Microscopic-macroscopic method for studying single-particle level density of superheavy nuclei

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Aims: -to investigate the nuclear level density of superheavy nuclei in the α-decay chains of $^{296,298,300}120$
-to establish the dependence of level density on shell effects in the region of superheavy nuclei.

Content:

- Scheme of Calculations
- Modified Two Center Shell Model
- Formalism of Level Density Calculations
- Results of calculations
- Conclusions

Scheme of the Calculations:

- Potential Energy Surface (PES) is obtained by the Strutinsky method using the modified Two Center Shell Model (TCShM).

- The single particle spectrum is obtained for the potentials corresponding to minimum of PES.

- The nuclear level density is defined using the statistical approach (saddle point method). The effects of pairing is included and studied in the BCS approximation.

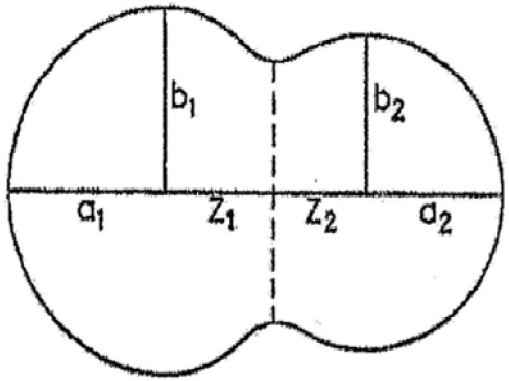
- The level density parameters are extracted by fitting the obtained numerical solution by the Fermi Gas expression.

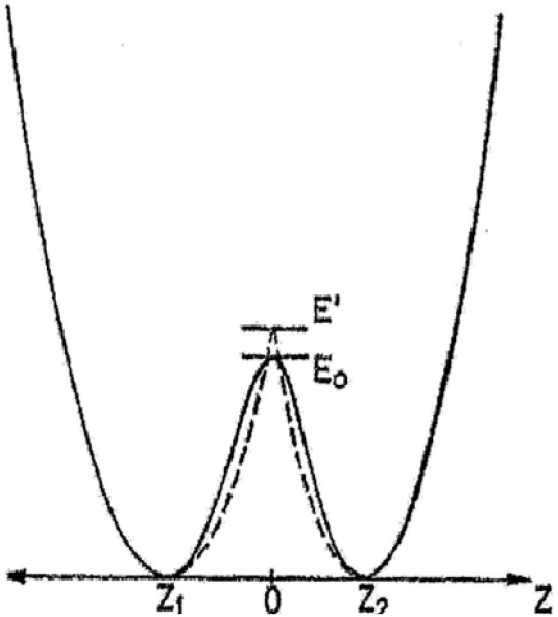
- The dependence of the level density parameters on the shell effects is determined.

- The dependence of the level density parameter on N and Z is established. The minima in this dependence correspond to the closed shells or sub-shell nuclei.

Two Center Shell Model

(J. Maruhn and W. Greiner, Z. Phys. 251 (1972) 431)





The model has five collective coordinates

$\frac{\omega_{\rho 2}}{\omega_{\rho 1}}$: mass asymmetry

$\frac{\omega_{zi}}{\omega_{\rho i}} = \frac{a_i}{b_i}$ ($i = 1,2$): ellipsoidal deformations of the fragments

$z_2 - z_1$: separation of the fragments (elongation)

$\frac{E_0}{E'}$: neck parameter, permitting a variable barrier height

$$\left(E' = \frac{1}{2} m_N \omega_{z1} z_1^2 = \frac{1}{2} m_N \omega_{z2} z_2^2 \right)$$

Modification of the two center shell model

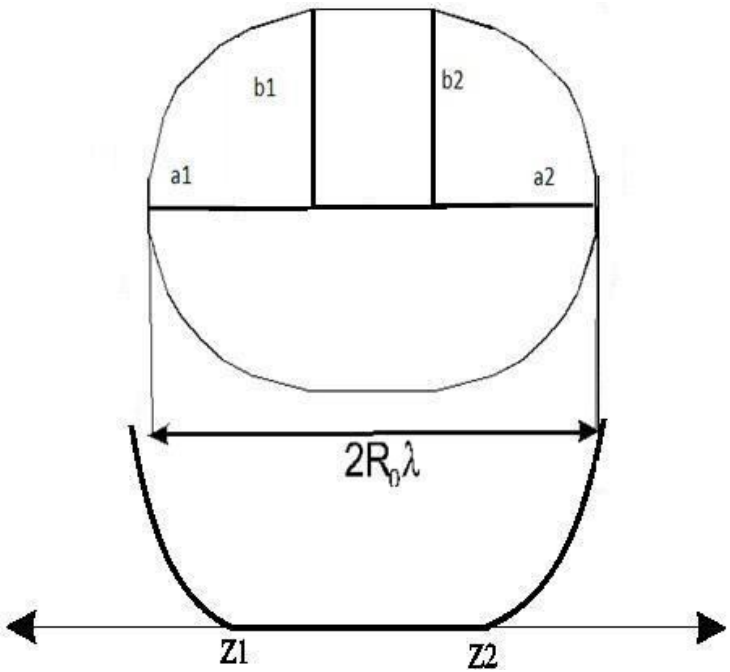
$$H = -\frac{\hbar^2 \nabla^2}{2m_0} + V(\rho, z) + V_{LS}(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{s}) + V_{L^2}(\boldsymbol{r}, \boldsymbol{l})$$

We choose the shape parametrization adopted in the TCSM

- the elongation $\lambda = L/2R_0$
- the case of deformation $\boldsymbol{\beta = a/b = \beta_1 = \beta_2}$
- the neck parameter $\boldsymbol{\varepsilon = E_0/E' = 0}$
- the mass asymmetry

$\boldsymbol{\eta = (A_1 - A_2)/(A_1 + A_2) = 0}$

Other variables are fixed.



Modification of the two center shell model

The potential energy is calculated as

$$U(Z, A, \lambda, \beta) = U_{LDM}(Z, A, \lambda, \beta) + \delta U_{mic}(Z, A, \lambda, \beta)$$

The first term is macroscopic energy (the Coulomb and surface energies) calculated with the liquid drop model. The second term is a microscopic energy. It contains the shell E_{sh} and pairing corrections.

The momentum-dependent part of the Hamiltonian consists of the sl- and l^2-like terms with the parameters $\kappa_{n,p}$ and $\mu_{n,p}$, respectively. In order to improve the description of the nuclear spins and parities, we introduce the weak dependence on (N−Z) in the $\kappa_{n,p}$ and $\mu_{n,p}$

$$\kappa_n = -0.076 + 0.0058\,(N-Z) - 6.53\times10^{-5}\,(N-Z)^2 + 0.002A^{1/3}$$
$$\mu_n = 1.598 - 0.0295\,(N-Z) + 3.036\times10^{-4}\,(N-Z)^2 - 0.095A^{1/3}$$

$$\kappa_p = 0.0383 + 0.00137\,(N-Z) - 1.22\times10^{-5}\,(N-Z)^2 - 0.003A^{1/3}$$
$$\mu_p = 0.335 + 0.01\,(N-Z) - 9.367\times10^{-5}\,(N-Z)^2 + 0.003A^{1/3}$$

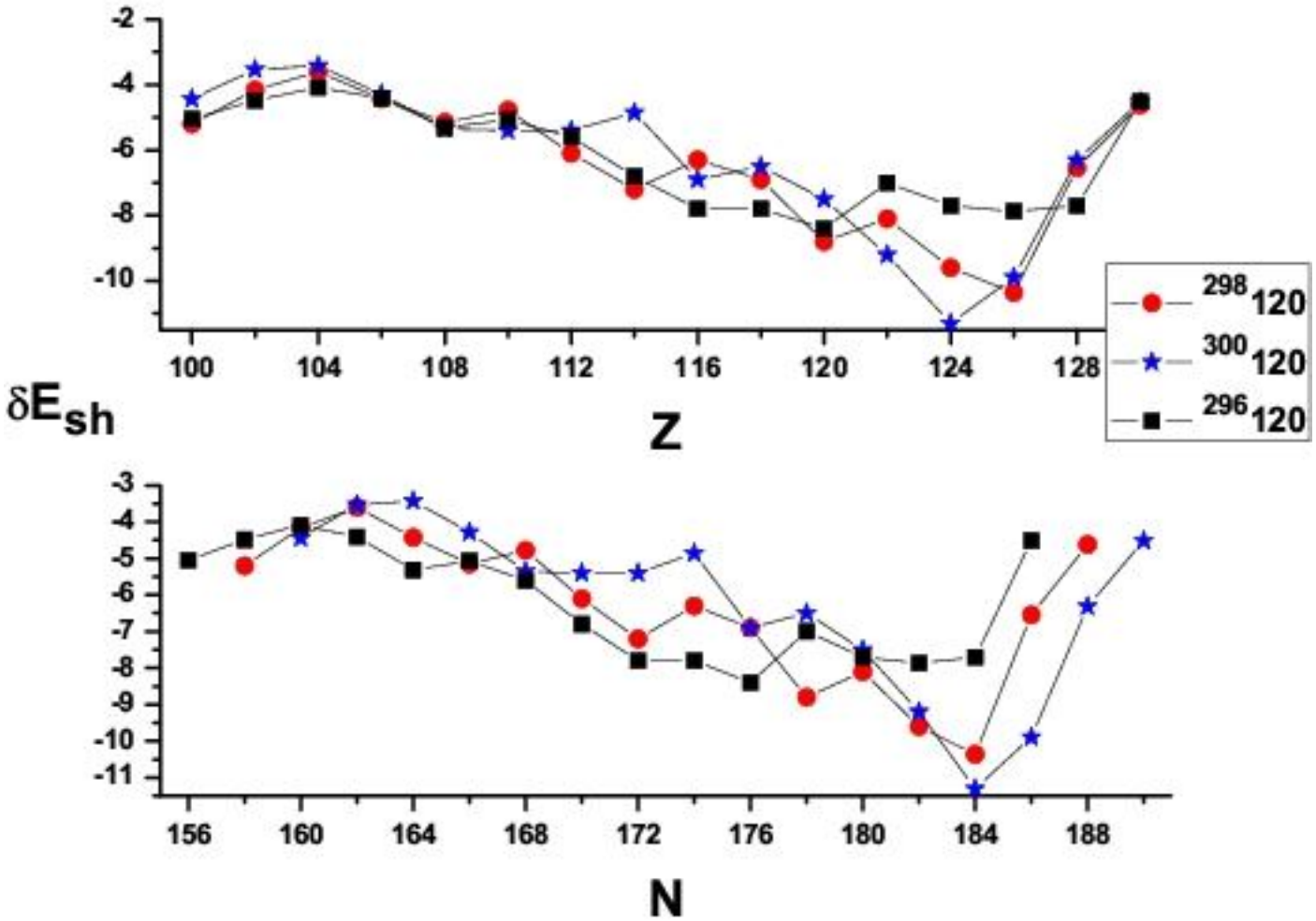
Results of TCSM Calculations for $^{300}120$ chain.

A	Z	E_sh	λ	β	λ_fp	Δ_p	λ_fn	Δ_n
260	100	-4.4373	1.18	1.28	42.12	0.52	50.08	0.67
264	102	-3.5330	1.18	1.28	42.33	0.62	50.05	0.67
268	104	-3.4229	1.16	1.22	42.39	0.63	50.03	0.65
272	106	-4.2856	1.14	1.08	42.22	0.48	49.92	0.59
276	108	-5.3513	1.12	1.00	42.40	0.45	49.75	0.58
280	110	-4.8611	1.10	0.96	42.58	0.45	49.60	0.58
284	112	-6.7046	1.06	0.86	42.20	0.46	49.37	0.45
288	114	-4.8579	1.02	1.04	41.92	0.45	48.67	0.65
292	116	-5.4617	1.04	1.06	42.94	0.45	49.24	0.57
296	118	-6.7534	1.04	1.02	42.77	0.45	49.25	0.45
300	120	-8.7417	1.02	0.92	42.63	0.45	49.30	0.45
304	122	-8.9565	1.00	0.92	43.24	0.62	49.41	0.45
308	124	-10.4585	0.98	0.90	43.00	0.45	49.08	0.45
312	126	-9.1472	0.98	0.90	43.38	0.45	49.44	0.45
316	128	-6.3110	0.98	0.90	43.50	0.62	49.39	0.45
320	130	-4.5176	0.98	0.90	43.49	0.75	49.42	0.45

prolate

oblate

Shell Corrections

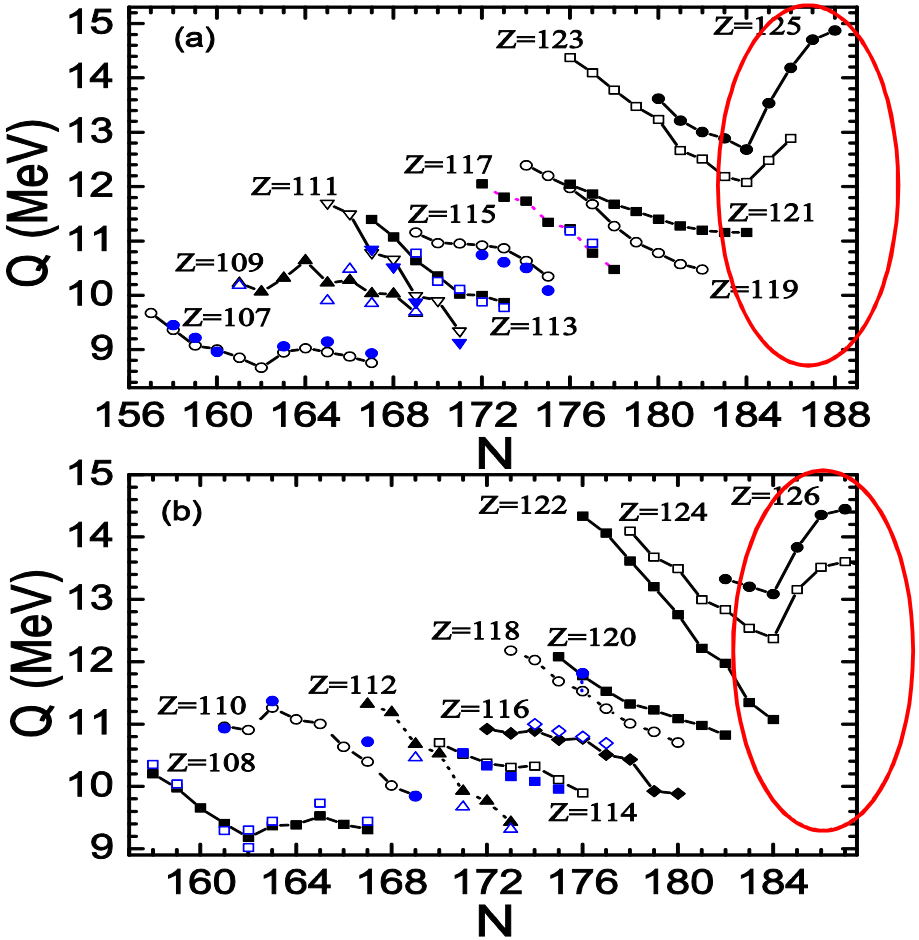


Strong shell effects at Z = 120-126 and N = 184.
Shell effects at Z = 108 and Z = 114 are weaker.

The Q_α values for α-emission for even-Z (b) and odd-Z (a)

*(A.N. Kuzmina (Bezbakh) et al., Phys. Rev. C**85**, 014319 (2012))*

$$Q_\alpha(Z,A) = B(Z,A) + 28.296 - B(Z-2,A-4)$$



❖**Z=114, N=172-176**

❖**N=184, Z=120-126**

The calculated Q_α are in a good, within 0.3 MeV, agreement with the available experimental data.

1.Yu.Ts. Oganessian, J. Phys. G **34**, R165 (2007) *et al*., Phys. Rev. Lett. **104**, 142502 (2010);

S. Hofmann *et al*., Eur. Phys. J. A **32**, 251 (2007) / Lec. Notes Phys. **764**, 203 (2009); Radiochim. Acta **99**, 405 (2011);

3.L. Stavsetra *et al*., Phys. Rev. Lett. **103**, 132502 (2009).

Formalism of Level Density Calculations

The level density is defined as

$$\omega(N, Z, E) = \sum_i \delta(N - N_i)\delta(Z - Z_i)\delta(E - E_i)$$

where

$$\hat{N}|i>= N_i|i>, \qquad \hat{Z}|i>= Z_i|i>, \qquad \hat{H}|i>= E_i|i>$$

The Laplace transformation of level density is the statistical sum

$$\begin{aligned} Q(\alpha_N, \alpha_Z, \beta) &= \int_0^\infty dN \int_0^\infty dZ \int_0^\infty dE\ \omega(N, Z, E) e^{-\beta E + \alpha_N N + \alpha_Z Z} \\ &= \sum_i e^{-\beta E_i + \alpha_N N_i + \alpha_Z Z_i} \equiv \sum_i < i|e^{-\beta \hat{H} + \alpha_N \hat{N} + \alpha_Z \hat{Z}}|i > \end{aligned}$$

The inverse Laplace transformation of statistical sum gives the level density

$$\omega(N, Z, E) = \frac{1}{(2\pi i)^3} \int_{\beta' - i\infty}^{\beta' + i\infty} d\beta \int_{\alpha'_N - i\infty}^{\alpha'_N + i\infty} d\alpha_N \int_{\alpha'_Z - i\infty}^{\alpha'_Z + i\infty} d\alpha_Z e^{S(\alpha_N, \alpha_Z, \beta)}$$

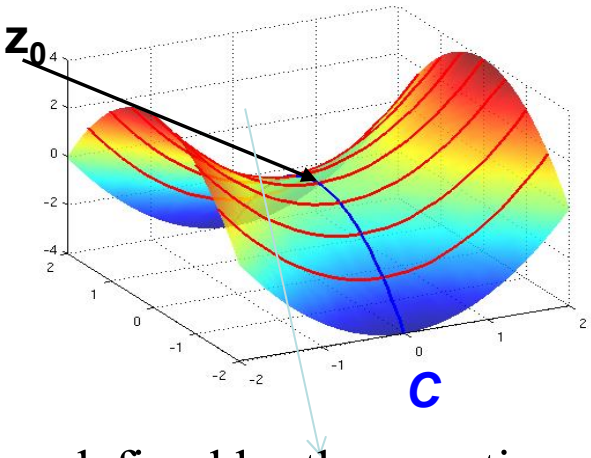
$$S(\alpha_N, \alpha_Z, \beta) = \beta E - \alpha_N N - \alpha_Z Z + \ln Q(\alpha_N, \alpha_Z, \beta)$$

Method of the saddle point (*method of steepest descent*)

is a technique used to approximate the integrals

$$F(\lambda) = \int_C f(\mathbf{z}) e^{\lambda S(\mathbf{z})} dz_1 dz_2 ... dz_n \approx \left(\frac{2\pi}{\lambda |S''(\mathbf{z_0})|} \right)^{n/2} e^{\lambda S(\mathbf{z_0})} f(\mathbf{z_0})$$

where $\mathbf{z_0}$ is the saddle point defined by the condition



$$S'(\mathbf{z_0}) = 0$$

and

$$S''(\mathbf{z_0}) = \det \left\{ \frac{\partial^2 S(\mathbf{z_0})}{\partial z_i \partial z_j} \right\}$$

In the case of level density the saddle point coordinates are defined by the equations:

$$\left.\begin{aligned} \frac{\partial S(\alpha_N, \alpha_Z, \beta)}{\partial \beta} &= 0 \\ \frac{\partial S(\alpha_N, \alpha_Z, \beta)}{\partial \alpha_N} &= 0 \\ \frac{\partial S(\alpha_N, \alpha_Z, \beta)}{\partial \alpha_Z} &= 0 \end{aligned}\right\} \Longrightarrow \boxed{\begin{aligned} E &=< E >= \frac{\sum_i E_i e^{-\beta E_i + \alpha_Z Z_i + \alpha_N N_i}}{\sum_i e^{-\beta E_i + \alpha_Z Z_i + \alpha_N N_i}} \\ N &=< N > \\ Z &=< Z > \end{aligned}}$$

Level Density in the Superfluid Nuclear Model

Nuclear level density for gas of noninteracting quasiparticles will be:

$$\rho(U) = (2\pi)^{-3/2} D^{-1/2} e^{S(\alpha_N, \alpha_Z, \beta)}$$

where:

$$D = \begin{vmatrix} \frac{\partial^2 S}{\partial \beta^2} & \frac{\partial^2 S}{\partial \beta \partial \mu_Z} & \frac{\partial^2 S}{\partial \beta \partial \mu_N} \\ \frac{\partial^2 S}{\partial \beta \partial \mu_Z} & \frac{\partial^2 S}{\partial \mu_Z^2} & 0 \\ \frac{\partial^2 S}{\partial \beta \partial \mu_N} & 0 & \frac{\partial^2 S}{\partial \mu_N^2} \end{vmatrix}$$

Entropy:

$$S(\alpha_N, \alpha_Z, \beta) = 2 \sum_{k=Z,N} \sum_{\nu} \left\{ \ln\left[1 + \exp(-\beta E_{\nu k})\right] + \frac{\beta E_{\nu k}}{1 + \exp(\beta E_{\nu k})} \right\}$$

Level Density in the Superfluid Nuclear Model

N - number of neutrons,
Z - number of protons,
$T=\beta^{-1}$ - temperature,
$\lambda_N=\alpha_N/\beta$ -chemical potential for neutrons,
$\lambda_Z=\alpha_Z/\beta$ -chemical potential for protons,
$\epsilon_{\nu N}, \epsilon_{\nu Z}$ -neutron, proton single- particle energies,
G_N, G_Z -pairing constants.

$E_{\nu k} = \sqrt{(\epsilon_{\nu k} - \lambda_k)^2 + \Delta_k^2}$ -quasiparticle energies of neutrons ($k=N$) and protons ($k=Z$).

Equations determining the saddle point:

$$N = \sum_\nu \left(1 - \frac{\epsilon_{\nu N} - \lambda_Z}{E_{\nu N}} \tanh \frac{\beta E_{\nu N}}{2}\right), \quad Z = \sum_\nu \left(1 - \frac{\epsilon_{\nu Z} - \lambda_Z}{E_{\nu Z}} \tanh \frac{\beta E_{\nu Z}}{2}\right)$$

$$\frac{2}{G_N} = \sum_\nu \frac{\tanh(\beta E_{\nu N}/2)}{E_{\nu N}}, \quad \frac{2}{G_Z} = \sum_\nu \frac{\tanh(\beta E_{\nu Z}/2)}{E_{\nu Z}}$$

$$E(T) = \sum_{k=N,Z} \left\{ \sum_\nu \epsilon_{\nu k} \left(1 - \frac{\epsilon_{\nu N} - \lambda_Z}{E_{\nu N}} \tanh \frac{\beta E_{\nu N}}{2}\right) - \frac{\Delta_k^2}{G_k} \right\}$$

Treating the Energy Gaps

- Depending on the number of basis single particle states the results of the solution of the energy gap equations are varying!
- For the given basis

88 proton levels and 121 neutron levels

it is necessary to determine the constants of pairing interaction G_N and G_Z by describing the experimental values of the pairing energies for protons and neutrons:

$$P_N(Z,N) = \frac{1}{2}\{2E_{Z,N-1}(0) + E_{Z,N}(0) - E_{Z,N-2}(0)\},$$
$$P_Z(Z,N) = \frac{1}{2}\{2E_{Z-1,N}(0) + E_{Z,N}(0) - E_{Z-2,N}(0)\}$$

•For our basis set we obtained

$$G_{\substack{N\\Z}} = (18 \mp 12\frac{N-Z}{A})A^{-1}\mathrm{MeV}$$

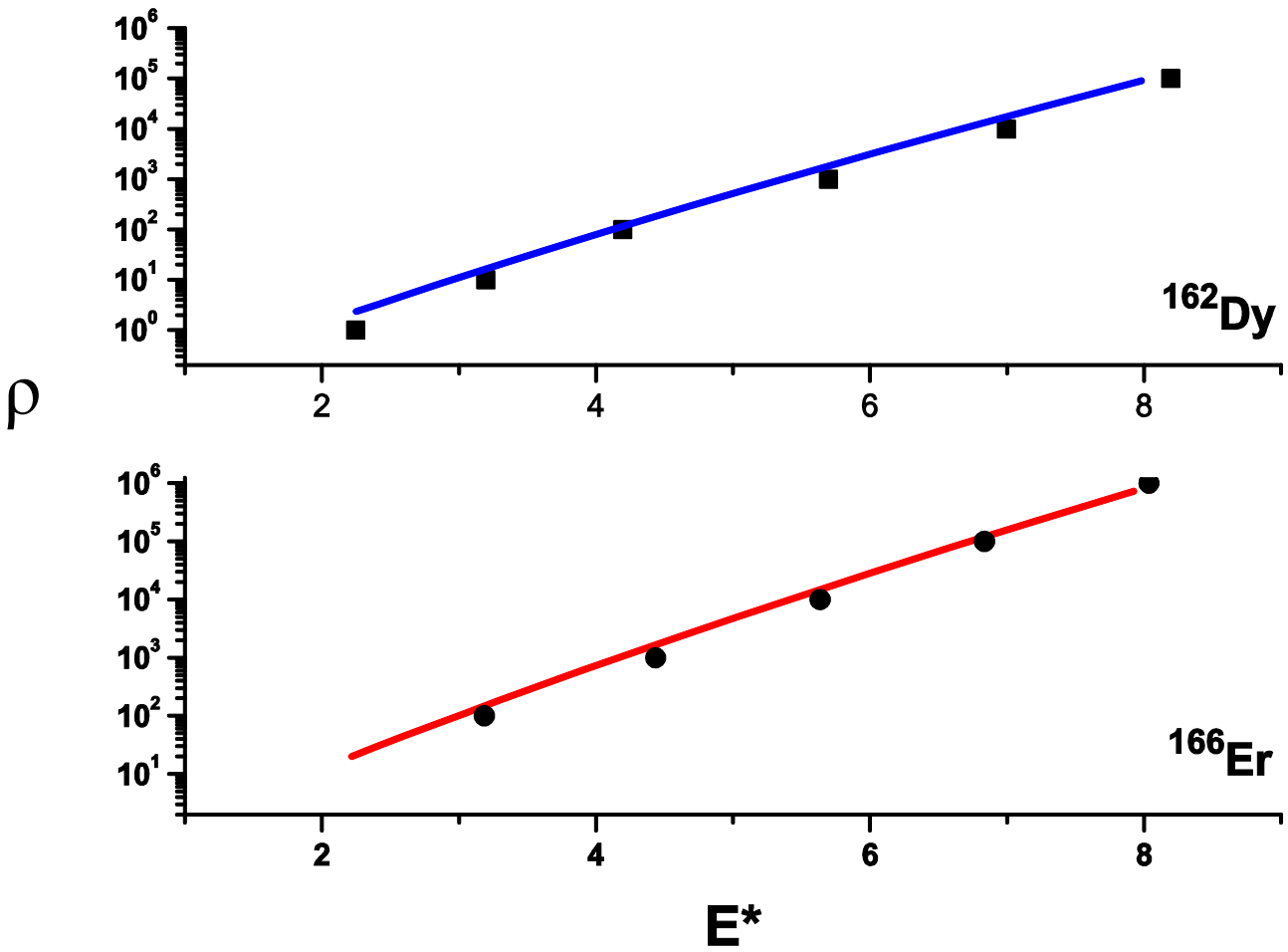
Comparison with TCSM

Results are near one to another → can use this method

A	Z	Δ_N	Δ_N^{TCSM}	Δ_Z	Δ_Z^{TCSM}	λ_N	$\lambda_N^{\mathrm{TCSM}}$	λ_Z	$\lambda_Z^{\mathrm{TCSM}}$
162	66	1.031	1.030	1.014	1.010	47.96	47.92	42.32	42.27
166	68	1.037	1.040	0.970	1.060	46.76	46.73	41.36	41.31
228	88	0.950	0.951	1.033	1.036	50.03	49.99	42.58	42.54
190	76	0.960	0.96	0.883	0.88	47.64	47.60	41.49	41.44
196	78	0.902	0.90	0.860	0.86	48.74	48.69	42.28	42.22
200	80	0.839	0.84	0.698	0.70	49.09	49.04	42.67	42.59
208	82	0.450	0.45	0.450	0.45	49.68	49.50	42.93	42.83
228	90	1.007	1.006	1.111	1.100	50.07	50.05	43.21	43.18
230	90	1.045	1.050	1.151	1.150	50.34	50.31	43.20	43.18

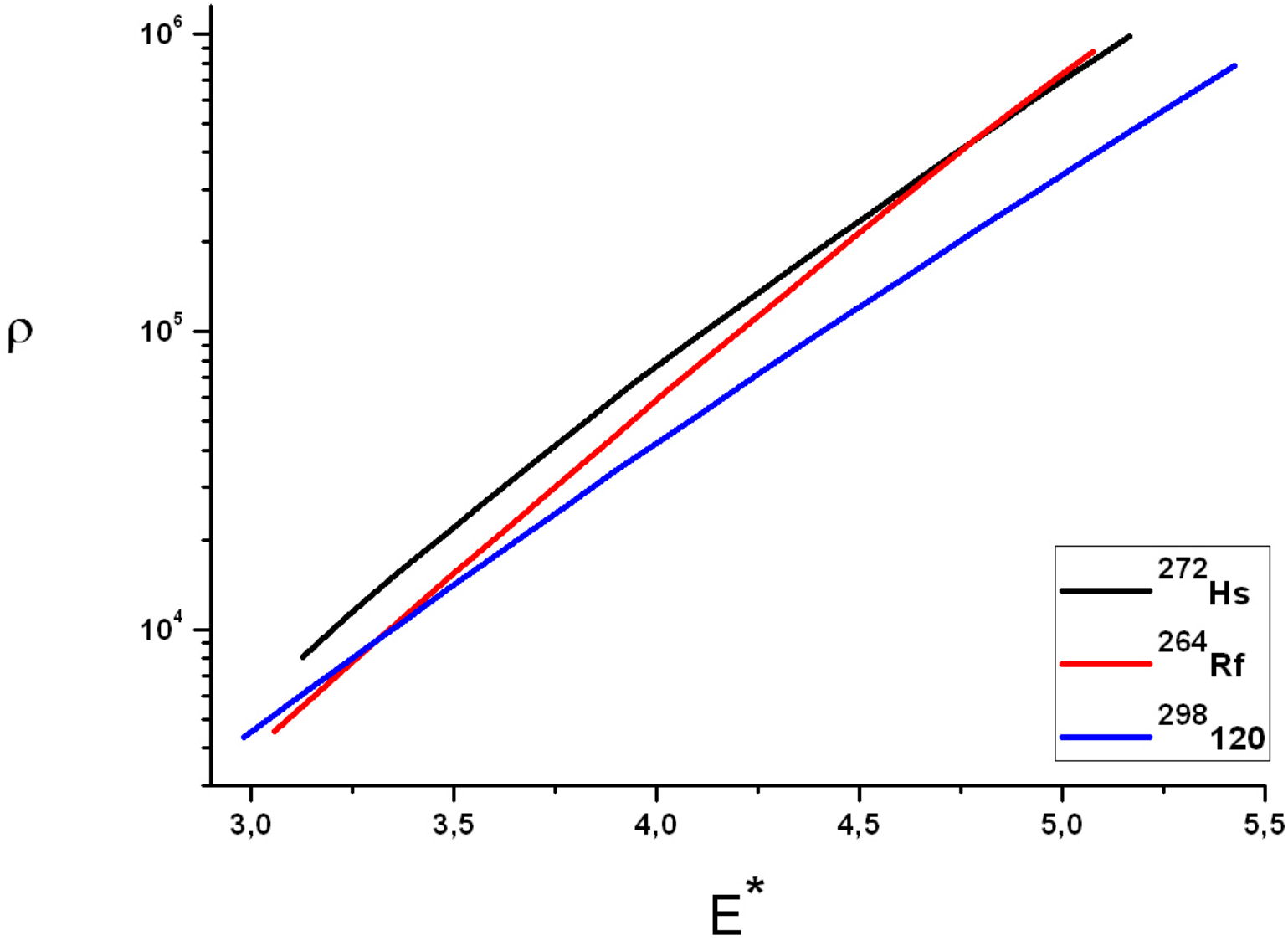
Validation of the proposed scheme

Energy back –shift: $U = E^{*} - 6.6A^{-0.32}$ MeV



Exp. data are taken from : E. Melby et al.,Phys. Rev. C 63, 044309

Level Density as a Function of Excitation Energy



Fermi Gas Expression for the Level Density

Treating nucleons as independent Fermi particles, one can obtain

$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{12a^{1/4}U^{5/4}} \exp\left[2\sqrt{aU}\right]$$

a -- the level density parameter.

$$U = aT^2, \qquad S = 2aT = 2\sqrt{aU}$$

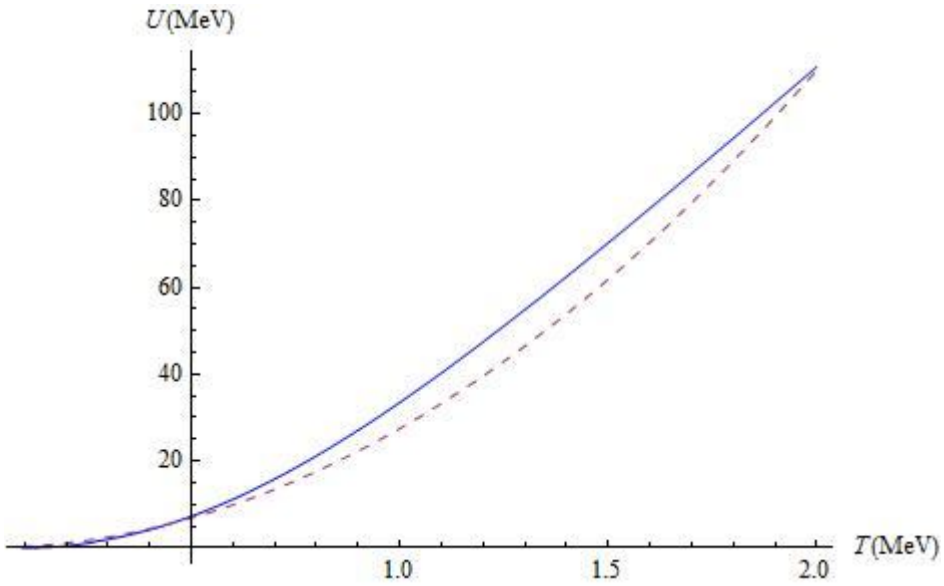
For particles moving in spherical potential well of radius $R = r_0 A^{1/3}$, $r_0 = 1.2$ fm, one can estimate:

$$a = \left(\frac{\pi}{3}\right)^{4/3} \frac{2m_N r_0^2}{\hbar^2} A \approx \frac{A}{13.5} \text{ MeV}^{-1}.$$

In the phenomenological calculations of surviving probabilities one usually takes

a=A/(10-12) MeV^{-1}

Temperature Dependence of the Excitation Energy (^{296}120 nucleus).

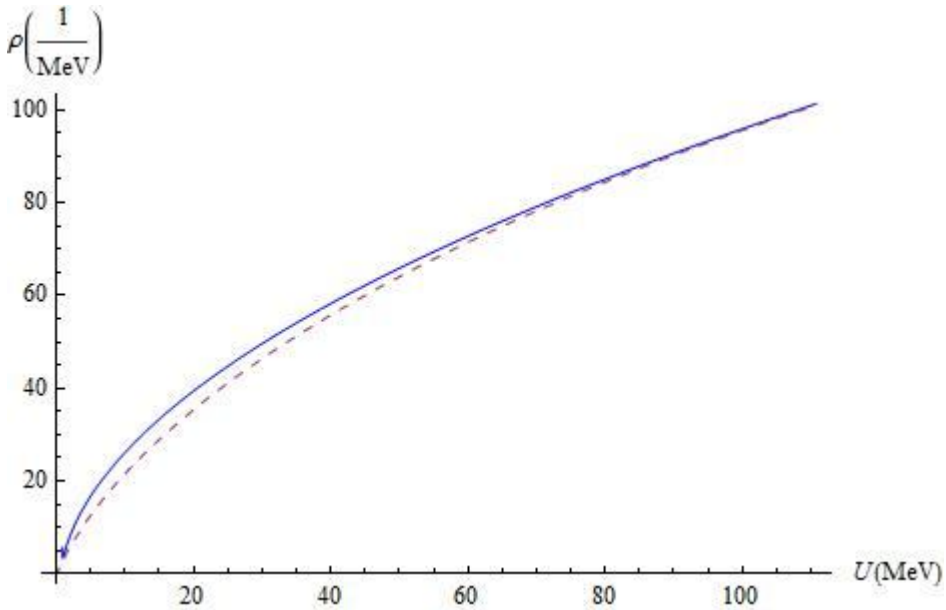


Excitation energy is fitted by the Fermi Gas expression

$$U = a\,T^2,$$

with

a=A/10.57 MeV^{-1}



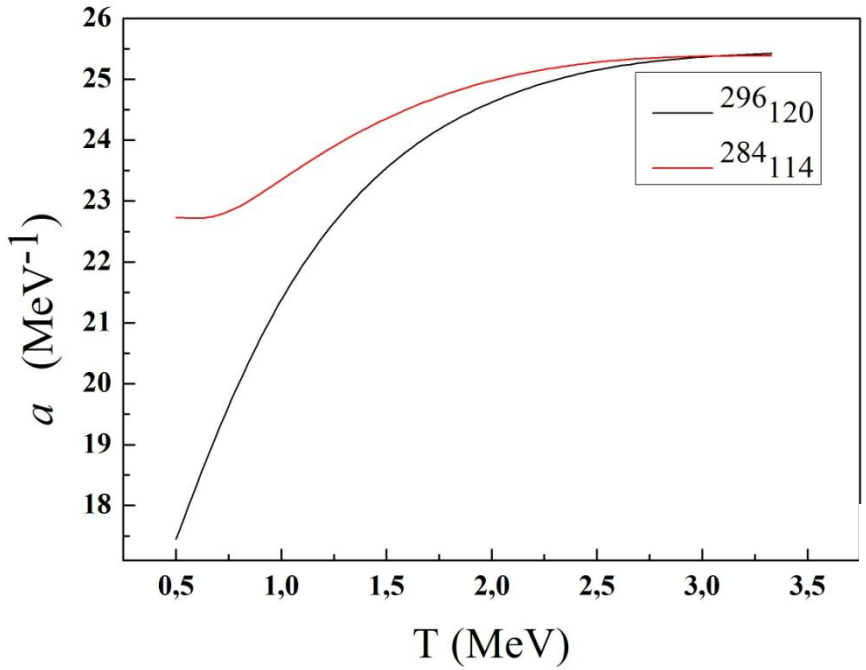
Level density is fitted by the Fermi Gas expression

$$\rho_{FG}(U) = \frac{\sqrt{\pi}}{12a^{1/4}U^{5/4}} \exp\left[2\sqrt{aU}\right]$$

with

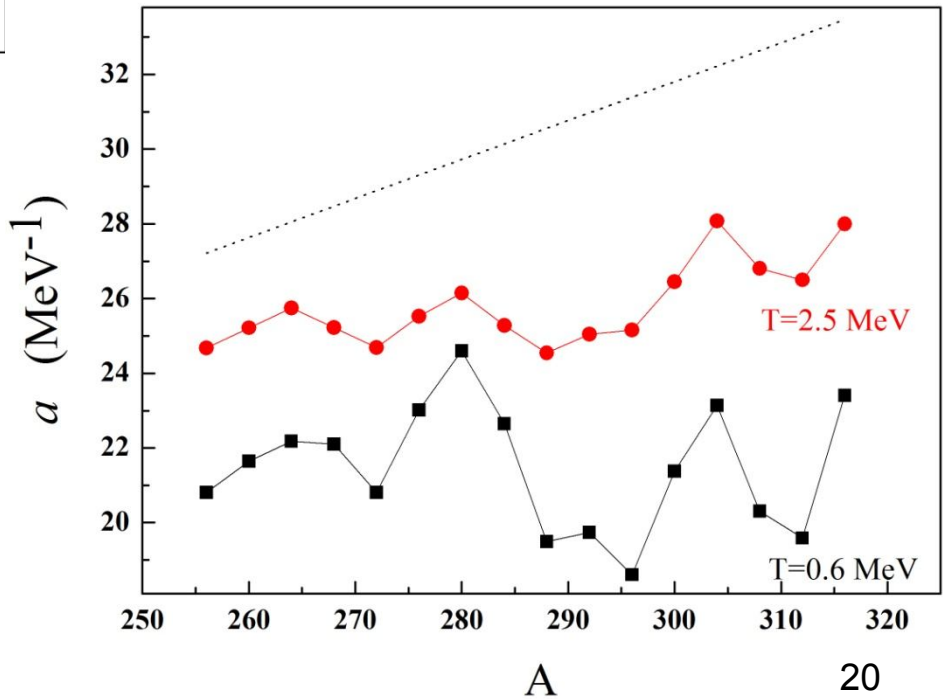
a=A/10.57 MeV^{-1}

Temperature Dependence of Level Density Parameter



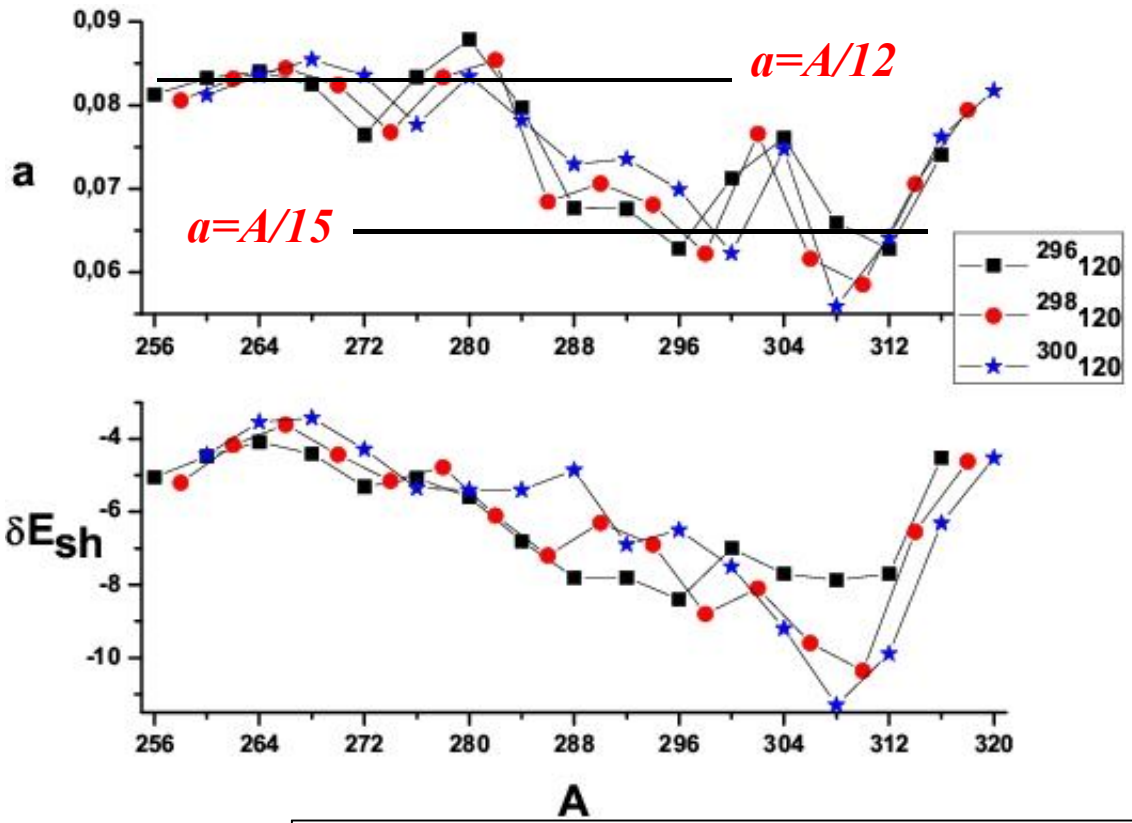
Level density parameter *a* decreases with temperature *T* achieving the asymptotic value at large *T*.

^{296}120 -α-chain



The influence of shell effects on the level density decreases with temperature increase.

Level Density Parameter and Shell Corrections



-For ***Z <116***, one can use the parametrization

a≅A/(12-14) MeV^{-1}

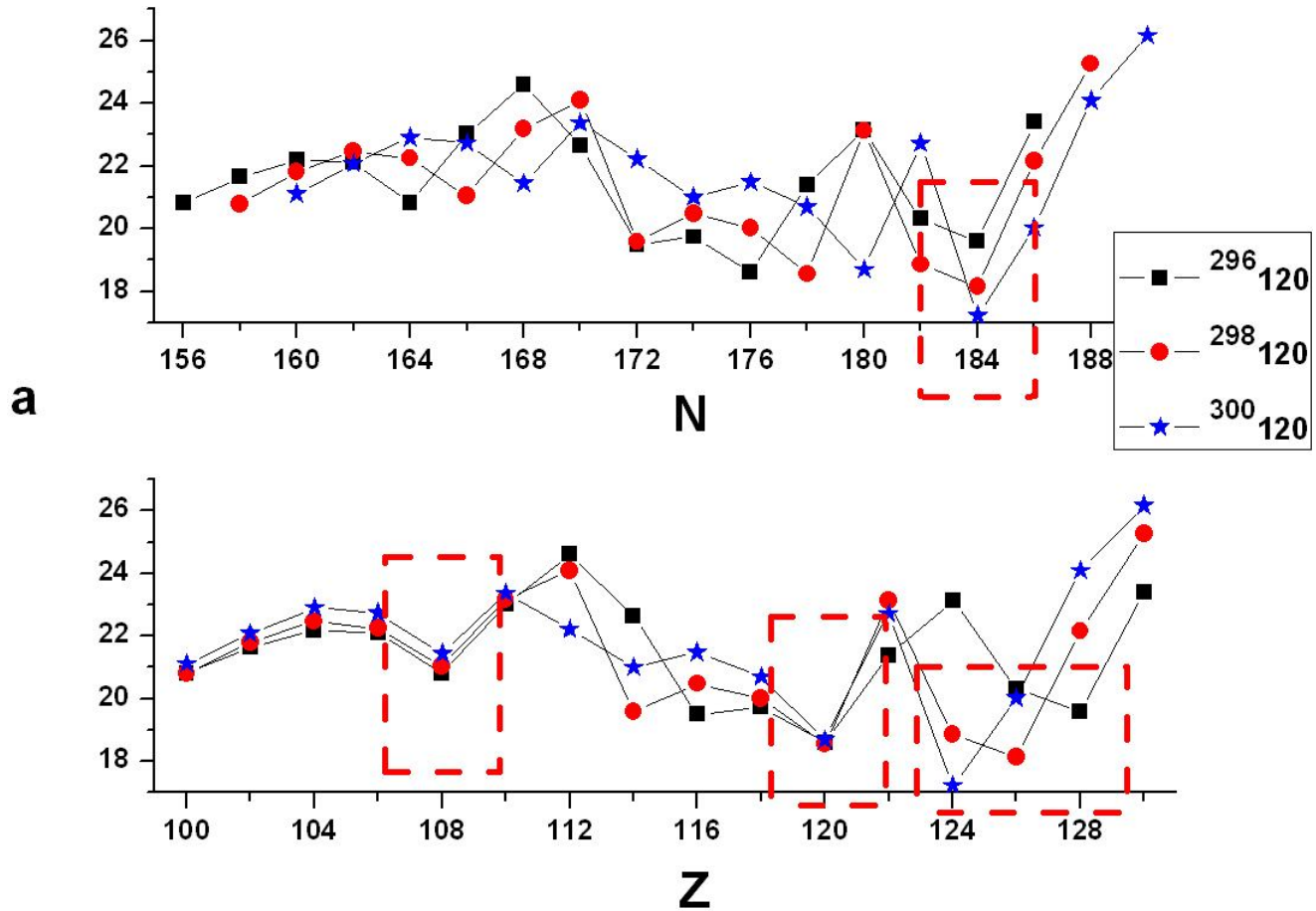
-For ***Z >116***, the better fit of the calculated values is

-

a≅A/15MeV^{-1}.

- $U = E^* - \Delta_Z - \Delta_N$ - for even-even nuclei
- $a(A,U) = \tilde{a}(A)\left[1 + \frac{1-\exp\{-\gamma U/E'_D\}}{U} E_{sh}\right]$
- $\tilde{a} = \alpha A + \beta A^2$
- $\alpha = 0.108, \beta = -4.4 \cdot 10^{-6}, \gamma = 0.053 MeV^{-1} \Longleftarrow$ 48 heavy nuclei.

Level Density Parameter and Closed Shells



- Strong shell effects at Z = 120-126 and N = 184,

- Shell effects at Z = 108 and Z = 114 are weaker.

Summary

• The calculation of PES of various nuclei with ***Z>104*** was performed with use of the Two-Center Shell Model (TCSM). The single-particle spectra corresponding to the minimum of the PES was obtained.

• The level densities of nuclei belonging to the alpha-chains leading to the $^{296,298,300}120$ nuclei were calculated using the statistical (saddle point) approach.

•For nuclei with **Z<116** at low excitation energies one can approximate the calculated level density with the Fermi Gas expression with level density parameter

***a=A/(12-14)*, Z<116,**

***a=A/15*, Z ≥ 116.**

•Energy dependence of the level density parameter can be approximated as

$$U = E^* - \Delta_Z - \Delta_N \text{ - for even-even nuclei}$$

$$a(A,U) = \tilde{a}(A)\left[1 + \frac{1-\exp\{-\gamma U/E'_D\}}{U} E_{sh}\right]$$

$$\tilde{a} = \alpha A + \beta A^2$$

$$\alpha = 0.108, \beta = -4.4 \cdot 10^{-6}, \gamma = 0.053 MeV^{-1}$$

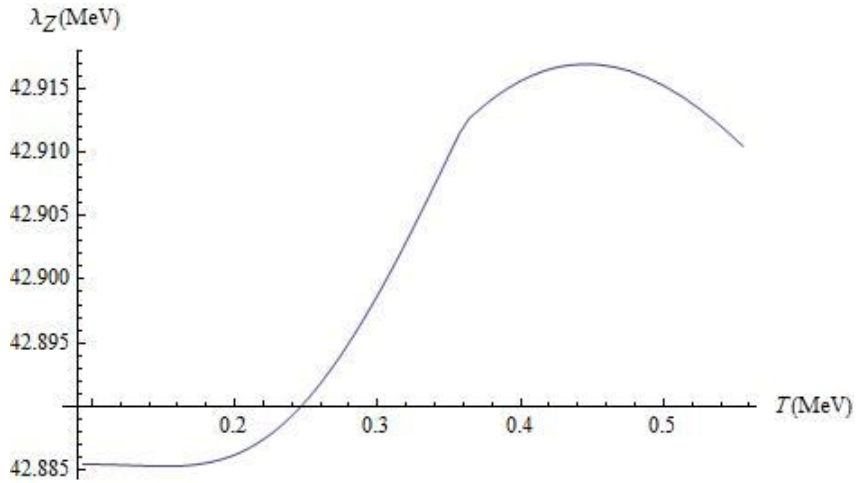
•Analysis of the level density parameters and shell corrections demonstrates that the next double magic nucleus beyond 208Pb is probably at

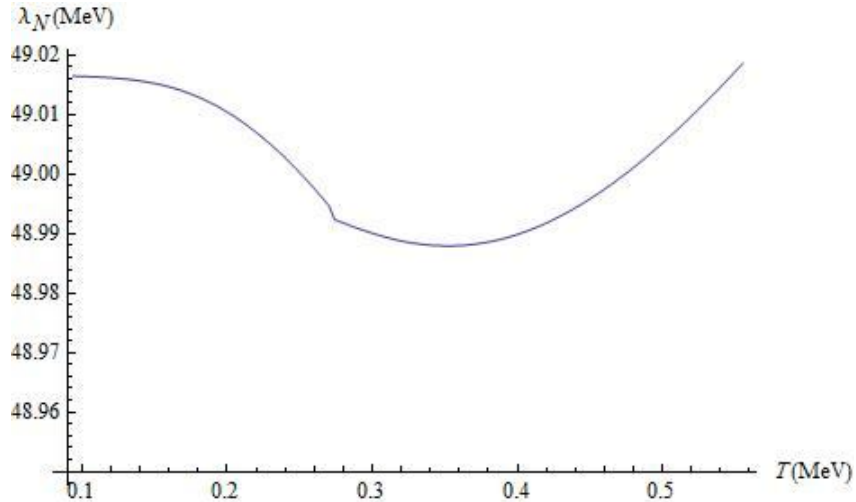
Z≥120, N=184

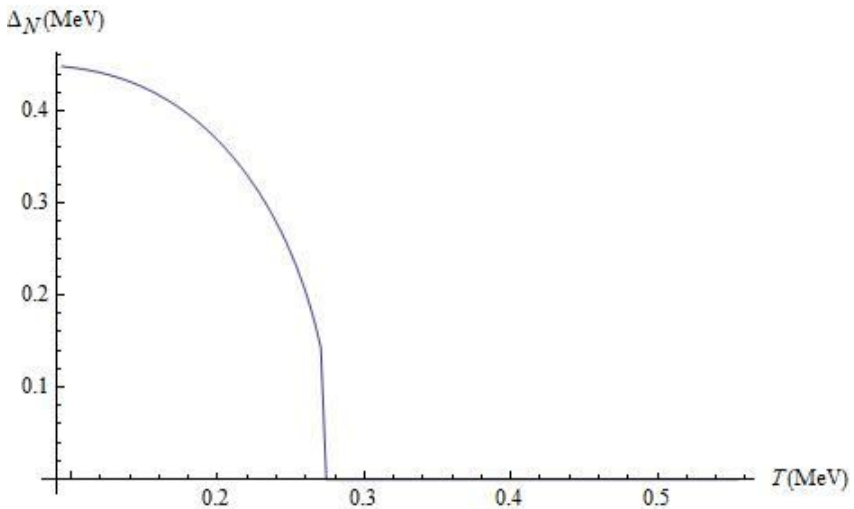
Thank you

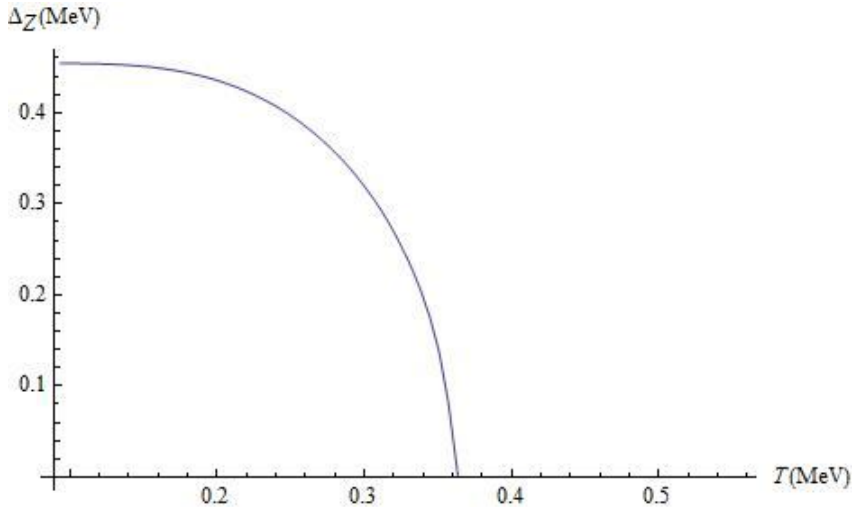
for your attention !

Temperature dependence of Chemical Potentials and Energy Gaps.









$^{296}120$