

Polyakov Loop Susceptibilities in Pure Gauge System

Pok Man Lo

GSI

FAIRNESS 2013

20-September 2013, Berlin

pmlo@gsi.de

In collaboration with...

Bengt Friman (*GSI*)

Chihiro Sasaki (*FIAS*)

Krzysztof Redlich (*U. of Wroclaw*)

Olaf Kaczmarek (*U. of Bielefeld*)

Content

Aspects of Deconfinement Phase Transition

- Z(3) center symmetry
- Polyakov loops
- Polyakov loops susceptibilities

Content

General Properties of PL Susceptibilities in SU(3) system

- Lattice results
- Ratios of PL SUS
- Implications to effective models

Content

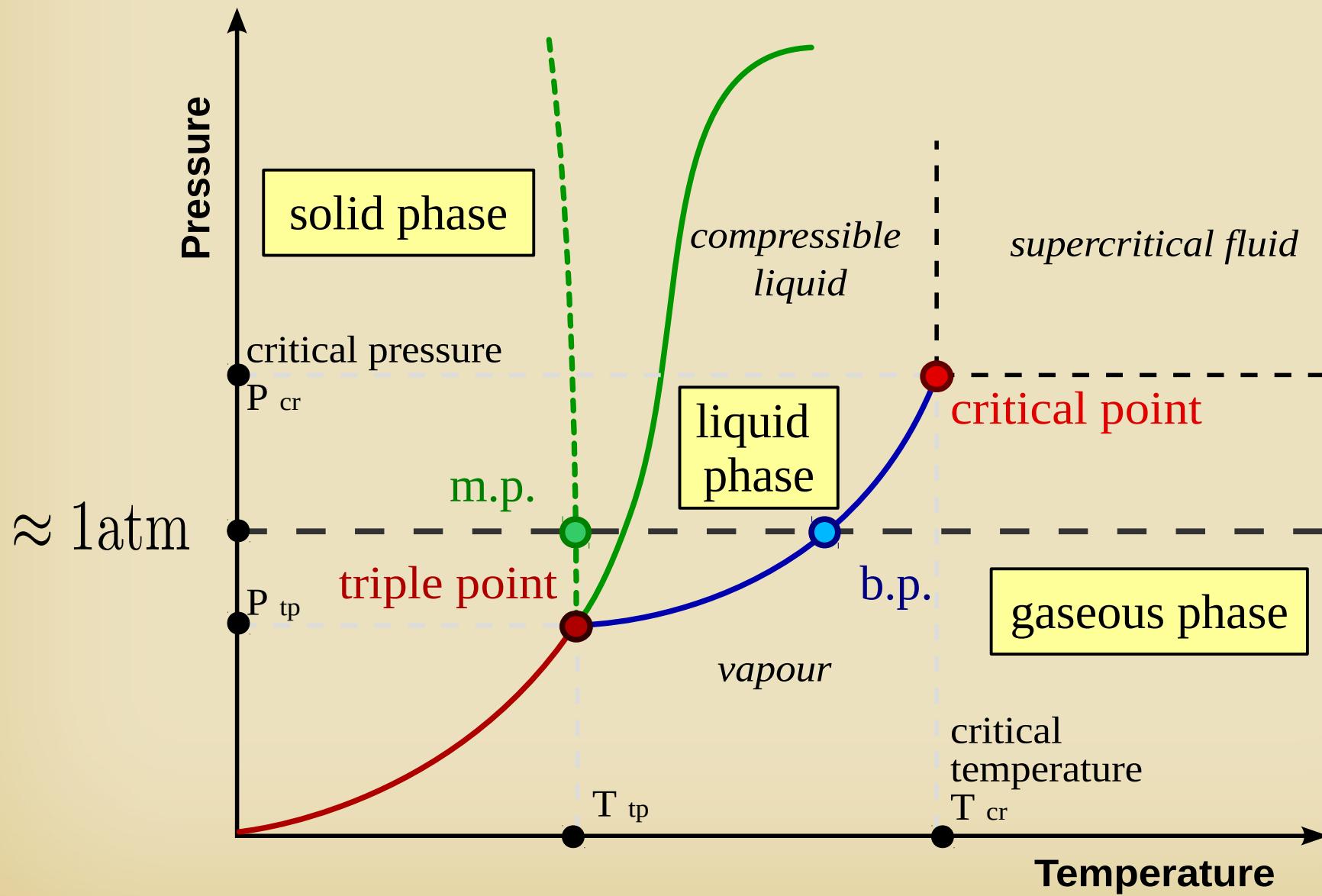
Explicit Z(3) Symmetry Breaking by Heavy Quarks

- PL-heavy quark coupling
- Critical quark mass and phase boundary
- Quark mass dependence of transition temperature

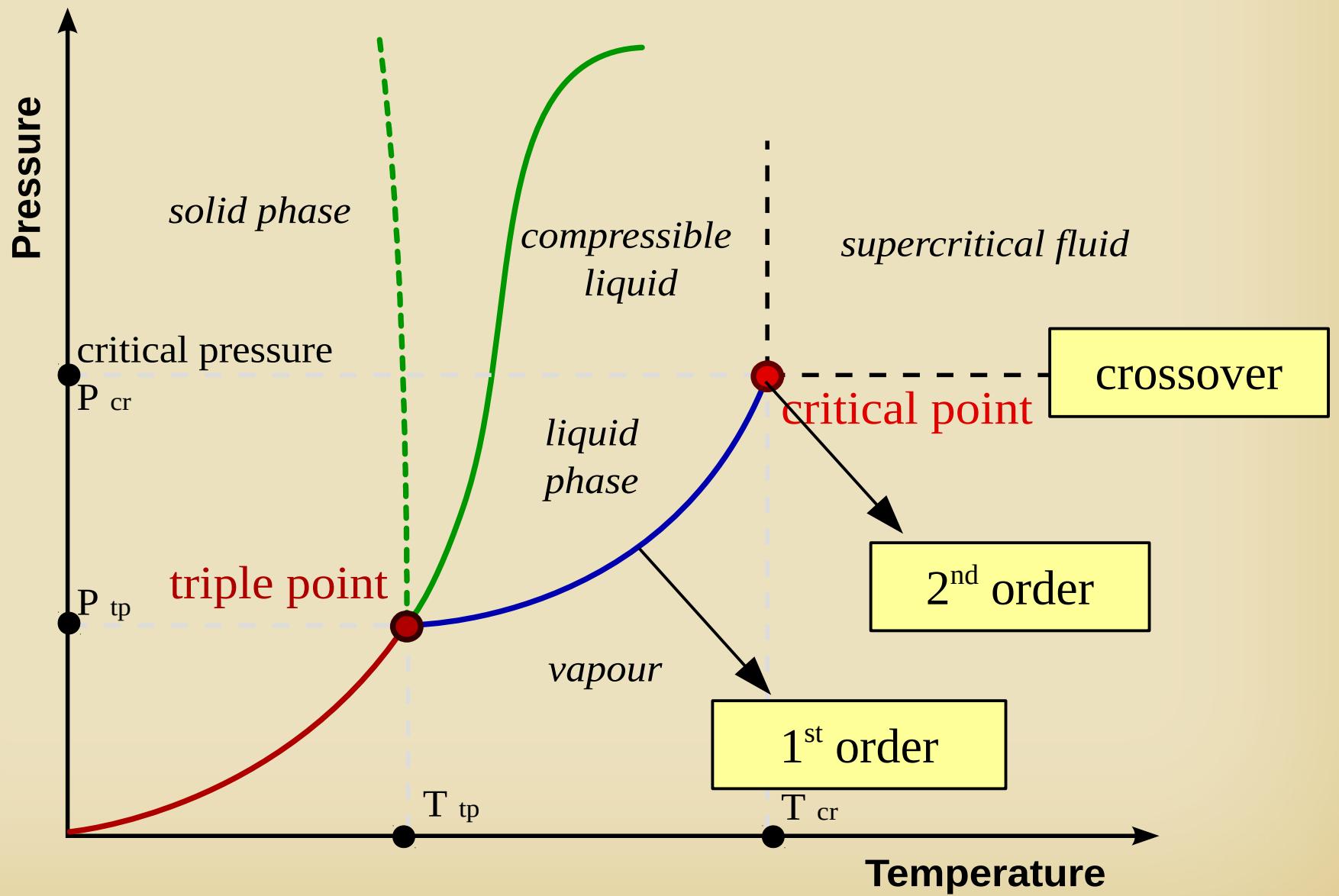
Aspects of Deconfinement Phase Transition

P.M. Lo, B. Friman, O. Kaczmarek,
K. Redlich & C. Sasaki
Phys. Rev. D 88, 014506 (2013)

Phase diagram of water



Phase diagram of water



Deconfinement phase transition

- Spontaneous breaking of $Z(3)$ center symmetry
- $SU(3)$ pure gauge system
 - exact $Z(3)$
 - string does not break at low T
 - infinitely heavy quarks

The Polyakov loop

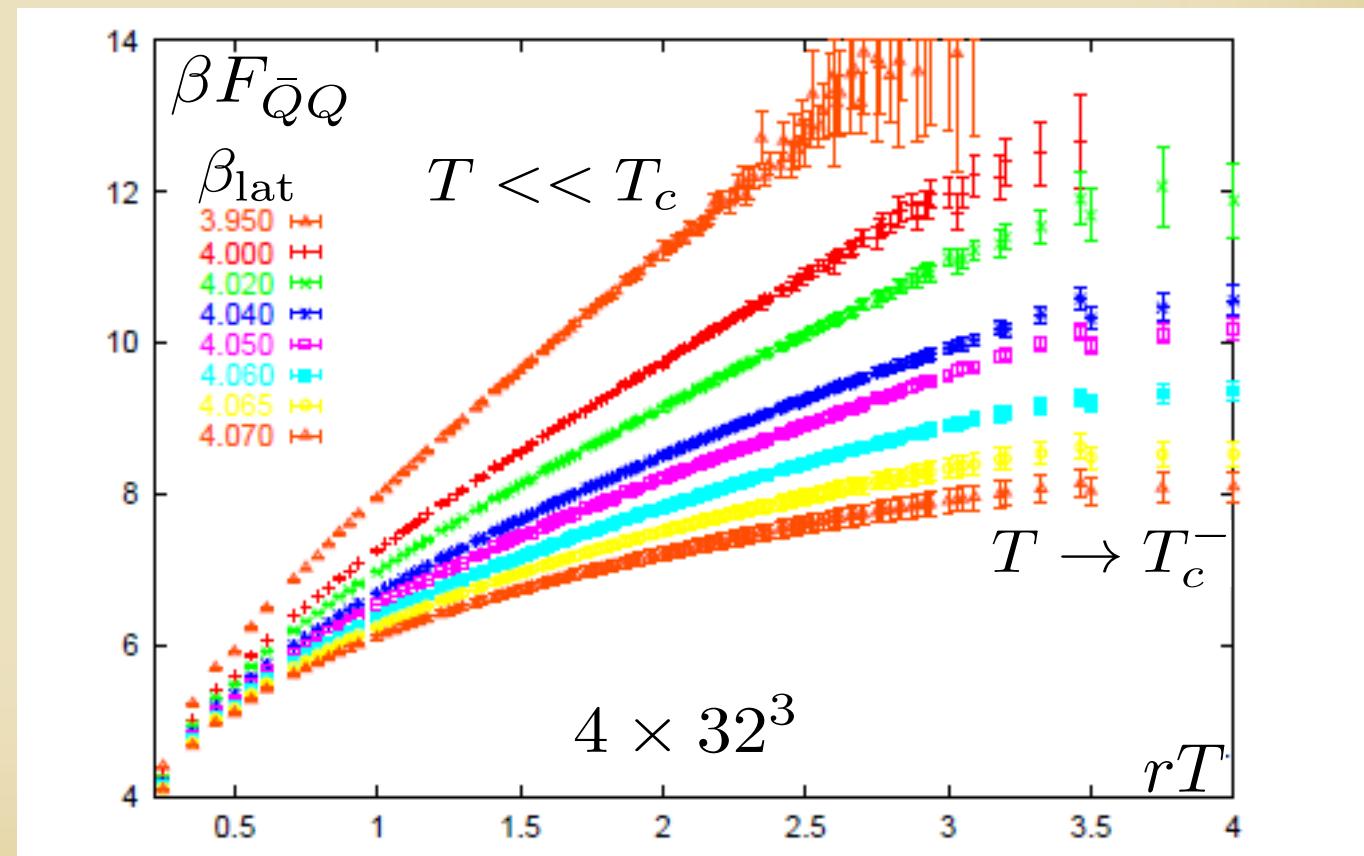
In the limit of infinitely heavy quarks

$$|\langle L \rangle|^2 = e^{-\beta F_{\bar{Q}Q}[r \rightarrow \infty, T]}$$

Kaczmarek *et. al.*

$T < T_c$
 $\langle L \rangle = 0$ Confined

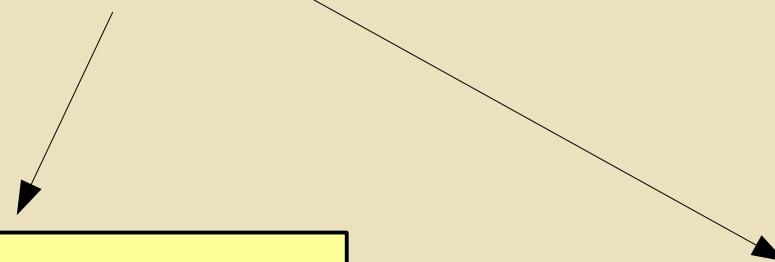
$T > T_c$
 $\langle L \rangle \neq 0$ Deconfined



Polyakov Loop

- Order parameter **characterizes** the state of the system

$$\langle L \rangle$$



Operator does **not** respect
the symmetry

$$L \rightarrow L e^{i2n\pi/3}$$

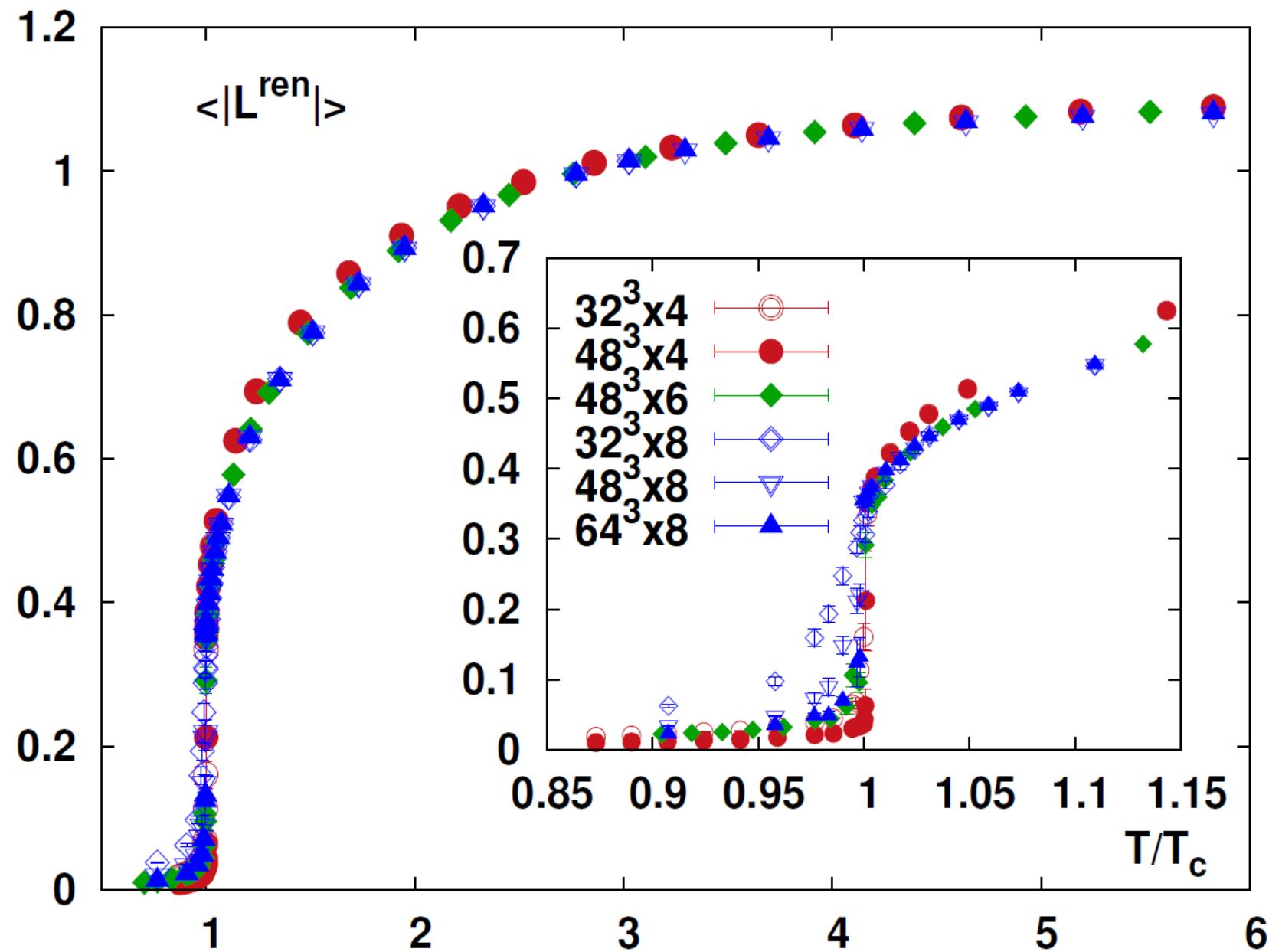
symmetric phase: $\langle L \rangle = 0$
broken phase: $\langle L \rangle \neq 0$

The Polyakov loop

- The effective theory for $\langle L \rangle$ has a global Z(3) symmetry

$$L \rightarrow e^{\frac{i2n\pi}{3}} L \quad S_{eff} \rightarrow S_{eff} \quad n = 0, 1, 2$$

- Confined phase = symmetry restored phase
 $\langle L \rangle = 0$
- Deconfined phase = Z(3) spontaneously broken phase
 $\langle L \rangle \neq 0$



Polyakov loop susceptibility

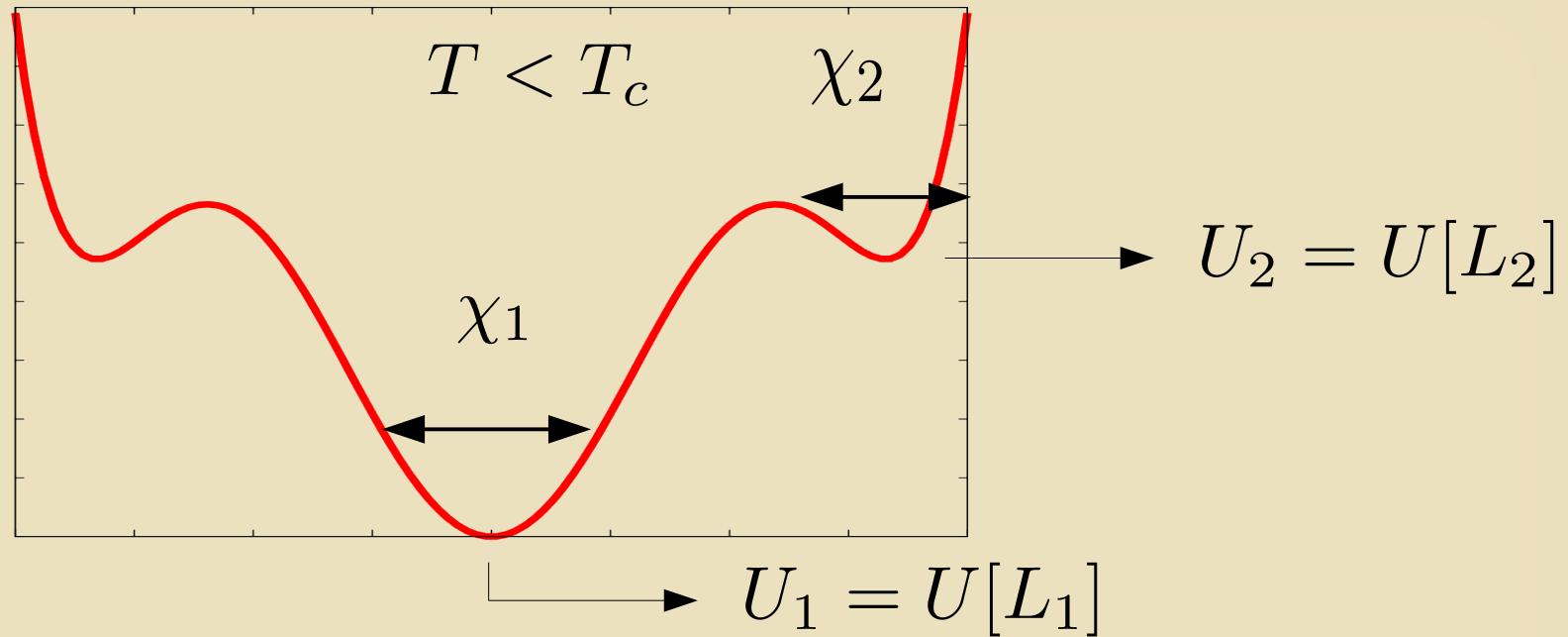
- Intensive susceptibility

$$\chi = \beta V < \hat{L} \hat{L} >_c = \frac{\partial}{\partial h} \langle L \rangle \Big|_{h \rightarrow 0}$$

- Sensitive to the **transition** of phases
features **peak** and **width** near the transition
- Effective potential: inverse of curvature

1st order phase transition

- Order parameter features a discontinuous jump
- Correlation length remains finite: $\xi \rightarrow \xi_0$
- Competing minima:
the system is “shocked” from one state to another
- Susceptibility has a delta function like behavior at transition due to phase coexistence

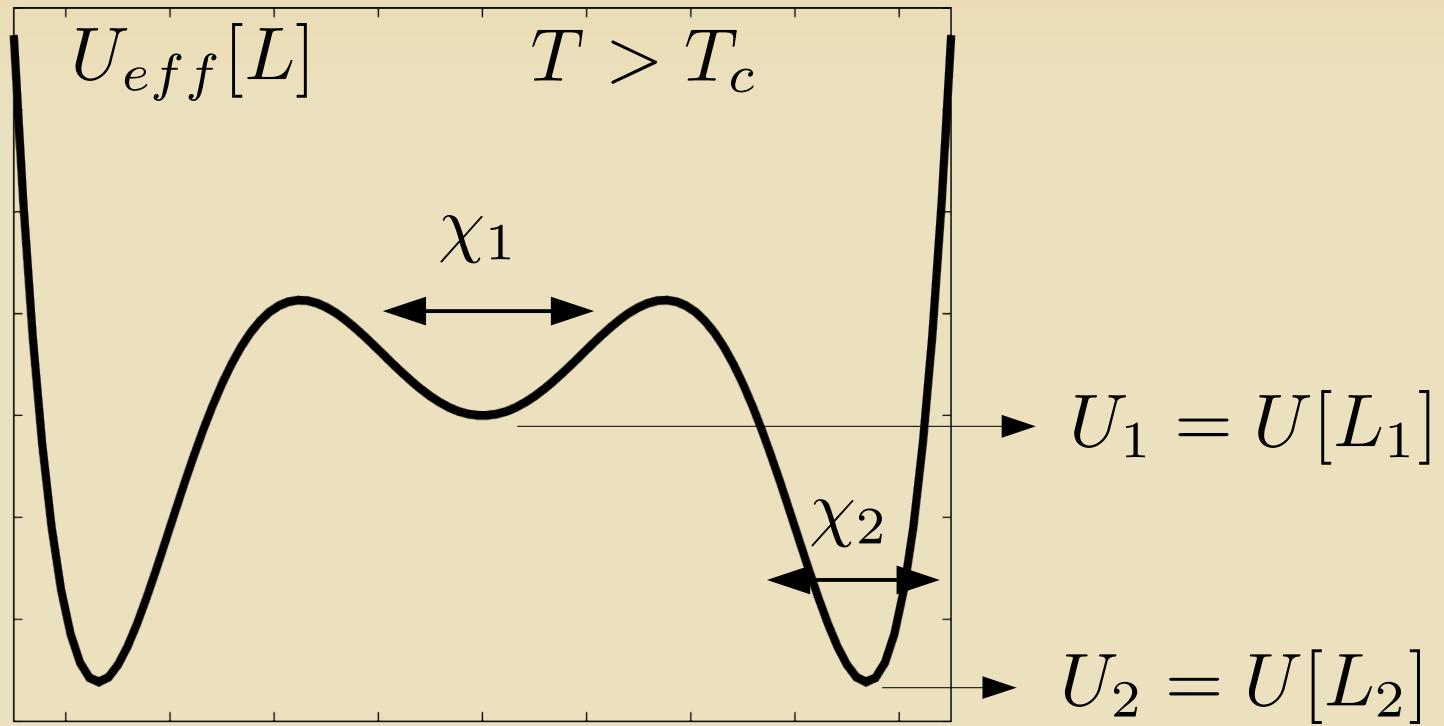
$$U_{eff}[L]$$


$$\frac{P_2}{P_1} \propto e^{-\beta V(U_2 - U_1)}$$

$$V \rightarrow \infty$$

$$\langle L \rangle \rightarrow L_1$$

$$\beta V \langle LL \rangle_c \rightarrow \chi_1$$

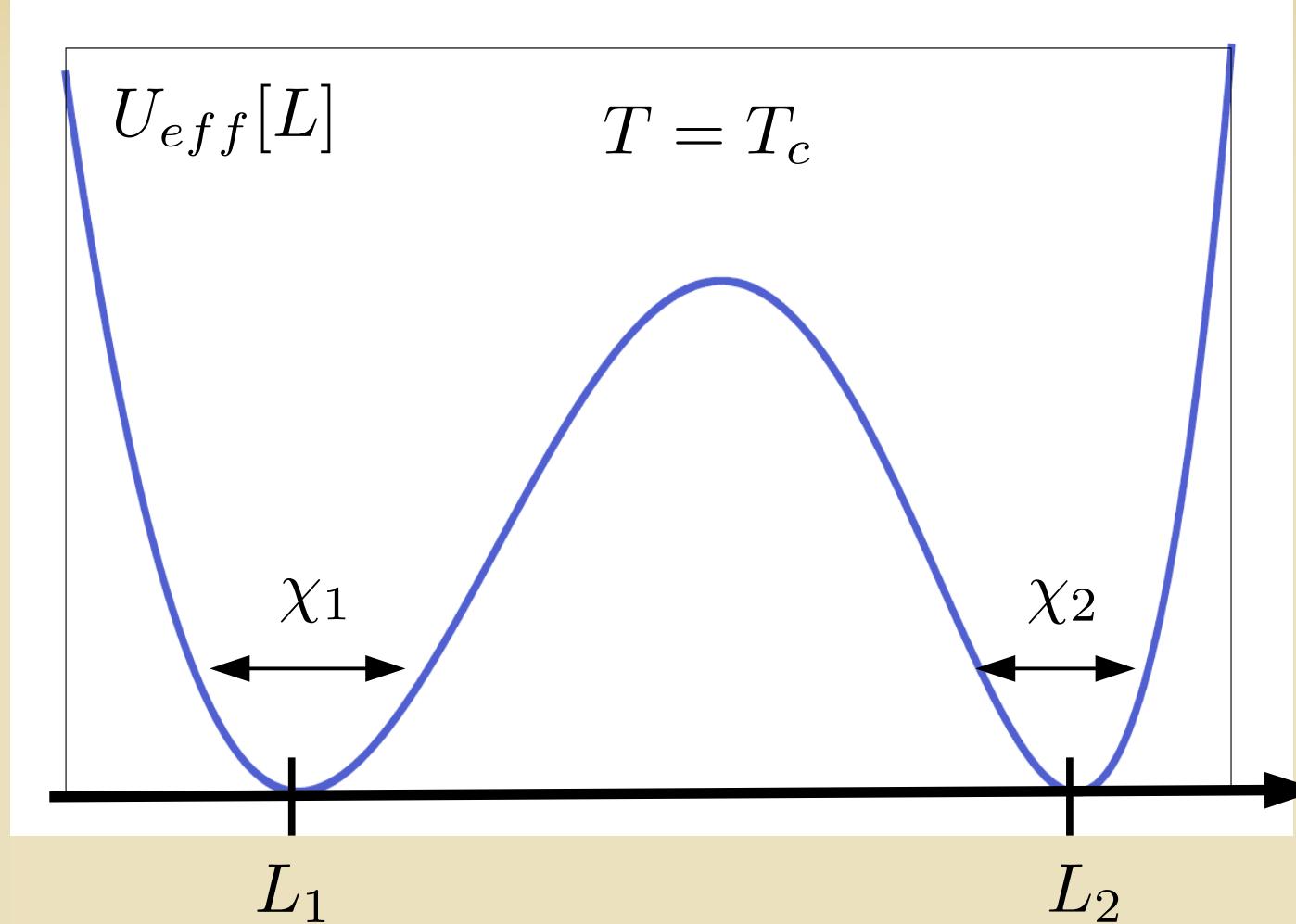


$$\frac{P_2}{P_1} \propto e^{-\beta V(U_2 - U_1)}$$

$$V \rightarrow \infty$$

$$\langle L \rangle \rightarrow L_2$$

$$\beta V \langle LL \rangle_c \rightarrow \chi_2$$



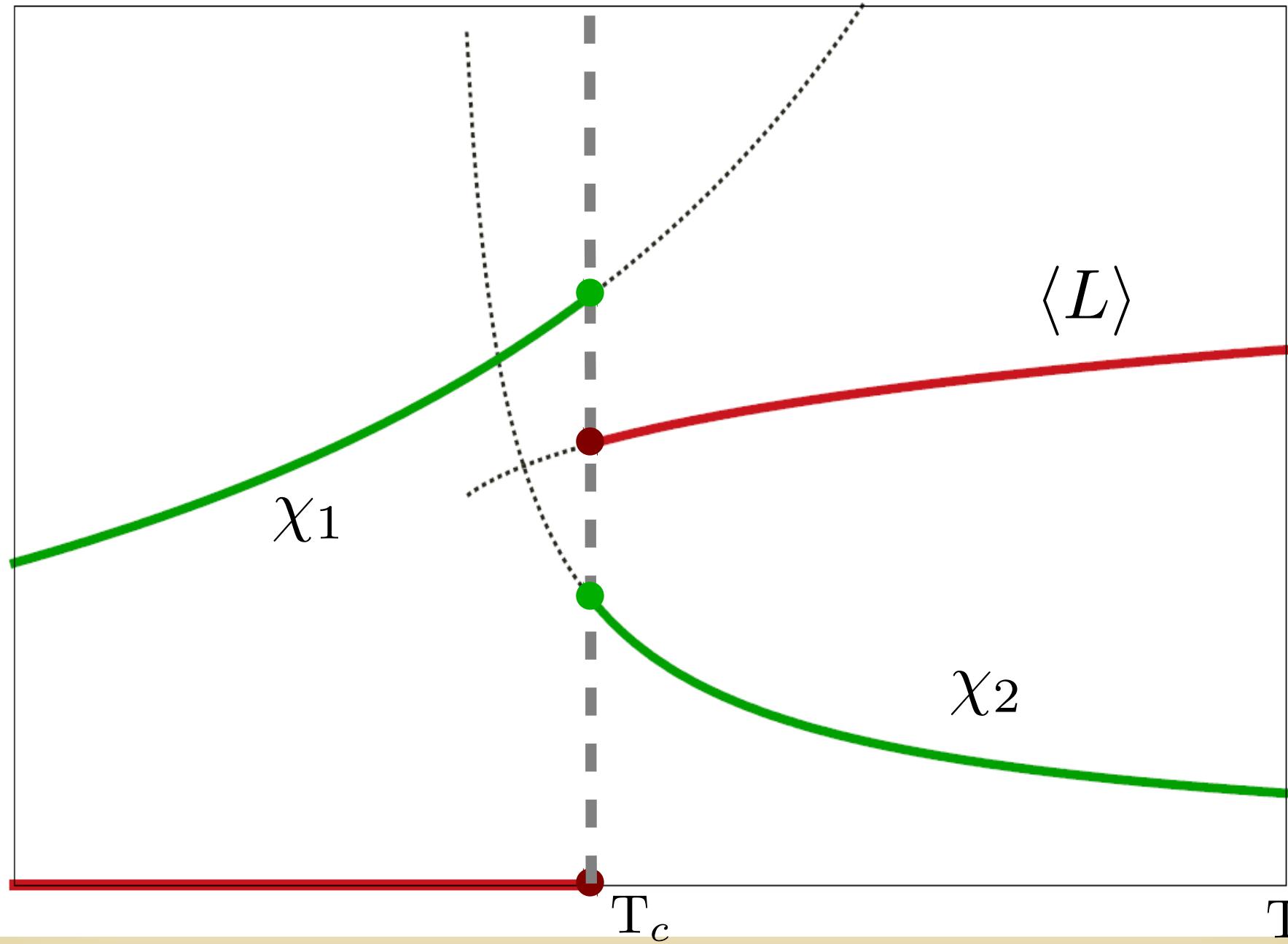
$$U_1 = U_2 \\ P_1 = P_2$$

$$V \rightarrow \infty$$

$$\langle L \rangle \rightarrow \{L_1, L_2\} ?$$

$$\beta V \langle LL \rangle_c \rightarrow \{\chi_1, \chi_2\} ?$$

Phase coexistence?



General Properties of PL Susceptibilities in SU(3) System

Lattice calculation

- L is complex-valued for SU(3)

$$L_L, L_T$$

- Fluctuations

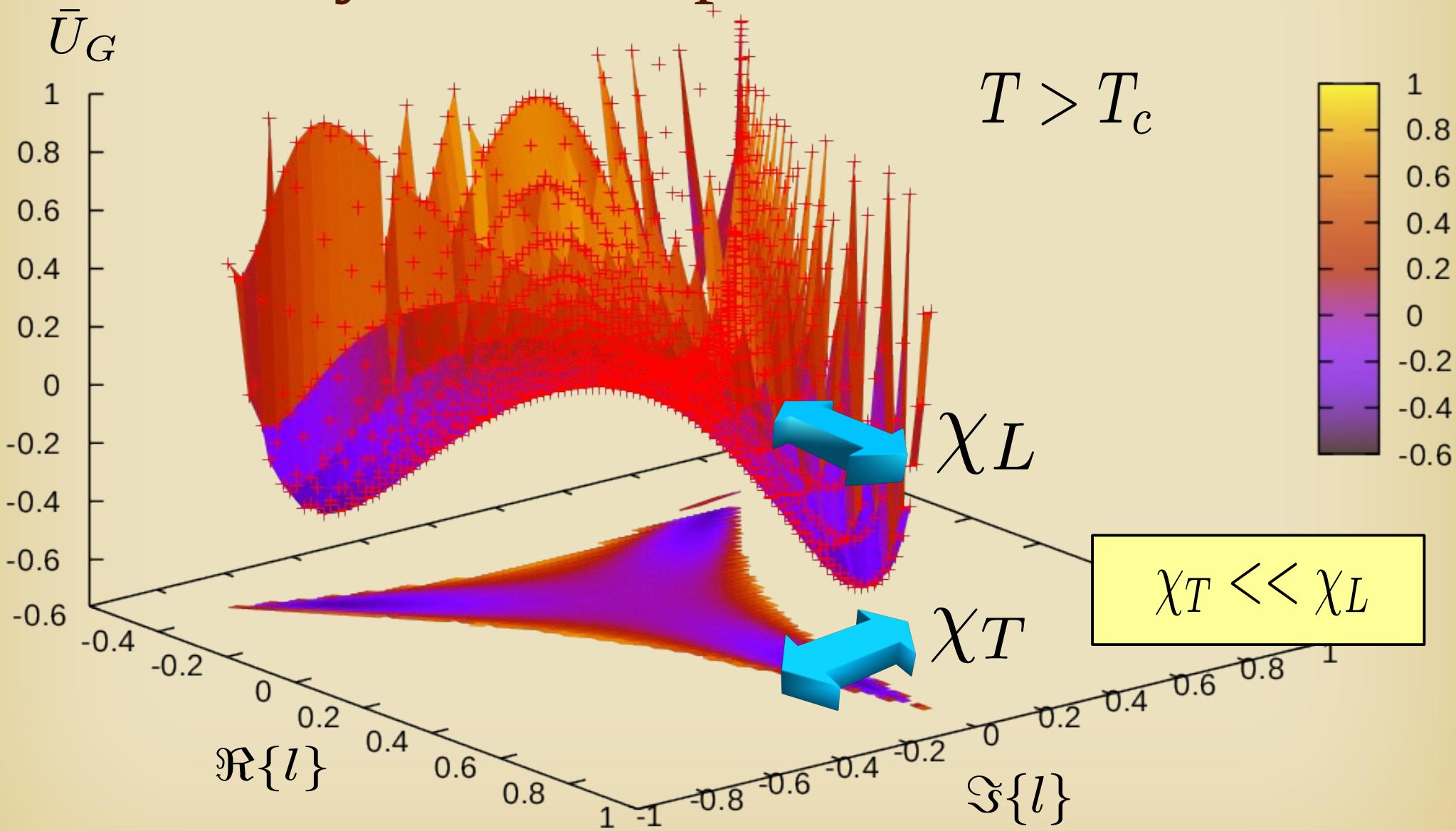
$$\chi_L = V(\langle L_L L_L \rangle - \langle L_L \rangle^2)$$

$$\chi_T = V(\langle L_T L_T \rangle - \langle L_T \rangle^2)$$

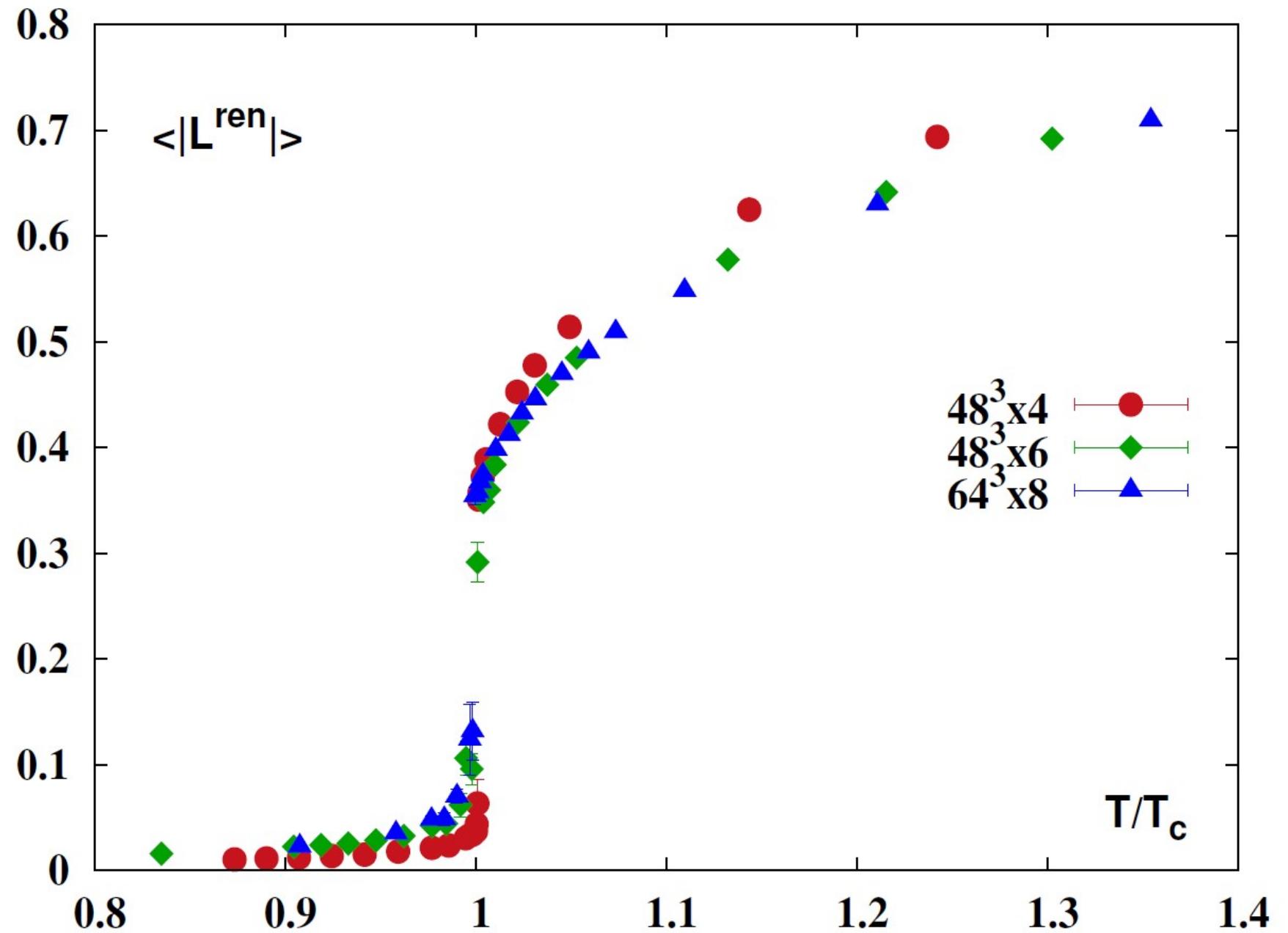
$$\chi_A = V(\langle |L| |L| \rangle - \langle |L| \rangle^2)$$

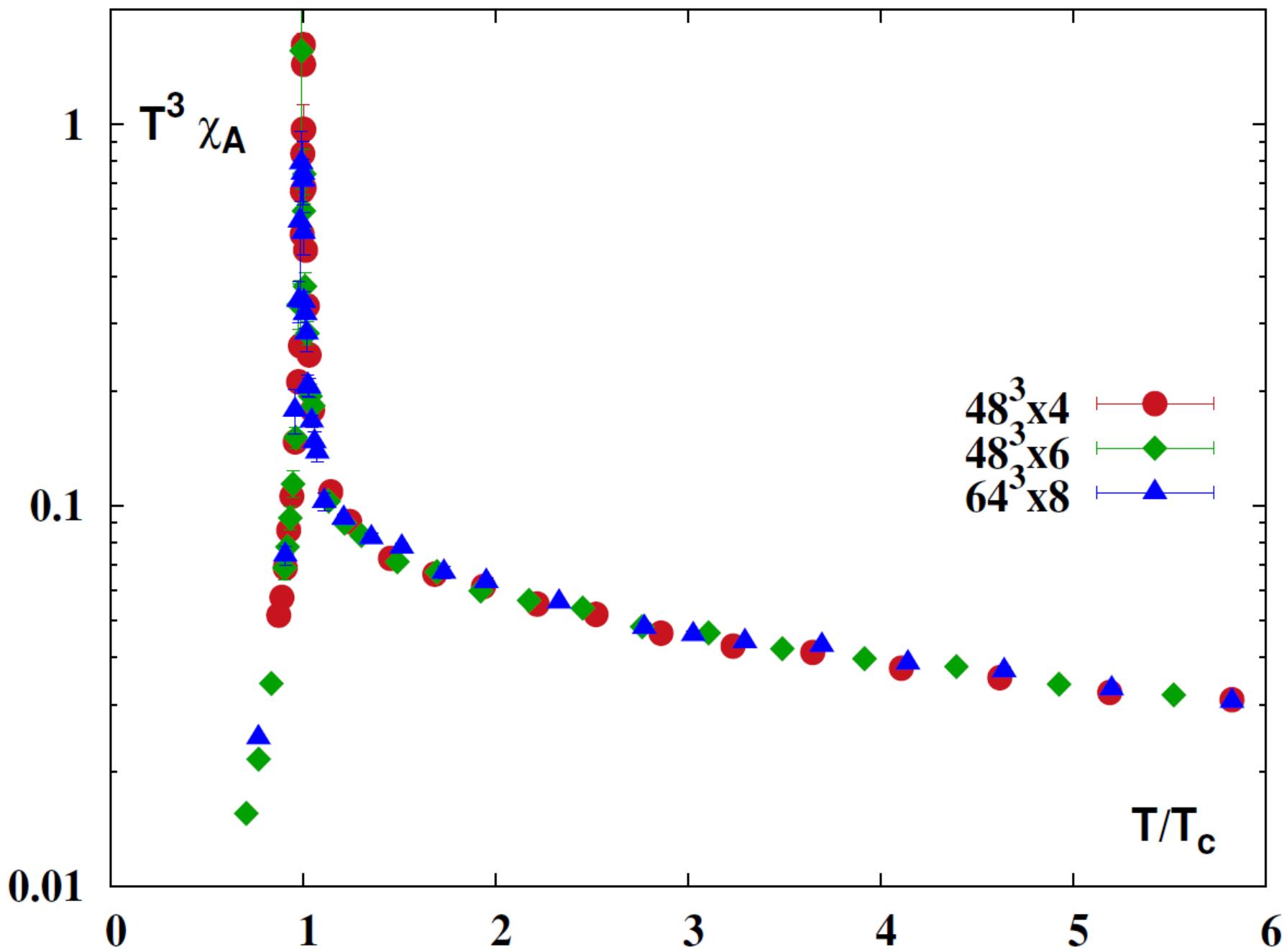
Note: $\chi_A \neq \chi_L + \chi_T$

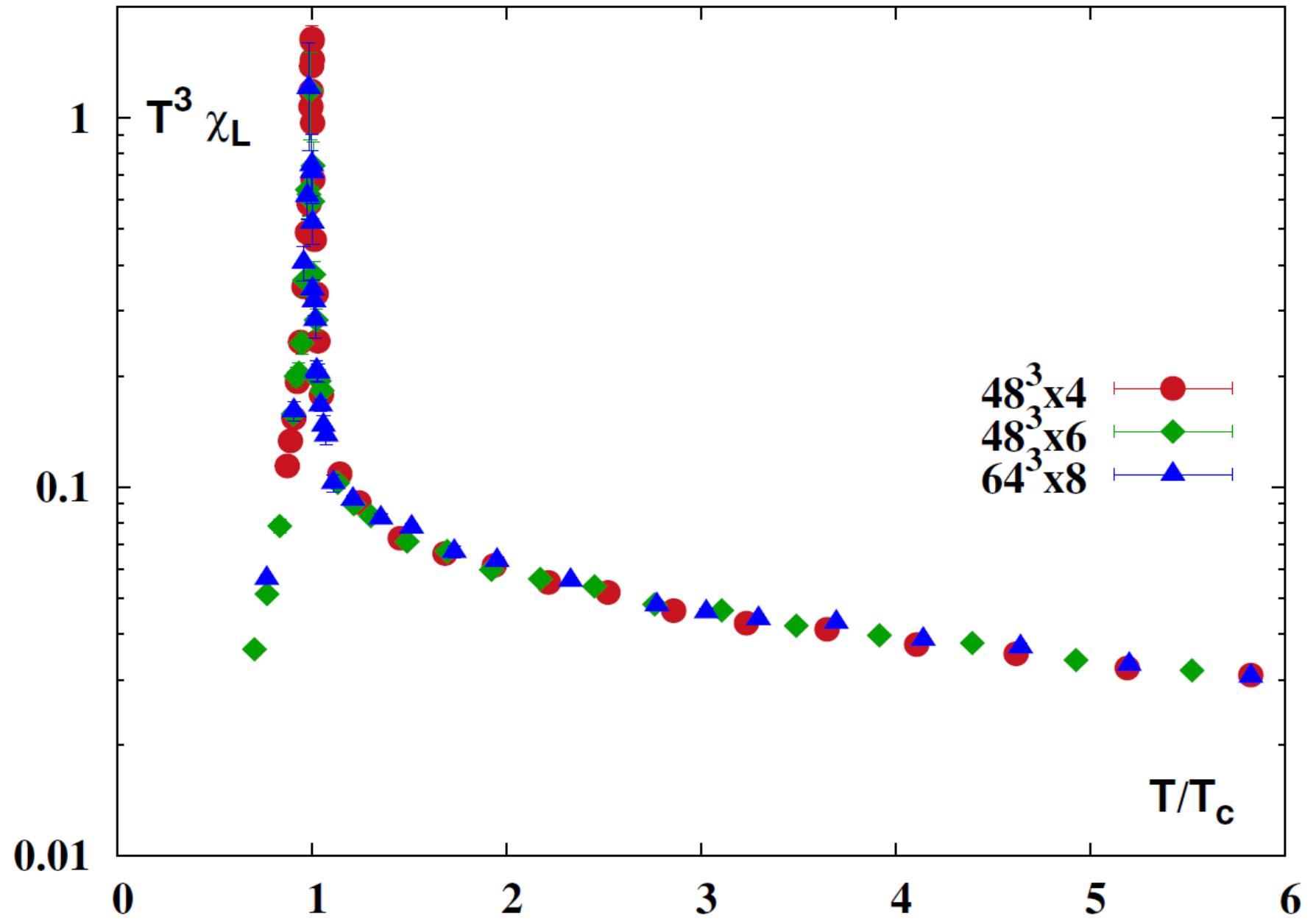
Polyakov Loop Eff. Potential

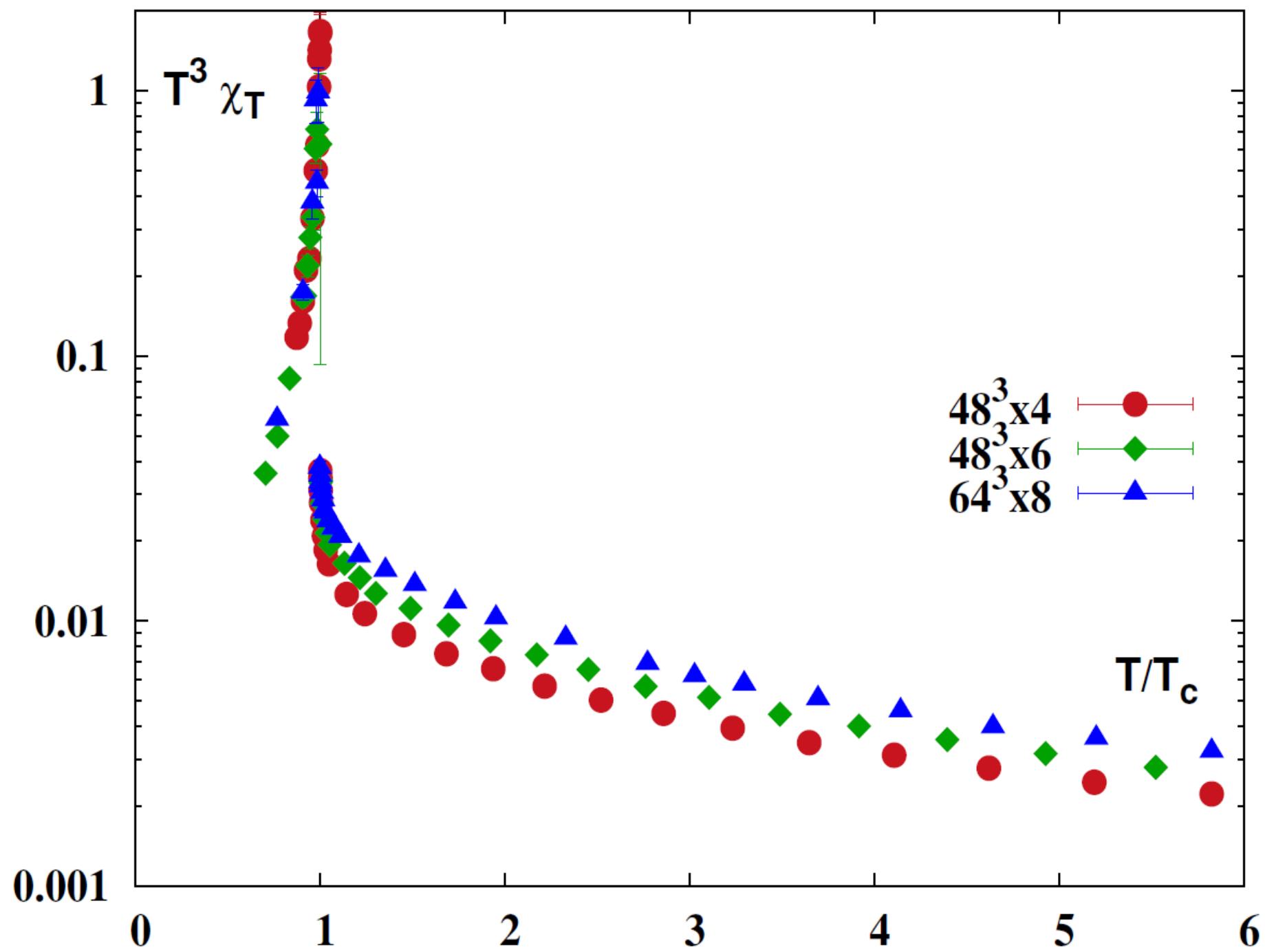


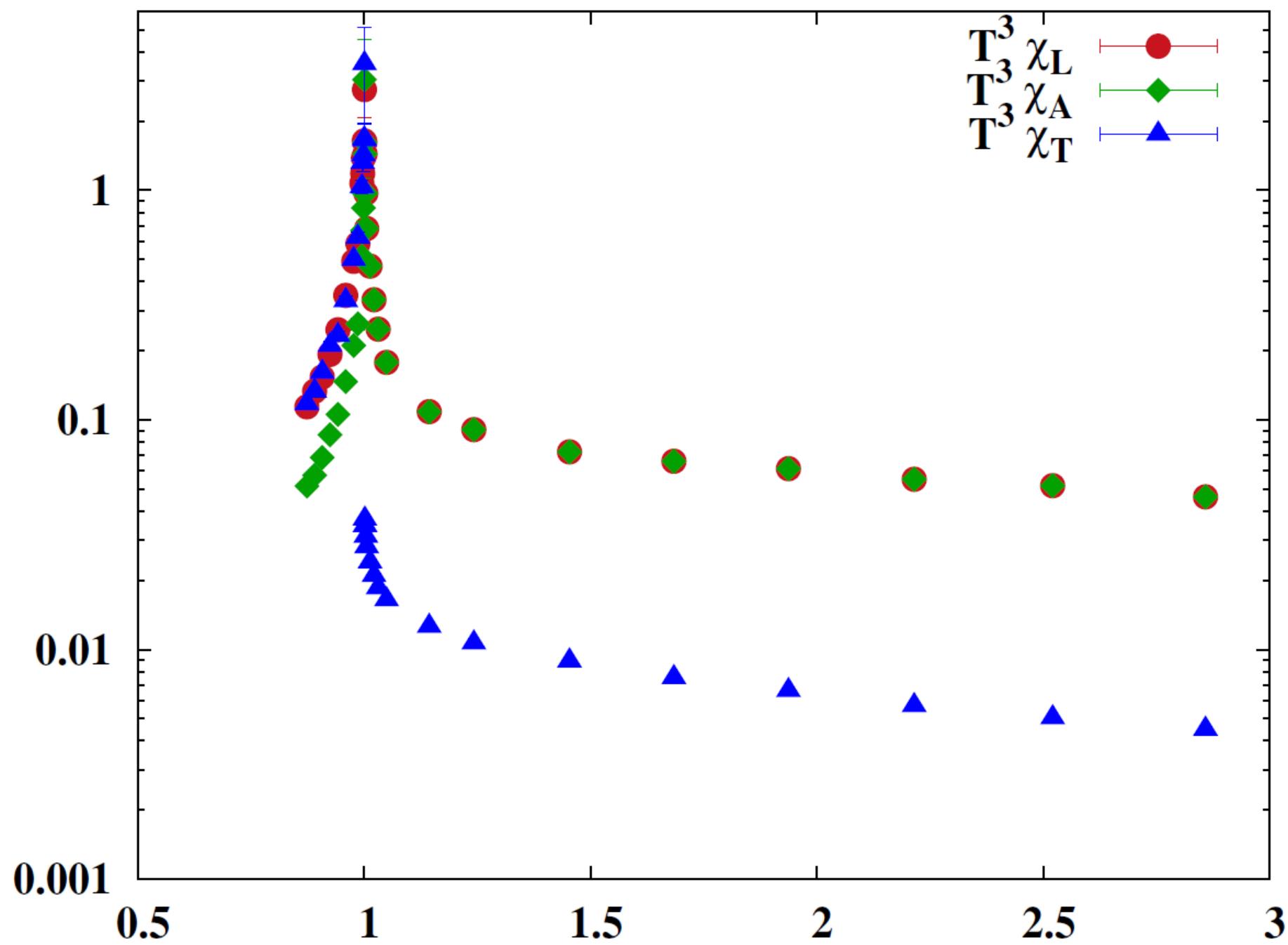
Lattice Results for SU(3) System

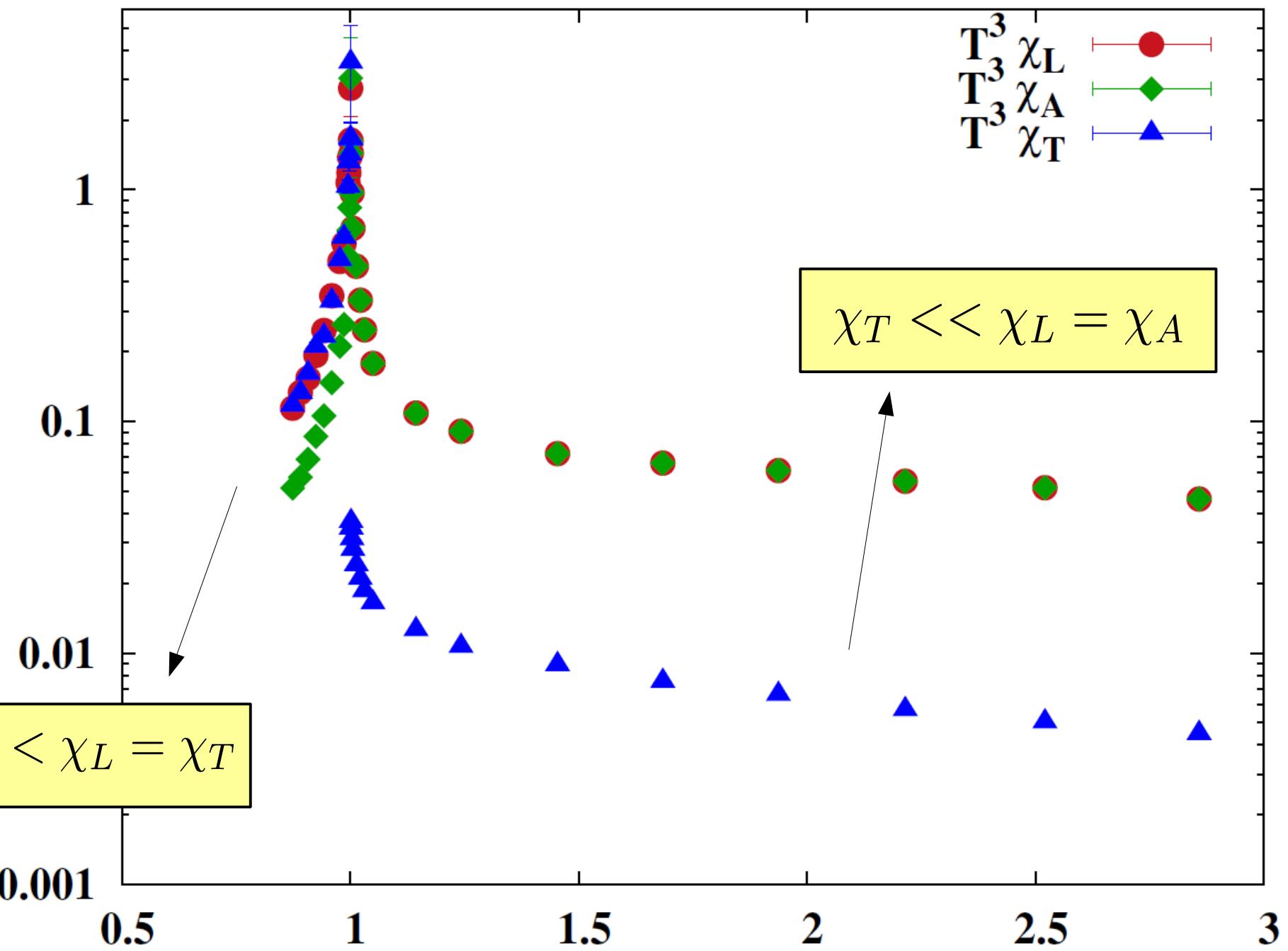












χ_A/χ_L

1

0.8

0.6

0.4

0.2

 χ_A/χ_L

0.8

0.9

1

1.1

1.2

1.3

1.4

 $48^3 \times 4$
 $48^3 \times 6$
 $64^3 \times 8$ 2- $\pi/2$ T/T_c

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

1

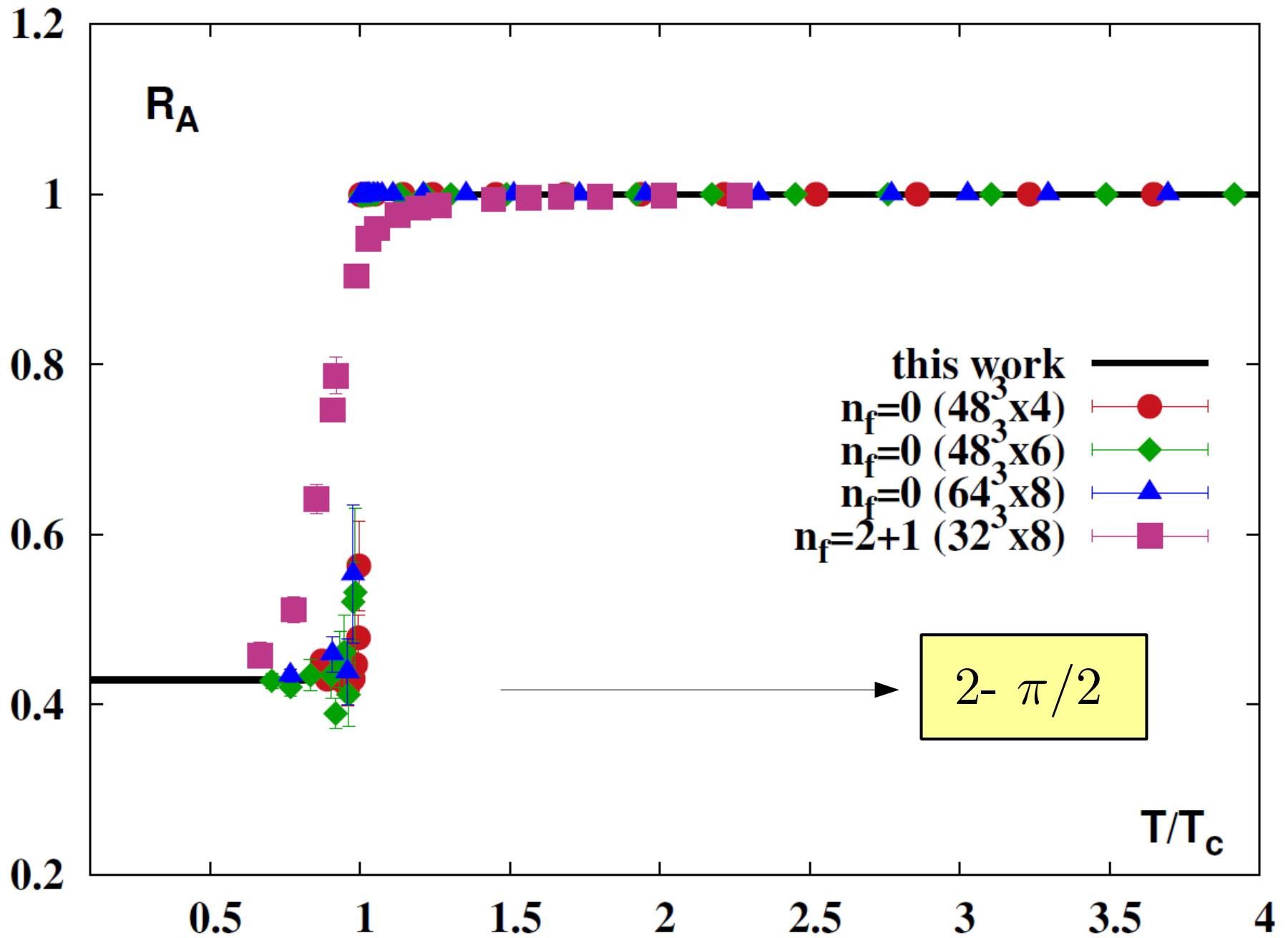
1

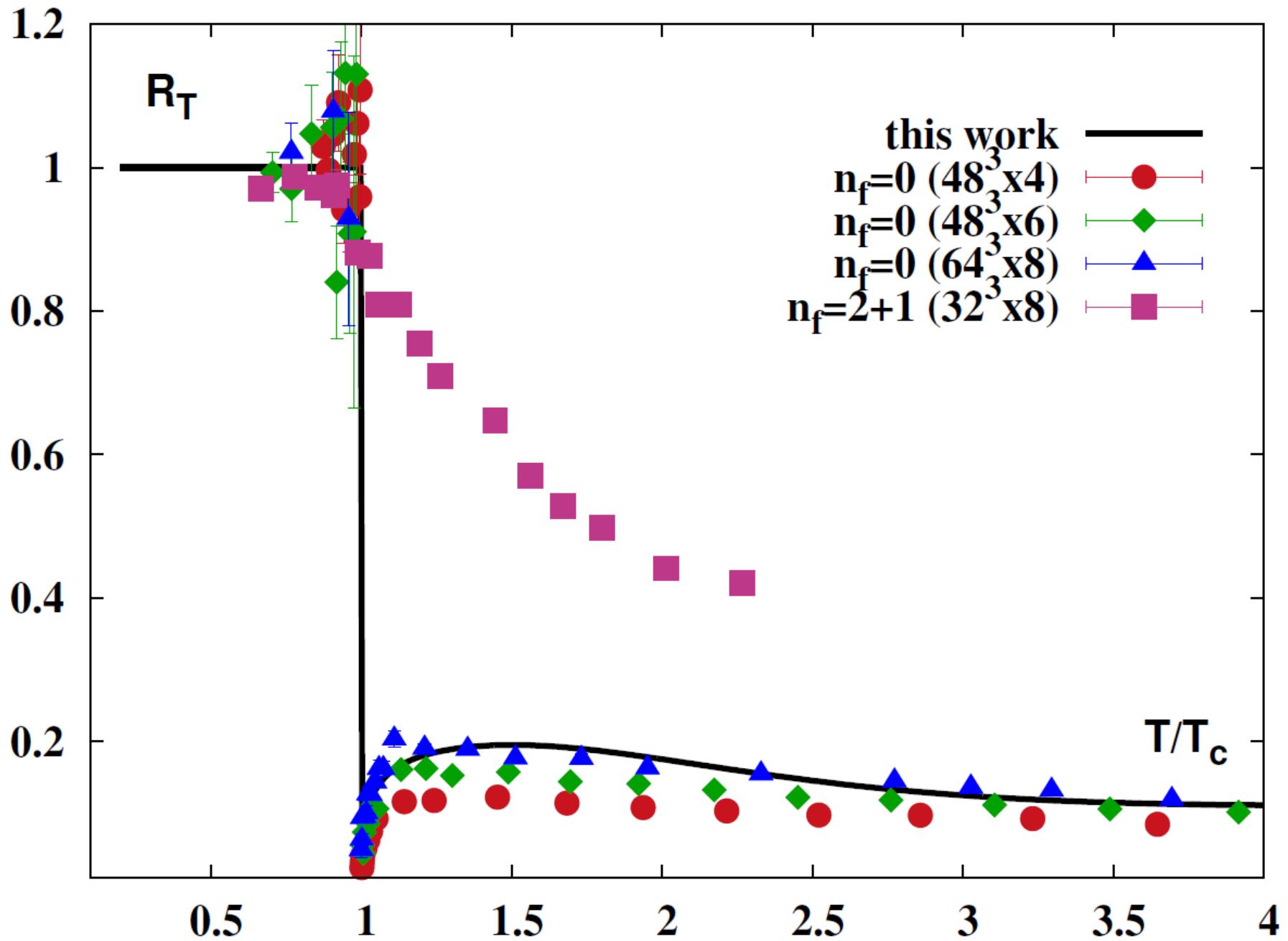
1

1

1

1





Gaussian distribution

- For 2D Gaussian distribution:

$$Z = \int dx dy e^{-VU[x,y]} \quad U[x, y] = x^2 + y^2$$

$$Z = \frac{\pi}{V}$$

$$\langle x \rangle = 0.$$

$$\langle y \rangle = 0.$$

$$\langle x^2 \rangle = \langle y^2 \rangle = \frac{1}{2V}$$

$$\hat{r} = \sqrt{\hat{x}^2 + \hat{y}^2}$$

$$\langle r^2 \rangle = \frac{1}{V}$$

$$\langle r \rangle = \frac{\sqrt{\pi}}{2} \frac{1}{\sqrt{V}}$$

Gaussian distribution

$$\chi_x = V \langle x^2 \rangle_c = V(\langle x^2 \rangle - \langle x \rangle^2)$$

$$\chi_y = V \langle y^2 \rangle_c = V(\langle y^2 \rangle - \langle y \rangle^2)$$

$$\chi_{abs} = V \langle r^2 \rangle_c = V(\langle r^2 \rangle - \langle r \rangle^2).$$



No symmetry breaking

$$\chi_x = 0.5$$

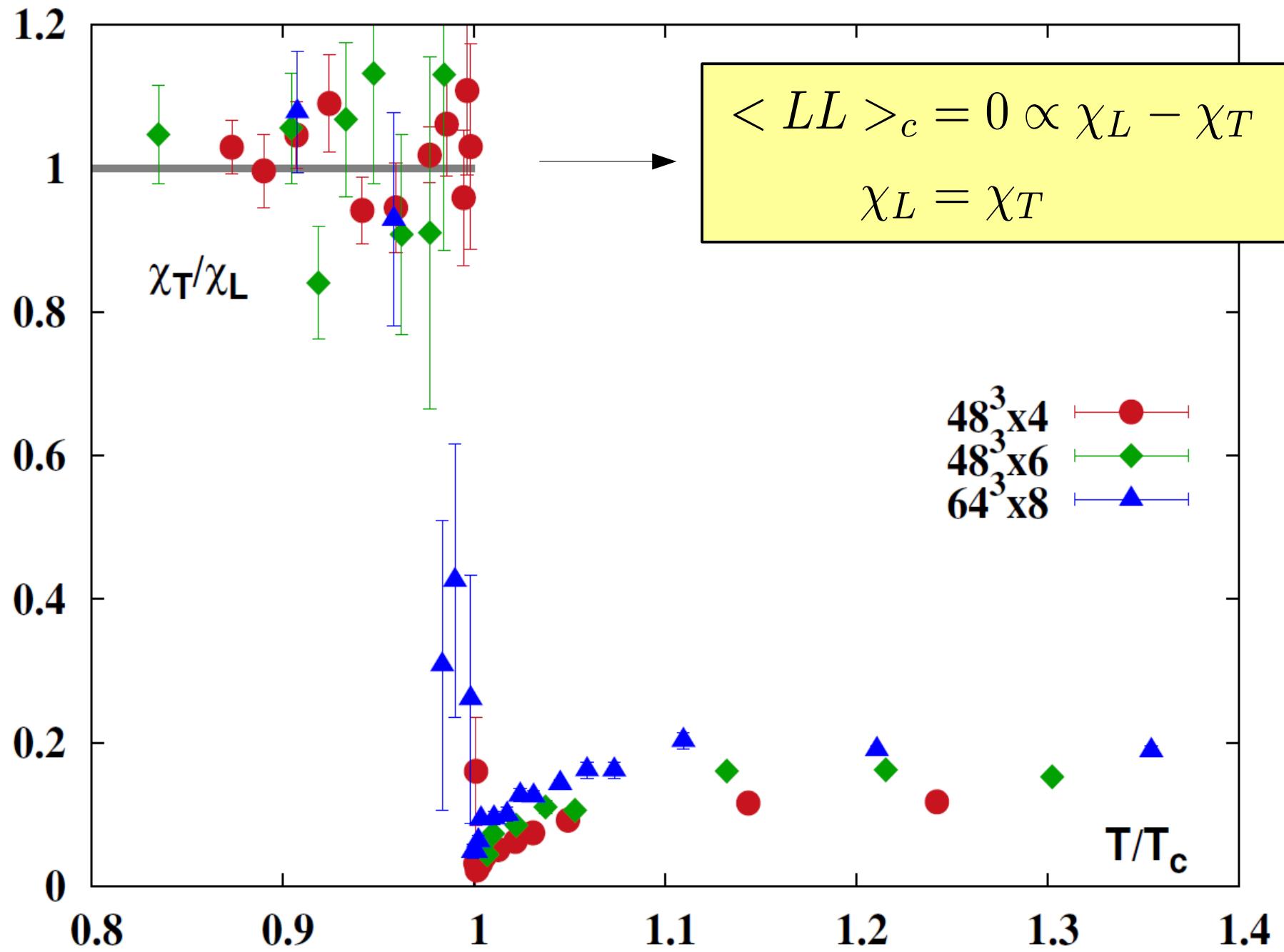
$$\chi_y = 0.5 = \chi_x$$

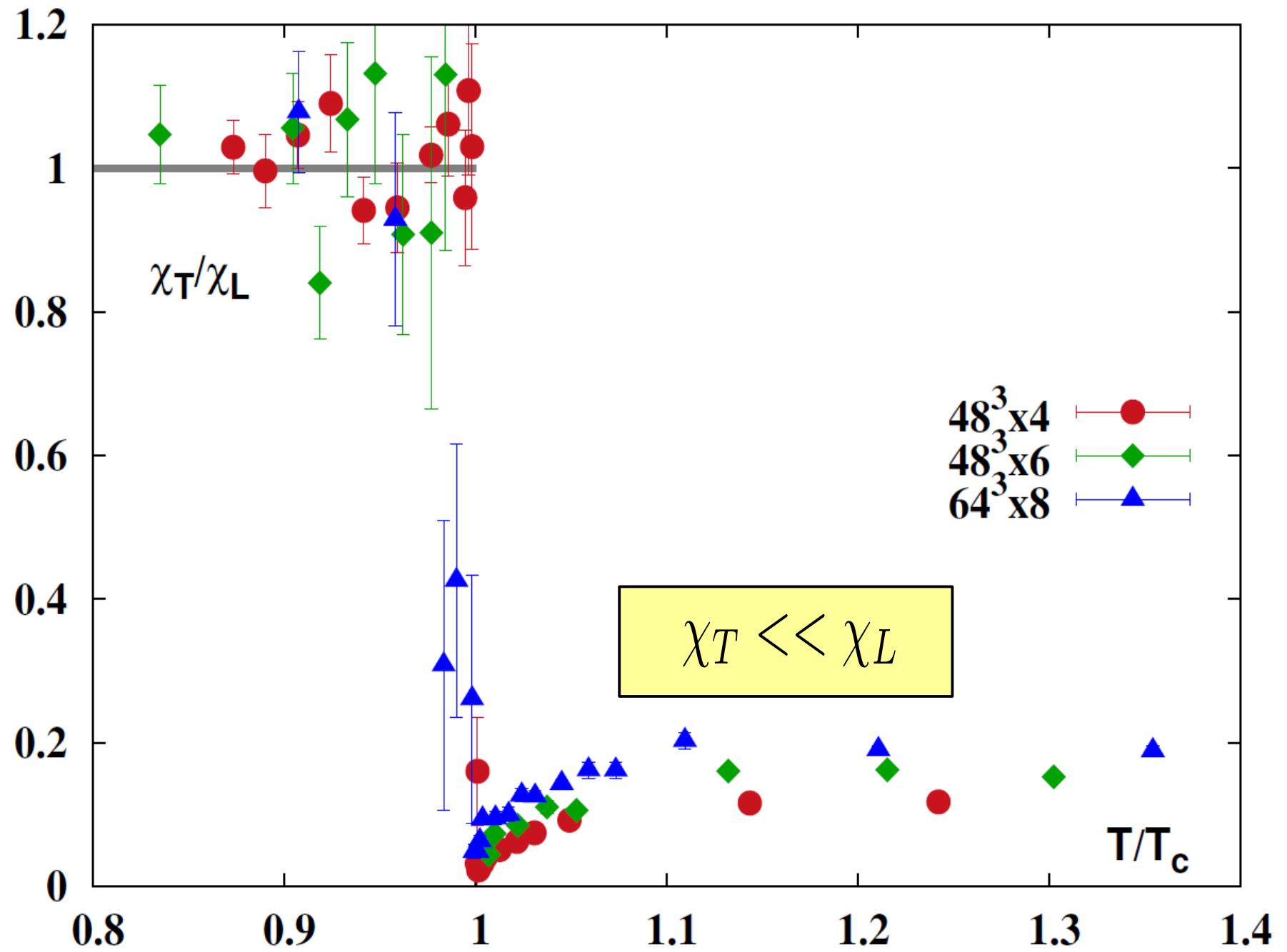
$$\chi_{abs} = 1. - \frac{\pi}{4} = \boxed{\chi_x (2. - \frac{\pi}{2})}.$$

The case for SU(2)

- For 2 color , PL is **real**, the Gaussian result:

$$\chi_A/\chi_L = 1 - 2/\pi$$

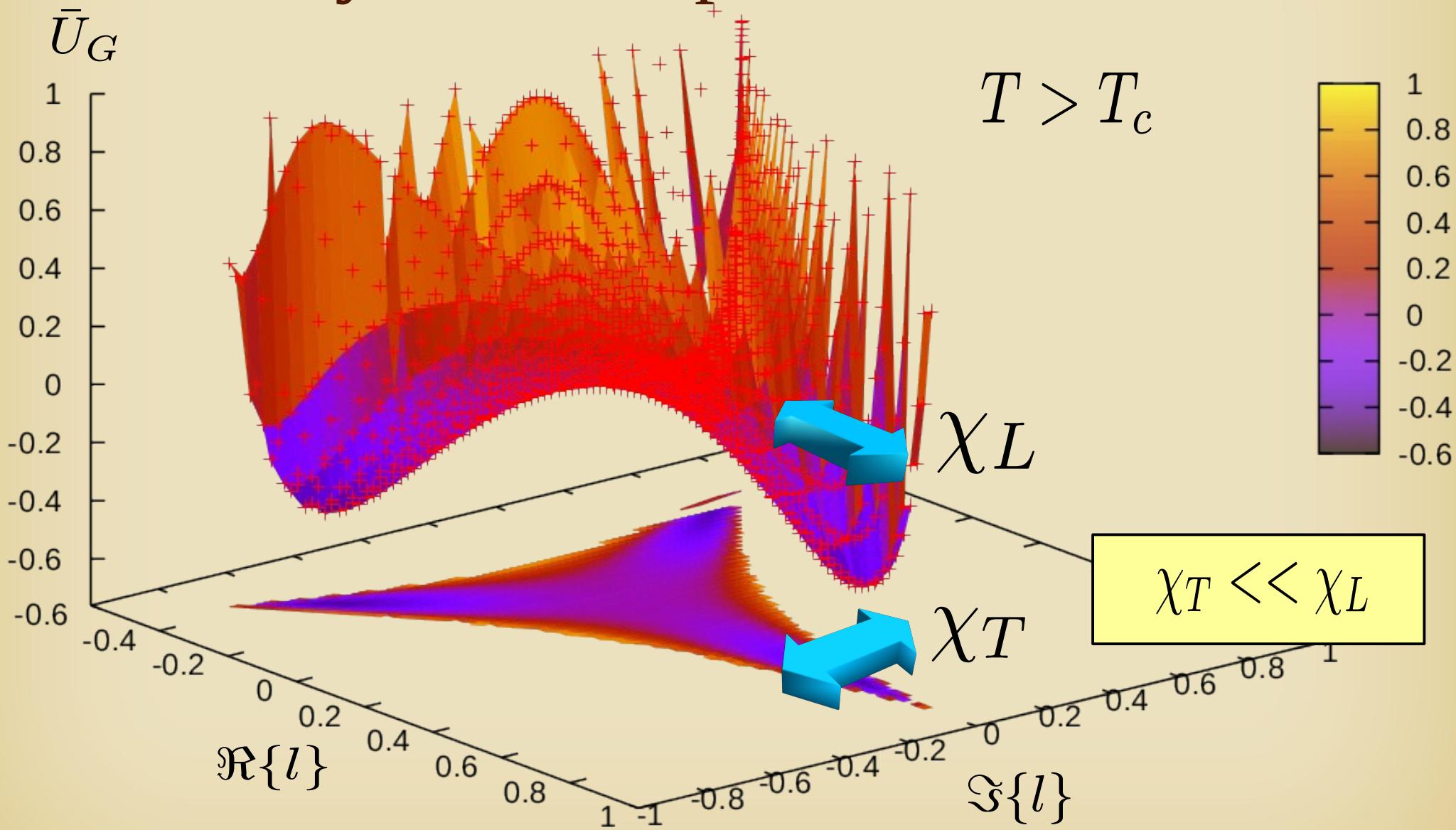




Implications to effective models

- $\chi_T/\chi_L \ll 1$ above T_c
 - Polynomial potential
 $\chi_T/\chi_L > 1$
 - Haar measure -type potential
 $\chi_T/\chi_L < 1$

Polyakov Loop Eff. Potential



Sigma model

$$U_{eff} = -\mu^2(\sigma^2 + \pi^2) + \lambda(\sigma^2 + \pi^2)^2$$

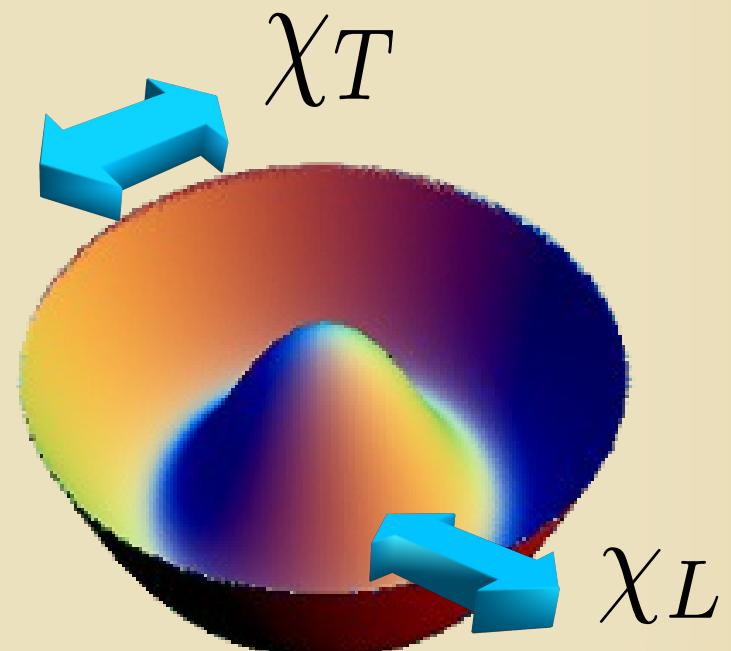


$$\langle \sigma \rangle = \sqrt{\frac{\mu^2}{2\lambda}}$$

$$\langle \pi \rangle = 0.$$

$$\chi_\sigma = \frac{1}{4\mu^2}$$

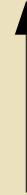
$$\chi_\pi = \frac{1}{0.} \gg \chi_\sigma$$



$$\chi_T \gg \chi_L$$

Construction of PL effective model

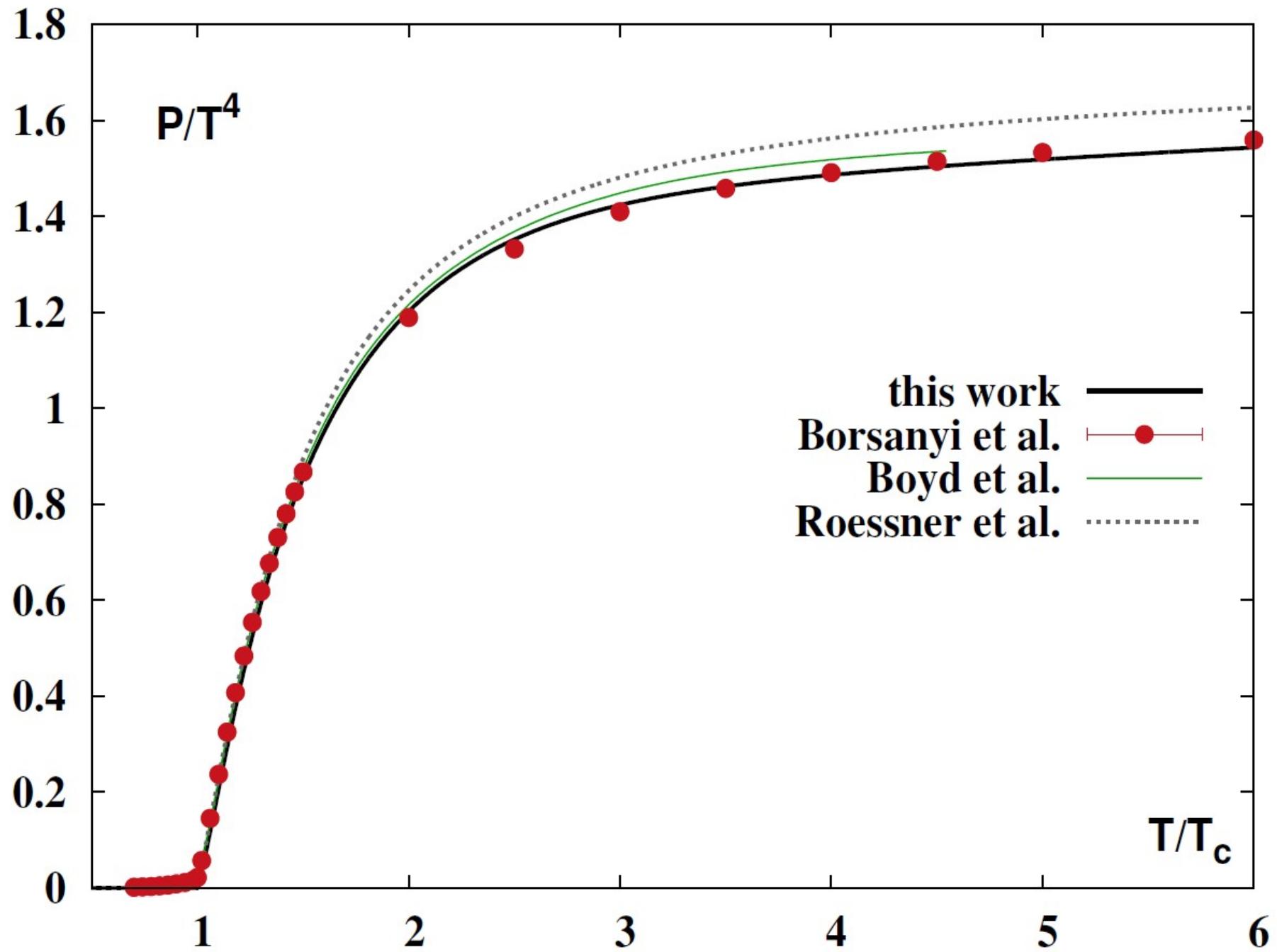
$$U_{eff}[L, \bar{L}] = -\frac{A[T]}{2} \bar{L}L + B[T] \ln M_{Haar} + \frac{1}{2} C[T] (L^3 + \bar{L}^3) + D[T] (\bar{L}L)^2$$

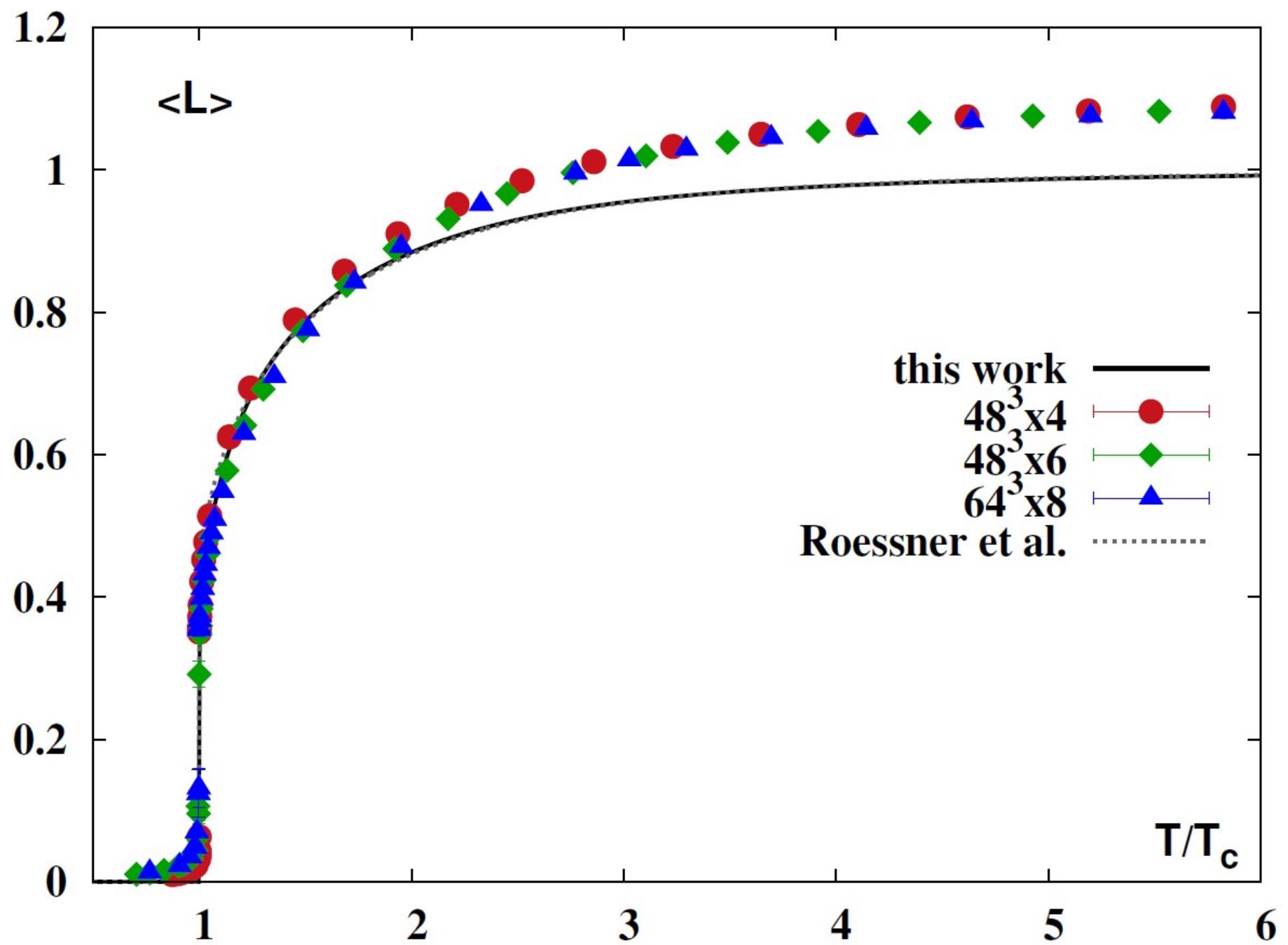


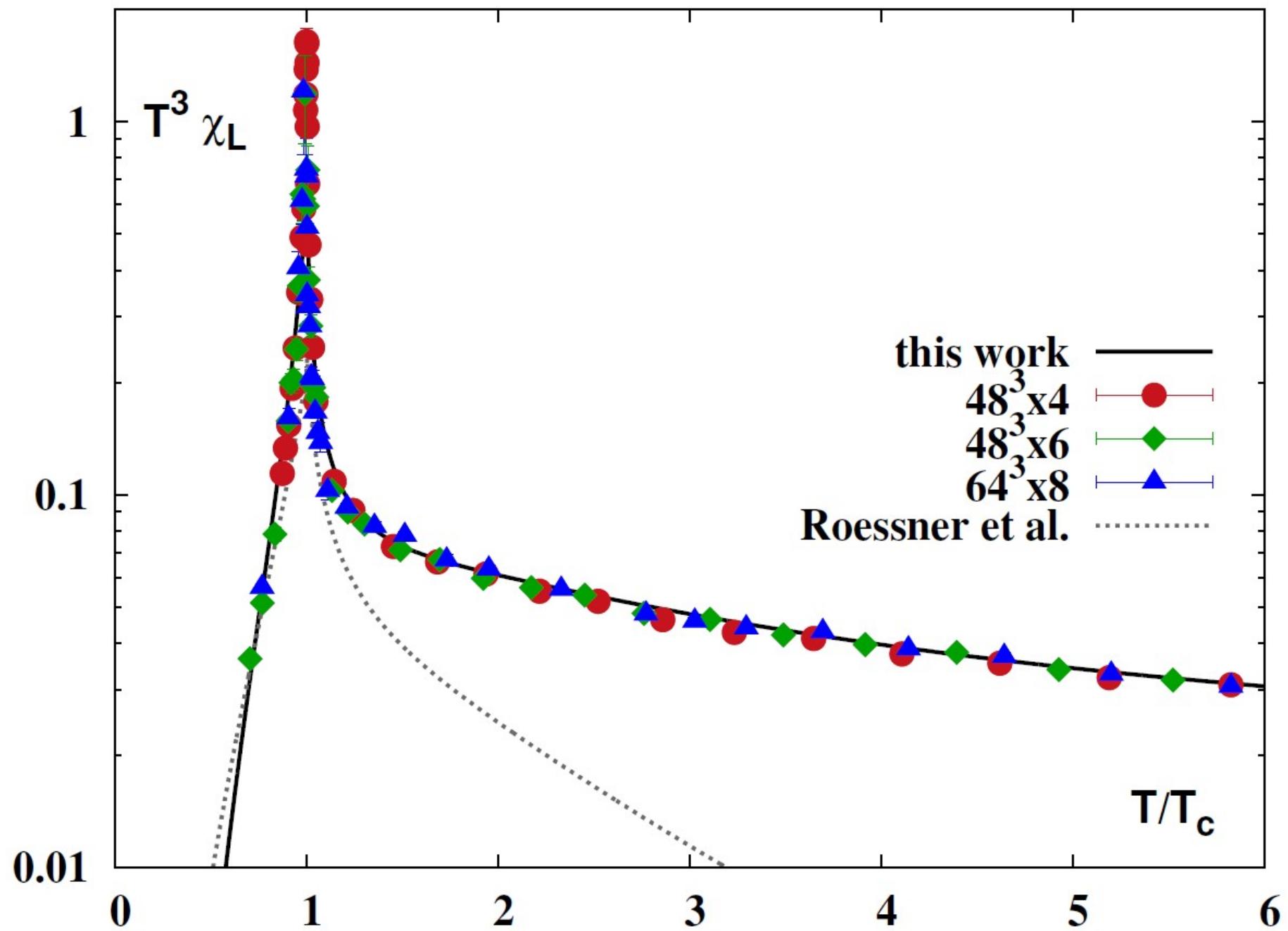
Lattice results

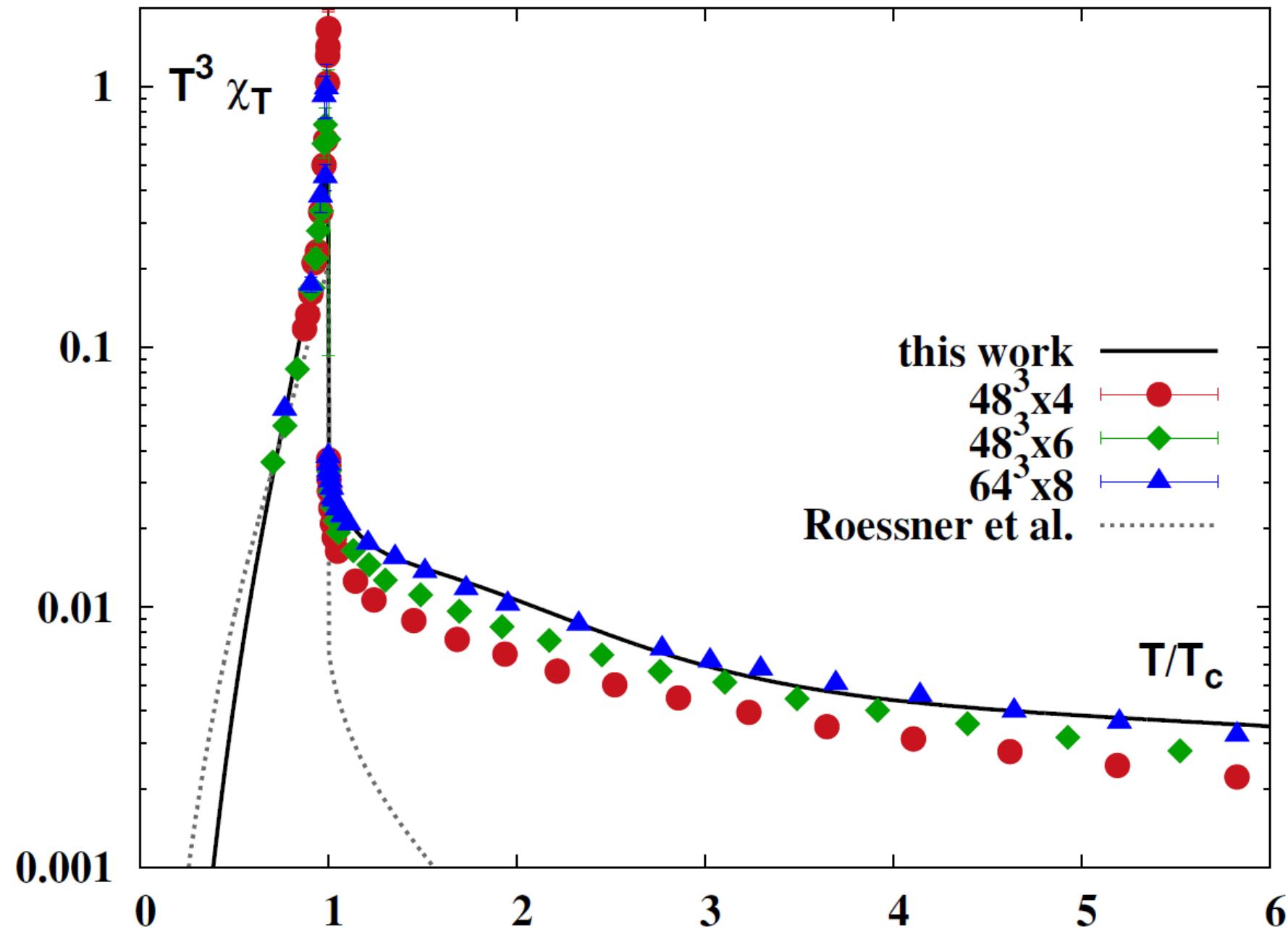
$P[T], \langle L \rangle, \chi_L, \chi_T$

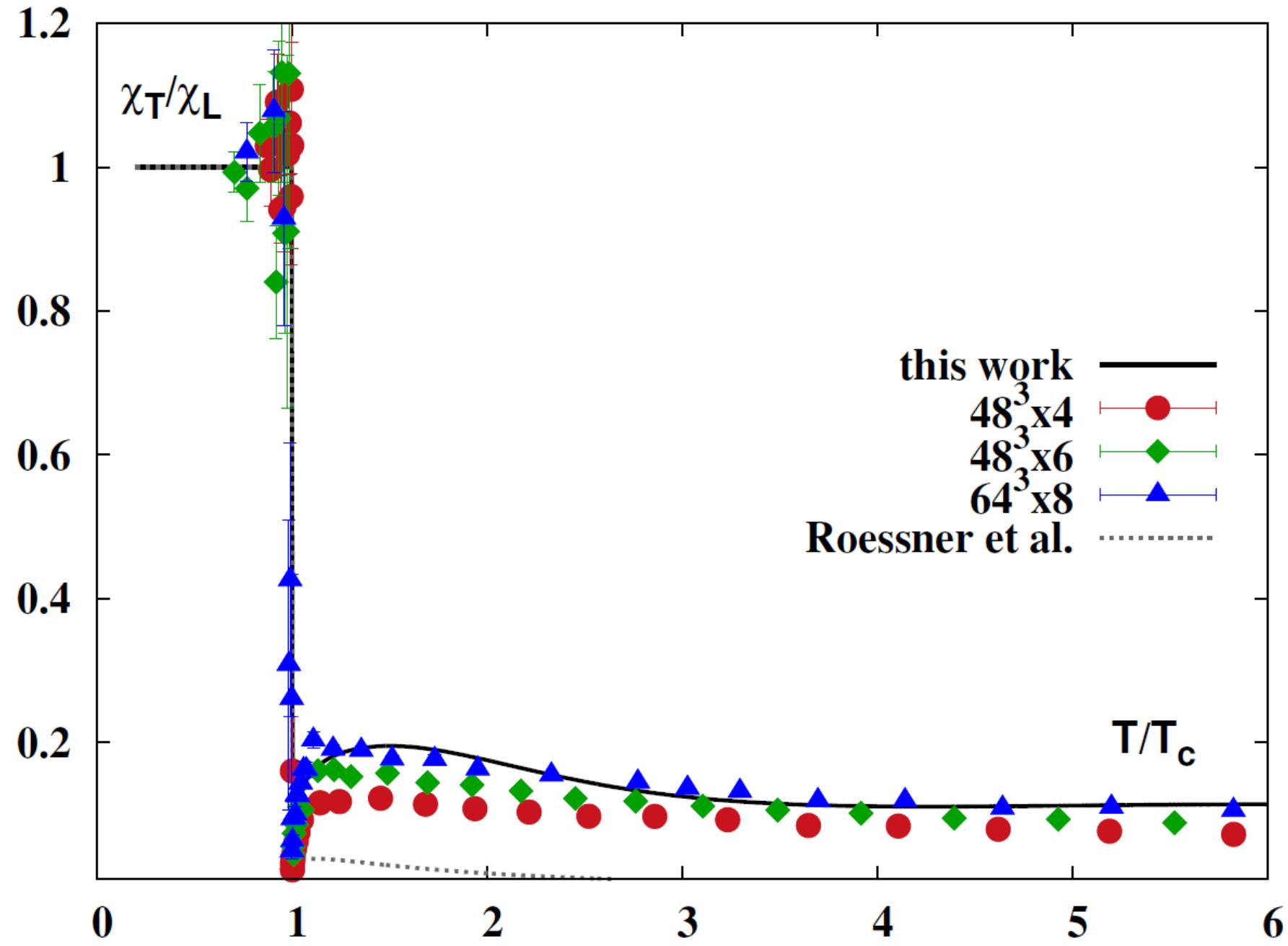
$Z(3)$ symmetry, Haar measure...





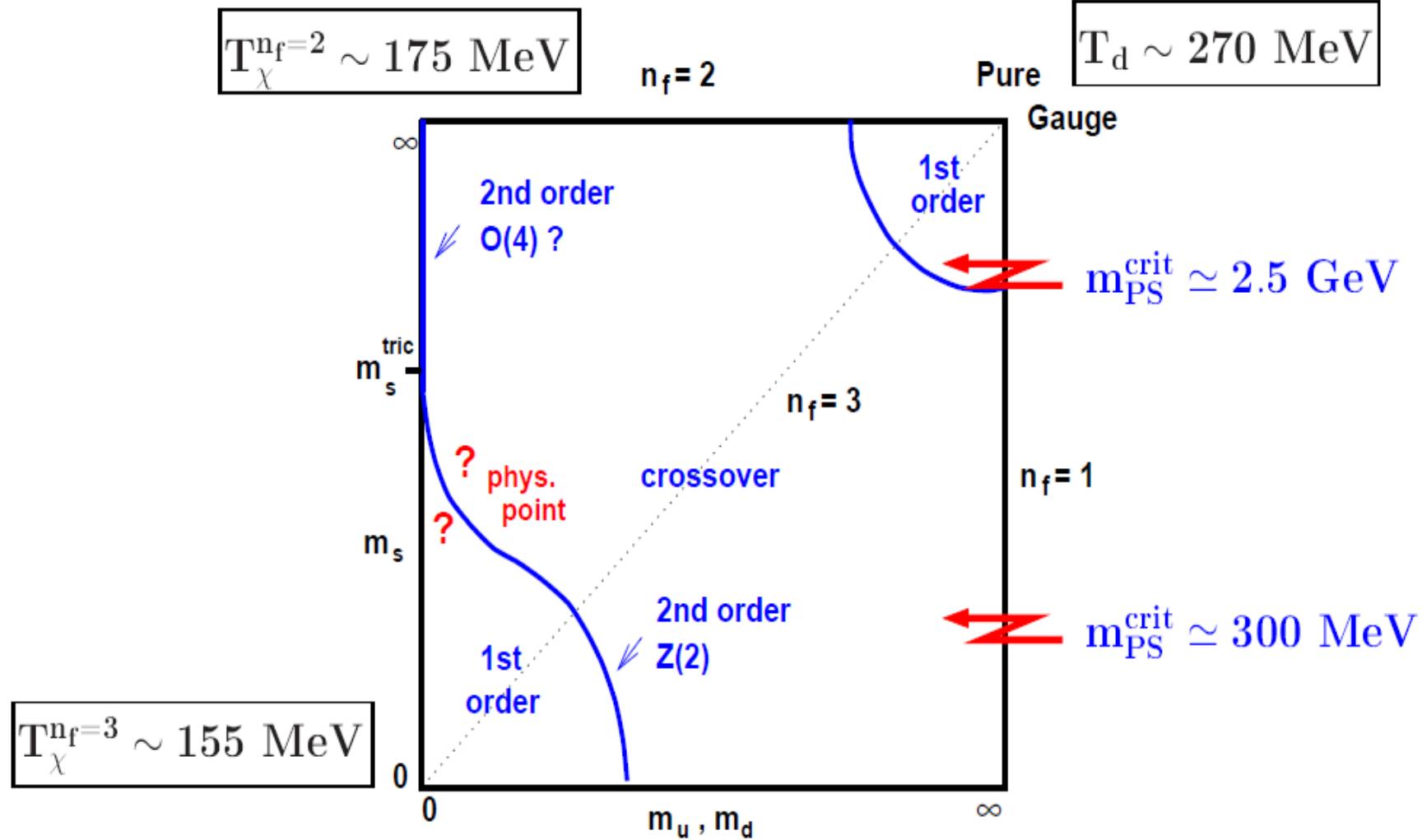






Explicit Z(3) Symmetry Breaking by Heavy Quarks

3-flavour phase diagram



F. Karsch, Lattice QCD at high temperature and density

PL-heavy quark coupling

- Fermionic determinant

$$Z = \int DLD\bar{L} \det[\hat{Q}_F] e^{-\beta VU_G[L, \bar{L}]}.$$

- Background field approach

$$\hat{Q}_F = (-\partial_\tau + \mu + igA_4)\gamma^0 + i\vec{\gamma} \cdot \nabla - M_Q$$

$$\hat{L} = e^{i\beta g A_4^a T^a}$$

$$L = \frac{1}{N_c} Tr_c \hat{L}$$

PL-heavy quark coupling

- Fermionic determinant (tree level)

$$\ln \det[\hat{Q}_F] = V2N_f \int \frac{d^3k}{(2\pi)^3} [3\beta E[k] + \ln g^+ \ln g^-]$$

$$g^\pm = 1 + 3\{L, \bar{L}\}e^{-\beta E^\pm} + 3\{\bar{L}, L\}e^{-2\beta E^\pm} + e^{-3\beta E^\pm}$$

$$E^\pm = E[k] \mp \mu$$

$$E[k] = (k^2 + M_Q^2)^{1/2}$$

PL-heavy quark coupling

- Effective potential

$$\ln \det[\hat{Q}_F] = -V T^3 \bar{U}_Q[L, \bar{L}; M_Q]$$

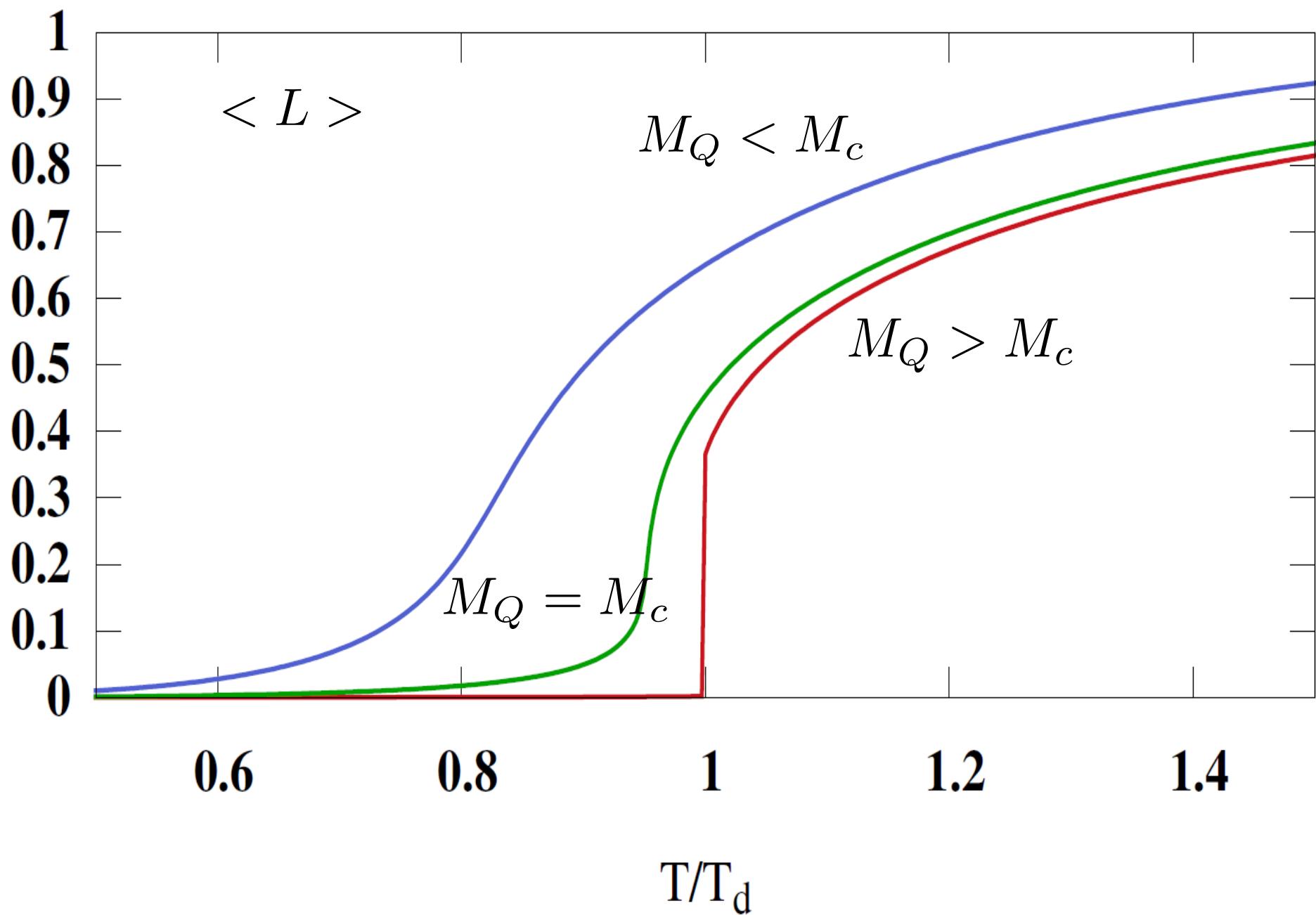
- Tree level result

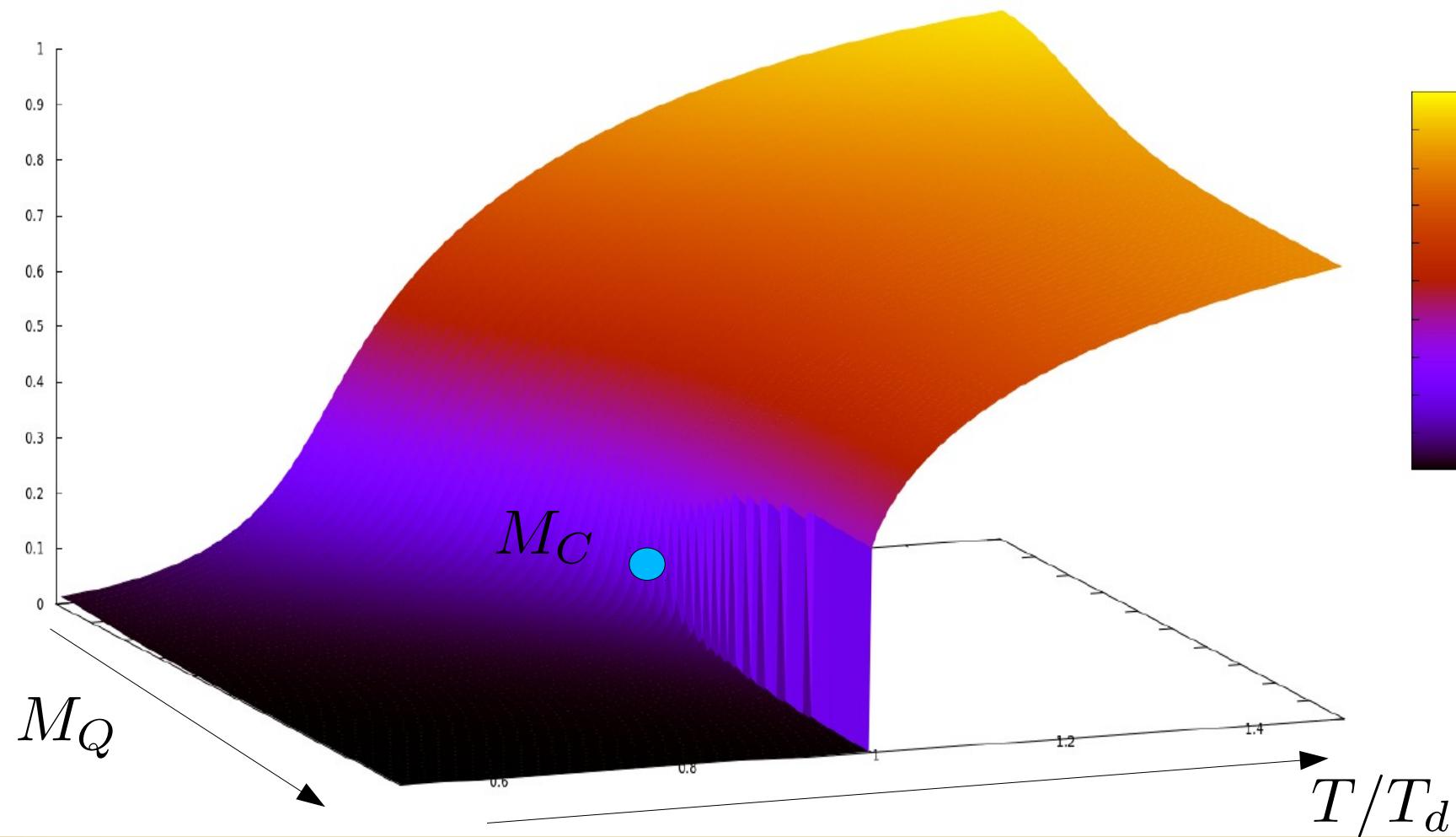
$$\bar{U}_Q[L, \bar{L}; M_Q] \approx -h_{\text{eff}}[M_Q/T] L_r + \dots$$

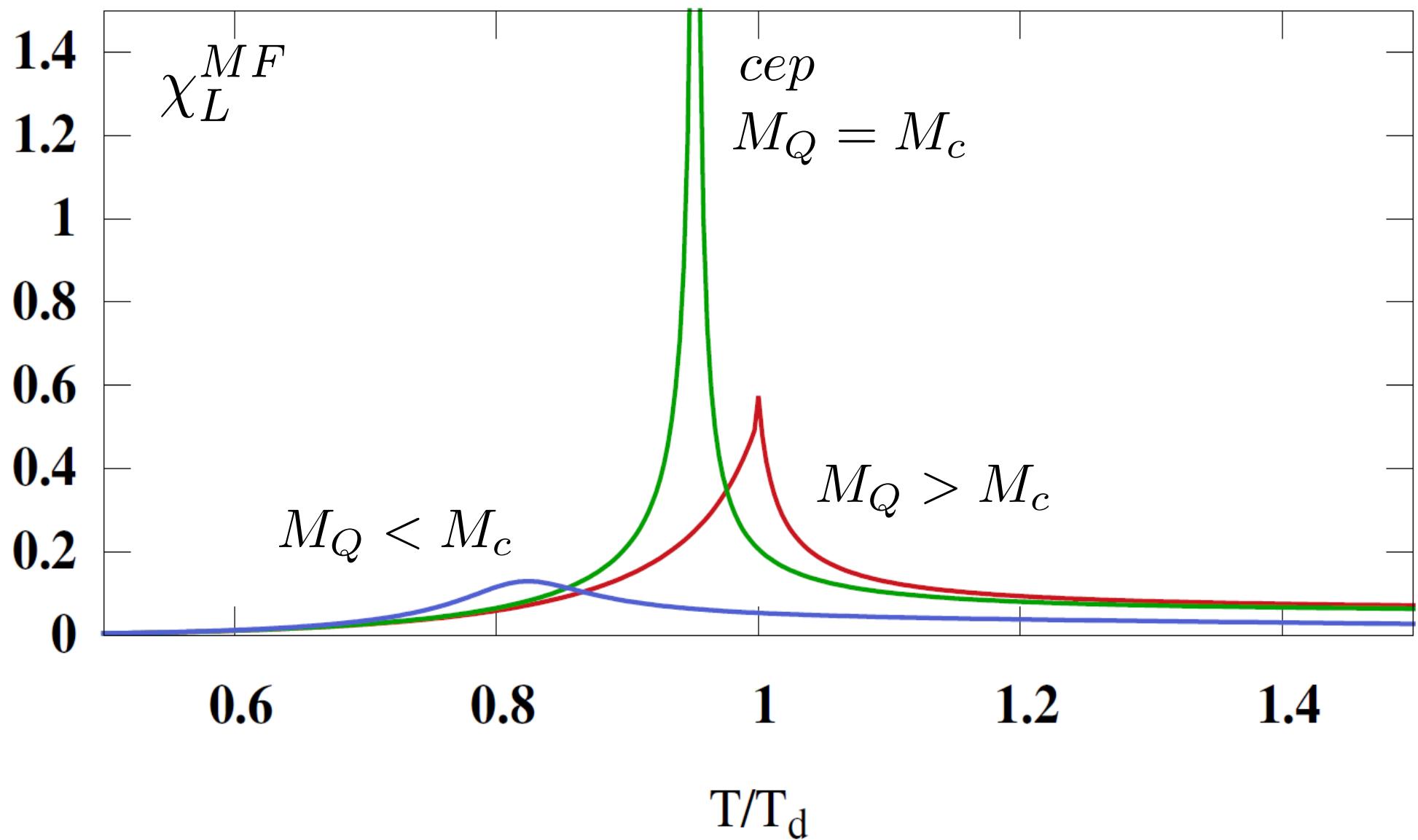
$$\bar{U}_G \rightarrow \bar{U}_G - h_{\text{eff}} L_r$$

$$h_{\text{eff}}[M_Q/T] \propto N_f e^{-M_Q/T}$$

$$M_Q/T \gg 1$$

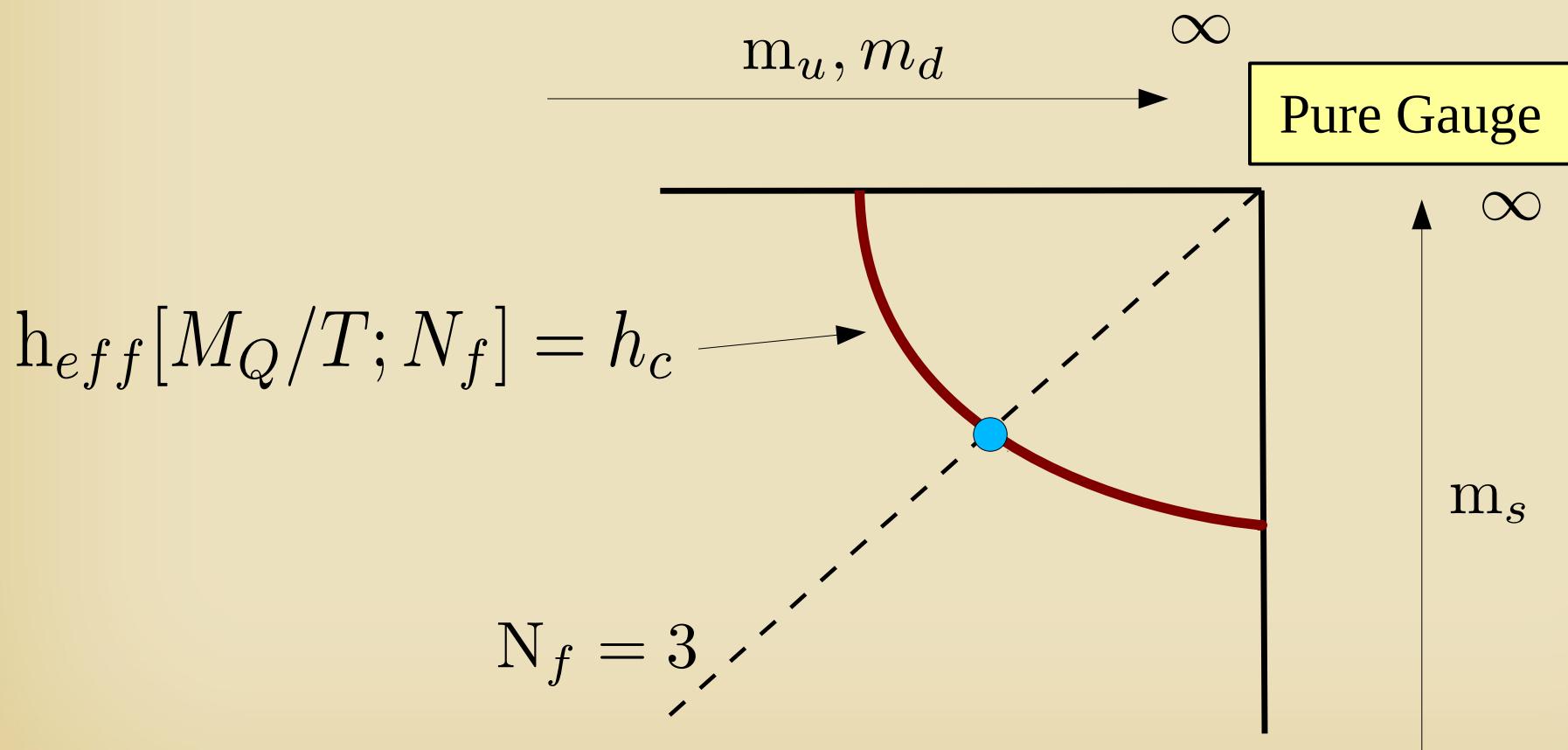


$< L > [T, M_Q]$ 



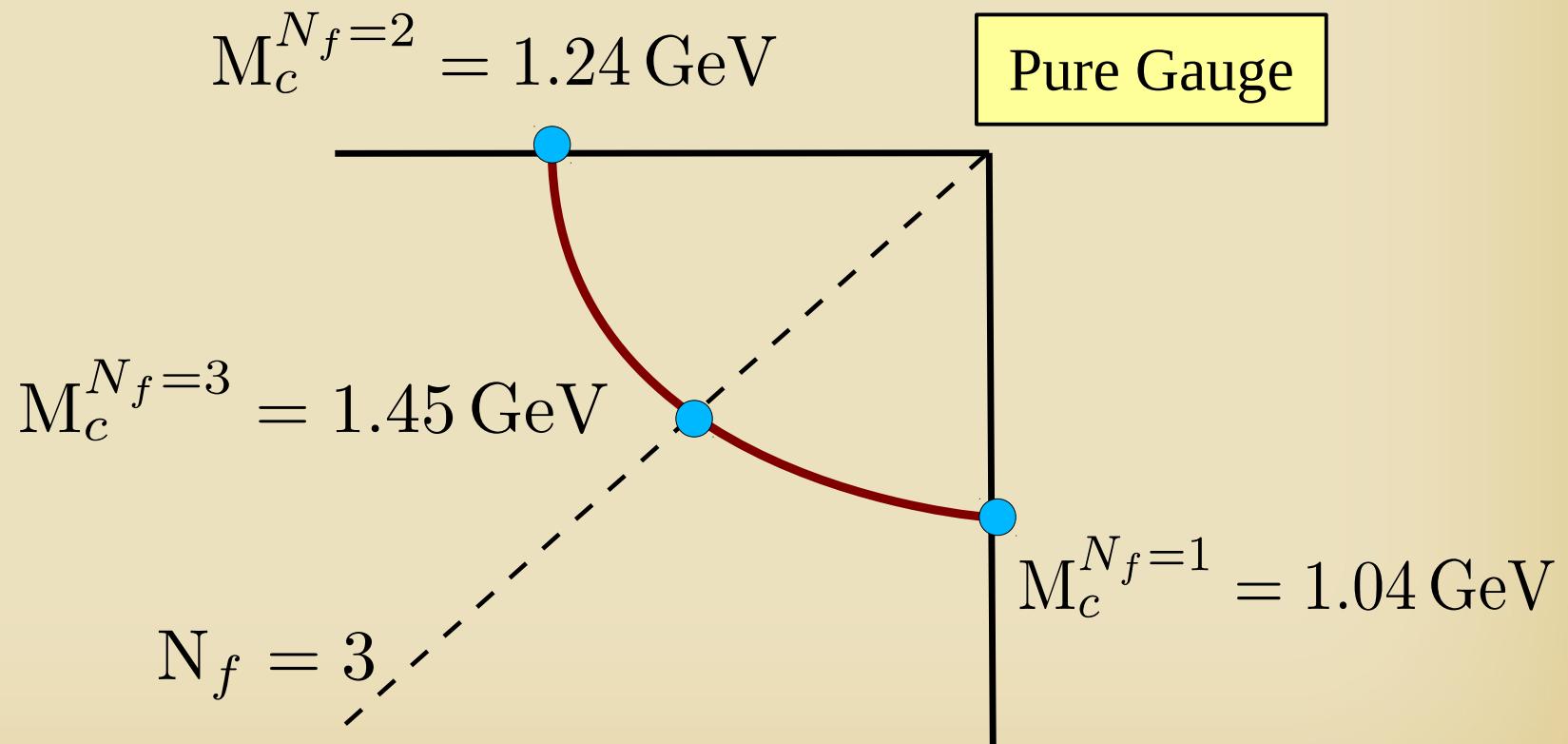
Critical quark mass

- Phase boundary of deconfinement phase transition



Critical quark mass

- Critical end point results from effective potential



Critical quark mass

- N_f dependence of critical quark mass

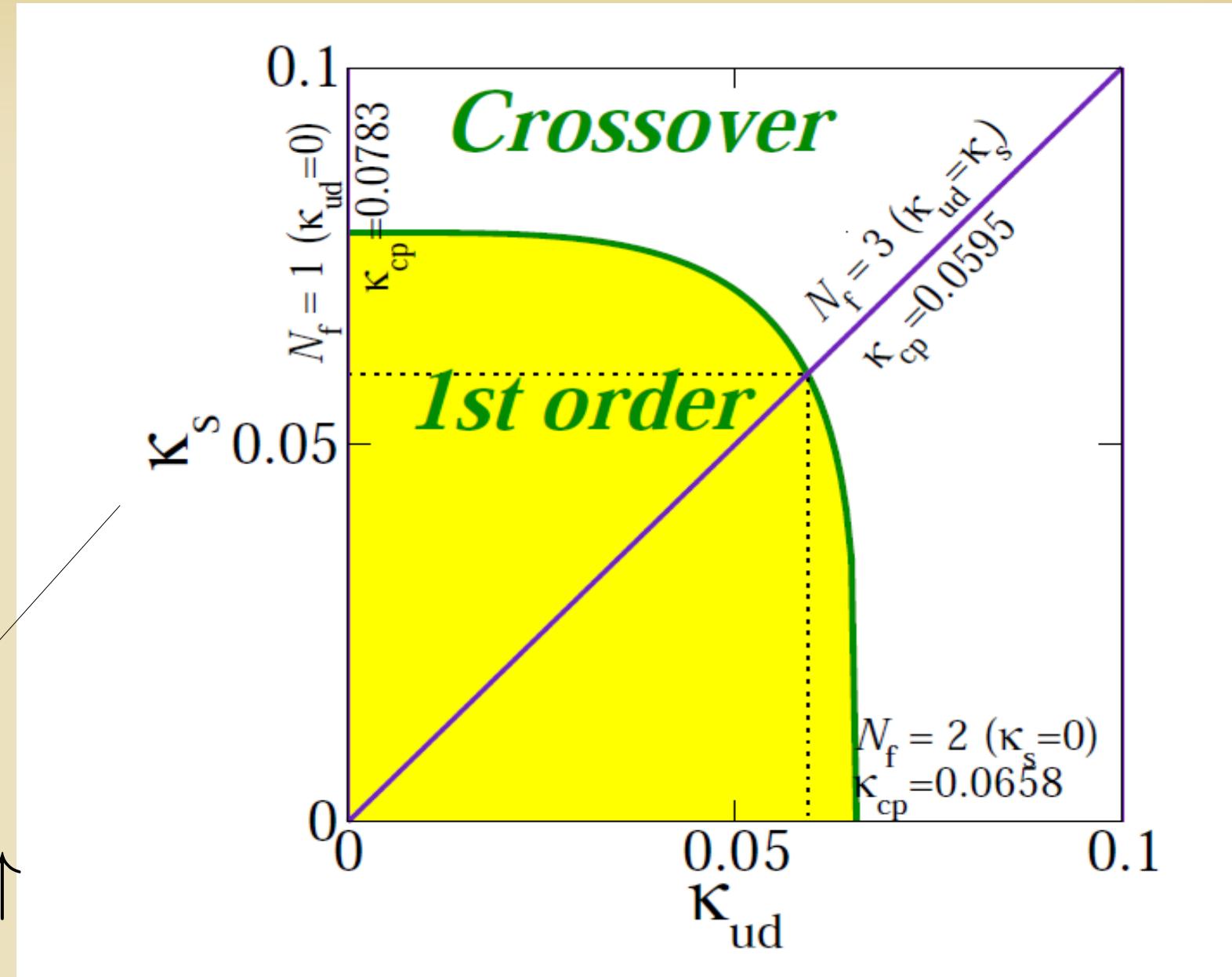
$$h_{eff}[M_Q/T; N_f] = h_c$$

$$h_{eff} \uparrow N_f \uparrow$$

$$h_{eff} \downarrow M_Q \uparrow$$

$$M_c^{N_f=3} > M_c^{N_f=2} > M_c^{N_f=1}$$

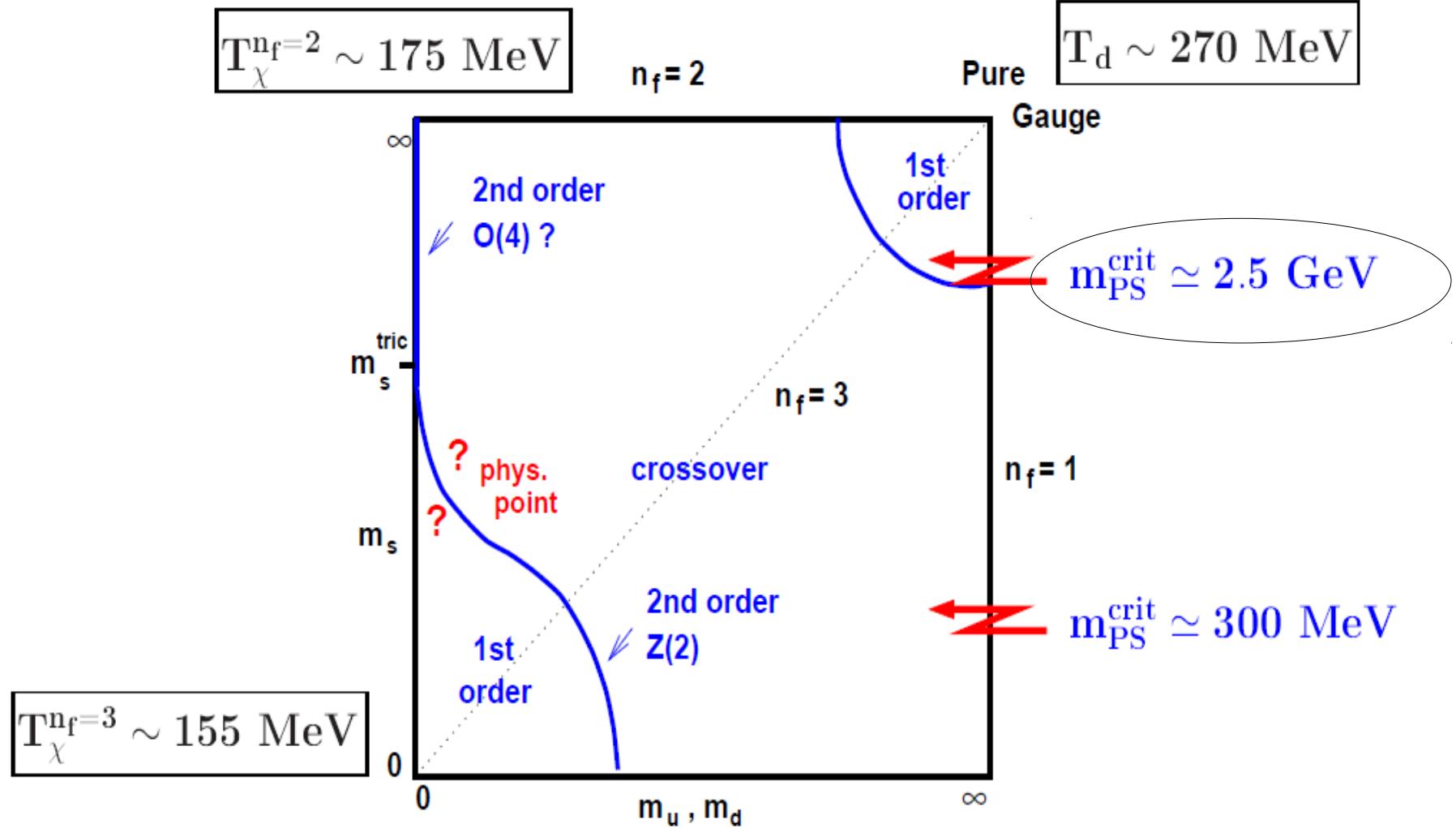
$\kappa \downarrow M_Q \uparrow$



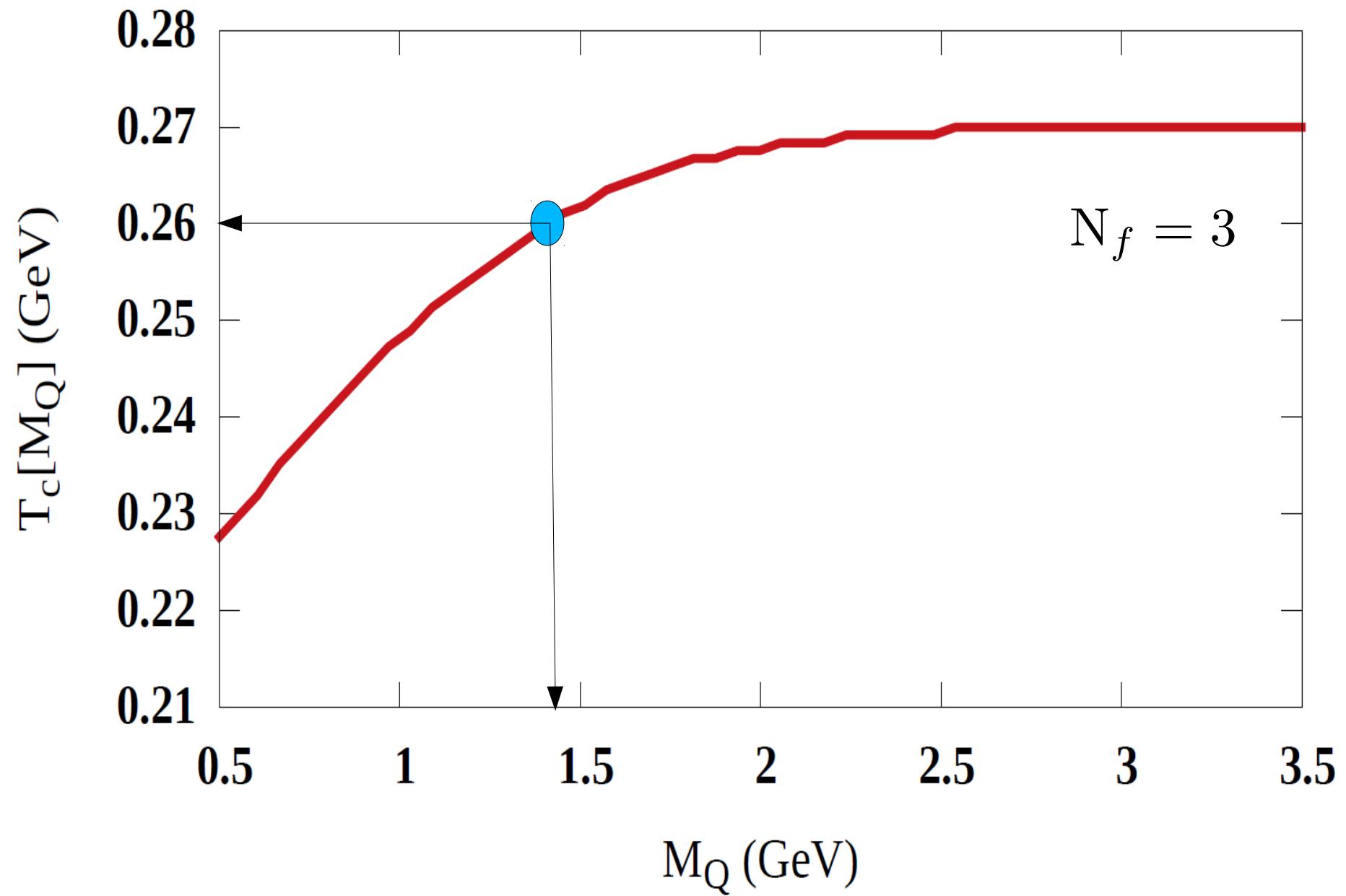
$$\kappa_c^{N_f=3} < \kappa_c^{N_f=2} < \kappa_c^{N_f=1}$$

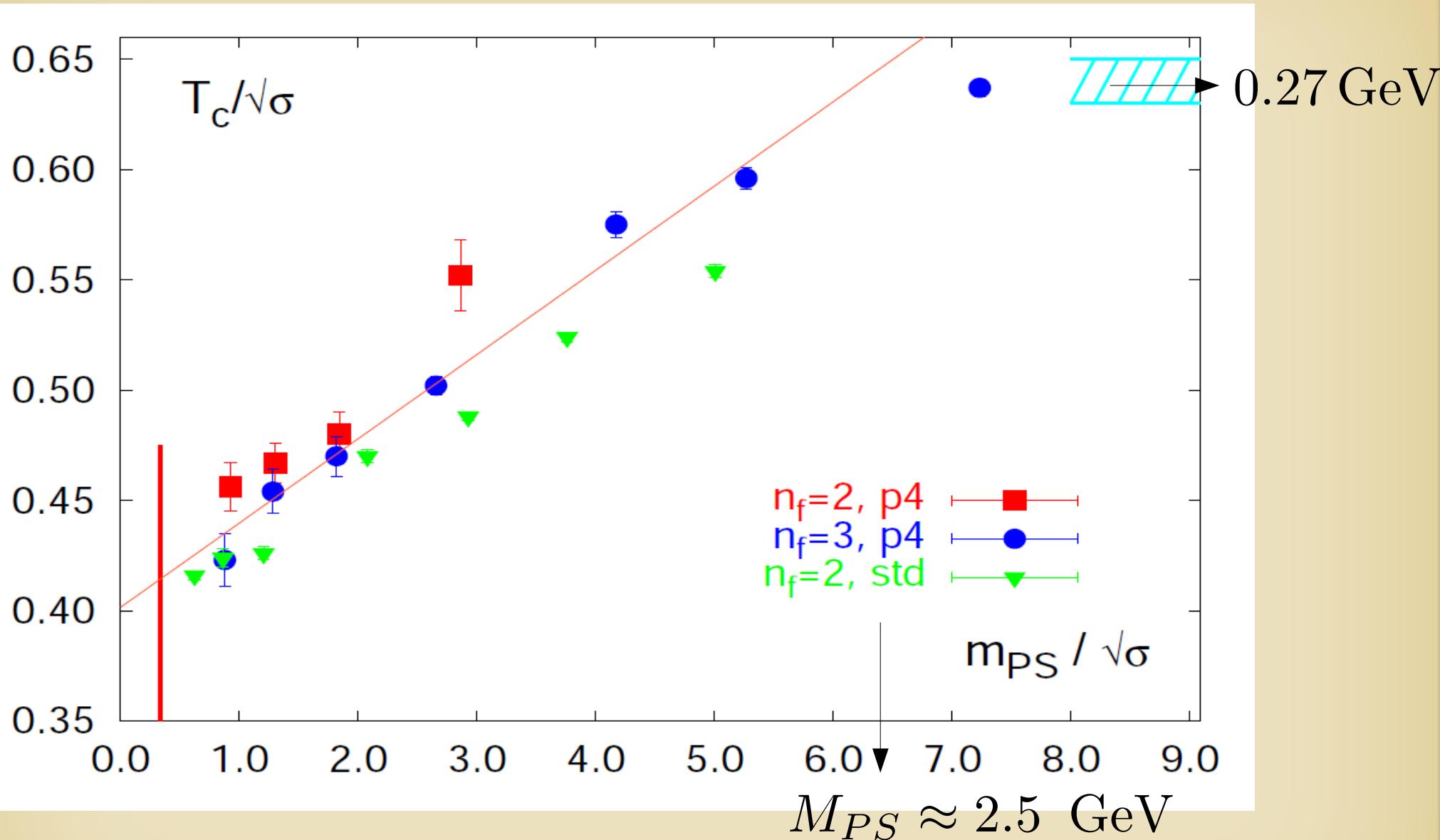
WHOT-QCD Collaboration (Saito, H. et al.)

3-flavour phase diagram



F. Karsch, Lattice QCD at high temperature and density





F. Karsch, Lattice QCD at high temperature and density

Heavy quark scale

- Typical heavy quarkonium mass scale:

$$m_c = 1.3 \text{ GeV}$$

$$m_{J/\psi} = 3.097 \text{ GeV}$$

$$m_{\eta_c} = 2.98 \text{ GeV}$$

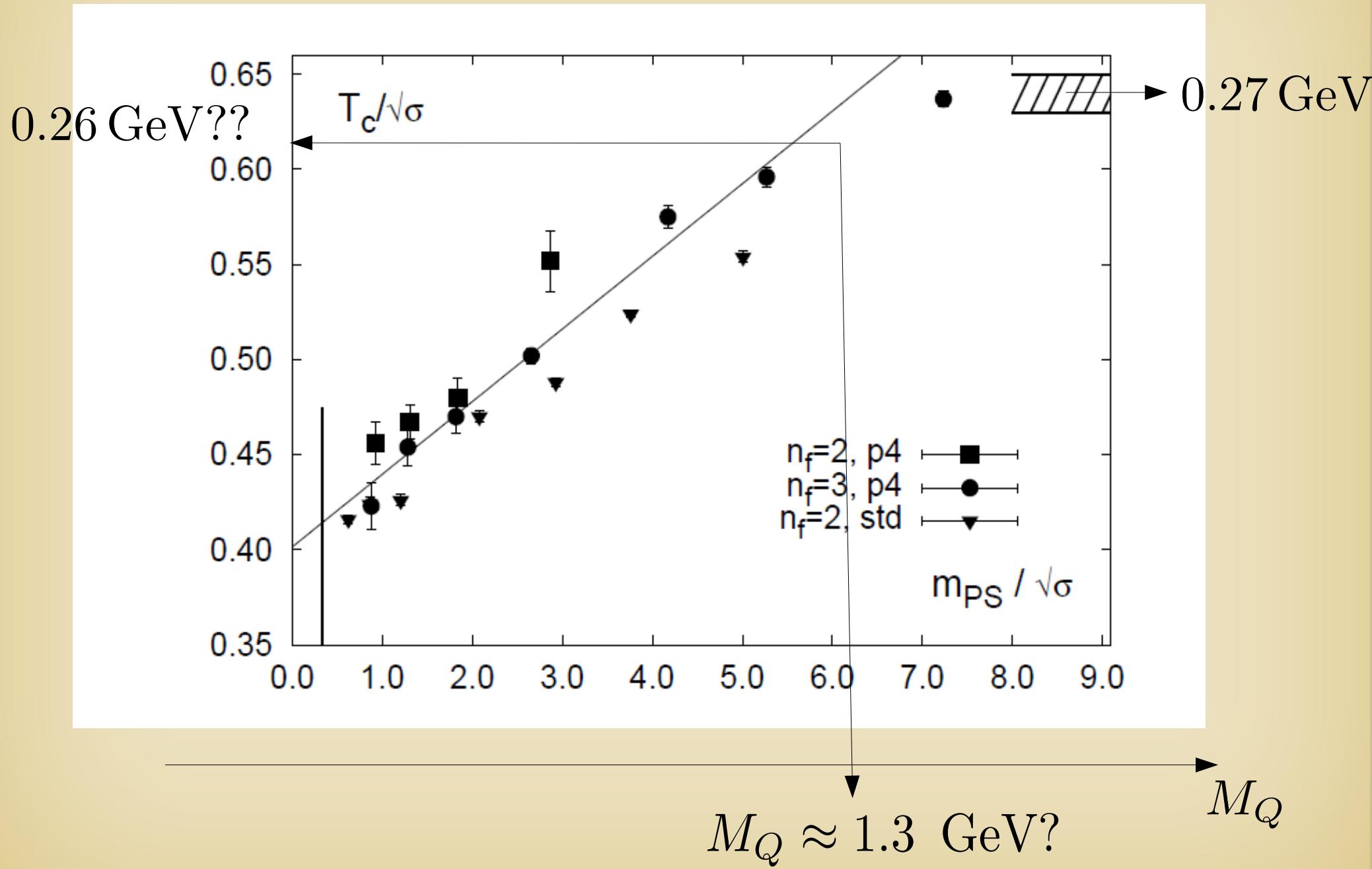
$$m_{PS}/m_V = 0.96$$

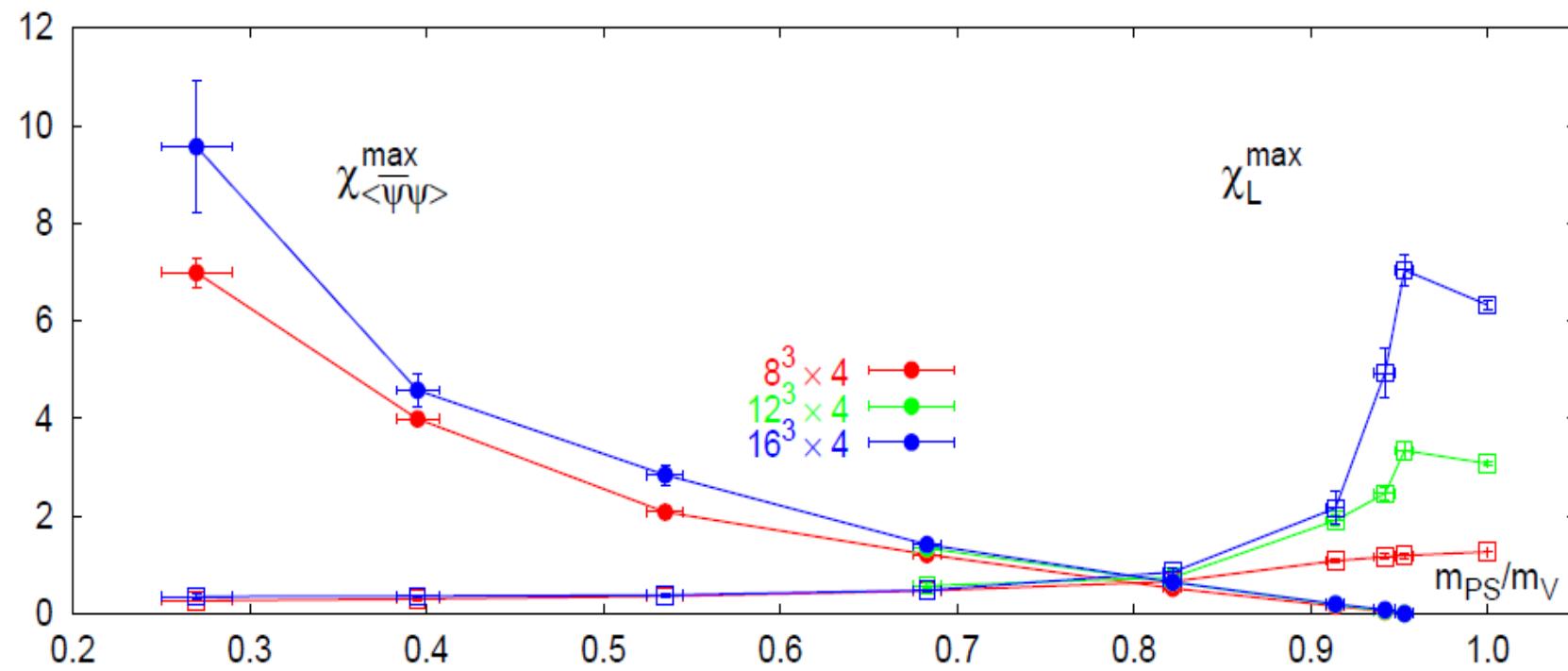
$$m_b = 4.2 \text{ GeV}$$

$$m_{\Upsilon} = 9.460 \text{ GeV}$$

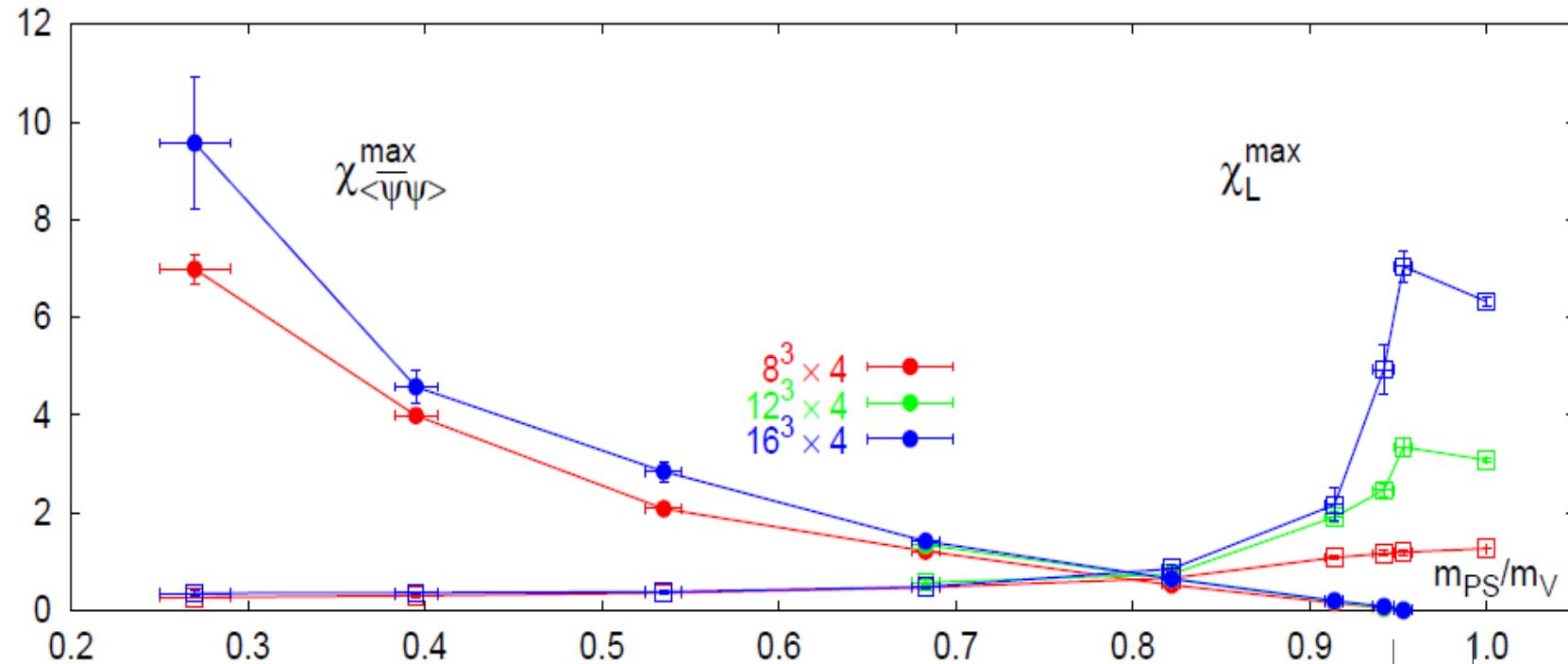
$$m_{\eta_b} = 9.391 \text{ GeV}$$

$$m_{PS}/m_V = 0.99$$





F. Karsch, Lattice QCD at high temperature
and density



The plot shows the behavior of the quark propagator χ as a function of the quark mass M_Q . The horizontal axis is labeled M_Q and ranges from 0.2 to 1.0. A vertical arrow points downwards from the horizontal axis to indicate the quark mass at approximately 1.3 GeV, where the physical pion mass m_{PS}/m_V is located. Another vertical arrow points downwards to indicate the quark mass at approximately 4.2 GeV.

Another test for PL model?

- Current model:

$$M_c^{N_f=3} \approx 1.45 \text{ GeV}$$

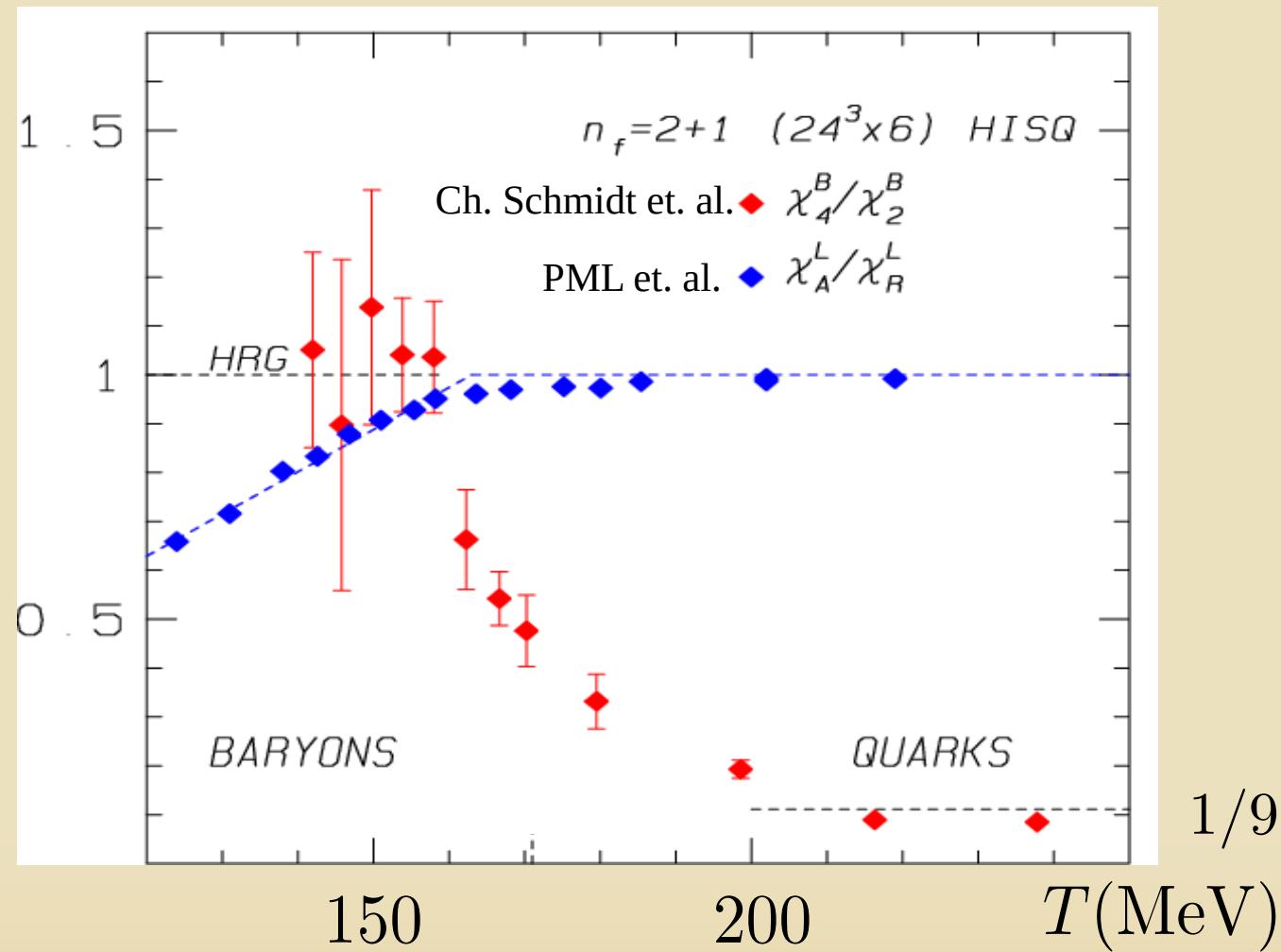
$$T_c^{\text{de}} \approx 0.26 \text{ GeV}$$

- Matrix model:

$$M_c^{N_f=3} \approx 2.5 \text{ GeV}$$

$$T_c^{\text{de}} \approx 0.27 \text{ GeV}$$

Recent QCD results



Summary

- Lattice calculation of PL susceptibilities within pure gauge system
- General properties of the ratios of PL susceptibilities
- Construction of effective potential

Summary

- Explicit Z(3) symmetry breaking by heavy quarks
- Critical end points

$$M_c^{N_f=3} \approx 1.45 \text{ GeV}$$

$$T_c^{\text{de}} \approx 0.26 \text{ GeV}$$

Thank You!

backup slides

Technical Aside

- Continuum quantities from lattice calculation:

$$\begin{aligned}\chi_{\text{lat.}} &= N_\sigma^3 \left\langle \sum_{i,j} \frac{1}{N_\sigma^3} \hat{L}_i \frac{1}{N_\sigma^3} \hat{L}_j \right\rangle_c \\ &= \frac{1}{N_\sigma^3} \left\langle \sum_{i,j} L_i L_j \right\rangle_c\end{aligned}$$



$$T^3 \chi_{\text{contin.}} = \frac{1}{N_\tau^3} \chi_{\text{lat}} = V T^3 \left\langle LL \right\rangle_c$$

Technical Aside

- Similar to the case for free energy density

$$T^{-4} f_{contin.} = N_\tau^4 \left(-\frac{\ln Z}{N_\sigma^3 N_\tau} \right)$$

$$T^3 \chi_{contin.} = \frac{1}{N_\tau^3} \chi_{lat} = VT^3 \langle LL \rangle_c$$

General features

- SU(3) matrix

Polyakov gauge:

$$\hat{L} = \frac{1}{N_c} Tr \mathcal{P} e^{ig \int_0^\beta d\tau A_4} = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{-i(\phi_1 + \phi_2)} \end{pmatrix}$$

$$L = \frac{1}{N_c} Tr_c \hat{L}.$$

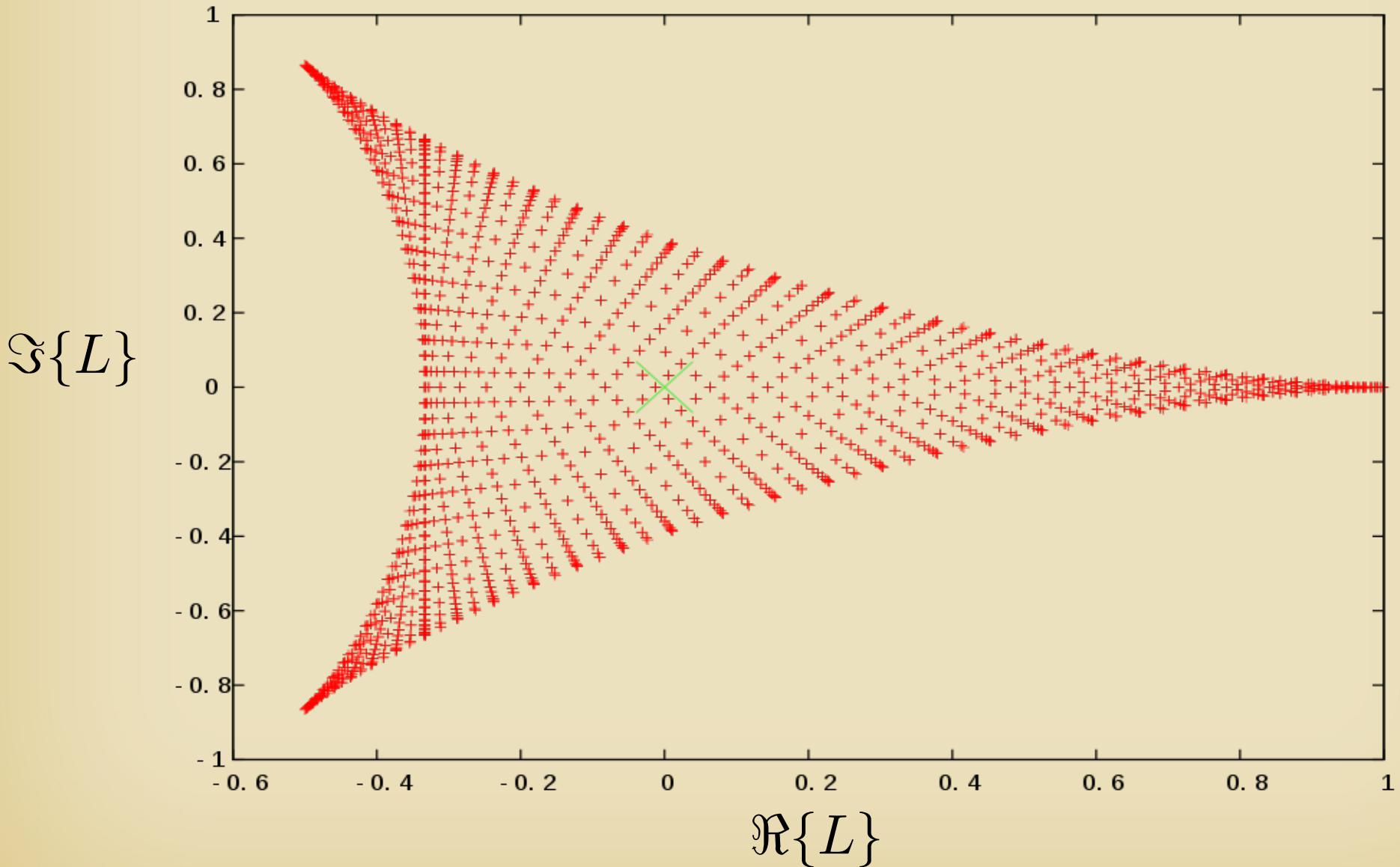
Target region

- Natural restriction on the range of complex values of L

$$\Re\{L\} = \frac{1}{3}(\cos(\phi_1) + \cos(\phi_2) + \cos(\phi_1 + \phi_2))$$

$$\Im\{L\} = \frac{1}{3}(\sin(\phi_1) + \sin(\phi_2) - \sin(\phi_1 + \phi_2))$$

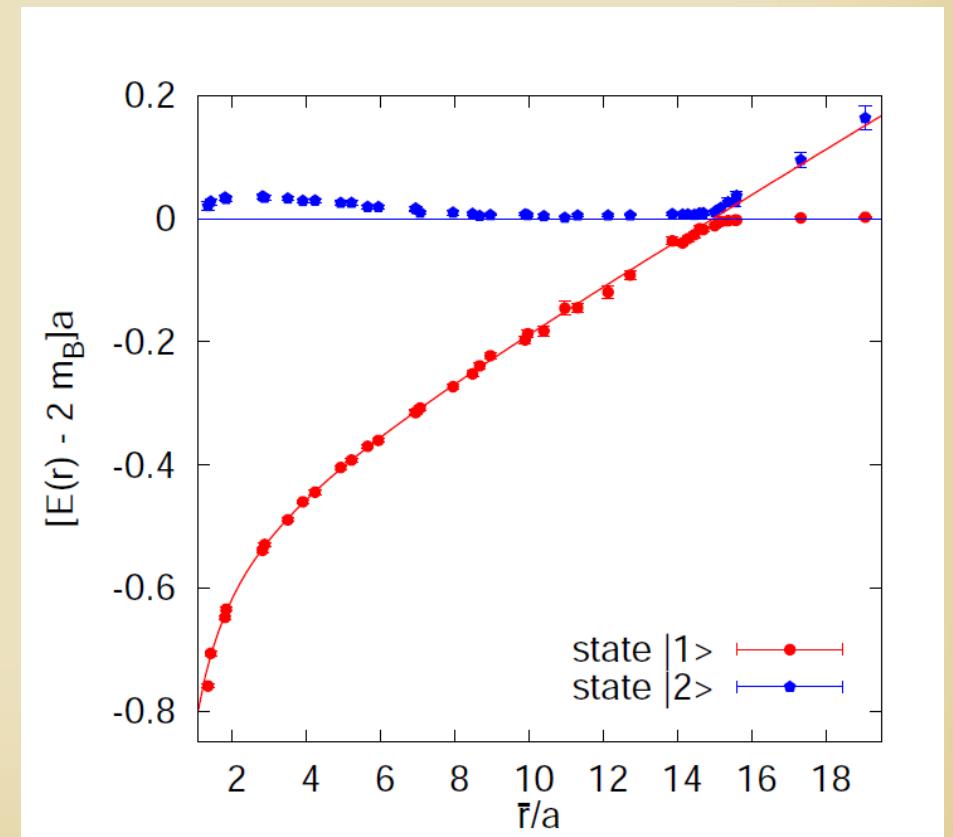
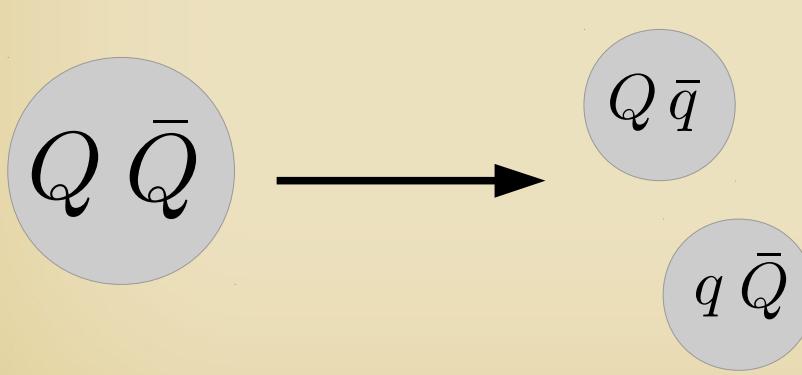
Target region



String breaking

- In the presence of dynamical quarks...

Bali *et. al.*



Group structure integration

- Order parameter as the effective field

$$Z[T, V] = \int dL P[L; T, V]$$

$$P[L; T, V] = e^{-\beta V U_{eff}[L]}$$

$$\langle \hat{O}[L] \rangle = \frac{1}{Z} \int dL O[L] e^{-\beta V U_{eff}[L]}$$

Matching Lattice data to PQM model

- Gap equation

$$\frac{\partial U_{eff}[l, T]}{\partial l} = 0 \longrightarrow \langle l \rangle$$

- Thermodynamics quantities

$$U_{eff}[l = \langle l \rangle, T] = -P[T]$$

- Two PL susceptibilities

$$\chi_{IJ} = \beta V \langle l_I l_J \rangle_c = \left(\frac{\partial^2 U_{\text{eff}}}{\partial l_I \partial l_J} \right)^{-1}$$

Field theoretical issues

Field theoretical issues

- Composite operators

$$l_{\vec{x}} = \langle \frac{1}{N_c} Tr \mathcal{P} e^{ig \int_0^\beta d\tau A^4[\tau, \vec{x}]} \rangle$$

$$C(\vec{x}) = \langle l_{\vec{x}} l_{\vec{0}} \rangle$$

$$\chi = \beta \int d^3x C(r)$$

- Effective field theory:
expansion in $\langle A^4 A^4 \rangle_c, \langle A^4 A^4 A^4 A^4 \rangle_c \dots$

Field theoretical issues

- Perturbation is **not** sufficient...

$$l = \left\langle \frac{1}{N_c} Tr \left(I_3 + ig\beta A^4 + \frac{1}{2} ig\beta i g\beta A^4[x] A^4[x] + \dots \right) \right\rangle$$

$$\approx 1. - \frac{1}{2N_c} g^2 \beta^2 Tr(T^a T^b) \left\langle A_a^4[x] A_b^4[x] \right\rangle .$$

$$\left\langle A_a^4[k] A_b^4[0] \right\rangle^P = \delta^{ab} \frac{1}{\beta} \frac{1}{k^2 + m_D^2}$$

$$l^P = 1 + \frac{N_c^2 - 1}{4N_c} \alpha_s \frac{m_D}{T}$$

$$l^P \approx 1 + \frac{N_c^2 - 1}{4N_c} \alpha_s \frac{m_D}{T}$$

$\ln l$

0.5

0

-0.5

-1

0

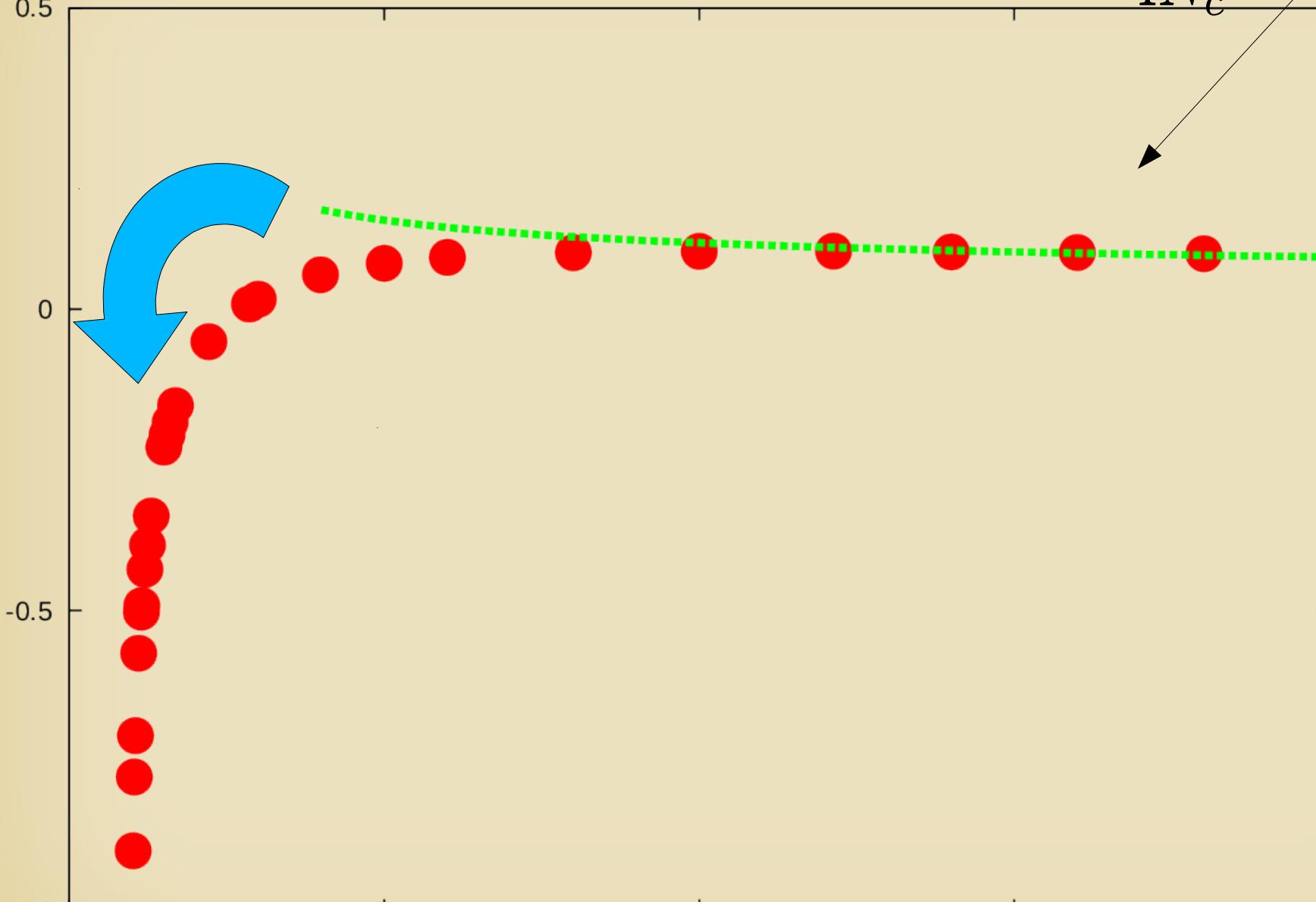
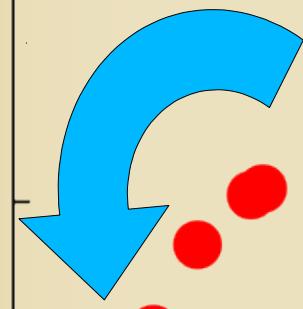
5

10

15

20

T/T_c



Field theoretical issues

- Ansatz for non-perturbative propagator (Megias *et al.*)

$$\langle A_a^4[x] A_b^4[0] \rangle = \delta^{ab} (D_{44}^P[\vec{x}] + D_{44}^{NP}[\vec{x}])$$

$$D_{44}^P[\vec{x}] = \frac{1}{\beta} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{k^2 + m_D^2} e^{i\vec{k}\cdot\vec{x}}$$

$$D_{44}^{NP}[\vec{x}] = \frac{1}{\beta} \int \frac{d^3 k}{(2\pi)^3} \frac{m_G^2}{(k^2 + m_D^2)^2} e^{i\vec{k}\cdot\vec{x}}$$

Field theoretical issues

- Zero temperature limit

$$g^2 (D_{44}^P[\vec{x}] + D_{44}^{NP}[\vec{x}]) = T \left(\frac{g^2}{4\pi |\vec{x}|} e^{-m_D |\vec{x}|} + \frac{g^2 m_G^2}{8\pi} \frac{e^{-m_D |\vec{x}|}}{m_D} \right)$$
$$\xrightarrow{m_D \rightarrow 0.} \delta(\tau) \left(\frac{g^2}{4\pi} \frac{1}{r} + -\frac{g^2 m_G^2}{8\pi} r + \text{Const.} \right)$$

- Effective string tension

$$b' = \frac{g^2 m_G^2}{8\pi}.$$

Field theoretical issues

To leading order...

$$l \approx 1 - \frac{1}{2N_c} g^2 \beta^2 \text{Tr}(T^a T^b) < A_a^4[x] A_b^4[x] >$$

$$C[r; T] \approx \frac{1}{4} \frac{1}{N_c^2} g^4 \beta^4 < \text{Tr}(A_4^2[x]) \text{Tr}(A_4^2[0]) >_c$$

$$\chi_l = \beta \int d^3x C(x)$$

$$l = l^P + l^{NP}$$

$$= \! 1 + \frac{N_c^2 - 1}{4 N_c} \, \alpha_s \, \frac{m_D}{T} - \frac{1}{T^2} \, \frac{N_c^2 - 1}{4 N_c} \, b' \, \frac{T}{m_D}$$

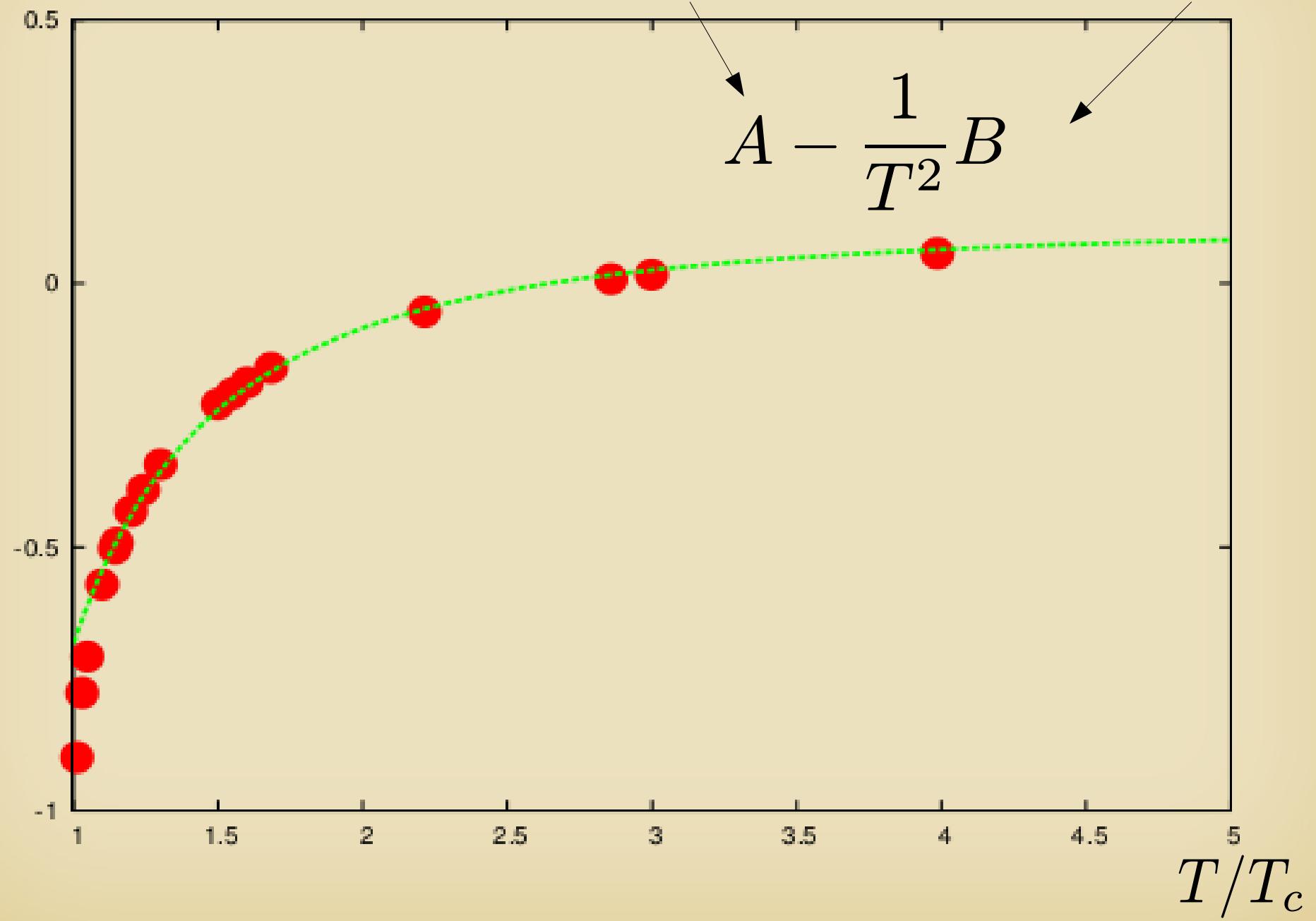
$$C[r;T] = C^P[r;T] + C^{NP}[r;T]$$

$$= \! \frac{N_c^2 - 1}{8 N_c^2} \alpha_s^2 \frac{e^{-2 r m_D}}{(r T)^2} + \frac{N_c^2 - 1}{8 N_c^2} \, {b'}^2 \, \frac{1}{m_D^2} \, \frac{e^{-2 m_D r}}{T^2}$$

$$\chi_l = \chi_l^P + \chi_l^{NP}$$

$$= \! \frac{N_c^2 - 1}{8 N_c^2} \, \alpha_s^2 \, \frac{2 \pi}{m_D T^3} + \frac{N_c^2 - 1}{8 N_c^2} \, \pi \, {b'}^2 \, \frac{1}{m_D^5 T^3}$$

$$\ln l \approx 1 + \frac{N_c^2 - 1}{4N_c} \alpha_s \frac{m_D}{T} - \frac{1}{T^2} \frac{N_c^2 - 1}{4N_c} b' \frac{T}{m_D}$$



Field theoretical issues

- Overall consistent description of lattice data in temperature range $1.1 T_c - 4 T_c$ with

$$g^2 \langle A_{0,a}^2 \rangle^{NP} = \frac{g^2(N_c^2 - 1)T m_G^2}{8\pi m_D} = 0.96 \text{ GeV}^2 = 13.2 T_c^2$$

- Similar analysis for trace anomaly

*Effective potential
order parameter
curvatures*

Field theoretical
quantities
correlation...

$$\begin{aligned} & \langle A_a^4[x]A_b^4[0] \rangle \\ & \langle A^4 A^4 A^4 A^4 \rangle_c \dots \end{aligned}$$

$$l_{\vec{x}} = \langle \frac{1}{N_c} \text{Tr } \mathcal{P} e^{ig \int_0^\beta d\tau A^4[\tau, \vec{x}]} \rangle$$

$$C(\vec{x}) = \langle l_{\vec{x}} l_{\vec{0}} \rangle$$

$$\chi = \beta \int d^3x C(r)$$

Lattice

*Effective potential
order parameter
curvatures*

Field theoretical
quantities
correlation...

$$\begin{aligned} & \langle A_a^4[x]A_b^4[0] \rangle \\ & \langle A^4 A^4 A^4 A^4 \rangle_c \dots \end{aligned}$$

$$l_{\vec{x}} = \langle \frac{1}{N_c} \text{Tr } \mathcal{P} e^{ig \int_0^\beta d\tau A^4[\tau, \vec{x}]} \rangle$$

$$C(\vec{x}) = \langle l_{\vec{x}} l_{\vec{0}} \rangle$$

$$\chi = \beta \int d^3x C(r)$$

Lattice

