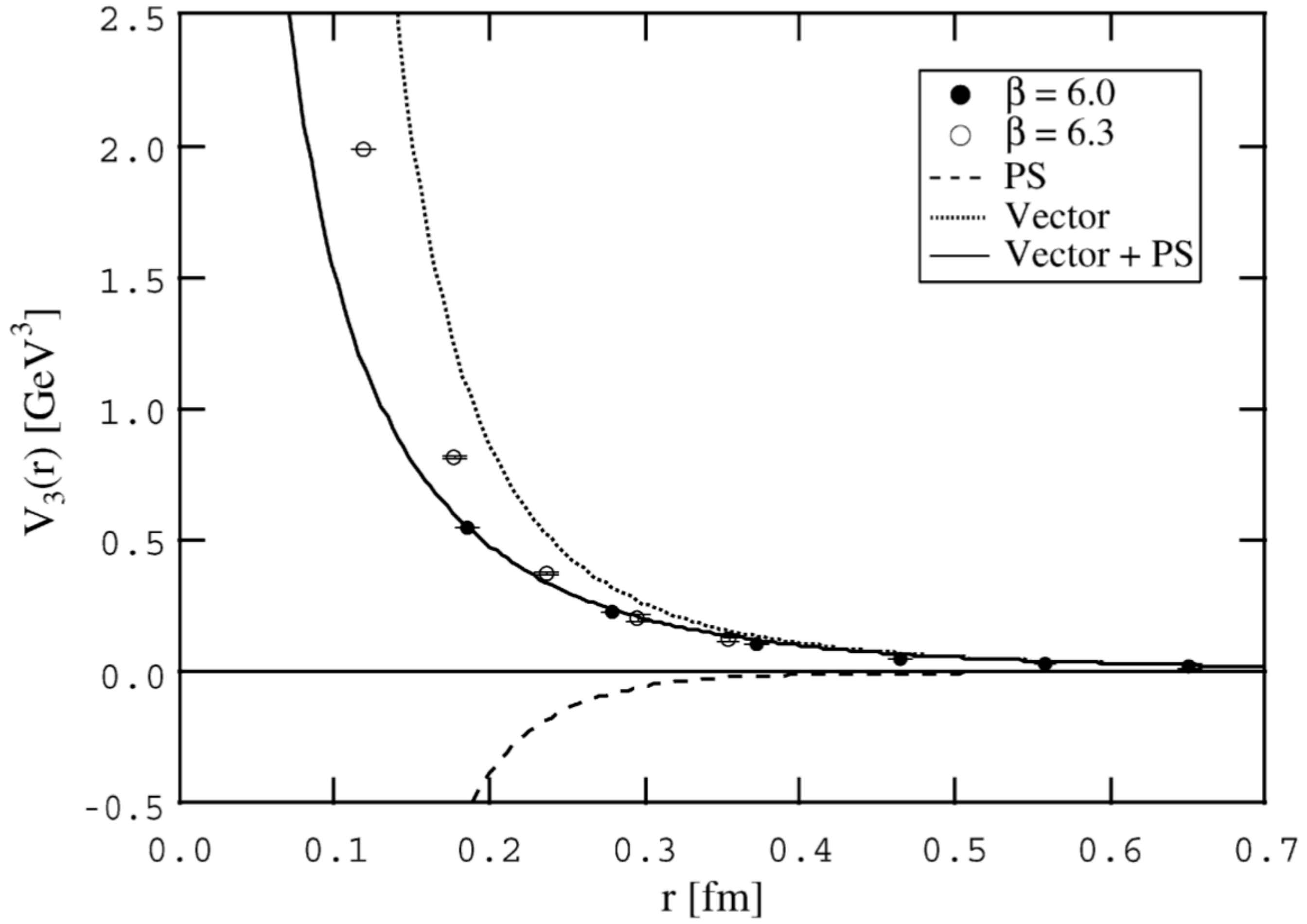
potential energy $O(1/m^2)$: Koma & Koma

lattice

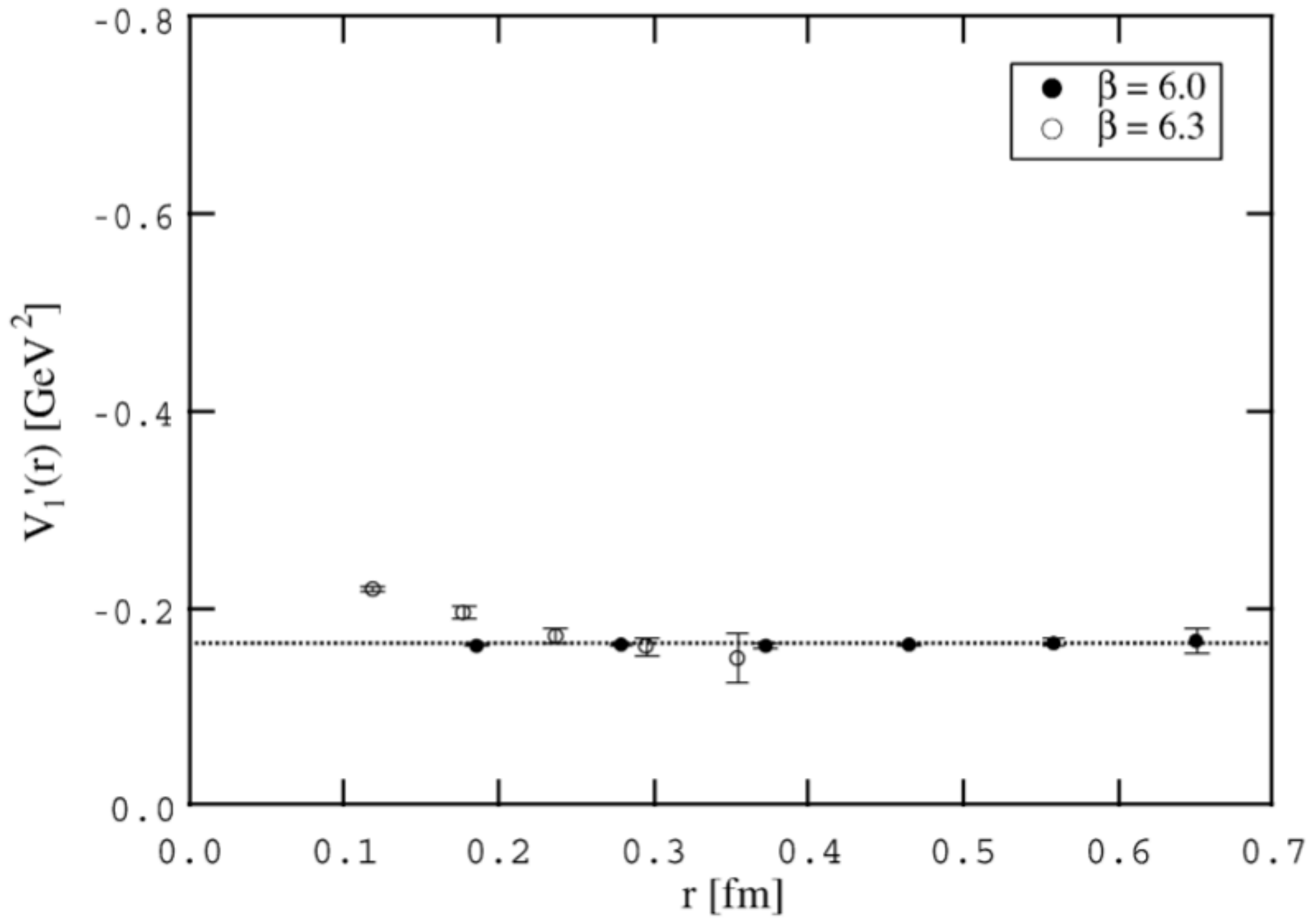
$$V_1'(r) = 0, \qquad V_2'(r) = \frac{c}{r^2}, \qquad V_3(r) = \frac{3c}{r^3}, \qquad V_4(r) = 8\pi c\,\delta^{(3)}(r),$$

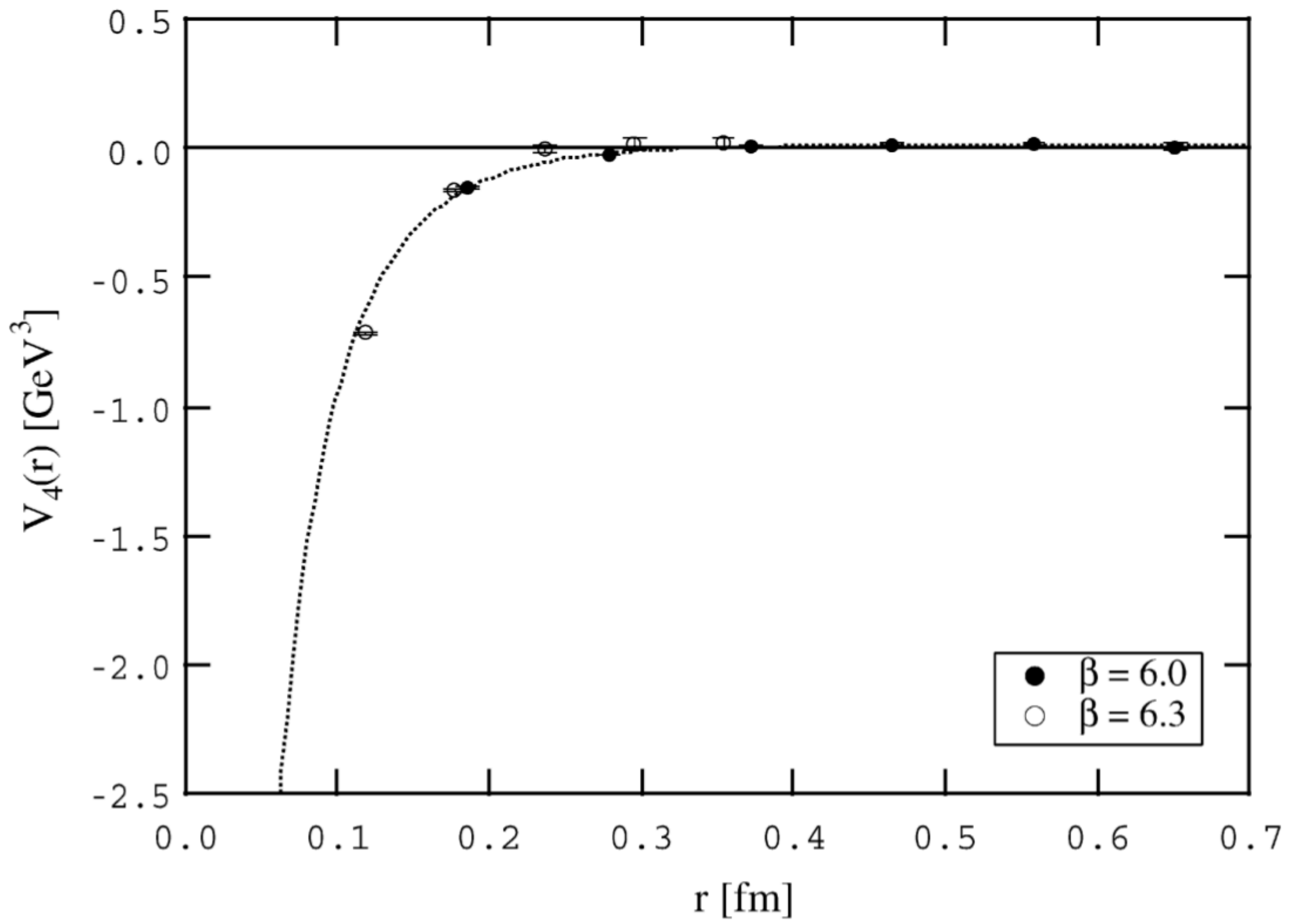




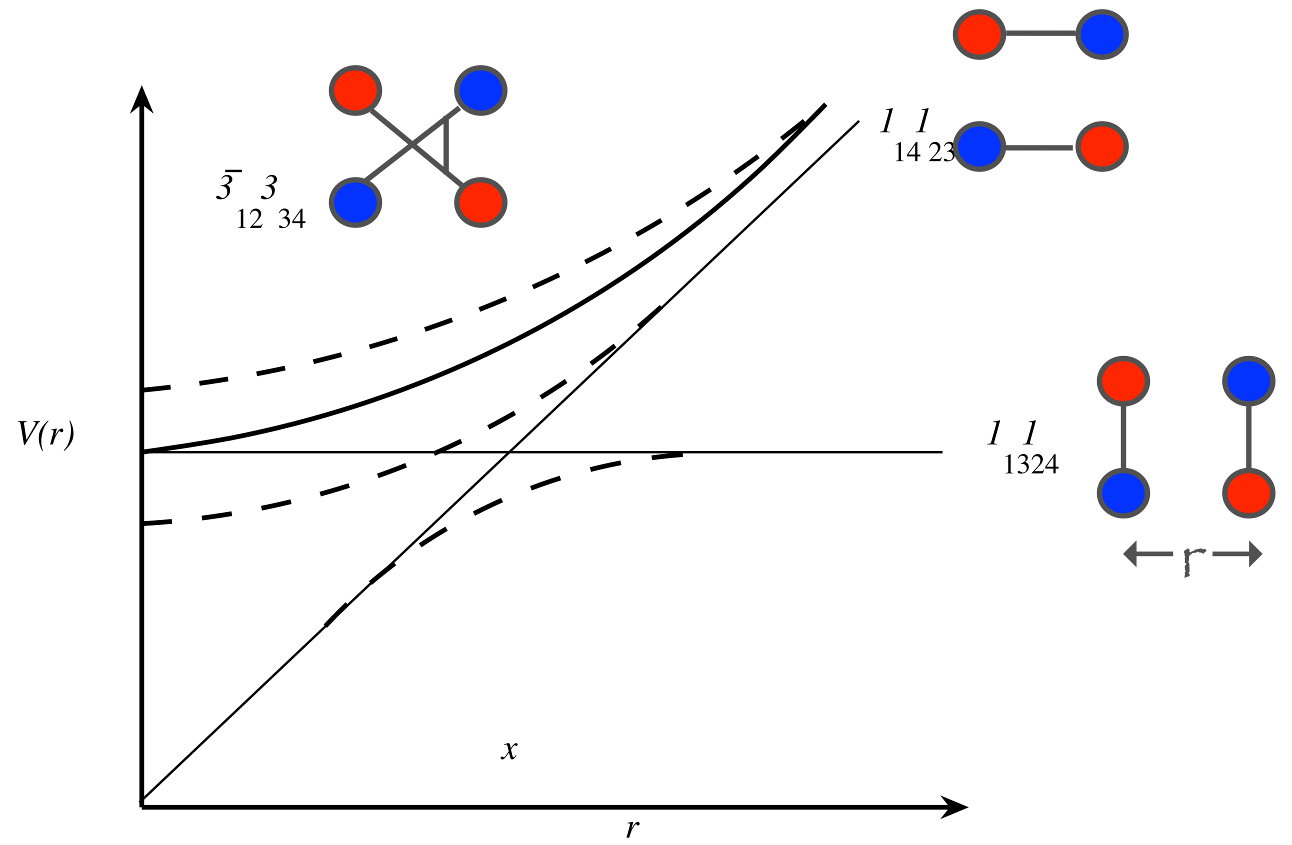


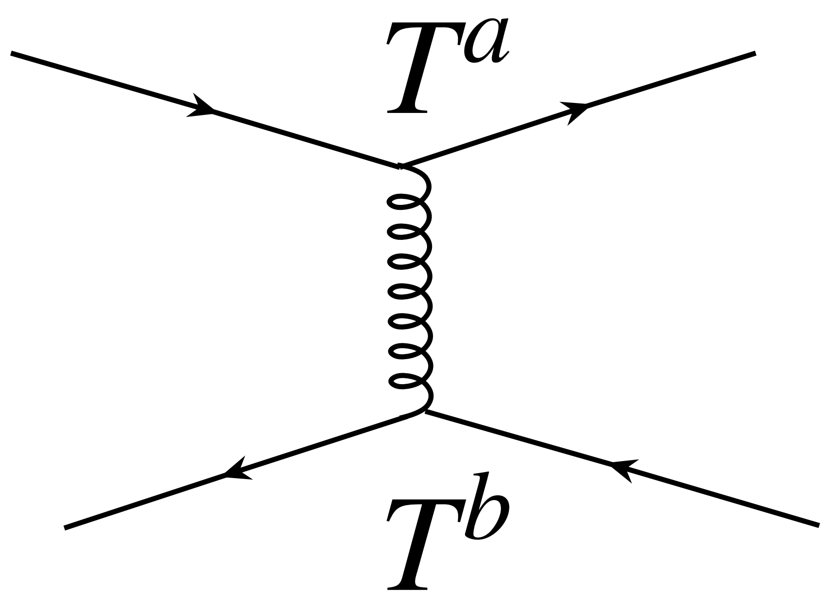
indication of mixed Dirac structure

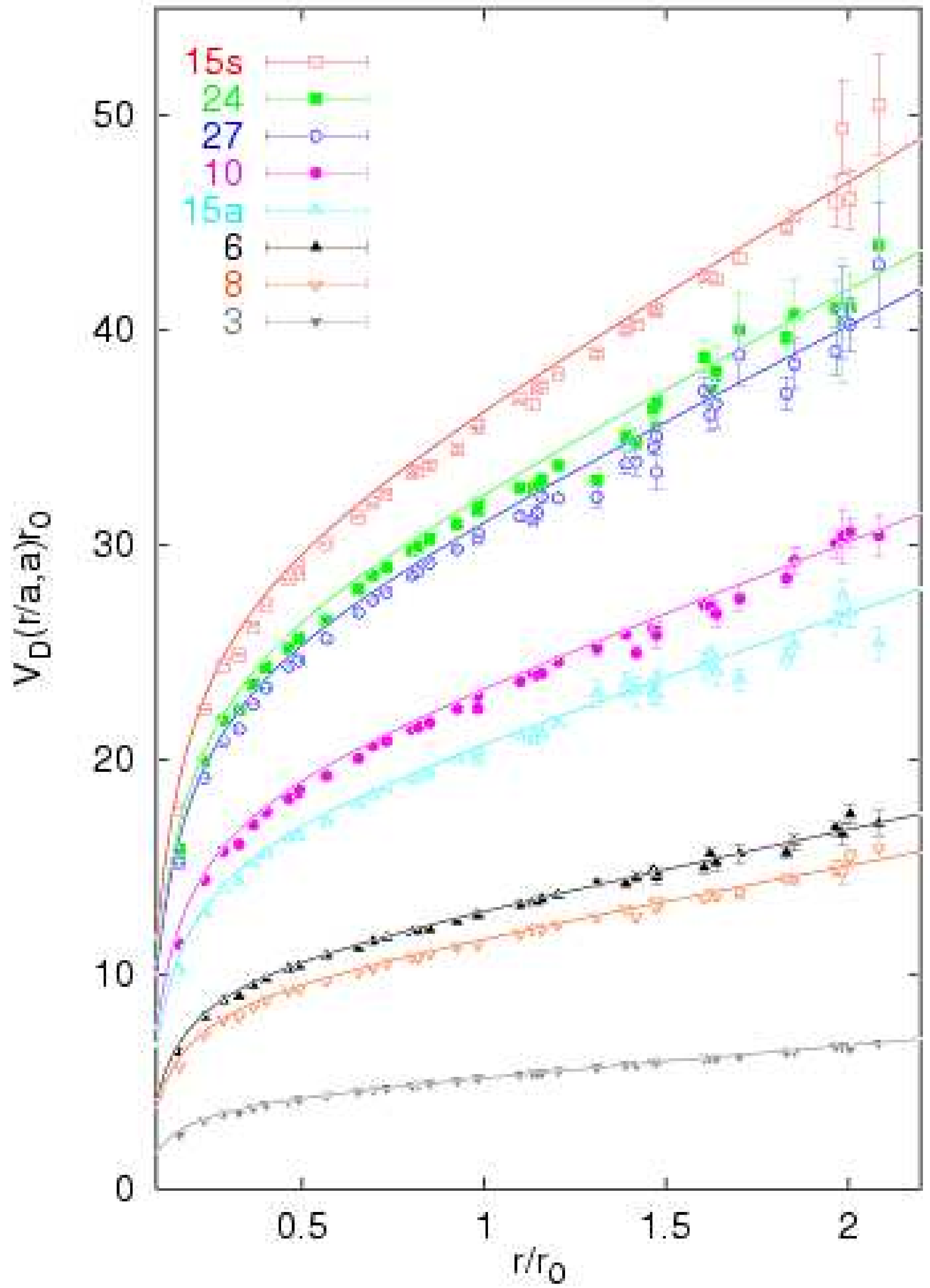




colour structure







⚠ very little is known about this...

a complete model:

$$H = \sum_i m_i + \sum_i \frac{p_i^2}{2m_i} + C + \sum_{i<j} \left[- \left(-\frac{\alpha_s}{r_{ij}} + \frac{3}{4} b r_{ij} \right) \vec{F}_i \cdot \vec{F}_j + V_{SD}^{oge}(r_{ij}) + V_{SD}^{conf}(r_{ij}) \right]$$

$$V_{hyp} = \frac{32\pi\alpha_s}{9m^2} \tilde{\delta}(r)\, \vec{S}_q \cdot \vec{S}_{\bar{q}}\, F_q \cdot F_{\bar{q}}$$

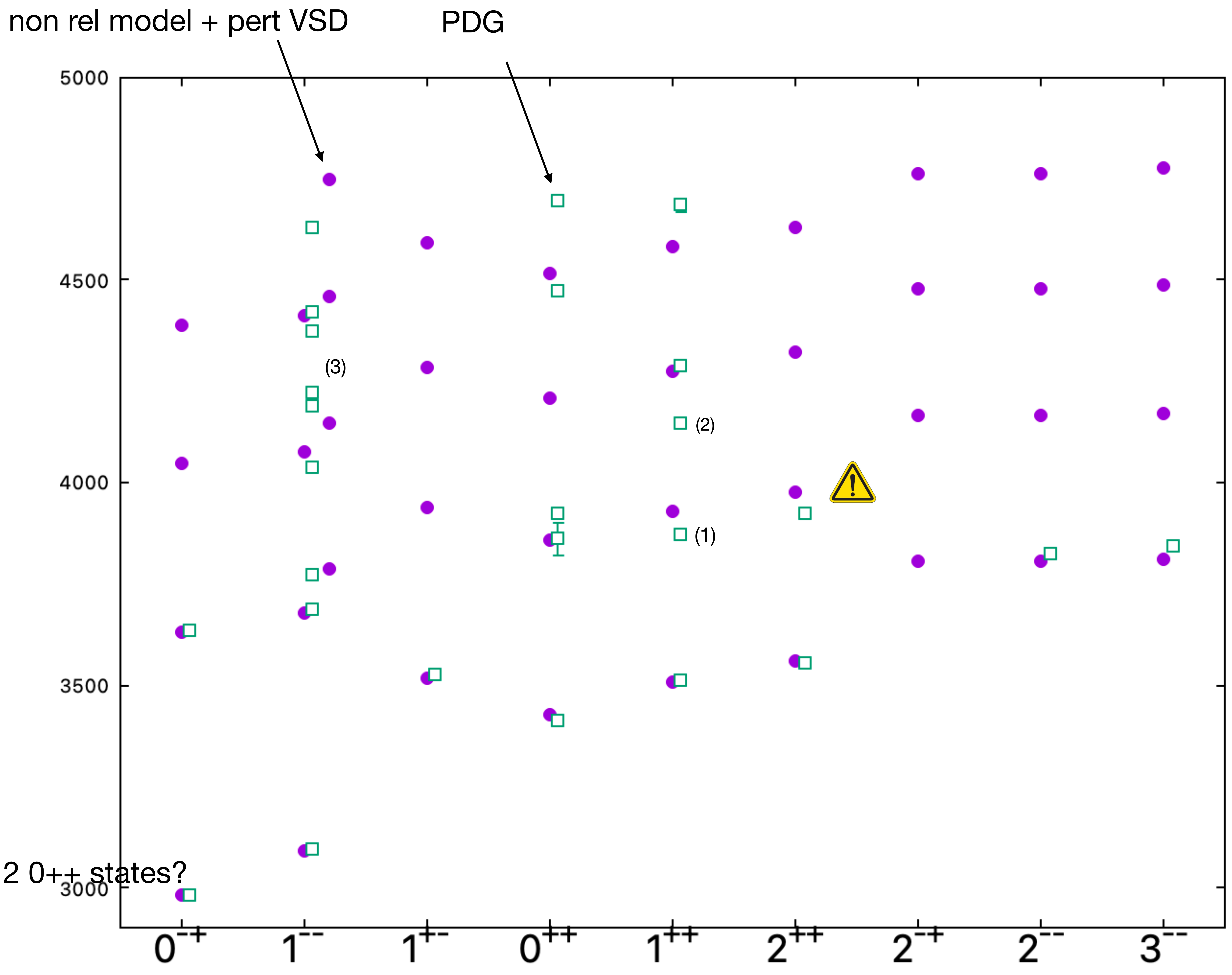
$$V_{spin-dep} = \frac{1}{m_c^2} \left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r} \right) \vec{\mathrm{L}} \cdot \vec{\mathrm{S}} + \frac{4\alpha_s}{r^3}\, \mathrm{T} \right] \cdot F_q \cdot F_{\bar{q}}$$

variants:

running coupling
smeared delta functions
relativized
perturbative corrections
Mercedes baryon potential
instanton potential
flip flop potential
apply to light quarks?

how does it do?

$c\bar{c}$



(1) 3872 = DD*, but 2++ is equally off, also 2 0++ states?

(2) could this be the 3P1(2P) shifted up?

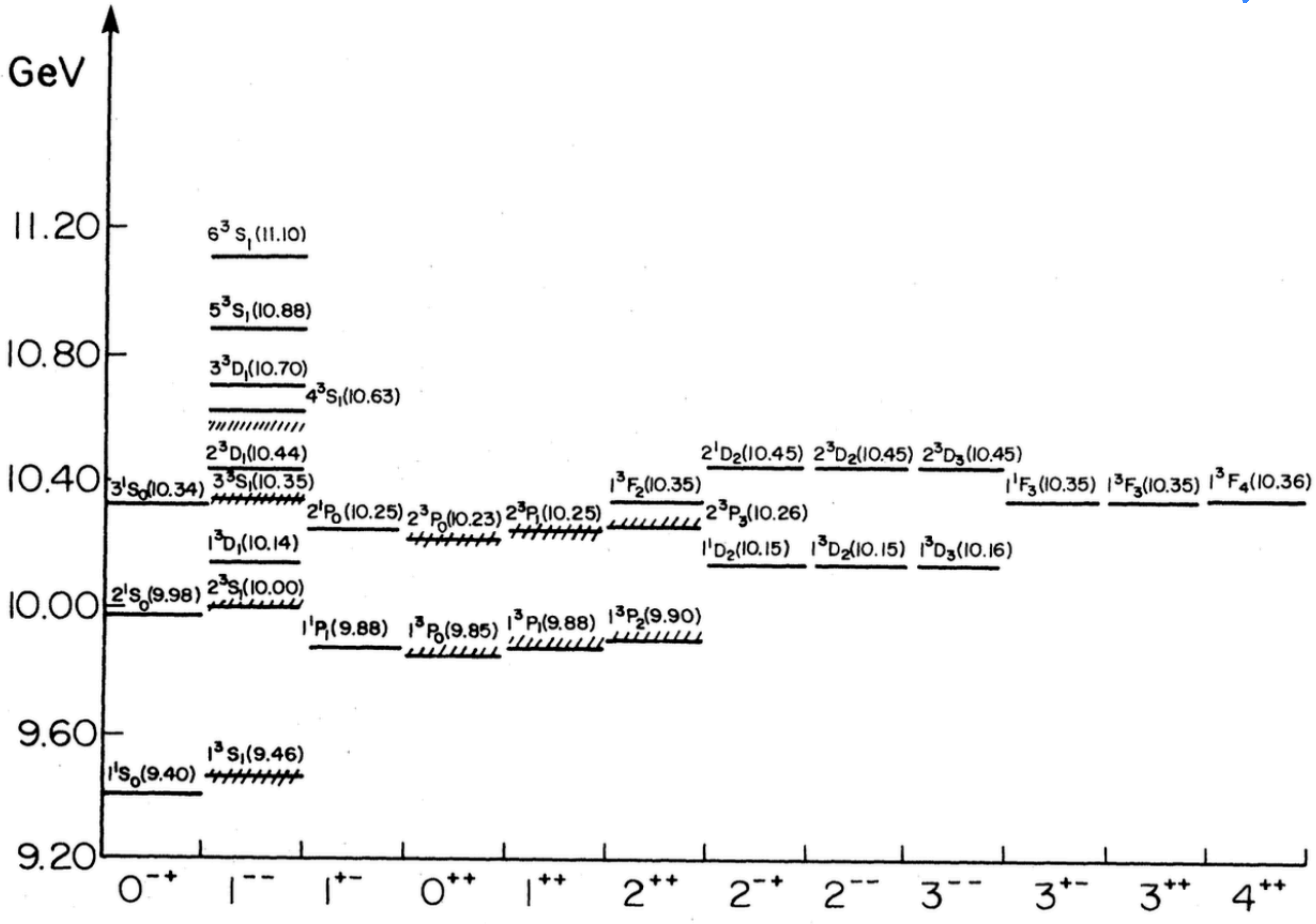
(3) something of a fiasco here

charmonia.dat / charmonia.plt

hybrids/spectrum/bb/fitBBc.cpp

$b\bar{b}$

Godfrey & Isgur



light quarks

problems with light quarks (?)

is the Fock space expansion sensible?

$$|\rho\rangle = |q\bar{q}\rangle + \sqrt{\alpha_s}\,|q\bar{q}g\rangle + \alpha_s\,|\pi\pi\rangle + \alpha_s^2\,|qqq\bar{q}\bar{q}\bar{q}\rangle + \ldots ?$$

does a "quark potential" exist?

is it sensible to think of the pion as a $|q\bar{q}\rangle$ state (rather than a quasiGoldstone boson)

relativistic effects ? $\langle q^2\rangle \sim m^2$

problems with light quarks (?)

is the Fock space expansion sensible? ***yes, if you are careful***

$$|\rho\rangle = |q\bar{q}\rangle + \sqrt{\alpha_s}\,|q\bar{q}g\rangle + \alpha_s\,|\pi\pi\rangle + \alpha_s^2\,|qqq\bar{q}\bar{q}\bar{q}\rangle + \ldots ?$$

does a "quark potential" exist? ***yes, if you are careful***

is it sensible to think of the pion as a $|q\bar{q}\rangle$ state (rather than a quasiGoldstone boson)

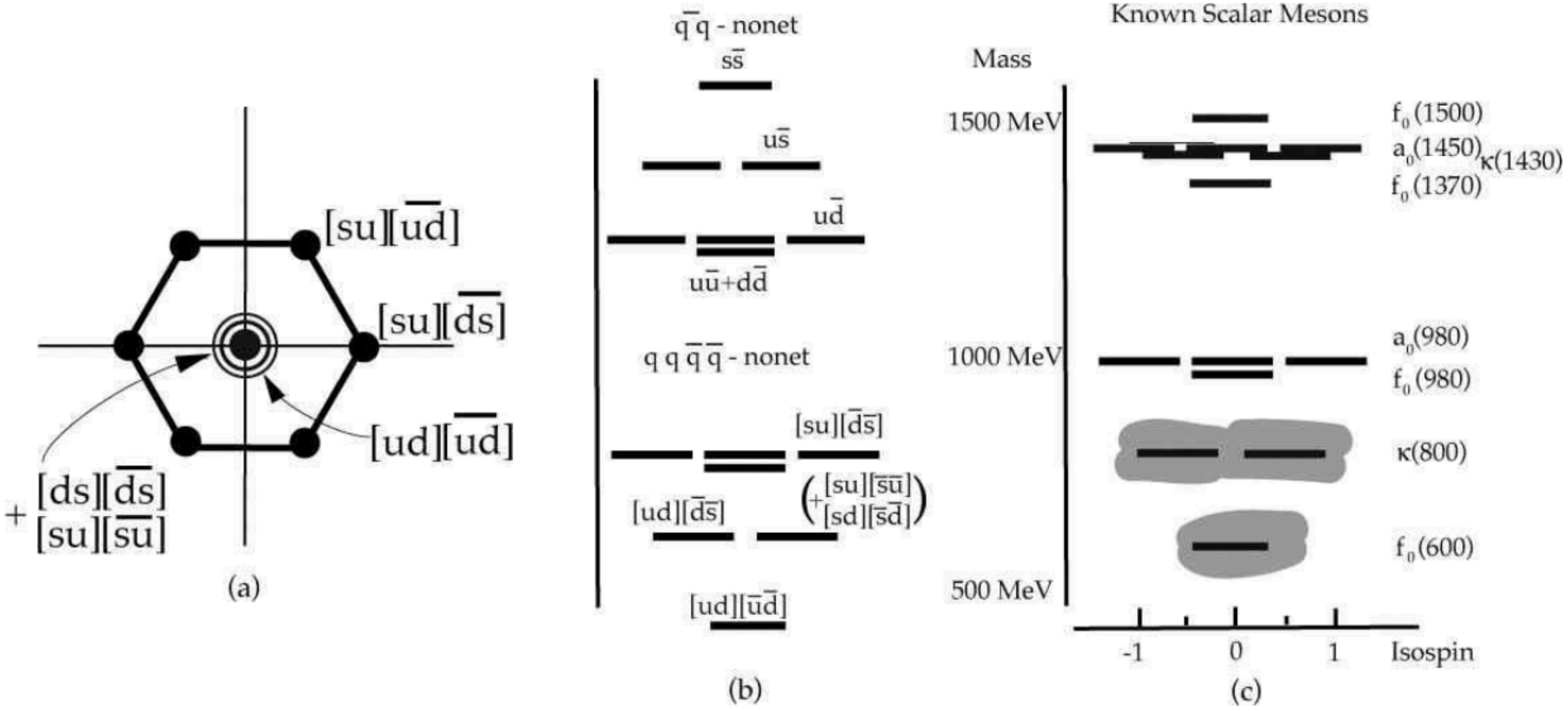
yes, if you are careful

relativistic effects ? $\langle q^2\rangle \sim m^2$

important for some observables

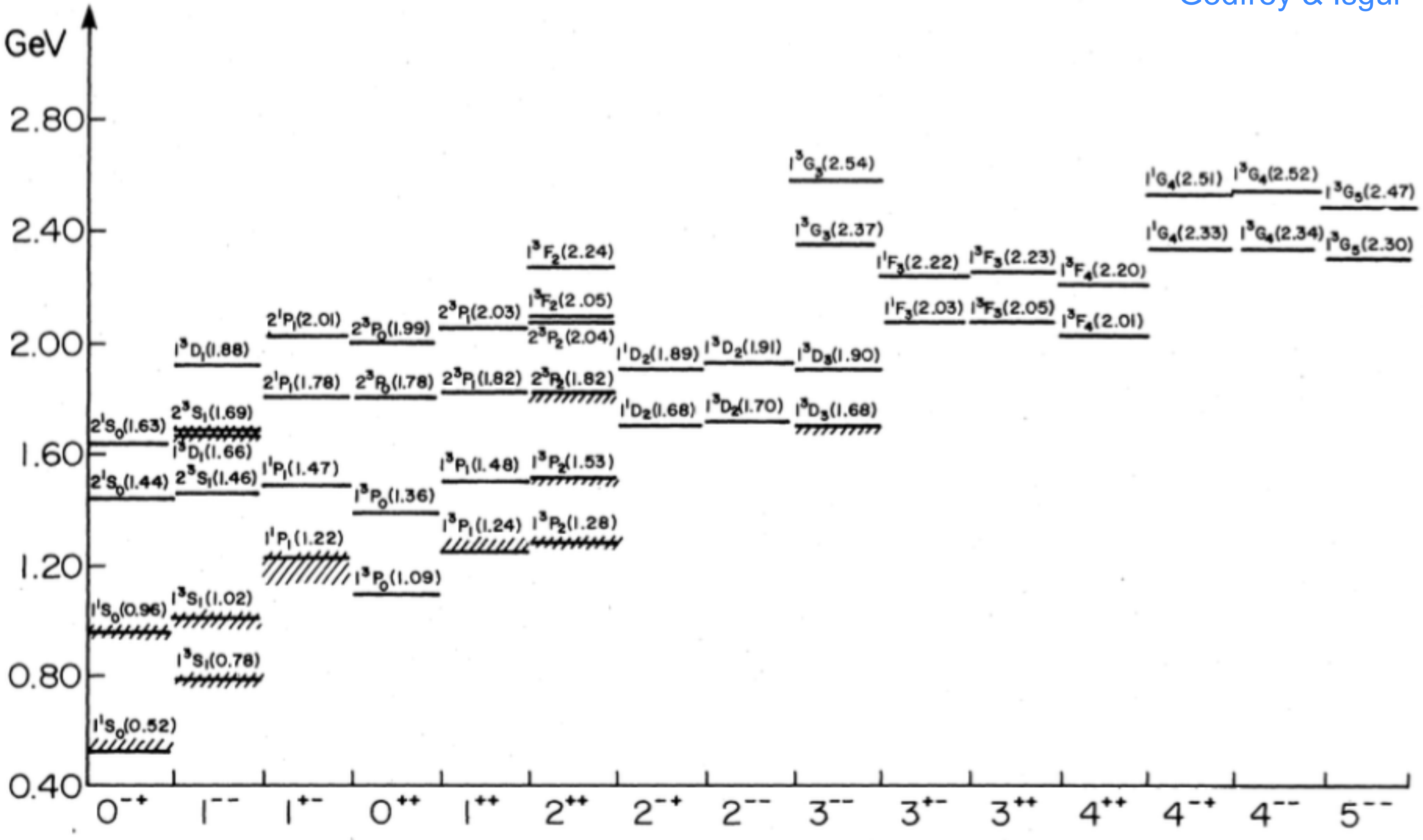
problems with light quarks (?)

light scalar nonet



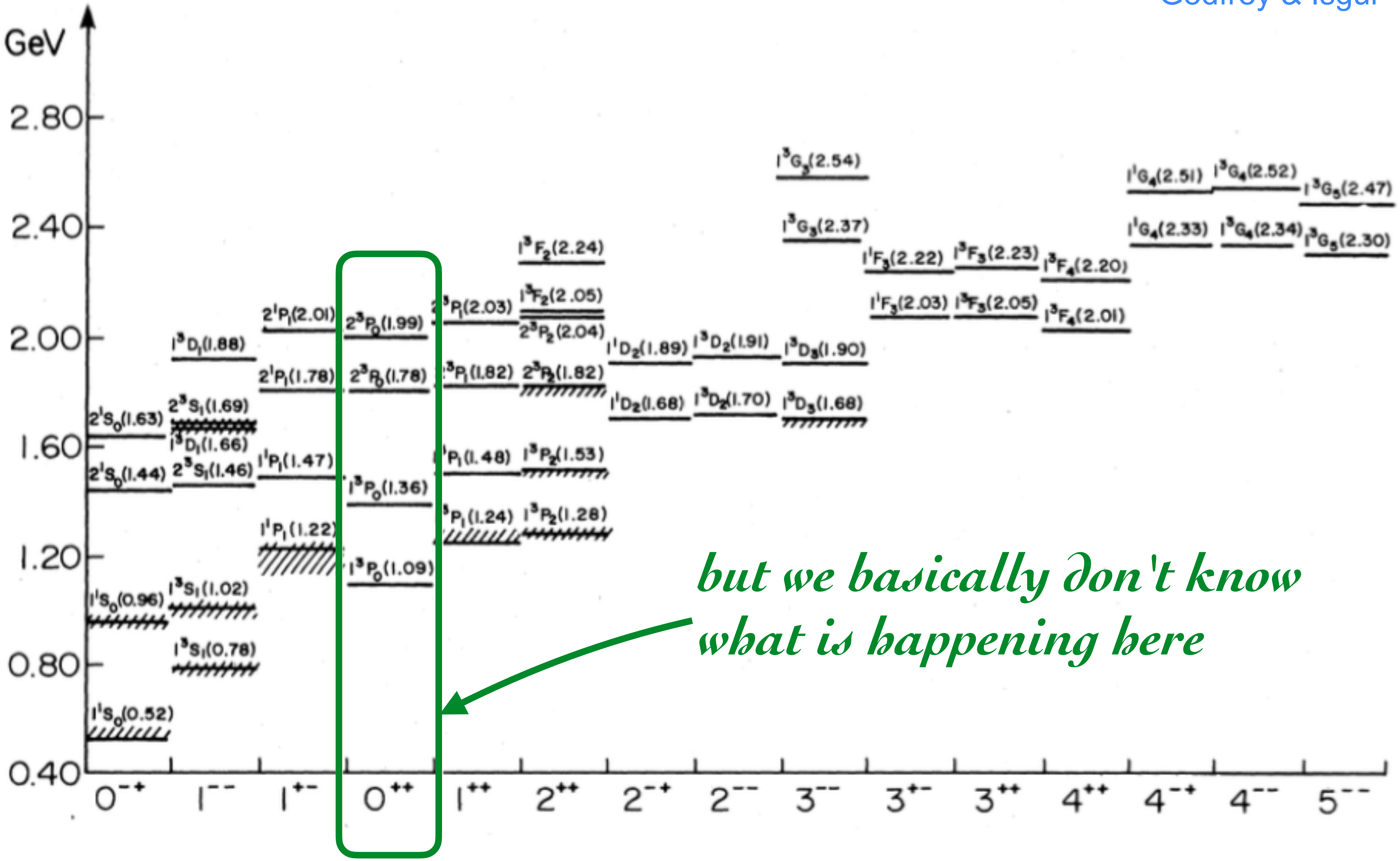
level ordering in the baryon spectrum

Godfrey & Isgur



isoscalar uu+dd+ss

Godfrey & Isgur



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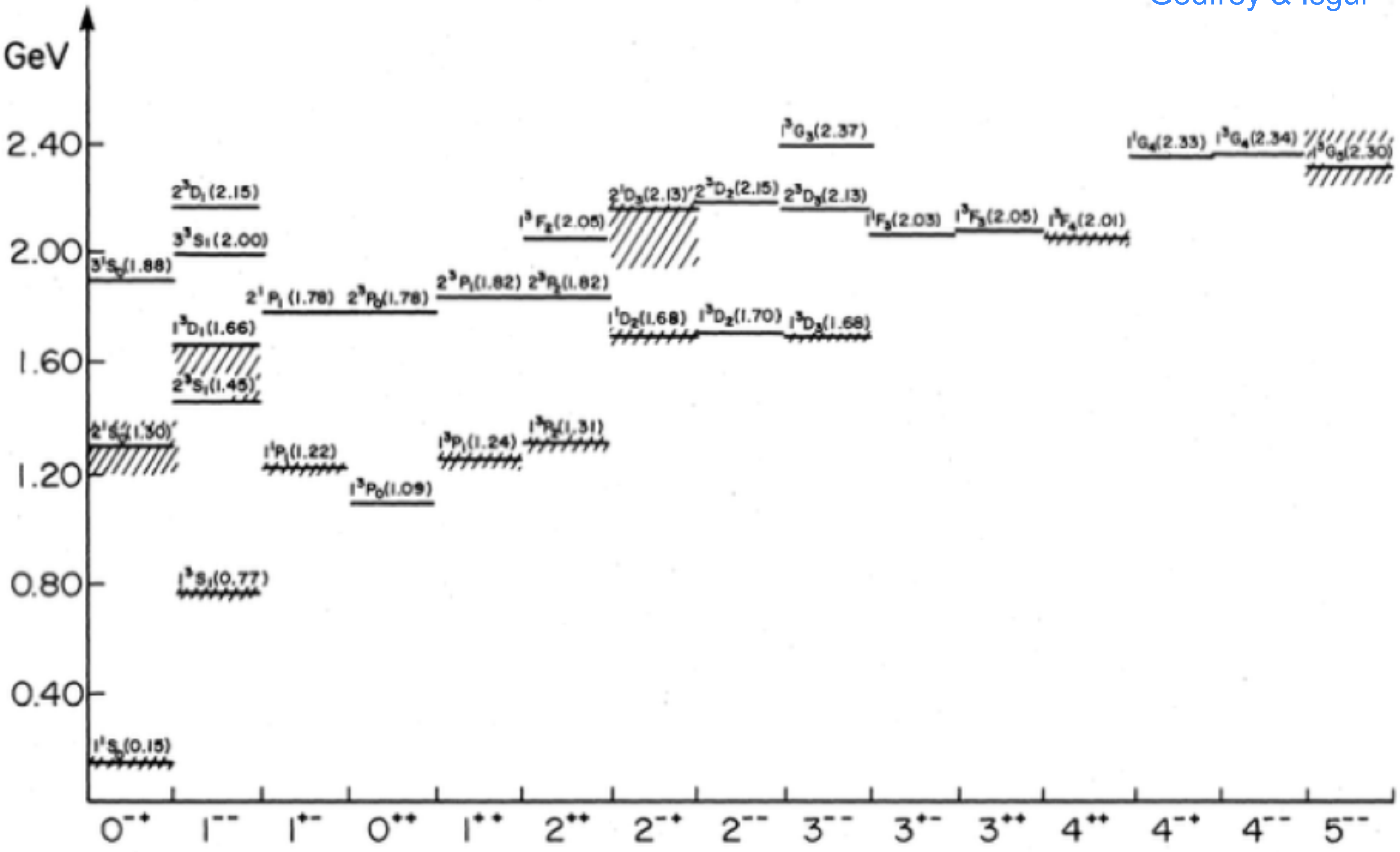
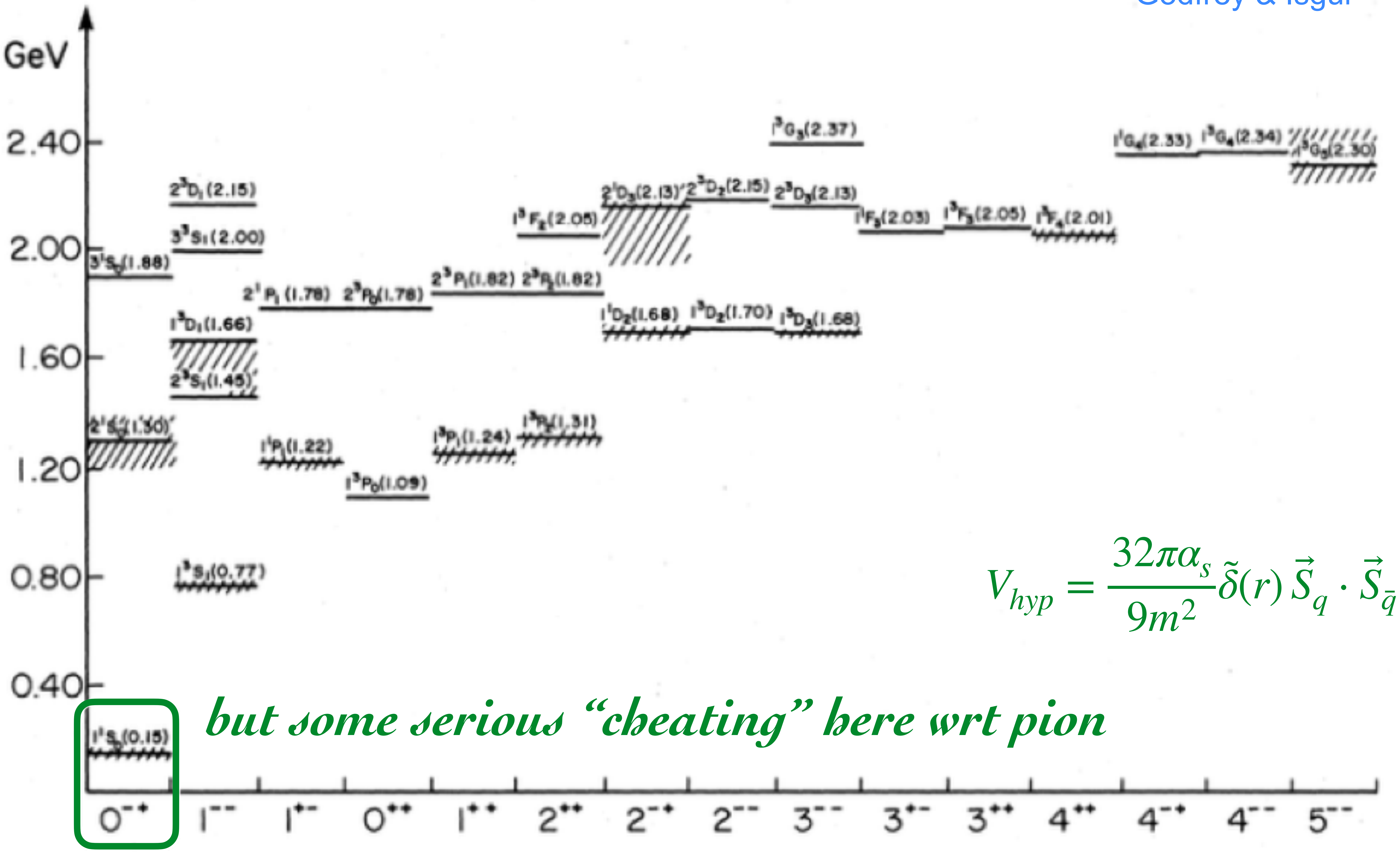
Godfrey & Isgur



isovector

Godfrey & Isgur



$$V_{hyp} = \frac{32\pi\alpha_s}{9m^2}\tilde{\delta}(r)\,\vec{S}_q \cdot \vec{S}_{\bar{q}}$$

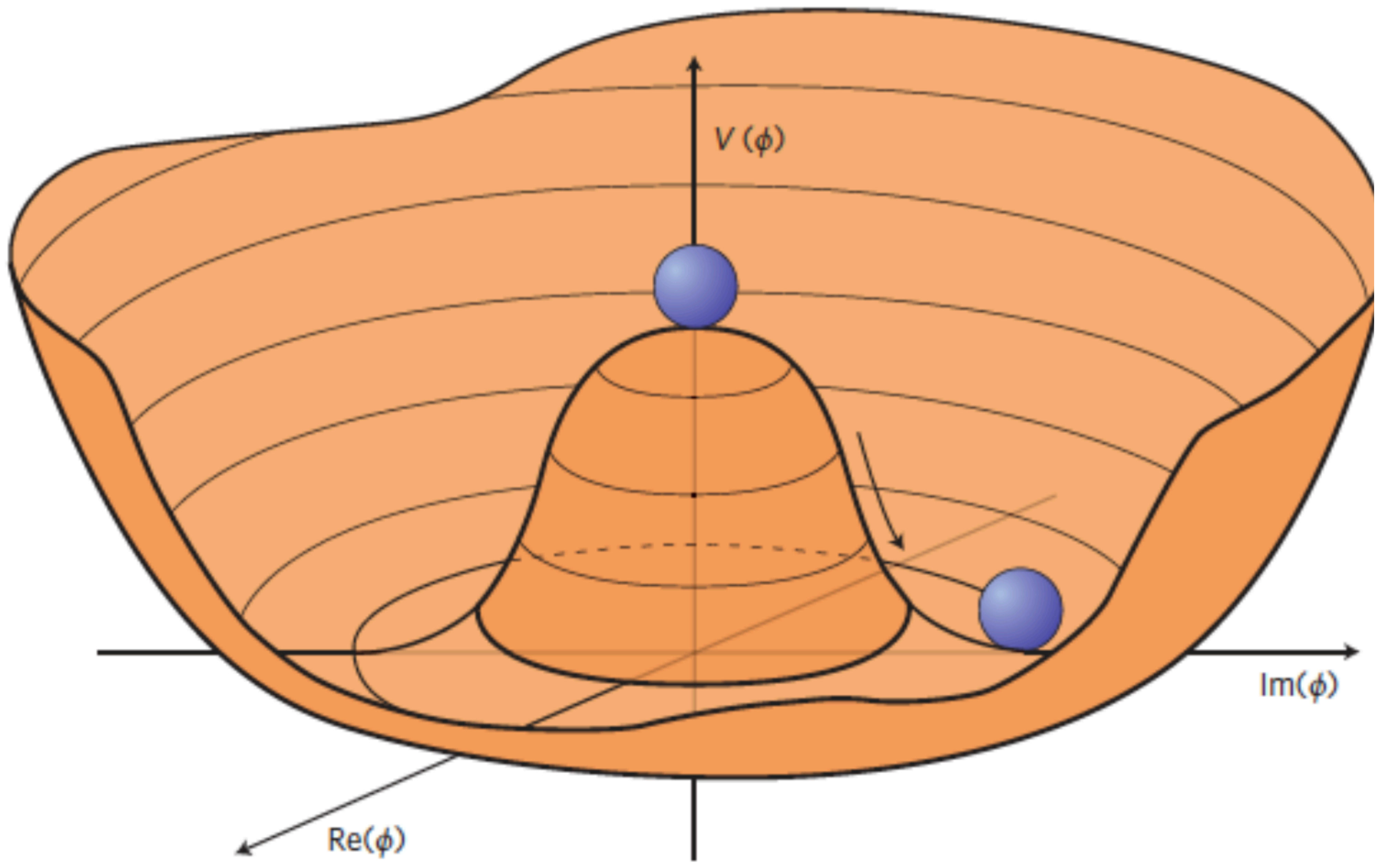
but some serious "cheating" here wrt pion

isovector

can we do better?

Goldstone bosons, pions, and constituent quarks

recall...



$$Q_5^a = \int d^3x\, \psi^\dagger \gamma_5 T^a \psi$$

$$\mathrm{e}^{i\theta^a Q_5^a}|0\rangle = |\theta\rangle \neq |0\rangle$$

$$H|\theta\rangle = H\mathrm{e}^{i\theta^a Q_5^a}|0\rangle = \mathrm{e}^{i\theta^a Q_5^a}H|0\rangle = E_0|\theta\rangle$$

[Higgs version -- we will generate this "symmetry breaking" dynamically]

Goldstone bosons and constituent quarks

an example:

Szczepaniak & Swanson, PRL87,072001 (01)

$$\mathcal{L} = \int d^4x \bar{\psi} \left(i\gamma^\mu \partial_\mu - m \right) \psi - \frac{\lambda}{2\Lambda^2} \int^\Lambda d^4x \, \psi^\dagger(x) T^a \psi(x) \psi^\dagger(x) T^a \psi(x)$$

$$\begin{aligned} H &= p\dot{q} - L \\ \gamma^\mu \partial_\mu &= \gamma^0 \partial_t - \gamma^i \partial_i \\ &= \beta \partial_t + \vec{\gamma} \cdot \nabla \\ &= \beta \partial_t + \beta \vec{\alpha} \cdot \nabla \end{aligned}$$

$$H = \int d^3x \psi^\dagger \left(-i\alpha \cdot \nabla + \beta m \right) \psi + \frac{\lambda}{2\Lambda^2} \int^\Lambda d^3x \, \psi^\dagger(x) T^a \psi(x) \psi^\dagger(x) T^a \psi(x)$$

Goldstone bosons and constituent quarks

$$\psi_{a,\alpha,f}(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}[u(\mathbf{k},s)_\alpha b_{a,s,f}(\mathbf{k}) + v(\mathbf{k},s)_\alpha d^\dagger_{a,s,f}(-\mathbf{k})\ ]\mathrm{e}^{-\mathbf{k}\cdot\mathbf{x}}$$

these operators are defined wrt a vacuum, what vacuum?

$$u(\mathbf{k},s) = \sqrt{\frac{1+s(k)}{2}}\begin{pmatrix} \chi_s \\ \frac{c(k)}{1+s(k)}\sigma\cdot\hat{k}\chi_s \end{pmatrix}$$

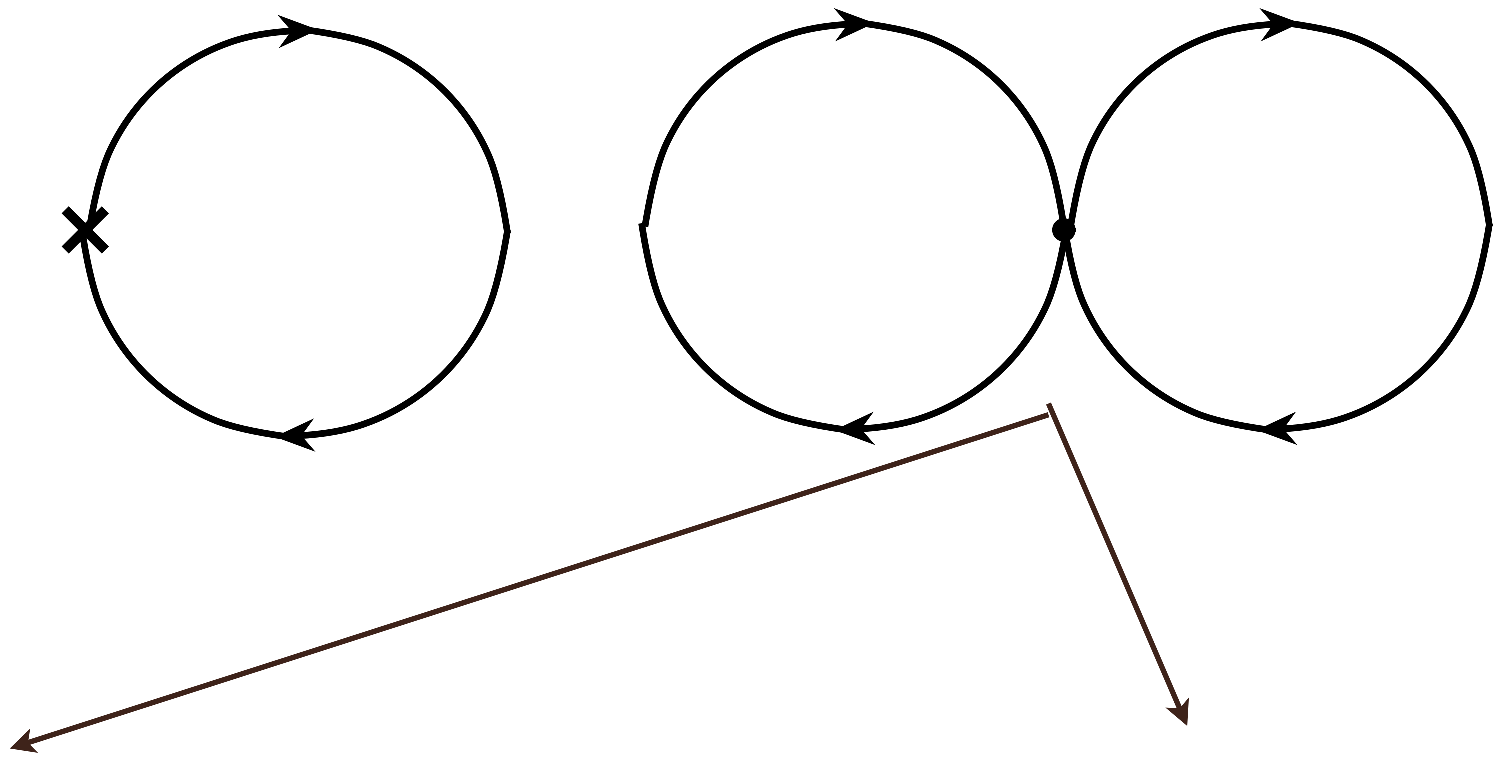
$$v(\mathbf{k},s) = \sqrt{\frac{1+s(k)}{2}}\begin{pmatrix} -\frac{c(k)}{1+s(k)}\sigma\cdot\hat{k}\tilde{\chi}_s \\ \tilde{\chi}_s \end{pmatrix}$$

unknown functions, to be determined

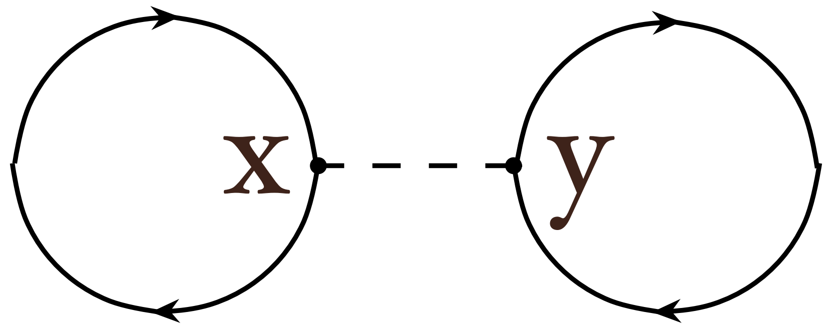
Goldstone bosons and constituent quarks

$$\langle H\rangle = \int d^3x\,\psi^\dagger\left(-i\alpha\cdot\nabla+\beta m\right)\psi +$$
$$\frac{\lambda}{2\Lambda^2}\int^\Lambda d^3x\,\psi^\dagger(x)T^a\psi(x)\psi^\dagger(x)T^a\psi(x)$$

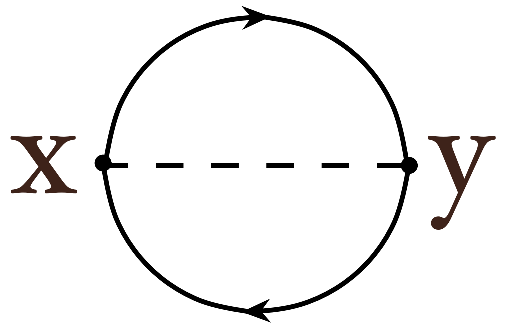
x x y y



Hartree



Fock



Goldstone bosons and constituent quarks

$$\langle H \rangle = -2N_c \int^{\Lambda} \frac{d^3k}{(2\pi)^3} \left(s(k)m + c(k)k\right) + \frac{\lambda}{2\Lambda^2} \frac{N_c^2-1}{2} \int^{\Lambda} \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left(1 - s(k)s(q) - c(k)c(q)\hat{k}\cdot\hat{q}\right)$$

$$s(k) = \sin\phi(k)$$

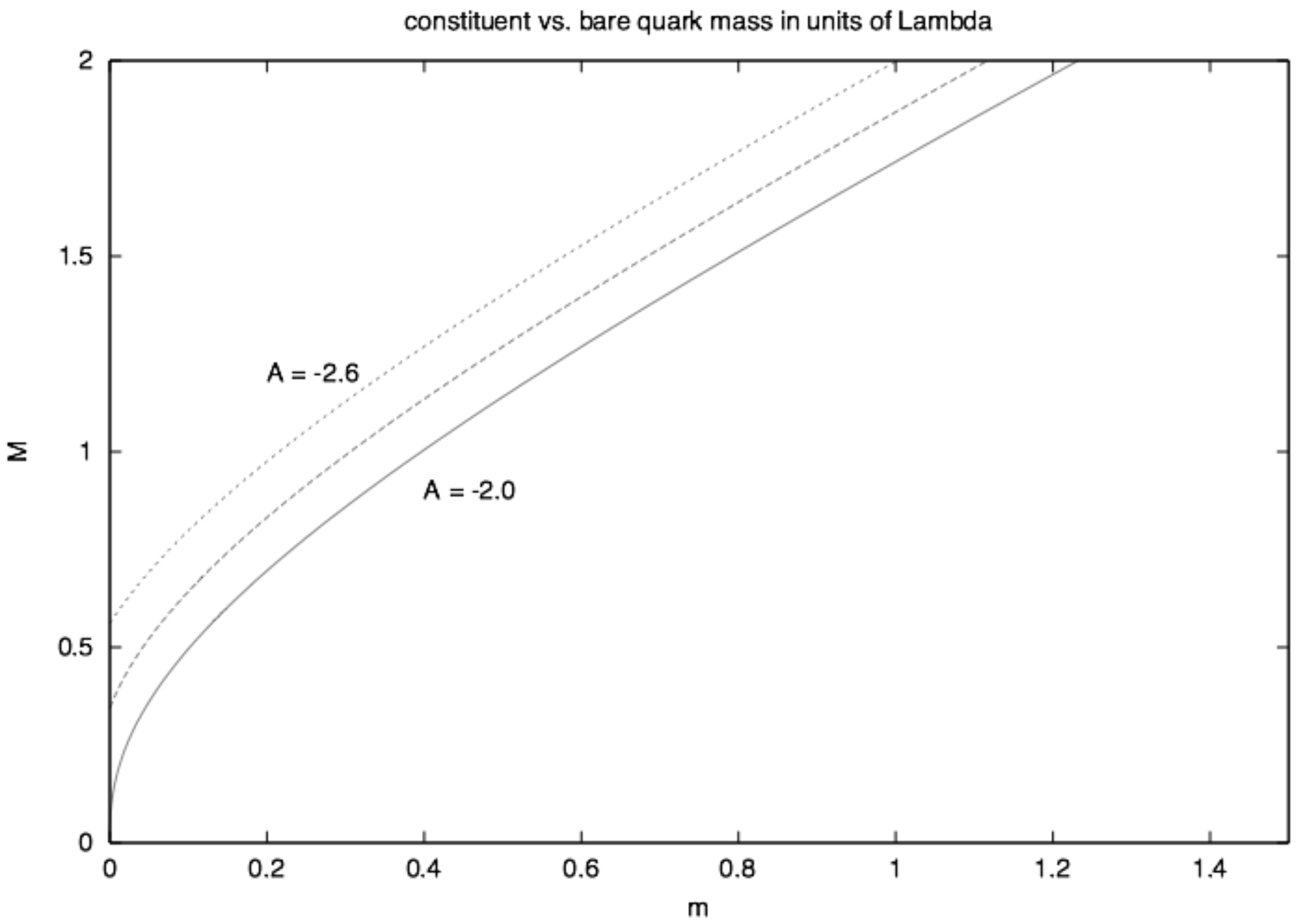
$$\frac{\delta}{\delta\phi}\langle H \rangle = 0$$

$$M(p) = \frac{ps(p)}{c(p)}$$

$$M(p) = m(\Lambda) + \frac{C_F\lambda}{4\pi^2\Lambda^2} \int^{\Lambda} q^2 dq \frac{M(q)}{\sqrt{M(q)+q^2}}$$

"gap equation"

Goldstone bosons and constituent quarks

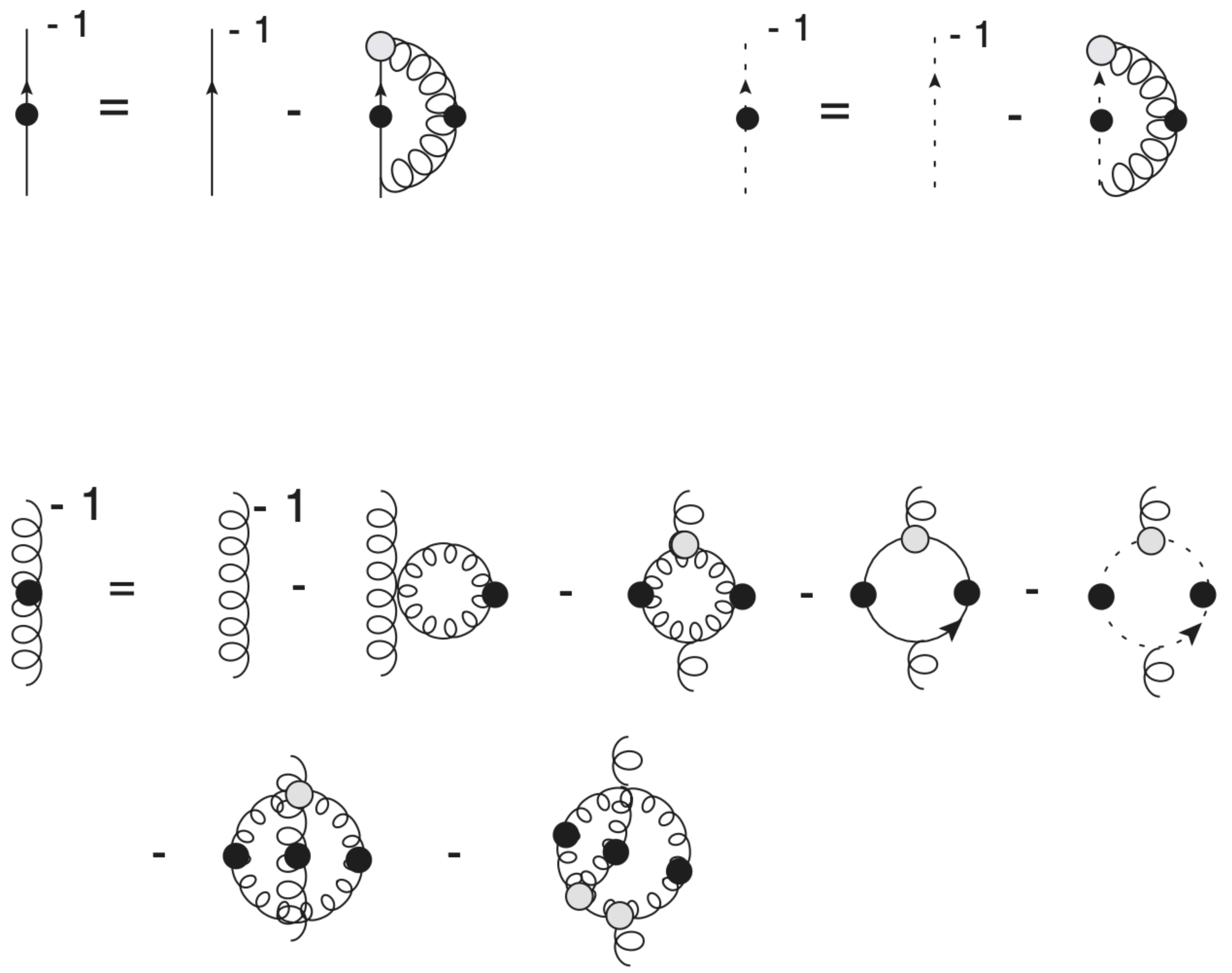


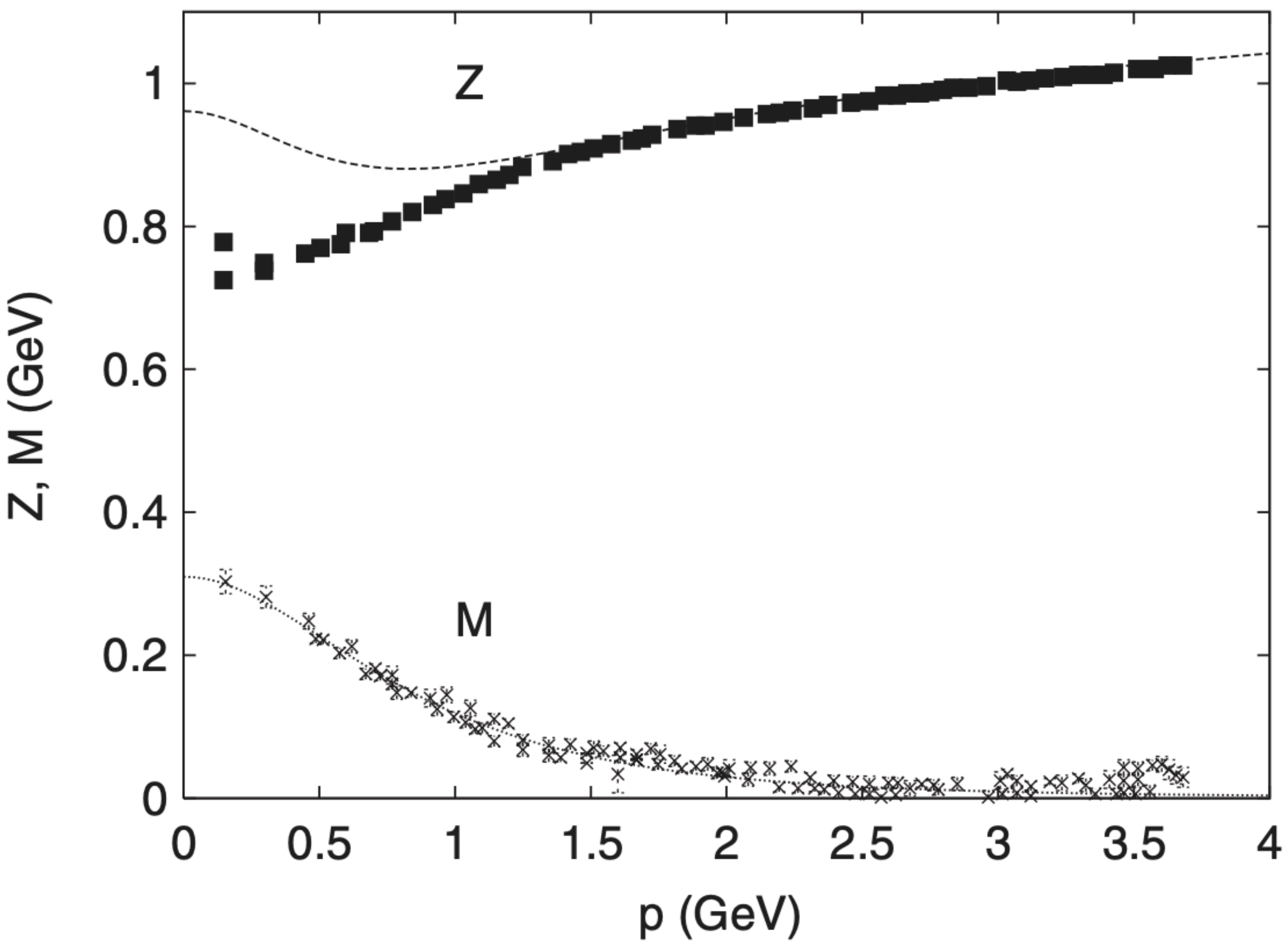
constituent quark mass

current quark mass

Goldstone bosons and constituent quarks

a more realistic model



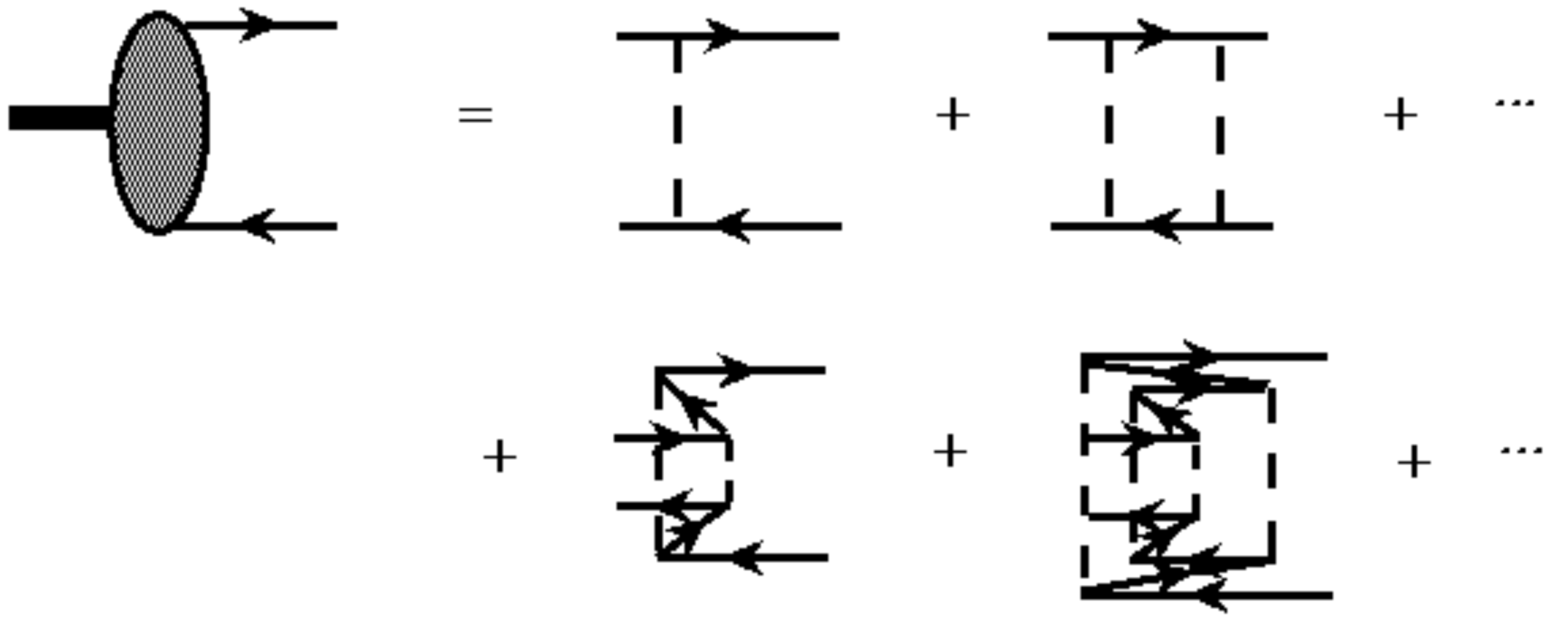


Goldstone bosons and constituent quarks

the "RPA" approximation

$$\langle M|[H, Q^{\dagger}_{M}]|BCS\rangle = (E_M - E_{BCS})\langle M|Q^{\dagger}_{M}|BCS\rangle$$

$$Q^{\dagger}_{M} = \Sigma_{\alpha\beta}(\psi^{+}_{\alpha\beta}B^{\dagger}_{\alpha}D^{\dagger}_{\beta} - \psi^{-}_{\alpha\beta}D_{\beta}B_{\alpha})$$



Goldstone bosons and constituent quarks

chiral symmetry breaking generates Goldstone bosons *and* constituent quarks

and underpins applicability of the NRCQM to light hadrons

skip>>

Nonrelativistic models

$$\langle p \rangle << m$$

L and S separately conserved

different parity corresponds to different waves

$$0^{-+} = {}^1S_0 \qquad\qquad 0^{++} = {}^3P_0$$

Relativistic models

$$\langle p \rangle >> m$$

L and S are *not* separately conserved

$$V(0^{++}) = V_0 c_p c_k + V_1 (1 + s_p s_k)$$

wave

$$V(0^{-+}) = V_0 (1 + s_p s_k) + V_1 c_p c_k$$

$$V(0^{++}) = V_0 c_p c_k + V_1(1 + s_p s_k)$$

$$V(0^{-+}) = V_0(1 + s_p s_k) + V_1 c_p c_k$$

NonRel

$$c_p = \frac{p}{E(p)} \to \frac{p}{m} \qquad s_p = \frac{\mu(p)}{E(p)} \to 1$$

$$V(0^{++}) \to 2V_1 + \mathcal{O}(\frac{1}{m^2})$$ P-wave

$$V(0^{-+}) \to 2V_0 + \mathcal{O}(\frac{1}{m^2})$$ S-wave

$$V(0^{++}) = V_0 c_p c_k + V_1 (1 + s_p s_k)$$

$$V(0^{-+}) = V_0 (1 + s_p s_k) + V_1 c_p c_k$$

Rel

$$c_p = \frac{p}{E(p)} \to 1 \qquad s_p = \frac{\mu(p)}{E(p)} \to 0$$

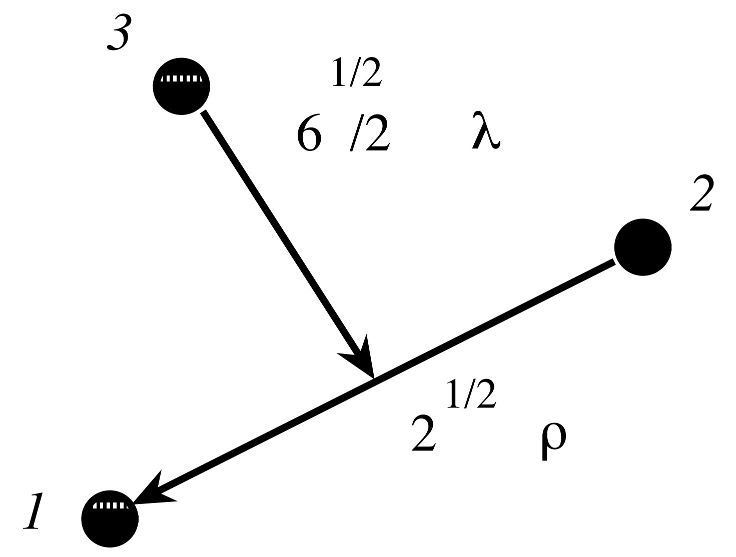
$$V(0^{++}) \to V_0 + V_1$$

$$V(0^{-+}) \to V_0 + V_1$$

baryons

Isgur-Karl Model

$$H_{IK} = \sum_{i=1}^{3}\left(m_i + \frac{p_i^2}{2m_i}\right) + \sum_{i<j}\frac{1}{2}kr_{ij}^2$$



$$H_{IK} = M_{tot} + \frac{P^2}{2M_{tot}} + \frac{p_\rho^2}{2m_\rho} + \frac{p_\lambda^2}{2m_\lambda} + \frac{3}{2}k\rho^2 + \frac{3}{2}k\lambda^2$$

$$m_\rho = m_1 = m_2 \qquad m_\lambda = 3\frac{m_1 m_3}{M_{tot}}$$

Isgur-Karl Model

$$E = (N_\rho + \frac{3}{2})\omega_\rho + (N_\lambda + \frac{3}{2})\omega_\lambda$$

$$\omega_\rho = \sqrt{\frac{3k}{m_\rho}} \qquad \omega_\lambda = \sqrt{\frac{3k}{m_\lambda}}$$

proton:

$$\Psi = C_A uud \left(\frac{\alpha_\rho \alpha_\lambda}{\pi}\right)^{3/2} \mathrm{e}^{-\frac{1}{2}(\alpha_\rho^2 \rho^2 + \alpha_\lambda^2 \lambda^2)} \chi$$

$$C_A = \frac{1}{\sqrt{6}} \left(rbg - brg + bgr - gbr + grb - rgb\right)$$

$$\chi = -\frac{1}{\sqrt{6}} \left(|\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle - 2|\uparrow\uparrow\downarrow\rangle\right)$$

Isgur-Karl Model

baryon flavour wavefunctions

State	++	+	0	-
N		uud	ddu	
Δ	uuu	uud	ddu	ddd
Λ			$\frac{1}{\sqrt{2}}$(ud -du)s	
Σ		uus	$\frac{1}{\sqrt{2}}$(ud+du)s	
Ξ			ssu	ssd
Ω				sss
Λ_c		$\frac{1}{\sqrt{2}}$(ud-du)c		
Σ_c	uuc	$\frac{1}{\sqrt{2}}$(ud+du)c	ddc	
Λ_b			$\frac{1}{\sqrt{2}}$(ud-du)b	
Σ_b		uub	$\frac{1}{\sqrt{2}}$(ud+du)b	ddb

Isgur-Karl Model

magnetic moments

$$\begin{aligned} \mu_p &= \langle \chi^{\lambda}_{1/2 1/2} | \sum_i \frac{e_i}{2m_i} \sigma^z_i | \chi^{\lambda}_{1/2 1/2} \rangle \\ &= \frac{4}{3}\mu_u - \frac{1}{3}\mu_d. \end{aligned}$$

$$\mu_n = 4/3\mu_d - 1/3\mu_u$$

$$\mu_u = -2\mu_d$$

$$\frac{\mu_n}{\mu_p} = -\frac{2}{3}$$

expt: -0.6849

Isgur-Karl Model

hyperfine splitting

$$\Delta m = \frac{4\pi\alpha_s}{9}|\psi(0)|^2 \sum_{i<j} \frac{\langle\vec{\sigma}_i \cdot \vec{\sigma}_j\rangle}{m_i m_j}.$$

$$\begin{aligned}\Delta N &= \frac{4\pi\alpha_s}{9m_u^2}(-3)|\psi(0)|^2 \equiv \frac{-3}{m_u^2}K \\ \Delta\Delta &= \frac{3}{m_u^2}K \\ \Delta\Sigma &= (\frac{1}{m_u^2} - \frac{4}{m_u m_s})K.\end{aligned}$$

K = 0.0066 GeV ^{3}

$$\begin{aligned}\langle\chi^\lambda|\vec{\sigma}_1 \cdot \vec{\sigma}_2|\chi^\lambda\rangle &= 1 \\ \langle\chi^\lambda|\vec{\sigma}_1 \cdot \vec{\sigma}_3|\chi^\lambda\rangle &= -2 \\ \langle\chi^\lambda|\vec{\sigma}_2 \cdot \vec{\sigma}_3|\chi^\lambda\rangle &= -2\end{aligned}$$

baryon(mass)	composition	$\Delta E/K$	predicted mass
N(939)	nnn	$-3/m_n^2$	939
Λ(1116)	nns	$-3/m_n^2$	1114
Σ(1193)	nns	$1/m_n^2 - 4/(m_n m_s)$	1179
Ξ(1318	nss	$1/m_s^2 - 4/(m_n m_s)$	1327
Δ(1232)	nnn	$3/m_n^2$	1239
Σ(1384)	nns	$1/m_n^2 + 2/(m_n m_s)$	1381
Ξ(1533)	nss	$1/m_s^2 + 2/(m_n m_s)$	1529
Ω(1672)	sss	$3/m_s^2$	1682

Isgur-Karl Model

Hyperfine Splitting in P-wave Baryons

S-wave P-wave

contact in λ, tensor in ρ

$$\begin{aligned} m_\Delta - m_N &= A\frac{8\pi}{3}\langle\psi_{00}|\delta(\vec{\rho})|\psi_{00}\rangle\left[\langle\chi^S_{3/2}|\vec{S}_1\cdot\vec{S}_2|\chi^S_{3/2} - \langle\chi^\lambda_{1/2}|\vec{S}_1\cdot\vec{S}_2|\chi^\lambda_{1/2}\right] \\ &= A\frac{8\pi}{3}\frac{\beta^3}{\pi^{3/2}}\left[\frac{3}{4}-\frac{-3}{4}\right] \\ &= 4A\frac{\beta^3}{\sqrt{\pi}} \\ &= 300\text{MeV}. \end{aligned}$$

notation

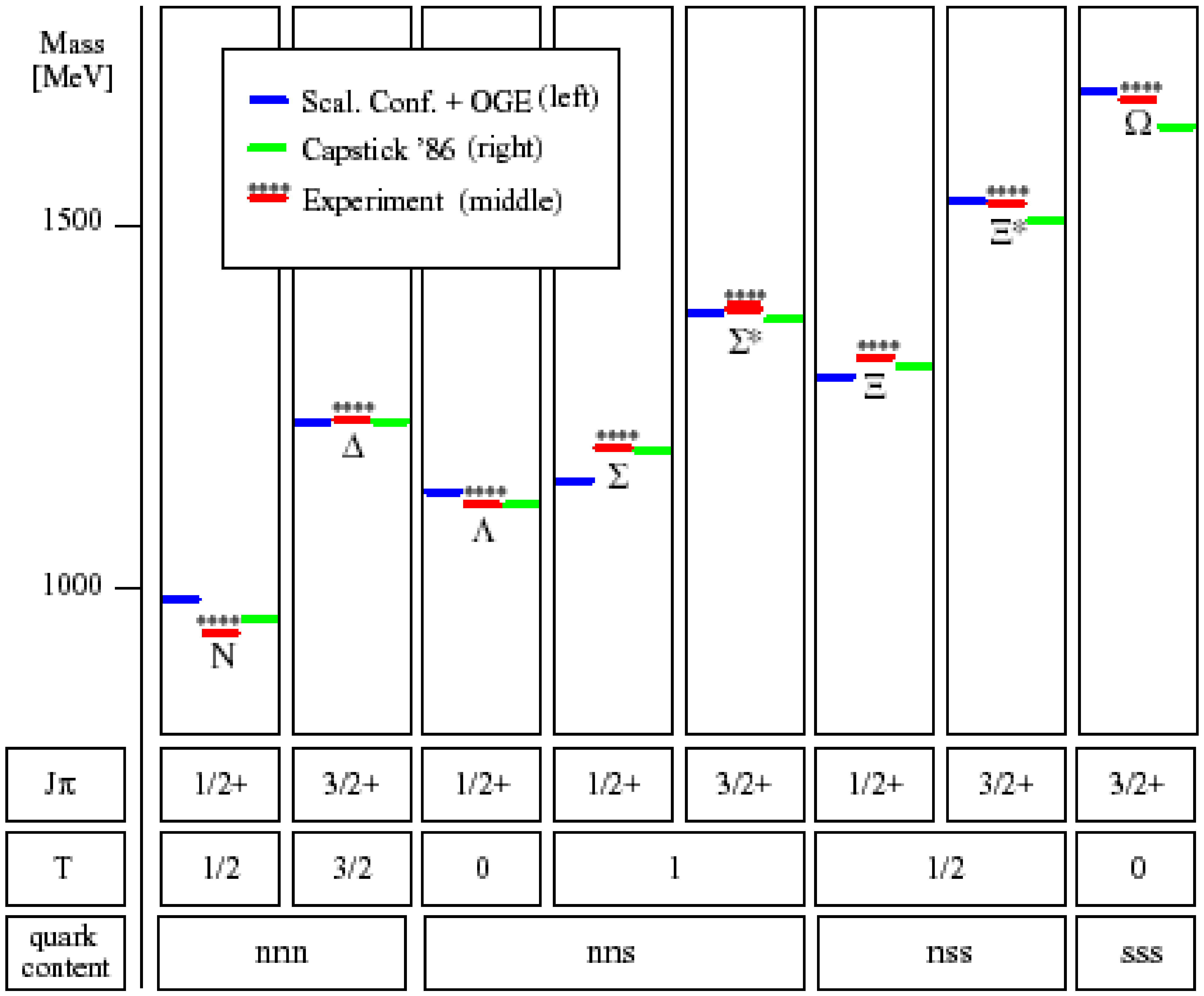
$|\Xi SLJ^P\rangle$

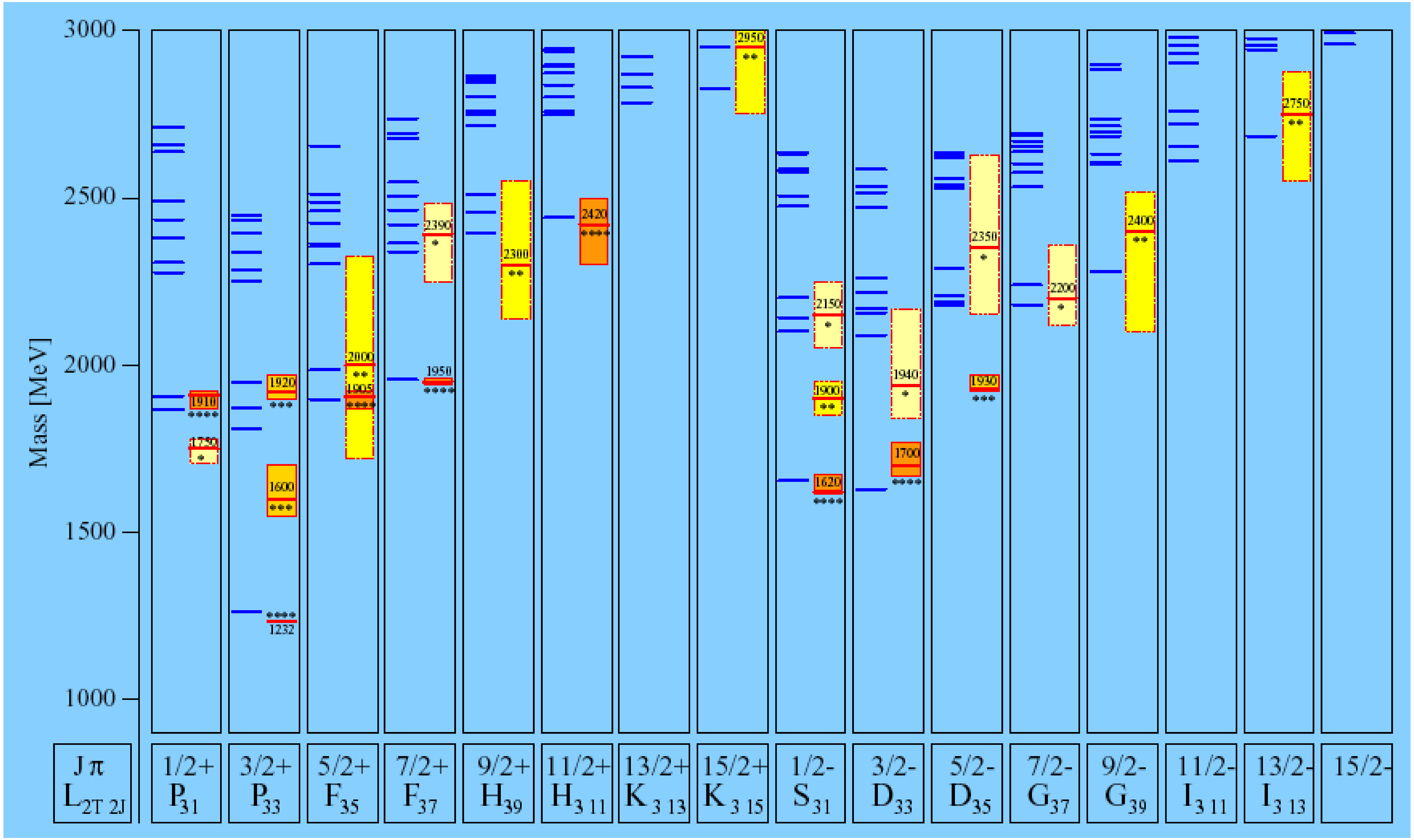
$$\begin{aligned} |N\,1/2\,P\,3/2^-\rangle &= C_A\frac{1}{2}\left[\chi^\rho_{1/21/2}\phi^\rho_N\psi^\lambda_{11} + \chi^\rho_{1/21/2}\phi^\lambda_N\psi^\rho_{11} + \chi^\lambda_{1/21/2}\phi^\rho_N\psi^\rho_{11} - \chi^\lambda_{1/21/2}\phi^\lambda_N\psi^\lambda_{11}\right] \\ |N\,3/2\,P\,5/2^-\rangle &= C_A\chi^S_{3/2}\frac{1}{\sqrt{2}}\left[\phi^\rho_N\psi^\rho_{11} + \phi^\lambda_N\psi^\lambda_{11}\right] \\ |\Delta\,1/2\,P\,3/2^-\rangle &= C_A\,\phi^S_\Delta\frac{1}{\sqrt{2}}\left[\chi^\rho_{1/21/2}\psi^\rho_{11} + \chi^\lambda_{1/21/2}\psi^\lambda_{11}\right]. \end{aligned}$$

$$\begin{aligned} \langle\Delta\,1\,1/2\,3/2|V_{hyp}|\Delta\,1\,1/2\,3/2\rangle &= 1 \\ \langle\Delta\,1\,1/2\,1/2|V_{hyp}|\Delta\,1\,1/2\,1/2\rangle &= 1 \\ \langle N\,1\,3/2\,5/2|V_{hyp}|N\,1\,3/2\,5/2\rangle &= \frac{4}{5} \end{aligned}$$

$$V_{hyp}\begin{pmatrix}|N\,1\,3/2\,3/2\rangle\\|N\,1\,1/2\,3/2\rangle\end{pmatrix} = \begin{pmatrix}\frac{9}{5} & \frac{1}{\sqrt{10}}\\ \frac{1}{\sqrt{10}} & -1\end{pmatrix}\begin{pmatrix}|N\,1\,3/2\,3/2\rangle\\|N\,1\,1/2\,3/2\rangle\end{pmatrix} \quad \Rightarrow\ \theta = 6.3^\circ \qquad \text{(expt)}\ \theta = 10^\circ$$

$$V_{hyp}\begin{pmatrix}|N\,1\,3/2\,1/2\rangle\\|N\,1\,1/2\,1/2\rangle\end{pmatrix} = \begin{pmatrix}0 & -1\\ -1 & -1\end{pmatrix}\begin{pmatrix}|N\,1\,3/2\,1/2\rangle\\|N\,1\,1/2\,1/2\rangle\end{pmatrix} \quad \Rightarrow\ \theta = -31.7^\circ \qquad \text{(expt)}\ \theta = -32^\circ$$


Mass
[MeV]
1500
1000
Scal. Conf. + OGE (left)
Capstick '86 (right)
Experiment (middle)
N
Δ
Λ
Σ
Σ*
Ξ
Ξ*
Ω
Jπ
1/2+
3/2+
1/2+
1/2+
3/2+
1/2+
3/2+
3/2+
T
1/2
3/2
0
1
1/2
0
quark content
nnn
nns
nss
sss



Mass [MeV]
3000
2500
2000
1500
1000
Jπ
$L_{2T\ 2J}$
1/2+ P_{31}
3/2+ P_{33}
5/2+ F_{35}
7/2+ F_{37}
9/2+ H_{39}
11/2+ $H_{3\ 11}$
13/2+ $K_{3\ 13}$
15/2+ $K_{3\ 15}$
1/2- S_{31}
3/2- D_{33}
5/2- D_{35}
7/2- G_{37}
9/2- G_{39}
11/2- $I_{3\ 11}$
13/2- $I_{3\ 13}$
15/2-
1910 ****
1750 *
1920 ***
1600 ***
1232 ****
2000 **
1905 ****
2390 *
1950 ****
2300 **
2420 ****
2950 **
2150 *
1900 **
1620 ****
1940 *
1700 ****
2350 *
1930 ***
2200 *
2400 **
2750 **

some problems...

- incorrect level ordering in the $N, \Delta, \Lambda, \Sigma$ spectra
- missing flavour dependence needed to describe level ordering in N and Δ spectra
- strong one gluon exhange spin-orbit interactions are not seen

✢ ÆRIC MEC HEHT GEWYRCAN