

How spinodal instabilities influence observables at FAIR

Christoph Herold
with M. Nahrgang, I. Mishustin and M. Bleicher

Frankfurt Institute for Advanced Studies

FAIRNESS, September 16, 2013



FIAS Frankfurt Institute
for Advanced Studies



HIC for **FAIR**
Helmholtz International Center

GSI

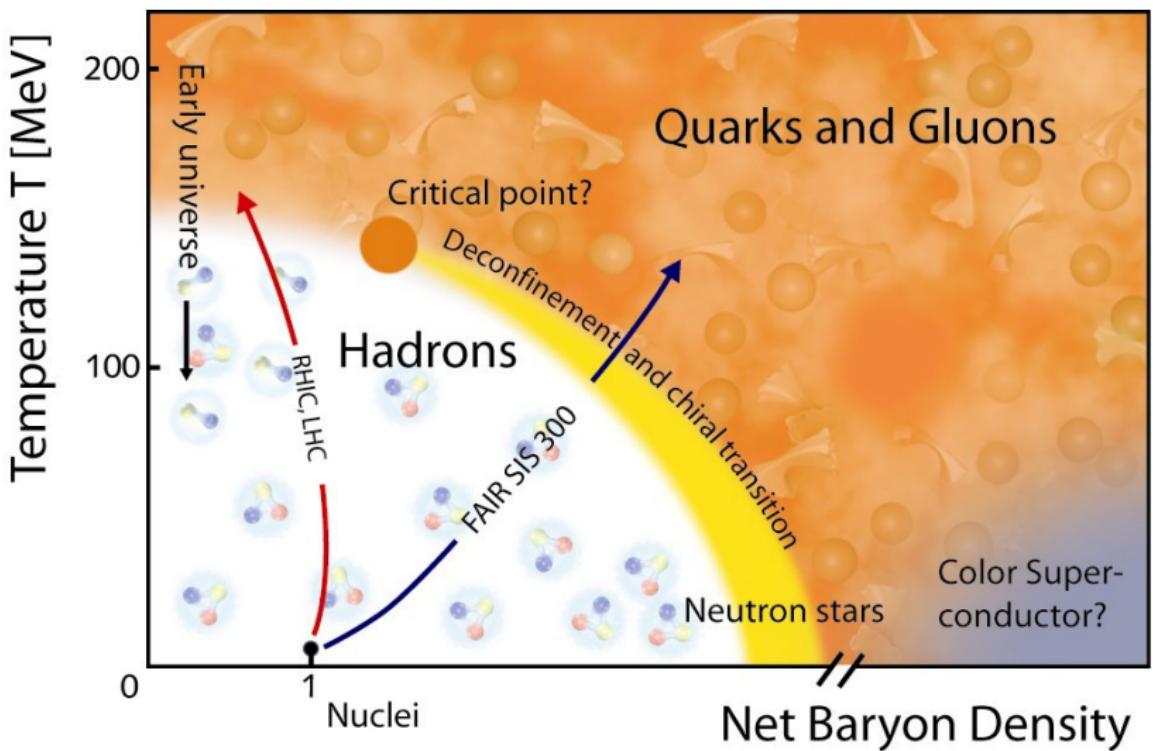
Motivation
●○○○

NP_X FD
○○○○

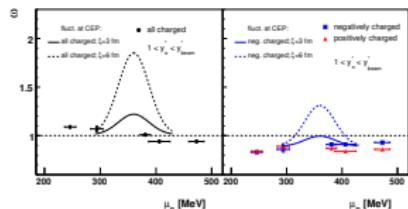
Zero μ
○○○○○○○

Finite μ
○○○○○○○

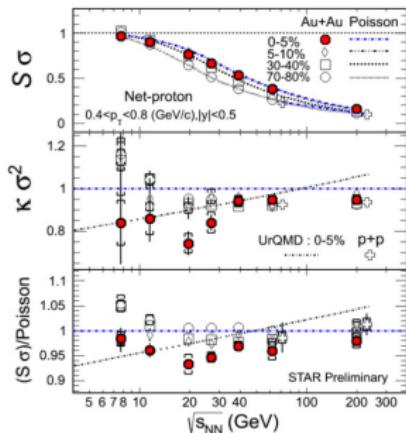
The QCD phase diagram



The critical point in heavy-ion collisions



(NA49 collaboration, Nucl. Phys. A 830 (2009))



(STAR collaboration, Quark Matter 2012)

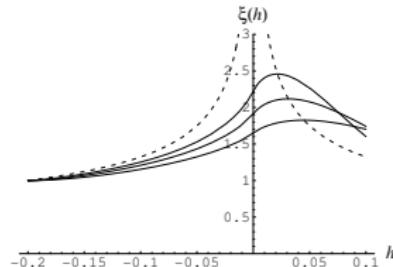
Moments/cumulants of particle distribution are sensitive to correlation length

$$\langle (\delta N)^2 \rangle \sim \xi^2$$

$$\langle (\delta N)^3 \rangle \sim \xi^{4.5}$$

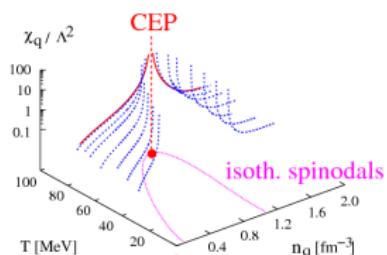
$$\langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2 \sim \xi^7$$

(Stephanov, PRL 102 (2009))



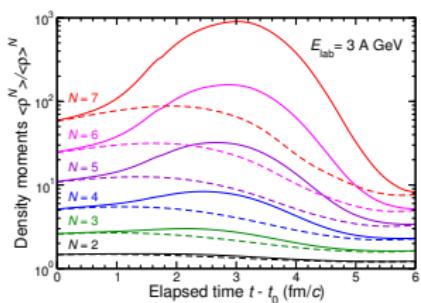
(Berdnikov, Rajagopal, PRD 61 (2000))

The first-order phase transition in heavy-ion collisions

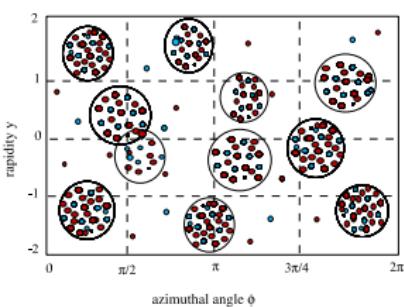


(Sasaki, Friman, Redlich, PRD 77 (2008))

- Formation of metastable phase
- Dynamical fragmentation
- Droplets
- Non-statistical multiplicity fluctuations



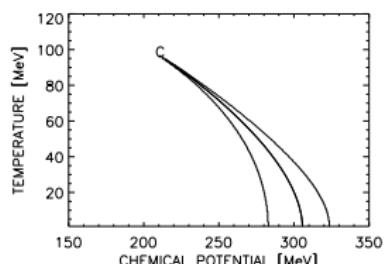
(Steinheimer, Randrup, PRL 109 (2012))



(Mishustin, PoS CPOD (2007))

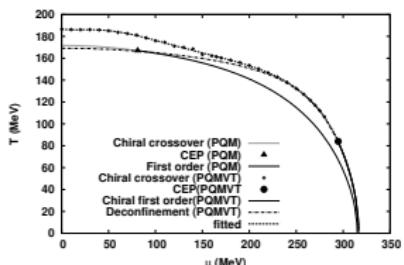
Effective models of QCD

Sigma model



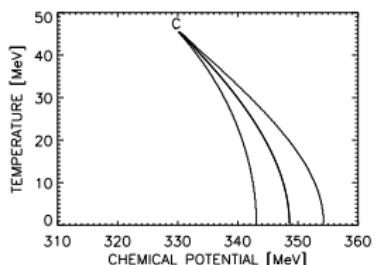
(Scavenius, Mocsy, Mishustin, Rischke, PRC **64** (2001))

Polyakov-quark-meson model



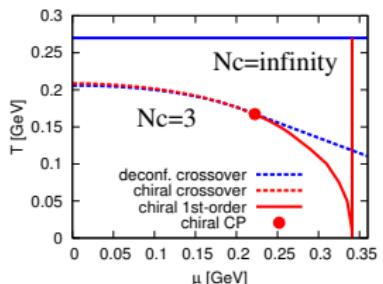
(Gupta, Tawari, arXiv:1107.1312v1 [hep-ph] (2011))

Nambu-Jona-Lasinio model



(Scavenius, Mocsy, Mishustin, Rischke, PRC **64** (2001))

Polyakov-NJL model



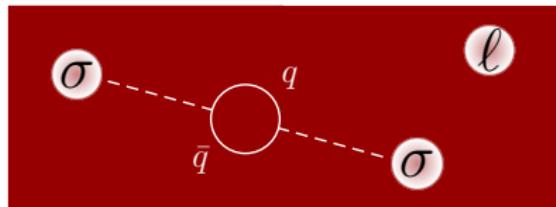
(C. Sasaki, APPS.3:659-668 (2010))

The NP χ FD model

Ideal quark fluid

- + Sigma field
- + Polyakov loop

Explicit Langevin
dynamics

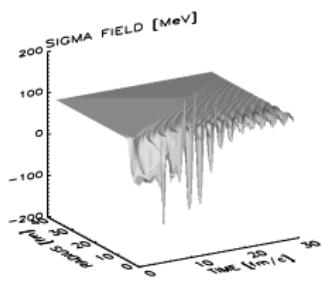


$$\mathcal{L} = \bar{q} [i(\gamma^\mu \partial_\mu - ig_s \gamma^0 A_0) - g\sigma] q + 1/2 (\partial_\mu \sigma)^2 - U(\sigma) - \mathcal{U}(\ell, \bar{\ell})$$

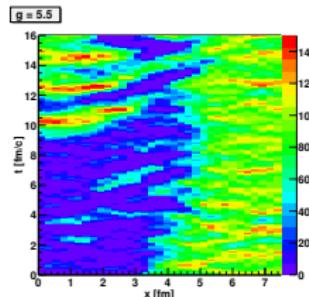
Fields and fluid are coupled via

- Mean-field potential $V_{\text{eff}} = U + \mathcal{U} + \Omega_{q\bar{q}}$
- Local pressure of quark fluid $p = p(\sigma, \ell, T, \mu)$
- Damping of sigma field due to interaction with quarks
- Energy-momentum exchange between fields and fluid

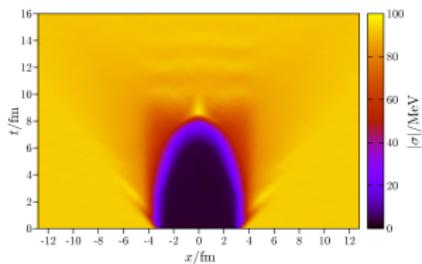
The Chiral fluid dynamics model



(I. N. Mishustin and O. Scavenius, Phys. Rev. Lett. **83** (1999))



(K. Paech, H. Stöcker and A. Dumitru, Phys. Rev. C **68** (2003))



(M. Nahrgang, C. H., S. Leupold, I. N. Mishustin and

M. Bleicher, arXiv:1105.1962v2)

- ➊ quark fluid coupled to chiral fields
- ➋ inclusion of fluctuations
- ➌ dissipation and noise

The coupled dynamics of fields and fluid

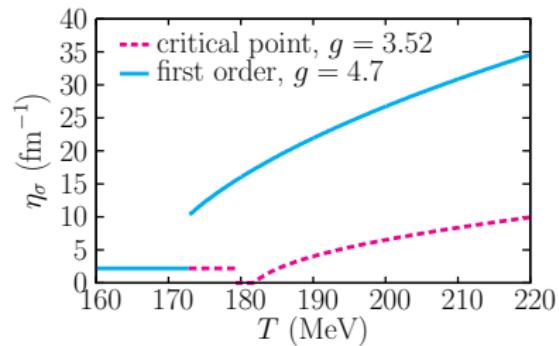
Langevin equation for the sigma field

$$\partial_\mu \partial^\mu \sigma + \eta_\sigma(T) \partial_t \sigma + \frac{\partial V_{\text{eff}}}{\partial \sigma} = \xi_\sigma, \quad \langle \xi_\sigma(t) \xi_\sigma(t') \rangle = \frac{1}{V} \delta(t - t') m_\sigma \eta_\sigma \coth\left(\frac{m_\sigma}{2T}\right)$$

(Nahrgang, Leupold, C. H., Bleicher, Phys. Rev. C **84** (2011))

Relaxation equation for the Polyakov loop, constant $\eta_\ell = 5/\text{fm}$

$$\begin{aligned} \eta_\ell \partial_t \ell + \frac{\partial V_{\text{eff}}}{\partial \ell} &= \xi_\ell \\ \langle \xi_\ell(t) \xi_\ell(t') \rangle T^2 &= \frac{1}{V} \delta(t - t') 2\eta_\ell T \end{aligned}$$



Energy momentum and baryon number conservation in ideal quark fluid

$$\partial_\mu T_q^{\mu\nu} = S_\sigma^\nu + S_\ell^\nu, \quad \partial_\mu N^\mu = 0$$

A dynamical model for the Polyakov loop

Phenomenological kinetic term and biquadratic coupling to chiral fields

$$\mathcal{L} = \mathcal{L}_\phi + \frac{N_c}{g^2} |\partial_\mu \ell|^2 T^2 - \mathcal{U}(\ell) - \frac{h^2}{2} \phi^2 |\ell|^2 T^2$$

(Dumitru, Pisarski, Phys. Lett. B 504 (2001))

Problem when $T = T(x^\mu)$
due to

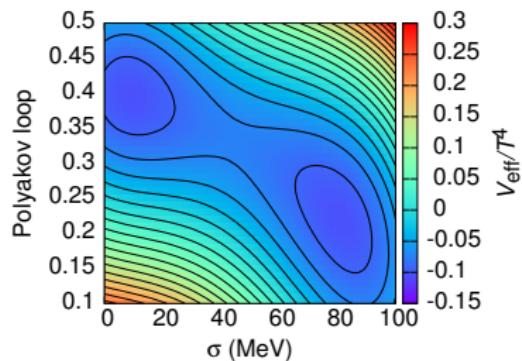
$$\frac{\partial}{\partial x^\mu} \left[\frac{N_c}{g^2} |\partial_\mu \ell|^2 T(x^\mu)^2 \right]$$

in Euler-Lagrange equation

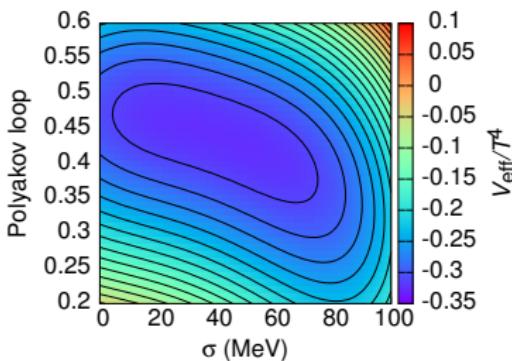
Possible solutions

- replace T by T_0
- Polyakov loop always in equilibrium
- use simple relaxation equation

Results at zero baryochemical potential



first order transition,
 $g = 4.7, T_c = 172.9 \text{ MeV}$

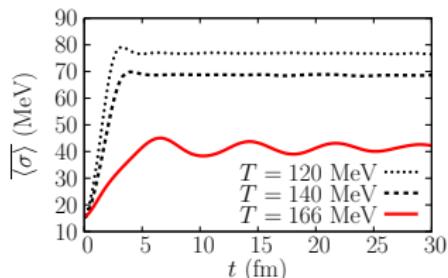


critical point,
 $g = 3.52, T_c = 180.5 \text{ MeV}$

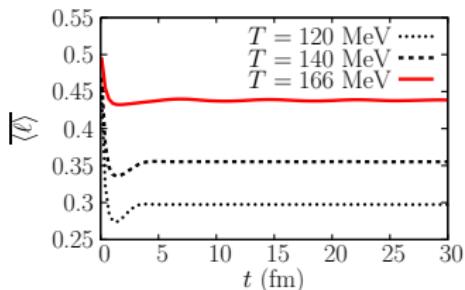
Study CP and FO by varying quark-meson coupling g

- Relaxational dynamics in a box
- Fluid dynamical expansion

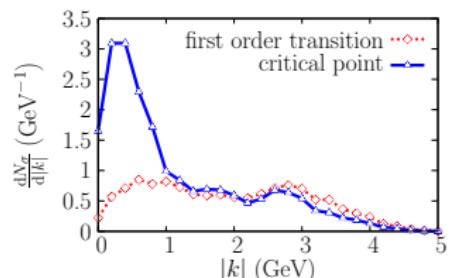
Box: Critical slowing down



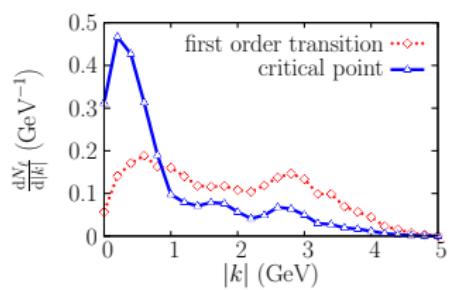
Relaxation of sigma field



Relaxation of Polyakov loop



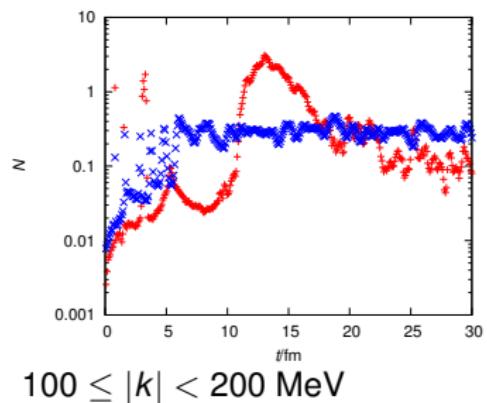
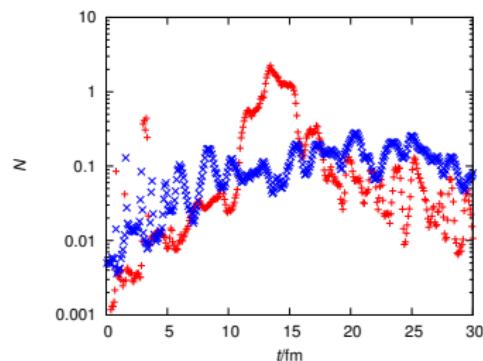
Sigma fluctuations



Polyakov loop fluctuations

Motivation
○○○○NP_χFD
○○○○Zero μ
○○●○○○○Finite μ
○○○○○○○

Box: Fourier analysis of sigma fluctuations

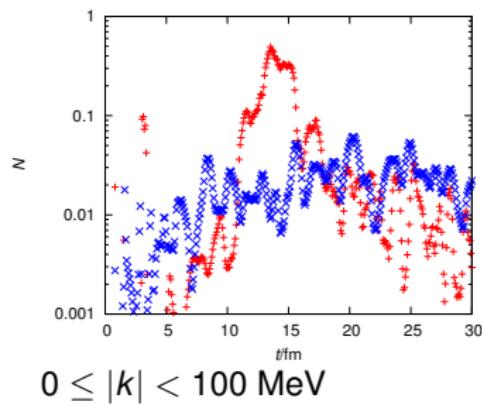


Intensity of sigma fluctuations:

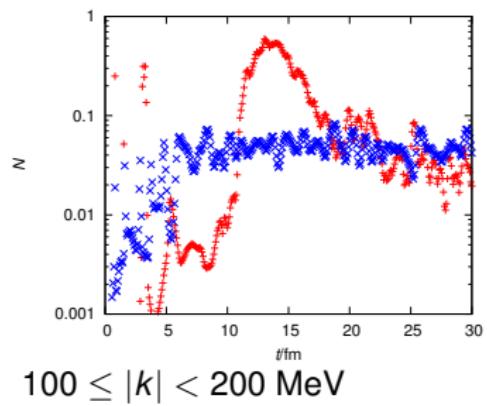
$$N = \int_{\Delta k} d^3k N_k = \int_{\Delta k} d^3k \frac{a_k^\dagger a_k}{(2\pi)^3 2\omega_k} = \int_{\Delta k} d^3k \frac{\omega_k^2 |\sigma_k|^2 + |\dot{\sigma}_k|^2}{(2\pi)^3 2\omega_k}$$

Motivation
○○○○NP χ FD
○○○○Zero μ
○○○●○○○Finite μ
○○○○○○○

Box: Fourier analysis of Polyakov loop fluctuations



$$0 \leq |k| < 100 \text{ MeV}$$



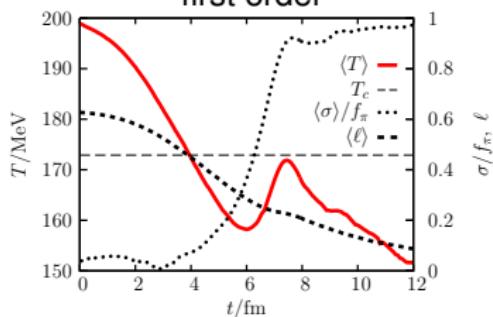
$$100 \leq |k| < 200 \text{ MeV}$$

Intensity of Polyakov loop fluctuations:

$$N = \int_{\Delta k} d^3k \, N_k = \int_{\Delta k} d^3k \frac{a_k^\dagger a_k}{(2\pi)^3 2\omega_k} = \int_{\Delta k} d^3k T^2 \frac{\omega_k^2 |\ell_k|^2 + |\dot{\ell}_k|^2}{(2\pi)^3 2\omega_k}$$

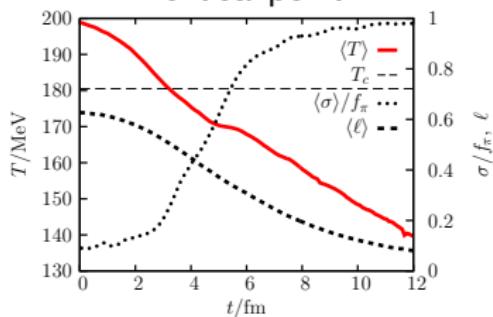
Expansion: first order vs. critical point

first order



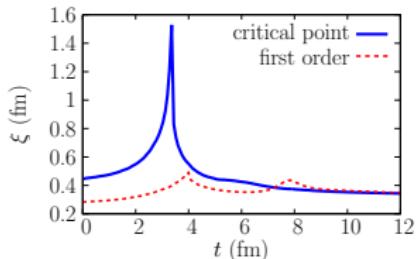
- Formation of supercooled phase
- Decay after ~ 2 fm
- Reheating of the quark fluid

critical point



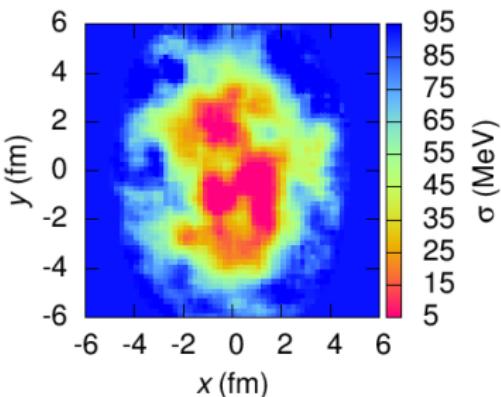
- Smooth transition
- Saddle point in $\langle T \rangle$ near T_c
- Slowing down

Expansion: Domain formation

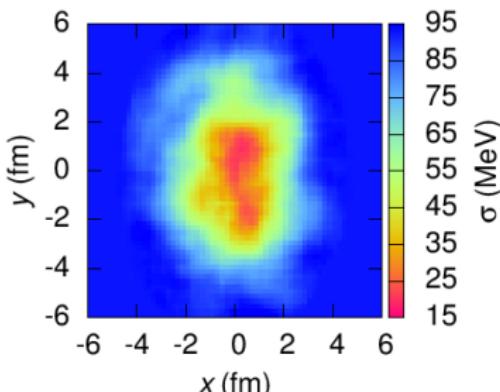


Correlate stochastic noise fluctuations over volume

$$V^{\text{corr}} = \xi^3 = \frac{1}{m(\langle T \rangle)^3}$$



at first-order



at CP

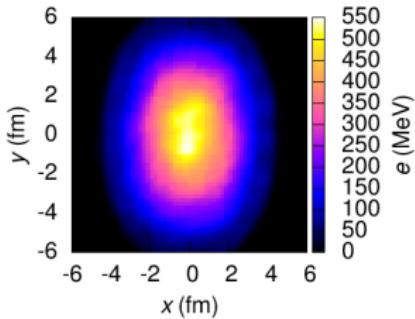
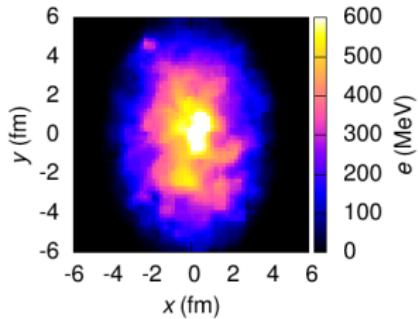
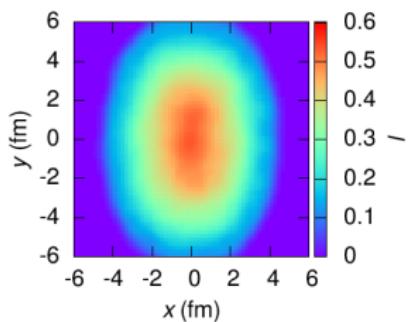
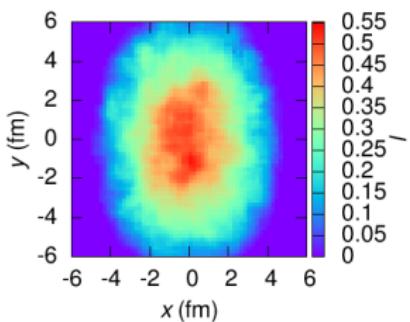
Motivation
○○○○

NP χ FD
○○○○

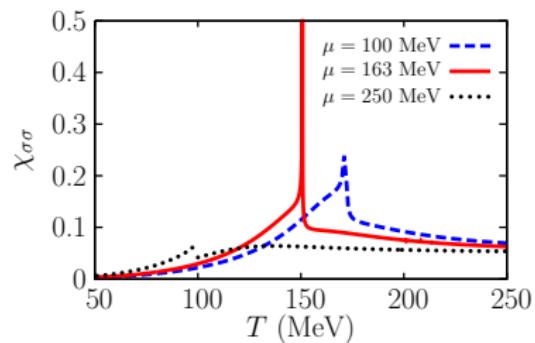
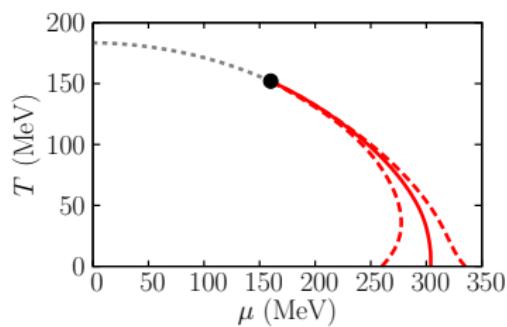
Zero μ
○○○○○○●

Finite μ
○○○○○○○

Free expansion: Domain formation

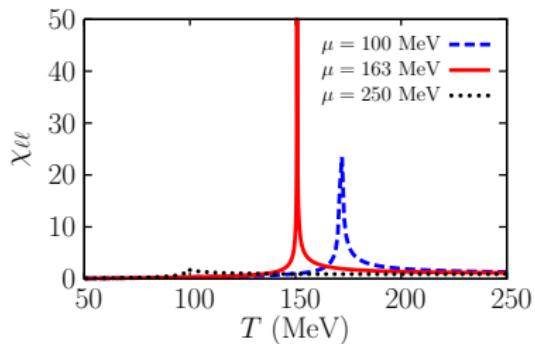


Results at finite baryochemical potential

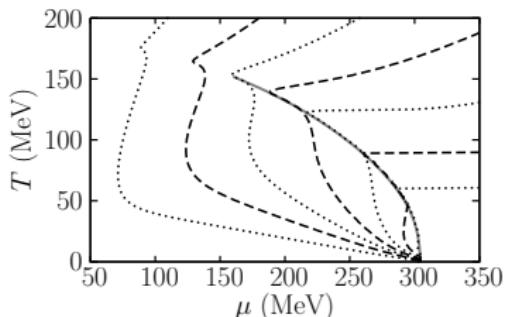
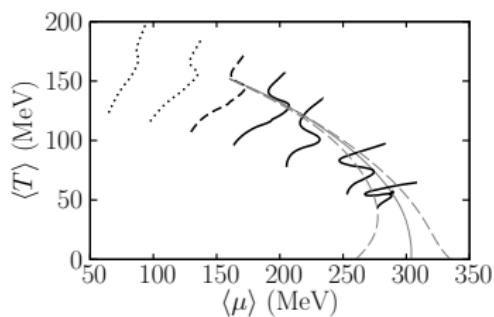


Phase diagram of Polyakov-Quark-meson model

- Common phase transition
- Common CEP



Expansion: Nonequilibrium trajectories



- Trajectories close to isentropes at crossover and CEP
- Trajectories influenced by nonequilibrium effects at first-order transition
- At high densities system remains in spinodal region for long time

Possibility for domain formation?

(CH, M. Nahrgang, I. Mishustin, M. Bleicher, in preparation)

Motivation
○○○○

NP χ FD
○○○○

Zero μ
○○○○○○○

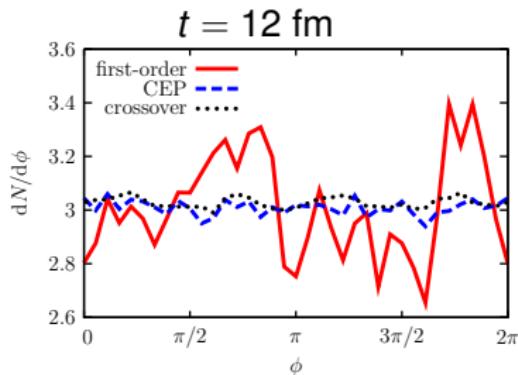
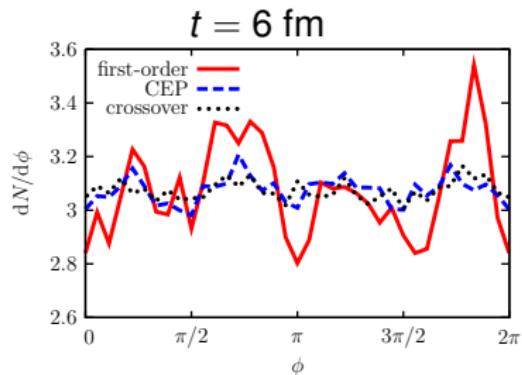
Finite μ
○○●○○○

Expansion: High-density domains

First-order

CEP

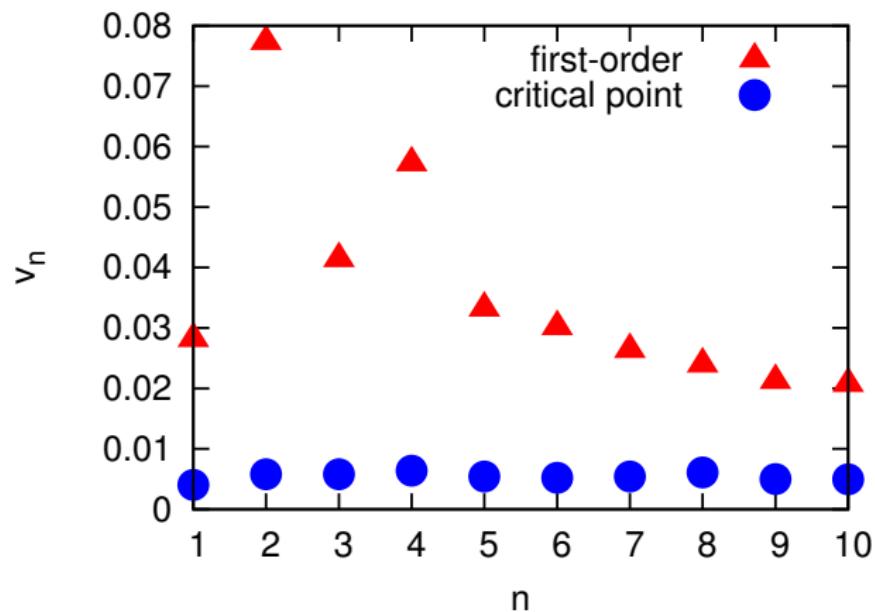
Expansion: High-density domains



Azimuthal distribution of net baryon number $\frac{dN}{d\phi}$

- Strong fluctuations at first-order transition
- Signal remains and is not washed out

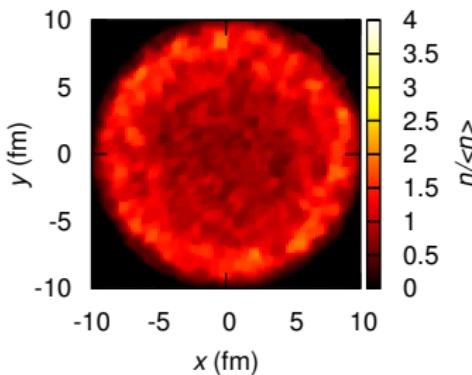
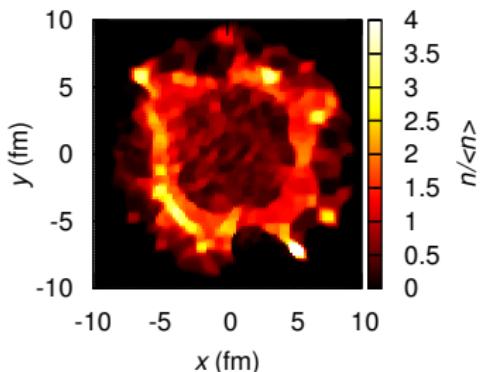
Expansion: High-density domains



Fourier coefficients $v_n = \langle \cos[n(\phi - \psi_n)] \rangle$

- Strong enhancement at first-order transition

Conclusions



- Nonequilibrium effects crucially influence a dynamical QCD phase transition
- Domain formation is observable in the order parameter fields
- Density profile evolve inhomogeneously
- Formation of baryon density domains may serve as experimental signal