RUHR UNIVERSITÄT BOCHUM RUB

Modern Techniques in Hadron Spectroscopy

July, 2024

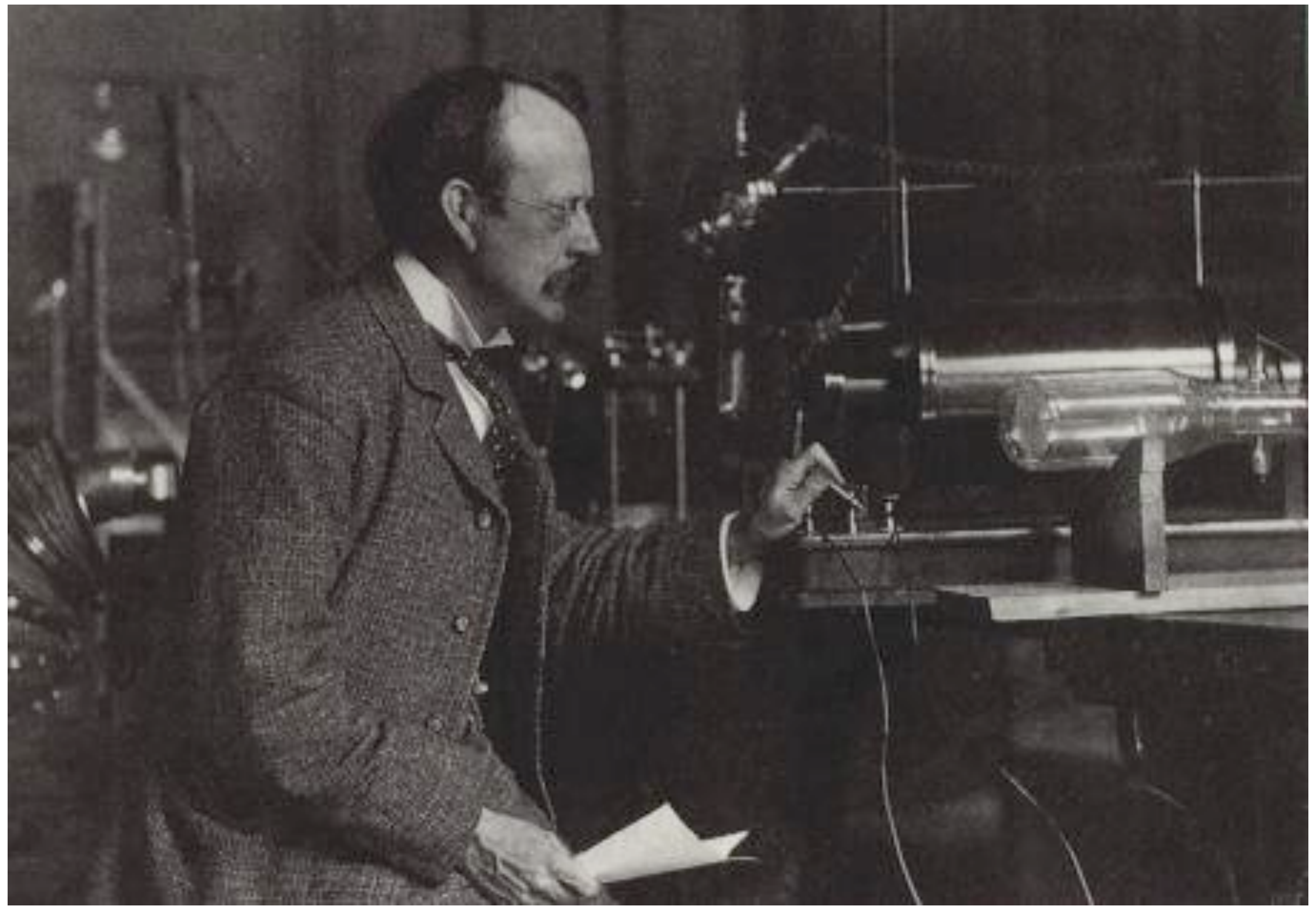
QCD Phenomenology -- Introduction

Eric Swanson

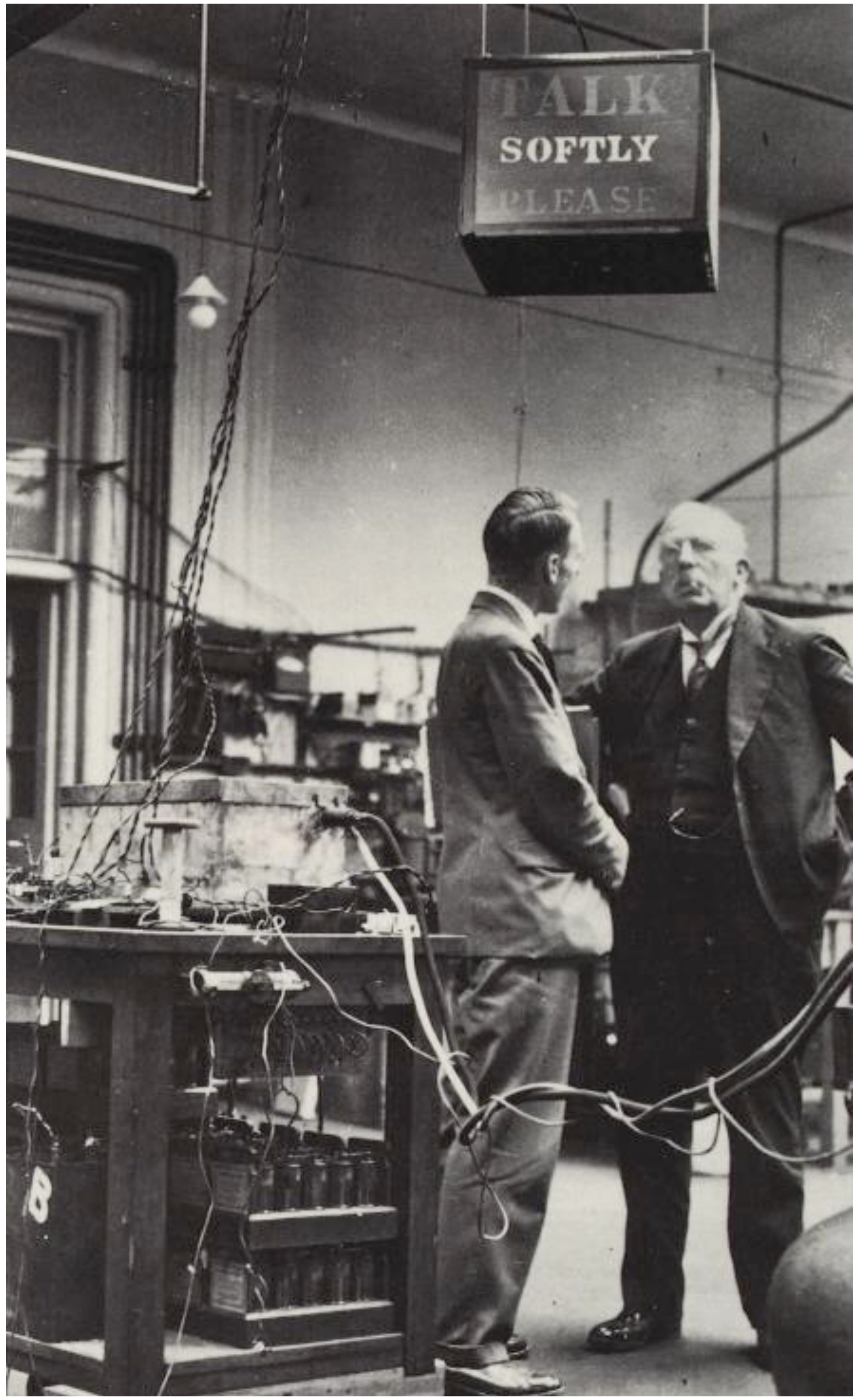
ExoHad
Exotic Hadrons Topical Collaboration

building the atom

1897-1932



electron

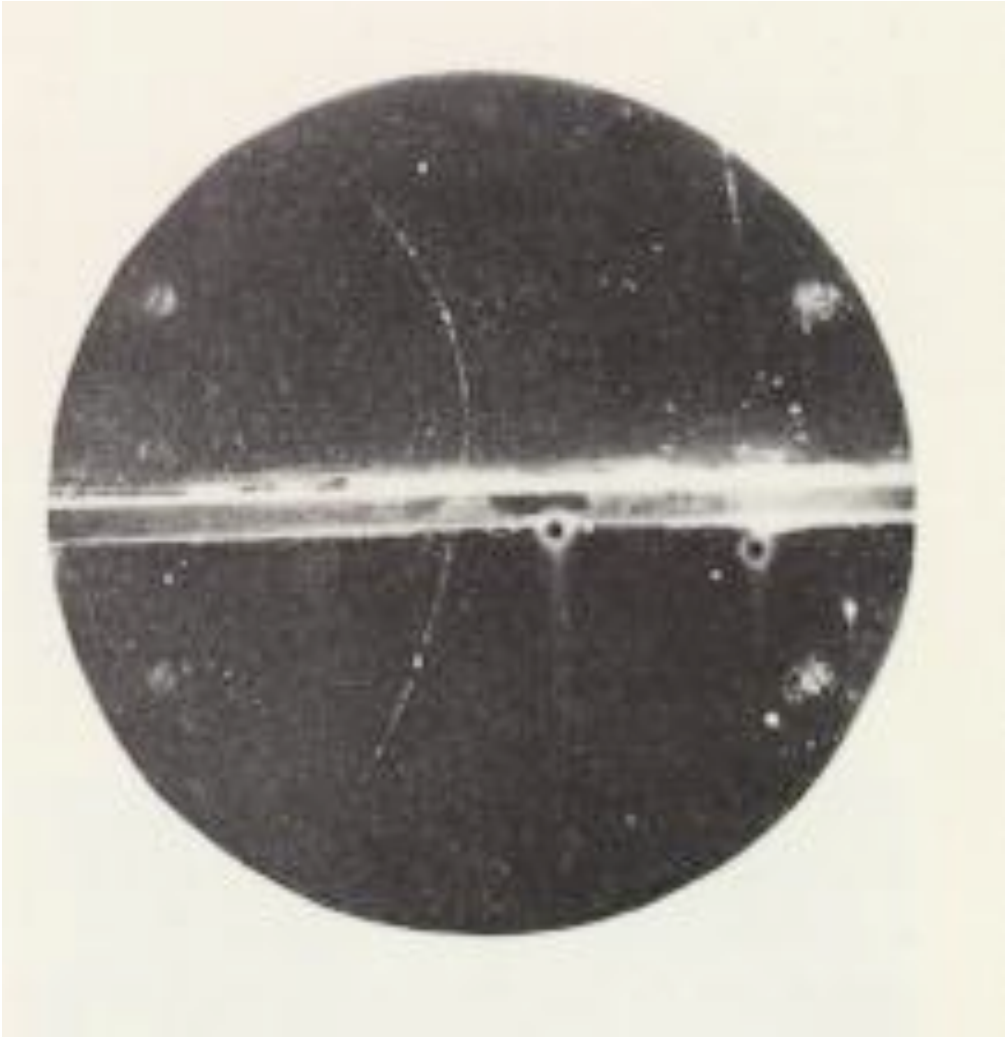


nucleus/proton

neutron

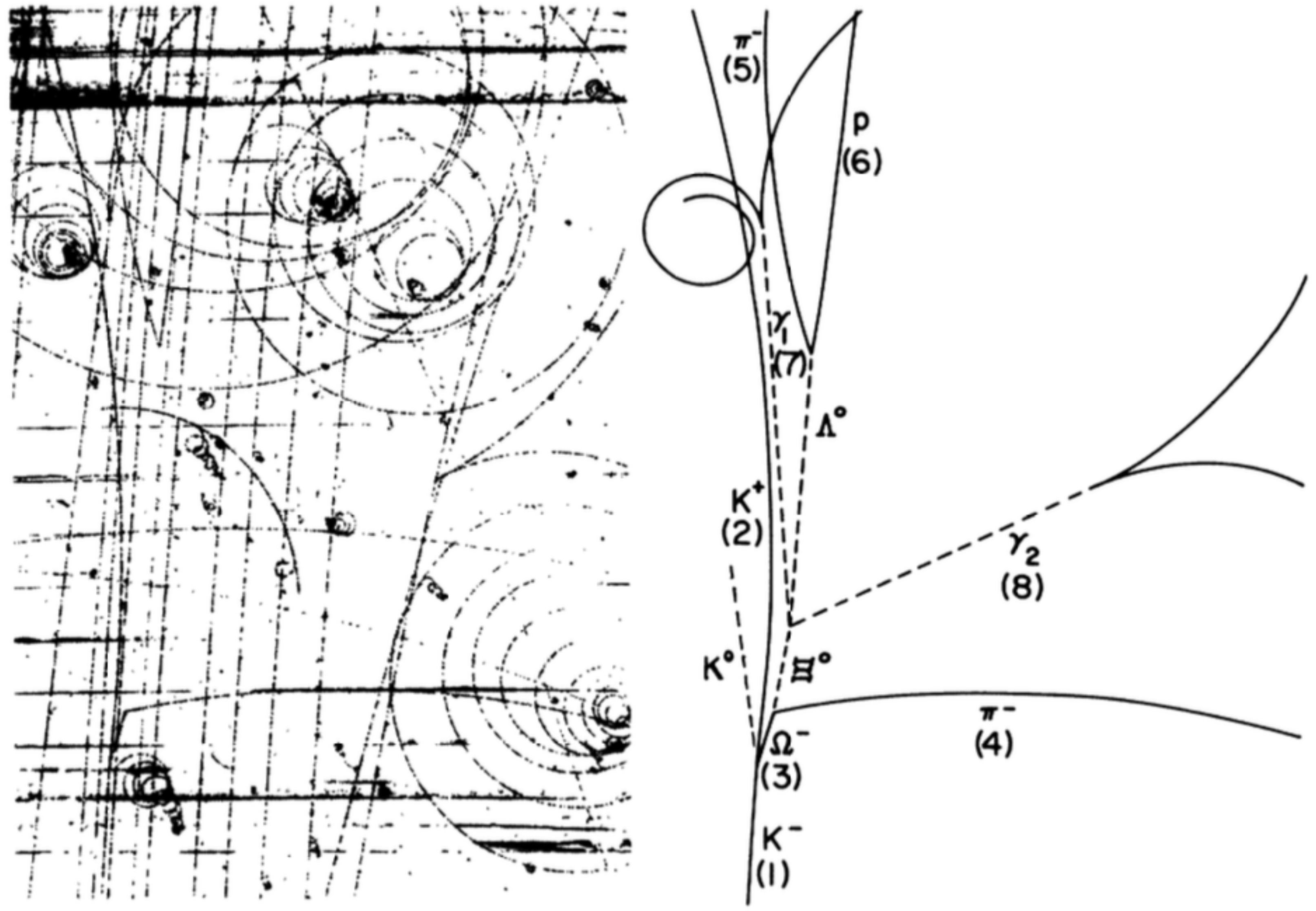
things are not so simple

1932-1964



positron

kaon



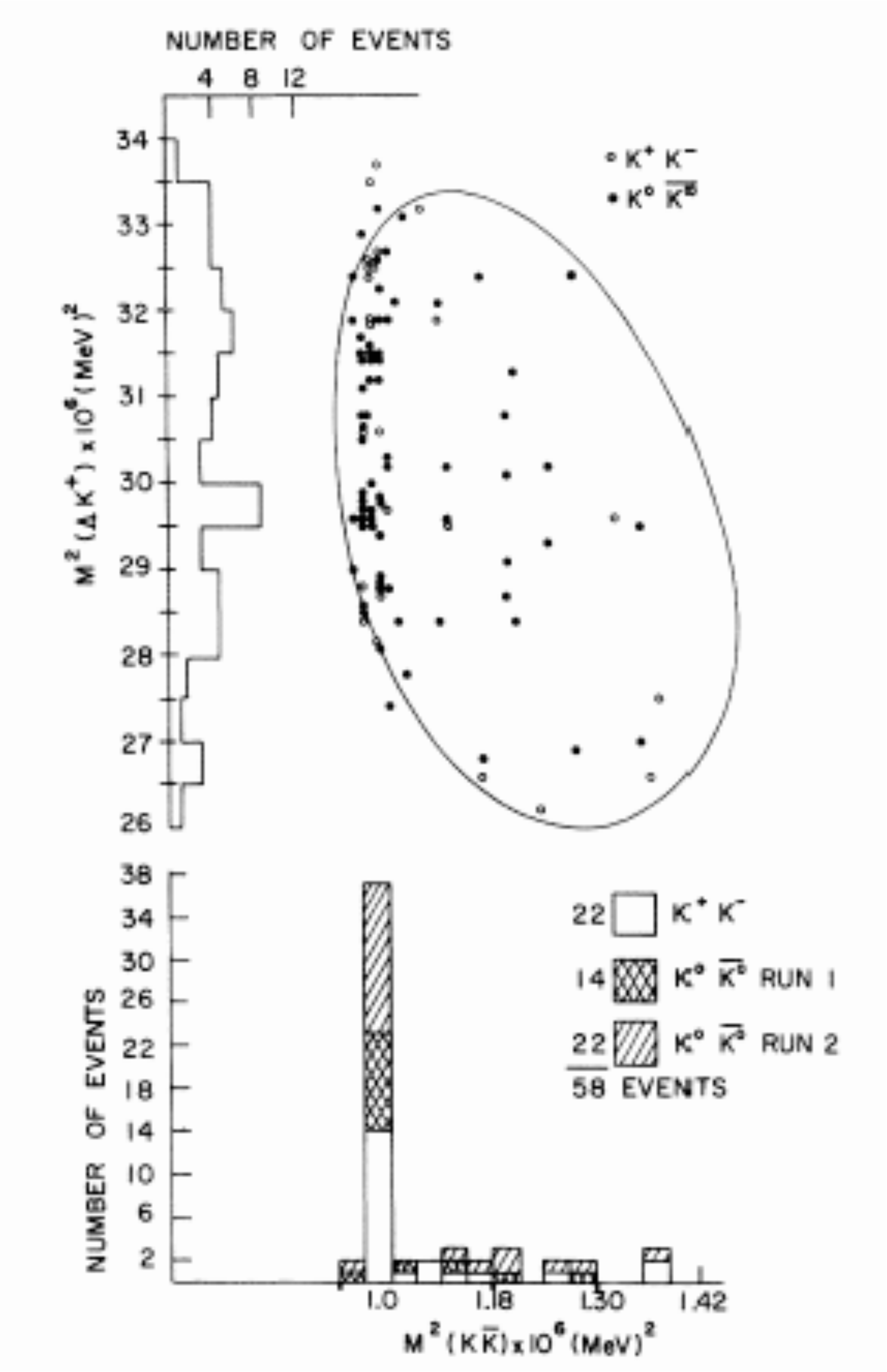
omega

Hideki (né Ogawa) Yukawa

(1907 – 1981)

quarks and the strong interaction

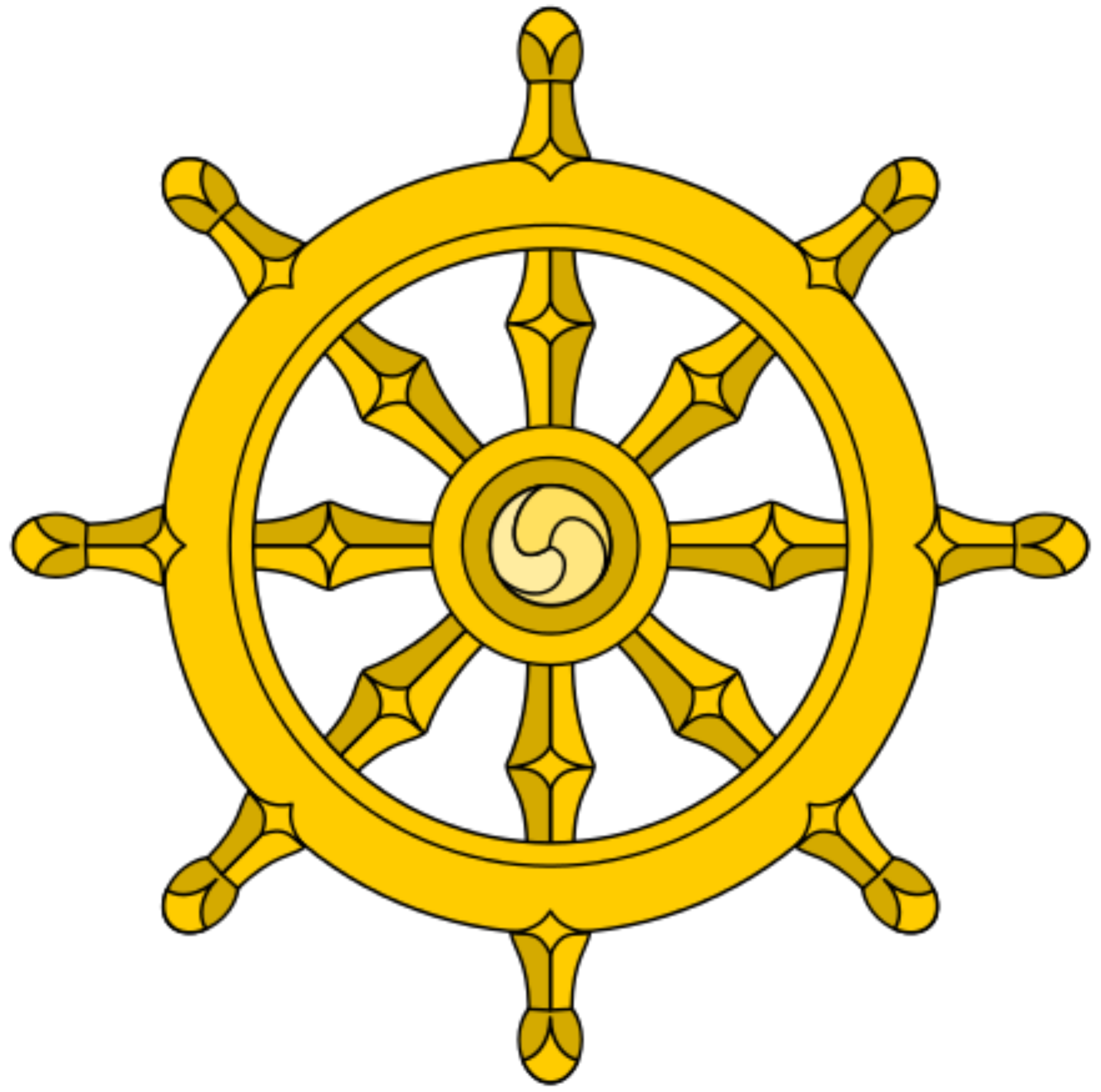
$$Kp \to \Lambda K \bar{K}$$



I remember being very surprised by Figure 1 …There was an enormous peak … right at the edge of phase space. The fact that the φ decayed predominantly into KK and not πρ was totally unintelligible. … Only conservation laws suppress reactions. Here was a reaction that was allowed but did not proceed! I had thought that hadrons probably have constituents and this experiment convinced me that they do, and that they are real. … This was a statement about dynamics which indicated that the constituents were not hypothetical objects carrying the symmetries of the theory, but real objects that moved in space-time from hadron to hadron."

George Zweig
(1937-)

the eightfold way



Eightfold Way

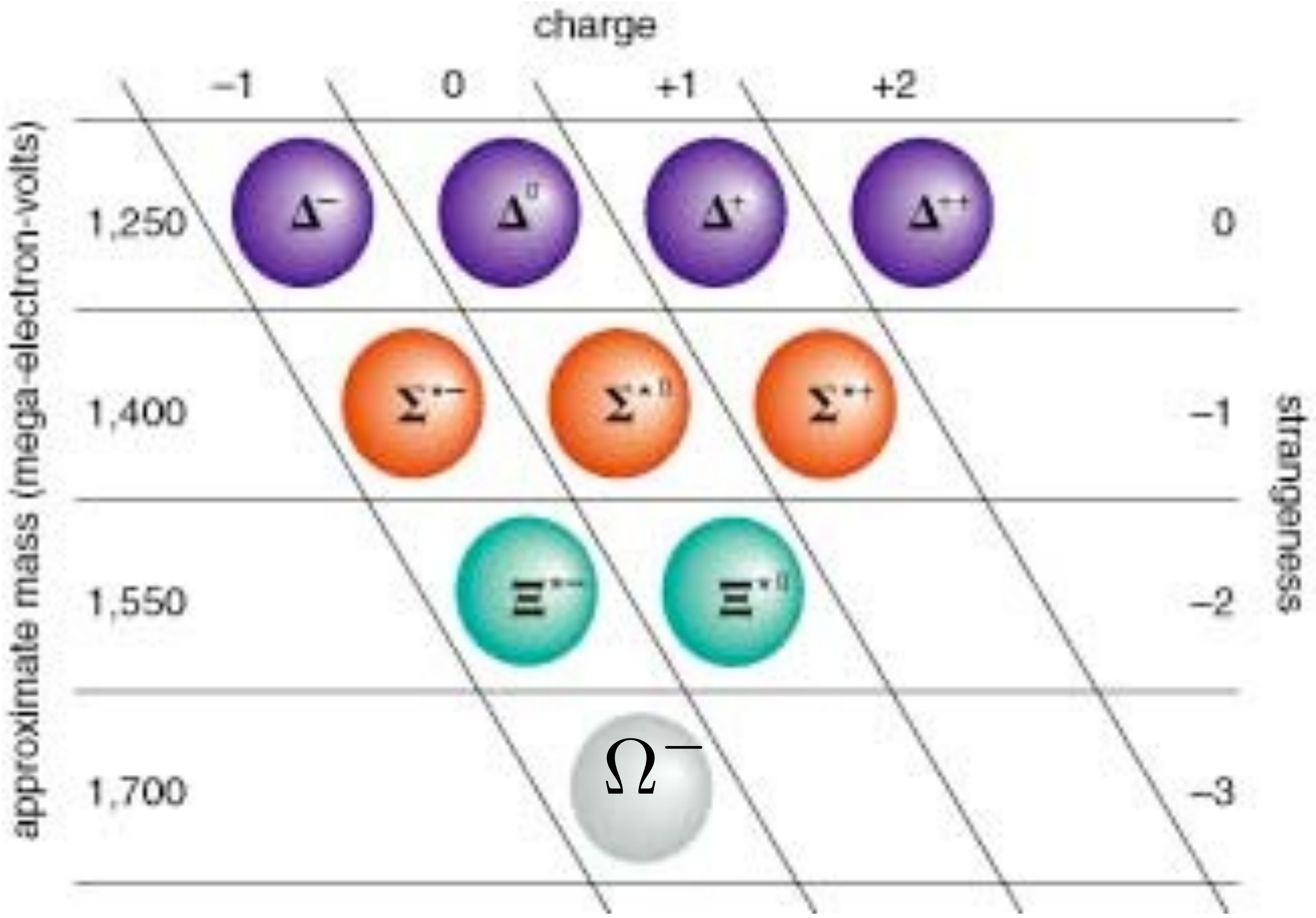
recall that a symmetry implies a degeneracy in the spectrum of a Hamiltonian:

$$[\vec{J}, H] = 0 \implies [J^+, H] = 0$$

$$H|jm\rangle = E_j|jm\rangle \qquad J^+H|jm\rangle = E_j J^+|jm\rangle \qquad H|jm+1\rangle = E_j|jm+1\rangle$$

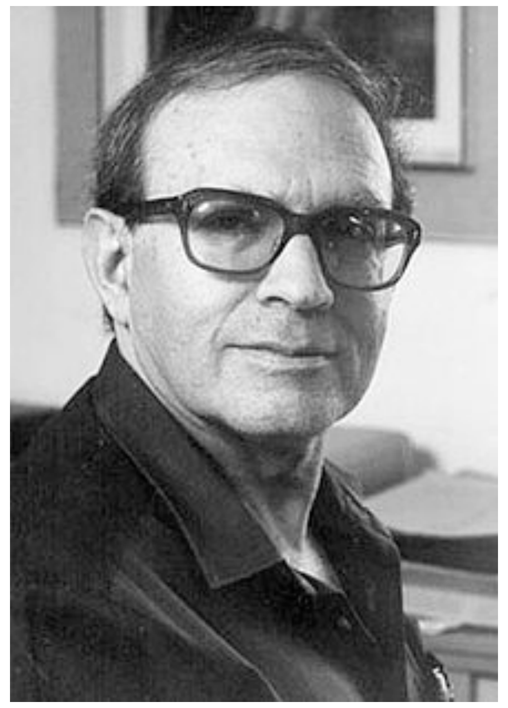
create a periodic table of hadrons:
organize according to the `Eightfold Way'

Eightfold Way



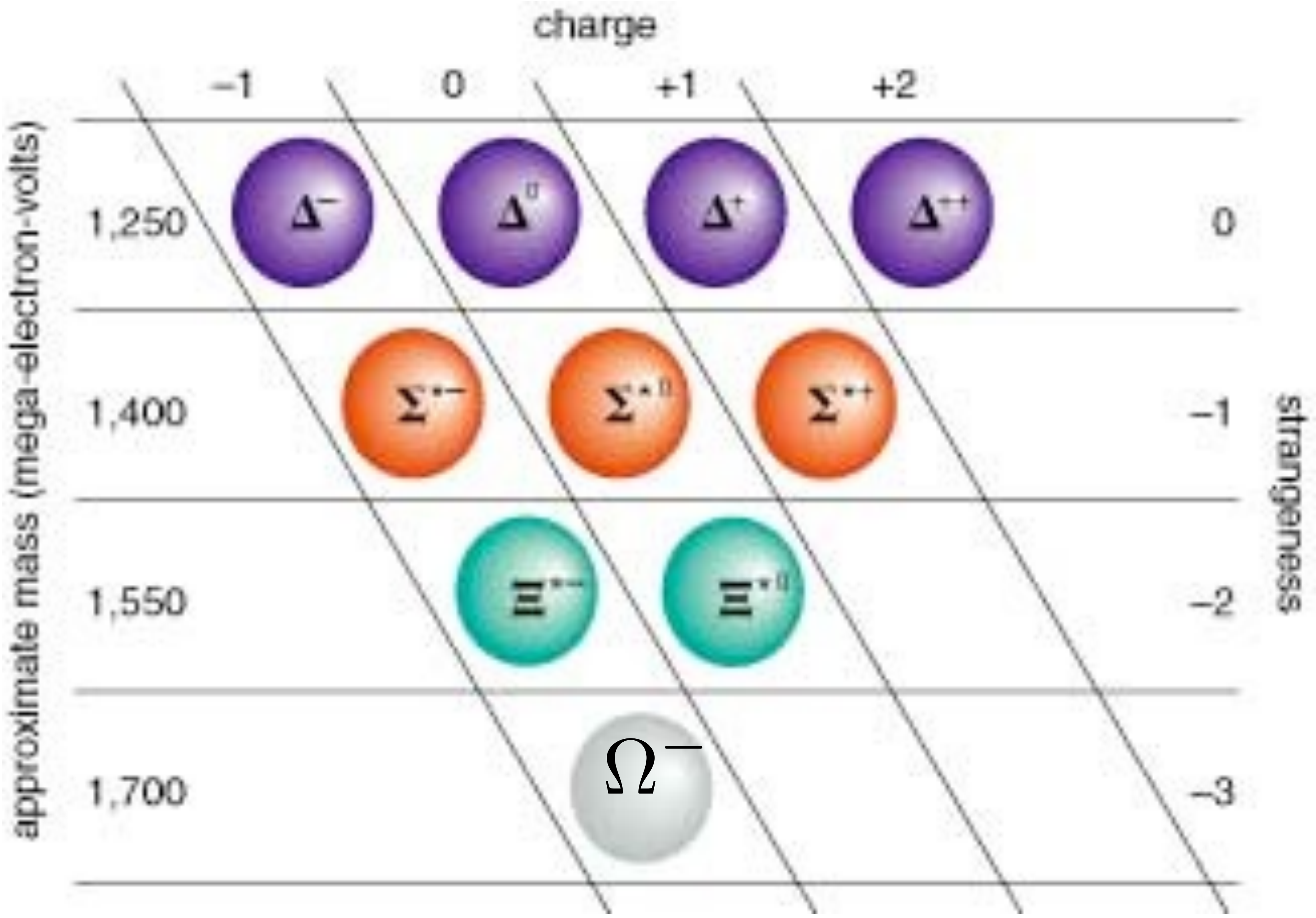
the quark model

Murray Gell-Mann
(1929-2019)



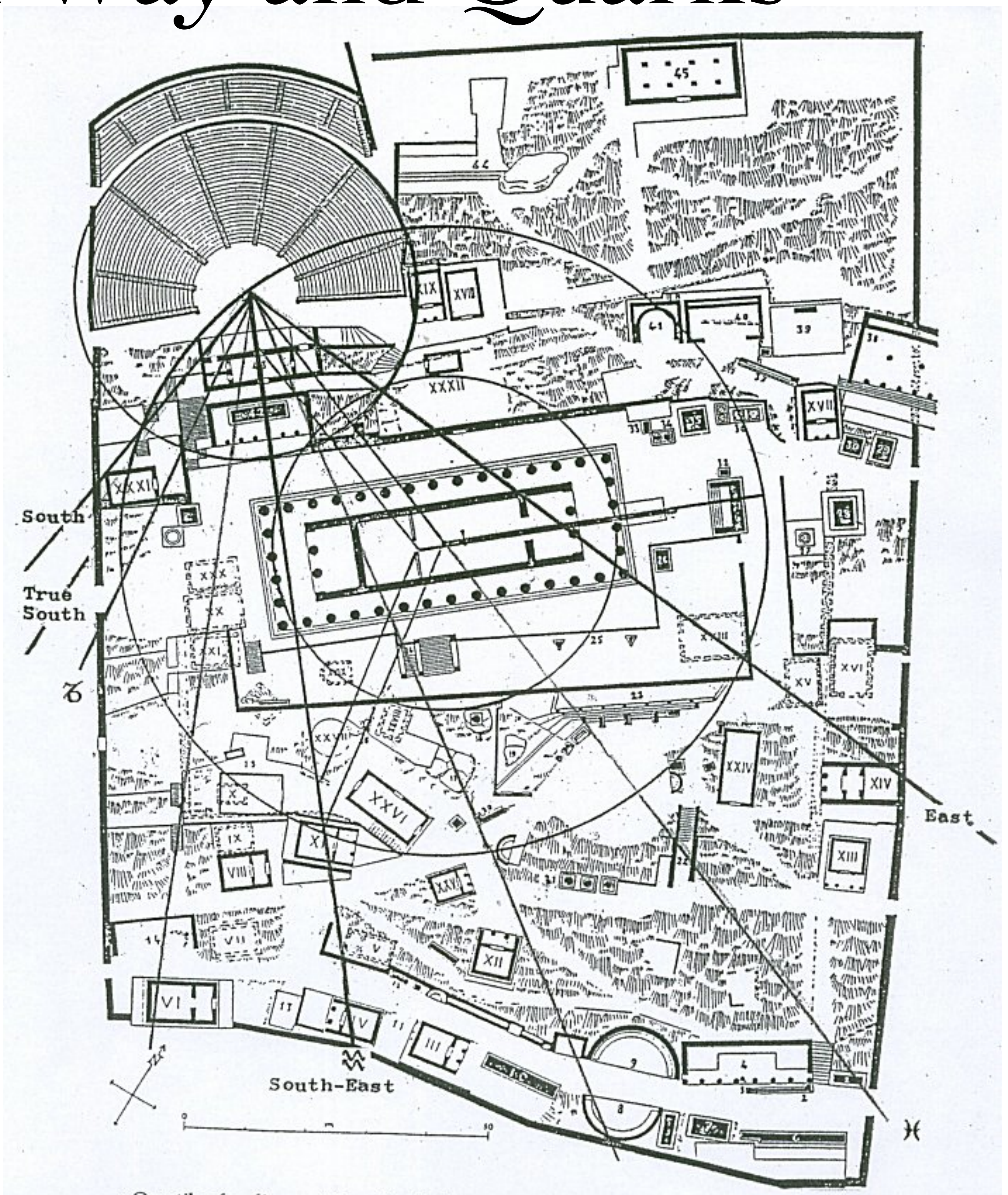
Yuval Ne'eman
(1925-2006)

Eightfold Way and Quarks

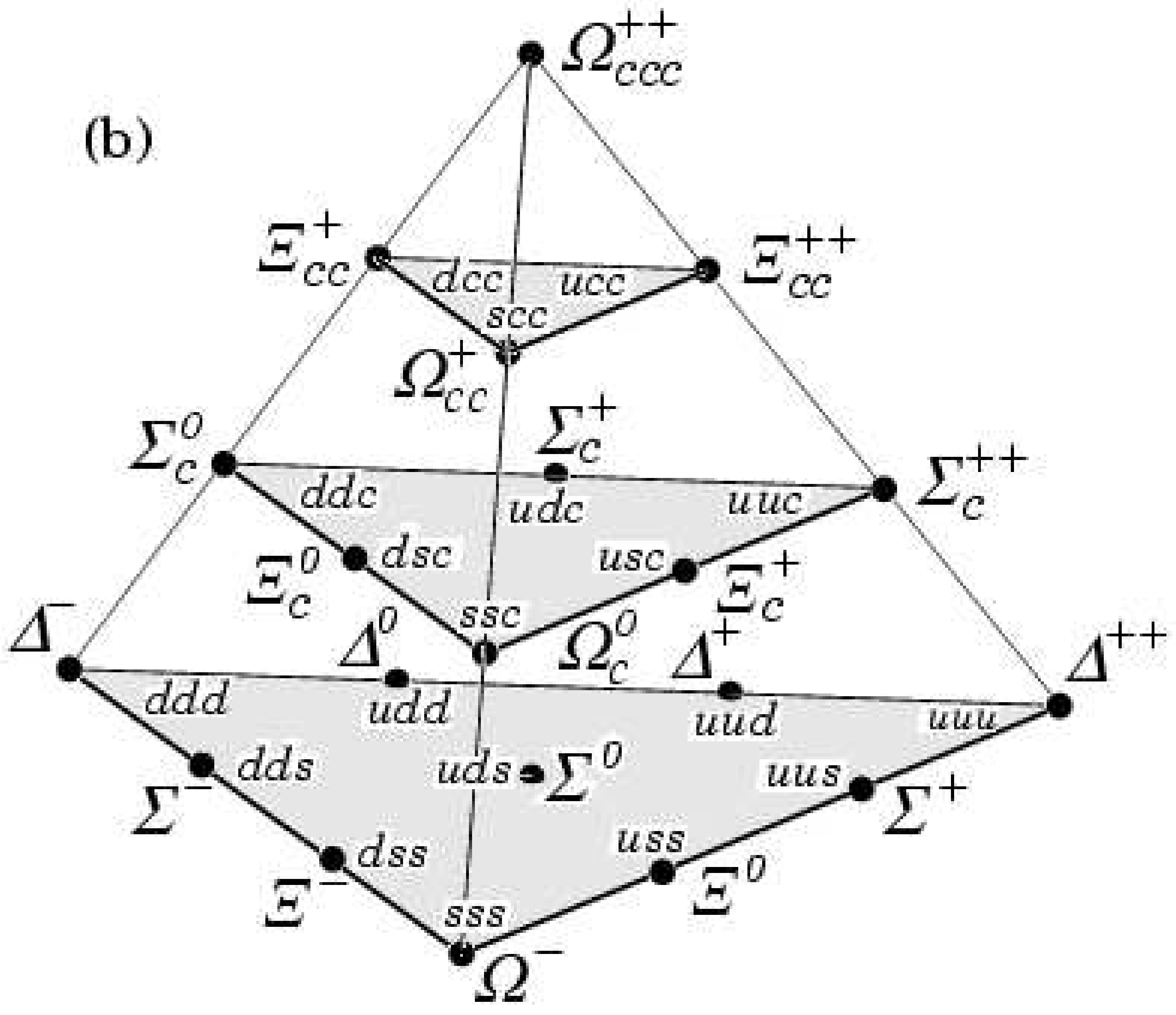


$$ddd \quad ddu \quad duu \quad uuu$$
$$sdd \quad sdu \quad suu$$
$$ssd \quad ssu$$
$$sss$$

Eightfold Way and Quarks



Quartile of zodiac, w. SE median linking Hestia, Gaia, and Athena. *Temenos* has deflection from true N of 7.5 degrees, 1/12th a 90 degree arc.



Late in researches into the mystery of spiritual renewal in Nature, while surfing the internet in such a quest, imagine my utter amazement when--in searching the site of the University of Pittsburgh's Physics Department, my article, "The Symmetry of Delphi" having appeared back in 1971 in the journal (Theatre Survey) then having its editorial office there--lo and behold!--there, on the small screen at fafnir,pitt,edu/particles, loomed an image of--the Tetractys! And, more remarkable still, it was not the planar 2-D Tetractys with which we're familiar. It was in 3-D! It was not the earth-bound planar diagram that conventionally plots the ten Pythagorean tetractyl points. It was a tetrahedron, the first 'solid' form, with Pythagoras' ten spheres/planets as its base, and with its apex an Apollonian arrow thrusting 'into' (and beyond!) space.

Eightfold Way and Quarks

(i) so where are they?

assumed very massive / superstrong forces

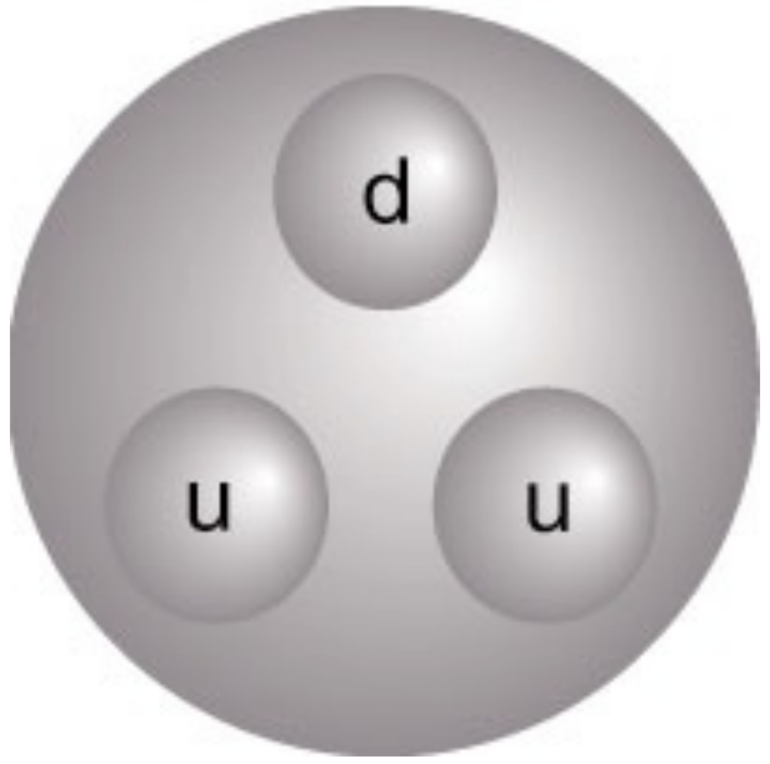
(ii) ”statistics problem”

$$\Delta^{++} = uuu\,(\uparrow\uparrow\uparrow)\,\psi$$

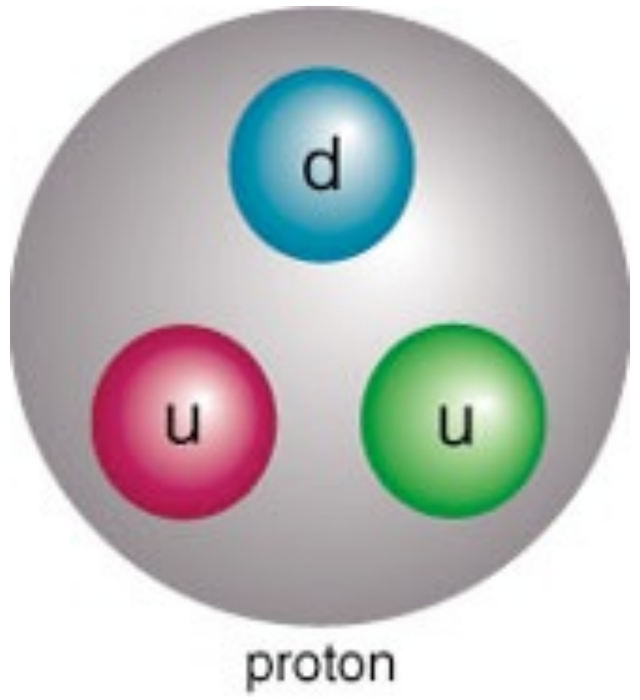
each is symmetric, yet the total wavefunction must be antisymmetric!

The Quark Model

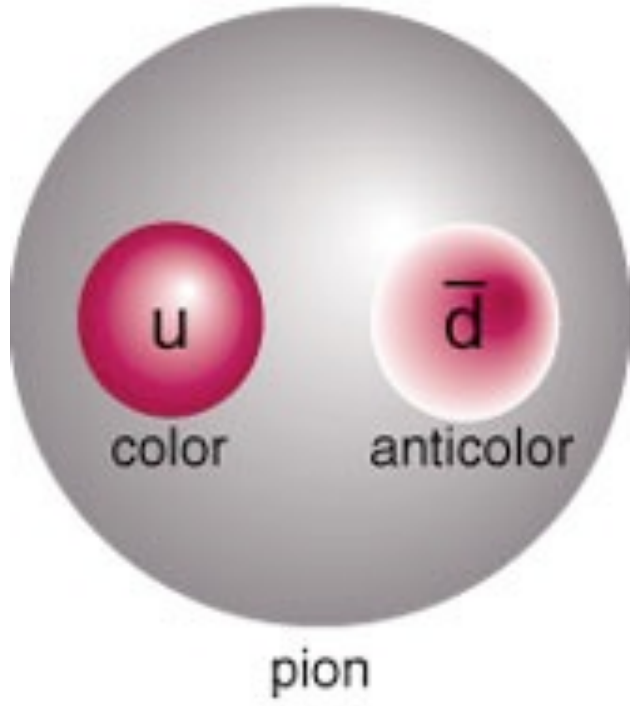
Assume that quarks have a new characteristic, or charge, called 'colour'.



proton (1964)

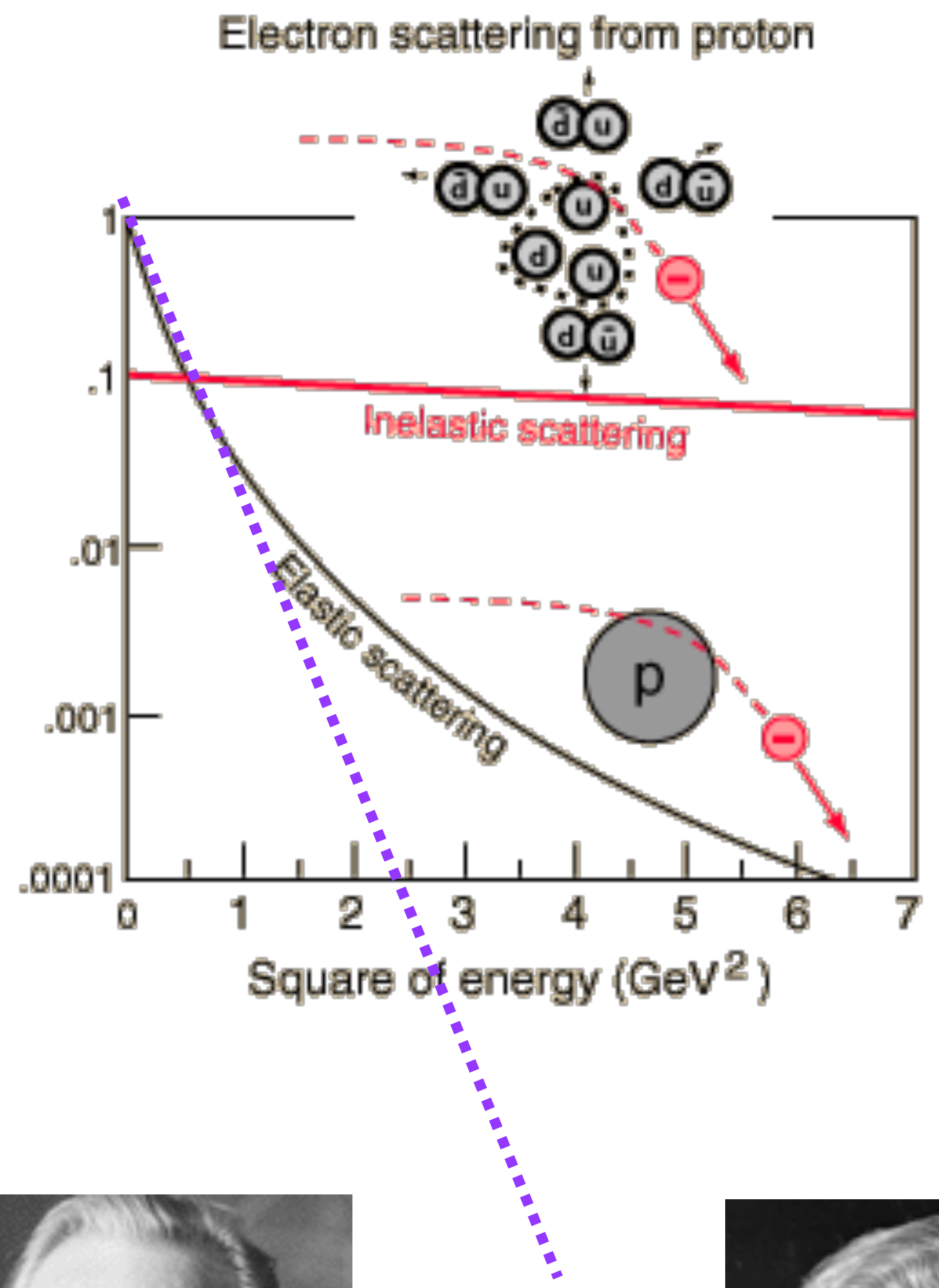


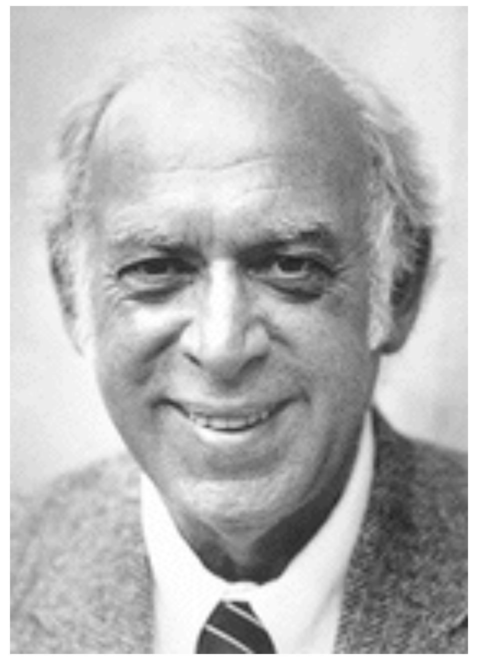
proton (1970)



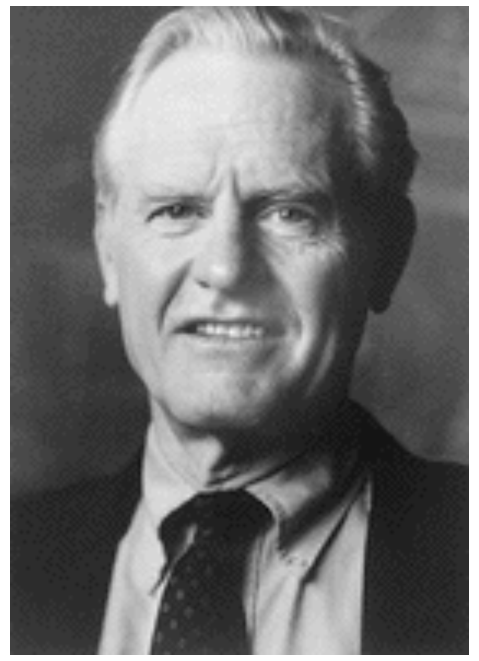
pion (1970)

Deep Inelastic Scattering -- Partons





Jerome Friedman
(1930-)



Henry Kendall
(1926-1999)

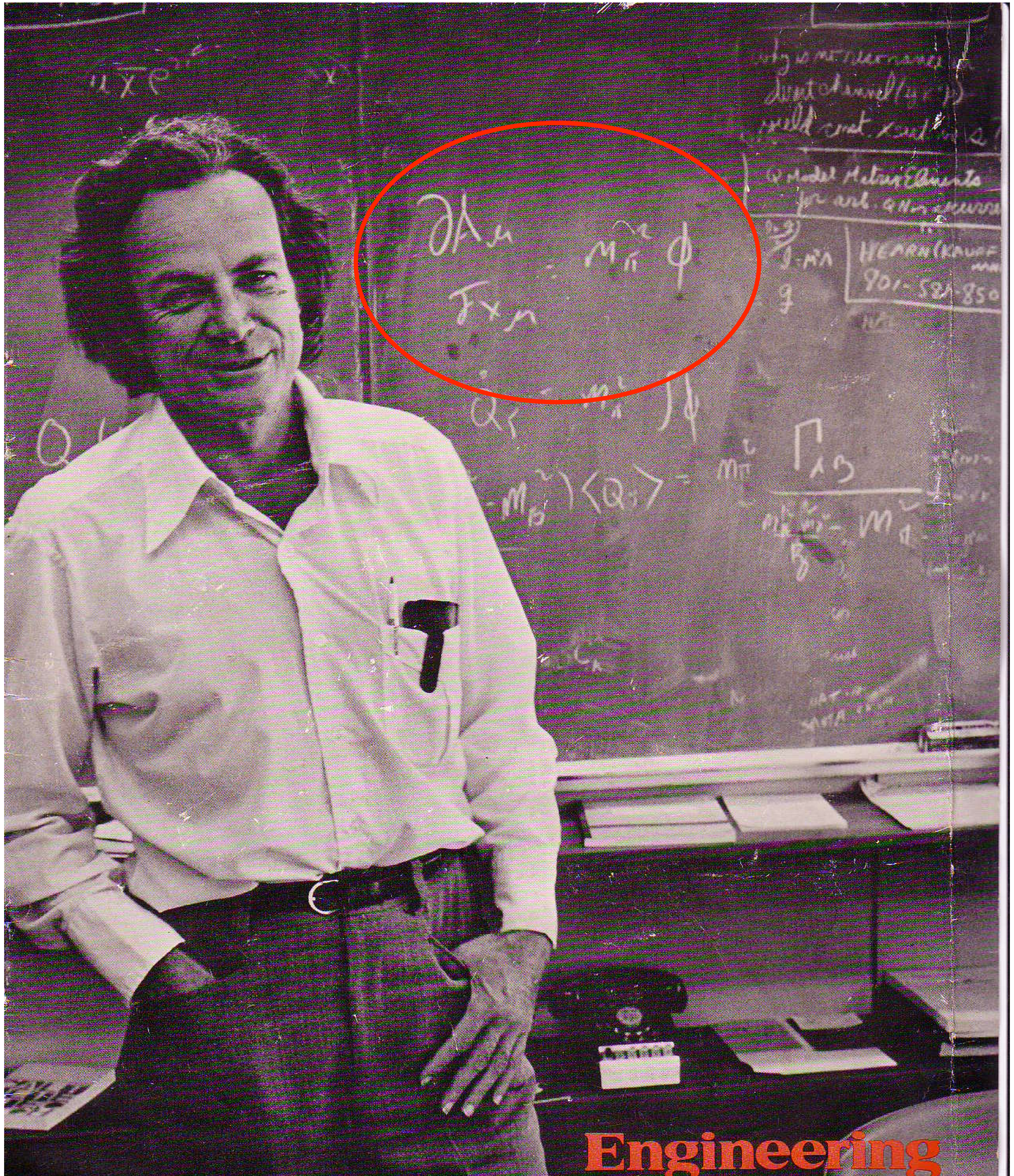
Richard Taylor
(1929-2018)

James Bjorken
(1934-)

Current Algebra -- PCAC

$$\langle 0|A^a_\mu(0)|\pi^b(p)\rangle = i\delta^{ab} f_\pi p_\mu$$

Jeffrey Goldstone
(1933-)



finally, a theory

Murray Gell-Mann
(1929-2019)

Heinrich Leutwyler
(1938-)

Harald Fritsch
(1943-2022)

Quantum Chromodynamics

we require a theory that

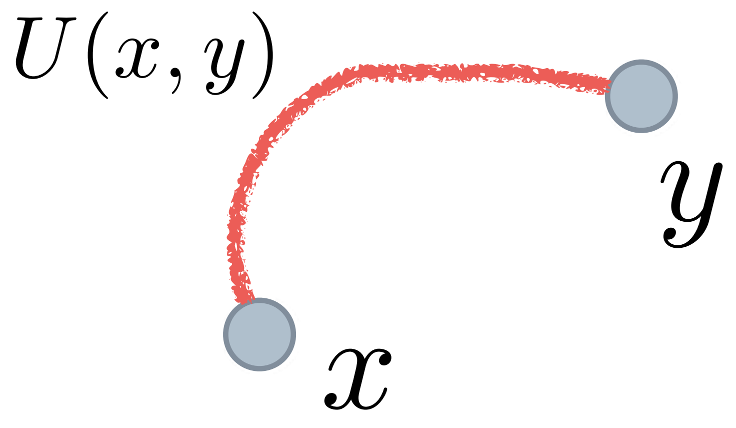
- has approximate chiral symmetry
- has approximate SU(3) flavour symmetry
- accounts for the parton model
- has colour
- and colour confinement
- is renormalizable

Quantum Chromodynamics

Quantum Electrodynamics is a "gauge theory"

local gauge transformation

$$\psi(x) \to \mathrm{e}^{i\alpha(x)}\psi(x)$$



$$\psi^\dagger(x)U(x,y)\psi(y) \to \psi^\dagger(x)\,\mathrm{e}^{-i\alpha(x)}U(x,y)\,\mathrm{e}^{i\alpha(y)}\psi(y)$$

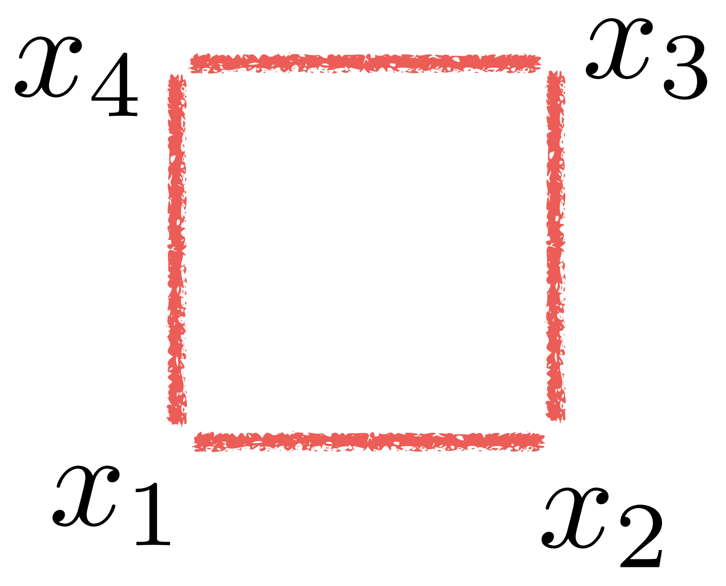
$$U(x,y) \to \mathrm{e}^{i\alpha(x)}U(x,y)\mathrm{e}^{-i\alpha(y)}$$

Quantum Chromodynamics

Quantum Electrodynamics is a “gauge theory”

a locally gauge invariant theory without electrons:

$$U(x_1, x_2)U(x_2, x_3)U(x_3, x_4)U(x_4, x_1) \to \text{ditto}$$



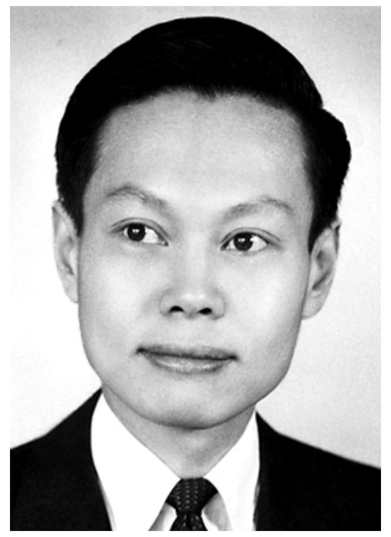
Quantum Chromodynamics

generalise to "non-Abelian" dynamics (Yang and Mills, 1954)

$$\psi(x) \to \psi_a(x)$$

$$U(x,y) \to U_{ab}(x,y)$$

there are $3 \cdot 3 - 1$ gluons



C.N. Yang
(1922 -)

Robert Mills
(1927 - 1999)

$$[\mathbf{U}(x), \mathbf{U}(y)] \neq 0$$

the gluons carry colour and are self-interacting

Quantum Chromodynamics

$$\mathcal{L}_{QCD} = \sum_f^{n_f} \bar{q}_f[i\gamma_\mu(\partial^\mu + igA^\mu) - m_f]q_f - \frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

$$A_\mu = A_\mu^a \frac{\lambda^a}{2}$$

$$[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}] = if^{abc}\frac{\lambda^c}{2}$$

$$\mathrm{Tr}(\lambda^a\lambda^b) = 2\delta^{ab}$$

flavour, colour, Dirac indices

$$\mathcal{L}_\theta = \theta \frac{g^2}{64\pi^2} F^{\mu\nu}\tilde{F}_{\mu\nu}$$

(

symmetries

symmetries in (classical) field theory

quark field

$$q \to f(q) \longrightarrow j_\mu \longrightarrow \partial_\mu j^\mu = 0$$

$$\frac{d}{dt}\int d^3x\, j_0 \equiv \frac{d}{dt}Q = \int d^3x\, \nabla \cdot \vec{j} = 0$$

symmetries in QCD

$U(1)_V$

$q \to \mathrm{e}^{-i\theta} q$	$j_\mu = \bar{q}\gamma_\mu q = \bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d$	$Q = \int d^3x \, (u^\dagger u + d^\dagger d)$
symmetry	current	charge

'baryon number conservation' $p \not\to e^+ \nu$

[violated by EW anomaly]

symmetries in QCD

$U(1)_A$ $\qquad m_u = m_d = 0$

$q \to \mathrm{e}^{-i\gamma_5\theta} q$
symmetry

$j_{\mu 5} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d$
current

$Q_5 = \int d^3x\, (u^\dagger\gamma_5 u + d^\dagger\gamma_5 d)$
charge

this symmetry does not exist in the quantum theory

$$\partial^\mu j_{\mu 5} = \frac{3\alpha_s}{8\pi} F\tilde{F}$$

symmetries in QCD

scale invariance $m_u = m_d = 0$

$$x \to \lambda x$$
$$q \to \lambda^{3/2} q(\lambda x)$$
$$A \to \lambda A(\lambda x)$$

$$j_\mu = x_\nu \Theta^{\mu\nu}$$

symmetry

current

$$\partial^\mu j_\mu = \Theta^\mu_\mu = 0$$

this symmetry does not exist in the quantum theory

$$\Theta^\mu_\mu = m\bar{q}q + \frac{\alpha_s}{12\pi} F^2$$

symmetries in QCD

SU(3)$_V$ $m_u = m_d$ isospin

$q \to e^{i\theta T_F^a} q$ $j_\mu^a = \bar{\psi}\gamma_\mu T_F^a \psi$ $Q^a = \int d^3x \psi^\dagger T_F^a \psi$

symmetry current charge

$$Q^+|\pi^-\rangle = \frac{1}{\sqrt{2}} \int d^3x \, \left(b^\dagger(\mathbf{x})\tau^+ b(\mathbf{x}) - d^\dagger(\mathbf{x})\tau^- d(\mathbf{x})\right) |\pi^-\rangle = |\pi^0\rangle$$

$$H|\pi^-\rangle = E_{\pi^-}|\pi^-\rangle; \quad Q^+ H|\pi^-\rangle = E_{\pi^-}|\pi^0\rangle; \quad H|\pi^0\rangle = E_{\pi^-}|\pi^0\rangle$$

this symmetry is explicitly broken by
quark mass and EW effects

symmetries in QCD

SU(3)$_A$

$m_u = m_d = 0$

$q \to \mathrm{e}^{-i\theta T^a \gamma_5} q$	$j_\mu^a = \bar{\psi}\gamma_\mu\gamma_5 T^a \psi$	$Q_5^a = \int d^3x \psi^\dagger \gamma_5 T^a \psi$
symmetry	current	charge

This symmetry is realised in the Goldstone mode.

transform the vacuum:

$$\mathrm{e}^{i\theta^a Q_5^a}|0\rangle = |0\rangle$$

$$Q_5^a|0\rangle = 0$$

} Wigner mode

symmetries in QCD

$SU(3)_A$

$$e^{i\theta^a Q_5^a}|0\rangle = |\theta\rangle \neq |0\rangle$$

} Goldstone mode

$$H|\theta\rangle = He^{i\theta^a Q_5^a}|0\rangle = e^{i\theta^a Q_5^a}H|0\rangle = E_0|\theta\rangle$$

so there is a continuum of states degenerate with the vacuum

Excitations of the vacuum may be interpreted as a particle. In this case fluctuations in θ are massless particles called Goldstone bosons.

symmetries in QCD

$\mathrm{SU(3)_A}$

Goldstone boson quantum numbers:

$$|\delta\theta\rangle = \theta^a Q_5^a |0\rangle$$

$$\sim \theta^a \int d^3x b^\dagger(\mathbf{x}) T_F^a d^\dagger(\mathbf{x})|0\rangle$$

spin singlet, spatial singlet, flavour octet $\Rightarrow$ the pion octet

gluons

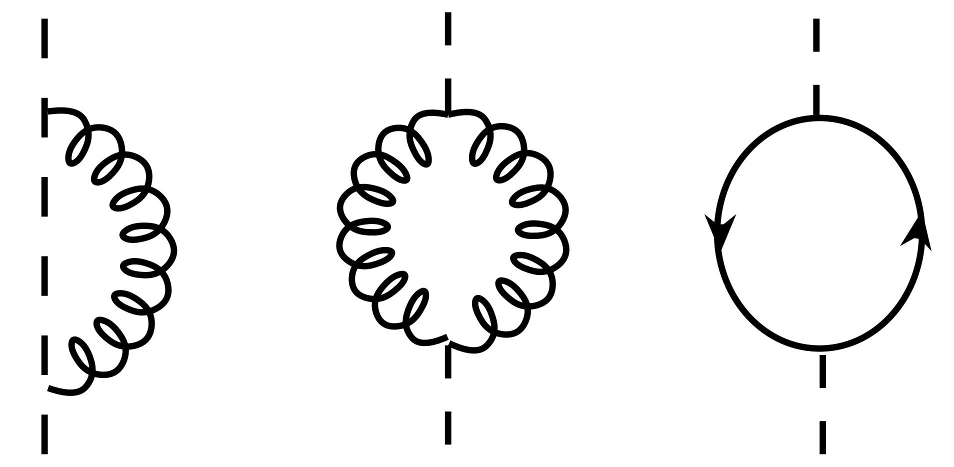
A three jet event at DESY. August, 1979

John Ellis, Mary Gaillard, Graham Ross

running coupling

running coupling

Khriplovich Yad. F. **10**, 409 (69)

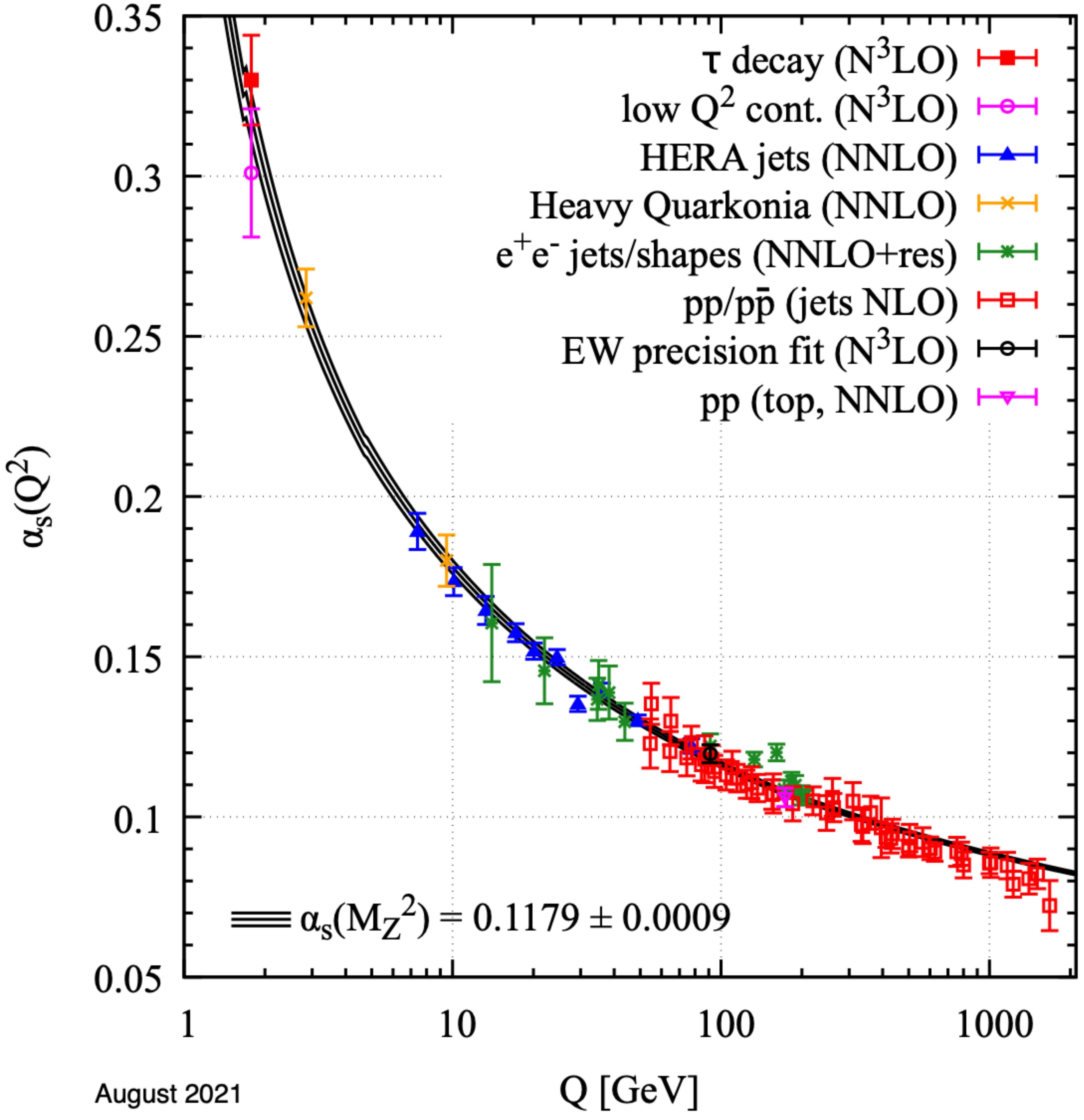


$\frac{12}{3}N_c$ $\quad -\frac{1}{3}N_c$ $\quad -\frac{2}{3}N_f$

$$\mu\frac{dg(\mu)}{d\mu} = -\frac{\beta_0}{(4\pi)^2}g^3(\mu)$$

$$\alpha_s(\mu^2) = \frac{4\pi}{(11-\frac{2}{3}n_f)\ln\mu^2/\Lambda^2_{QCD}}$$

running coupling



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