RUB

Evgeny Epelbaum, RUB

Modern Techniques in Hadron Spectroscopy
RUB, 15-27.07.2024

# Chiral Dynamics of QCD

## Concepts

Effective field theory, chiral perturbation theory, renormalization, predictive power, KSW vs Weinberg, power counting…

## Methods

Effective Lagrangian, heavy-baryon expansion, perturbative calculation of the amplitude, methods to derive nuclear forces (and currents), …

DFG Deutsche Forschungsgemeinschaft
Bundesministerium für Bildung und Forschung
erc European Research Council

# Syllabus

## Today

— brief introduction to EFT

EFT philosophy, renormalization, power counting, construction principles…

## Thursday

— chiral perturbation theory (ChPT)

Chiral symmetry, effective Lagrangian, chiral expansion, loops, inclusion of nucleons, …

# Part I: Brief Introduction to EFT

1. Main idea using a classical example
2. Basic QFT terminology
3. First example of an EFT

Some lecture notes (free access)

— Antonio Pich, Effective Field Theory, hep-ph/9806303

— Ira Rotstein, TASI lectures on effective field theories, hep-ph/0308266

— David Kaplan, Five lectures on effective field theory, nucl-th/0510023

— Aneesh Manohar, Introduction to Effective Field Theories, arXiv:1804.05863 [hep-ph]

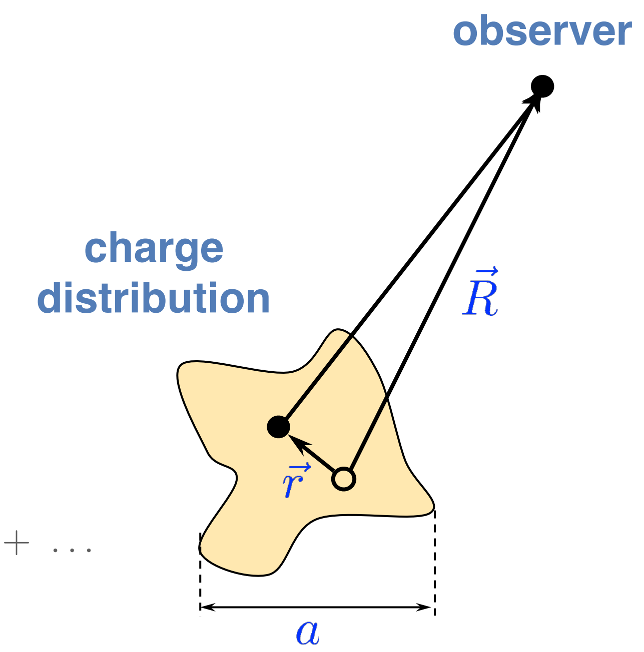
— Matthias Neubert, Renormalization Theory and EFTs, arXiv:1901.06573 [hep-ph]

# What is an effective theory?

## 1. Main idea using a classical example

The goal: compute electric potential generated by a localized charge distribution $\rho(\vec{r})$

The answer is $V(\vec{R}) \propto \int d^3r \, \frac{\rho(\vec{r})}{|\vec{R}-\vec{r}|}$




An effective theory for $R \gg a$: The Top-Down approach

$$\frac{1}{|\vec{R}-\vec{r}|} = \frac{1}{R} + r_i \left[\frac{\partial}{\partial r_i}\frac{1}{|\vec{R}-\vec{r}|}\right]_{\vec{r}=0} + \frac{1}{2!} r_i r_j \left[\frac{\partial^2}{\partial r_i \partial r_j}\frac{1}{|\vec{R}-\vec{r}|}\right]_{\vec{r}=0} + \ldots$$

$$= \frac{1}{R} + \frac{R_i}{R^3} r_i + \frac{1}{2!}\frac{R_i R_j}{R^5}(3 r_i r_j - r^2 \delta_{ij}) + \ldots$$

$$\Rightarrow \boxed{V(\vec{R}) = \frac{q}{R} + \frac{R_i}{R^3} P_i + \frac{1}{2}\frac{R_i R_j}{R^5} Q_{ij} + \ldots}$$

with $q = \int d^3r \, \rho(\vec{r}), \qquad P_i = \int d^3r \, \rho(\vec{r})\, r_i, \qquad Q_{ij} = \int d^3r \, \rho(\vec{r})\,(3 r_i r_j - r^2 \delta_{ij})$

We have just „integrated out“ short-distance physics. For $R \gg a$, the only information needed about $\rho(\vec{r}\,)$ is hidden in the moments $q$, $P_i$, $Q_{ij}$, …

# What is an effective theory?

## An effective theory for $R \gg a$: The Bottom-Up approach

What if we cannot „integrate out" short-distance physics or don't even now $\rho(\vec{r})$, apart from the fact that it is localized in the volume $\sim a^3$?

**Solution**: Write down the **most general expression for $V$** using the **long-distance DoF** (i.e., $\vec{R}$) compatible with the **symmetry principles** (rotational invariance)

$$V(\vec{R}) = \sum \left[ \begin{array}{l} \textit{rotational tensors} \\ \textit{constructed from } \vec{R} \end{array} \right] \cdot \left[ \begin{array}{l} \textit{rotational tensors characterizing} \\ \textit{the system, independent of } \vec{R} \end{array} \right]$$

symmetric and traceless (otherwise redundant structures)

$$= \underbrace{\frac{1}{R}}_{[V] = \text{length}^{-1}} \text{const} + \frac{1}{R^3} R_i \underbrace{X_i}_{\sim a \text{ (NDA)}} + \frac{1}{R^5} R_i R_j \underbrace{X_{ij}}_{\sim a^2 \text{ (NDA)}} + \ldots$$

The $(2n+1)$ components of $X_{i_1 \ldots i_n}$ are called in the EFT language LECs and can be determined from experimental data.

$\Rightarrow$ systematically improvable approximation for $V(\vec{R})$ at $R \gg a$ without knowing $\rho(\vec{r})$!

# Quantum Field Theory

## 2. Basic QFT terminology

— canonical (or path integral) quantization of classical field theories

— main objects to calculate are Green's functions:

$$\underbrace{G_l(x_1,\ldots,x_l)}_{\substack{\Rightarrow \text{ S-matrix (LSZ)} \\ {}_{\text{out}}\langle \vec{k}_1 \ldots \vec{k}_n | \vec{p}_1 \ldots \vec{p}_m \rangle_{\text{in}}}} \equiv \underbrace{\langle \Omega | T\{\hat{\phi}(x_1)\ldots\hat{\phi}(x_l)\} | \Omega \rangle}_{\text{Heisenberg-picture operators}} = \underbrace{\frac{\langle 0 | T\left\{\hat{\phi}_I(x_1)\ldots\hat{\phi}_I(x_l) e^{i\int d^4x \hat{\mathcal{H}}^I_{\text{int}}}\right\} | 0 \rangle}{\langle 0 | T e^{i\int d^4x \hat{\mathcal{H}}^I_{\text{int}}} | 0 \rangle}}_{\substack{\text{basis for perturbation theory, can be cast into a set} \\ \text{of rules (Feynman diagrams)}}}$$

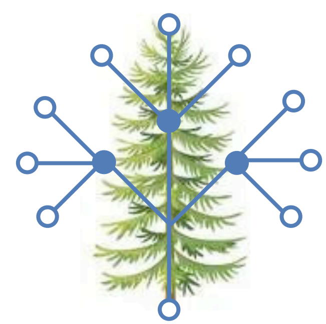
Those not familiar see: K. Kumeric, *Feynman Diagrams for Beginners*, arXiv:1602.04182 [physics.ed-ph]

— two types of diagrams: Trees and loops

Tree-level diagrams emerge when (pertirbatively) solving the EOM in classical field theory

Loop diagrams represent quantum corrections:

loop expansion = expansion in $\hbar$

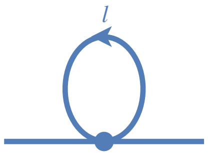


tree-level diagram

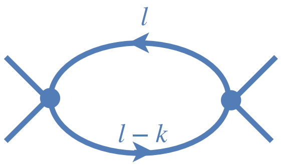
loop diagram

# UV divergences

Loop diagrams are typically UV divergent. E.g., for $\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$



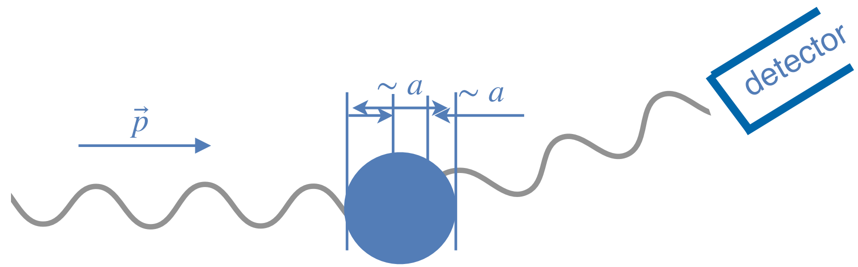

$$\frac{1}{2}(-i\lambda)\int\frac{d^4l}{(2\pi)^4}\,\frac{i}{l^2-m^2+i\epsilon}$$ ⟵ quadratically divergent




$$\frac{1}{2}(-i\lambda)^2\int\frac{d^4l}{(2\pi)^4}\,\frac{i}{l^2-m^2+i\epsilon}\,\frac{i}{(l-k)^2-m^2+i\epsilon}$$ ⟵ logarithmically divergent

### What is the origin of UV divergences?

Consider quantum mechanical scattering off some potential $V(r)$, e.g. $V(r) = e^{-r^2/(2a^2)}$




potential $V(r)$

At $p \ll 1/a$ can approximate: $V(r) \propto \delta^3(r)$

$\Rightarrow$ $V(q) = \text{const} \equiv C$

$\Rightarrow$ the Lippmann-Schwinger eq. becomes divergent: $T = V + \underbrace{VG_0V}_{\int\frac{d^3l}{(2\pi)^3}C\,\frac{m}{\vec{p}^2-\vec{l}^2+i\epsilon}\,C} + VG_0VG_0V + \ldots$

The basic principles of a QFT (causality, unitarity, relativity & cluster separability) require local Lagrangian densities…

# Regularization, renormalization and all that…

### How to deal with UV divergences in QFT?

1. Regularize (DimReg, Pauli-Villars, cutoff, lattice, …)
2. Renormalize: express the (generally infinite) bare parameters in $\mathscr{L}$ (masses, fields, coupling constants) in terms of finite, physical quantities. Notice: this is ambiguous
   $\Rightarrow$ dependence on renormalization conditions/subtraction scales.
   (an inappropriate choice may spoil convergence of the loop expansion…)
3. Remove the regulator to restore the original theory (optional for EFTs)

**Example: the $\phi^4$-theory** $\mathcal{L} = \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 - \frac{1}{2}m_0^2\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4$

— rewrite $\mathscr{L}$ using renormalized quantities $\phi_0 =: \sqrt{Z}\phi,\ Zm_0^2 =: Z_m m^2$ and $Z^2\lambda_0 =: Z_\lambda\lambda$:

$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4}_{\textit{renormalized Lagrangian}} + \underbrace{\frac{1}{2}\delta_Z\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\delta_m\phi^2 - \frac{\delta_\lambda}{4!}\phi^4}_{\textit{counter terms } (\Delta\mathscr{L})}$$

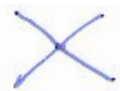
$Z-1$ $\quad m^2(Z_m-1)$ $\quad \lambda(Z_\lambda-1)$

Notice: counter terms are <u>not</u> free parameters (and not observable) and determined from the requirement to cancel the UV divergences: $\delta_i = \hbar\delta_i^{(1)} + \hbar^2\delta_i^{(2)} + \ldots$

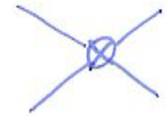
# Regularization, renormalization and all that…

Feynman rules:

 : $\frac{i}{p^2 - m^2}$

 : $-i\lambda$

 : $i\,(p^2 \delta_Z - \delta_m)$

 : $-i\,\delta_\lambda$

*renormalized Lagrangian* *counter terms ($\Delta\mathscr{L}$)*

2-point function to 1 loop: $-i\Sigma(p^2)\;=$ 

$=$  $+$ 

Using e.g. cutoff regularization one finds: $\Sigma_{\text{loop}}(p^2) \;=\; \alpha\Lambda^2 + \beta m^2 \ln\frac{\Lambda}{m} + \gamma m^2$

Dressed propagator: 


$$= \frac{i}{p^2 - m^2 - \Sigma(p^2)} \overset{!}{=} \frac{i}{p^2 - m^2} + \text{non pole terms}$$

$$\Rightarrow \quad \Sigma(p^2)\Big|_{p^2=m^2} = 0, \quad \frac{d}{dp^2}\Sigma(p^2)\Big|_{p^2=m^2} = 0$$

*on-shell renormalization conditions*
*(renorm. m = physical mass)*

$\Rightarrow$ read off: $\delta_Z^{(1)} = 0, \quad \delta_m^{(1)} = -(\alpha\Lambda^2 + \beta m^2 \ln\frac{\Lambda}{m} + \gamma m^2)$

$\delta_X^{(n)}$ depend on both the regulator and renorm. cond., while renormalized result is unambiguous…

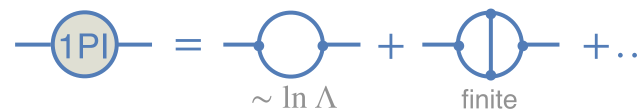
# Regularization, renormalization and all that...

For $\phi^4$-theory in 4 dimensions, $\forall$ divergences in n-point functions are cancelled by $\delta_Z$, $\delta_m$ and $\delta_\lambda$ at any loop order, so that the theory is renormalizable. (Perturbative) renormalizability is generally determined by the mass dimension $[\lambda]$, ($\lambda \sim \text{mass}^{[\lambda]}$) of the coupling.

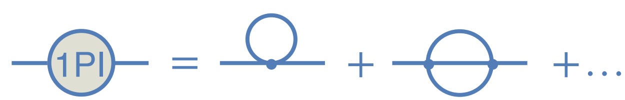
Consider e.g. $\mathscr{L}_{\text{int}} = \dfrac{\lambda}{3!}\phi^3$ in 4 dimensions: $[S] = 0 \;\Rightarrow\; [\mathscr{L}] = 4 \;\Rightarrow [\phi] = 1 \;\Rightarrow\; [\lambda] = 1$

Using NDA, one can show:

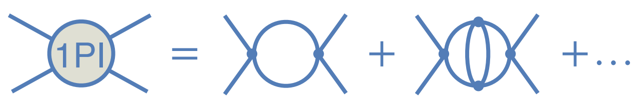
$[\lambda] > 0$: super-renormalizable (only few divergent diagrams)



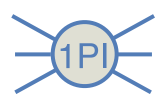

$[\lambda] = 0$: renormalizable (QED, QCD)




$\longleftarrow$ divergent ( $\sim \Lambda^2$)




$\longleftarrow$ divergent ( $\sim \ln \Lambda$)




$\longleftarrow$ convergent

$[\lambda] < 0$: non-renormalizable (starting from some loop order, $G_n$ become divergent for all $n$)

Notice: obviously, only a very limited number of possible interactions in 4 dimensions are renormalizable!

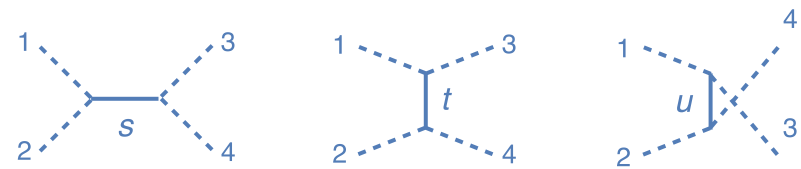
# Example of an EFT

## 3. First example of an EFT

Consider a QFT for two scalar fields ($M \gg m$) interacting with a Yukawa-like coupling:

$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2}_{\textit{light}} + \underbrace{\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M^2\Phi^2}_{\textit{heavy}} - \frac{\lambda}{2}\phi^2\Phi$$

$\phi\phi \to \phi\phi$ scattering at LO (i.e., $\mathcal{O}(\lambda^2)$):




$$i\mathcal{A} = -i\lambda^2\left(\frac{1}{s-M^2} + \frac{1}{t-M^2} + \frac{1}{u-M^2}\right)$$ where $s = (p_1+p_2)^2, \quad t = (p_1-p_3)^2, \quad u = (p_1-p_4)^2$

At low energies ($s \sim t \sim u \sim m^2 \sim E^2 \ll M^2$): $i\mathcal{A} \approx -i\lambda^2\left(-\frac{1}{M^2}\right)\left(3 + \frac{4m^2}{M^2} + \frac{s^2+t^2+u^2}{M^4} + \dots\right)$

But this looks like the tree-level amplitude, obtained from the effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{l_1}{4!}\phi^4 - \frac{l_2}{4}(\partial_\mu\phi)(\partial^\mu\phi)\phi^2 + \underbrace{\dots}$$

$l_1 = -\frac{3\lambda^2}{M^2}$ $\quad l_2 = \frac{\lambda^2}{2M^4}$

*an infinite tower of non-renormalizable interactions suppressed by powers of $M$*

# Example of an EFT

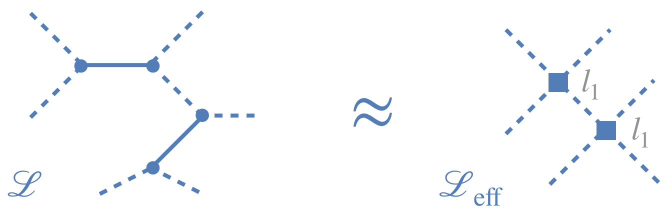
What if we were not able to determine $\mathscr{L}_{\text{eff}}$ by matching (e.g., the underlying theory not known or non-perturbative)?

$\Rightarrow$ write down all possible terms in $\mathscr{L}_{\text{eff}}(\phi)$ compatible with the symmetries (why not a $\phi^3$-interaction?) and fix LECs from experimental data

What about predictive power?

— at tree level, $\mathscr{A}(s,t)$ is determined by a single LEC from $l_1/(4!)\,\phi^4$ (up to corrections $\sim E^2/M^2$)

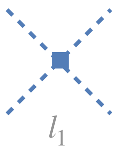
— this interaction also determines the LO contribution to processes with more $\phi$'s, e.g.:



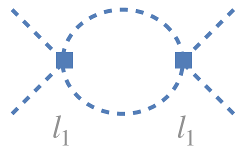

— obviously, contributions of terms with derivatives (e.g., $l_2\phi^2\,\Box\,\phi^2$) are suppressed, at tree level, by powers of $M$ („irrelevant" interactions). But inside loop diagrams, we integrate over arbitrarily high momenta! Can one expect irrelevant operators be suppressed beyond tree level?

# Example of an EFT

Let's do power counting (NDA):



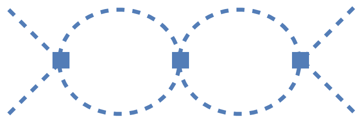

$\sim \text{const}$



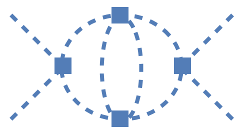

$$\sim \int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2 - m^2)((l-p)^2 - m^2)} \sim \mathcal{O}(1)$$

(we count powers of **soft scales** $Q$ like $p \sim m \sim \mu_i$)

Similarly:

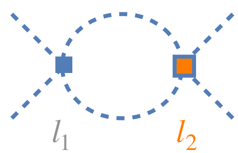


$\sim$



$\sim \mathcal{O}(1)$

On the other hand:




$$\sim \int \frac{d^4l}{(2\pi)^4} \frac{l^2}{(l^2 - m^2)((l-p)^2 - m^2)} \sim \mathcal{O}(Q^2)$$

(after renormalization!)

The suppression appears automatically using DR, but it also holds in general (e.g., using $\Lambda$) for proper renormalization conditions (all subtraction scales $\mu_i \sim Q$).

$\Rightarrow$ power counting:

LO ( $\sim Q^0$): $\forall$ diagrams made out of $l_1$-vertices

NLO ( $\sim Q^2$): $\forall$ diagrams made out of $l_1$-vertices and 1 insertion of dim-6 vertex ($l_2$)

…

The birth of ChPT (and an EFT in general): Steven Weinberg, Phenomenological Lagrangians, Physica A96 (79) 327 (about 4000 citations…)

# EFT vs Multipole Expansion

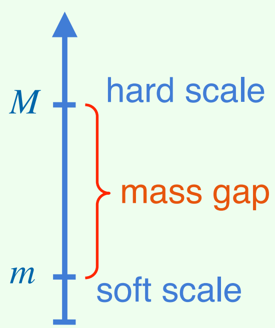
## Effective Field Theory

- Most general effective Lagrangian for light DoF compatible with the symmetries of the underlying theory

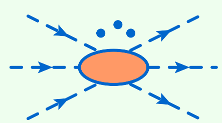
$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{l_1}{4!}\phi^4 - \frac{l_2}{4}(\partial_\mu\phi)(\partial^\mu\phi)\phi^2 + \ldots$$

- The size of (renormalized) LECs governed by the hard scale M. LECs carry information about short-range dynamics. They can be calculated from matching or determined from experiment

- Separation of scales: [soft] $Q \sim m \ll M$ [hard]




- Energy expansion of the amplitude (Feynman graphs, power counting, renormalization)



## Electric potential

Most general expression for the electric potential (rotational invariance)

LECs (multipoles) governed by the size $a$ of $\rho(\vec{r})$, they can be calculated or determined from exp.

[soft] $1/R \ll 1/a$ [hard]

Multipole expansion for $V(\vec{R})$ in powers of $a/R$

# The principles of an EFT

## Construction of QFTs (~1930 … 1980)

1) Construct the action respecting some symmetries. E.g., gauge invariance of QED:

$$\Psi \to \Psi' = e^{-i\alpha(x)}\Psi, \quad A_\mu \to A'_\mu = A_\mu - \partial_\mu \alpha(x)$$

~~2) Retain only renormalizable interactions ($D \leq 4$), e..g. in QED:~~

$$\underbrace{\bar{\Psi}\gamma_\mu(\partial^\mu + ieA^\mu)\Psi}_{D=4}, \quad \underbrace{\bar{\Psi}S_{\mu\nu}\Psi F^{\mu\nu}}_{D=5}, \quad \underbrace{(F_{\mu\nu}F^{\mu\nu})^2}_{D=8}, \quad \ldots$$

3) Quantize, compute the amplitude 

4) Fix parameters from data (in QED, only $e$ and fermion masses) and make predictions…

## Modern view is based on Weinberg's Theorem:

„if one writes down the most general possible Lagrangian, including all terms consistent with the assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition, and the assumed symmetry properties“

S. Weinberg, Physica 96A (1979) 327; see also H. Leutwyler, Annals Phys. (1994) 165