Finite density equation of state of QCD by means of resummed perturbation theory

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Fairness II

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Main references:
Jens O. Andersen, S.M., Nan Su and Aleksi Vuorinen; PRD 87, 074003 [arXiv:1210.0912]
S.M., Jens O. Andersen, Michael Strickland, Nan Su and Aleksi Vuorinen; submitted to JHEP [arXiv:1307.8098]

Why hot and dense QCD via resummed perturbation theory?

- Connect lattice region to asymptotically high energies
- Qualitative understanding of plasma properties, even at large μ_f/T
- Show usefulness of weak coupling methods for finite μ_f problems

Why diagonal quark number susceptibilities?

- Natural quantities for direct comparison with lattice
- Carries information about the response of a system to nonzero μ_f

A few reminders:

- Implementing $\mu_f \neq 0$ in (imaginary time formalism) thermal pQCD:
  $k_0^f = (2n+1)\pi T \to (2n+1)\pi T - i\,\mu_f$
- Quark number susceptibilities (QNS) as derivatives of the pressure:

$$\chi_{u_i\, d_j\, s_k \ldots}(T) \equiv \left.\frac{\partial^{i+j+k+\ldots}\; p(T,\{\mu_f\})}{\partial\mu_u^i\,\partial\mu_d^j\,\partial\mu_s^k\ldots}\right|_{\{\mu_f\}=0}$$

HTL gauge invariant effective action: Frenkel, Taylor [Nucl Phys B **334**, 199] and Braaten, Pisarski [Nucl Phys B **337**, 569]

- Reorganization of the perturbative series of thermal pQCD:

$$\mathcal{L}_{\mathrm{HTLpt}} = \left(\mathcal{L}_{\mathrm{QCD}} + \mathcal{L}_{\mathrm{HTL}}\right)\Big|_{g\to\sqrt{\delta}g} + \Delta\mathcal{L}_{\mathrm{HTL}}$$

- With the gauge invariant HTL improvement term:

$$\mathcal{L}_{\mathrm{HTL}} = -\frac{1}{2}(1-\delta)\, m_D^2 \mathrm{Tr}\left(F_{\mu\alpha}\left\langle \frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y F^\mu_\beta\right)$$
$$+(1-\delta)\, i \sum_f^{N_f} m_{q_f}^2\, \bar{\psi}_f\, \gamma^\mu \left\langle \frac{y_\mu}{y\cdot D}\right\rangle_y \psi_f$$

- δ: Formal expansion parameter, $\delta = 1$ in the end $\to$ QCD!
  Count the # of HTL dressed loops
  $\Delta\mathcal{L}_{\mathrm{HTL}}$: HTL counterterm(s); y^μ: Lightlike 4-vector
  m_D/m_{q_f}: Debye/quark thermal mass parameters
- Adding $\mathcal{L}_{\mathrm{HTL}}$: "SHIFTS" the ground state to an ideal gas of thermal (massive!) quasiparticles

- At 1-loop, Debye m_D and quark thermal m_{q_f} mass parameters identified with their weak coupling values: $m_D \sim m_{q_f} \sim gT$
- 1-loop truncated pressure through $\mathcal{O}\big((m_D/T)^5, (m_{q_f}/T)^5\big)$:
  J.O.A., S.M., N.S., A.V. [PRD **87**, 074003]
  2-loop truncated pressure through $\mathcal{O}\big((m_D/T)^5, (m_{q_f}/T)^5\big)$, $\mathcal{O}\big((\mu_f/T)^4\big)$
  and corresponding 2nd and 4th order QNS:
  Haque, Mustafa, Strickland [PRD **87**, 105007] and [JHEP **07**, 184]
- Here and in [arXiv:1307.8098]:
  Full 1-loop results, accurate to all orders in m_D/T and m_{q_f}/T, without any truncations in μ_f/T
  Hence:
  1. No approximations $\rightarrow$ Exact (numerical) result!
  2. Refines the comparison between the truncated 1-loop HTLpt results and lattice data, plus...
  3. ...gives access to higher order QNS as well as the full μ_f-dependence of the pressure itself!
  4. Probes quantitatively the convergence of the truncation respect to the full results (pressure and QNS)

Dimensional reduction of QCD: Braaten, Nieto [PRD **51**, 6990]

- Separation of the pressure into different contributions:

$$p_{\rm QCD} \;=\; p_{\rm hard}(g) + T\, p_{\rm EQCD}(m_{\rm E}, \lambda_{\rm E}, g_{\rm E}, \zeta)$$

Lagrangian density of the effective 3d Yang-Mills plus adjoint Higgs theory i.e. Electrostatic QCD (EQCD):

$$\begin{aligned}\mathcal{L}_{\rm EQCD} \;&\equiv\; \frac{1}{2}\,{\rm Tr}\left[G_{ij}^2\right] + {\rm Tr}\left[(D_i\,A_0)^2\right] + m_{\rm E}^2\,{\rm Tr}\left[A_0^2\right] \\ &+\; i\zeta\,{\rm Tr}\left[A_0^3\right] + \lambda_{\rm E}\,{\rm Tr}\left[A_0^4\right] + \delta\mathcal{L}_{\rm E}\,,\end{aligned}$$

where $\delta\mathcal{L}_{\rm E}$ includes higher order (non-renormalizable) operators.

- Note that:
  1. $p_{\rm hard}$: From hard modes ($\propto 2\pi T$), via strict loop-expansion in the 4d theory
  2. $p_{\rm EQCD}$: From soft modes ($\propto gT$), via effective EQCD

$\Rightarrow$ Gives the correct momentum scale contributions

- Known 4-loop pressure from A.V. [PRD **68**, 054017]
- Natural resummation scheme:
  Suggested by Blaizot, Iancu, Rebhan [PRD **68**, 025011]
  Implemented at $\mu_f = 0$ by Laine, Schröder [PRD **73**, 085009]
  Here ([PRD **87**, 074003] and [arXiv:1307.8098]), we apply it at $\mu_f \neq 0$

  $\Rightarrow$ Keep the EQCD parameters unexpanded. While having the proper terms through $g^6 \log g$, allow the explicit scale dependence in p_{hard} to cancel the one of p_{soft} at $\mathcal{O}\left(g^6\right)$

$$p_{\text{QCD}} = p_{\text{HARD}}(g_{\text{E}}) + T\, p_{\text{SOFT}}(m_{\text{E}}, g_{\text{E}}, \zeta)$$

  Freedom in the reorganization of the higher order contributions, and straightforward application at finite μ_f. Consequences:

  1. Substantially reduces the renormalization scale dependence!
     (by a factor larger than 5 for the 2nd order QNS)
  2. Resums an important class of higher order contributions
- No fitting: Unknown $\mathcal{O}\left(g^6\right)$ term in the pressure set to zero!

- $N_c = 3$ with two and three flavors
- Perturbative running for g: HTLpt/DR $\rightarrow$ 1/2-loop
- QCD scale $\Lambda_{\overline{\text{MS}}}$:
  Matching 1/2-loop running to lattice value at reference scale 1.5 GeV from Bazavov et al. [PRD **86**, 114031]
  Gives $\Lambda_{\overline{\text{MS}}}^{\text{HTLpt/DR}} \rightarrow$ 176/283 MeV ($N_f = 3$); 204/324 MeV ($N_f = 2$), plus a variation of $\pm$30MeV
- Renormalization scale $\bar{\Lambda}$:
  Central value obtained by optimizing the 3d gauge coupling of EQCD, see Kajantie et al. [Nucl. Phys. B **503**, 357]
  Gives (at $\mu_f = 0$) $\bar{\Lambda}_{\text{opt}} \approx 1.44 \times 2\pi T$ ($N_f = 3$); $1.29 \times 2\pi T$ ($N_f = 2$)
  $\bar{\Lambda}$ is then varied by a factor of 2 around the central value

Diagonal QNS...

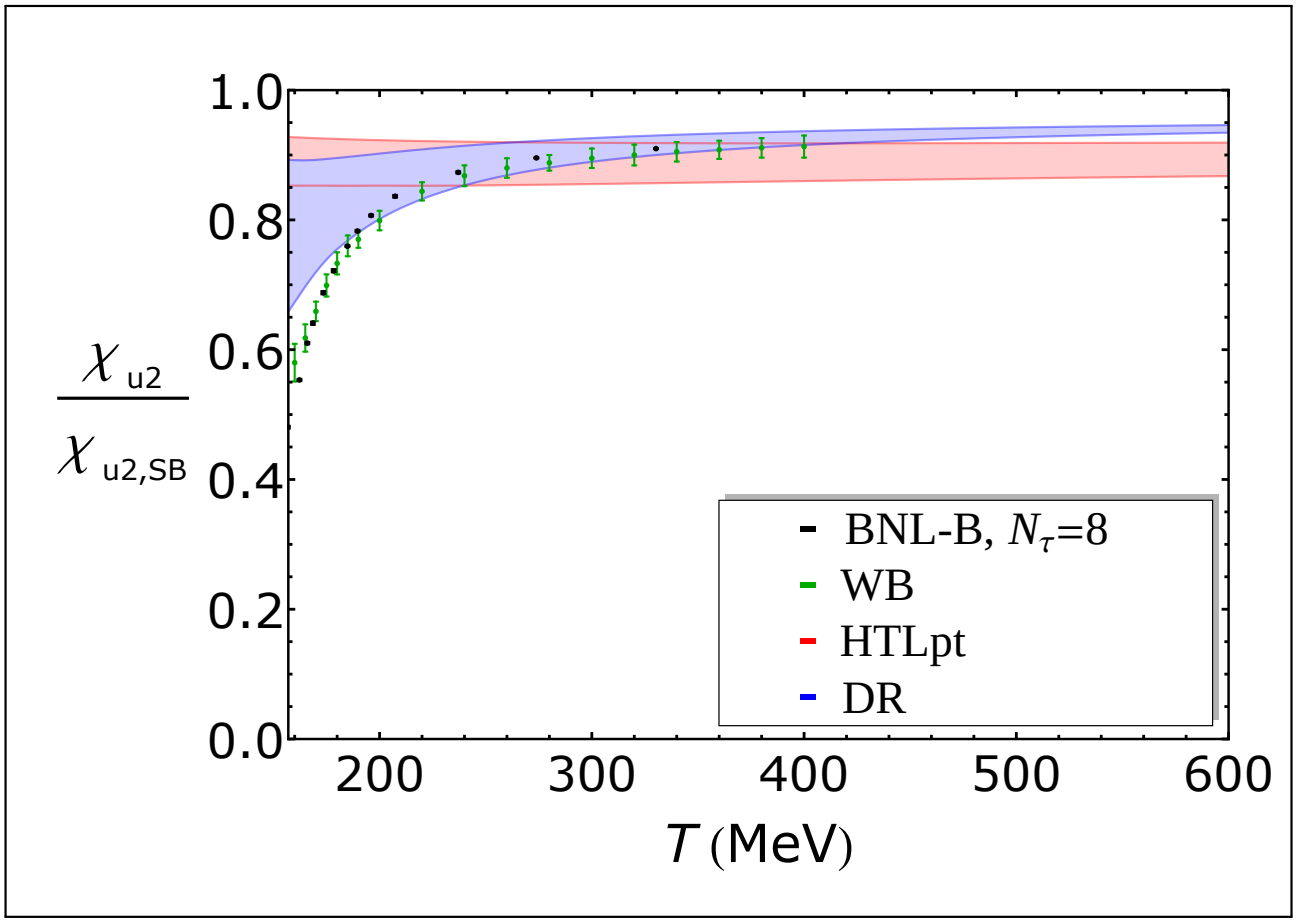

Figure : Ref: [arXiv:1307.8098]. Lattice data from Borsányi et al. [JHEP **01**, 138]; Schmidt [J.Phys. Conf. Ser. **432**, 012013v]; Schmidt [Nucl. Phys. A **904-905**, 865c]

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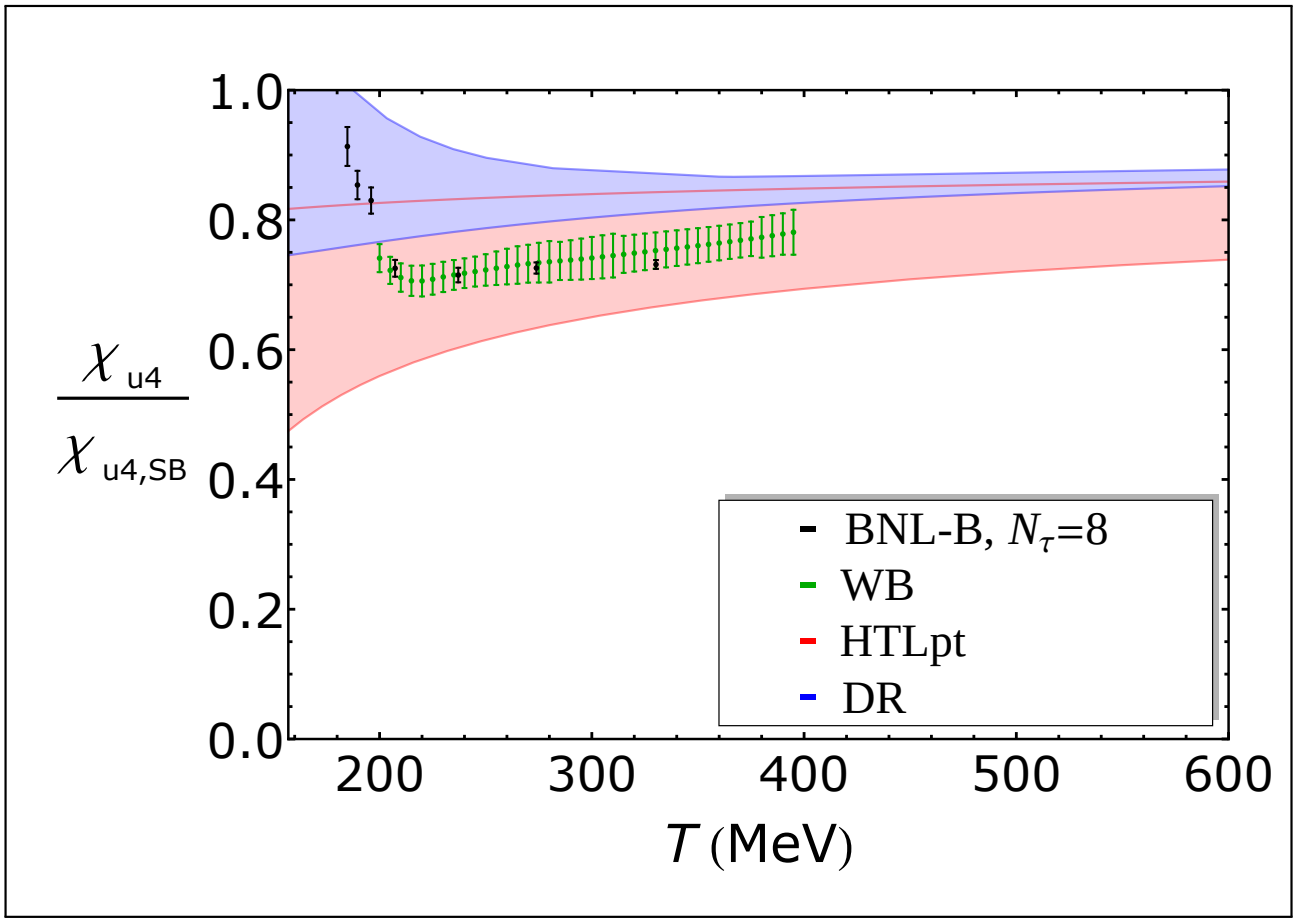

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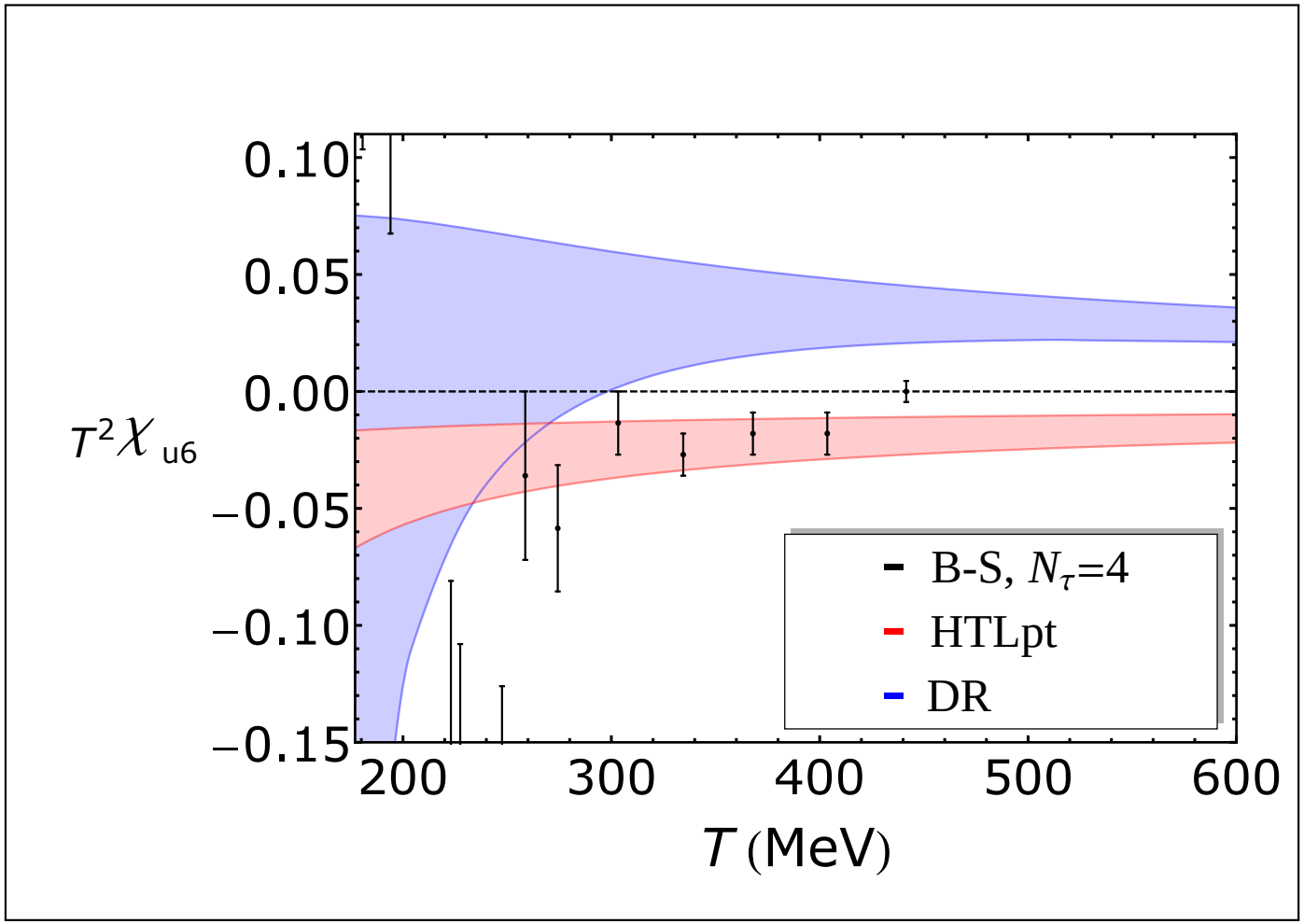
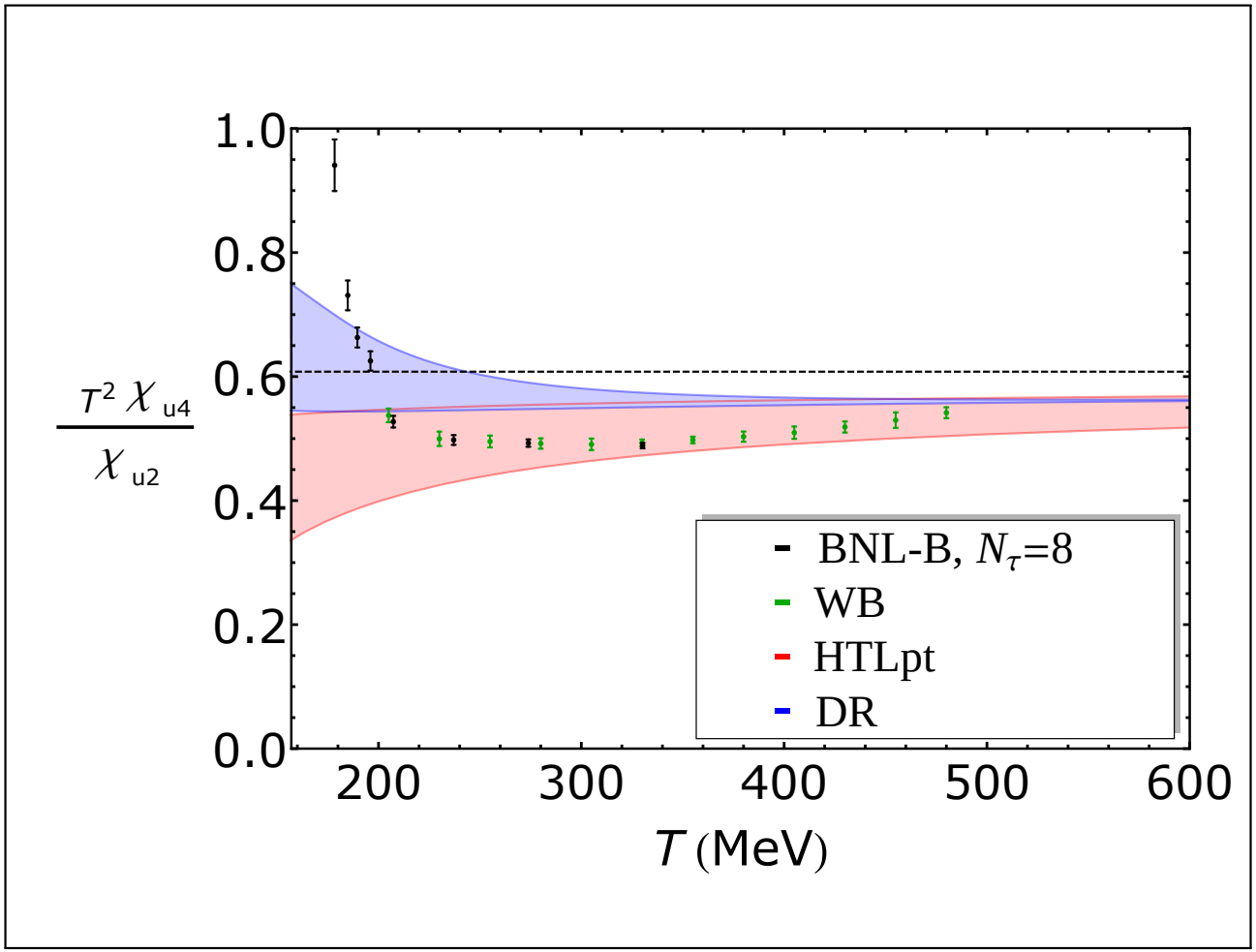


Figure : Ref: [arXiv:1307.8098]. Lattice data from Allton et al. [PRD **71**, 054508]

...and kurtosis



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$\Delta p \equiv p(\mu_f \neq 0) - p(\mu_f = 0)$

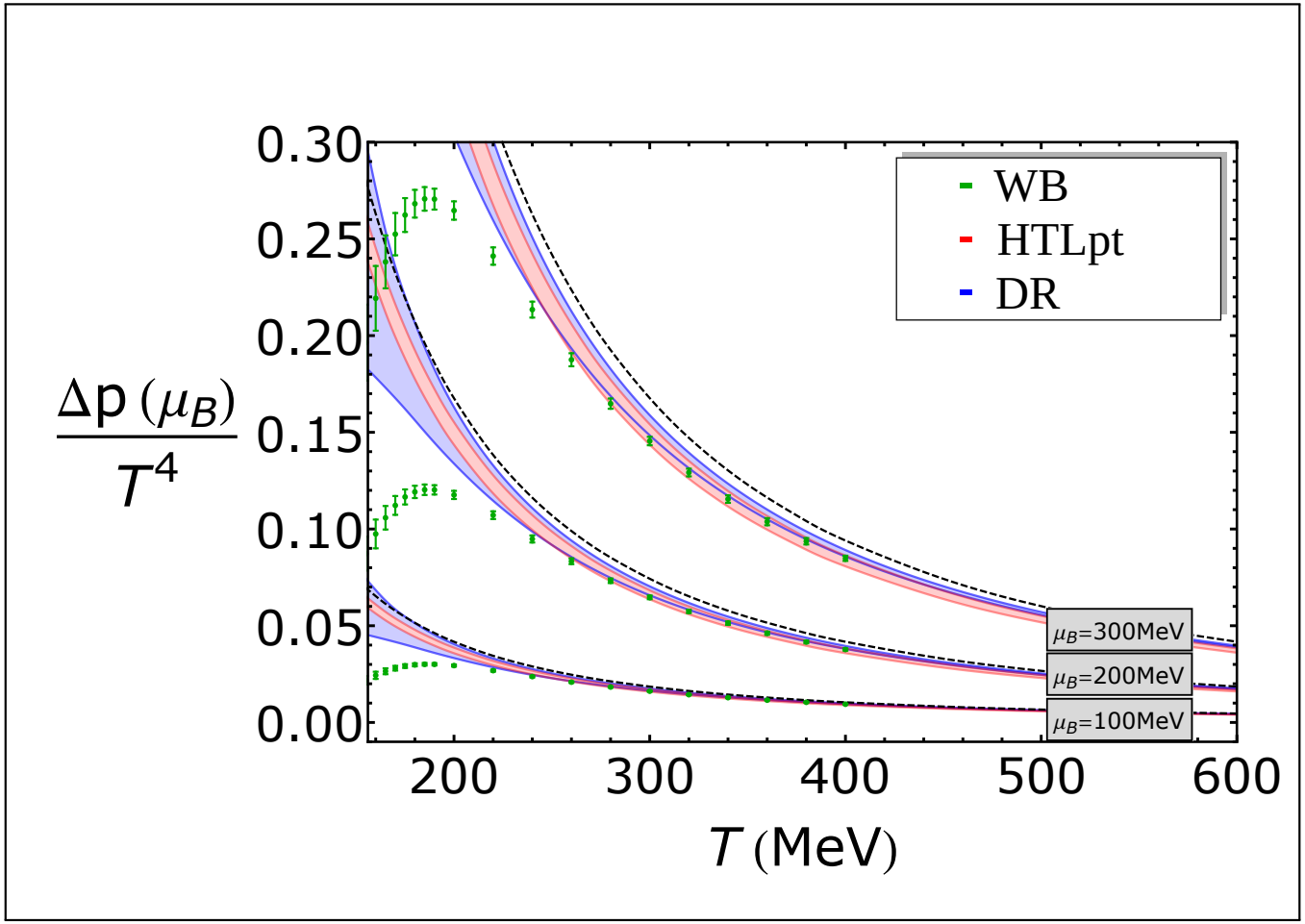
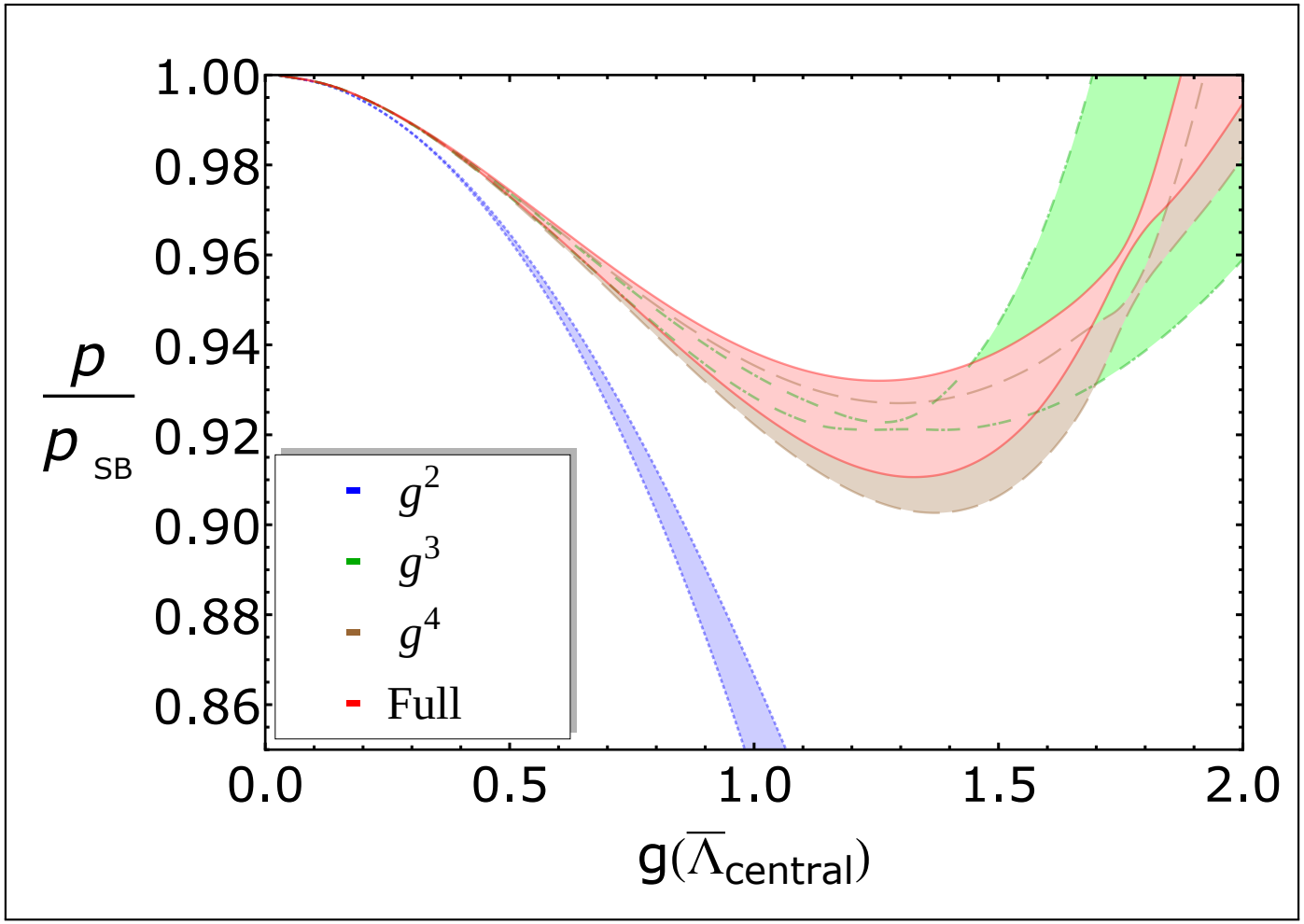


Figure : Ref: [arXiv:1307.8098]. Lattice data from Borsányi et al. [JHEP **08**, 053]

1-loop full vs truncated HTLpt results



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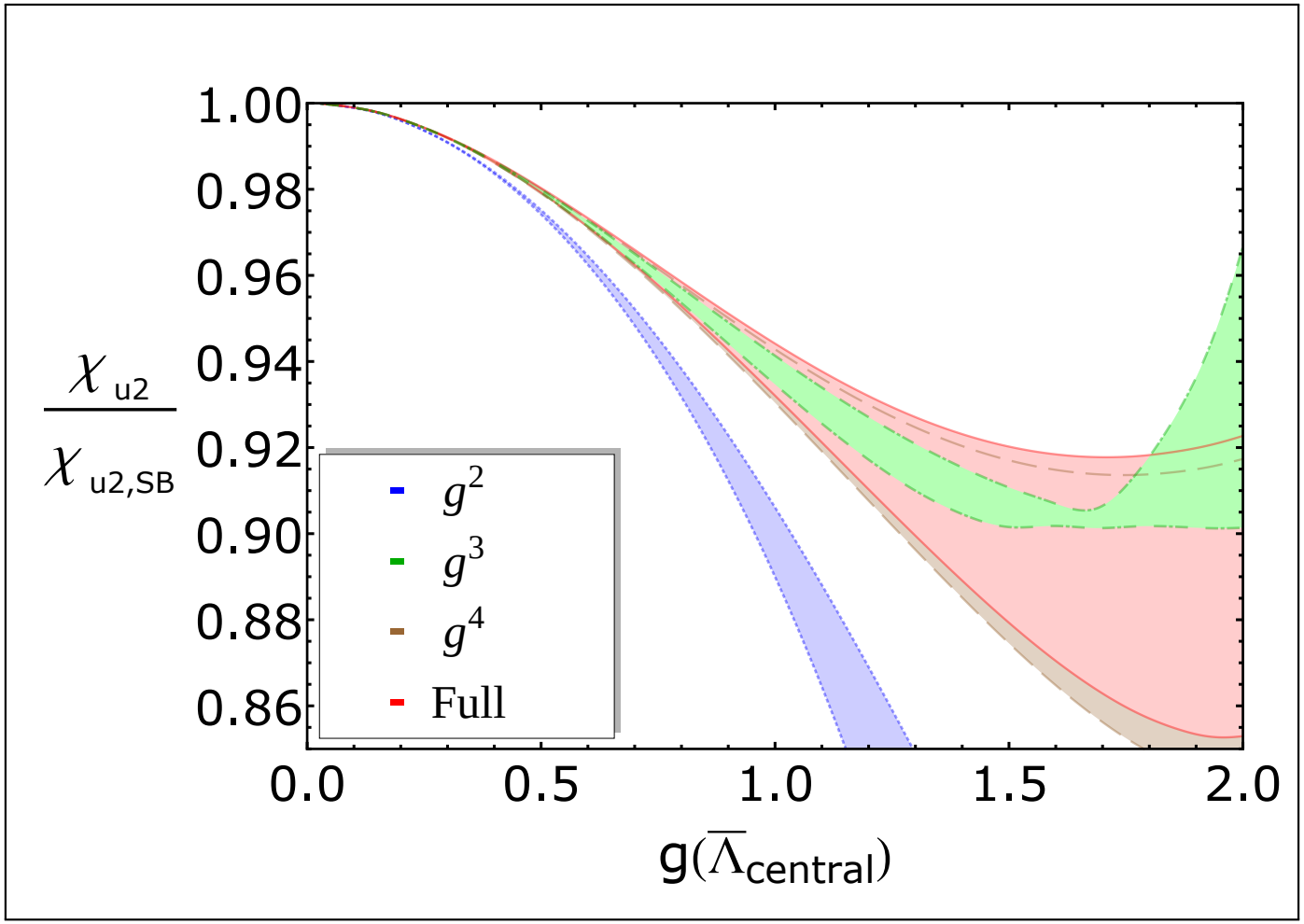

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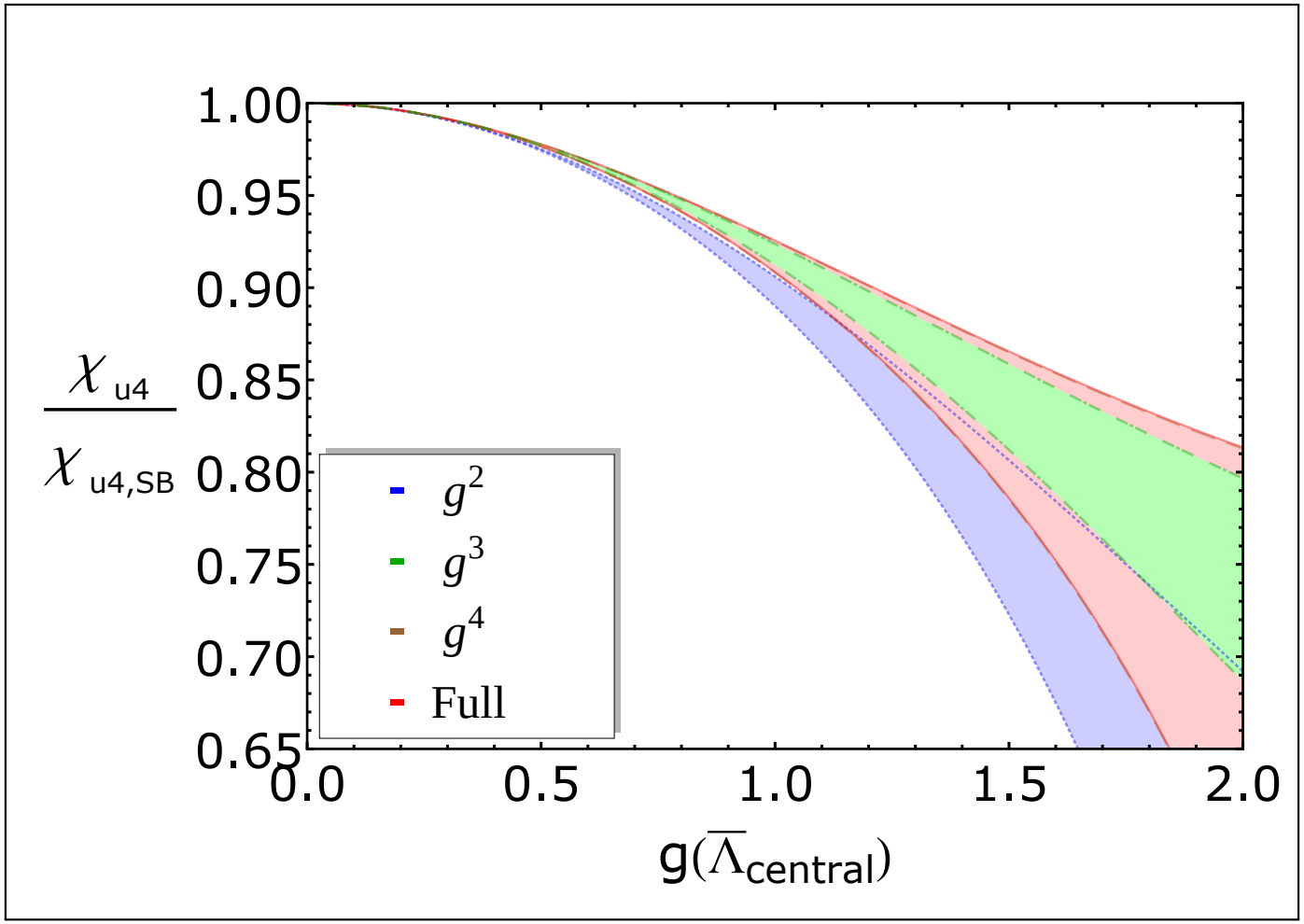

- Lattice data agree with full 1-loop HTLpt for 2nd–6th order QNS...
  ...over a wide range of temperatures, down to $T \approx 200$ MeV!

However,

1. 2-loop truncated results seems to ruin the agreement. Need the 3-loop correction to conclude since good convergence of the truncation
2. Recent high-T lattice data by Bazavov et al. [arXiv:1309.2317] seem not to agree with the 1-loop truncated HTLpt results of [JHEP **07**, 184], already at $T \approx 400 - 1000$ MeV...

- Slight but visible discrepancy between lattice and DR for the 4th order QNS; severe disagreement with full 1-loop HTLpt as wel as lattice for the 6th order QNS!

However,

1. Possible artefact of the resummation which drastically reduce the theoretical uncertainty
2. Possible sizable effect of the unknown $\mathcal{O}\left(g^6\right)$ term in the pressure...

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- THANKS FOR YOUR ATTENTION!

- Type of full 1-loop sum-integral to compute, via branch cuts:

$$
\begin{aligned}
p_{\mathrm{q_f}}(T,\boldsymbol{\mu}) &= 2\sum\!\!\!\!\!\!\!\int_{\widetilde{K}_f} \log\left[A_{\mathrm{S}}^2(\widetilde{K}_f) - A_0^2(\widetilde{K}_f)\right] \\
A_0(\widetilde{K}_f) &\equiv i\,\widetilde{\omega}_n + \mu_f - \frac{m_{q_f}^2}{i\,\widetilde{\omega}_n + \mu_f}\;\widetilde{\mathcal{T}}_{\mathrm{K}}(i\,\widetilde{\omega}_n + \mu_f, k) \\
A_{\mathrm{S}}(\widetilde{K}_f) &\equiv k + \frac{m_{q_f}^2}{k}\left[1 - \widetilde{\mathcal{T}}_{\mathrm{K}}(i\,\widetilde{\omega}_n + \mu_f, k)\right] \\
\widetilde{\mathcal{T}}_{\mathrm{K}} &\equiv \frac{\Gamma\left(\frac{3}{2}-\epsilon\right)}{\Gamma\left(\frac{3}{2}\right)\Gamma\left(1-\epsilon\right)}\int_0^1 \mathrm{d}c\,\left(1-c^2\right)^{-\epsilon}\frac{(i\,\widetilde{\omega}_n + \mu_f)^2}{(i\,\widetilde{\omega}_n + \mu_f)^2 - k^2c^2} \\
&= {}_2F_1\left(\frac{1}{2}, 1; \frac{3}{2}-\epsilon; \frac{k^2}{(i\,\widetilde{\omega}_n + \mu_f)^2}\right)
\end{aligned}
$$

where:

$$\widetilde{\omega}_n \equiv (2n+1)\pi T$$

- 1-loop HTLpt mass parameters prescription:
  Identification to Debye and quark thermal mass weak coupling values

$$
\begin{aligned}
m_D^2 &\equiv \frac{g^2}{3}\left[\left(N_c+\frac{N_f}{2}\right)T^2+\frac{3}{2\pi^2}\sum_g^{N_f}\mu_g^2\right] \\
m_{q_f}^2 &\equiv g^2\,\frac{N_c^2-1}{16\,N_c}\left(T^2+\frac{\mu_f^2}{\pi^2}\right)
\end{aligned}
$$