

Nonequilibrium dynamics and transport near the chiral phase transition of a quark-meson model

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in collaboration with C. Wesp, H. van Hees and C. Greiner

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Outline

Motivation:

phase diagram, critical phenomena

Model:

lagrangian

spontaneously / explicitly broken chiral symmetry

mean-field and Vlasov equation

Equilibrium results

sigma and sigma mass spectrum

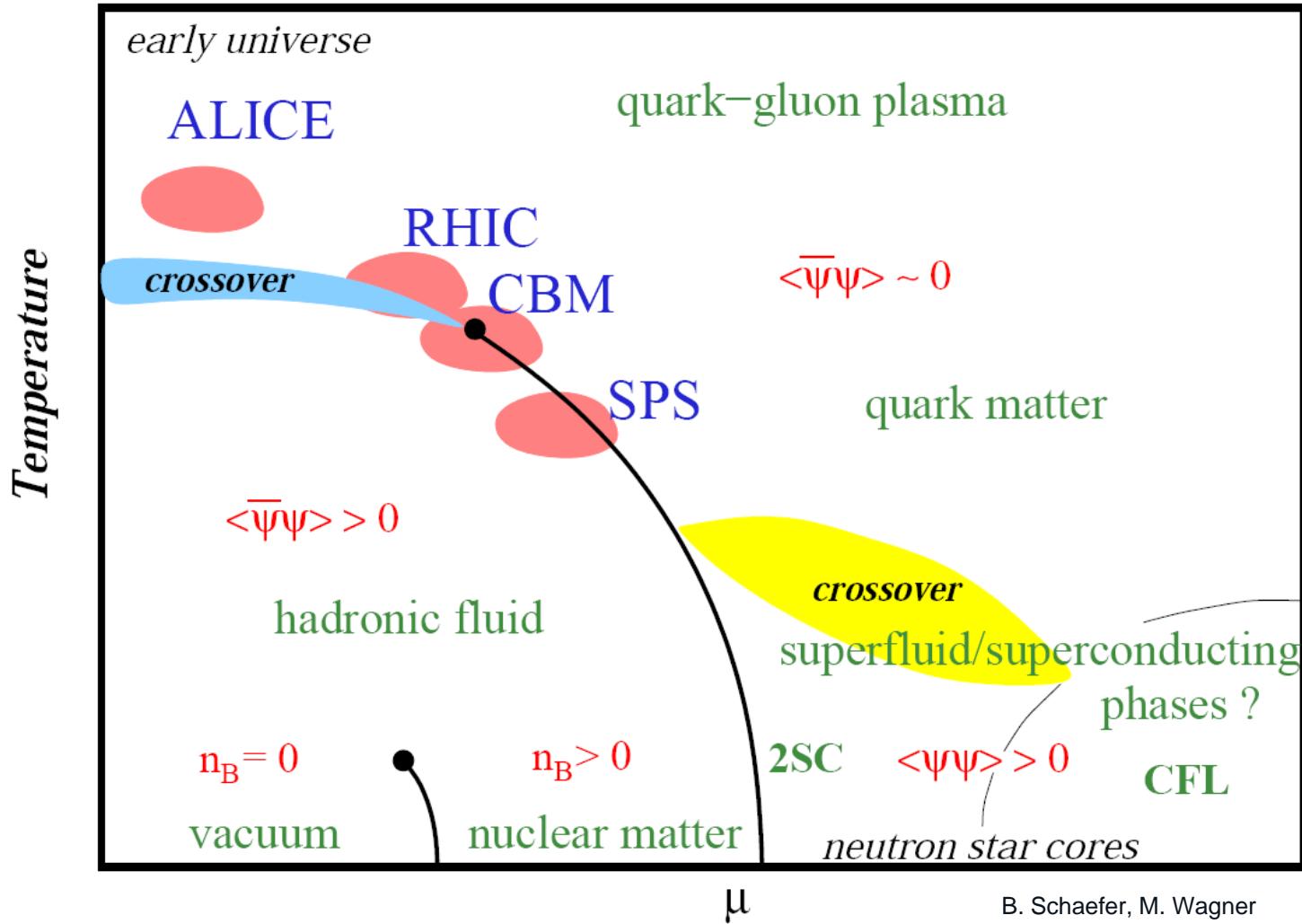
thermal blob scenario

Nonequilibrium

2PI effective action

dissipation kernel

Motivation

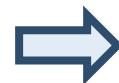


B. Schaefer, M. Wagner
arXiv:0812.2855

Motivation

critical phenomena at the phase transition

- order of the phase transition
- fluctuations of observables
 - behavior of correlation functions in momentum and space
- critical point (T_c ?):
 - behavior of soft modes, IR sector
 - critical slowing down
 - correlation lengths



How strong are these effects?

Model: Lagrangian

$$\mathcal{L} = \bar{\psi} [i\partial - g (\sigma + i\vec{\pi} \cdot \vec{\tau}\gamma_5)] \psi - \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi}) - U(\sigma, \vec{\pi})$$

Yukawa coupling

O(N) theory / Φ^4 coupling

$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - \nu^2)^2 - f_\pi m_\pi^2 \sigma + U_0$$

parameters:

$$\lambda = 20$$

self coupling

$$g = 3\dots 6$$

quark-sigma coupling

$$\nu^2 = f_\pi^2 - m_\pi^2/\lambda$$

field shift term

$$f_\pi = 93 \text{ MeV}$$

pion decay constant

$$m_\pi = 138 \text{ MeV}$$

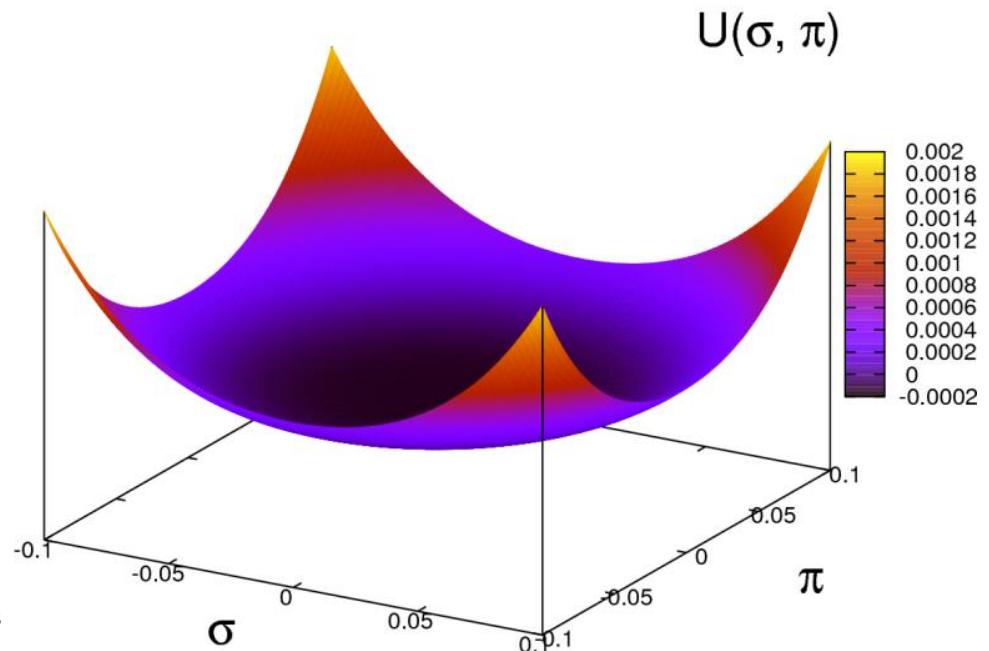
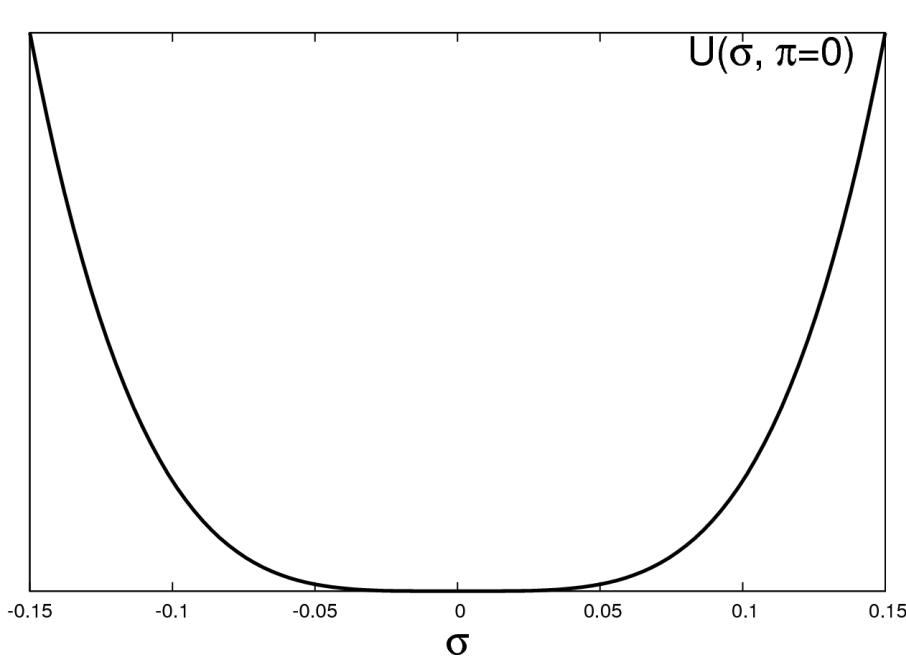
pion mass

$$U_0 = m_\pi^4/(4\lambda) - f_\pi^2 m_\pi^2$$

ground state

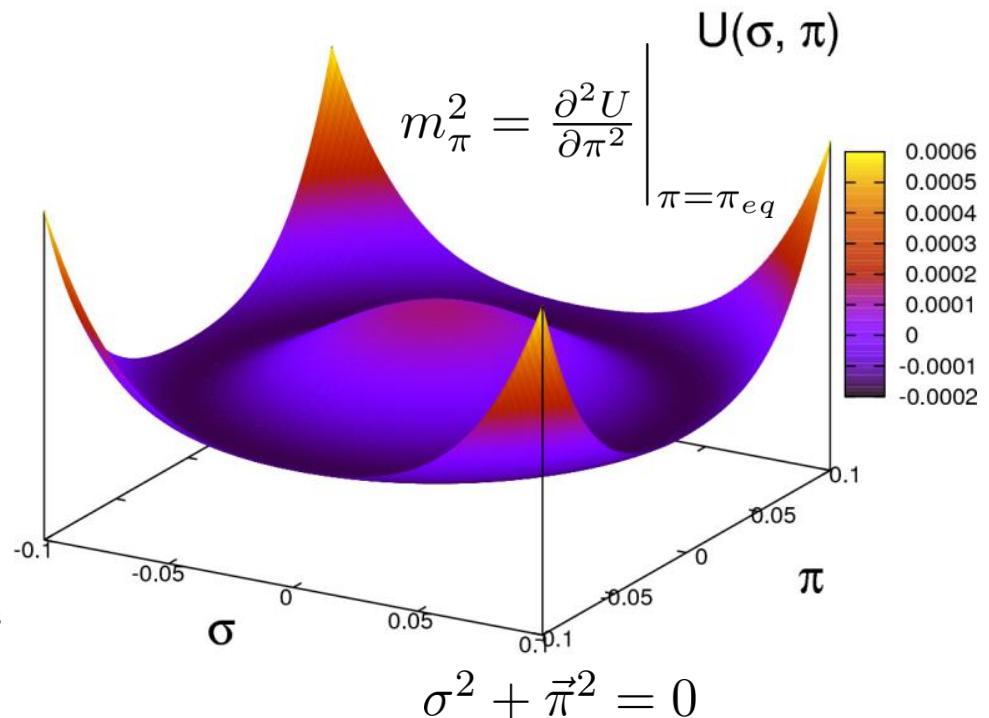
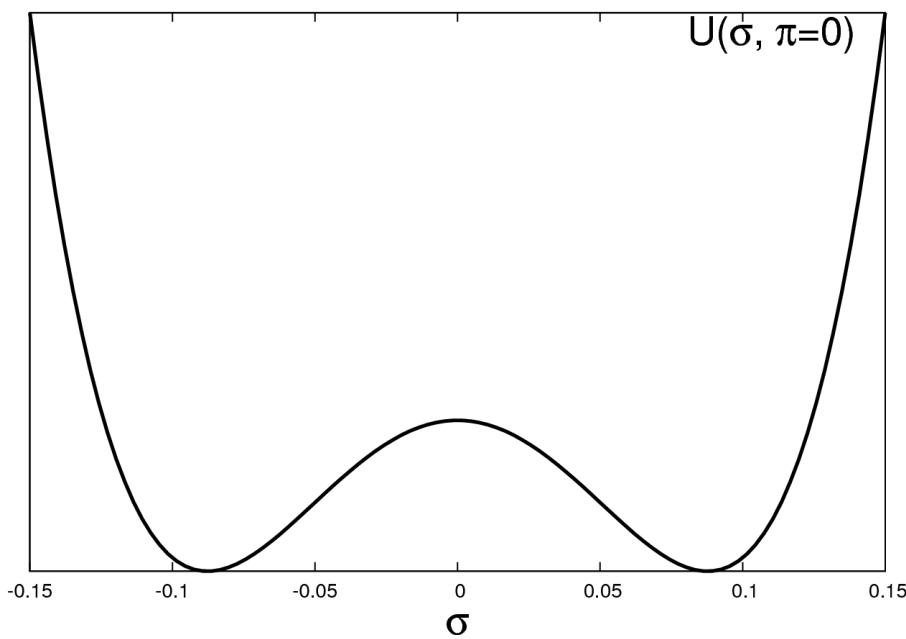
Symmetric potential

$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - \nu^2)^2 - f_\pi m_\pi^2 \sigma + U_0$$



Spontaneously broken symmetry

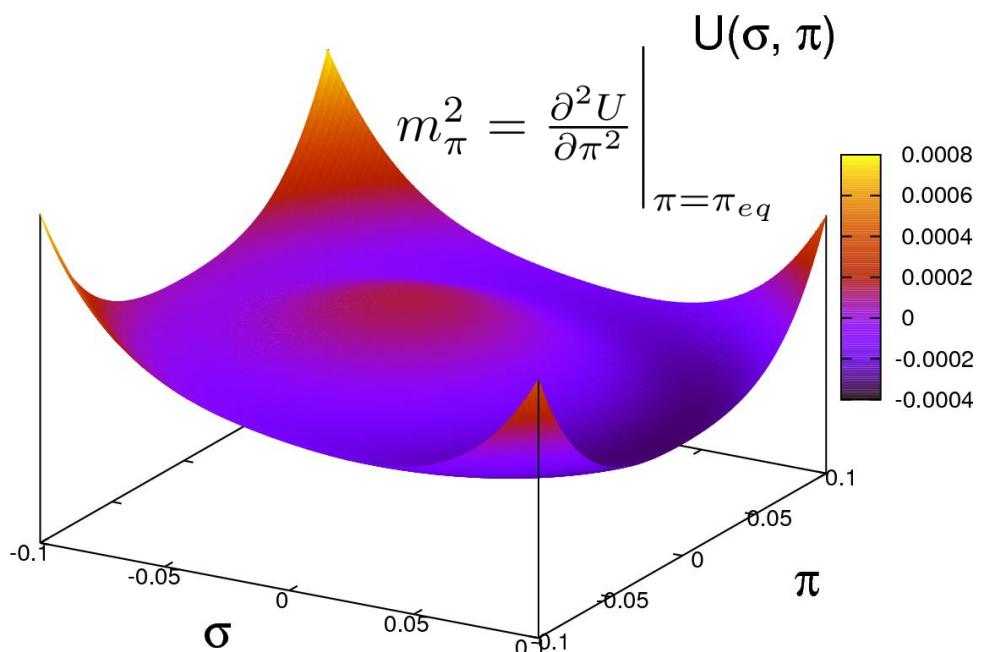
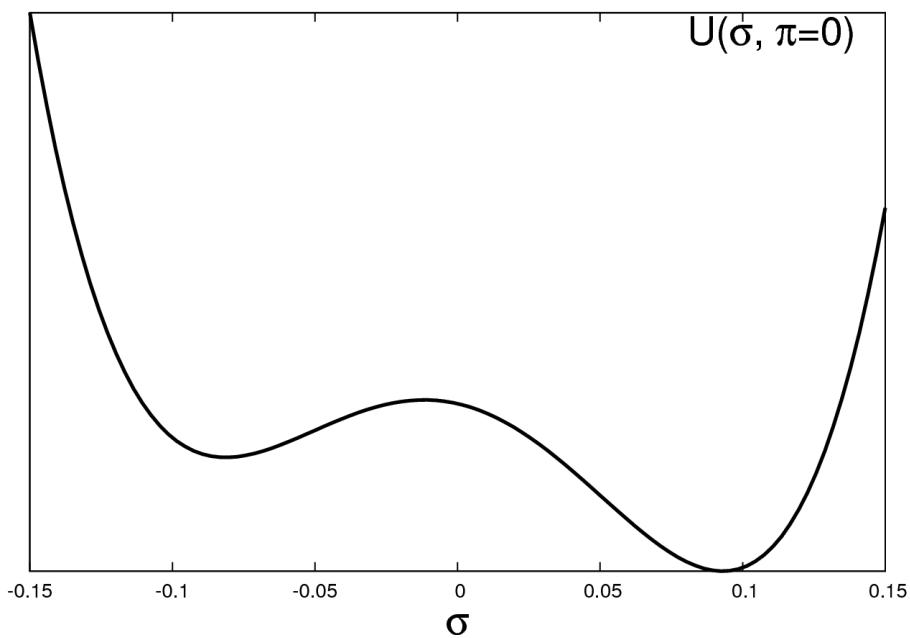
$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - \nu^2)^2 - f_\pi m_\pi^2 \sigma + U_0$$



$$\sigma^2 + \vec{\pi}^2 = 0$$

Explicitly broken symmetry

$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4} (\sigma^2 + \vec{\pi}^2 - \nu^2)^2 - f_\pi m_\pi^2 \sigma + U_0$$



Mean-field EoM (meson fields)

$$S = \int d^4x \mathcal{L}, \quad \frac{\delta S[\sigma, \vec{\pi}]}{\delta \sigma(x)} \stackrel{!}{=} 0, \quad \frac{\delta S[\sigma, \vec{\pi}]}{\delta \vec{\pi}(x)} \stackrel{!}{=} 0$$

3D+1 simulation

Klein-Gordon equations:

$$\partial_\mu \partial^\mu \sigma + \lambda (\sigma^2 + \vec{\pi}^2 - \nu^2) \sigma - f_\pi m_\pi^2 + g \langle \bar{\psi} \psi \rangle = 0$$
$$\partial_\mu \partial^\mu \vec{\pi} + \lambda (\sigma^2 + \vec{\pi}^2 - \nu^2) \vec{\pi} + g \langle \bar{\psi} i\gamma_5 \vec{\tau} \psi \rangle = 0$$

with scalar and pseudo-scalar quark densities:

$$\langle \bar{\psi} \psi \rangle (t, \vec{x}) = g d_q \sigma (t, \vec{x}) \int d^3 \vec{p} \frac{f_q (t, \vec{x}, \vec{p}) + f_{\bar{q}} (t, \vec{x}, \vec{p})}{E (t, \vec{x}, \vec{p})}$$

$$\langle \bar{\psi} i\gamma_5 \vec{\tau} \psi \rangle (t, \vec{x}) = g d_q \vec{\pi} (t, \vec{x}) \int d^3 \vec{p} \frac{f_q (t, \vec{x}, \vec{p}) + f_{\bar{q}} (t, \vec{x}, \vec{p})}{E (t, \vec{x}, \vec{p})}$$

Vlasov equation (quarks)

3D+1 simulation

Vlasov equation for quarks as test particles:

$$\left[\partial_t + \frac{\vec{p}}{E(t, \vec{x}, \vec{p})} \cdot \nabla_{\vec{x}} - \nabla_{\vec{x}} E(t, \vec{x}, \vec{p}) \nabla_{\vec{p}} \right] f_q(t, \vec{x}, \vec{p}) = 0$$

$$E(t, \vec{x}, \vec{p}) = \sqrt{\vec{p}^2(t) + M^2(t, \vec{x})}$$

$$M^2(t, \vec{x}) = g^2 [\sigma^2(t, \vec{x}) + \vec{\pi}^2(t, \vec{x})]$$

with a test particle distribution function:

$$f(t, \vec{x}, \vec{p}) = \frac{1}{N_{test}} \sum_i \delta^3(\vec{x} - \vec{x}_i(t)) \delta^3(\vec{p} - \vec{p}_i(t))$$

Equilibrium results

Equilibrium values from self-consistent equations:

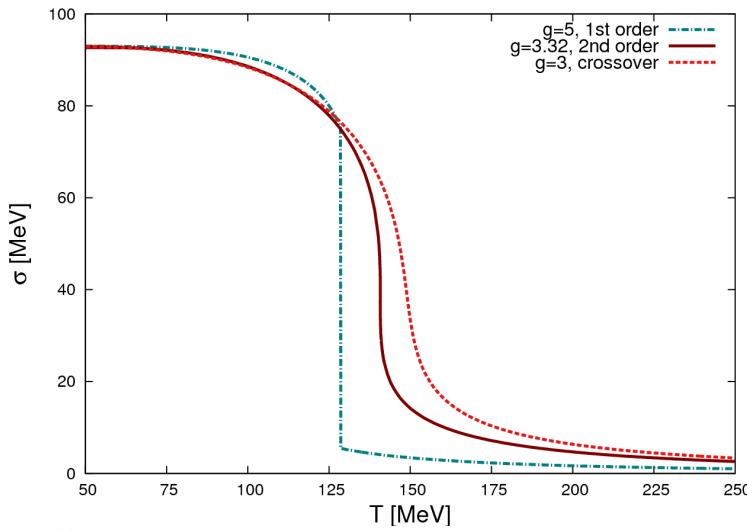
$$\partial_\mu \partial^\mu \sigma + \boxed{\lambda (\sigma^2 + \vec{\pi}^2 - \nu^2) \sigma - f_\pi m_\pi^2 + g \langle \bar{\psi} \psi \rangle} = 0$$
$$\partial_\mu \partial^\mu \vec{\pi} + \lambda (\sigma^2 + \vec{\pi}^2 - \nu^2) \vec{\pi} + g \langle \bar{\psi} i\gamma_5 \vec{\tau} \psi \rangle = 0$$



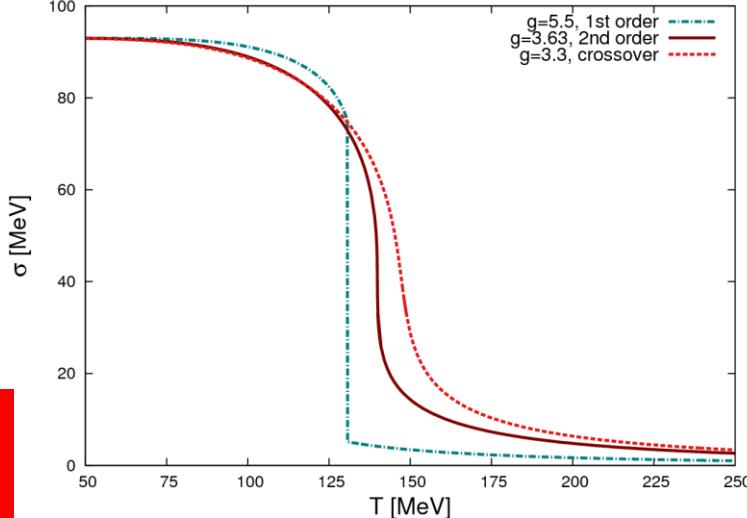
$$\partial_\mu \partial^\mu \sigma = 0, \vec{\pi} = 0 \quad \Rightarrow \quad \frac{\partial \Omega}{\partial \sigma} = \boxed{\lambda (\sigma^2 - \nu^2) \sigma - f_\pi m_\pi^2 + g \langle \bar{\psi} \psi \rangle} = 0$$
$$m_\sigma^2 = \left. \frac{\partial^2 \Omega}{\partial \sigma^2} \right|_{\sigma=\sigma_{eq}}$$

Equilibrium results

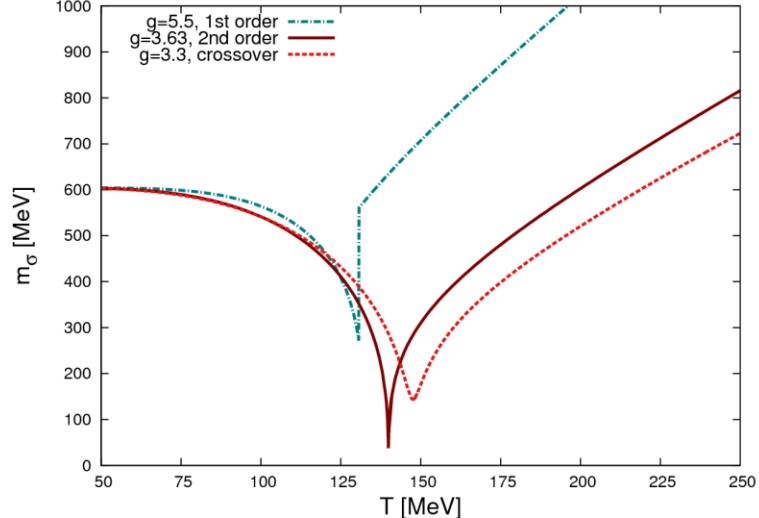
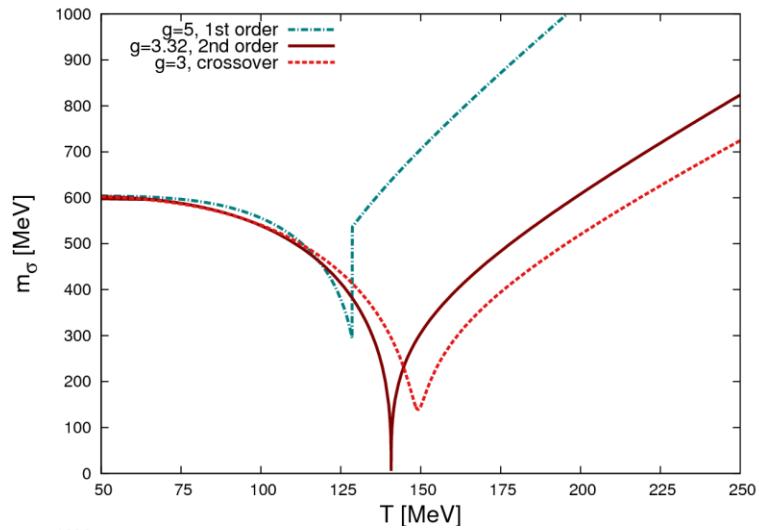
boltzmann



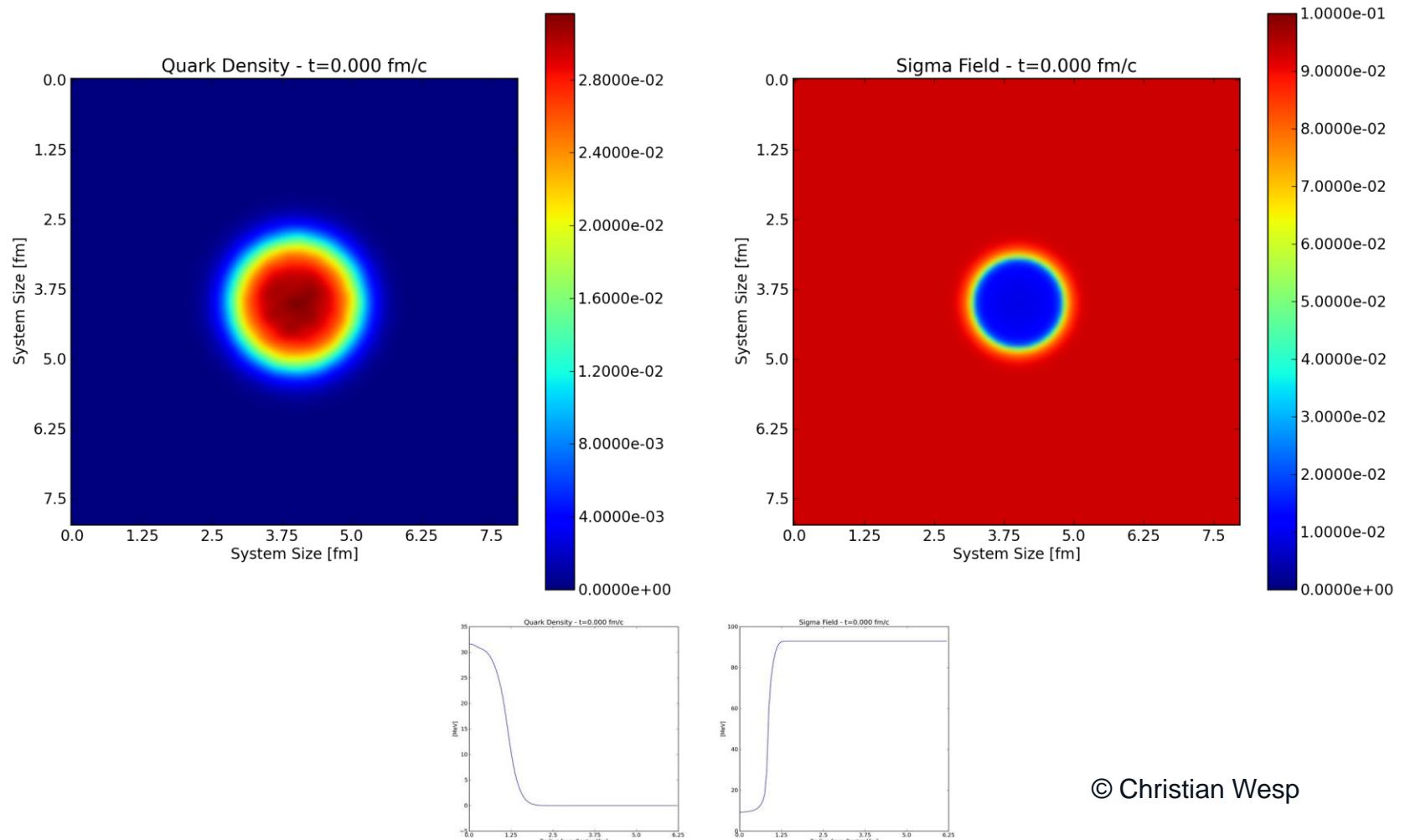
fermi



remark:
different values for g ,
best choice for 2nd
order transition

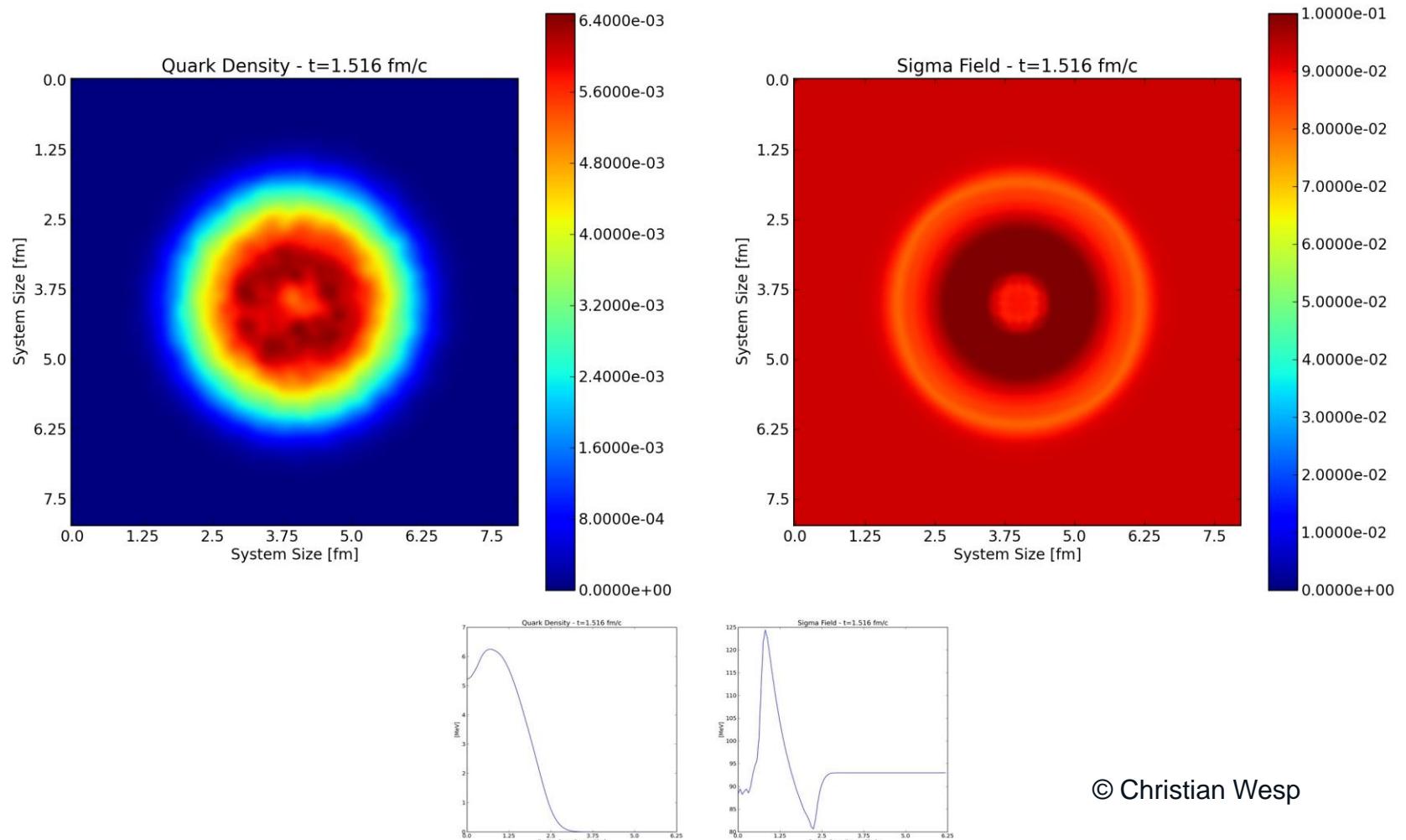


Thermal blob scenario



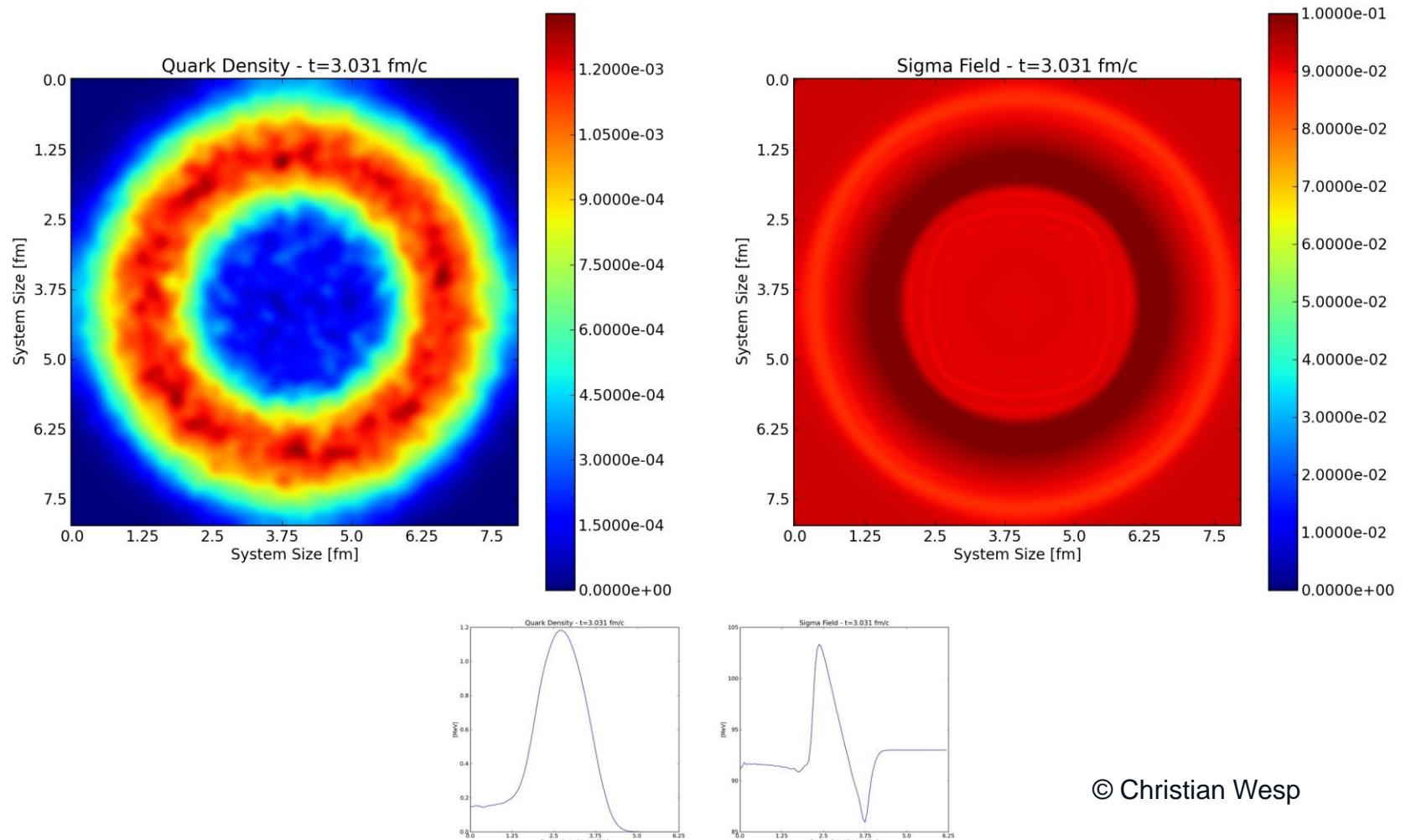
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Thermal blob scenario



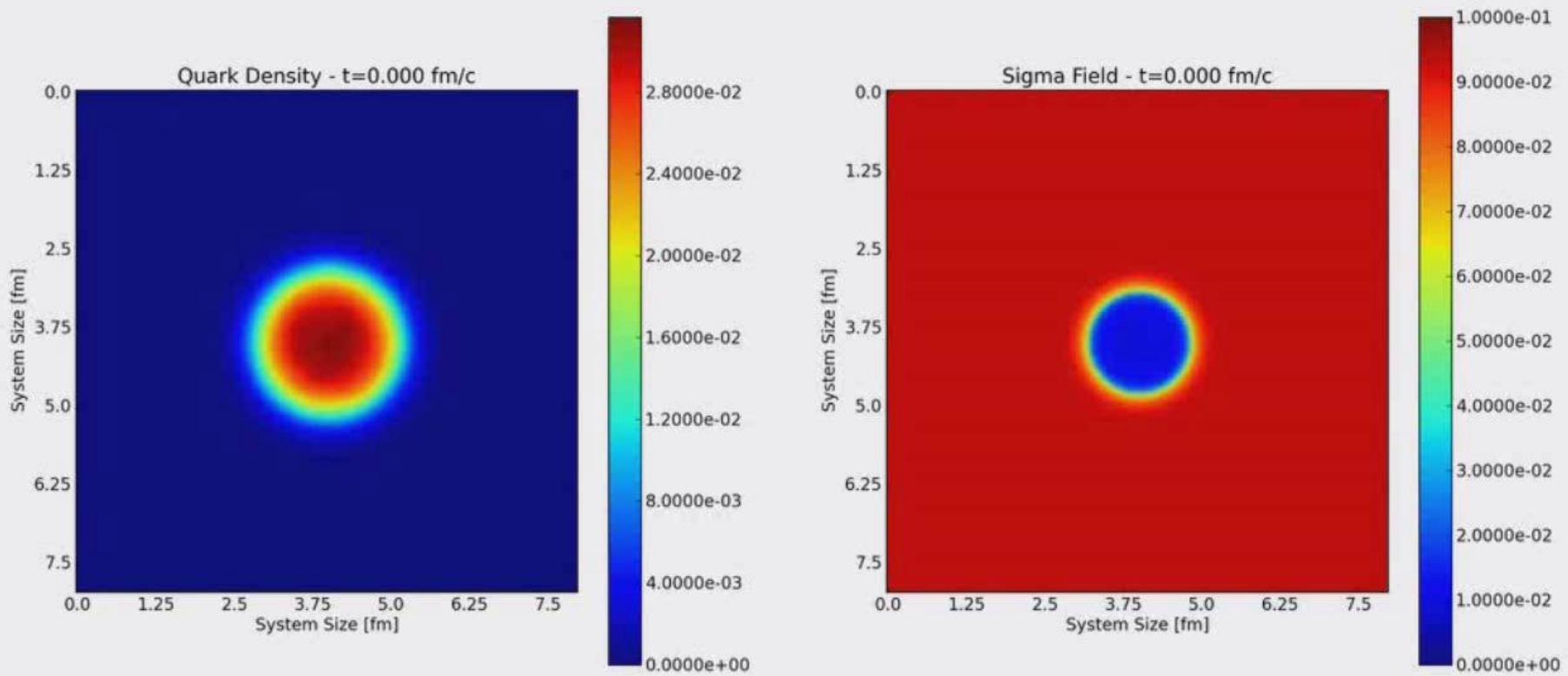
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Thermal blob scenario



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Thermal blob scenario

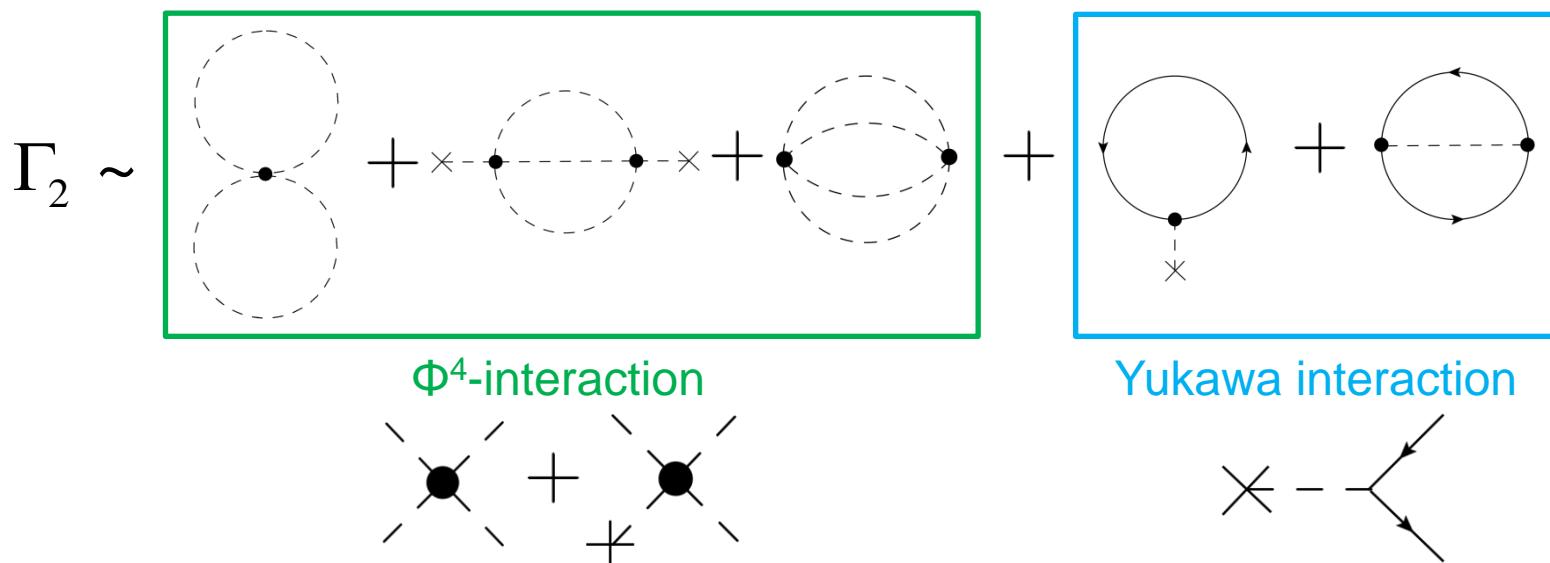


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2PI effective action

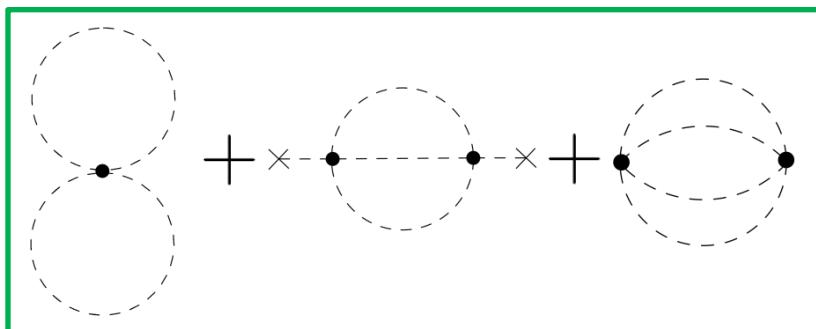
$$\begin{aligned}\Gamma[\phi, \psi, \bar{\psi}, G, D] = S_{cl}[\phi, \psi, \bar{\psi}] + \frac{i}{2} \cdot Tr \log G^{-1} + \frac{i}{2} \cdot Tr G_0^{-1} G \\ - i \cdot Tr \log D^{-1} - i \cdot Tr D_0^{-1} D + \Gamma_2[\phi, \psi, \bar{\psi}, G, D]\end{aligned}$$

relevant 2PI-diagrams (3 loops):

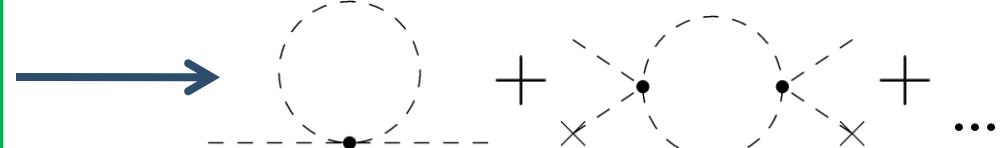


2PI contributions – self energies

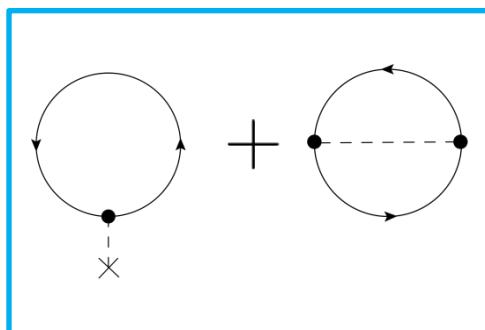
additional contributions beyond the mean-field:



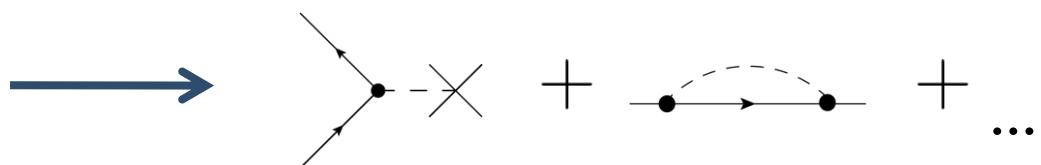
Φ^4 -interaction



self-energies (sigma and pion)



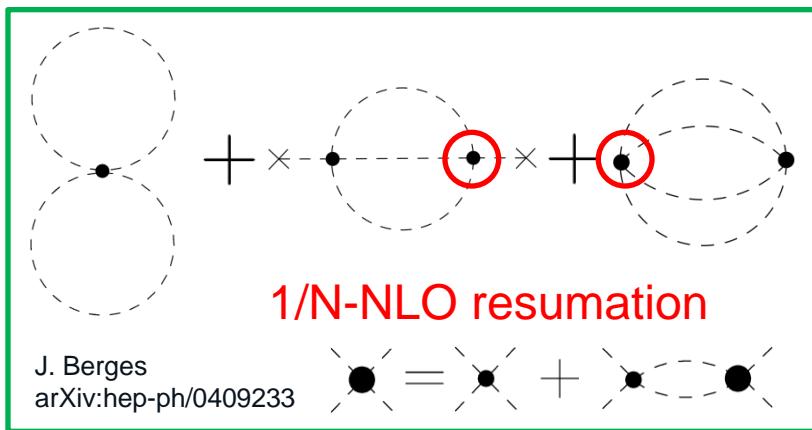
Yukawa interaction



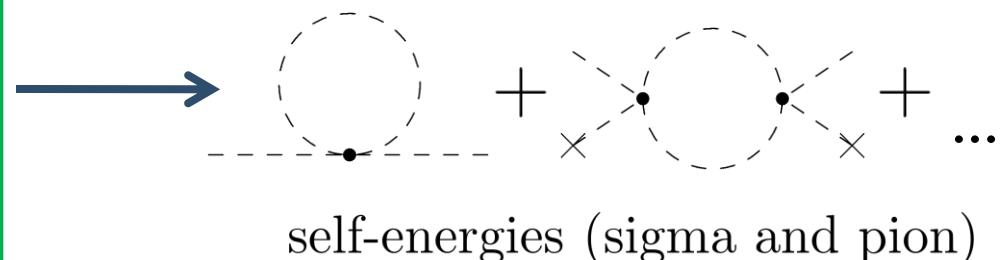
self-energies (quarks and sigma/pion)

2PI contributions – self energies

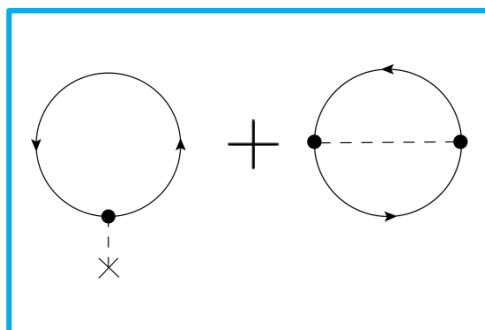
additional contributions beyond the mean-field:



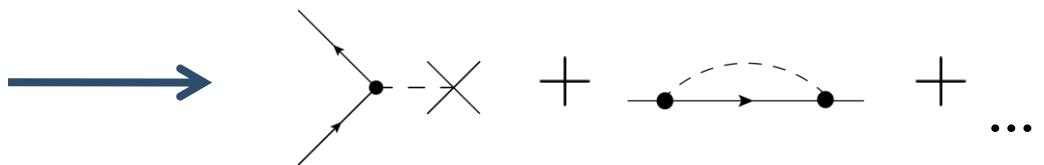
Φ^4 -interaction



self-energies (sigma and pion)



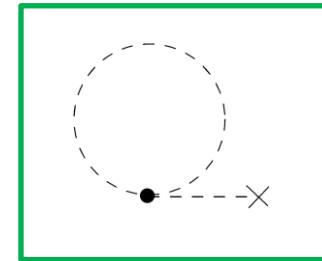
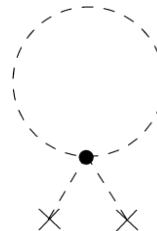
Yukawa interaction



self-energies (quarks and sigma/pion)

2PI effective action – mean field

$$\Gamma \sim \frac{i}{2} \cdot Tr G_0^{-1} G \sim$$



$$\partial_\mu \partial^\mu \sigma + \lambda \left(\sigma^2 + \vec{\pi}^2 - \nu^2 + \boxed{\frac{3}{2}G_{\sigma\sigma} + \frac{3}{2}G_{\pi\pi}} \right) \sigma - f_\pi m_\pi^2 + g \langle \bar{\psi}\psi \rangle = 0$$

$$\partial_\mu \partial^\mu \vec{\pi} + \lambda \left(\sigma^2 + \vec{\pi}^2 - \nu^2 + \boxed{\frac{1}{2}G_{\sigma\sigma} + \frac{5}{2}G_{\pi\pi}} \right) \vec{\pi} + g \langle \bar{\psi} i\gamma_5 \vec{\tau} \psi \rangle = 0$$

$$G_{\phi\phi}(t, \vec{x}, \vec{p}) = \frac{1}{2} \int \frac{d^3 p}{(2\pi)^2} \frac{1 + 2N_\phi(t, \vec{x}, \vec{p})}{\sqrt{\vec{p}^2 + M_\phi^2(t, \vec{x})}} \quad \text{with} \quad \phi \in \{\sigma, \pi\}$$

Equilibrium results

$$\partial_\mu \partial^\mu \sigma + \left[\lambda \left(\sigma^2 + \vec{\pi}^2 - \nu^2 + \frac{3}{2} G_{\sigma\sigma} + \frac{3}{2} G_{\pi\pi} \right) \sigma - f_\pi m_\pi^2 + g \langle \bar{\psi} \psi \rangle = 0 \right]$$

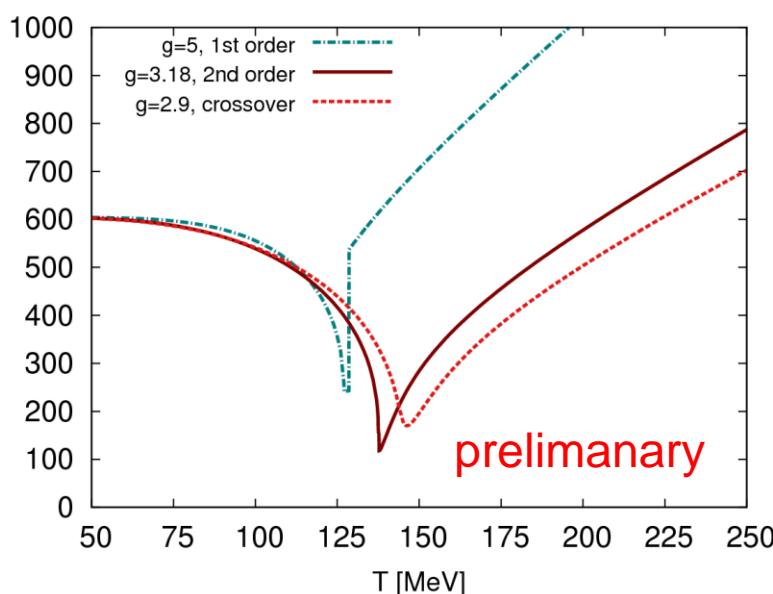
$$\partial_\mu \partial^\mu \vec{\pi} + \lambda \left(\sigma^2 + \vec{\pi}^2 - \nu^2 + \frac{1}{2} G_{\sigma\sigma} + \frac{5}{2} G_{\pi\pi} \right) \vec{\pi} + g \langle \bar{\psi} i\gamma_5 \vec{\tau} \psi \rangle = 0$$

$$\partial_\mu \partial^\mu \sigma = 0, \vec{\pi} = 0, G_{\pi\pi} = 0 \Rightarrow$$



$$\frac{\partial \Omega}{\partial \sigma} = \lambda \left(\sigma^2 - \nu^2 + \frac{3}{2} G_{\sigma\sigma} \right) \sigma - f_\pi m_\pi^2 + g \langle \bar{\psi} \psi \rangle = 0$$

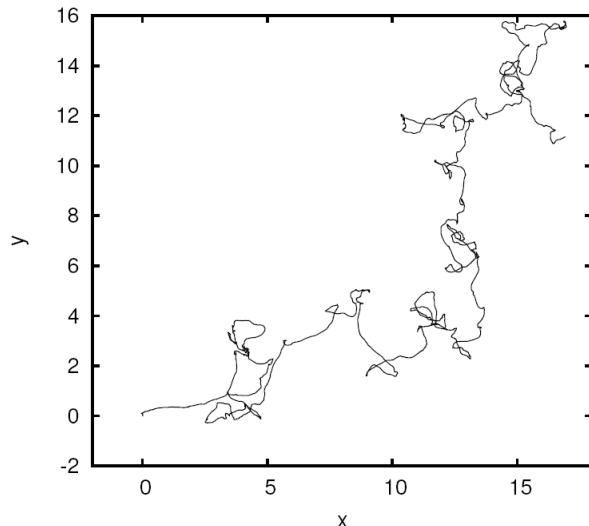
$$m_\sigma^2 = \left. \frac{\partial^2 \Omega}{\partial \sigma^2} \right|_{\sigma=\sigma_{eq}}$$



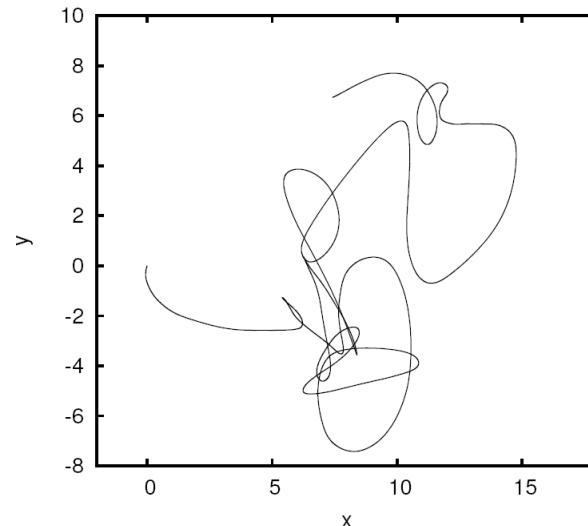
Dissipation kernel - motivation

Langevin equation (1D classical case):

$$m\ddot{x}(t) + 2 \int_0^t dt' \Gamma(t-t') \dot{x}(t') - F_{ext}(x) = \xi(t)$$



(a) Simulation with white noise.



(b) Simulation with coloured noise.

Dissipation kernel

Langevin equation (1D classical case):

$$m\ddot{x}(t) + \left[2 \int_0^t dt' \Gamma(t-t') \dot{x}(t') \right] - F_{ext}(x) = \xi(t)$$

Mean-field equation with a dissipation kernel:



$$\partial_\mu \partial^\mu \sigma + D(t, \vec{x}) + \lambda \left(\sigma^2 + \vec{\pi}^2 - \nu^2 + \frac{3}{2} G_{\sigma\sigma} + \frac{3}{2} G_{\pi\pi} \right) \sigma - f_\pi m_\pi^2 + g \langle \bar{\psi} \psi \rangle = 0$$

$$D(t, \vec{x}) \sim \int \frac{d^3 k}{(2\pi)^3} e^{i \vec{k} \cdot \vec{x}} \int \frac{dk^0}{2\pi} \frac{\mathcal{M}(t, \vec{x}, \vec{k})}{E_k} \partial_t \sigma(t, \vec{k}) \pi \delta(E_k - k^0)$$

Summary and outlook

- linear sigma model with Yukawa coupling to quarks
- numerics for mean-field dynamics and quarks as quasi particles exists already, 3D+1 simulation
- equilibrium and nonequilibrium initial conditions
 - dynamic evolution with a pseudo phase transition
 - particle creation and annihilation is needed

Outlook:

- on-shell approximation of collision terms
 - Boltzmann dynamics for the distribution functions
 - sigma, pion and quarks
 - dissipation kernel for the mean-field equation
- long term:
 - solution of kinetic quantum transport equations

Thank you
for your attention!



realtime-formalism

$$\langle O(t) \rangle = Tr \rho(t) O(t) = \frac{1}{Z} Tr \{ U(t_i - i\beta, t_i) U(t_i, t_f) U(t_f, t) O(t) U(t, t_i) \}$$

time-ordered Green's functions:

$$iG(x, y) = \langle T_C \varphi(x) \varphi(y) \rangle$$

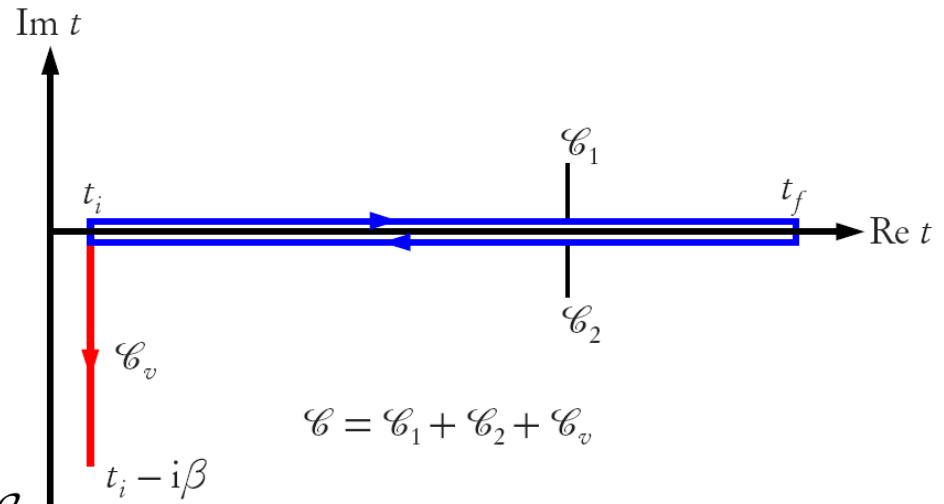
$$i\hat{G}_C = i \begin{pmatrix} G^{11} & G^{12} \\ G^{21} & G^{22} \end{pmatrix}$$

$$iG^{11} = \langle T^c \varphi(x) \varphi(y) \rangle \quad \text{for } x^0, y^0 \in \mathcal{C}_1$$

$$iG^{12} = \langle \varphi(y) \varphi(x) \rangle \quad \text{for } x^0 \in \mathcal{C}_1, y^0 \in \mathcal{C}_2$$

$$iG^{21} = \langle \varphi(x) \varphi(y) \rangle \quad \text{for } x^0 \in \mathcal{C}_2, y^0 \in \mathcal{C}_1$$

$$iG^{22} = \langle T^a \varphi(x) \varphi(y) \rangle \quad \text{for } x^0, y^0 \in \mathcal{C}_2$$



2PI effective action

$$Z[J, K] = \exp[iW[J, K]] = \int \mathcal{D}\Phi \exp \left[i \left(S[\Phi] + \int_{\mathcal{C}} J\Phi + \frac{1}{2} \int \int_{\mathcal{C}} \Phi K \Phi \right) \right]$$

$$\frac{\delta W[J, K]}{\delta J^i(x)} = \langle \Phi^i \rangle = \phi^i(x), \quad \frac{\delta W[J, K]}{\delta K^{ij}(x, y)} = \frac{1}{2} (\phi^i(x) \phi^j(y) + G^{ij}(x, y))$$

$$\Gamma[\phi, G] = W[J, K] - \int_{\mathcal{C}} J\phi - \frac{1}{2} \int \int_{\mathcal{C}} K(\phi\phi + G)$$

explicitly to the 2PI contribution (two-particle irreducible):

$$\begin{aligned} \Gamma[\phi, \psi, \bar{\psi}, G, D] &= S_{cl}[\phi, \psi, \bar{\psi}] + \frac{i}{2} \cdot Tr \log G^{-1} + \frac{i}{2} \cdot Tr G_0^{-1} G \\ &\quad - i \cdot Tr \log D^{-1} - i \cdot Tr D_0^{-1} D + \Gamma_2[\phi, \psi, \bar{\psi}, G, D] \end{aligned}$$

Transport equations

$$\left. \frac{\delta \Gamma [\phi, G]}{\delta \phi^i (x)} \right|_{J=0} = 0, \quad \left. \frac{\delta \Gamma [\phi, G]}{\delta G^{ij} (x, y)} = \frac{i}{2} \Pi^{ij} (x, y) + \frac{\Gamma_2 [\phi, G]}{\delta G^{ij} (x, y)} \right|_{J=K=0} = 0$$

with $\Pi (x, y) = G_0^{-1} (x, y) - G^{-1} (x, y)$

Schwinger-Dyson and Kadanoff-Baym equations:

$$-\left(\partial_\mu \partial^\mu + m^2 + \frac{\lambda}{2} \phi^2 + \Pi^{loc} \right) G^{ret,adv} - \Pi^{ret,adv} \odot G^{ret,adv} = \delta$$

$$-\left(\partial_\mu \partial^\mu + m^2 + \frac{\lambda}{2} \phi^2 + \Pi^{loc} \right) G^{<,>} - \Pi^{ret} \odot G^{<,>} = \Pi^{<,>} \odot G^{adv}$$

with $G^{ret} = \theta (x^0 - y^0) (G^> - G^<) , \quad G^{adv} = -\theta (y^0 - x^0) (G^> - G^<)$
 $G^< = G^{12} , \quad G^> = G^{21}$

Gradient expansion

Wigner transformation with $\Delta x = x - y, \quad X = \frac{x + y}{2}$

$$\tilde{A}(p, X) = \int d^4 \Delta x e^{ip_\mu \Delta x^\mu} A\left(X + \frac{\Delta x}{2}, X - \frac{\Delta x}{2}\right)$$

$$\int d^4 \Delta x e^{ip_\mu \Delta x^\mu} \int d^4 z A(x, z) B(z, y) = e^{-\frac{i}{2}(\partial_X^A \partial_p^B - \partial_X^B \partial_p^A)} \{\tilde{A}(p, X)\} \{\tilde{B}(p, X)\}$$

SD-equations $\xrightarrow{\text{WT}}$ dynamics of the spectral function (Breit-Wigner form)

KB-equations \longrightarrow kinetic quantum transport equations

$$2p_\mu \partial_X^\mu \left(i\tilde{G}^{<,>} \right) + \{\tilde{M}^2 + \text{Re } \tilde{\Pi}^{\text{ret}}, i\tilde{G}^{<,>}\} + \{i\tilde{\Pi}^{<,>}, \text{Re } \tilde{G}^{\text{ret}}\} = i\tilde{\Pi}^{<} i\tilde{G}^{>} - i\tilde{\Pi}^{>} i\tilde{G}^{<}$$

driftVlasovoff-shellgain/loss