Medium-Modification of D Mesons

Four-Quark Condensates and Wilson Coefficients

Extending QCD Sum Rules for D Mesons

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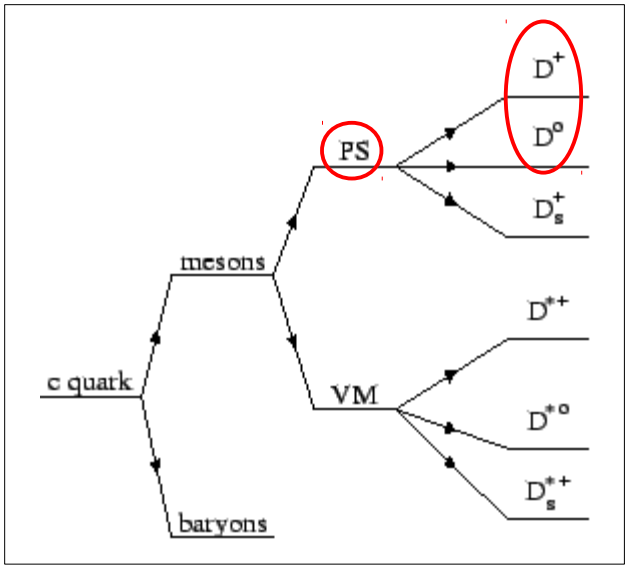
Motivation
D Mesons

quark contents:

$$D^+ \ (c\bar{d}) \,, \quad D^- \ (\bar{c}d)$$
$$D^0 \ (c\bar{u}) \,, \quad \bar{D}^0 \ (\bar{c}u)$$
$$D_s^+ \ (c\bar{s}) \,, \quad D_s^- \ (\bar{c}s)$$



[modified figure from desy.de/~ameyer/hq/node38.html]

$$m_{D^+} = 1870 \text{ GeV}$$
$$m_{D^0} = 1865 \text{ GeV}$$
$$m_{D_s^+} = 1968 \text{ GeV}$$
$$m_{D^{*+}} = 2010 \text{ GeV}$$
$$m_{D^{*0}} = 2007 \text{ GeV}$$
$$m_{D_s^{*+}} = 2112 \text{ GeV}$$

Why D mesons ?

- exact spectral properties as input for investigations of exotic charmed mesons (X, Y, Z, etc.) in vacuum and medium
- serve as probes of hot and dense nuclear matter via medium modifications
  - recent interest [Blaschke et al., PRD 85 (2012)], [He et al., PRL 110 (2013)], [Tolos et al., arXiv:1306.5426 (2013)], [Yasui et al., PRC 87 (2013)]
  - evidence for chiral restoration ?

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Motivation

Chiral Symmetry Breaking / Restoration in Medium

vacuum – (spontaneous) chiral symmetry breaking:

order parameter $\langle \bar{q}q \rangle \neq 0$

further chirally odd condensates (e.g. certain four-quark condensates)

➔ mass splitting of chiral partner mesons

medium modifications:

non-zero temperature (T) and baryon density (n)

$$\langle \bar{q}q \rangle_{T,n} = \langle \bar{q}q \rangle \left(1 - \frac{T^2}{8 f_\pi^2} - \frac{\sigma_N \, n}{m_\pi^2 f_\pi^2} \right)$$

➔ signal of chiral symmetry restoration

impact of chirally odd four-quark condensates ?

ρ meson [Hilger et al., PLB 709 (2012)]

➔ translation to **D mesons** is possible
[Hilger et al., PRC 84 (2011)]

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Motivation

Medium Dependence of D Mesons, CBM & Panda @ FAIR

Why four-quark condensates ?

- change QCD sum rules (e.g. ρ meson)

  more precise spectral properties:

  D meson mass and **width**

- medium modifications

  evidence for chiral restoration if odd

  four-quark condensates vanish

→ enlighten investigations of medium modifications of D mesons by the CBM & Panda experiments @ FAIR

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QCD Sum Rules

causal current-current correlator $$\Pi(q) = \int d^4x\, e^{iqx} \langle \mathrm{T}\left[j(x) j^\dagger(0)\right]\rangle$$

quark d.o.f.

$u \quad d \quad s$
$c \quad b$

QCD

Sum Rule

hadronic d.o.f.

$\pi \quad \rho \quad \omega \quad J/\psi \quad a_1 \quad D \quad B$
$p \quad n \quad \Delta \quad \Lambda$

phenomenology

dispersion relation (from analyticity of $\Pi(q)$) $$\Pi(q^2) = \frac{1}{\pi} \int ds \frac{\mathrm{Im}\Pi(s)}{s - q^2}$$

$$\boxed{\Pi_{\mathrm{OPE}}(q^2)} \quad = \quad \boxed{\int ds \frac{\text{spectral density}\,(s)}{s - q^2}}$$

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In-Medium OPE

determined [Hilger et al., PRC 97 (2009)]

$$\Pi_{\mathrm{OPE}}(q) = \sum_n C_n(q)\langle O_n\rangle$$

$$= C_0(q)\mathbb{1} + C_3(q)\langle\bar{q}q\rangle + C_4(q)\langle G^2\rangle + C_5(q)\langle\bar{q}Gq\rangle$$

$$+ C_{6,q}(q)\langle\bar{q}q\bar{q}q\rangle + C_{6,G}(q)\langle G^3\rangle + \ldots$$

determined for vacuum situations
[Nikolaev, Radyushkin, NPB 213 (1983)]

determined for light mesons [Thomas et al., PRL 95 (2005)]

HERE: qQ mesons

additional condensates containing heavy quarks: e.g. $\langle\bar{q}q\bar{Q}Q\rangle$

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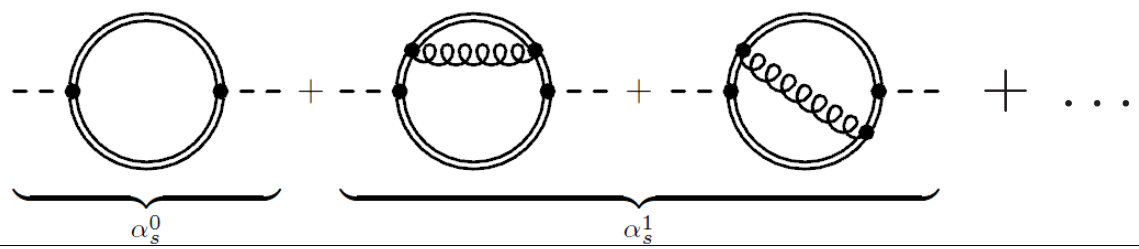
OPE: loop expansion

$$\Pi_{\mathrm{OPE}}(q) = \int d^4x\, e^{ipx} \langle \mathrm{T}\Big[j(x) j^\dagger(0) \Big(\mathbb{1} + \\ + \frac{(i)^2}{2!} \int d^4y_1 d^4y_2 \mathcal{L}_{\mathrm{int}}(y_1) \mathcal{L}_{\mathrm{int}}(y_2) + \dots \Big) \Big] \rangle \\ = \Pi_{\alpha_s^0}(q) + \Pi_{\alpha_s^1} + \dots$$

with reduced interaction Lagrangian:

$$\mathcal{L}_{\mathrm{int}}(y) = g \bar{q}(y) \gamma^\mu t^A q(y) G^A_\mu(y) + (q \longrightarrow Q)$$

$\Pi_{\mathrm{OPE}}(q) =$



$+ \dots$

OPE: QCD quark propagator

background field method in Fock-Schwinger gauge

$$S(p) = \sum_{i=0}^{\infty} S^{(i)}(p) \qquad S^{(i)}(p) = -S^{(i-1)}(p)\gamma^{\mu}\tilde{A}_{\mu}S^{(0)}(p)\,,\ i \geq 1$$

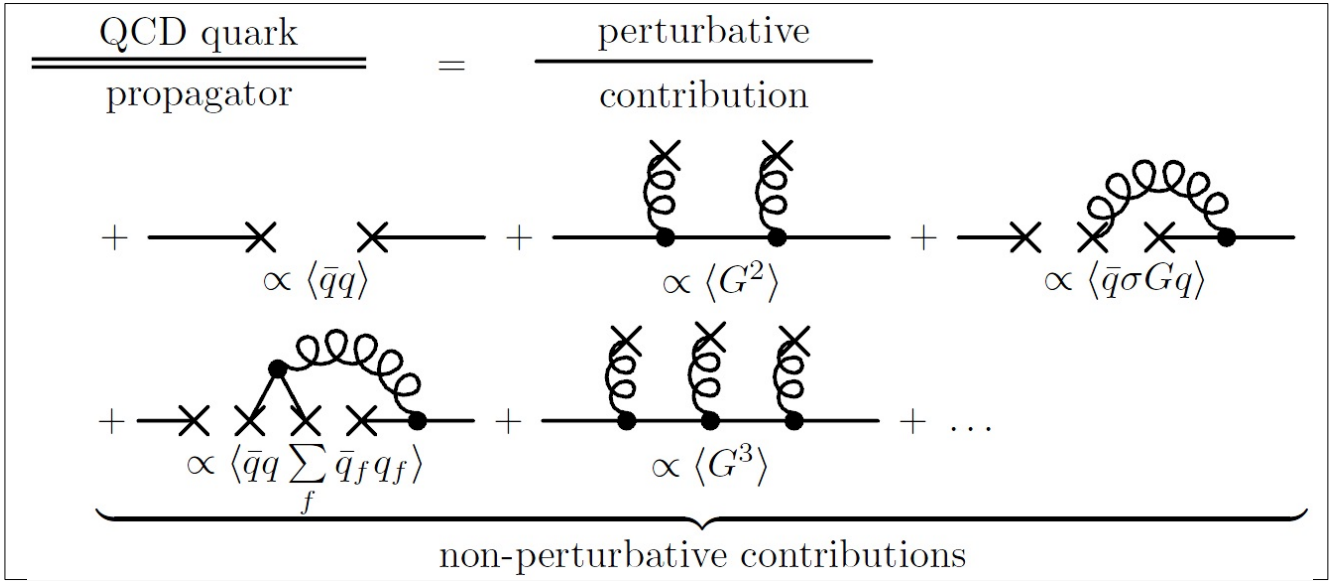
free quark propagator

with derivative operator

$$\tilde{A}_{\mu} = -\sum_{j=0}^{\infty} g\frac{(-i)^j}{j!(j+2)} D_{\vec{\alpha}_j} G_{\mu\nu}\partial^{\nu}\partial^{\vec{\alpha}_j}$$

and 2 Wick uncontracted non-local quark operators

construction of condensates



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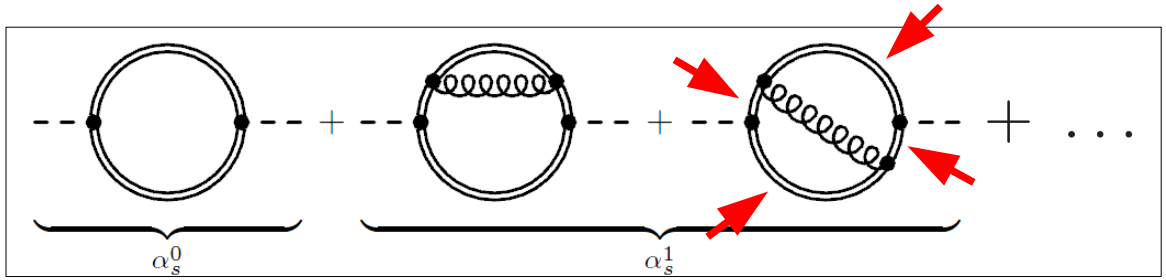
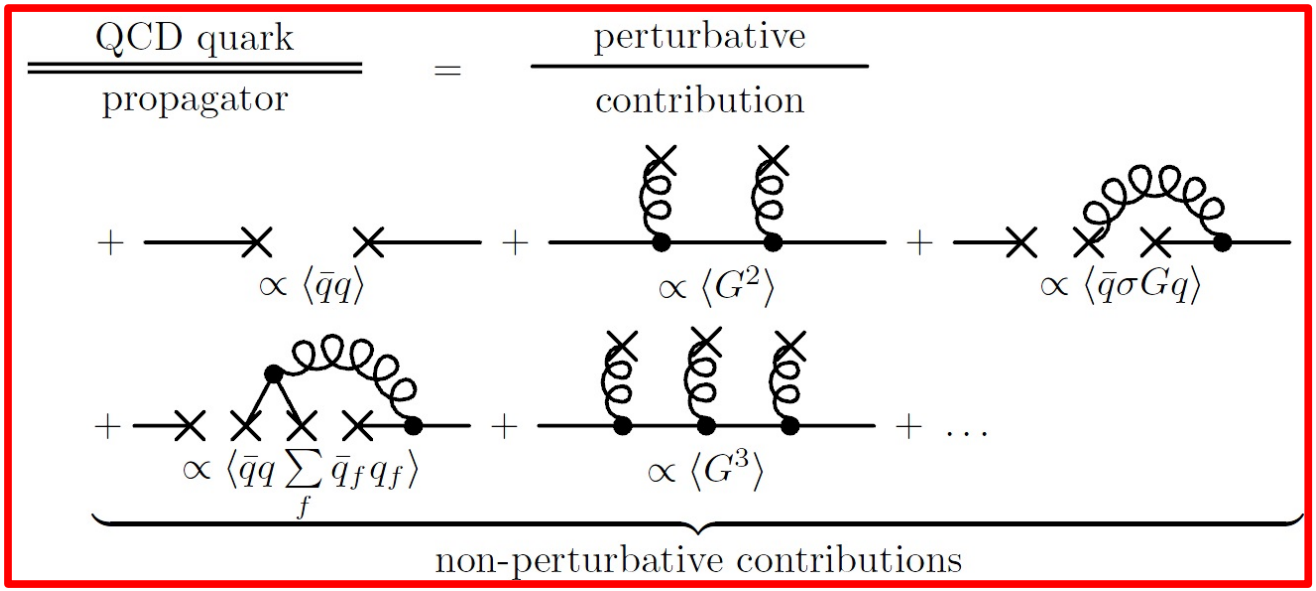
OPE

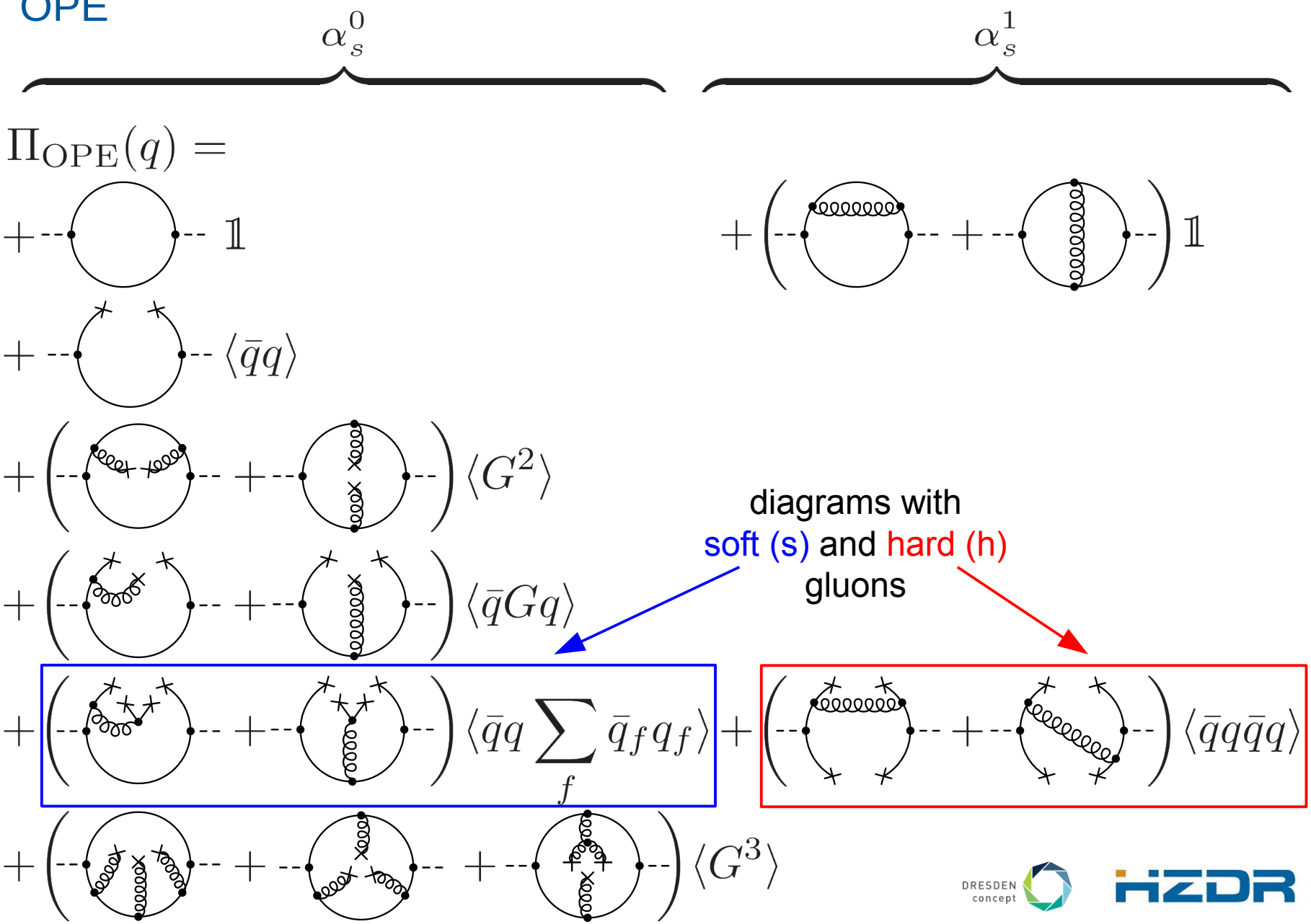
$\Pi_{\mathrm{OPE}}(q) =$



$\otimes$

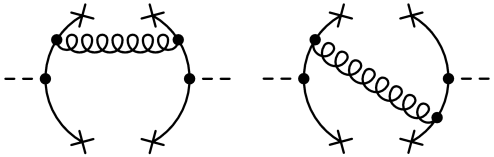


OPE


α_s^0
α_s^1
$\Pi_{\mathrm{OPE}}(q) =$
$\mathbb{1}$
$\langle \bar{q}q \rangle$
$\langle G^2 \rangle$
$\langle \bar{q}Gq \rangle$
$\langle \bar{q}q \sum_f \bar{q}_f q_f \rangle$
$\langle \bar{q}q\bar{q}q \rangle$
$\langle G^3 \rangle$
diagrams with
soft (s) and hard (h)
gluons

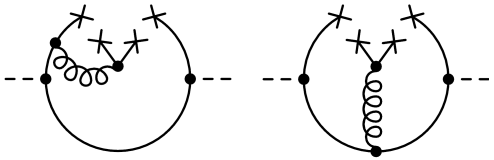
Four-Quark Condensates

$$\Pi_{\mathrm{OPE}}(q) = \sum_k C_k(q) \boxed{\langle \mathcal{O}_k \rangle}$$



$$\langle \mathcal{O}_k \rangle^h = 4\langle \mathcal{O}_k \rangle^h_{(\mathbb{1})} - 3\langle \mathcal{O}_k \rangle^h_{(t^A)}$$

k	$\langle \mathcal{O}_k \rangle^h_{(\mathbb{1})}$	$\langle \mathcal{O}_k \rangle^h_{(t^A)}$
1	$\langle : \bar{q} q \bar{Q} Q : \rangle$	$\langle : \bar{q} t^A q \bar{Q} t^A Q : \rangle$
2	$\langle : \bar{q} \gamma_\nu q \bar{Q} \gamma^\nu Q : \rangle$	$\langle : \bar{q} \gamma_\nu t^A q \bar{Q} \gamma^\nu t^A Q : \rangle$
3	$\langle : \bar{q} \sigma_{\nu\rho} q \bar{Q} \sigma^{\nu\rho} Q : \rangle$	$\langle : \bar{q} \sigma_{\nu\rho} t^A q \bar{Q} \sigma^{\nu\rho} t^A Q : \rangle$
4	$\langle : \bar{q} \gamma_5 \gamma_\nu q \bar{Q} \gamma_5 \gamma^\nu Q : \rangle$	$\langle : \bar{q} \gamma_5 \gamma_\nu t^A q \bar{Q} \gamma_5 \gamma^\nu t^A Q : \rangle$
5	$\langle : \bar{q} \gamma_5 q \bar{Q} \gamma_5 Q : \rangle$	$\langle : \bar{q} \gamma_5 t^A q \bar{Q} \gamma_5 t^A Q : \rangle$
6	$\langle : \bar{q} \not{v} q \bar{Q} \not{v} Q : \rangle / v^2$	$\langle : \bar{q} \not{v} t^A q \bar{Q} \not{v} t^A Q : \rangle / v^2$
7	$\langle : \bar{q} \sigma^{\sigma\omega} q \bar{Q} \sigma^{\nu\rho} Q : \rangle g_{\nu\omega} v_\sigma v_\rho / v^2$	$\langle : \bar{q} \sigma^{\sigma\omega} t^A q \bar{Q} \sigma^{\nu\rho} t^A Q : \rangle g_{\nu\omega} v_\sigma v_\rho / v^2$
8	$\langle : \bar{q} \gamma_5 \not{v} q \bar{Q} \gamma_5 \not{v} Q : \rangle / v^2$	$\langle : \bar{q} \gamma_5 \not{v} t^A q \bar{Q} \gamma_5 \not{v} t^A Q : \rangle / v^2$
9	$\langle : \bar{q} \not{v} q \bar{Q} Q : \rangle$	$\langle : \bar{q} \not{v} t^A q \bar{Q} t^A Q : \rangle$
10	$\langle : \bar{q} q \bar{Q} \not{v} Q : \rangle$	$\langle : \bar{q} t^A q \bar{Q} \not{v} t^A Q : \rangle$
11	$\langle : \bar{q} \sigma^{\sigma\omega} q \bar{Q} \gamma_5 \gamma^\nu Q : \rangle \varepsilon_{\alpha\nu\sigma\omega} v^\alpha$	$\langle : \bar{q} \sigma^{\sigma\omega} t^A q \bar{Q} \gamma_5 \gamma^\nu t^A Q : \rangle \varepsilon_{\alpha\nu\sigma\omega} v^\alpha$
12	$\langle : \bar{q} \gamma_5 \gamma^\nu q \bar{Q} \sigma^{\sigma\omega} Q : \rangle \varepsilon_{\alpha\nu\sigma\omega} v^\alpha$	$\langle : \bar{q} \gamma_5 \gamma^\nu t^A q \bar{Q} \sigma^{\sigma\omega} t^A Q : \rangle \varepsilon_{\alpha\nu\sigma\omega} v^\alpha$



k	$\langle \mathcal{O}_k \rangle^s$
1	$\langle : \bar{q} \gamma^\nu t^A q \sum_f \bar{q}_f \gamma_\nu t^A q_f : \rangle$
2	$\langle : \bar{q} \not{v} t^A q \sum_f \bar{q}_f \not{v} t^A q_f : \rangle / v^2$
3	$\langle : \bar{q} t^A q \sum_f \bar{q}_f \not{v} t^A q_f : \rangle$
4	$\langle : \bar{Q} \gamma^\nu t^A Q \sum_f \bar{q}_f \gamma_\nu t^A q_f : \rangle$
5	$\langle : \bar{Q} \not{v} t^A Q \sum_f \bar{q}_f \not{v} t^A q_f : \rangle / v^2$
6	$\langle : \bar{Q} t^A Q \sum_f \bar{q}_f \not{v} t^A q_f : \rangle$

medium

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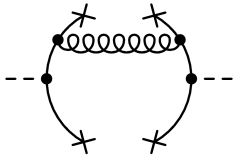
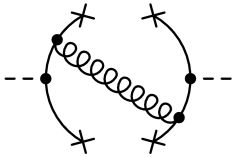
Wilson Coefficients

$$\Pi_{\mathrm{OPE}}(q) = \sum_k \boxed{C_k(q)} \langle \mathcal{O}_k \rangle$$

$$C_k(q) = \sum_l \mathcal{C}_{kl} L_{kl}$$



	$\mathcal{C}^h_{kl}$		L^h_{kl}	
k	$l=1$	$l=2$	$l=1$	$l=2$
1	$\frac{1}{9}\frac{1}{q^2}\left(\frac{q^2+m_Q^2}{(q^2-m_Q^2)^2}+\frac{1}{q^2}\right)$	0	1	—
2	$-\frac{1}{36}\frac{1}{q^2}\left(\frac{q^2-2m_Q^2}{(q^2-m_Q^2)^2}+\frac{1}{q^2}\right)$	$\frac{1}{108}\frac{1}{q^2}\left(\frac{1}{(q^2-m_Q^2)^2}+\frac{1}{q^4}\right)$	1	$q^2-4\frac{(vq)^2}{v^2}$
4	$\frac{1}{36}\frac{1}{q^2}\left(\frac{q^2+2m_Q^2}{(q^2-m_Q^2)^2}+\frac{1}{q^2}\right)$	$-\frac{1}{108}\frac{1}{q^2}\left(\frac{1}{(q^2-m_Q^2)^2}+\frac{1}{q^4}\right)$	1	$q^2-4\frac{(vq)^2}{v^2}$
5	$\frac{1}{9}\frac{1}{q^2}\left(\frac{1}{q^2-m_Q^2}+\frac{1}{q^2}\right)$	0	1	—
6	0	$-\frac{1}{27}\frac{1}{q^2}\left(\frac{1}{(q^2-m_Q^2)^2}+\frac{1}{q^4}\right)$	—	$q^2-4\frac{(vq)^2}{v^2}$
8	0	$\frac{1}{27}\frac{1}{q^2}\left(\frac{1}{(q^2-m_Q^2)^2}+\frac{1}{q^4}\right)$	—	$q^2-4\frac{(vq)^2}{v^2}$
9	0	$-\frac{2}{9}\frac{m_Q}{q^2(q^2-m_Q^2)^2}$	—	$\frac{(vq)}{v^2}$
10	0	$-\frac{1}{9}\frac{m_Q}{q^2(q^2-m_Q^2)^2}$	—	$\frac{(vq)}{v^2}$
11	0	$-\frac{1}{18}\frac{m_Q}{q^2(q^2-m_Q^2)^2}$	—	$\frac{(vq)}{v^2}$



	$\mathcal{C}^h_{kl}$		L^h_{kl}	
k	$l=1$	$l=2$	$l=1$	$l=2$
1	$\frac{1}{9}\frac{1}{q^2(q^2-m_Q^2)}$	0	1	—
2	$\frac{1}{18}\frac{1}{q^2(q^2-m_Q^2)}$	$-\frac{1}{54}\frac{1}{q^4(q^2-m_Q^2)}$	1	$q^2-4\frac{(vq)^2}{v^2}$
3	$-\frac{1}{18}\frac{1}{q^2(q^2-m_Q^2)}$	0	1	—
4	$-\frac{1}{18}\frac{1}{q^2(q^2-m_Q^2)}$	$\frac{1}{54}\frac{1}{q^4(q^2-m_Q^2)}$	1	$q^2-4\frac{(vq)^2}{v^2}$
5	$\frac{1}{9}\frac{1}{q^2(q^2-m_Q^2)}$	0	1	—
6	0	$\frac{2}{27}\frac{1}{q^4(q^2-m_Q^2)}$	—	$q^2-4\frac{(vq)^2}{v^2}$
8	0	$-\frac{2}{27}\frac{1}{q^4(q^2-m_Q^2)}$	—	$q^2-4\frac{(vq)^2}{v^2}$
9	0	$-\frac{1}{9}\frac{m_Q}{q^4(q^2-m_Q^2)}$	—	$\frac{(vq)}{v^2}$
10	0	$-\frac{2}{9}\frac{m_Q}{q^4(q^2-m_Q^2)}$	—	$\frac{(vq)}{v^2}$
11	0	$-\frac{1}{18}\frac{m_Q}{q^4(q^2-m_Q^2)}$	—	$\frac{(vq)}{v^2}$

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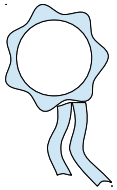
How to handle heavy quarks in four-quark condensates?

4 Approaches:

quality of the sum rule

- neglecting condensates containing heavy quarks, factorization
- factorization and subsequent heavy-quark expansion
- heavy-quark expansion and subsequent factorization
- lattice calculations 

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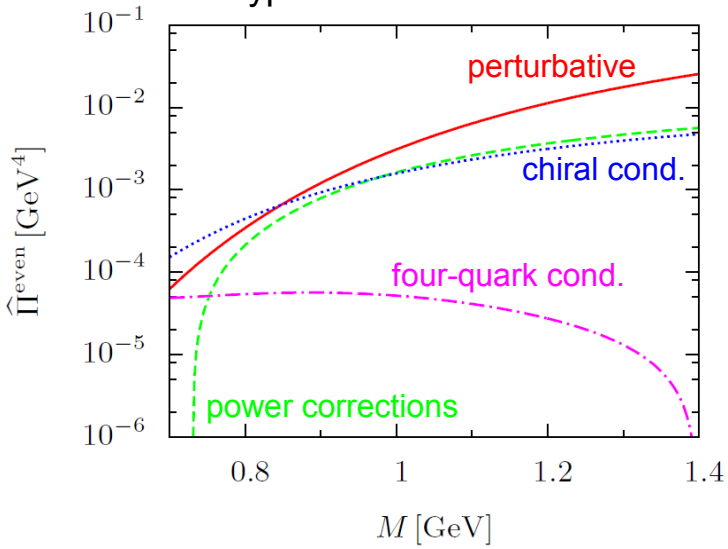
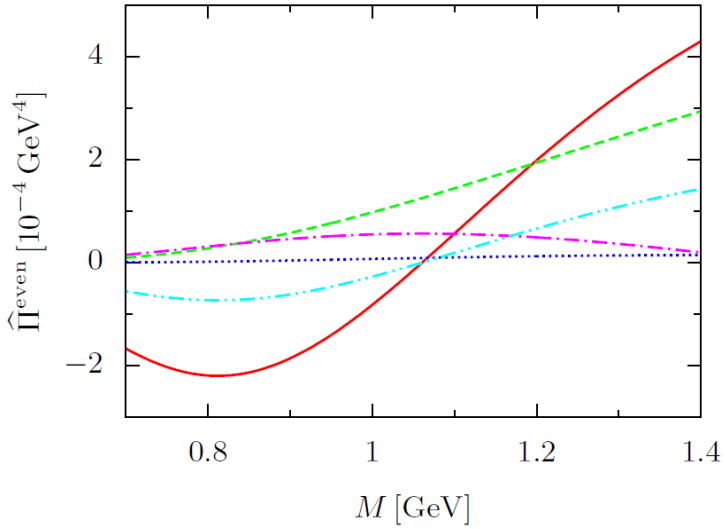
Numerical Evaluation

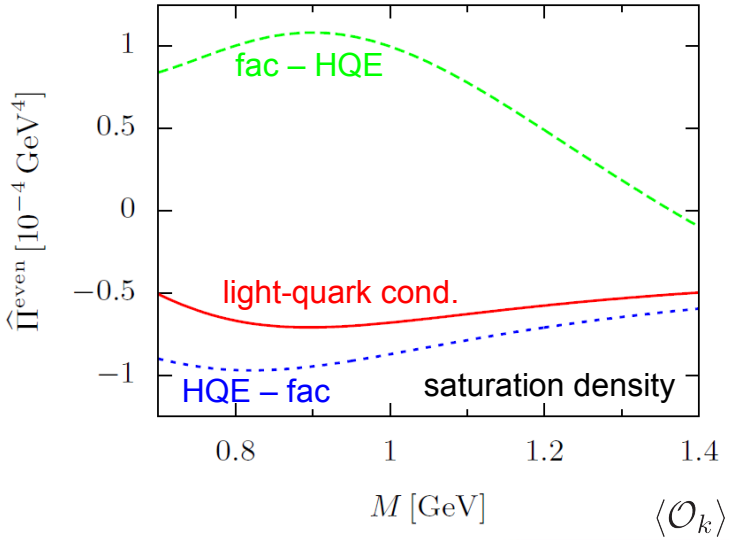
scale of typical OPE contributions

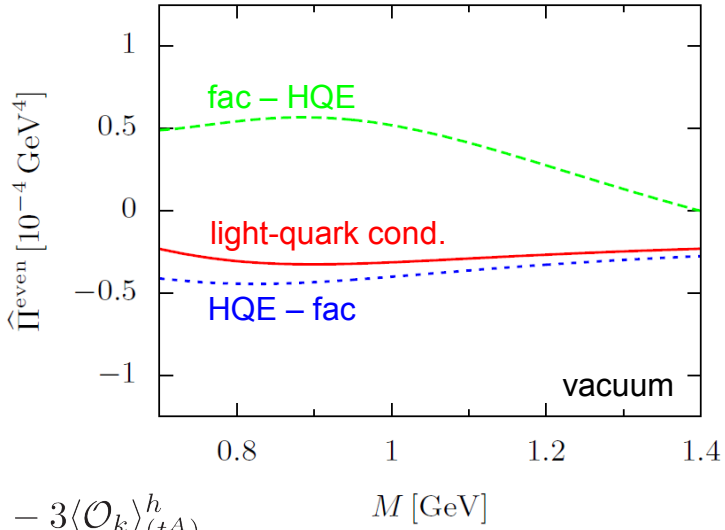


OPE contributions of mass dimension 4 and 5 condensates



OPE contributions of four-quark condensates in three different approaches





$$\langle \mathcal{O}_k \rangle^h = 4\langle \mathcal{O}_k \rangle^h_{(\mathbb{1})} - 3\langle \mathcal{O}_k \rangle^h_{(t^A)}$$

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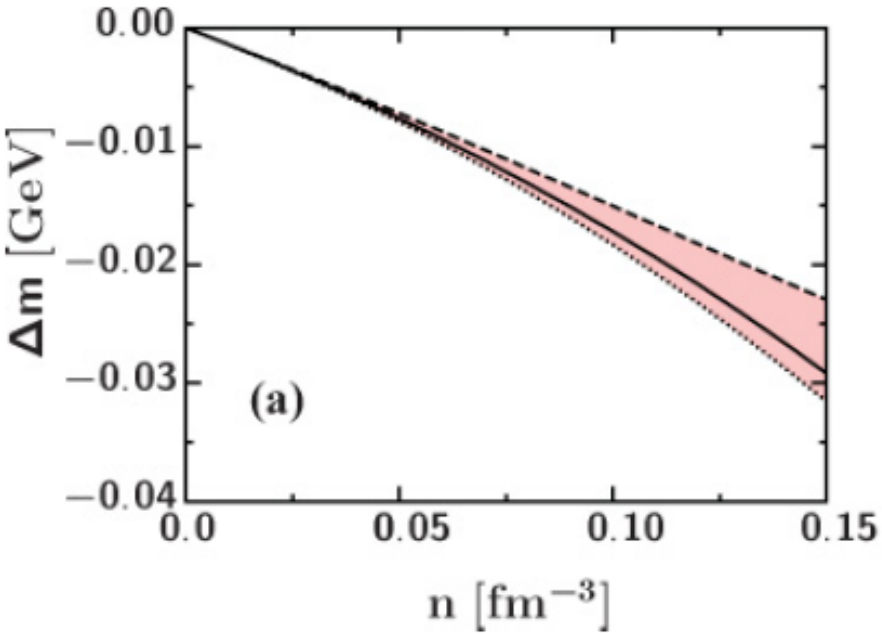
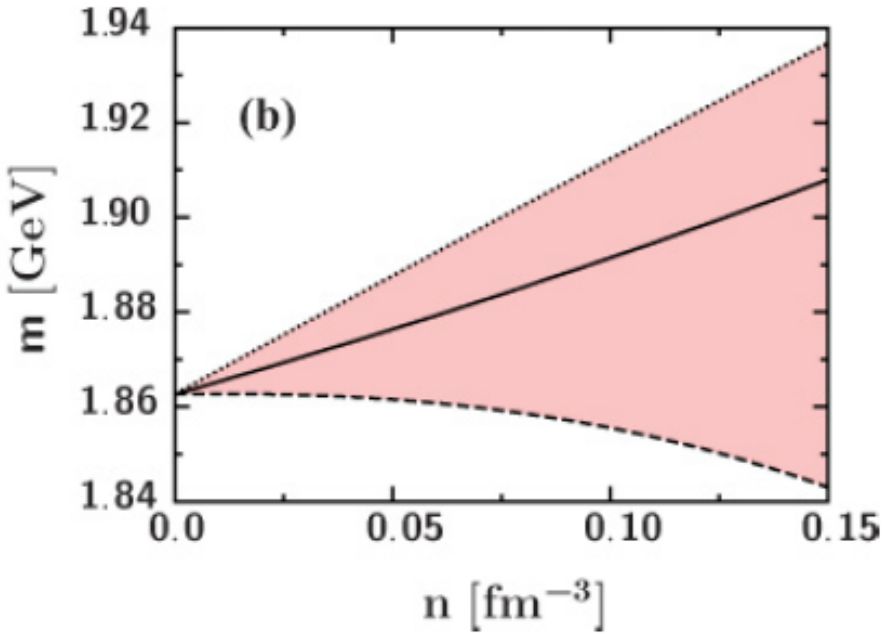
Numerical Evaluation

chiral condensate dominates OPE → [Hilger et al., PRC 79 (2009)] holds





Summary

- Chiral condensate contributions dominate OPE / sum rules of D mesons

- error-prone HQE and factorization four-quark condensate results do not change OPE significantly either in vacuum or in medium

➔ spectral properties obtained in [Hilger et al., PRC 79 (2009)] hold

- four-quark condensates as order parameters of chiral symmetry restoration addressed by chiral partner sum rules (Weinberg-type sum rules) in future work

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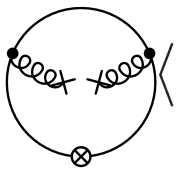
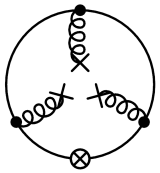
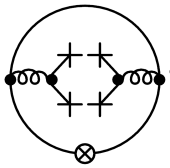
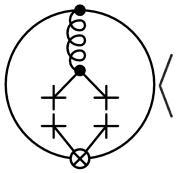
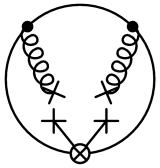
Heavy-Quark Expansion (HQE)

heavy two-quark condensate: [Generalis, Broadhurst, PLB139 (1984)]

$\langle \bar{Q}Q \rangle =$  $\langle G^2 \rangle +$  $\langle G^3 \rangle +$  $\langle \sum_f \bar{q}_f q_f \sum_{f'} \bar{q}_{f'} q_{f'} \rangle + \ldots$

leading order: $$\langle \bar{Q}Q \rangle = -\frac{g^2}{48\pi^2 m_Q} \langle G^2 \rangle + \mathcal{O}(1/m_Q^3)$$

heavy-light four-quark condensate:

$\langle \bar{q}Aq\bar{Q}BQ \rangle =$  $\langle \bar{q}q \sum_f \bar{q}_f q_f \rangle +$  $\langle \bar{q}qG^2 \rangle + \ldots$

leading order:

$$\langle \bar{q}\gamma_\nu t^A q \sum_f \bar{q}_f \gamma^\nu t^A q_f \rangle, \langle \bar{q}\not{v} t^A q \sum_f \bar{q}_f \not{v} t^A q_f \rangle / v^2, \langle \bar{q} t^A q \sum_f \bar{q}_f \not{v} t^A q_f \rangle$$

with HQE coefficiens of order $1/m_Q^0$

Factorization of Four-Quark Condensats

colorless hadronic states
and the QCD vacuum

vacuum

$$\langle \bar{q}\Gamma_1 t^A q \bar{q}\Gamma_2 t^A q\rangle = \sum_n c_n(\Gamma_1,\Gamma_2,t^A)\langle \bar{q}q|n\rangle\langle n|\bar{q}q\rangle$$
$$\approx c_0(\Gamma_1,\Gamma_2,t^A)\langle \bar{q}q\rangle^2$$

medium

reduction of light four-quark condensates:

$$\langle \bar{q}\Gamma_1 t^A q \bar{q}\Gamma_2 t^A q\rangle = a\langle \bar{q}q\rangle^2 + b\langle \bar{q}q\rangle\langle \bar{q}\not{v}q\rangle + c\langle \bar{q}\not{v}q\rangle^2$$

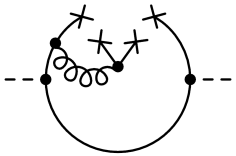
reduction of heavy-light four-quark condensates:

$$\langle \bar{q}\Gamma_1 t^A q \bar{Q}\Gamma_2 t^A Q\rangle = A\langle \bar{q}q\rangle\langle \bar{Q}Q\rangle + B\langle \bar{q}\not{v}q\rangle\langle \bar{Q}Q\rangle$$
$$+ C\langle \bar{q}q\rangle\langle \bar{Q}\not{v}Q\rangle + D\langle \bar{q}\not{v}q\rangle\langle \bar{Q}\not{v}Q\rangle$$

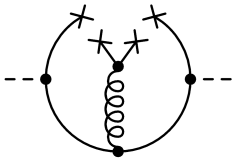
Wilson Coefficients

$$\Pi_{\mathrm{OPE}}(q) = \sum_k \boxed{C_k(q)} \langle \mathcal{O}_k \rangle$$

$$C_k(q) = \sum_l \mathcal{C}_{kl} L_{kl}$$



	$\mathcal{C}^s_{kl}$			L^s_{kl}		
k	$l=1$	$l=2$	$l=3$	$l=1$	$l=2$	$l=3$
1	$-\frac{1}{3}\frac{1}{(q^2-m_Q^2)^2}\left(1-\frac{q^2}{q^2-m_Q^2}\right)^2$	$\frac{2}{9}\frac{1}{(q^2-m_Q^2)^3}$	$-\frac{8}{3}\frac{1}{(q^2-m_Q^2)^4}$	1	$q^2-4\frac{(vq)^2}{v^2}$	$\frac{3}{8}q^4-2\frac{q^2(vq)^2}{v^2}+\frac{(vq)^4}{v^4}$
2	0	$-\frac{4}{9}\frac{1}{(q^2-m_Q^2)^3}$	$\frac{8}{3}\frac{1}{(q^2-m_Q^2)^4}$	—	$q^2-4\frac{(vq)^2}{v^2}$	$q^4-7\frac{q^2(vq)^2}{v^2}+6\frac{(vq)^4}{v^4}$
3	0	$-\frac{4}{3}\frac{m_Q}{(q^2-m_Q^2)^3}$	$\frac{8}{3}\frac{m_Q}{(q^2-m_Q^2)^4}$	—	$\frac{(vq)}{v^2}$	$\frac{(vq)}{v^2}\left(q^2-\frac{(vq)^2}{v^2}\right)$
4	0	$\frac{2}{9}\frac{1}{q^6}$	$-\frac{8}{3}\frac{1}{q^8}$	—	$q^2-4\frac{(vq)^2}{v^2}$	$\frac{3}{8}q^4-2\frac{q^2(vq)^2}{v^2}+\frac{(vq)^4}{v^4}$
5	0	$-\frac{4}{9}\frac{1}{q^6}$	$\frac{8}{3}\frac{1}{q^8}$	—	$q^2-4\frac{(vq)^2}{v^2}$	$q^4-7\frac{q^2(vq)^2}{v^2}+6\frac{(vq)^4}{v^4}$



	$\mathcal{C}^s_{kl}$		L^s_{kl}	
k	$l=1$	$l=2$	$l=1$	$l=2$
1	$\frac{1}{3}\frac{1}{(q^2-m_Q^2)^2}\left(2-\frac{q^2}{q^2-m_Q^2}\right)$	$-\frac{1}{9}\frac{1}{(q^2-m_Q^2)^3}$	1	$q^2-4\frac{(vq)^2}{v^2}$
2	0	$\frac{2}{9}\frac{1}{(q^2-m_Q^2)^3}$	—	$q^2-4\frac{(vq)^2}{v^2}$
3	0	$\frac{2}{3}\frac{m_Q}{(q^2-m_Q^2)^3}$	—	$\frac{(vq)}{v^2}$
4	$\frac{1}{3}\frac{1}{q^4}$	$-\frac{1}{9}\frac{1}{q^6}$	1	$q^2-4\frac{(vq)^2}{v^2}$
5	0	$\frac{2}{9}\frac{1}{q^6}$	—	$q^2-4\frac{(vq)^2}{v^2}$

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