Zero temperature properties of mesons and baryons from an extended linear sigma-model

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Motivation

Our ultimate goal: finite temperature/chemical potential investigations $\longrightarrow$ determine the position of the chiral phase boundary in the $T - \mu_B$ plane / determine existence of the CEP in

- an effective model based on global symmetries of QCD
- with all the lightest multiplets (scalar, pseudoscalar, vector, axialvector, baryon octet, baryon decuplet) to give a good approximation

At first $\longrightarrow$ need to describe the zero temperature spectrum as good as possible
$\longrightarrow$ through the parametrization of the model

included particles:

- scalars, pseudoscalars (2 nonets, 18 particles)
- vectors, axialvectors (2 nonets, 18 particles)
- octet baryons, decuplet baryons (1 octet, 1 decuplet; 18 particles)

Problem in the scalar sector

	Mass (MeV)	width (MeV)	decays
$a_0(980)$	(980 ± 20)	$50 - 100$	$\pi\pi$ dominant
$a_0(1450)$	(1474 ± 19)	(265 ± 13)	$\pi\eta$, $\pi\eta'$, $K\bar{K}$
$K_0^\star(800) = \kappa$	(676 ± 40)	(548 ± 24)	$K\pi$
$K_0^\star(1430)$	(1425 ± 50)	(270 ± 80)	$K\pi$ dominant
$f_0(500) = \sigma$	$550 - 1200$	$400 - 700$	$\pi\pi$ dominant
$f_0(980)$	(990 ± 20)	$40 - 100$	$\pi\pi$ dominant
$f_0(1370)$	$1200 - 1500$	$200 - 500$	$\pi\pi \approx 250$, $K\bar{K} \approx 150$
$f_0(1500)$	(1505 ± 6)	(109 ± 7)	$\pi\pi \approx 38$, $K\bar{K} \approx 9.4$
$f_0(1710)$	(1720 ± 6)	(135 ± 8)	$\pi\pi \approx 30$, $K\bar{K} \approx 71$

according to the table: 2 a_0's, 2 K_0's, 5 f_0's $\leftrightarrow$

in our model: 1 scalar nonet with 1 a_0's, 1 K_0's, 2 f_0's

other possible scalar states: meson-meson molecules, glueballs, four quarks

Chiral symmetry

If the quark masses are zero (chiral limit) $\Longrightarrow$ QCD invariant under the following global transformation (chiral symmetry):

$$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A = SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$$

$U(1)_V$ term $\longrightarrow$ baryon number conservation

$U(1)_A$ term $\longrightarrow$ broken through axial anomaly

$SU(3)_A$ term $\longrightarrow$ broken down by any quark mass

$SU(3)_V$ term $\longrightarrow$ broken down to $SU(2)_V$ if $m_u = m_d \neq m_s$ (isospin symmetry)
$\longrightarrow$ totally broken if $m_u \neq m_d \neq m_s$ (realized in nature)

Since QCD is very hard to solve $\longrightarrow$ low energy effective models can be set up $\longrightarrow$ reflecting the global symmetries of QCD $\longrightarrow$ degrees of freedom: observable particles instead of quarks and gluons

Linear realization of the symmetry $\longrightarrow$ linear sigma model
(nonlinear representation $\longrightarrow$ chiral perturbation theory (ChPT))

Extended linear sigma model (meson part)

(based on: chiral symmetry)

$$
\begin{aligned}
\mathcal{L}_{\text{ext}} = {} & \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\
& - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \mathrm{Tr}[\hat{\epsilon}(\Phi + \Phi^\dagger)] \\
& + c_1(\det\Phi - \det\Phi^\dagger)^2 + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger). \\
& + g_3[\mathrm{Tr}(L_\mu L_\nu L^\mu L^\nu) + \mathrm{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4[\mathrm{Tr}\,(L_\mu L^\mu L_\nu L^\nu) + \mathrm{Tr}\,(R_\mu R^\mu R_\nu R^\nu)] \\
& + g_5\mathrm{Tr}\,(L_\mu L^\mu)\ \mathrm{Tr}\,(R_\nu R^\nu) + g_6[\mathrm{Tr}(L_\mu L^\mu)\,\mathrm{Tr}(L_\nu L^\nu) + \mathrm{Tr}(R_\mu R^\mu)\,\mathrm{Tr}(R_\nu R^\nu)] \\
& + \mathcal{L}_{\text{baryon}},
\end{aligned}
$$

where

$$D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA^\mu[T_3, \Phi]$$

$$\Phi = \sum_{i=0}^{8} (\sigma_i + i\pi_i) T_i$$

$$R^\mu = \sum_{i=0}^{8} (\rho_i^\mu - b_i^\mu) T_i$$

$$L^\mu = \sum_{i=0}^{8} (\rho_i^\mu + b_i^\mu) T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA^\nu [T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA^\nu [T_3, R^\mu]\}$$

$$\hat{\epsilon} = \sum_{i=0}^{8} \varepsilon_i T_i \qquad U(3) \text{ generators: } T_0 := \frac{1}{\sqrt{6}}\mathbf{1},\ T_i = \frac{\lambda_i}{2}\ \ i = 1 \ldots 8$$

determinant breaks $U_A(1)$ symmetry

explicit symmetry breaking: external fields $\varepsilon_0, \varepsilon_8 \neq 0 \Longleftrightarrow m_u = m_d \neq 0, m_s \neq 0$ or $\varepsilon_0, \varepsilon_3, \varepsilon_8 \neq 0 \Longleftrightarrow m_u \neq m_d \neq 0, m_s \neq 0$

non strange – strange base:

$$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$$

$$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \qquad \varphi \in (\sigma, \pi, \varepsilon)$$

broken symmetry: non-zero condensates $\langle\sigma_N\rangle, \langle\sigma_S\rangle \longleftrightarrow \phi_N, \phi_S$

Pseudoscalar- and scalarmeson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_S = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$

Scalars: $a_0(980 \text{ or } 1450), K_0^\star(800 \text{ or } 1430)$,

(σ_N, σ_S) : 2 of $f_0(500, 980, 1370, 1500, 1710)$

Vector- and axialvectormeson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star-} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A_V^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(892), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

Extended linear sigma model (baryon part)

$$
\begin{aligned}
\mathcal{L}_{\text{baryon}} &= \text{Tr}\left[\bar{B}\left(i\not{D} - M_{(8)}\right)B\right] \\
&- \text{Tr}\left\{\bar{\Delta}_\mu\left[\left(i\not{D} - M_{(10)}\right)g^{\mu\nu} - i\left(\gamma^\mu D^\nu + \gamma^\nu D^\mu\right)\gamma^\mu\left(i\not{D} + M_{(10)}\right)\gamma^\nu\right]\Delta_\nu\right\} \\
&+ C\text{Tr}\left[\bar{\Delta}^\mu \cdot \left(-\frac{1}{f}(\partial_\mu - ieA^e_\mu[T_3,\Phi]) - \frac{g_1}{f}[\Phi, V_\mu] + g_1 A_\mu\right)B\right] + \text{h. c.} \\
&- \xi_1 \text{Tr}\left(\bar{B}B\right)\text{Tr}\left(\Phi^\dagger\Phi\right) - \xi_2 \text{Tr}\left(\bar{B}\{\{\Phi,\Phi^\dagger\},B\}\right) - \xi_3 \text{Tr}\left(\bar{B}[\{\Phi,\Phi^\dagger\},B]\right) \\
&- \xi_4\left(\text{Tr}\left(\bar{B}\Phi\right)\text{Tr}\left(\Phi^\dagger B\right) + \text{Tr}\left(\bar{B}\Phi^\dagger\right)\text{Tr}\left(\Phi B\right)\right) - \xi_5 \text{Tr}\left(\bar{B}\{[\Phi,\Phi^\dagger],B\}\right) \\
&- \xi_6 \text{Tr}\left(\bar{B}[[\Phi,\Phi^\dagger],B]\right) - \xi_7\left(\text{Tr}\left(\bar{B}\Phi\right)\text{Tr}\left(\Phi^\dagger B\right) - \text{Tr}\left(\bar{B}\Phi^\dagger\right)\text{Tr}\left(\Phi B\right)\right) \\
&- \xi_8\left(\text{Tr}\left(\bar{B}\Phi B\Phi^\dagger\right) - \text{Tr}\left(\bar{B}\Phi^\dagger B\Phi\right)\right) + \chi_1 \text{Tr}\left(\bar{\Delta}\cdot\Delta\right)\text{Tr}\left(\Phi^\dagger\Phi\right) \\
&+ \chi_2 \text{Tr}\left((\bar{\Delta}\cdot\Delta)\{\Phi,\Phi^\dagger\}\right) + \chi_3 \text{Tr}\left((\bar{\Delta}\cdot\Phi)(\Phi^\dagger\cdot\Delta) + (\bar{\Delta}\cdot\Phi^\dagger)(\Phi\cdot\Delta)\right) \\
&+ \chi_4 \text{Tr}\left((\bar{\Delta}\cdot\Delta)[\Phi,\Phi^\dagger]\right)
\end{aligned}
$$

note: mass terms based on every possible invariants
which contains $2\ B$ (or Δ) and $2\ \Phi$ fields $\longrightarrow$ only blue terms contribute

Baryon octet, decuplet

$$B = \sqrt{2}\sum_{i=0}^{8} b_a T_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ -\Xi^- & \Xi^0 & -\frac{2}{\sqrt{2}}\Lambda^0 \end{pmatrix}$$

$$\Delta_\mu^{111} = \Delta_\mu^{++}, \quad \Delta_\mu^{112} = \frac{1}{\sqrt{3}}\Delta_\mu^{+}, \quad \Delta_\mu^{122} = \frac{1}{\sqrt{3}}\Delta_\mu^{0}, \quad \Delta_\mu^{222} = \Delta_\mu^{-},$$

$$\Delta_\mu^{113} = \frac{1}{\sqrt{3}}\Sigma_\mu^{\star+}, \quad \Delta_\mu^{123} = \frac{1}{\sqrt{6}}\Sigma_\mu^{\star 0}, \quad \Delta_\mu^{223} = \frac{1}{\sqrt{3}}\Sigma_\mu^{\star-},$$

$$\Delta_\mu^{133} = \frac{1}{\sqrt{3}}\Xi_\mu^{\star 0}, \quad \Delta_\mu^{233} = \frac{1}{\sqrt{3}}\Xi_\mu^{\star-},$$

$$\Delta_\mu^{333} = \Omega_\mu^{-}$$

Particle content:

Octet baryons: $p/n(938), \Sigma(1193), \Xi(1315), \Lambda(1116)$

Decuplet baryons: $\Delta(1232), \Sigma^\star(1385), \Xi^\star(1530), \Omega(1672)$

Spontaneous symmetry breaking and particle mixing

SSB $\longrightarrow$ through Higgs mechanism generates particle masses $\longrightarrow$ since vacuum has zero quantum numbers $\longrightarrow$ only $\sigma_0, \sigma_8, \sigma_3$ (equivalently $\sigma_N, \sigma_S, \sigma_3$) can have non-zero vev ($\sigma_3 \longrightarrow$ isospin violation $\longrightarrow$ neglected)

note: pion/kaon condensates $\longrightarrow$ even other σ's have non-zero expectation values ($\longrightarrow$ parity, charge violation)

shifting with vev in the Lagrangian: $\sigma_i \to \sigma_i + \phi_i$ ($\longrightarrow$ mass generation)

- For (pseudo)scalars this shifting results in particle mixing in the $N - S$ sector $\longrightarrow \sigma_N/\pi_N, \sigma_S/\pi_S$ fields are not mass eigenstates $\longrightarrow$ orthogonal transformations needed to resolve
- For (axial)vectors $\longrightarrow$ mixing between different nonets $\longrightarrow$ resolved by certain field shiftings $\longrightarrow$ results in: field renormalization constants
- For baryons there is no mixing

Tree-level meson masses

Pseudoscalar mass squares:

$$m_\pi^2 = Z_\pi^2 \left[m_0^2 + \Lambda_N \Phi_N^2 + \lambda_1 \Phi_S^2 \right]$$

$$m_K^2 = Z_K^2 \left[m_0^2 + \Lambda_N \Phi_N^2 - \frac{\lambda_2}{\sqrt{2}} \Phi_N \Phi_S + \Lambda_S \Phi_S^2 \right]$$

$$m_{\eta_N}^2 = Z_\pi^2 \left[m_0^2 + \Lambda_N \Phi_N^2 + \lambda_1 \Phi_S^2 + c_1 \, \Phi_N^2 \Phi_S^2 \right]$$

$$m_{\eta_S}^2 = Z_{\eta_S}^2 \left[m_0^2 + \lambda_1 \Phi_N^2 + \Lambda_s \Phi_S^2 + \frac{c_1}{4} \Phi_N^4 \right]$$

$$m_{\eta_{NS}}^2 = Z_\pi Z_{\pi_S} \frac{c_1}{2} \Phi_N^3 \Phi_S$$

Scalar mass squares:

$$m_{a_0}^2 = m_0^2 + \Lambda'_N \Phi_N^2 + \lambda_1 \Phi_S^2$$

$$m_{K_S}^2 = Z_{K_S}^2 \left[m_0^2 + \Lambda_N \Phi_N^2 + \frac{\lambda_2}{\sqrt{2}} \Phi_N \Phi_S + \Lambda_S \Phi_S^2 \right]$$

$$m_{\sigma_N}^2 = m_0^2 + 3\Lambda_N \Phi_N^2 + \lambda_1 \Phi_S^2$$

$$m_{\sigma_S}^2 = m_0^2 + \lambda_1 \Phi_N^2 + 3\Lambda_s \Phi_S^2$$

$$m_{\sigma_{NS}}^2 = 2\lambda_1 \Phi_N \Phi_S$$

Mass square eigenvalues for σ and π in the $N-S$ sector

$$m^2_{f_0^H/f_0^L} = \frac{1}{2}\left[m^2_{\sigma_N} + m^2_{\sigma_S} \pm \sqrt{(m^2_{\sigma_N} - m^2_{\sigma_S})^2 + 4m^2_{\sigma_{NS}}}\right]$$

$$m^2_{\eta'/\eta} = \frac{1}{2}\left[m^2_{\eta_N} + m^2_{\eta_S} \pm \sqrt{(m^2_{\eta_N} - m^2_{\eta_S})^2 + 4m^2_{\eta_{NS}}}\right]$$

Vector mass squares:

$$m^2_\rho = m_1^2 + \frac{1}{2}(h_1 + h_2 + h_3)\Phi_N^2 + \frac{h_1}{2}\Phi_S^2 + 2\delta_N$$

$$m^2_{K^\star} = m_1^2 + H_N\Phi_N^2 + \frac{1}{\sqrt{2}}\Phi_N\Phi_S(h_3 - g_1^2) + H_S\Phi_S^2 + \delta_N + \delta_S$$

$$m^2_{\omega_N} = m^2_\rho$$

$$m^2_{\omega_S} = m_1^2 + \frac{h_1}{2}\Phi_N^2 + \left(\frac{h_1}{2} + h_2 + h_3\right)\Phi_S^2 + 2\delta_S$$

Axialvector meson mass squares:

$$m^2_{a_1} = m_1^2 + \frac{1}{2}(2g_1^2 + h_1 + h_2 - h_3)\Phi_N^2 + \frac{h_1}{2}\Phi_S^2 + 2\delta_N$$

$$m^2_{K_1} = m_1^2 + H_N\Phi_N^2 - \frac{1}{\sqrt{2}}\Phi_N\Phi_S(h_3 - g_1^2) + H_S\Phi_S^2 + \delta_N + \delta_S$$

$$m^2_{f_{1N}} = m^2_{a_1}$$

$$m^2_{f_{1S}} = m_1^2 + \frac{h_1}{2}\Phi_N^2 + \left(2g_1^2 + \frac{h_1}{2} + h_2 - h_3\right)\Phi_S^2 + 2\delta_S$$

Tree-level baryon masses

Octet masses:

$$m_p = m_n = \frac{1}{2}\xi_1(\Phi_N^2 + \Phi_S^2) + \frac{1}{2}\xi_2(\Phi_N^2 + 2\Phi_S^2) + \frac{1}{2}\xi_3(\Phi_N^2 - 2\Phi_S^2)$$

$$m_\Xi = \frac{1}{2}\xi_1(\Phi_N^2 + \Phi_S^2) + \frac{1}{2}\xi_2(\Phi_N^2 + 2\Phi_S^2) - \frac{1}{2}\xi_3(\Phi_N^2 - 2\Phi_S^2)$$

$$m_\Sigma = \frac{1}{2}\xi_1(\Phi_N^2 + \Phi_S^2) + \xi_2\Phi_N^2$$

$$m_\Lambda = \frac{1}{2}\xi_1(\Phi_N^2 + \Phi_S^2) + \frac{1}{3}\xi_2(\Phi_N^2 + 4\Phi_S^2) + \frac{1}{3}\xi_4(\Phi_N - \sqrt{2}\Phi_S)^2$$

Decuplet masses:

$$m_\Delta = \frac{1}{2}\chi_1(\Phi_N^2 + \Phi_S^2) + \frac{1}{2}\chi_2\Phi_N^2$$

$$m_{\Sigma^\star} = \left(\frac{1}{2}\chi_1 + \frac{1}{3}\chi_2\right)(\Phi_N^2 + \Phi_S^2) + \frac{1}{6}\chi_3(\Phi_N - \sqrt{2}\Phi_S)^2$$

$$m_{\Xi^\star} = \frac{1}{2}\chi_1(\Phi_N^2 + \Phi_S^2) + \frac{1}{6}\chi_2(\Phi_N^2 + 4\Phi_S^2) + \frac{1}{6}\chi_3(\Phi_N - \sqrt{2}\Phi_S)^2$$

$$m_\Omega = \frac{1}{2}\chi_1(\Phi_N^2 + \Phi_S^2) + \chi_2\Phi_N^2$$

Decay widths

For a $A \to BC$ decay process the decay with is:

$$\Gamma_{A\to BC} = \frac{k_A}{8\pi m_A^2} |\mathcal{M}_{A\to BC}|^2$$

$k_A \longrightarrow$ three momentum of the produced particles in the rest frame of A
$\mathcal{M}_{A\to BC} \longrightarrow$ transition matrix element

- If A vectormeson and $C = B^\dagger$ (pseudo)scalarmeson:

  $$|\mathcal{M}_{A\to BB^\dagger}|^2 = \tfrac{4}{3} k_A^2 V_\mu V^{\mu\star}$$

  $V_\mu \longrightarrow$ vertex function directly followed from the three-coupling terms of $\mathcal{L}$

- If A vectormeson, B scalarmeson and $C = \gamma$ (photon):

  $$|\mathcal{M}_{A\to B\gamma}|^2 = \tfrac{1}{3}\left(g^{\alpha\beta} - \frac{k_A^\alpha k_A^\beta}{m_A^2}\right) V_{\alpha\alpha'} V_\beta^{\star\alpha'}$$

- If A vectorspinor B pseudoscalar and C spinor:

  $$|\mathcal{M}_{A\to BC}|^2 = \tfrac{2}{3}|G|^2 k_A^2 m_A (m_C + E_C)$$

  where the vertex function from the Lagrangian $V^\mu = iGk_B^\mu$

Some decay widths

- The $\rho \to \pi\pi$ decay width:

$$\Gamma_{\rho\to\pi\pi} = \frac{m_\rho^5}{48\pi m_{a_1}^4}\left[1-\left(\frac{2m_\pi}{m_\rho}\right)^2\right]^{3/2}\left[g_1 Z_\pi^2 - \frac{g_2}{2}\left(Z_\pi^2-1\right)\right]^2$$

The experimental value from the PDG: $\Gamma^{(\text{exp})}_{\rho\to\pi\pi} = (149.1 \pm 0.8)$ MeV

- The $a_1 \to \pi\gamma$ decay width:

$$\Gamma_{a_1\to\pi\gamma} = \frac{e^2 g_1^2 \Phi_N^2}{96\pi m_{a_1}} Z_\pi^2 \left[1-\left(\frac{m_\pi}{m_{a_1}}\right)^2\right]^3$$

The experimental value: $\Gamma^{(\text{exp})}_{a_1\to\pi\gamma} = (0.640 \pm 0.246)$ MeV

- The $\Delta \to \pi p$ decay width:

$$\Gamma_{\Delta\to\pi p} = \frac{k_\Delta^3}{24 m_\Delta}(m_p + E_p) C^2 Z_\pi^2 (\frac{1}{f^2} + g_1^2 w_{a_1}^2)$$

The experimental value: $\Gamma^{(\text{exp})}_{\Delta\to\pi p} \approx 110$ MeV

Parametrization: general considerations

In order to make predictions $\longrightarrow$ unknown constants of the model must be determined

$\Longrightarrow$ choose a set of (well known) physical quantities/conditions for fitting procedure

For instance:

- PartiallyConservedAxialCurrent $\longrightarrow$ fix the condensates (2 parameter)
- Particle masses (which can be compared with PDG)
- Decay widths (which can be compared with PDG)

Finding a good parameter set $\longrightarrow$ non-trivial task (usually there are lots of solutions, but non of them is perfect)

The parameters are determined in several steps (first: mesons without f_0, second: f_0 mesons, third: baryons)

Parametrization

21 unknown parameters $\longrightarrow$ Determined by the minimalization of the χ^2:

$$\chi^2(x_1,\ldots,x_N) = \sum_{i=1}^{M}\left[\frac{Q_i(x_1,\ldots,x_N) - Q_i^{\text{exp}}}{\delta Q_i^{\text{exp}}}\right]^2,$$

where $(x_1,\ldots,x_N) = (m_0,\lambda_1,\lambda_2,\ldots)$, $Q_i(x_1,\ldots,x_N)$ calculated from the model, while $Q_i^{\text{exp}} \pm \delta Q_i^{\text{exp}}$ taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow 2$ physical quantities: f_π, f_K
- Tree-level masses $\rightarrow 22$ physical quantities:
  - mesons: m_π, m_η, $m_{\eta'}$, m_K, m_ρ, m_Φ, $m_{K^\star}$, m_{a_1}, $m_{f_1^H}$, m_{K_1}, m_{a_0}, m_{K_s}, $m_{f_0^L}$, $m_{f_0^H}$
  - baryons: m_p, m_Ξ, m_Σ, m_Λ, m_Δ, $m_{\Sigma^\star}$, $m_{\Xi^\star}$, m_Ω
- Decay widths $\rightarrow 16$ physical quantities:
  - mesons: $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
  - baryons: $\Gamma_{\Delta\to\pi p}, \Gamma_{\Sigma^\star\to\pi\Lambda}, \Gamma_{\Sigma^\star\to\pi\Sigma}, \Gamma_{\Xi^\star\to\pi\Xi}$

Results

- First run $\rightarrow$ which pairs of $a_0, K_0^\star$ give acceptable fits
  f_0 mesons were left out $\rightarrow$ their properties are very uncertain (Different analyses give different results)

  13 parameters to fit 28 measured quantities

Pair	χ^2	χ^2_{red}
$a_0(1450)/K_0^\star(1430)$	12.33	1.23
$a_0(980)/K_0^\star(800)$	129.36	11.76
$a_0(980)/K_0^\star(1430)$	22.00	2.00
$a_0(1450)/K_0^\star(800)$	242.27	24.23

  The best χ^2 is given by the pair: $a_0(1450), K_0^\star(1430)$

- Second run $\rightarrow$ which pair of f_0's gives a better fit
  Detailed analysis shows $\longrightarrow$ $(f_0(1370), f_0(1710))$ are favored

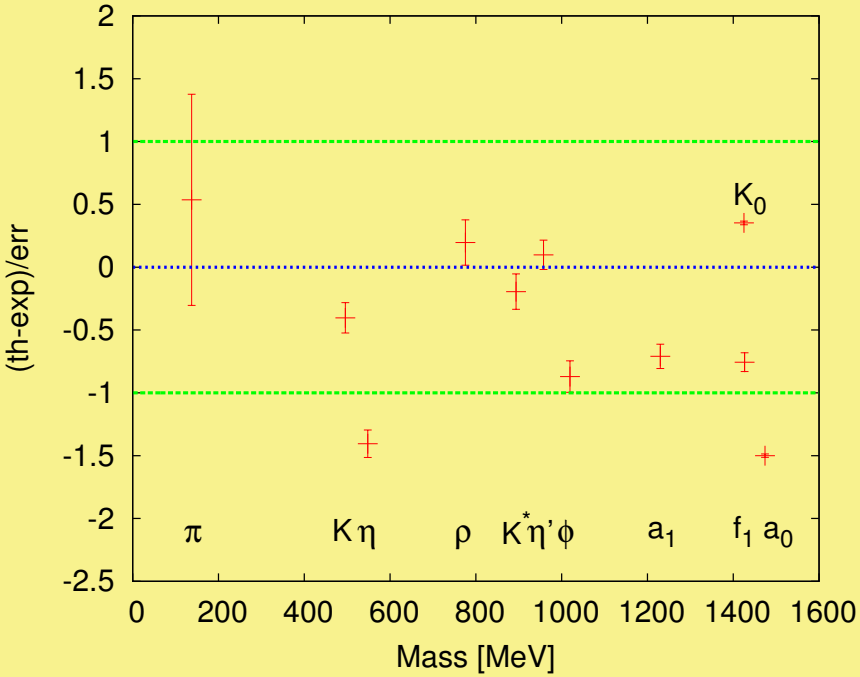
- Third run $\rightarrow$ Describe the baryon octet and decuplet masses and decuplet decays $\rightarrow$ $+8$ parameters for $+12$ physical quantities
  $\rightarrow$ results in a good description of the baryons

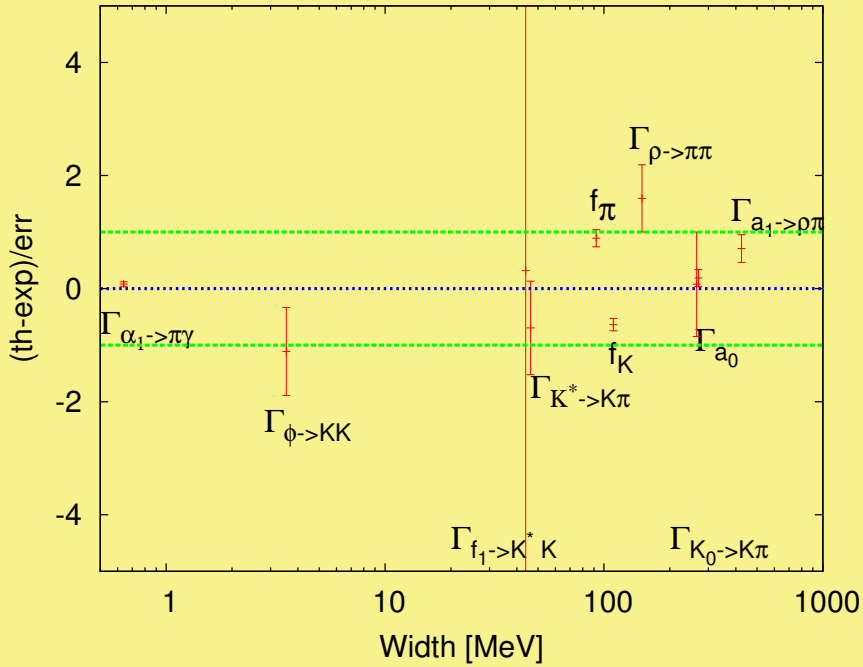
Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
$m_{K^\star}$	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^\star}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho\to\pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi\to\bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1\to\rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1\to\pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^\star\to K\pi}$	285 ± 12	270 ± 80

Observable	Fit [MeV]	χ^2	Experiment [MeV]
m_p	939.0	0.0	939.0 ± 47.0
m_Λ	1116.0	0.0	1116.0 ± 55.8
m_Σ	1193.0	0.0	1193.0 ± 59.7
m_Ξ	1318.0	0.0	1318.0 ± 65.9
m_Δ	1231.9	$5.0 \cdot 10^{-6}$	1232.0 ± 61.6
$m_{\Sigma^\star}$	1385.5	$6.1 \cdot 10^{-5}$	1385.0 ± 69.3
$m_{\Xi^\star}$	1532.3	$7.4 \cdot 10^{-5}$	1533.0 ± 76.7
m_Ω	1672.3	$1.0 \cdot 10^{-5}$	1672.0 ± 83.6
$\Gamma_{\Delta \to p\pi}$	67.3	15.1	110.0 ± 11.0
$\Gamma_{\Sigma^\star \to \Lambda\pi}$	27.0	2.4	32.0 ± 3.2
$\Gamma_{\Sigma^\star \to \Sigma\pi}$	4.9	2.0	4.3 ± 0.4
$\Gamma_{\Xi^\star \to \Xi\pi}$	11.2	3.1	9.5 ± 1.0

- 4 octet masses with 4 parameter $\longrightarrow$ perfect fit (not so surprising)
- 4 decuplet masses with 3 parameters $\longrightarrow$ perfect fit (more surprising)
- 4 decuplet decays with 1 parameter (their ratios are purely kinematical) $\longrightarrow$ acceptable fit

Comparison of theory and experiment for observables





More detail in: Phys. Rev. D **87**, 014011 (2013), [arXiv:1208.0585 [hep-ph]]

$(f_0(1370), f_0(1500), f_0(1710))$: mixing of glueball and the 2 scalar nonet states
$(f_0(500), f_0(980))$: ? molecular states, tetraquark

Summary

- Vacuum phenomenology was presented within the framework of an extended linear sigma model with the lowest lying particle multiplets including scalars, pseudoscalars, vectors, axialvectors, octet baryons, and decuplet baryons

- We used multiparametric χ^2 minimalization for the determination of Lagrangian parameters

- In the meson sector we found that the $a_0(q\bar{q})$ must be assigned to $a_0(1450)$, while the $K_S(q\bar{q})$ to $K_0^\star(1430)$, while the two $f_0(q\bar{q})$ should be assigned to $f_0(1370)$ and $f_0(1710)$

- In the baryon sector we see that the octet and decuplet masses can be described with extremely good precision, while the decuplet decay width with acceptable precision

- The establishment of the vacuum phenomenology gives a good background to future finite temperature/density investigations