D-meson diffusion in hadronic matter

in collaboration with L. Abreu, D. Cabrera,
F.J. Llanes-Estrada and L. Tolos

Juan M. Torres-Rincon

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Bellaterra, Spain

Talk at FAIRNESS 2013 workshop
Berlin. Sept. 18, 2013

FAIRNESS
16-21 September 2013
Berlin

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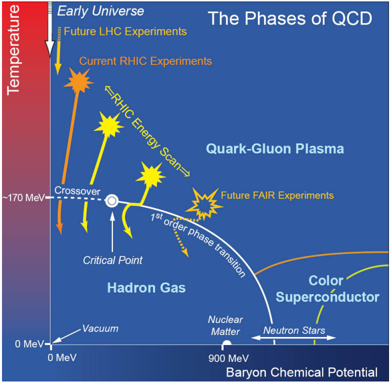

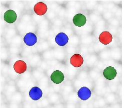

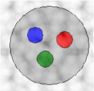

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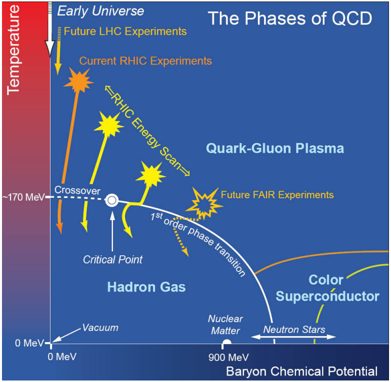

QGP is the system we would like to understand

HG is the phase we can more easily access

Why focus on heavy (D-) mesons in a relativistic ion collision?

D mesons

They are one of the cleanest probes of the early stages of the collision.

The relaxation time of heavy quarks is larger than the one for light quarks (D. Teaney and G.D. Moore, *Phys. Rev.C.*71, 064904 (2005))

$$\tau_{rel}^{H} \sim \frac{M}{T} \, \tau_{rel}^{light} \; ; \qquad \frac{M}{T} \sim 6 - 20$$

In spite of suffering several collisions with the lighter hadrons, the D mesons cannot completely relax to the equilibrium state at the decoupling (kinetic freeze-out) time,

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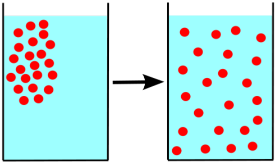
I will focus on D mesons and study their dynamics through the hot and dense hadronic medium created after a heavy-ion collision.

This medium –the thermal bath– will mainly contain light hadrons like pions, kaons...

D-meson diffusion in hadronic matter

- Focus on a central collision at the LHC, $\mu_B \sim 0$
- Consider a D meson after hadronization, $m_D \gg T_{had}$.
- It will interact with the most populated species in the medium i.e. pions which we consider already equilibrated.
- Charm quantum number is conserved by the strong interaction $\rightarrow$ diffusion coefficient of D mesons.

$$\vec{j} = -D_x \vec{\nabla} n$$



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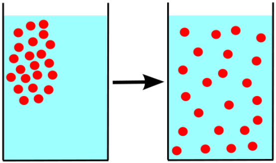
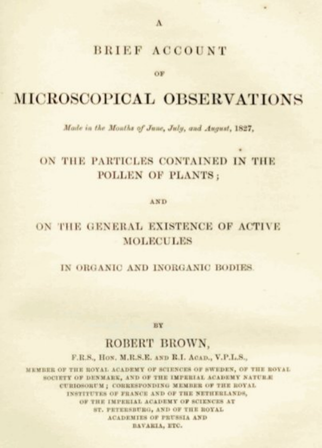
Due to their large mass they can be treated as Brownian particles

$$m_D \gg m_\pi$$





A

BRIEF ACCOUNT

OF

MICROSCOPICAL OBSERVATIONS

Made in the Months of June, July, and August, 1827,

ON THE PARTICLES CONTAINED IN THE POLLEN OF PLANTS;

AND

ON THE GENERAL EXISTENCE OF ACTIVE MOLECULES

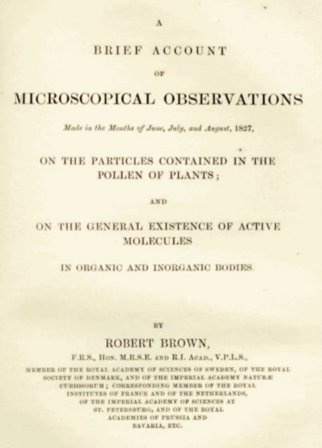
IN ORGANIC AND INORGANIC BODIES.

BY

ROBERT BROWN,

F.R.S., HON. M.R.S.E. AND R.I. ACAD., V.P.L.S.,

MEMBER OF THE ROYAL ACADEMY OF SCIENCES OF SWEDEN, OF THE ROYAL SOCIETY OF DENMARK, AND OF THE IMPERIAL ACADEMY NATURÆ CURIOSORUM; CORRESPONDING MEMBER OF THE ROYAL INSTITUTES OF FRANCE AND OF THE NETHERLANDS, OF THE IMPERIAL ACADEMY OF SCIENCES AT ST. PETERSBURG, AND OF THE ROYAL ACADEMIES OF PRUSSIA AND BAVARIA, ETC.



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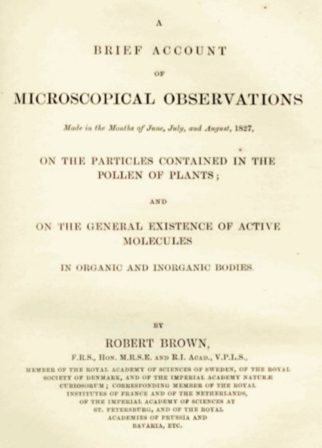
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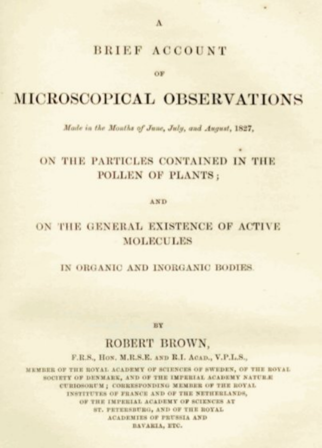
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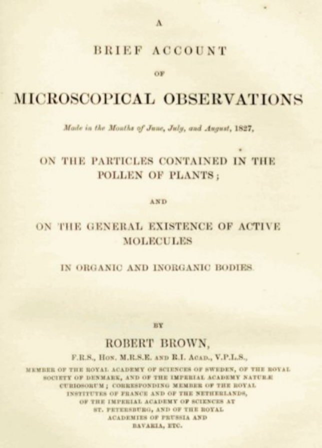

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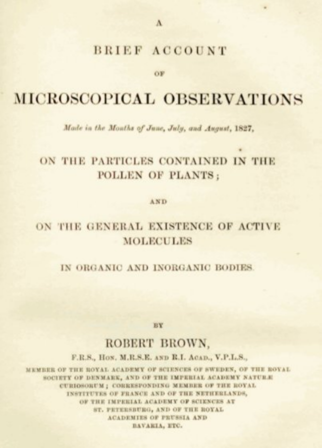
3. Raise the temperature several orders of magnitude

D-meson diffusion in hadronic matter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4. ...et Voilà

Diffusion

One can start by the Langevin equation to solve the D meson dynamics

$$\dot{p} = -F(p)\ p + \xi(t)$$

▸ more on this...

but better use a kinetic description in terms of the distribution function

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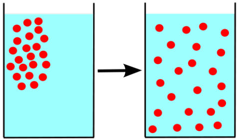
Fick's diffusion law

$$\vec{j} = -D_x \vec{\nabla} n$$

where

$$\vec{j} = \int \frac{d^3p}{E_p}\ \vec{p}\ f(t,p)$$

f being the one-particle distribution function
(in equilibrium: the Bose-Einstein distribution function)



Fokker-Planck equation

For a Brownian particle $M_D \gg M_{light}$ the kinetic equation is (equivalent to the Langevin equation)

Fokker-Planck equation

$$\frac{\partial f(t,p)}{\partial t} = \frac{\partial}{\partial p_i} \left\{ F_i(p) f(t,p) + \frac{\partial}{\partial p_j} \left[\Gamma_{ij}(p) f(t,p) \right] \right\}$$

where $i = 1, 2, 3$ denote the spatial direction.

$F_i(p)$ and $\Gamma_{ij}(p)$ are the relevant transport coefficients.

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In an isotropic gas, there are three different coefficients

Fokker-Planck equation

Their expressions:

Drag force

$$F(p) = \int d\mathbf{k} \; w(\mathbf{p}, \mathbf{k}) \frac{\mathbf{k} \cdot \mathbf{p}}{p^2}$$

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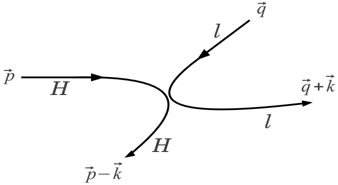
$$\begin{aligned} w(\mathbf{p}, \mathbf{k}) \quad = \quad & g_l \int \frac{d\mathbf{q}}{(2\pi)^9} \; f_l(\mathbf{q}) \; \frac{1}{2E_q^l} \frac{1}{2E_p^H} \frac{1}{2E_{q+k}^l} \frac{1}{2E_{p-k}^H} \\ & \times (2\pi)^4 \; \delta(E_p^H + E_q^l - E_{p-k}^H - E_{q+k}^l) \; \overline{|T|^2} \end{aligned}$$

where $\overline{|T|^2}$ is the scattering amplitude squared.



Fokker-Planck equation

Diffusion coefficients (transverse and longitudinal)

$$\Gamma_0(p) = \frac{1}{4} \int d\mathbf{k} \;\; w(\mathbf{p}, \mathbf{k}) \;\; \left[k^2 - \frac{(\mathbf{k} \cdot \mathbf{p})^2}{p^2} \right]$$

$$\Gamma_1(p) = \frac{1}{2} \int d\mathbf{k} \;\; w(\mathbf{p}, \mathbf{k}) \;\; \frac{(\mathbf{k} \cdot \mathbf{p})^2}{p^2}$$

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If $p \to 0$ (static limit), the Einstein relation is recovered

Einstein Relation

$$F = \frac{\Gamma_0}{m_D T} = \frac{\Gamma_1}{m_D T}$$

Other coefficients

Other related coefficients that are of physical interest:

Spatial diffusion coefficient

$$D_x = \lim_{p \to 0} \frac{\Gamma_0}{m_H^2 F^2}$$

Relaxation time

an argument for this

$$\tau_R = F^{-1}$$

D-meson diffusion in hadronic matter

The last piece to complete the computation is $|T|^2$

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Focus on the case $\mu_B \simeq 0$ for the high energetic ion-ion collisions at the LHC or RHIC.
The bath is mainly populated by π, K ... $\rightarrow$
chiral effective theory for the D meson interacting with light mesons

See talk by Daniel Cabrera this morning!

$D-$hadron interaction

Light mesons

$$\pi \quad K \quad \bar{K} \quad \eta$$

Effective Lagrangian based on chiral and heavy-quark symmetries see it
-L.S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise
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Potential or perturbative amplitude

$$\begin{aligned} V &= \frac{C_0}{2F^2}(p_1 \cdot p_2 - p_1 \cdot p_4) + \frac{2C_1}{F^2} h_1 + \frac{2C_2}{F^2} h_3 (p_2 \cdot p_4) \\ &+ \frac{2C_3}{F^2} h_5 [(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)] \end{aligned}$$

Isospin coefficients, Low-energy constants

$D-$hadron interaction II

Baryons at finite μ_B

$N \quad \Delta$

We use an effective model based on a Weinberg-Tomozawa interaction (eventually an *s*-wave contact term).

T.Mizutani, A.Ramos *Phys.Rev.C74, 065201 (2006)*

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Tree Level Amplitude

$$V_{ij} = D_{ij} \frac{2\sqrt{s} - M_i - M_j}{4 f_i f_j} \sqrt{\frac{E_i + M_i}{2M_i}} \sqrt{\frac{E_j + M_j}{2M_j}} \ ,$$

D_{ij} coefficients depending on the channel ($IJSC$) similar to the C_i for the light mesons.
f_i are decay constants, analogous to the F in the light-meson sector.

Unitarization

The perturbative amplitudes V are polynomials in energy. They increase arbitrarily, eventually violating the $S-$matrix unitarity.

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We need to "unitarize" the amplitude to restores unitarity and be able to describe resonances.

- On-shell unitarization:
  - J.A.Oller and E. Oset *Nucl.Phys.A620 (1997) 438*
  - L. Roca, E. Oset and J. Singh *Phys.Rev.D72 (2005) 014002*
- Based on $s-$wave $D-\pi$ resummation to construct a Bethe-Salpeter equation

$$T = V + VGV + VGVGV + \cdots = V + VGT$$

Unitarization

Unitarized amplitude

$$T = \frac{V}{1 - GV}$$

Notice that being a rational function it can describe the existence of resonances as poles of the amplitude.

$$\det\,(1 - GV) = 0$$

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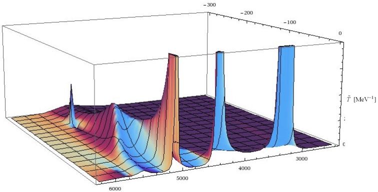
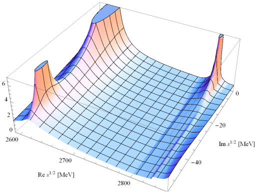
Heavy meson-π scattering with I=1/2.



$DN \to DN$ with $I = 1, J = 1/2$



Results

RESULTS

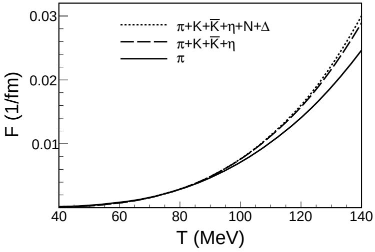
Results in the static limit $p = 100$ MeV

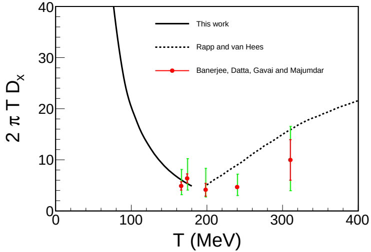
Results at $\mu_B = 0$!

-L.M. Abreu, D. Cabrera, F.J. Llanes-Estrada and JMTR

Annals Phys. 326 (2011) 2737-2772 (pion gas)

-L. Tolos and JMTR, arxiV: 1306.5426 (meson+baryon gas)

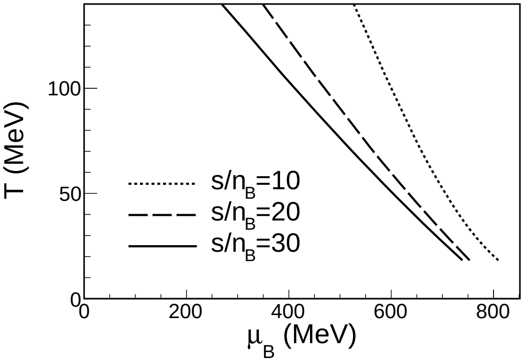




Nice agreement around the cross over temperature at $\mu_B = 0$!

Results at finite chemical potential

For FAIR physics (with $\sqrt{s} = 5 - 40 A\text{GeV}$) we consider hadronic trajectories in the phase diagram and assume that they follow adiabatic expansions (with $S/N_B = 10 - 30$).

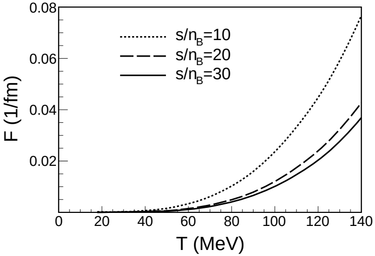


Adiabatic expansion is an idealization!
But it works pretty well, see Zabrodin et al. *J.Phys.* *G*36 (2009) 064065

$$s = \left(\frac{\partial P}{\partial T}\right)_{\mu_B} \qquad n_B = \left(\frac{\partial P}{\partial \mu_B}\right)_T$$

Finite chemical potential

For finite chemical potential the baryon contribution can be large.



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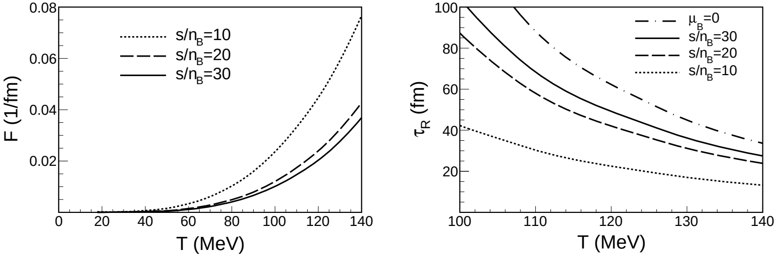
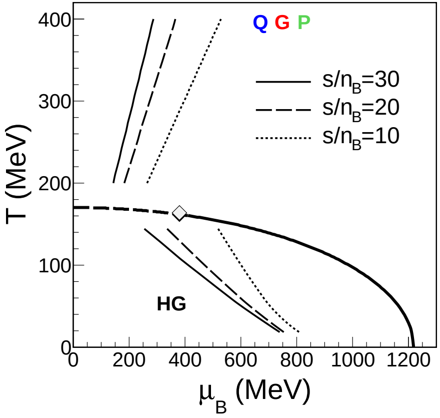


For $T = m_\pi$:

	μ_B (MeV)	n/n_0	τ_R (fm)
$\mu_B = 0$	0	0	33.0
$s/n_B = 30$	286	0.11	27.2
$s/n_B = 20$	361	0.20	23.5
$s/n_B = 10$	536	0.68	13.2

Matching with pQCD



Adiabatic FAIR trajectories following an ideal gas of massless quarks and gluons before T_{had}.

$$P = \frac{7\pi^2 T^4}{60} + \sum_f^{N_f} \left(\frac{\mu_f^2 T^2}{2} + \frac{\mu_f^4}{4\pi^2} \right)$$

Phase boundary parametrized like in J. Kapusta and JMT-R, *Phys.Rev. C86 (2012) 054911*, but not relevant for this study.

Matching with pQCD

For the perturbative plasma we choose the handy result from G.D.Moore and D.Teaney, *Phys. Rev.C.*71, 064904 (2005) and generalize it to finite quark chemical potential:

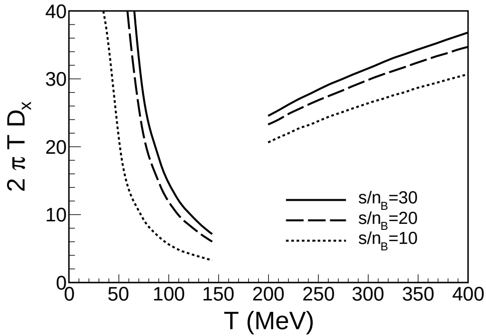
$$\Gamma = \frac{8}{9\pi}\alpha_s^2 \int_0^\infty dk k^2 \int_0^{2k} \frac{dq\ q^3}{[q^2 + m_D^2(T,\mu_f)]^2} \left[\sum_f^{N_f} \frac{e^{(k-\mu_f)/T}}{(e^{(k-\mu_f)/T}+1)^2} \left(2 - \frac{q^2}{2k^2}\right) + 3\frac{e^{k/T}}{(e^{k/T}-1)^2}\left(2 - \frac{q^2}{k^2} + \frac{q^4}{4k^4}\right)\right]$$

with a Debye mass dependent on both T and μ_f

$$m_D^2 = g^2 \left[\left(1 + \frac{N_f}{6}\right) T^2 + \sum_f^{N_f} \frac{\mu_f^2}{2\pi^2} \right] .$$

The spatial diffusion coefficient simply reads: $D_x = T^2/\Gamma$.

Matching with pQCD



- Similar dependence on nuclear density
- We do NOT expect a continuous matching for two different reasons:
  - Perturbative QCD not valid around T_c (only a crude approximation to gain some insight)
  - Evolution is potentially crossing a first-order transition line: discontinuity in transport coefficients

Thanks!

Vielen Dank !

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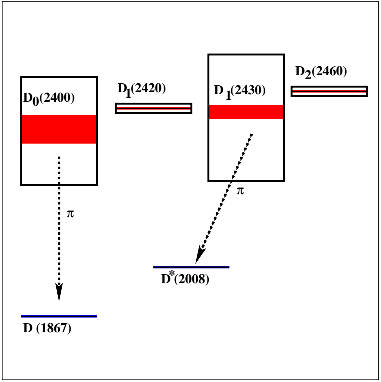
Auxiliar slides

D-meson spectrum



- D meson: fundamental degree of freedom, stable under strong force.
- D^* meson is effectively stable and the HQ partner of D meson.
- $D_0(2400)$ and $D_1(2430)$ are broad resonances decaying to the ground state in s-wave.
- Pions are the most abundant hadrons in the bath.
- K, $\bar{K}$ and η mesons will also be introduced in the same representation.
- Eventually nucleons and Δs to describe baryonic matter at finite μ_B.

Langevin equation

It is an alternative (but equivalent) description of the Fokker-Planck equation.

$$\begin{cases} \frac{dx^i}{dt} & = & \frac{p^i}{m_H} \\ \frac{dp^i}{dt} & = & -F^i(p) + \xi^i(t) \end{cases}$$

where $F^i(p)$ is a deterministic drag force and with ξ^i is a stochastic Gaussian force

$$\langle \xi^i(t) \rangle = 0$$
$$\langle \xi^i(t) \xi^j(t') \rangle = \Gamma^{ij}(p) \; \delta(t-t') \; ,$$

with F^i and Γ^{ij} related through the fluctuation-dissipation theorem

go to diffusion...

Relaxation time

Consider Newton's law (with $F^i = Fp^i$)

$$\frac{dp_i}{dt} = -F\ p^i$$

Assuming constant F one can solve the equation for $p(t)$

$$p(t) = p(0)\ e^{-t/F}$$

The inverse of F plays the role of a relaxation time τ_R

$$\tau_R \ = \ 1/F$$

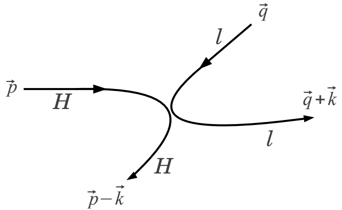
go back

Kinetic Equation

Boltzmann-Uehling-Uhlenbeck Equation

$$\frac{\partial f(\mathbf{p})}{\partial t} = \int d\mathbf{k} \ f(\mathbf{p}+\mathbf{k}) w(\mathbf{p}+\mathbf{k},\mathbf{k}) \ [1+f(\mathbf{p})] - f(\mathbf{p}) w(\mathbf{p},\mathbf{k}) \ [1+f(\mathbf{p}-\mathbf{k})]$$

where $w(\mathbf{p},\mathbf{k})$ represents the probability of a particle with momentum $\mathbf{p}$ of having a collision and loosing a momentum $\mathbf{k}$.



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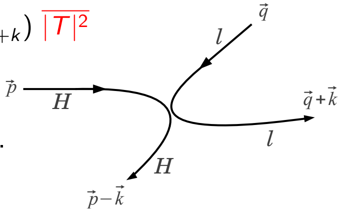
$$w(\mathbf{p},\mathbf{k}) = g_l \int \frac{d\mathbf{q}}{(2\pi)^9} \ f_l(\mathbf{q}) \ \frac{1}{2E^l_q} \frac{1}{2E^H_p} \frac{1}{2E^l_{q+k}} \frac{1}{2E^H_{p-k}} \times (2\pi)^4 \ \delta(E^H_p + E^l_q - E^H_{p-k} - E^l_{q+k}) \ \overline{|T|^2}$$

where $\overline{|T|^2}$ is the scattering amplitude squared.



Fluctuation-Dissipation Theorem

$F(p)$ is a deterministic drag force that causes energy loss (dissipation) whereas the diffusion coefficients are related to the strength of a stochastic (or fluctuating) force.

The fluctuation-dissipation theorem relates the 3 coefficients:

Fluctuation-Dissipation Theorem

$$F(p) + \frac{1}{p}\frac{\partial \Gamma_1(p)}{\partial p} + \frac{2}{p^2}\left[\Gamma_1(p) - \Gamma_0(p)\right] = \frac{\Gamma_1(p)}{m_H T}$$

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In the static limit, i.e. when $p \to 0$ the two diffusion coefficients become degenerate and the Einstein relation is recovered

Einstein Relation

$$F = \frac{\Gamma}{m_H T}$$

$D-\pi$, $D^*-\pi$ interaction

go to interaction details...

Effective Lagrangian: L.S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise *Phys.Rev.D82,05422 (2010)*

Chiral symmetry (NLO) + Heavy Quark symmetry (LO)

$$\mathcal{L}^{(1)} = Tr[\nabla^\mu D \nabla_\mu D^\dagger] - M_D^2 Tr[DD^\dagger] - Tr[\nabla^\mu D^{*\nu} \nabla_\mu D_\nu^{*\dagger}] + M_{D^*}^2 Tr[D^{*\mu} D_\mu^{*\dagger}]$$

$$+ igTr\left[\left(D^{*\mu} u_\mu D^\dagger - Du^\mu D_\mu^{*\dagger}\right)\right] + \frac{g}{2M_D} Tr\left[\left(D_\mu^* u_\alpha \nabla_\beta D_\nu^{*\dagger} - \nabla_\beta D_\mu^* u_\alpha D_\nu^{*\dagger}\right)\epsilon^{\mu\nu\alpha\beta}\right]$$

$$\mathcal{L}^{(2)} = -h_0 Tr[DD^\dagger] Tr[\chi_+] + h_1 Tr[D\chi_+ D^\dagger] + h_2 Tr[DD^\dagger] Tr[u^\mu u_\mu] + h_3 Tr[Du^\mu u_\mu D^\dagger]$$

$$+ h_4 Tr[\nabla_\mu D \nabla_\nu D^\dagger] Tr[u^\mu u^\nu] + h_5 Tr[\nabla_\mu D\{u^\mu, u^\nu\}\nabla_\nu D^\dagger] + \{D \to D^\mu\}$$