The relation between cross-section, decay width and imaginary potential of heavy quarkonium in a quark-gluon plasma

Miguel A. Escobedo

Physik-Department T30f. Technische Universität München

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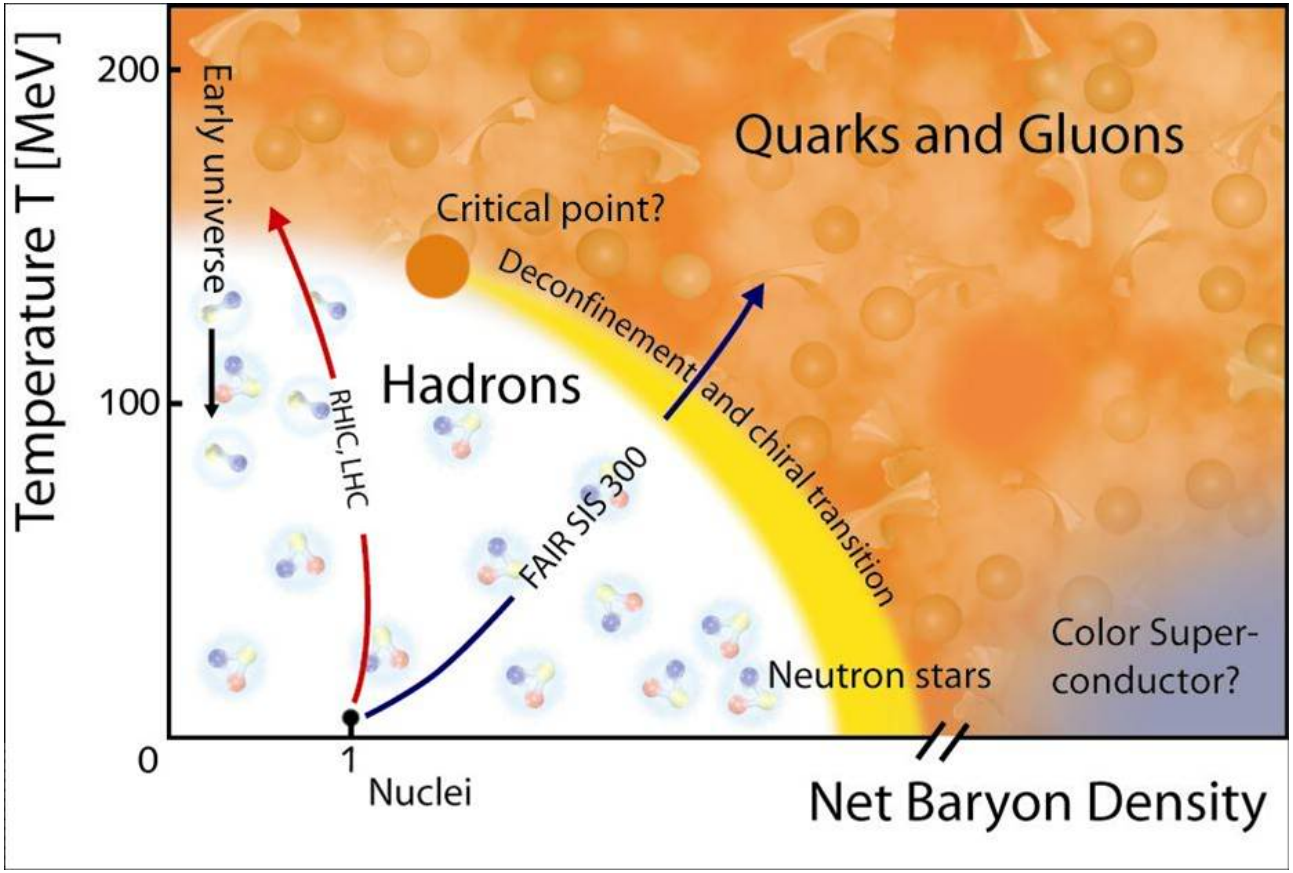
Nora Brambilla, MAE, Jacopo Ghiglieri, Antonio Vairo, JHEP12(2011)116 and JHEP1305(2013)130.
Fairness 2013, Berlin

Outline

1. Introduction
2. Gluo-dissociation
3. Quasi-free dissociation
4. Conclusions

Introduction

QCD phase diagram



What is heavy quarkonium?

A meson formed by heavy quarks $m_Q \gg \Lambda_{QCD}$.

- An important part of the physics is dominated by the scale m_Q rather to Λ_{QCD} (as opposite to pions). It is a good system to use perturbative computations.
- It is a non-relativistic system. The velocity of the heavy quarks around the center of mass is small. QCD hydrogen atom. It also complicates perturbative computations.

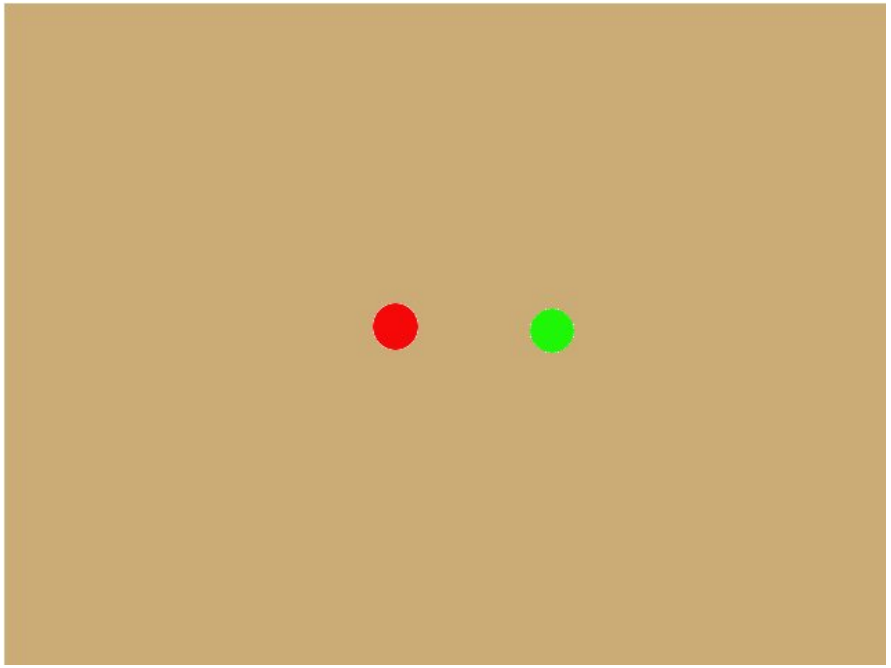
The original idea of Matsui and Satz (1986)

- Quarkonia is quite stable in the vacuum.
- Need a high energy to create a $Q\bar{Q}$ pair.
- Dissociation is due to colour screening, quantities measurable in Lattice QCD at finite temperature (static) support this. For example Polyakov loop.
- Dissociation of heavy quarkonium in heavy-ion collisions due to colour screening signals the creation of a quark-gluon plasma.

Colour screening

$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}$$

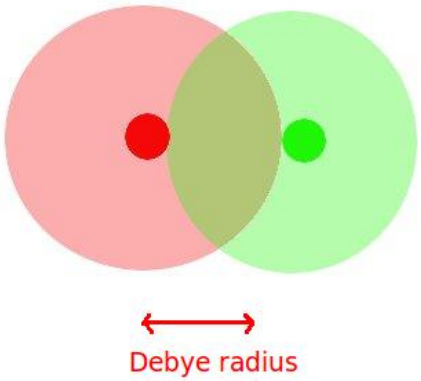
In the vacuum



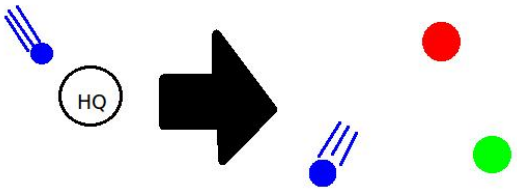
Colour screening

$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}$$

At finite temperature



Another mechanism, the decay width



- This effect makes the peak in the spectral function broader. It can arrive to a point where it is so broad that it does not make sense to speak of a bound state anymore.

Laine et al. perturbative potential (2007)

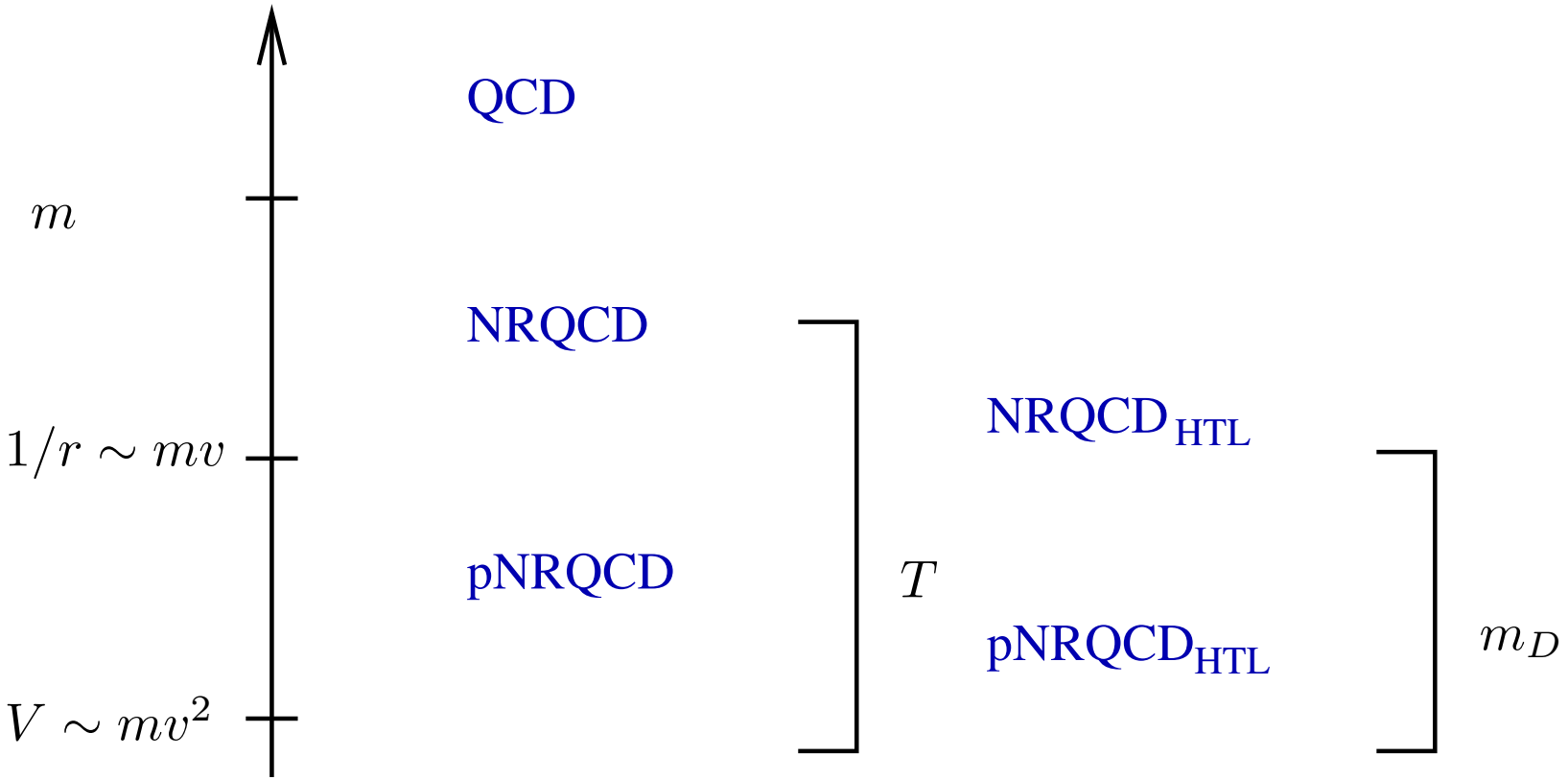
$$V(r) = -\alpha_s C_F \left[m_D + \frac{e^{-m_D r}}{r} \right] - i\alpha_s T C_F \phi(m_D r)$$

with

$$\phi(x) = 2 \int_0^\infty \frac{dzz}{(z^2+1)^2} \left(1 - \frac{\sin(zx)}{zx} \right)$$

- This potential was obtained through the Wilson loop in Minkownski space at finite temperature.
- It has an imaginary part that has to be related with a decay width.

Effective field theories



(Brambilla, Ghiglieri, Petreczky And Vairo, M. A. E and Soto)

Heavy quarkonium is non-relativistic

In a perturbative computation of the binding energy.

$$E = m_Q \alpha_s \sum_{n=0}^{\infty} \alpha_s^n A_n(v)$$

because v is small we can not know the size of $A_n(v)$, for example, it could go like $1/v$.
If we use EFT the computation is an expansion in both v and α_s.

$$E = m_Q \alpha_s v^2 \sum_{n,m} \alpha_s^n v^m B_{n,m}$$

now $B_{n,m}$ is of order 1.
In perturbation theory $v \sim \alpha_s$.

What has been found until now using EFTs in Quarkonia?

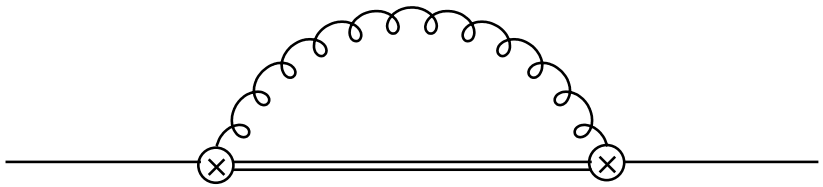
In the temperature regime relevant for dissociation $T \gg \frac{1}{r} \sim m_D$.

- We recover the perturbative potential with an imaginary part found by Laine, Philipsen, Romatschke and Tassler (2007).
- This is caused by the fact that gluons lose energy by Landau damping (property of gluon propagator).

What has been found until now using EFTs in Quarkonia?

In the low temperature regime $\frac{1}{r} \gg T$.

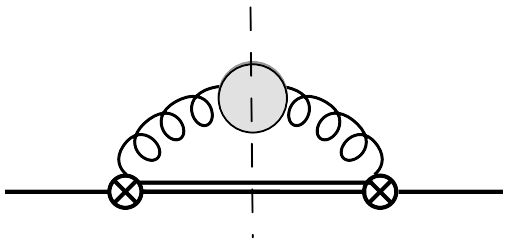
- We get corrections to the decay width that can not always be encoded in an correction to the potential.
- If $E \gg m_D$ the decay width is dominated by the imaginary part of the following diagram. Breaking of the singlet into an octet due to the absorption of a gluon.



What has been found until now using EFTs in Quarkonia?

In the low temperature regime $\frac{1}{r} \gg T$.

- We get corrections to the decay width that can not always be encoded in an correction to the potential.
- If $m_D \gg E$ the decay width is dominated by the imaginary part of the following diagram. Landau damping.



Other approach to quarkonia decay width

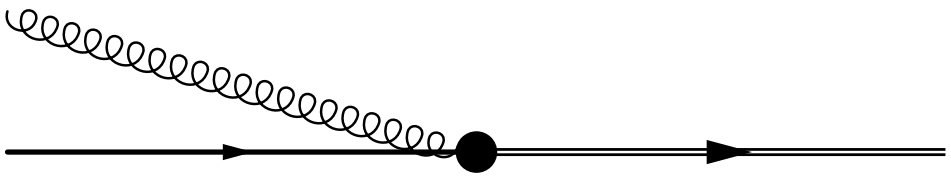
- Use a cross-section computed at $T = 0$, $\sigma(k)$.
- Convolute with the thermal distribution

$$\Gamma = \int \frac{d^3k}{(2\pi)^3} f(k)\sigma(k)$$

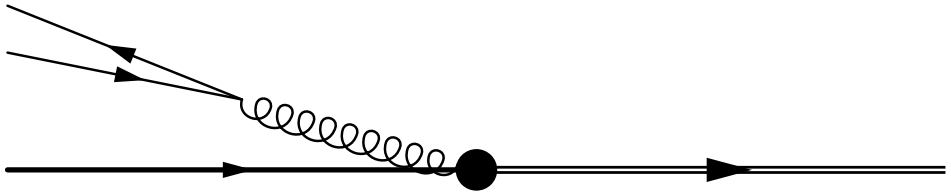
- These cross-sections are computed in perturbation theory and later they are "adapted" to strong coupling by using $\alpha_{\rm s}$ as a free parameter, introducing thermal masses...
- This information is used as an input to predict the observed suppression in nowadays experiments. See for example Zhao and Rapp (2010).

Perturbative computations of cross-section for quarkonia in the literature

Gluo-dissociation



Bhanot and Peskin (1979)
Quasi-free dissociation



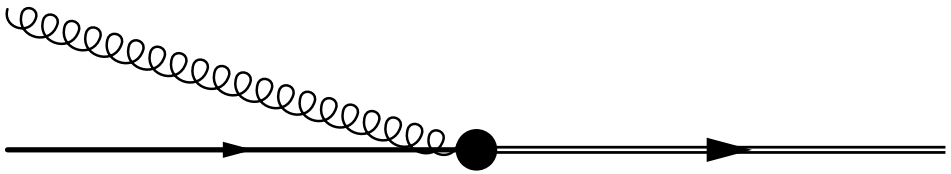
Combridge (1978), Park, Kim, Song, Lee and Wong (2007)

Motivation

- Understand the physical process behind the imaginary part of the potential and the corrections to the decay width.
- Translate the EFT results that have been found to cross-sections convoluted with distribution function "language".
- Analyze the assumptions made by previous perturbative computations and check if they agree or disagree with the EFT framework.

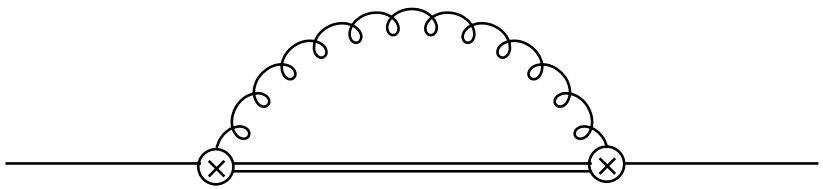
Gluo-dissociation

Gluo-dissociation in Bhanot and Peskin



- They use OPE. The interaction between the singlet, the octet and the gluon is a color dipole interaction.
- This approximation is convenient because the gluo-dissociation is the dominant dissociation mechanism only for $E \gg m_D$. It is very similar to what is done in pNRQCD.
- They use the large N_c limit approximation. In this limit $V_o = 0$ and computations are simplified.
- We are going to see that the large N_c limit is a good approximation for $T \gg E$ but not for $T \sim E$.

Gluo-dissociation in pNRQCD



- Computed for $T \gg E$ in HQ. Brambilla, MAE, Ghiglieri, Soto and Vairo (2010)

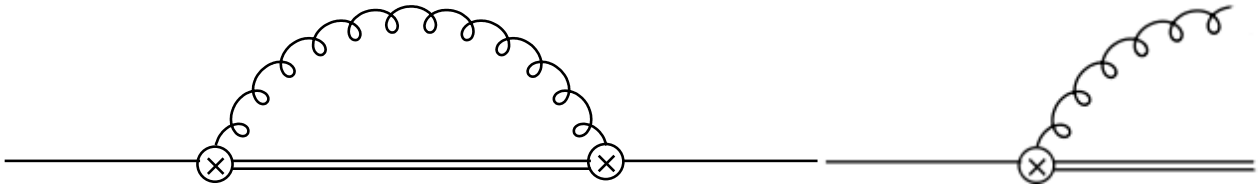
$$\delta\Gamma_n = \frac{1}{3} N_C^2 C_F \alpha_s^3 T - \frac{16}{3m} C_F \alpha_s T E_n + \frac{4}{3} N_C C_F \alpha_s^2 T \frac{2}{m n^2 a_0}$$

where E_n is the binding energy and a_0 the Bohr radius.

- Computed for $T \sim E$ in the hydrogen atom. MAE and Soto (2008).

$$\delta\Gamma_n = \frac{4}{3} \alpha_s C_F T \langle n | r_i \frac{|E_n - h_o|^3}{e^{\beta|E_n - h_o|} - 1} r_i | n \rangle$$

Cutting rules at finite temperature

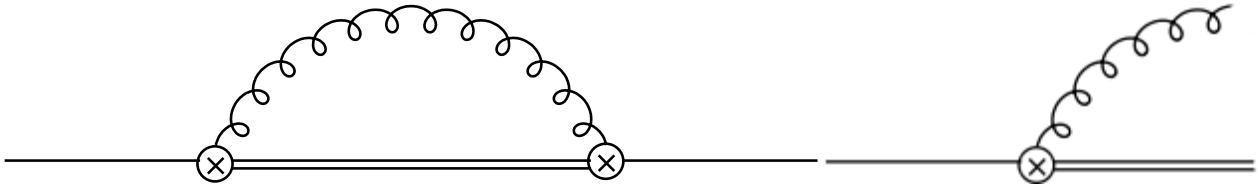


Similar to what is found at $T = 0$.

- Multiply by $n_B(k)$ ($n_F(k)$) for in-coming bosons (fermions).
- Multiply by $1 + n_B(k)$ ($1 - n_F(k)$) for out-going bosons (fermions).

Kobes and Semenoff (1986)

Cutting rules at finite temperature



Similar to what is found at $T = 0$.

- Multiply by $n_B(k)$ ($n_F(k)$) for in-coming bosons (fermions).
- Multiply by $1 + n_B(k)$ ($1 - n_F(k)$) for out-going bosons (fermions).

In this case we get a structure

$$\delta\Gamma_n = \int \frac{d^3k}{(2\pi)^3} n_B(k) \langle n | h_\sigma(r, p, k) | n \rangle$$

pNRQCD gluo-dissociation σ_{gd} for $1S$

- If we do the same approximations as Bhanot and Peskin (large N_c limit) we recover their result.
- Without doing this approximation we get

$$\sigma_{gd}(k) = \frac{8\pi^2 C_F \alpha_s m a_0^2 k}{3} |\langle 1S|r_i|ma_0^2(k+E_1)\rangle_o|^2 \Theta(k+E_1)$$

$|\epsilon\rangle_o$ are the octet wave function taking into account the octet potential.

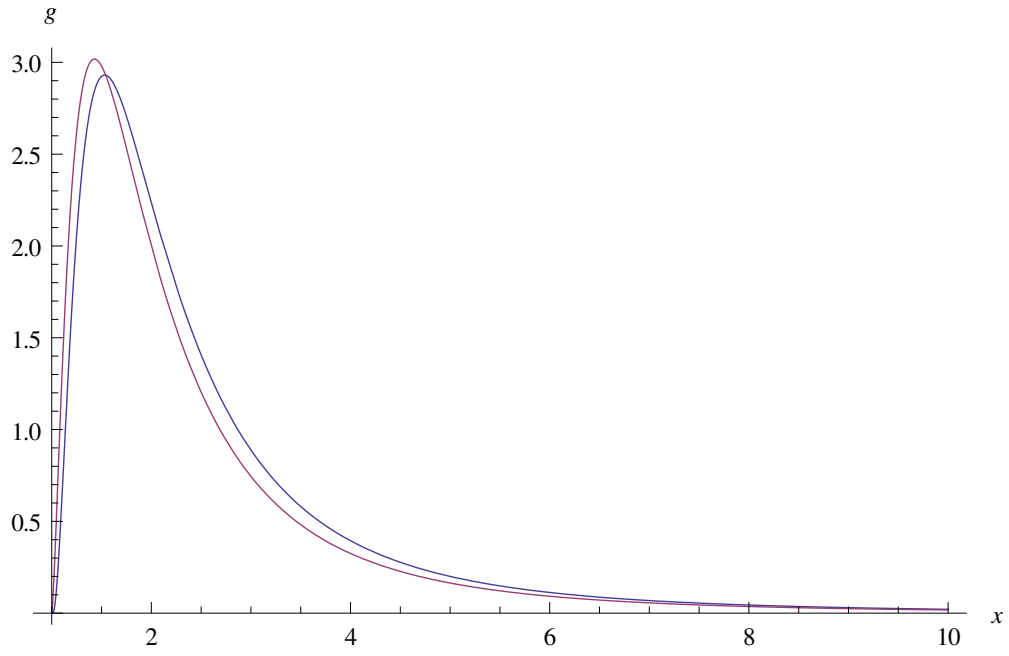
$$\int_0^\infty d\epsilon \langle\epsilon|\epsilon\rangle_o = 1$$

Agrees with Brezinski and Wolschin (2011)

Comparison between Bhanot and Peskin and pNRQCD gluo-dissociation

$$\sigma_{gd}(k) = \sigma_R g(x)$$

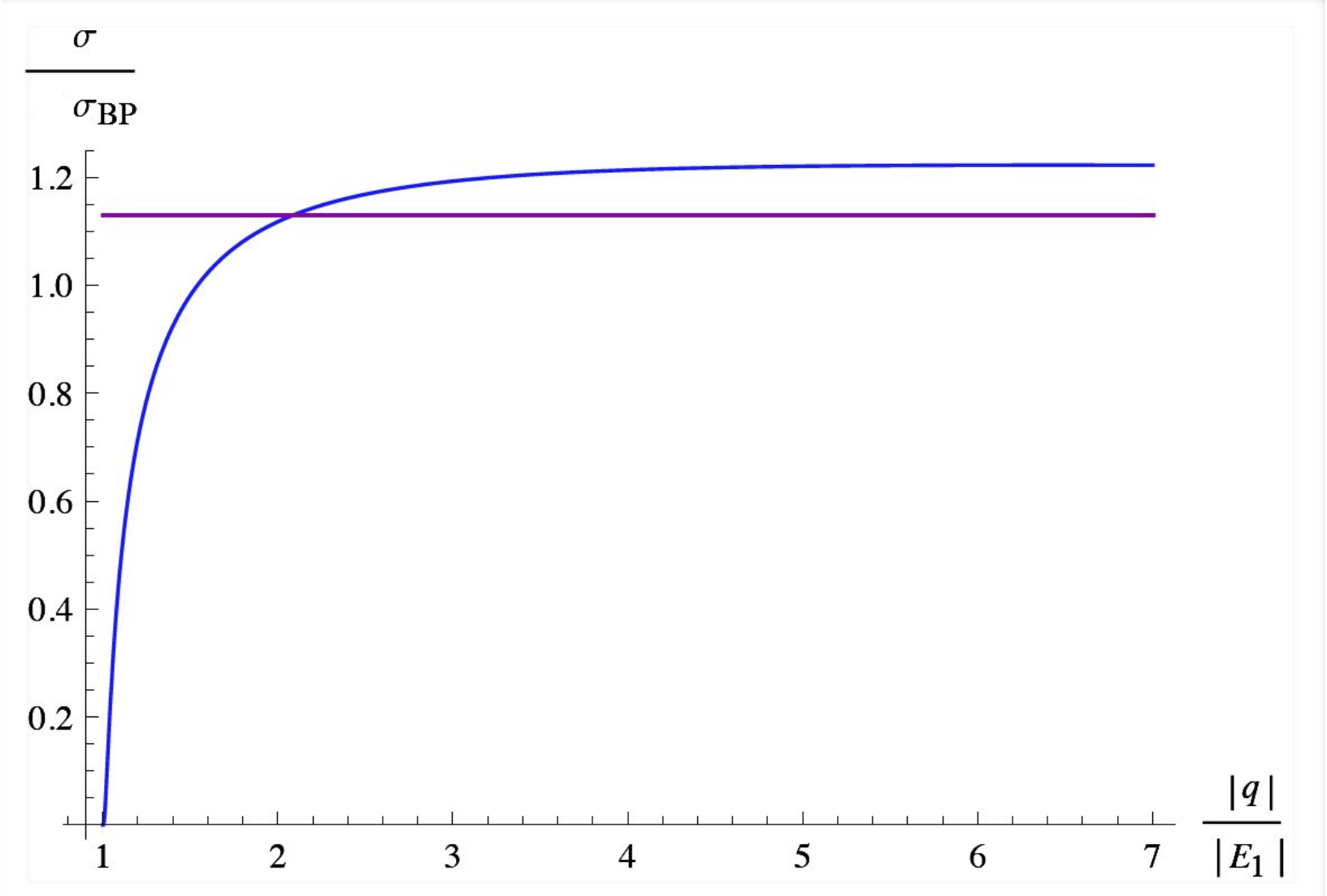
with $\sigma_R = \frac{32\pi C_F \alpha_s a_0^2}{3}$ and $x = \frac{k}{|E_1|}$



Bhanot and Peskin large N_c limit

pNRQCD

Comparison between Bhanot and Peskin and pNRQCD gluo-dissociation



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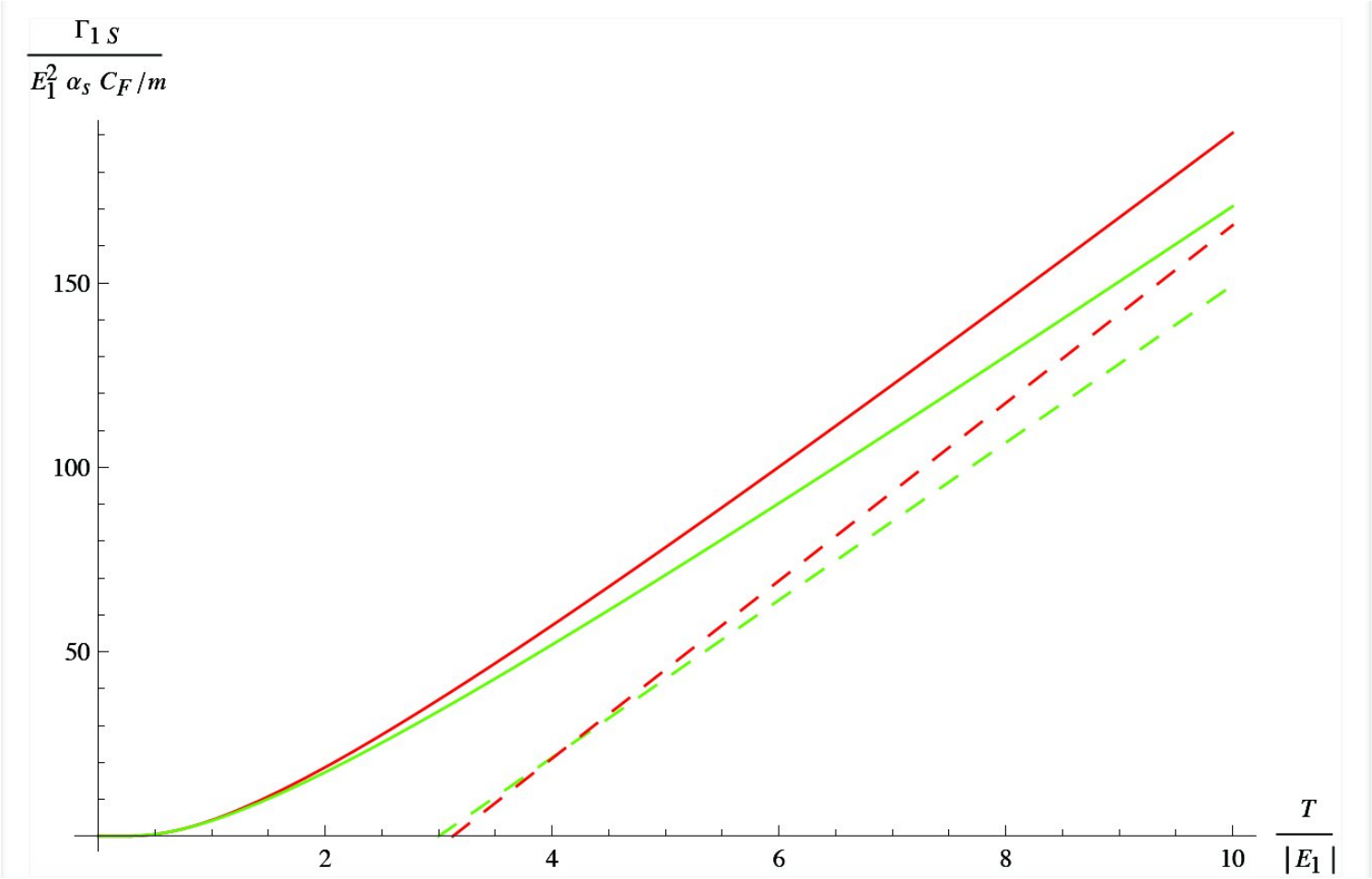
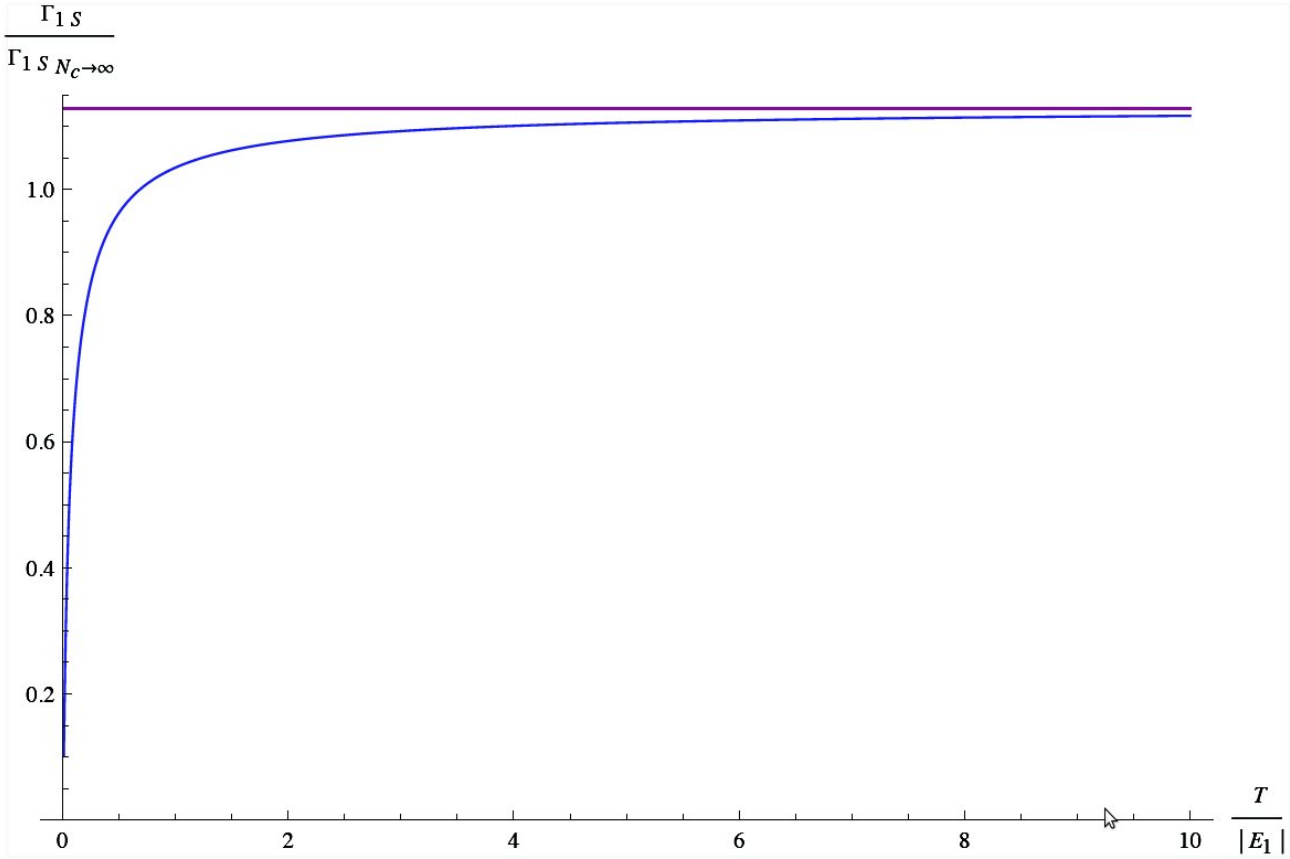


Bhanot and Peskin, pNRQCD

Comparison between Bhanot and Peskin and pNRQCD gluo-dissociation

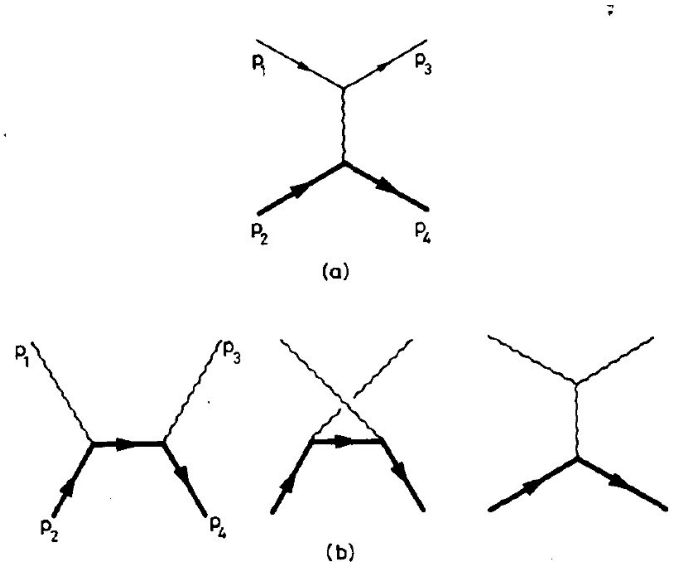


Quasi-free dissociation

(Or Landau damping)

Quasi-free in Combridge

He computed the process $qc \to qc$ for a charm quark, no information of the bound state is included.



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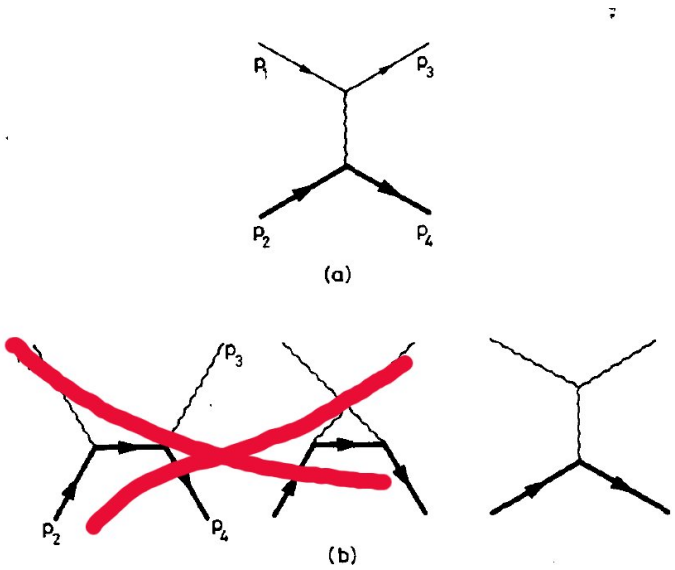
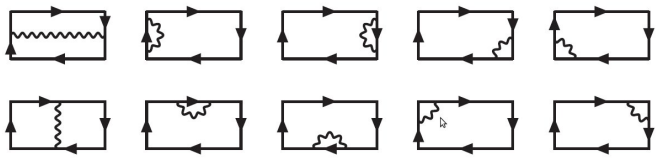


Note that in NRQCD (valid for $m_Q \gg T$) and using the Coulomb gauge the crossed diagrams are subleading.

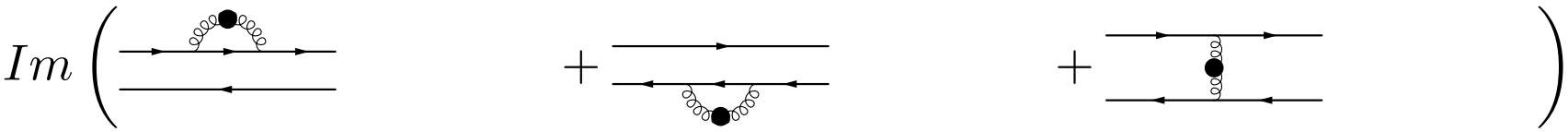
HQ potential for $T \gg \frac{1}{r} \sim m_D$

Laine, Philipsen, Romatschke and Tassler

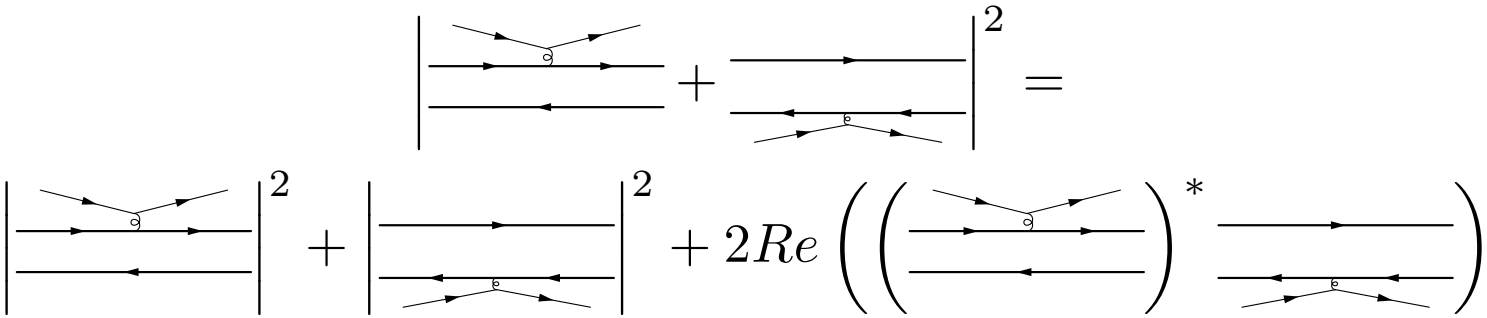


pNRQCD

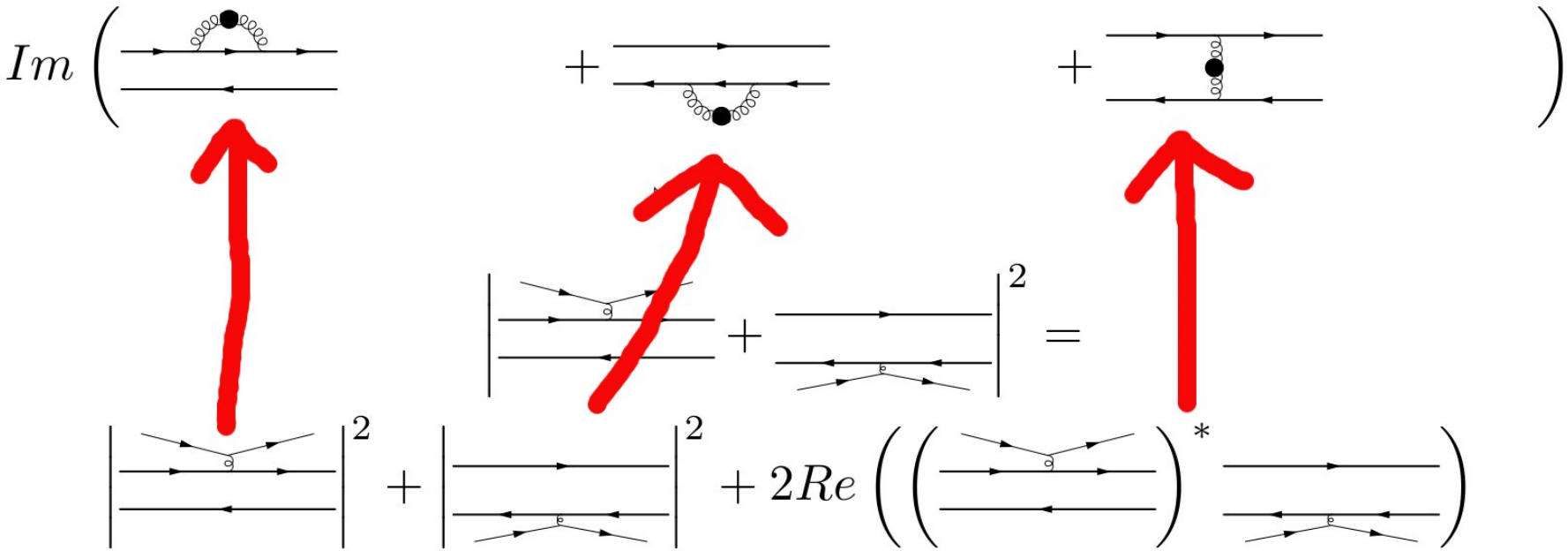
Imaginary part of the potential



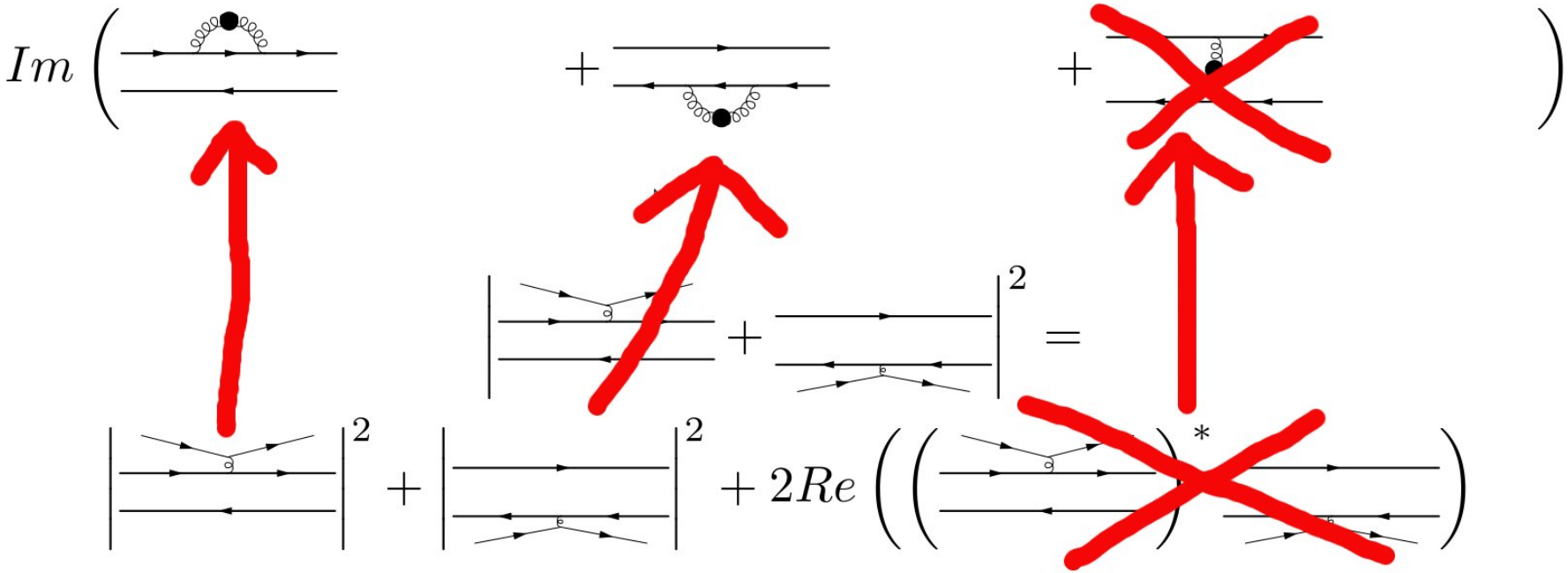
By the optical theorem



Imaginary part of the potential

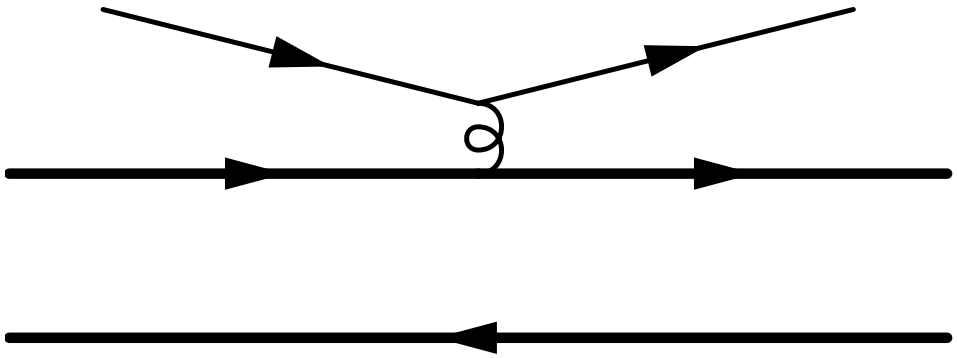


Combridge approximation



Interference term is neglected. Good approximation for $T, m_D \gg \frac{1}{r}$.

From the cross-section to the decay width



- Apart from the heavy quarks that are not thermalized, there is an in-coming parton and an out-going parton.
- The decay width then has the structure.

$$\Gamma = \int \frac{d^3k}{(2\pi)^3} f(k)(1+f(k))\sigma(k)$$

CAUTION Resummation

- All the previous discussions assume that the imaginary part is a perturbation. We are using quantum mechanical perturbation theory starting from the Coulomb potential. The aim was to illustrate the relation between the two approaches.
- In fact for $T \gg \frac{1}{r} \sim m_D$ what the pNRQCD power counting tell us is to solve the Schrödinger equation taking into account this imaginary part.
- Equivalent to a ladder resummation of the previously discussed diagram.
- This is not included in the computations based on Combridge cross-section.

Cross-section for the $1S$ state

We proceed in a similar way to what is done for the gluo-dissociation. We start by our previous EFT computations and "translate" them

- In gluo-dissociation only a energy scale was relevant. This is not the case now.
- As we need information of the scale m_D the HTL has to be performed at some part of the computation. σ is going to depend also on the temperature due to this.

Some notation

$$\sigma(k, m_D) = \sigma_R f(x, y)$$

where

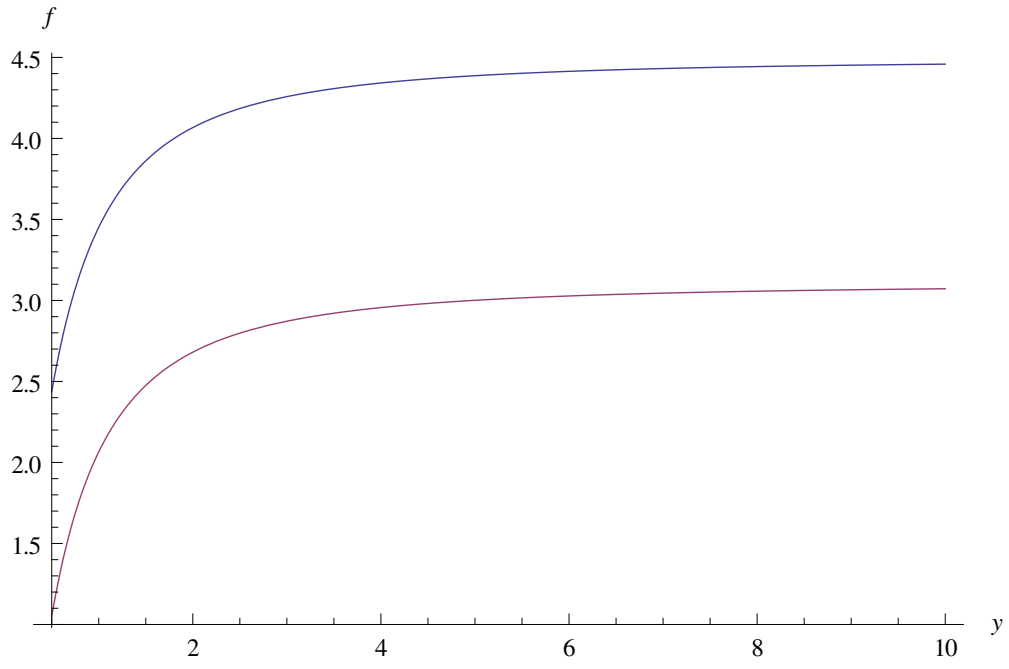
$$\sigma_R = 8\pi C_F \alpha_s^2 N_F a_0^2$$
$$x = m_D a_0$$
$$y = k a_0$$

I will only show the result for the fermion part, the boson part is quantitatively and qualitatively very similar.

$T \sim \frac{1}{r} \gg m_D$ cross-section for $1S$

$$f(x,y) = -\frac{3}{2} + 2\log\left(\frac{2}{x}\right) + \log\left(\frac{y^2}{1+y^2}\right) - \frac{1}{y^2}\log(1+y^2)$$

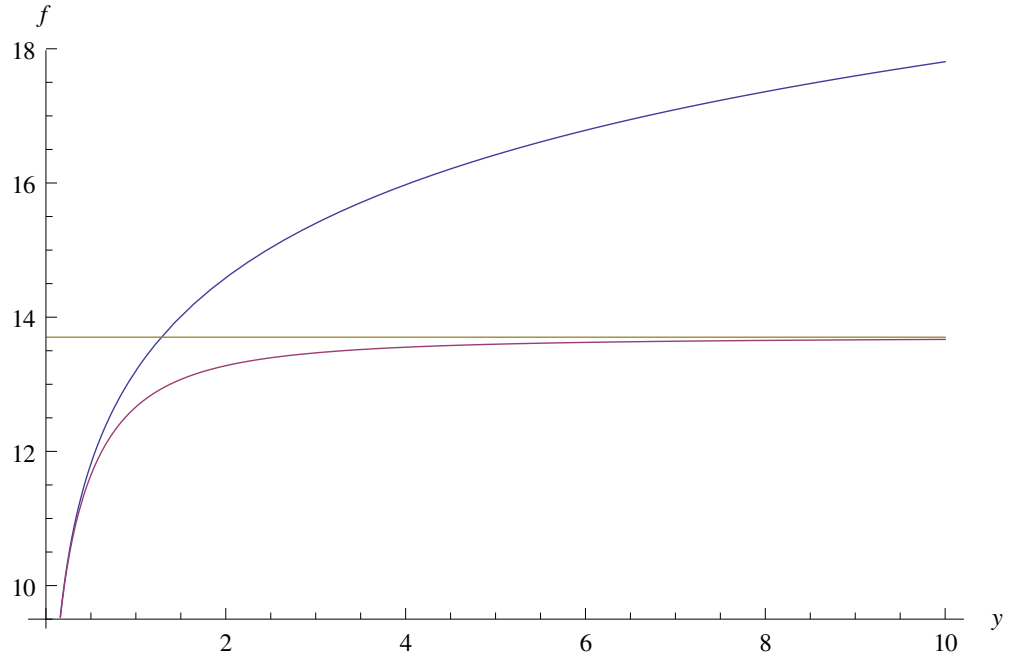
$x \ll 1$ and $y \sim 1$.



$m_D a_0 = 0.1$ and $m_D a_0 = 0.2$.

Summary cross-section for $1S$

$m_D a_0 = 0.001$



$\frac{1}{r} \gg T \gg m_D$, $T \sim \frac{1}{r} \gg m_D$ and $T \gg \frac{1}{r} \sim m_D$. Discrepancy between blue and red lines signals a failure of color dipole approximation.

Conclusions

- The Bhanot and Peskin result that is normally used correspond to the large N_c limit of pNRQCD result. This is a good approximation for $T \gg E$ but it is not so good for $T \sim E$.
- The imaginary part of the potential and the quasi-free dissociation describe the same physical process at different temperatures.
- EFT improve in considering bound state properties in quasi-free dissociation.