Superconducting Accelerator Magnets

How Numerical Field Computation is used during Design, Optimization, Measurement, Quality Assurance, and Commissioning

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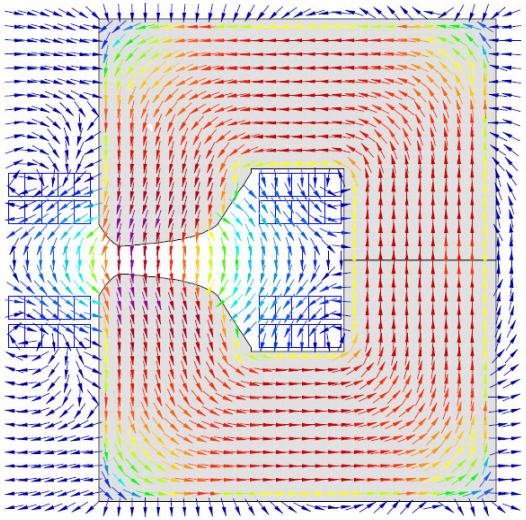
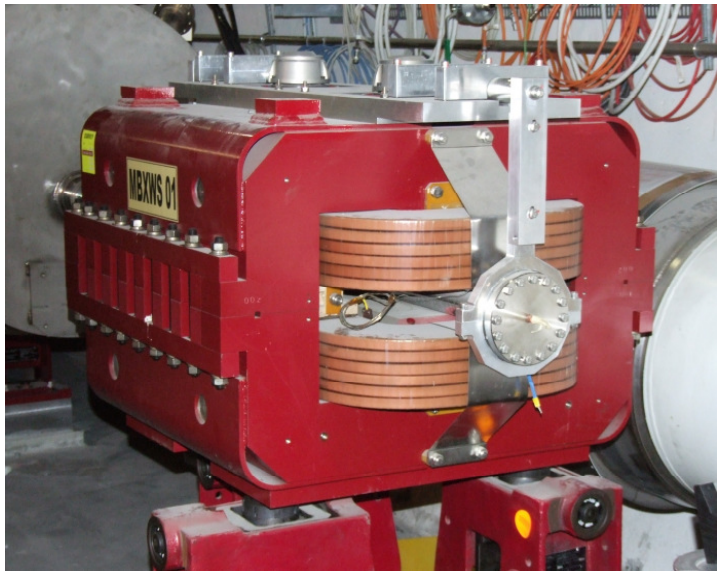
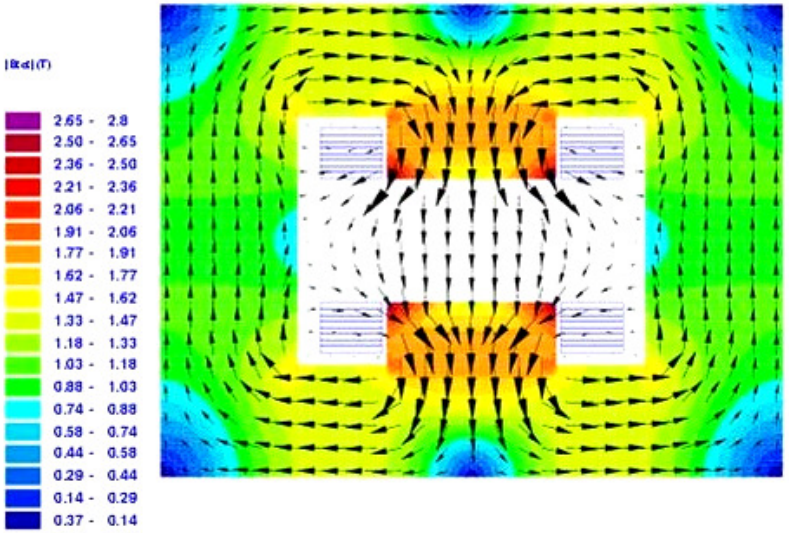
Iron Dominated Magnets

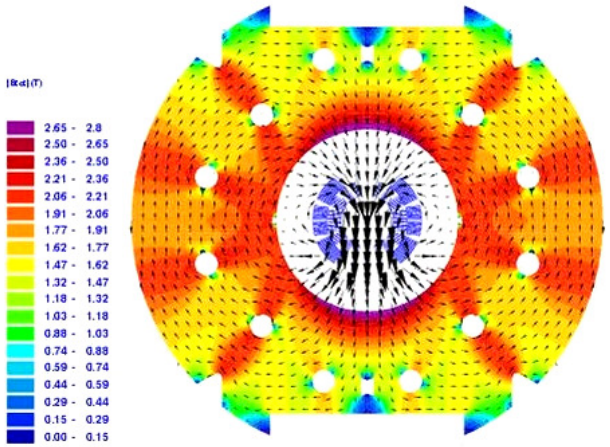




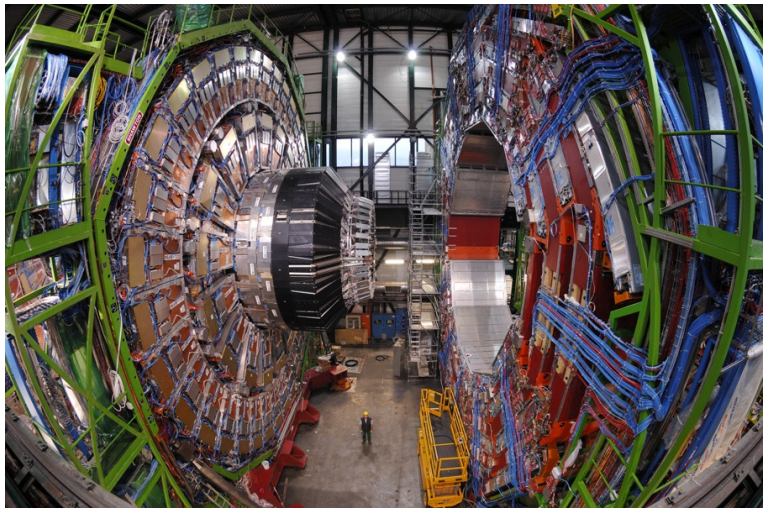


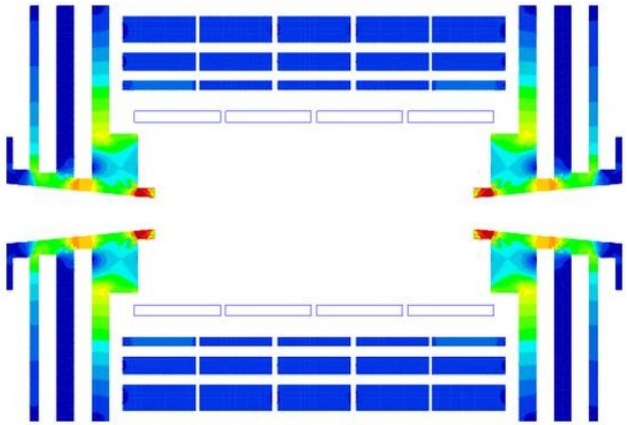
$N \cdot$ $N \cdot I = 96000$ **A** $B_1 = 1.18$ **T** $B_s = 0.26$ **T**

Coil Dominated Magnets



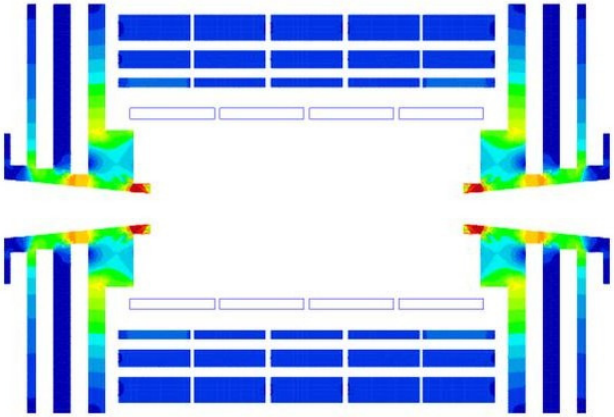
$B = 8.33$ T $\quad B_s = 7.77$ T

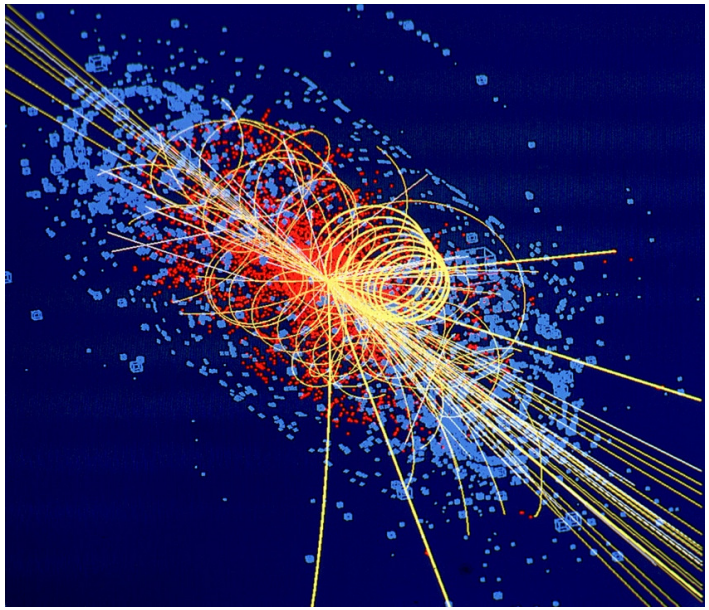


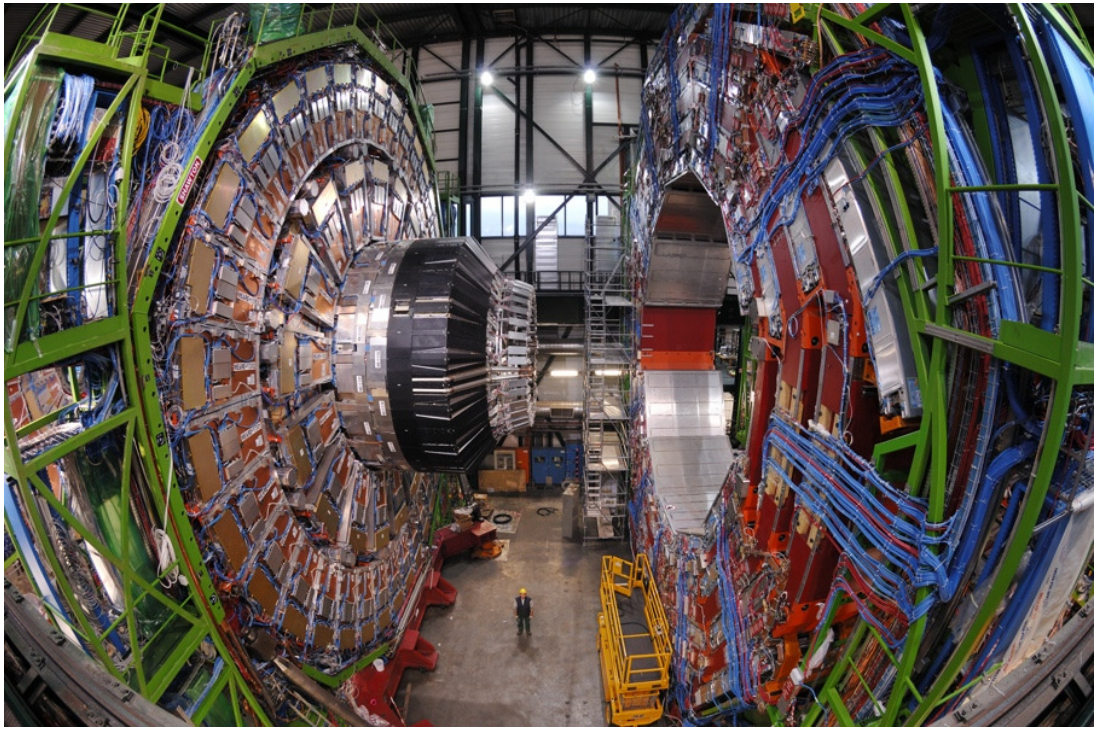


$B = 4$ T $\quad B_s = 3.69$ T

CMS (Class 1 Magnets)



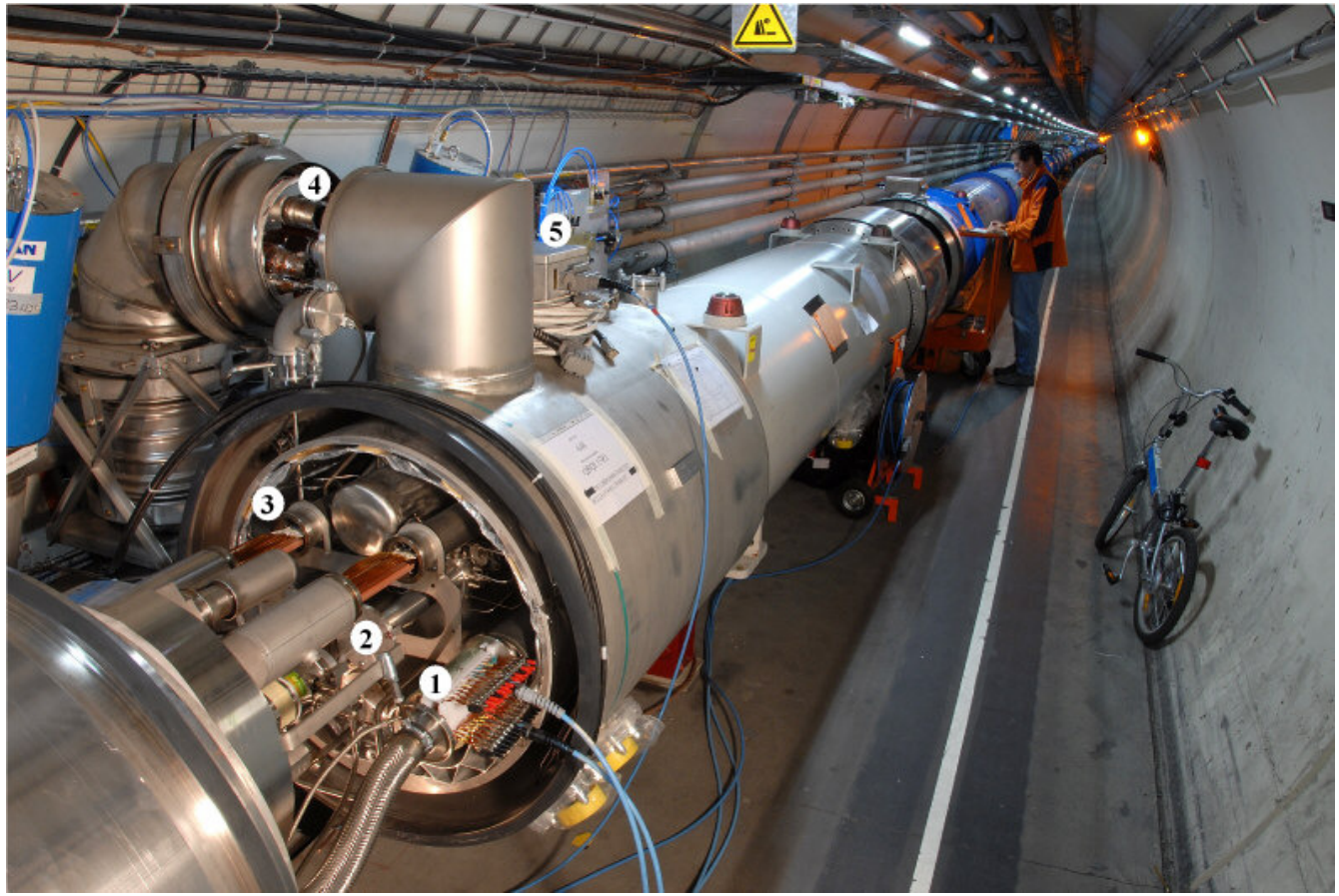




$$S = R(1 - \cos\frac{\alpha}{2}) \approx \frac{R\alpha^2}{8} = \frac{QBL^2}{8p}$$

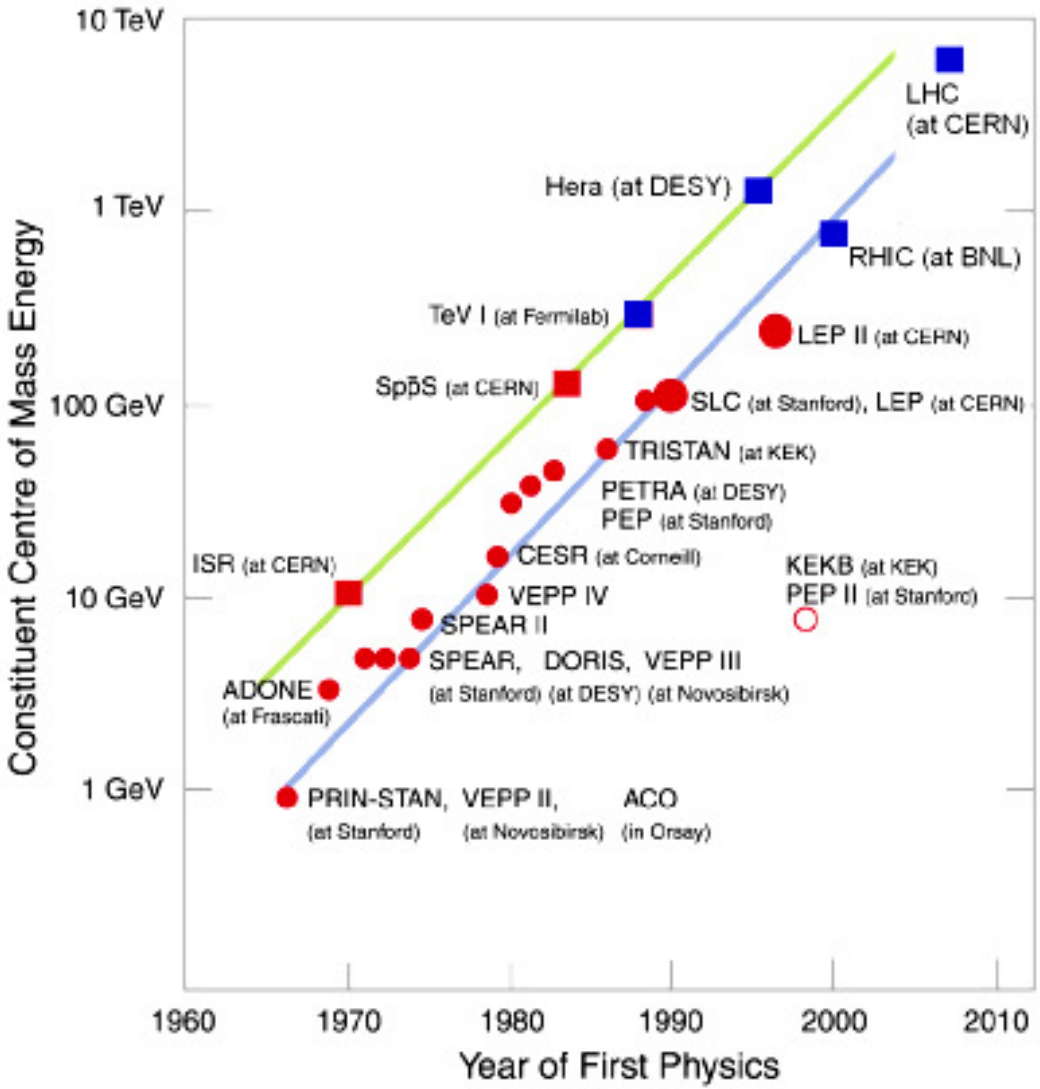
String of LHC Magnets in the Tunnel (Class 2 Magnets)

$$\{p\}_{\mathrm{GeV/c}} \approx 0.3\{Q\}_e\{R\}_{\mathrm{m}}\{B_0\}_{\mathrm{T}}$$

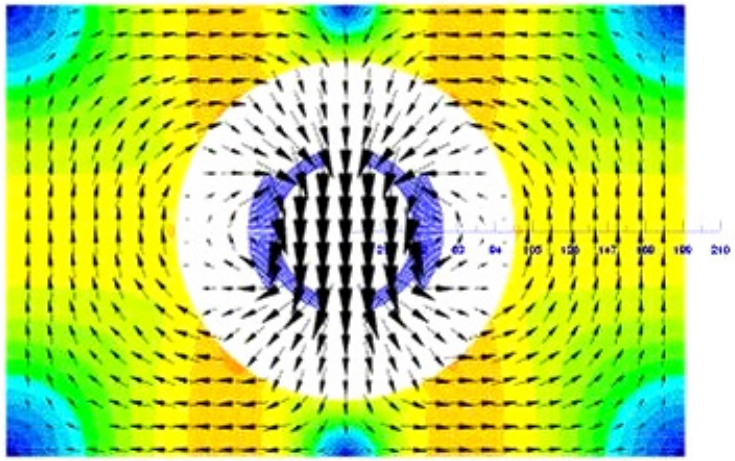


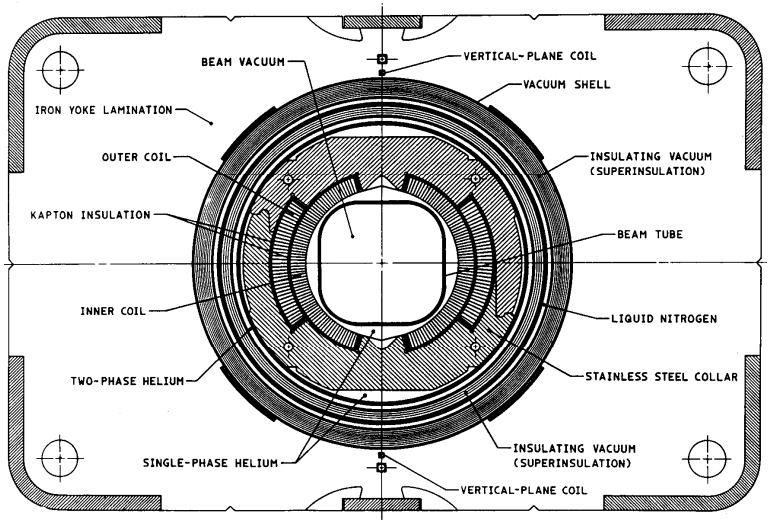
High field and high current density

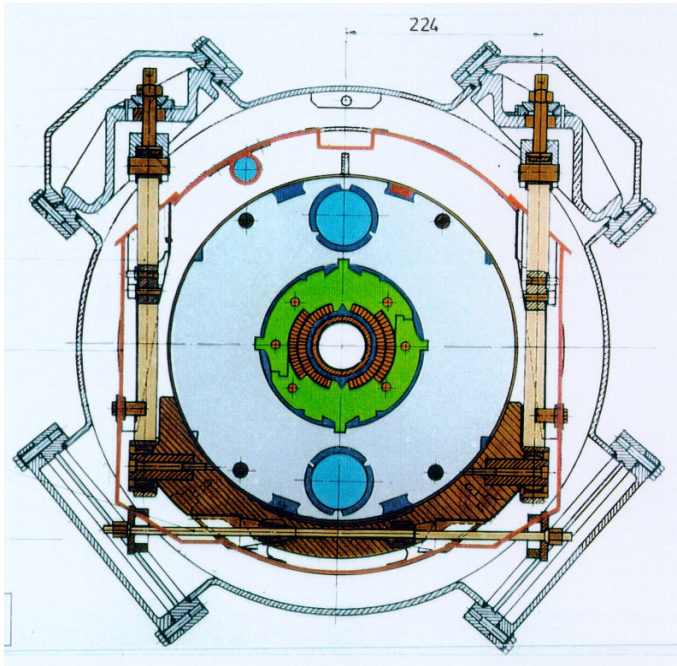
Livingston Plot

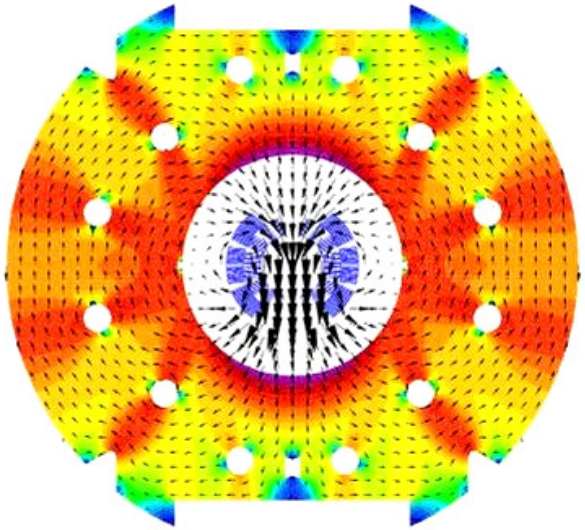


Cos θ Tevatron, HERA, Rhic, LHC Single Aperture Models

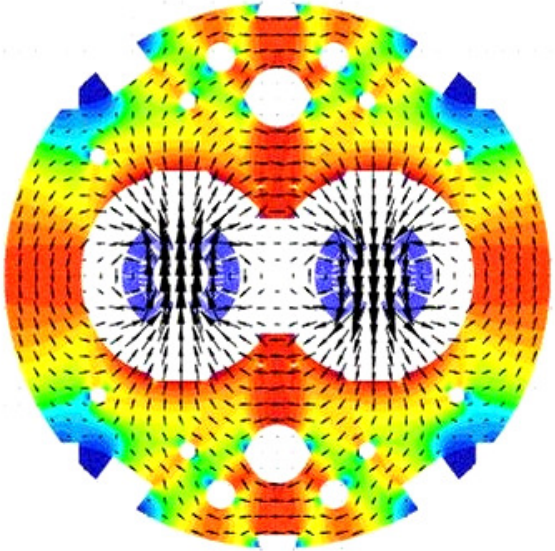


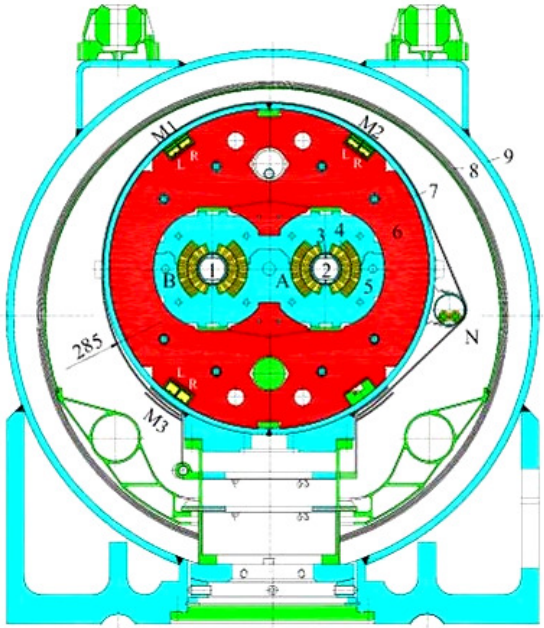






Cross-section of the LHC Cryodipole





Conventional and Superconducting (Class 2) Magnets

➔ Conventional magnets

- Important ohmic losses require water cooling
- Field is defined by the iron pole shape (max 1.5 T)
- Easy electrical and beam-vacuum interconnections
- Voltage drop over one coil of the MBW magnets = 22 V

➔ Superconducting magnets

- Field is defined by the coil layout
- Maximum field limited to 10 T (NbTi), 12 T (Nb_3Sn)
- Enormous electromagnetic forces (400 tons/m in MB for LHC)
- Quench protection system required
- Cryogenic installation (1.8 K)
- Electrical interconnections in cryo-lines
- Voltage drop on LHC magnet string (154 MB) 155 V

Comparison (EM-Design)

- Conventional magnets
  - Ideal pole shape known from potential theory
  - One-dimensional (analytical) field computation for main field
  - Commercial FEM software can be used as a black box (hysteresis modeling)

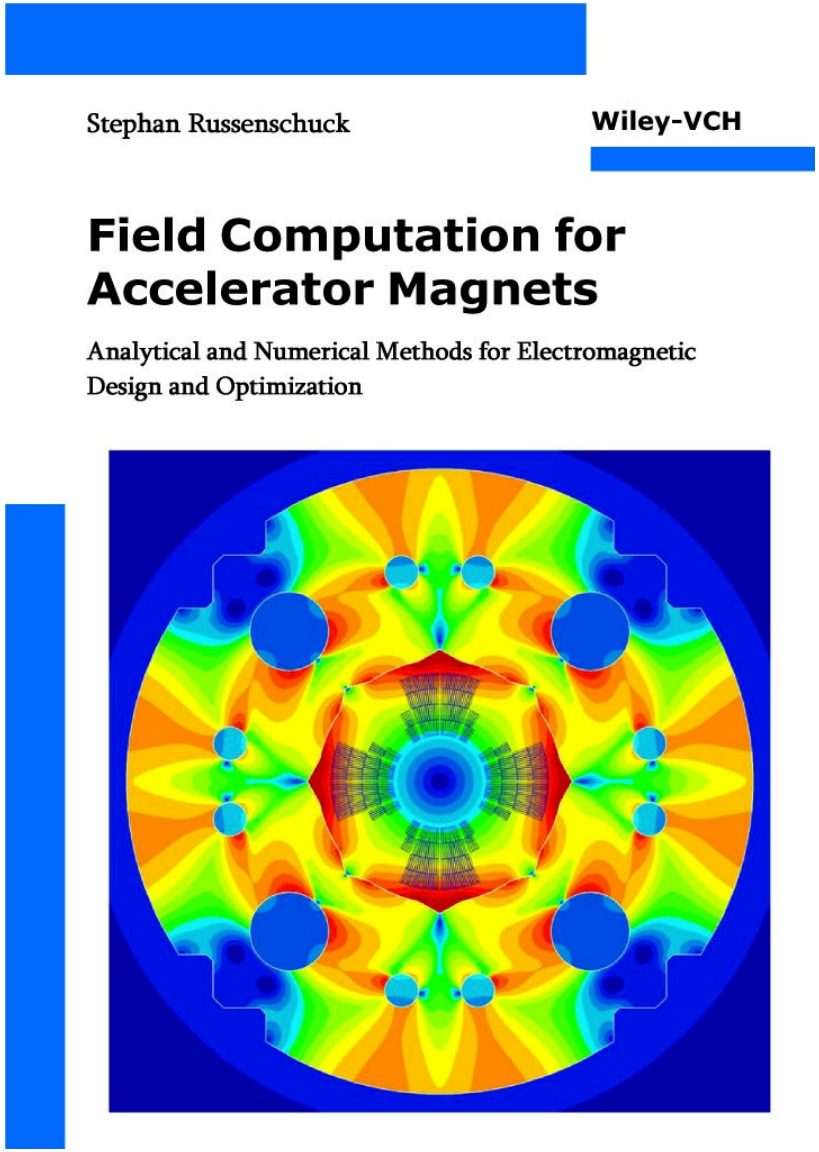
- Superconducting magnets
  - Decoupling of coil and yoke optimization
  - Accuracy of the field solution
  - Modeling of the coils
  - Filament magnetization
  - Quench simulations

A Multiphysics Problem

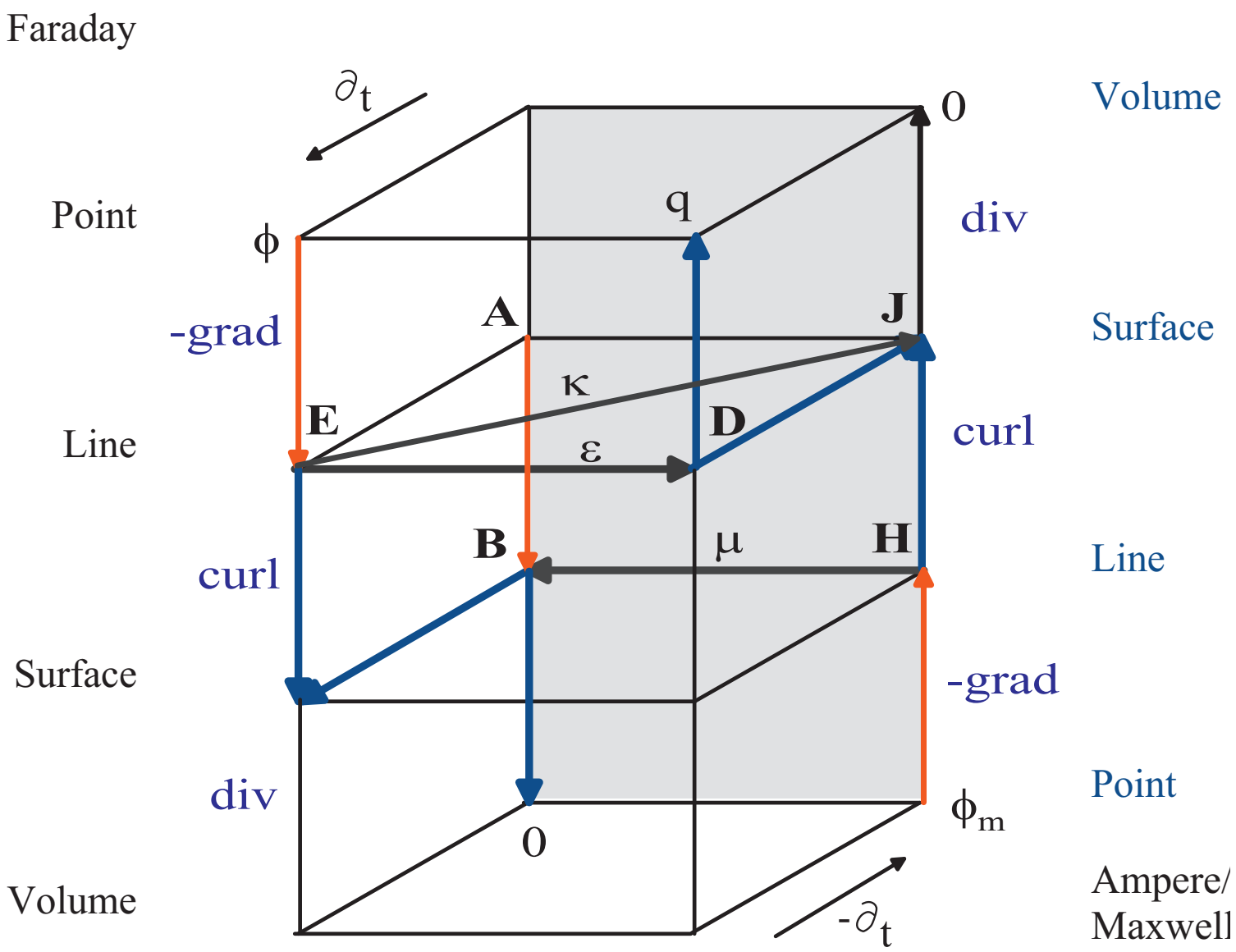
- Beam physics
- Material science: Superconducting cable, Steel, Insulation
- Mechanics and large-scale mechanical engineering
- Vacuum technology
- Cryogenics (Superfluid helium)
- Metrology and alignment
- Field measurements
- Electrical engineering (Power supplies, leads, buswork, quench detection and magnet protection
- Analytical and numerical field computation

Analytical and Numerical Field Computation

- Linear algebra
- Vector analysis
- Harmonic fields
- Green's functions and the method of images
- Complex analysis
- Differential geometry
- Numerical field computation
- Hysteresis modeling
- Coupled (thermo, magnetic, electric) systems
- Mathematical optimization



Maxwell's Facade



Maxwell's Facade

$$\mathrm{curl}\,\frac{1}{\mu}\mathrm{curl}\,\mathbf{A} = \mathbf{J}$$

$$\frac{1}{\mu_0}\mathrm{curl}\,\mathrm{curl}\,\mathbf{A} = \mathbf{J}$$

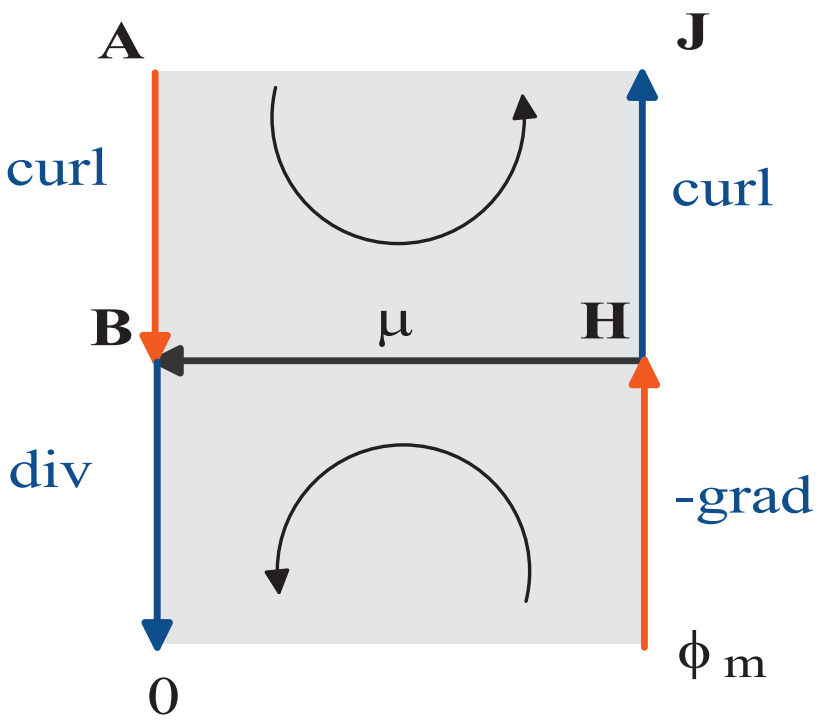
$$\nabla^2\mathbf{A} - \mathrm{grad}\,\mathrm{div}\,\mathbf{A} = 0$$

$$\nabla^2 A_z = 0$$

$$\mathrm{div}\,\mu\mathrm{grad}\,\phi_\mathrm{m} = 0$$

$$\mu_0\mathrm{div}\,\mathrm{grad}\,\phi_\mathrm{m} = 0$$

$$\nabla^2\phi_\mathrm{m} = 0$$



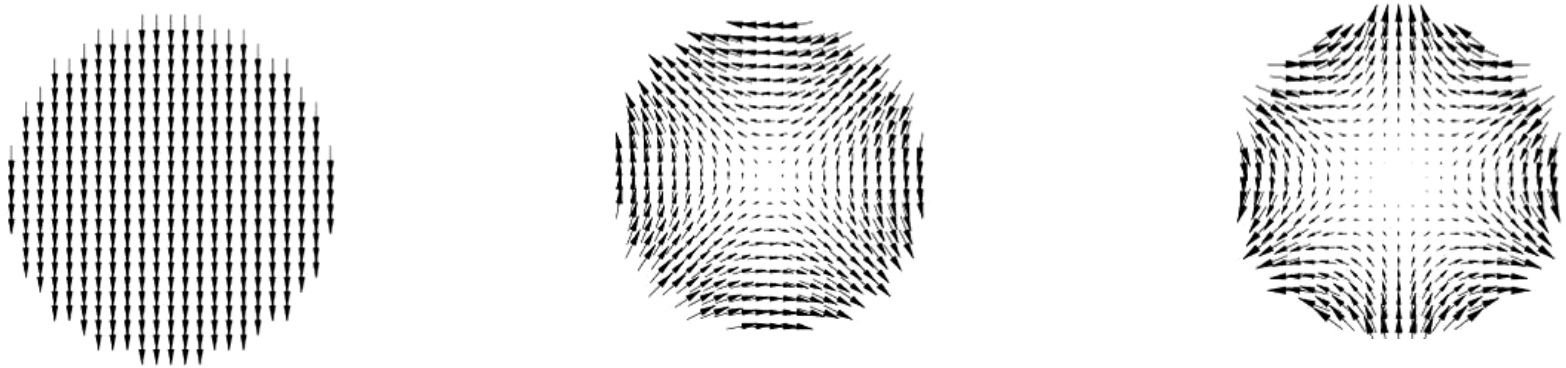
Eigensolutions of the Laplace Equation

$$A_z(r,\varphi) = \sum_{n=1}^{\infty} r^n (\mathcal{A}_n \sin n\varphi + \mathcal{B}_n \cos n\varphi).$$

The field components can then be expressed as

$$B_r(r,\varphi) = \frac{1}{r}\frac{\partial A_z}{\partial \varphi} = \sum_{n=1}^{\infty} n r^{n-1} (\mathcal{A}_n \cos n\varphi - \mathcal{B}_n \sin n\varphi),$$

$$B_\varphi(r,\varphi) = -\frac{\partial A_z}{\partial r} = -\sum_{n=1}^{\infty} n r^{n-1} (\mathcal{A}_n \sin n\varphi + \mathcal{B}_n \cos n\varphi),$$



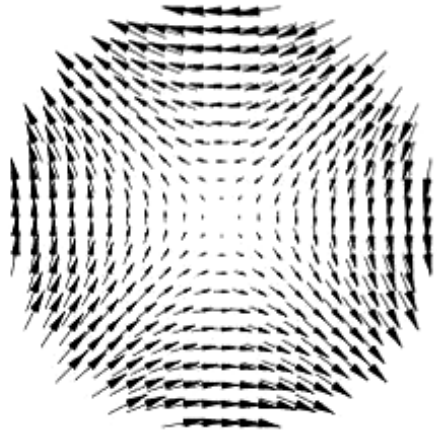
Harmonic Analysis (Solving the BVP)

$$B_r(r_0,\varphi) = \sum_{n=1}^{\infty} (B_n(r_0)\sin n\varphi + A_n(r_0)\cos n\varphi)$$

$$A_n(r_0) \approx \frac{2}{N}\sum_{k=0}^{N-1} B_r(r_0,\varphi_k)\cos n\varphi_k$$

$$B_n(r_0) \approx \frac{2}{N}\sum_{k=0}^{N-1} B_r(r_0,\varphi_k)\sin n\varphi_k.$$

$$\varphi_k = \frac{2\pi k}{N}, \qquad k = 0,1,2,\ldots,N-1$$

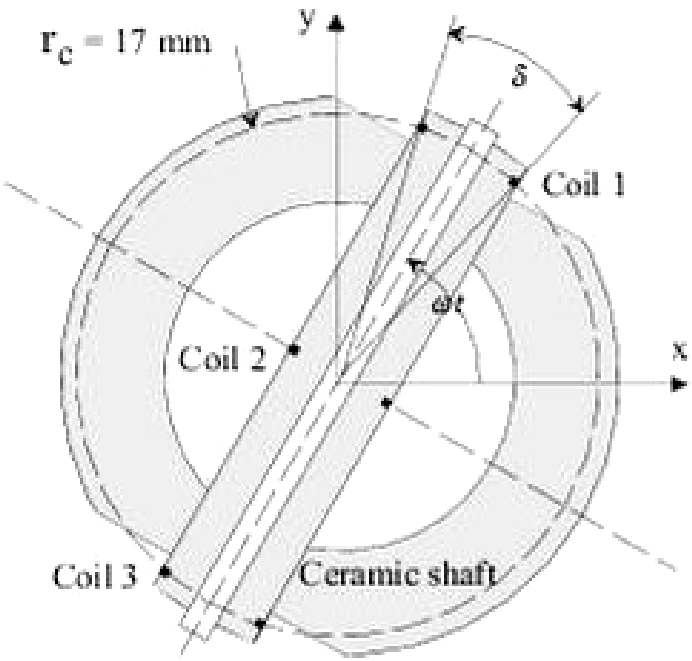


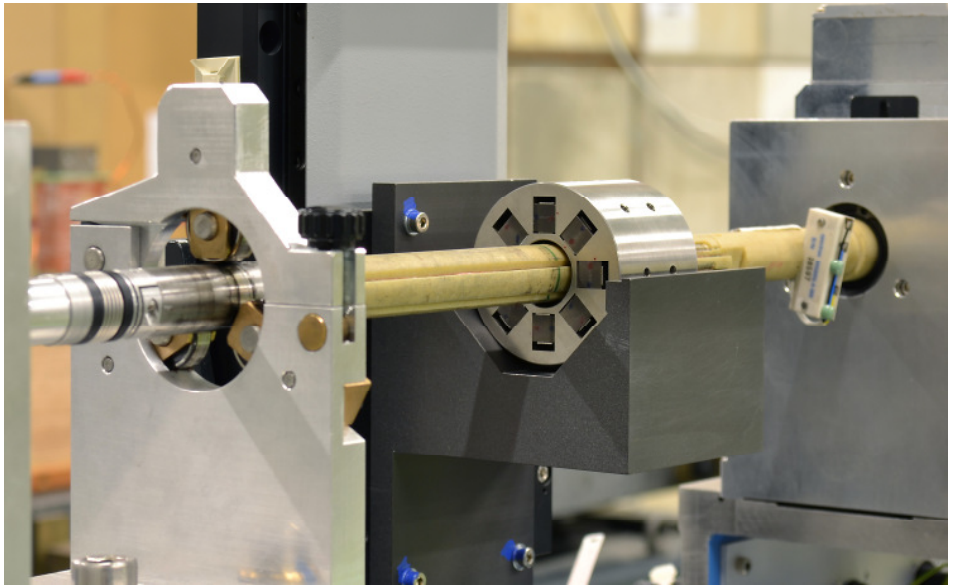
Don't bother with the Fast Fourier Transform, use discrete Fourier transform, but consider to expand the vector-potential (calculated or from flux measurements).

$$\mathcal{A}_n = \frac{1}{n\, r_0^{n-1}} A_n(r_0), \qquad \mathcal{B}_n = \frac{-1}{n\, r_0^{n-1}} B_n(r_0).$$

And please: Never use holomorphic continuation, it's just to inaccurate

Rotating Coil Measurements





$$\Phi = N\ell \int_{\varphi-\delta/2}^{\varphi+\delta/2} B_r(r_c, \varphi)\, r_c \, \mathrm{d}\varphi$$

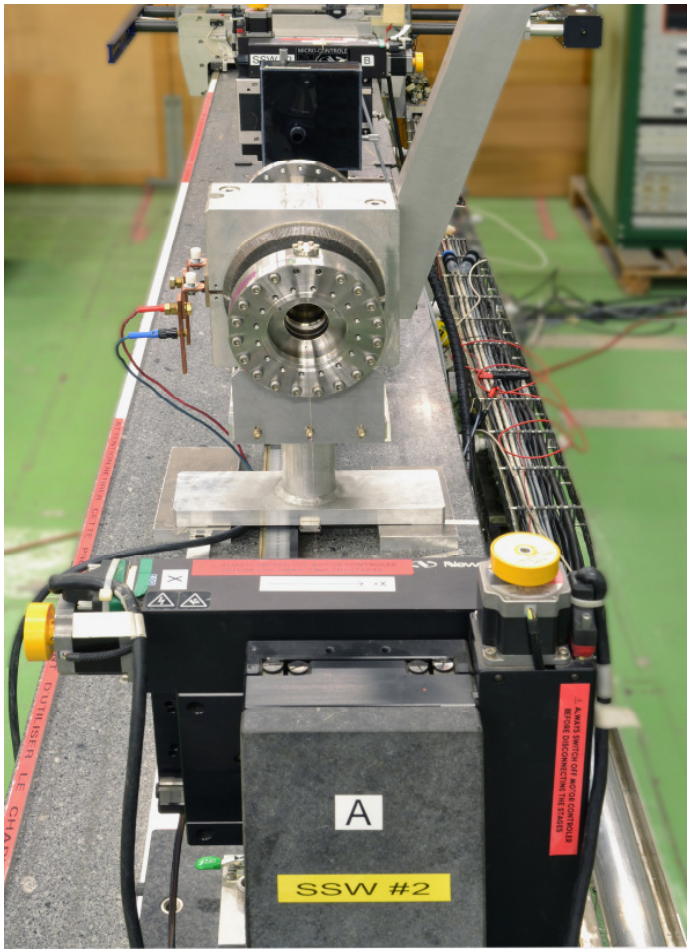
$$= \sum_{n=1}^{\infty} K_n^{\tan} \frac{\ell}{r_0^{n-1}} \left[B_n(r_0) \sin(n\omega t + n\Theta) + A_n(r_0) \cos(n\omega t + n\Theta) \right],$$

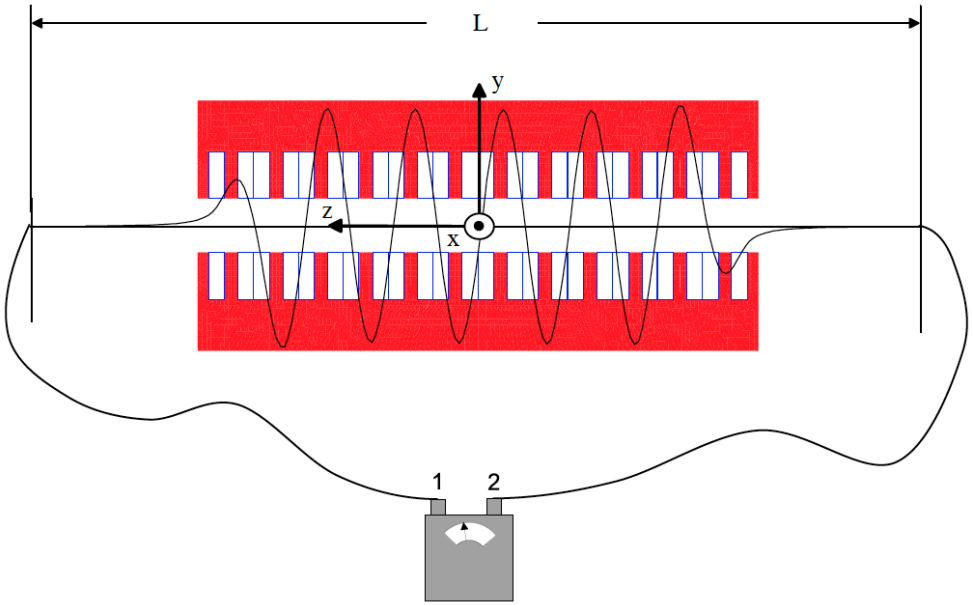
where

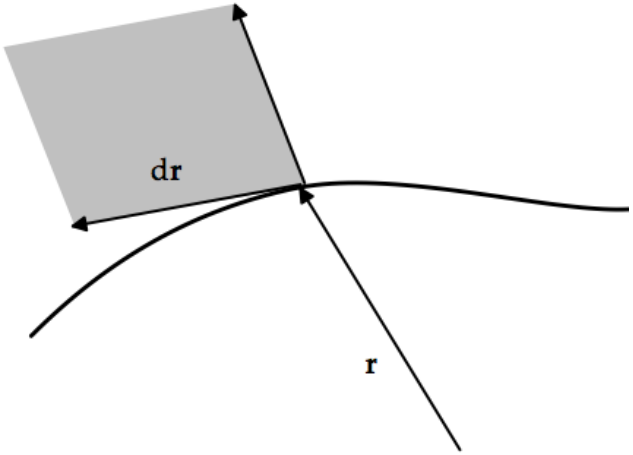
$$K_n^{\tan} = \frac{2N}{n} r_c^n \sin\left(\frac{n\delta}{2}\right),$$

Series Measurements of the LHC Magnets

Single Stretched Wire Measurements (Motional EMF)

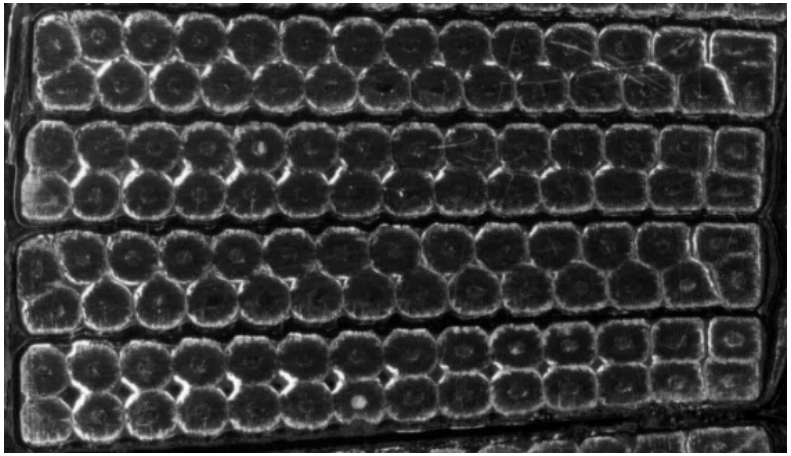


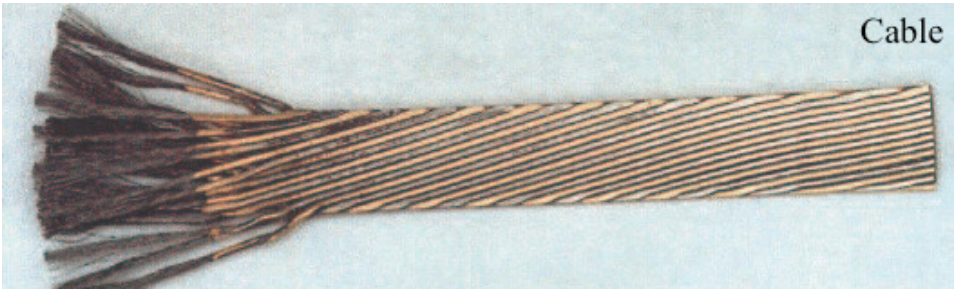


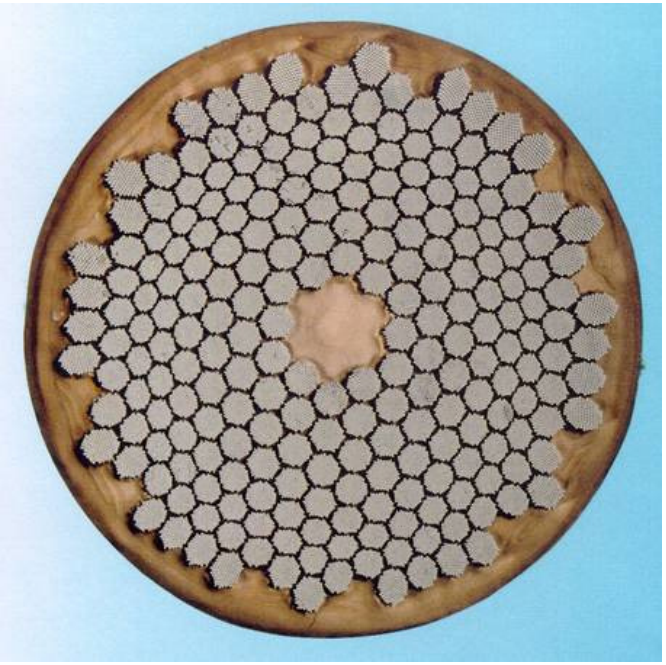


$$U = \int_{\mathscr{S}} (\mathbf{v} \times \mathbf{B}) \cdot \mathrm{d}\mathbf{r} = -\int_{\mathscr{S}} \mathbf{B} \cdot (\mathbf{v} \times \mathrm{d}\mathbf{r}) = -\frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathscr{S}} \mathbf{B} \cdot \mathrm{d}\mathbf{a}$$

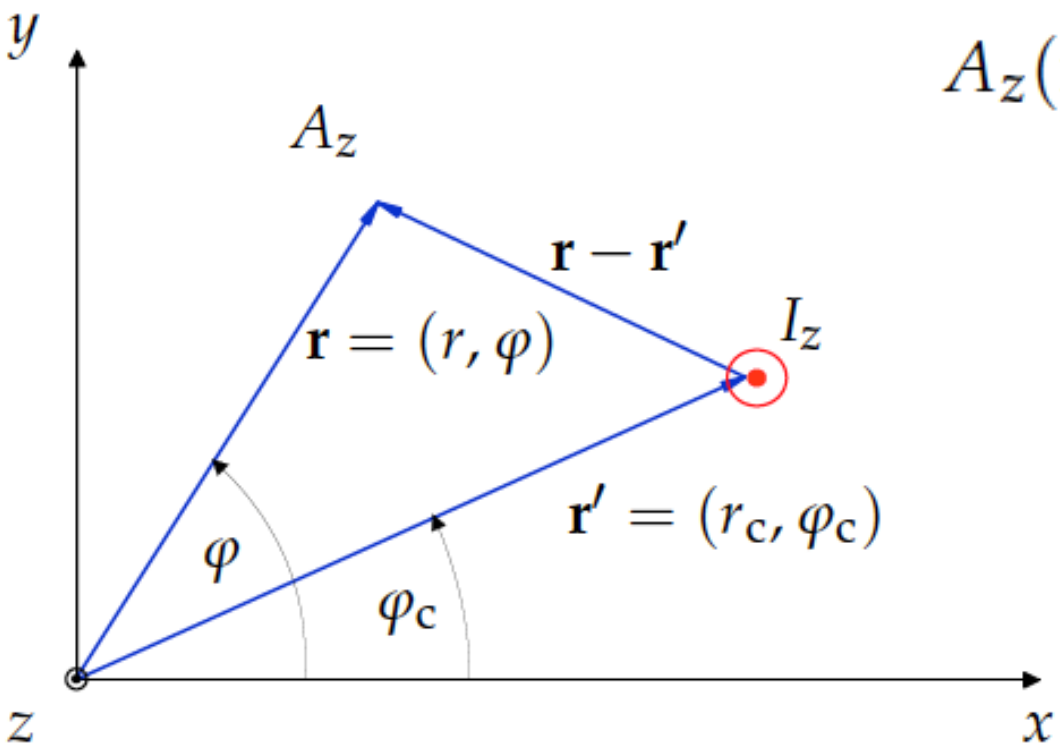
Rutherford (Roebel) Kabel, Strand, Nb-Ti Filament







The Field of a Line Current



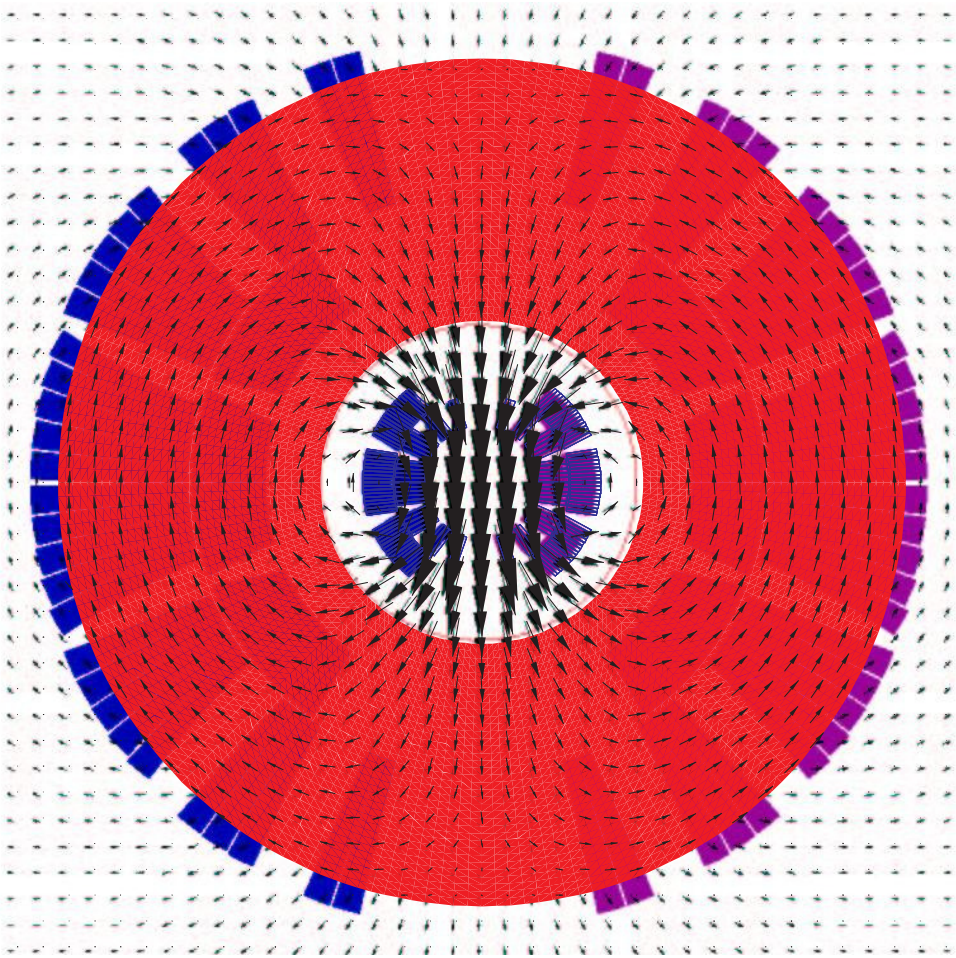
$$A_z(\mathbf{r}) = -\frac{\mu_0 I}{2\pi}\ln\left(\frac{|\mathbf{r}-\mathbf{r}'|}{r_{\mathrm{ref}}}\right)$$

$$A_z(r,\varphi) = -\frac{\mu_0 I}{2\pi}\ln\left(\frac{r_c}{r_{\mathrm{ref}}}\right) + \frac{\mu_0 I}{2\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{r}{r_c}\right)^n \cos n(\varphi-\varphi_c)$$

$$B_n(r_0) = -\frac{\mu_0 I}{2\pi r_c}\left(\frac{r_0}{r_c}\right)^{n-1}\cos n\varphi_c, \qquad A_n(r_0) = \frac{\mu_0 I}{2\pi r_c}\left(\frac{r_0}{r_c}\right)^{n-1}\sin n\varphi_c.$$

Imaging Method

$$B_n(r_0) = -\sum_{k=1}^{K} \frac{\mu_0 I_k}{2\pi} \frac{r_0^{n-1}}{r_{c,k}^n} \left(1 + \lambda_\mu \left(\frac{r_{c,k}}{r_y}\right)^{2n}\right) \cos n\varphi_{c,k}\,,$$



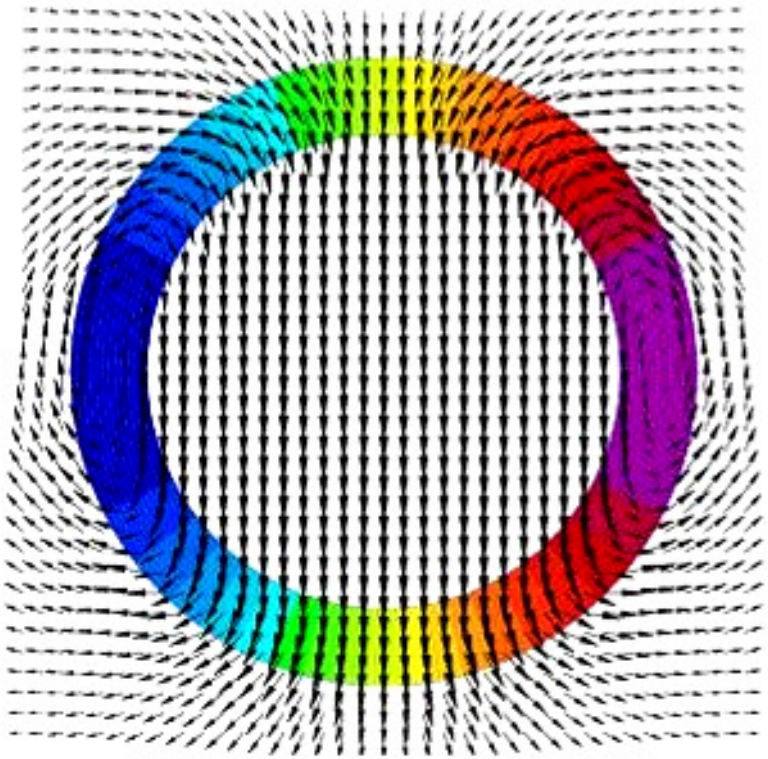
$$\lambda_\mu I := \frac{\mu_r - 1}{\mu_r + 1} I\,.$$

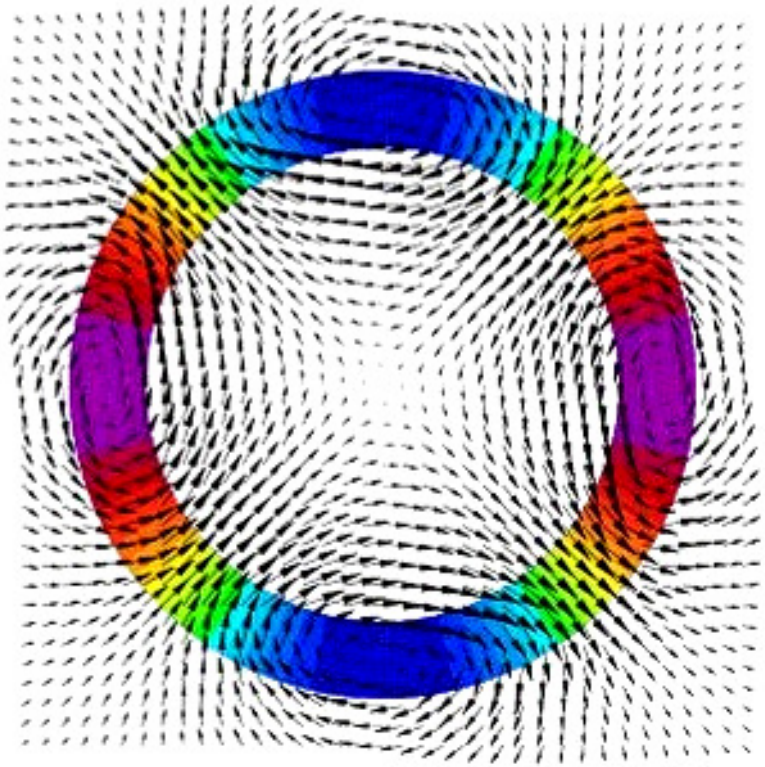
$$\frac{B_N^{\text{imag}}}{B_N + B_N^{\text{imag}}} \approx \left(1 + \left(\frac{r_y}{r}\right)^{2N}\right)$$

Ideal Current Distributions

$$B_n(r_0) = \int_{r_a}^{r_e} \int_0^{2\pi} -\frac{\mu_0 J_E}{2\pi} \quad J_c(B) = d\,(B_{c2} - B) \qquad \cos n\varphi_c\, r_c \mathrm{d}\varphi_c\, \mathrm{d}r_c$$

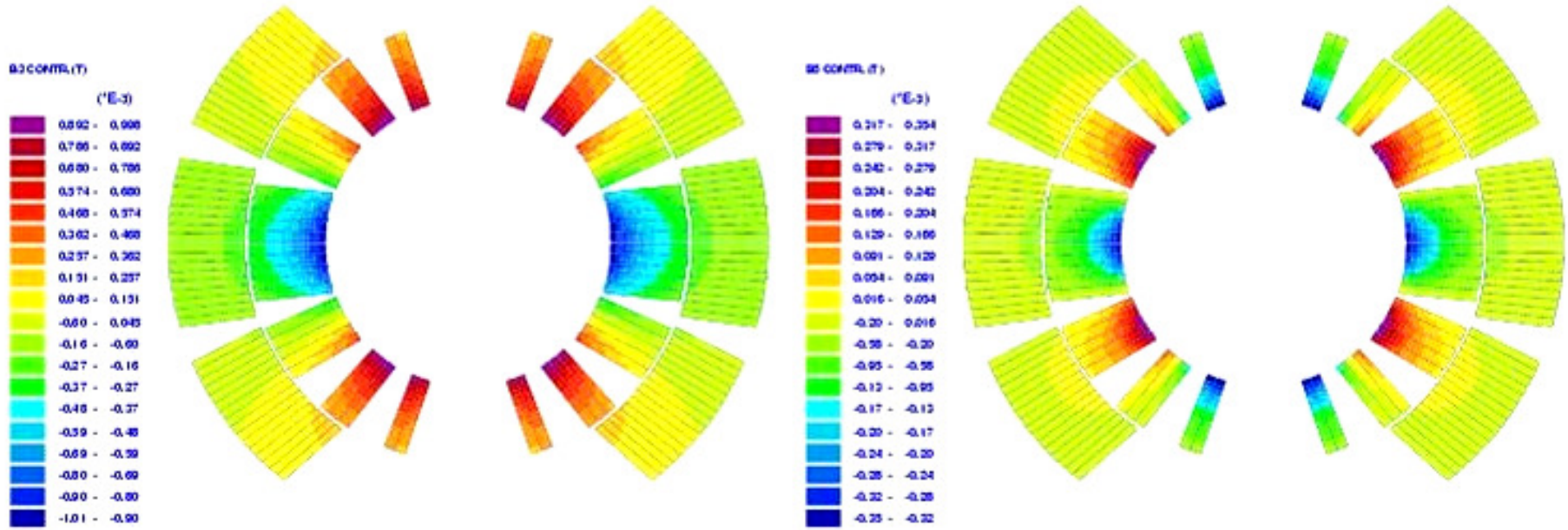
$$B = \frac{\mu_0}{2}\lambda_{tot}\, J_c\,(r_e - r_a) = \frac{\mu_0}{2}\lambda_{tot}\, d\,(B_{c2} - B)\,(r_e - r_a)\,,$$





Coil Block Approximation

$$B_n(r_0) = -\sum_{k=1}^{K} \frac{\mu_0 I_k}{2\pi} \frac{r_0^{n-1}}{r_{c,k}^n} \left(1 + \lambda_\mu \left(\frac{r_{c,k}}{r_y}\right)^{2n}\right) \cos n\varphi_{c,k} \,,$$



Sensitivity to Manufacturing Errors

$$B_n(r_0) = -\sum_{k=1}^{K} \frac{\mu_0 I_k}{2\pi} \frac{r_0^{n-1}}{r_{c,k}^n} \left(1 + \lambda_\mu \left(\frac{r_{c,k}}{r_y}\right)^{2n}\right) \cos n\varphi_{c,k},$$

$$\frac{\partial B_n(r_0)}{\partial \varphi_c} = -\frac{\mu_0 I_k}{2\pi} \frac{n r_0^{n-1}}{r_c^n} \left(1 + \left(\frac{r_c}{r_y}\right)^{2n}\right) \sin n\varphi_c,$$

$$\frac{\partial B_n(r_0)}{\partial r_c} = \frac{\mu_0 I_k}{2\pi} \frac{n r_0^{n-1}}{r_c^{n+1}} \left(1 - \left(\frac{r_c}{r_y}\right)^{2n}\right) \cos n\varphi_c.$$

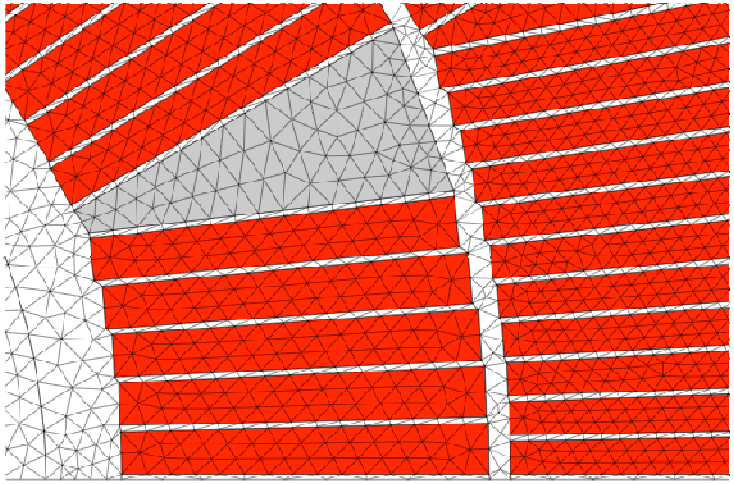
Increase of the azimuthal coil size by 0.1 mm produces (in units of 10^{-4}):

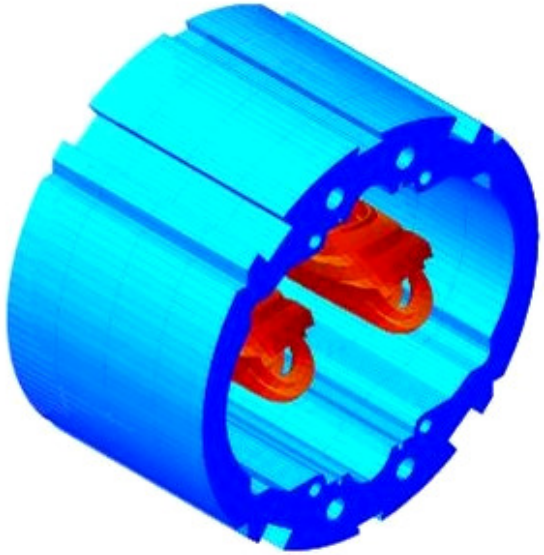
$b_1 = -14.$ $b_3 = 1.2$ $b_5 = 0.03$

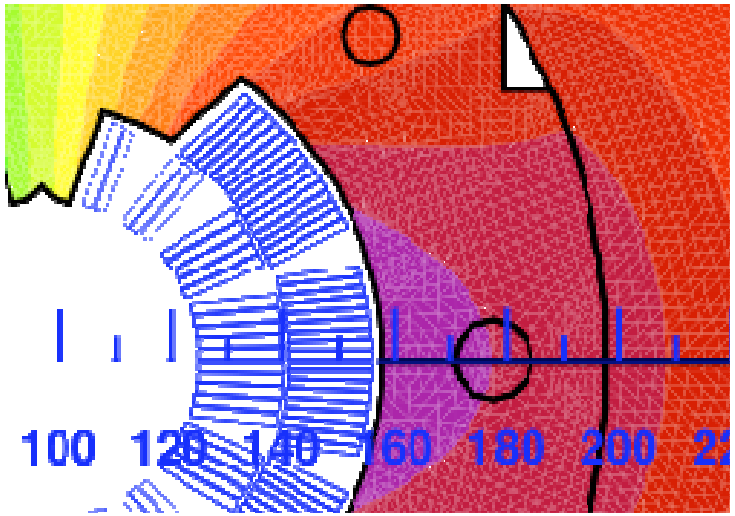
Specified tolerances on coils: ± 0.025 mm

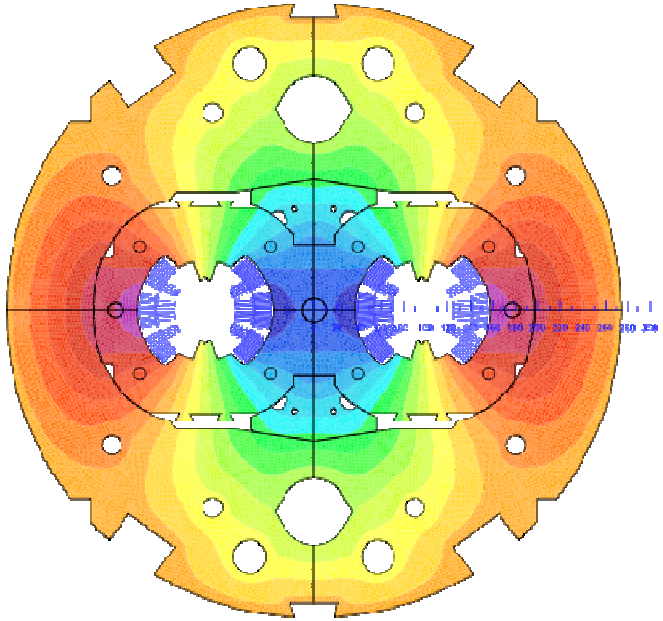
Coil Winding and Curing

Finite-Element / Boundary-Element Coupling

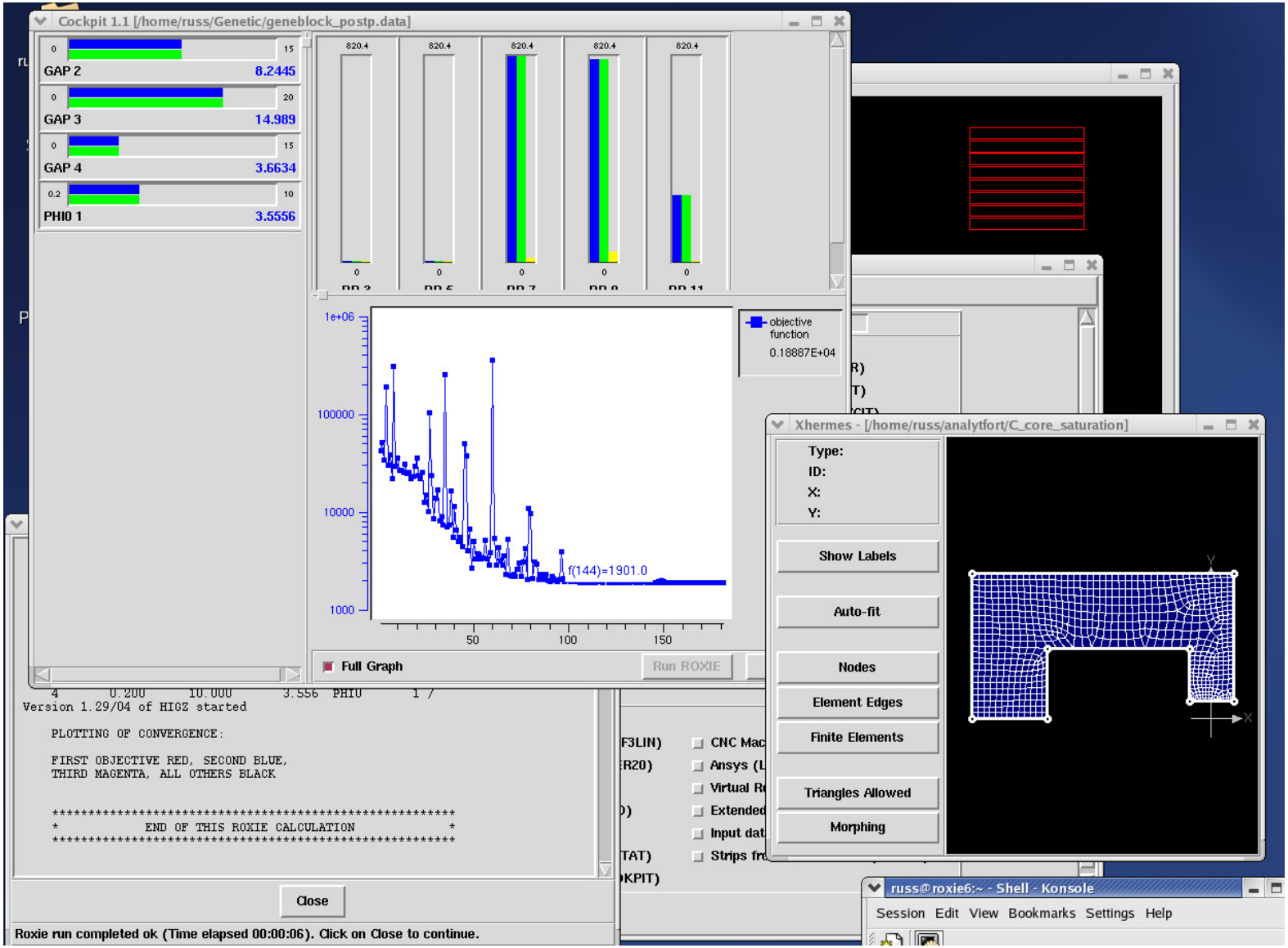








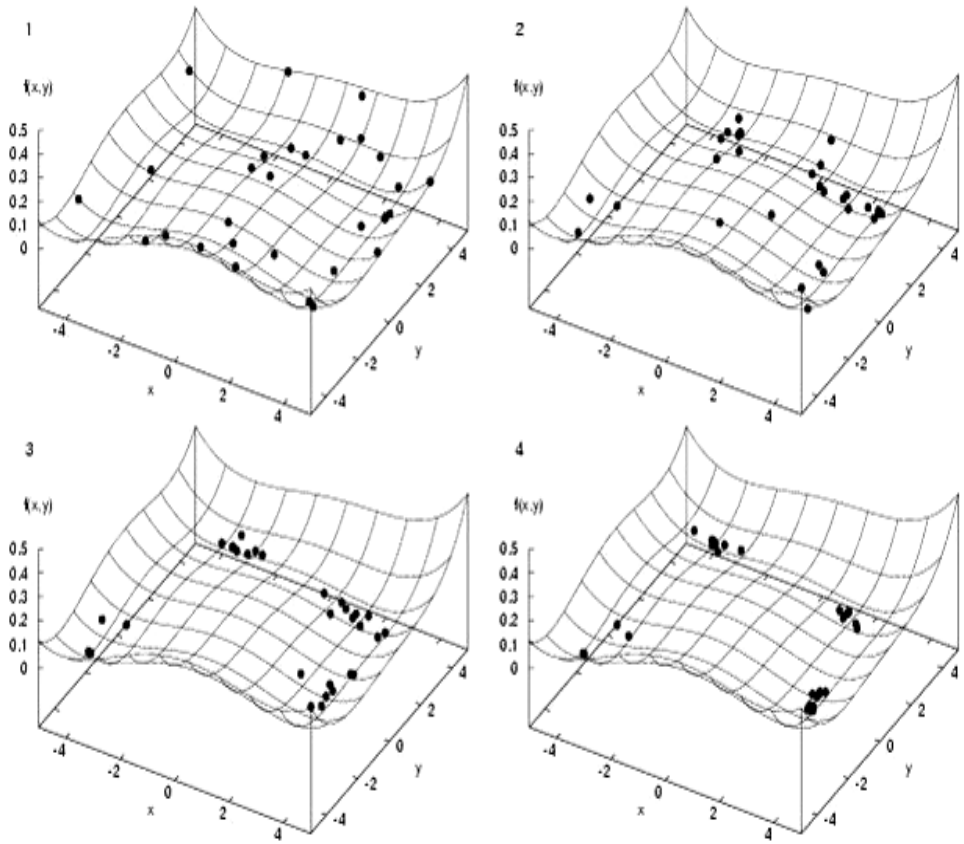
The CERN Field Computation Program ROXIE

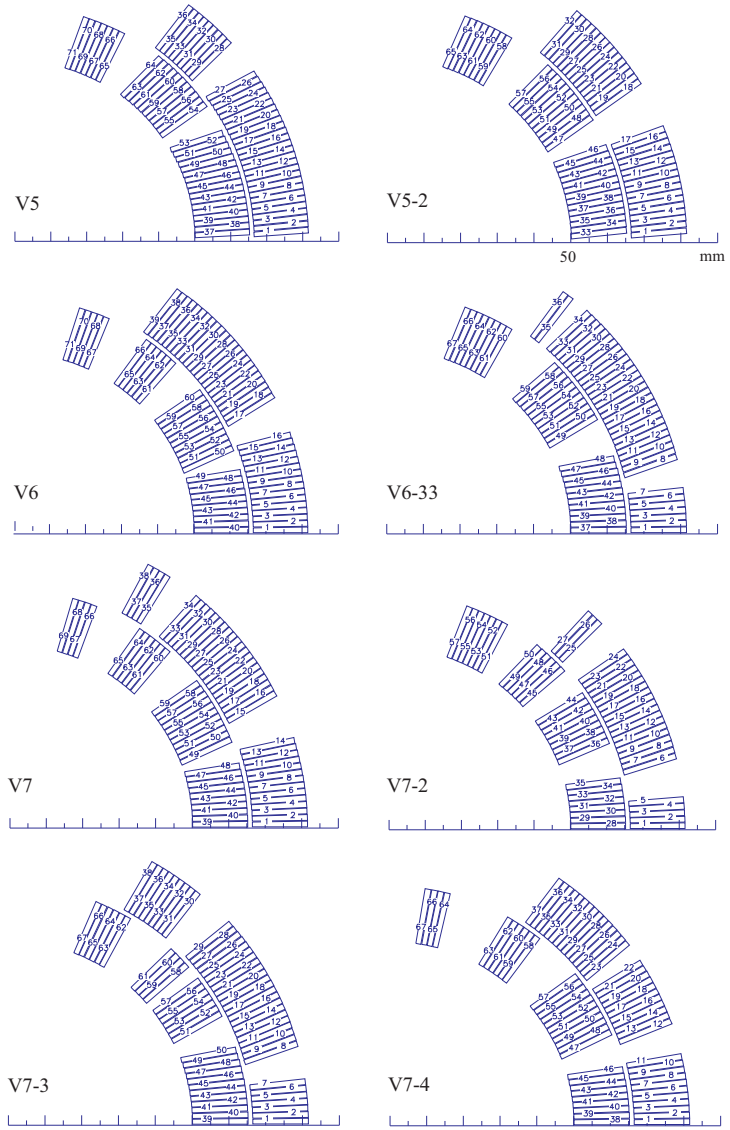


Integrated Design

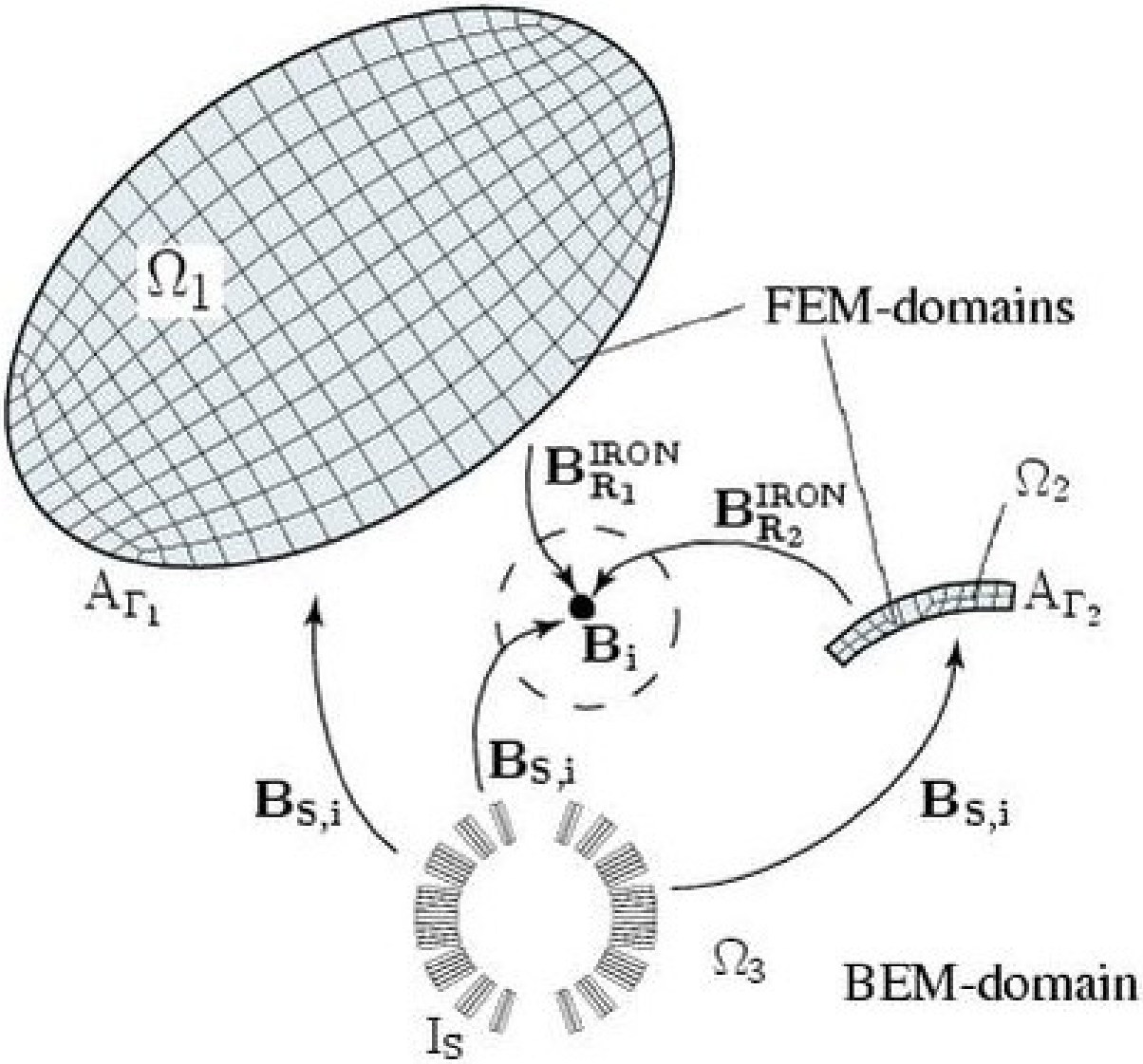
- Feature based geometry modeling
- Conceptual design using genetic optimization
- Minimization of iron saturation effects (BEM-FEM)
- Calculation of superconductor magnetization
- Eddy-currents in Rutherford cables
- Quenchsimulation
- 3D-Coil geometry and yoke optimization
- Sensitivity analysis
- Making of drawings, rapid prototyping
- Inverse field computation

Optimization Algorithms (Designs for SIS 300 (6 T) Dipoles



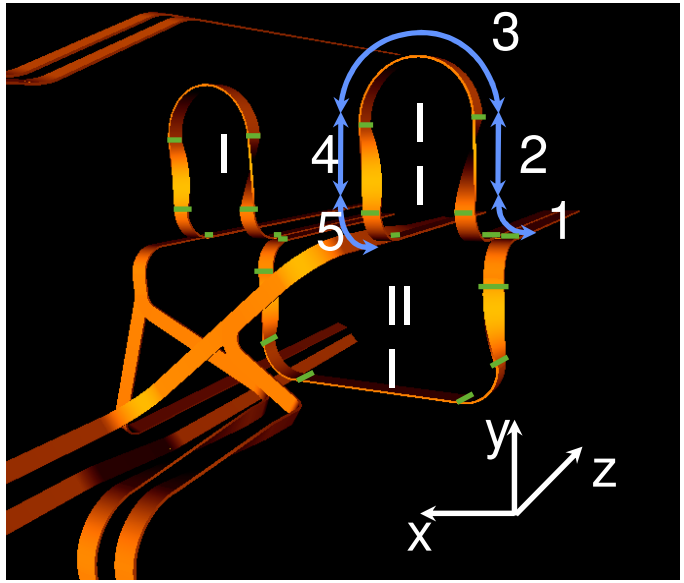


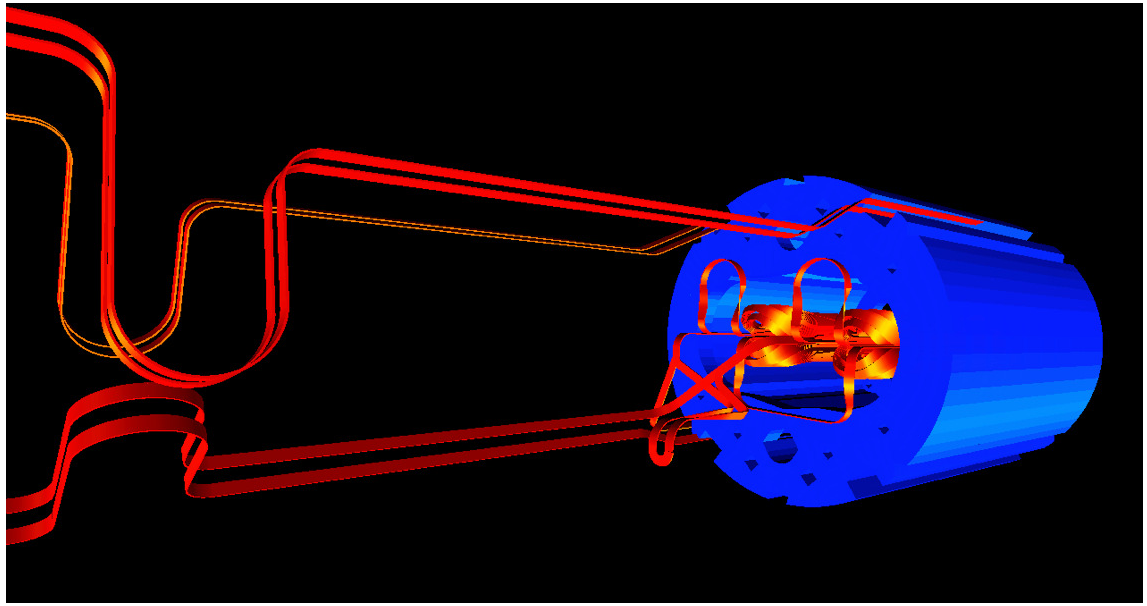
BEM-FEM Coupling



Forces (N) in the Connection Ends of the LHC Main Dipole

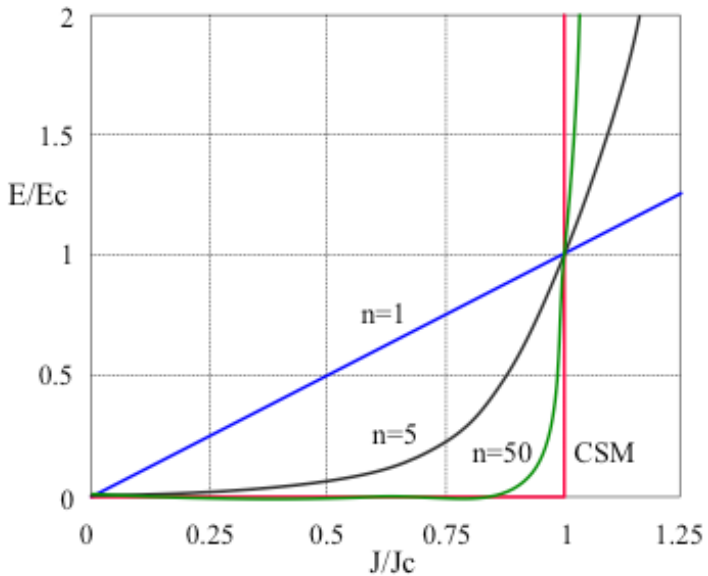
I	Fx	Fy	Fz
1	-39.7	-44.0	-45.4
2	-6.5	3.7	-41.7
3	-6.1	88.3	-38.2
4	1.25	3.9	-28.5
5	48.1	-46.7	-48.5
Sum	-2.95	5.2	-202.3

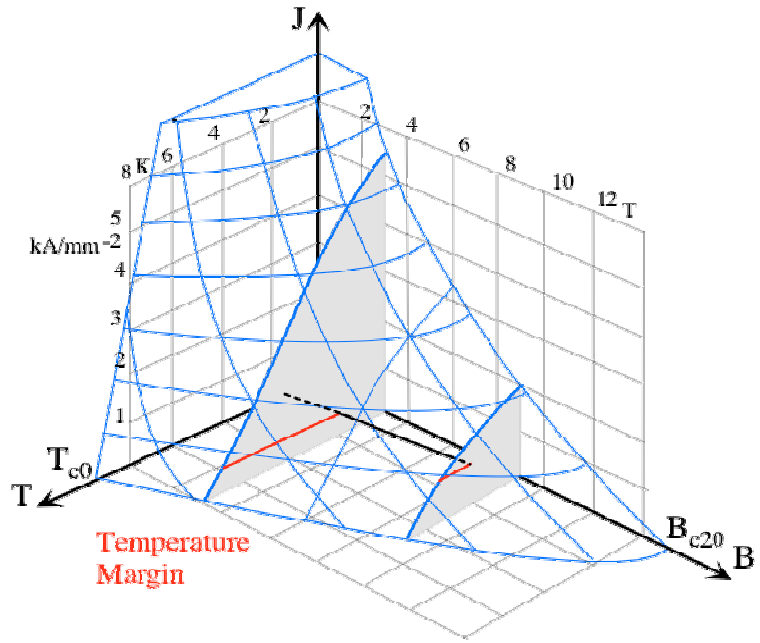




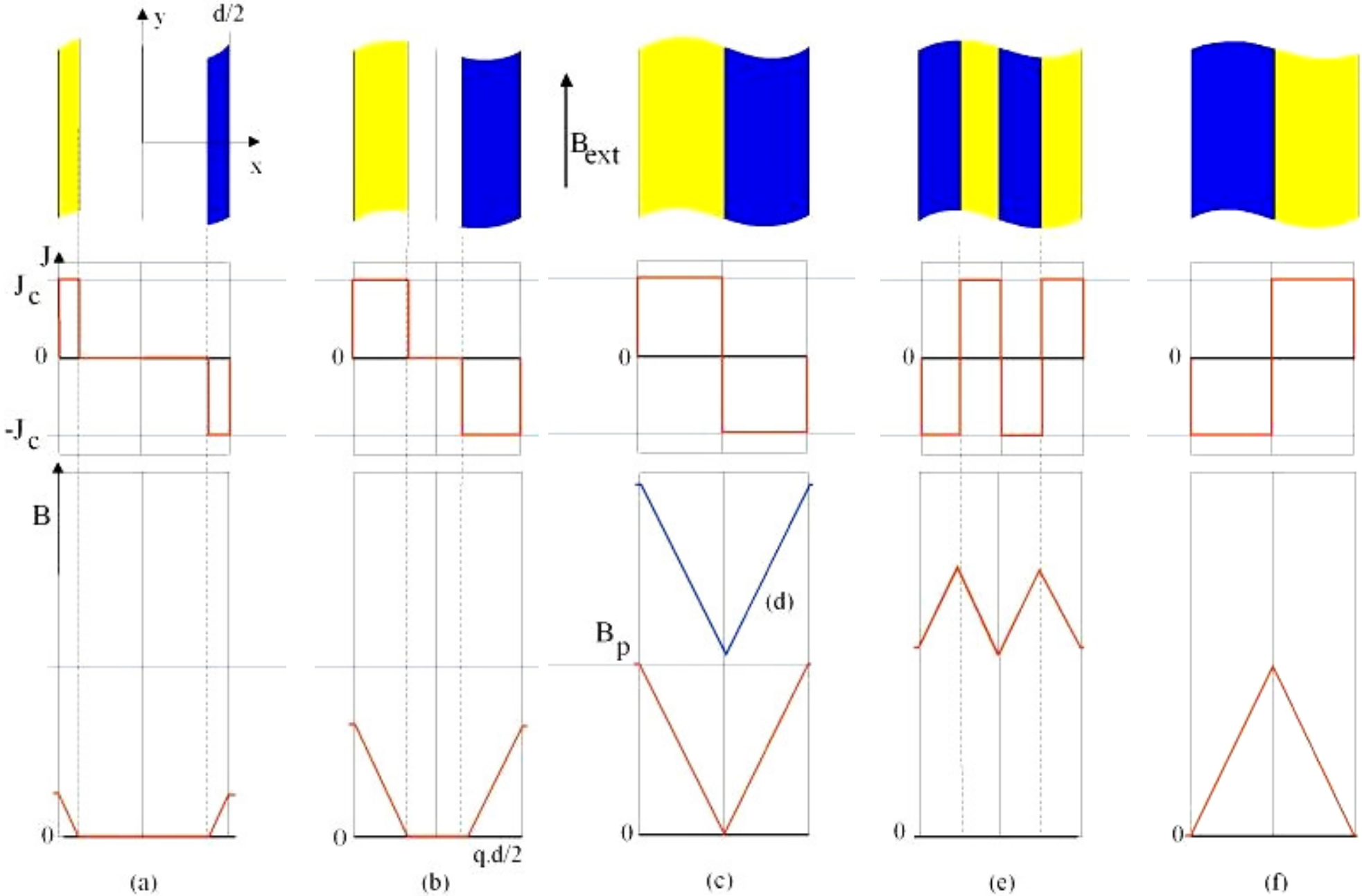
Superconductor Properties

- Hard Superconductors (Type 2)
  - Magnetic field can penetrate
  - Magnetization with hysteresis
- Critical current density J_c
  - Current density at spec. electric field (E_c = 1 µV/cm)
- Critical surface
  - Dependence of J_c on T and B

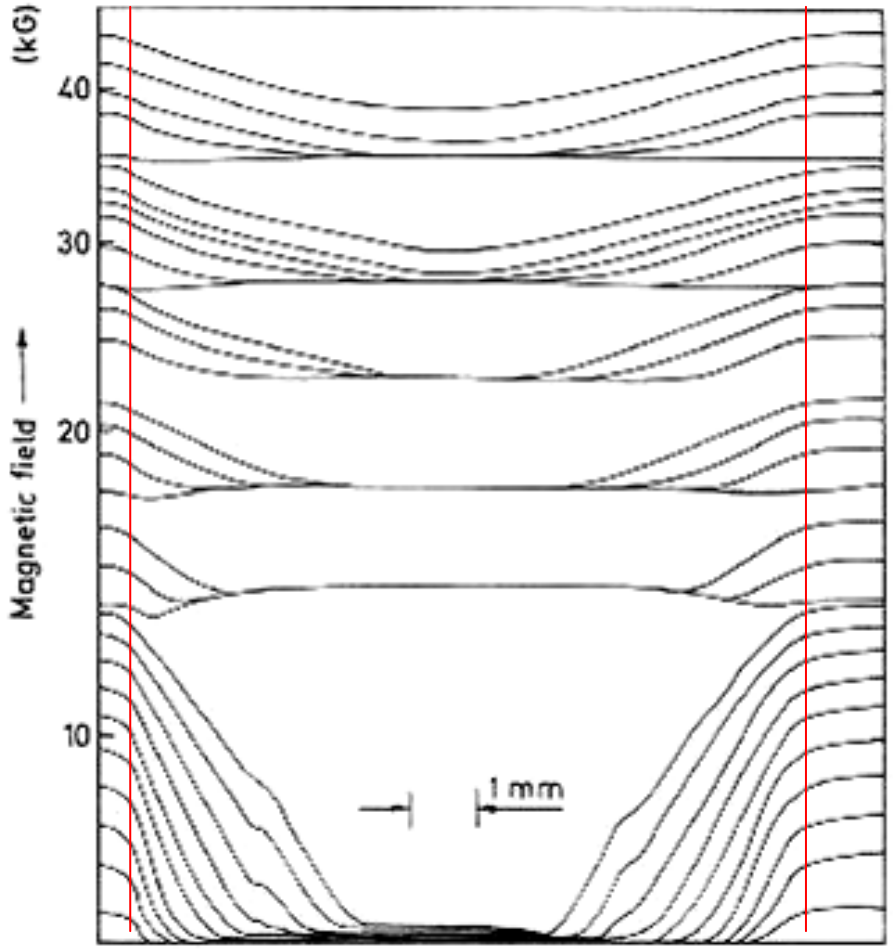




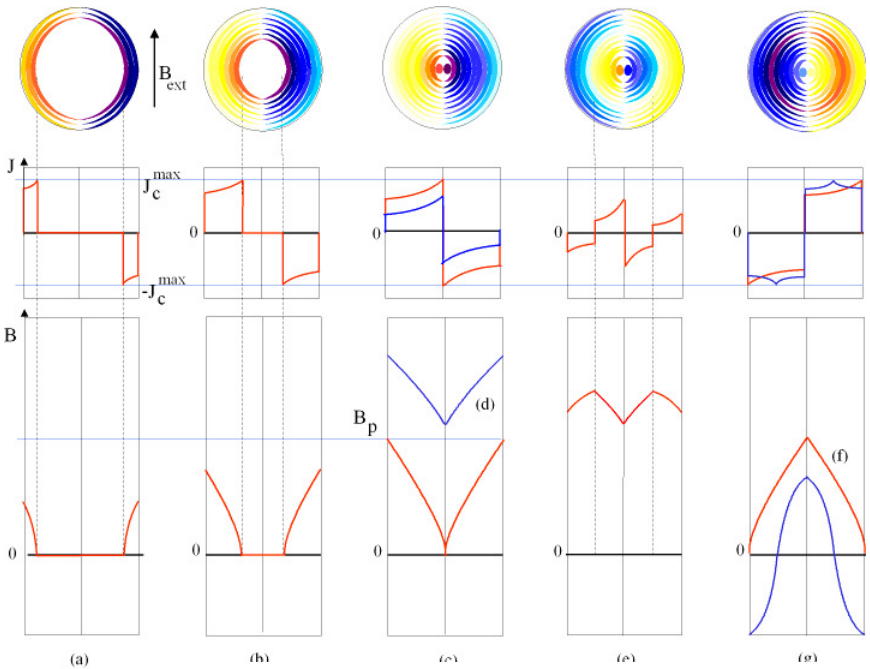
Bean's Critical State Model (CSM)

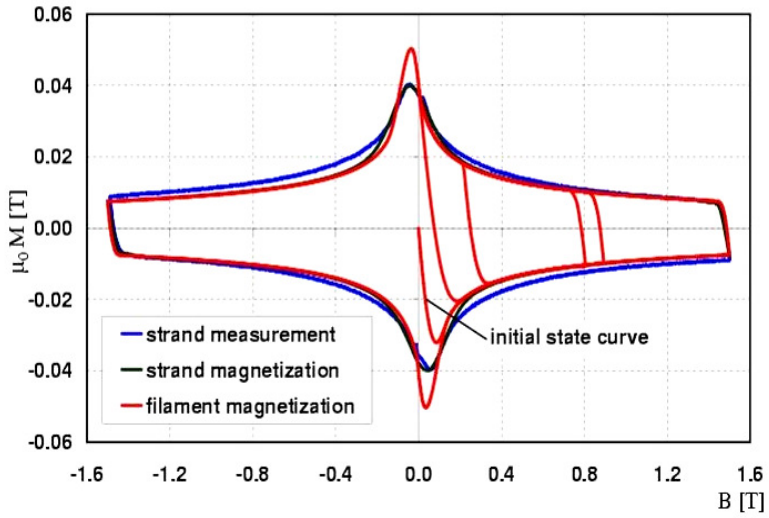


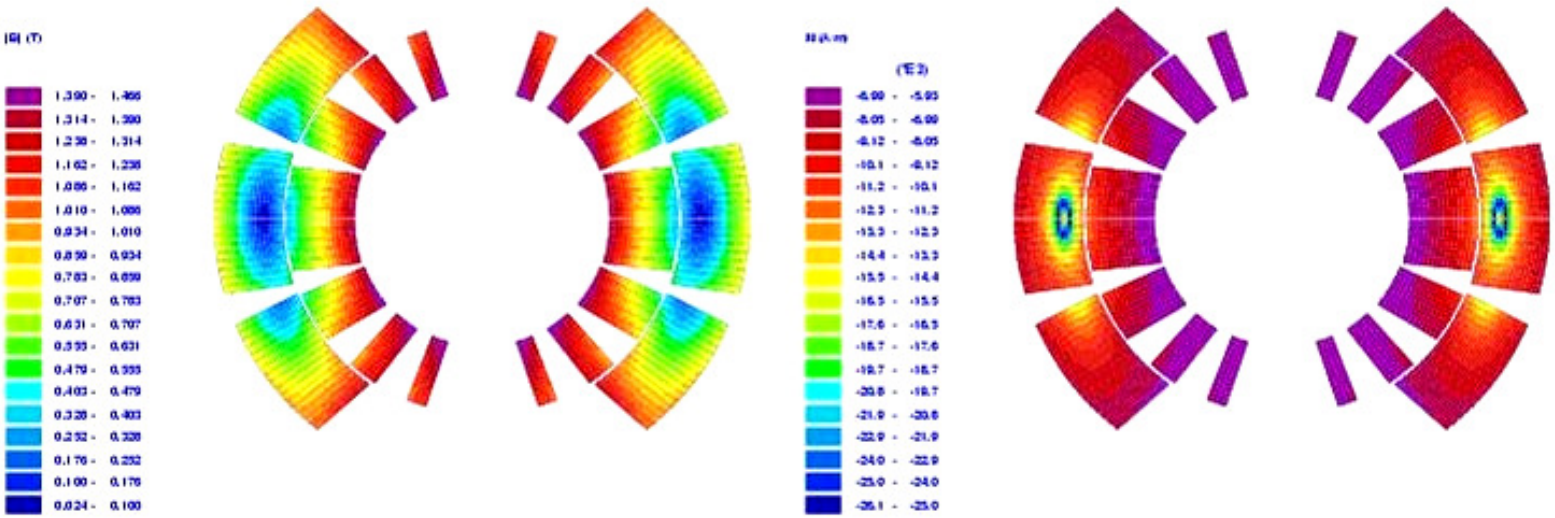
Screening Field in a Slab



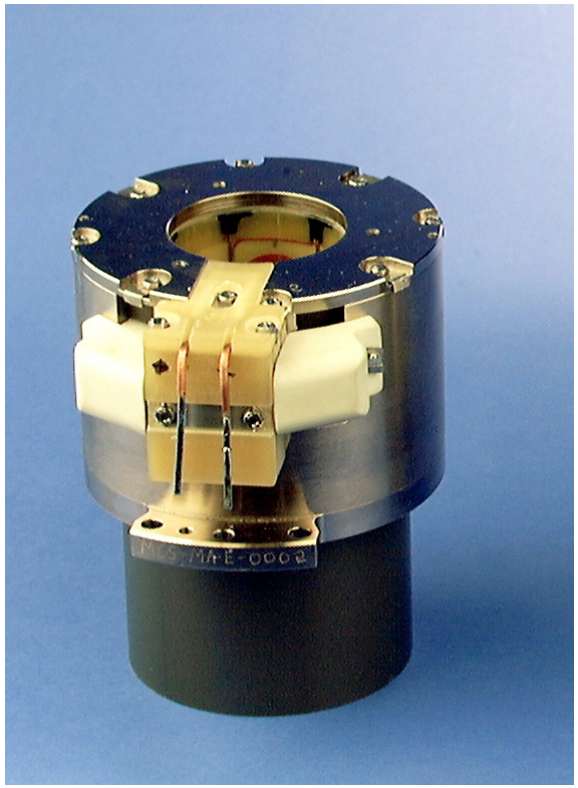
Superconducting Magnetization (Hysteresis Model)





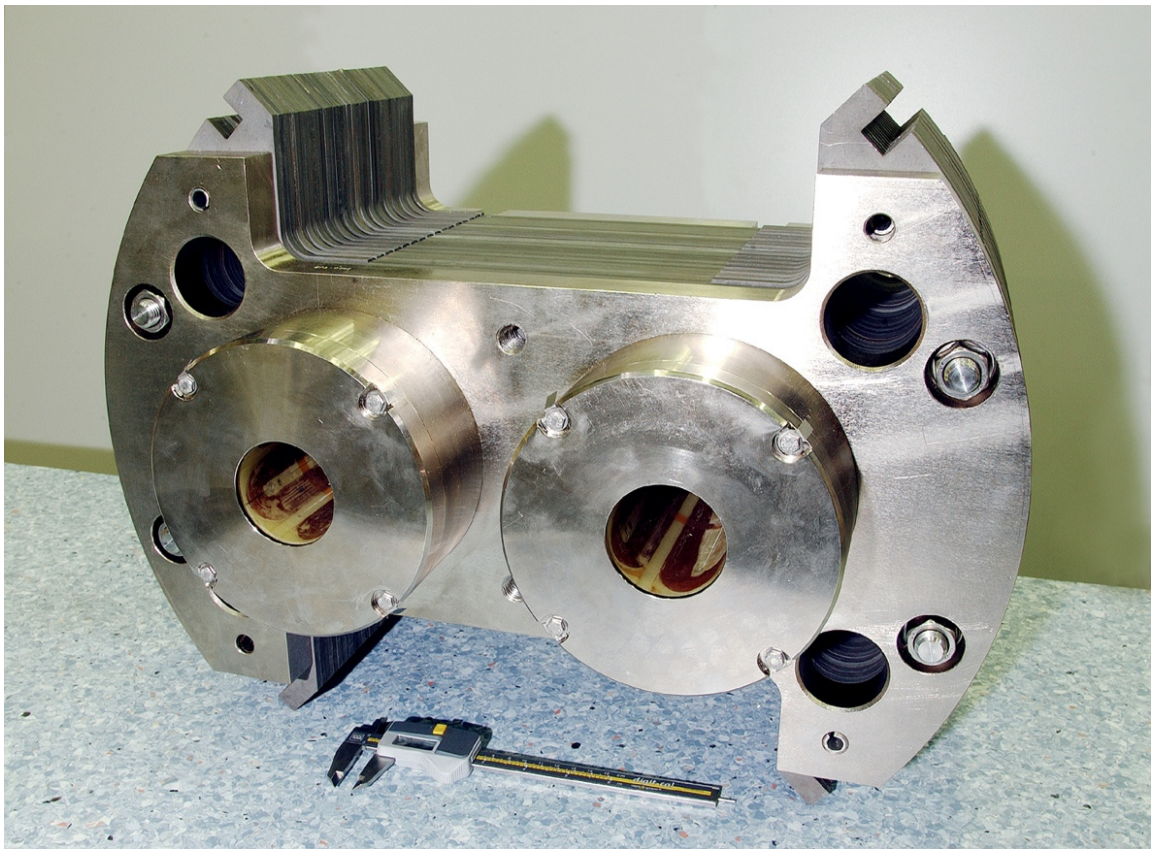


Corrector Magnets

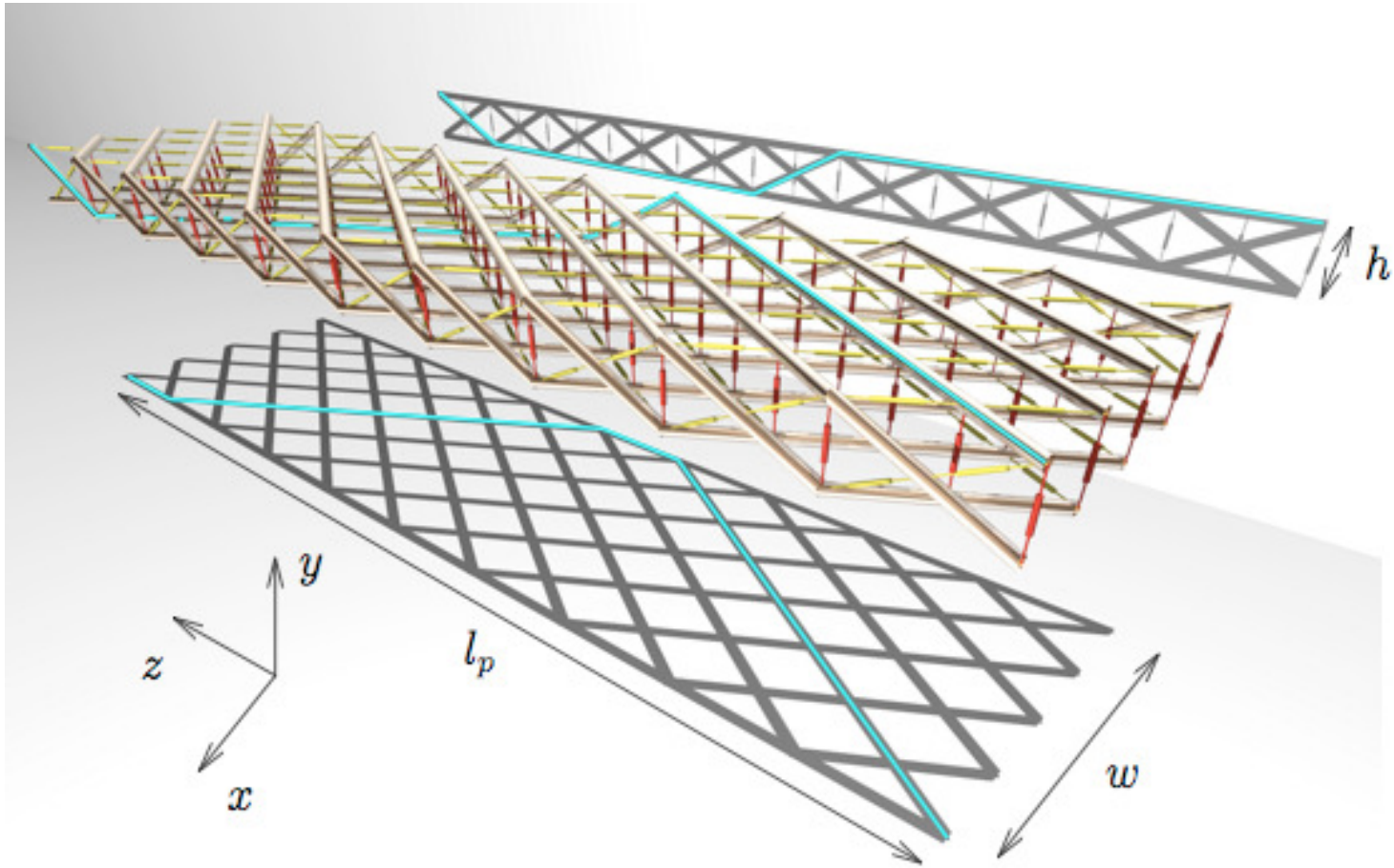


Sextupole-spool pieces

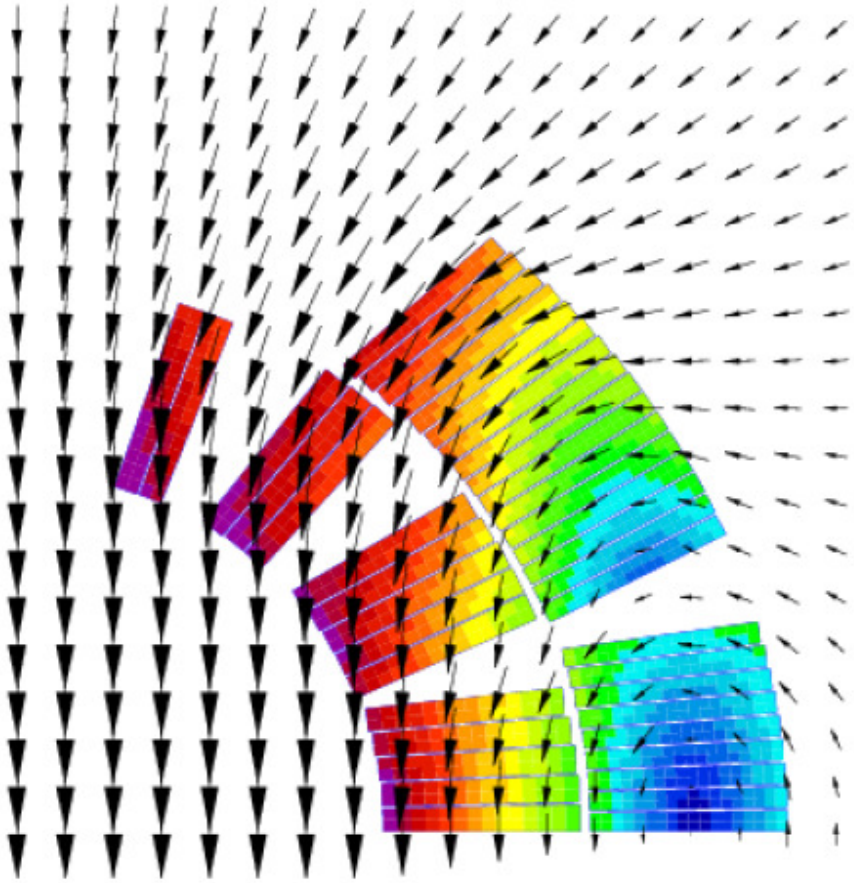
Octupole

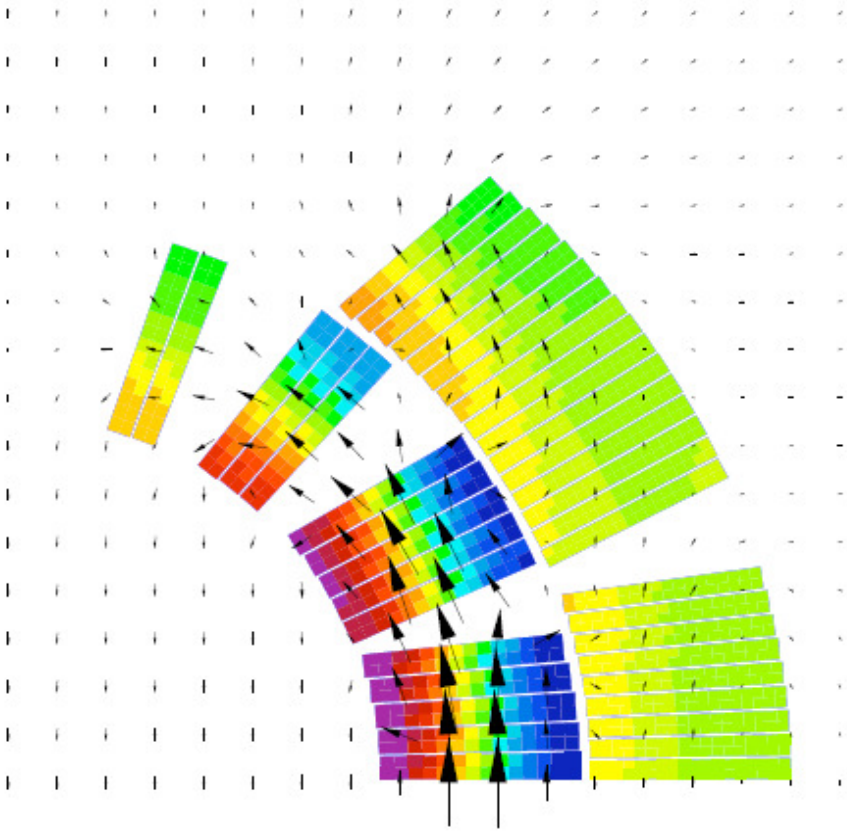


Eddy Currents in Rutherford Cables (Algebraic Topology)

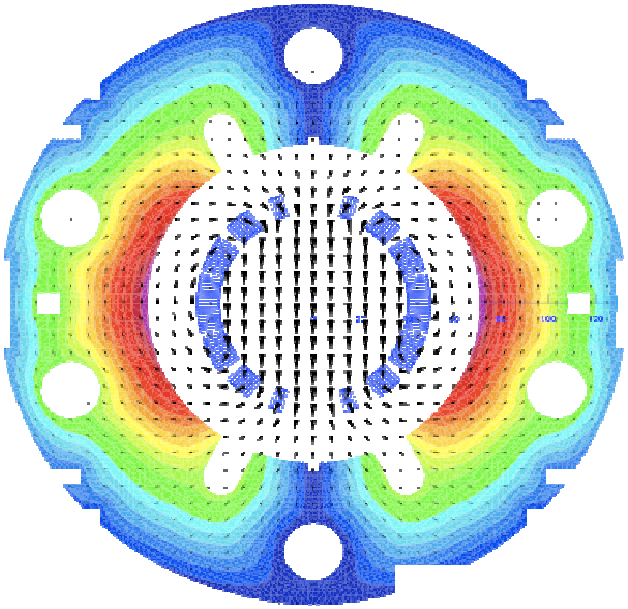


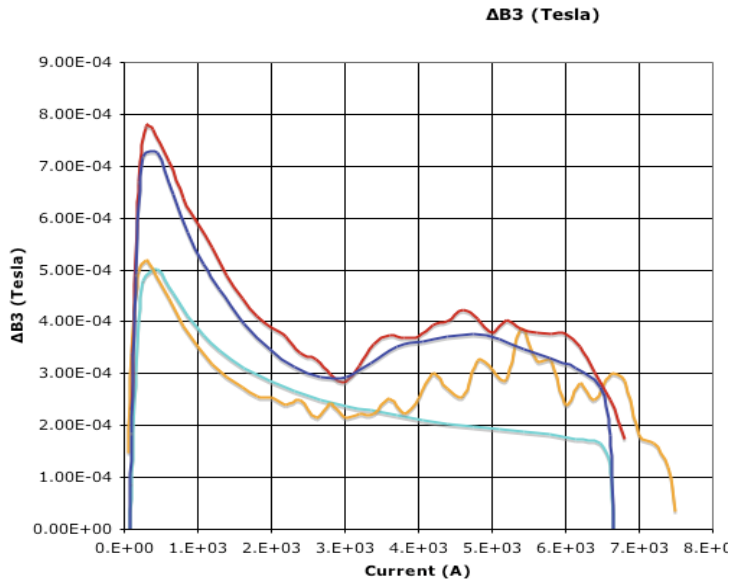
Field Generated by ISCC

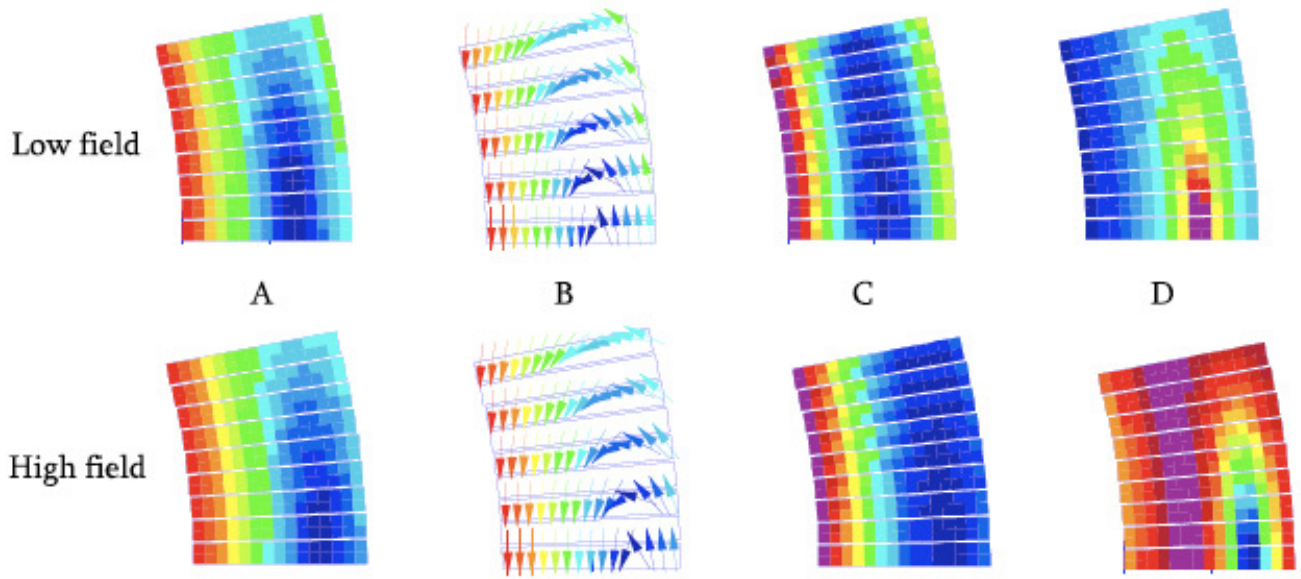




2-D Transient Field Computation for GSI-001

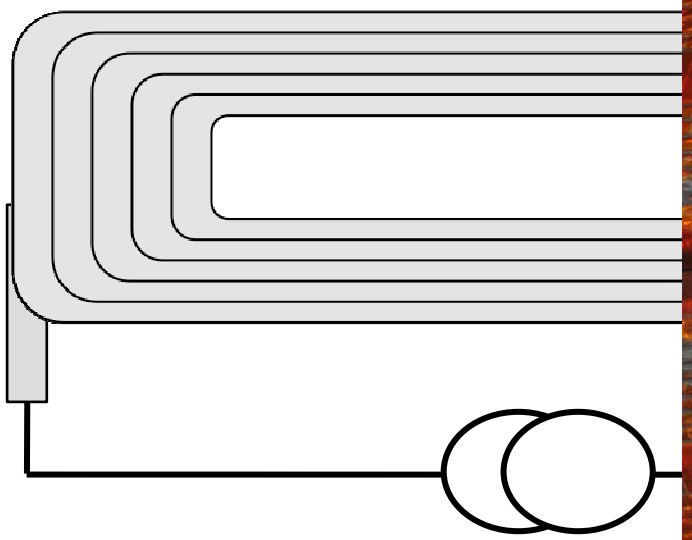


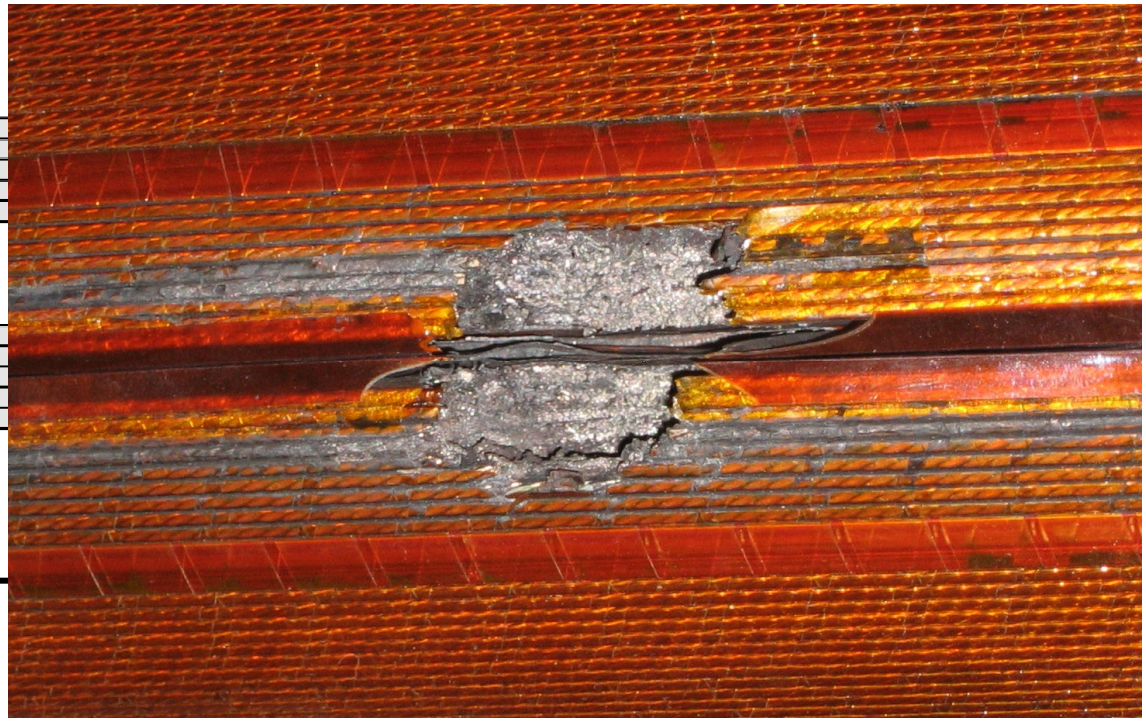




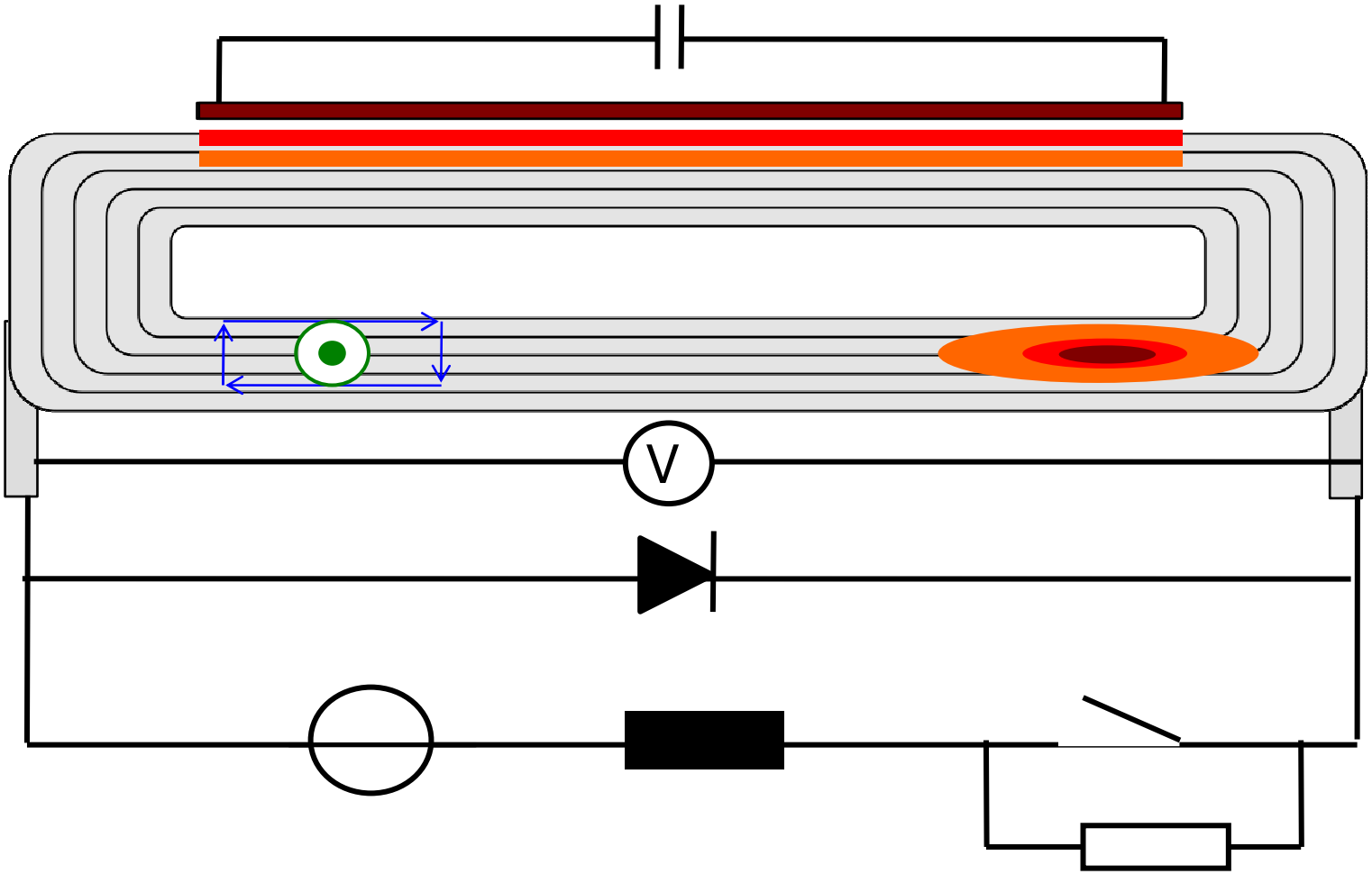
Quench

- Quench: Transition from SC to normal conducting state caused by beam losses, conductor movement, eddy currents etc.
- Propagation:
  - Normal conducting zone generates Ohmic heat
  - Quench- und temperature distribution determined by loss-mechanisms and cooling capacity

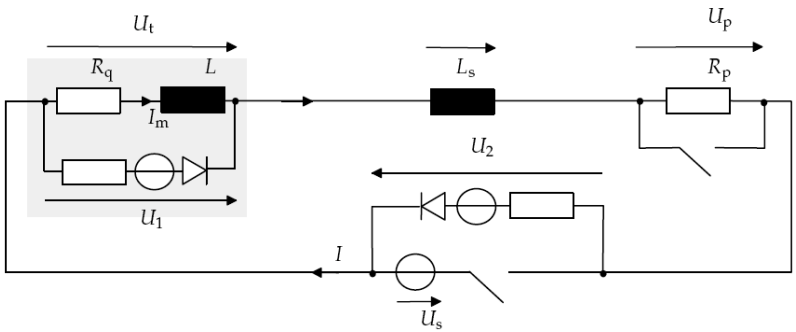


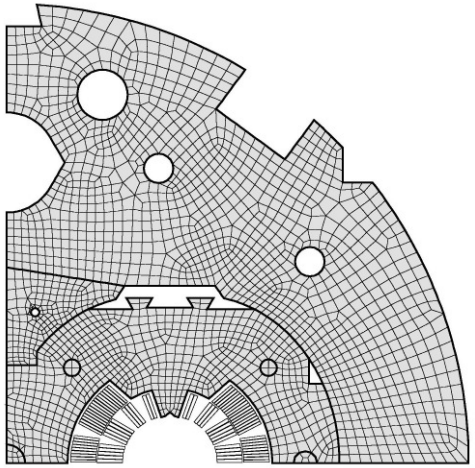


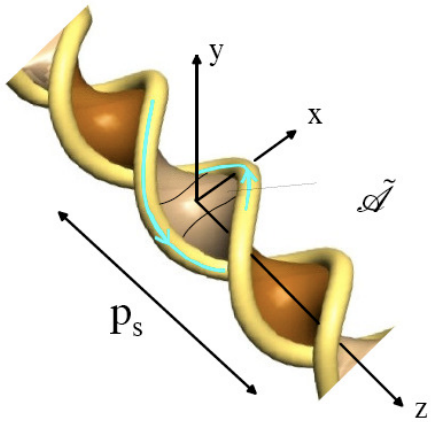
Quench Mechanism and Magnet Protection

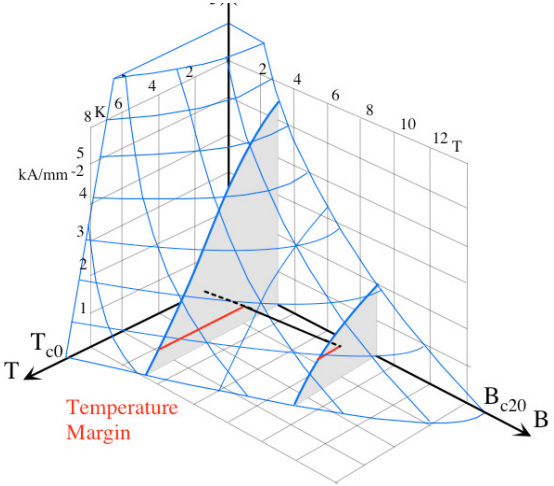


Quench Simulation (Multi-Physics, Multi-Scale)

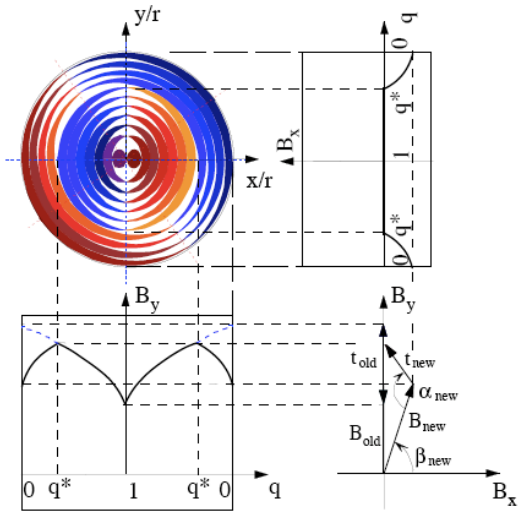


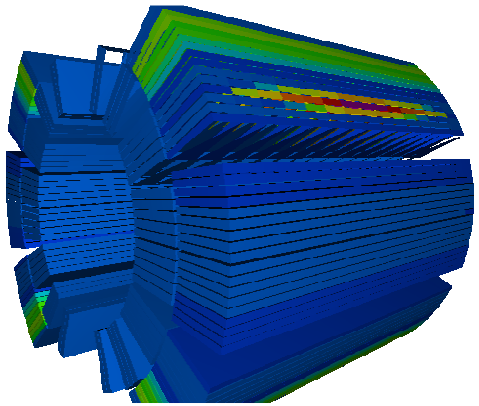


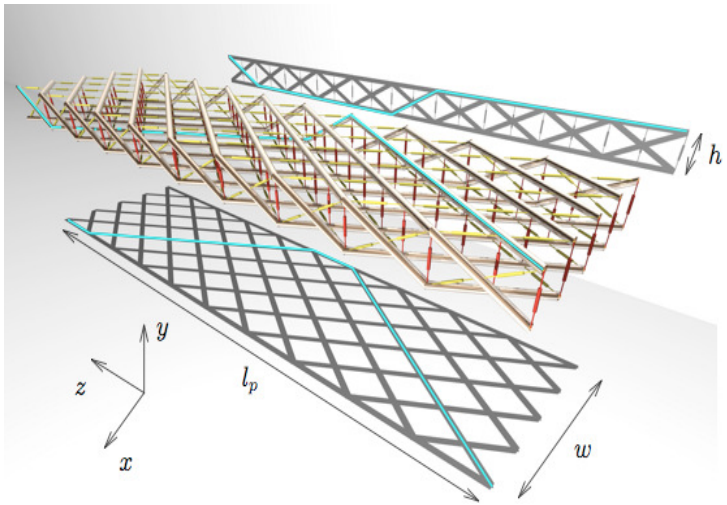




Quench Simulation in ROXIE



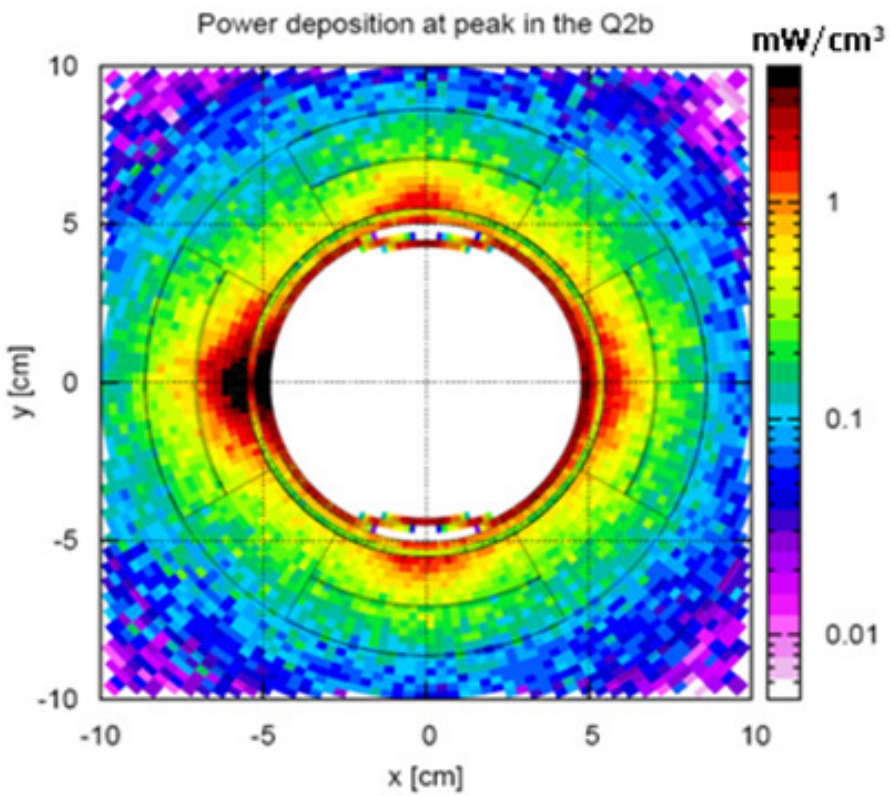




Protection Against Particle Debris

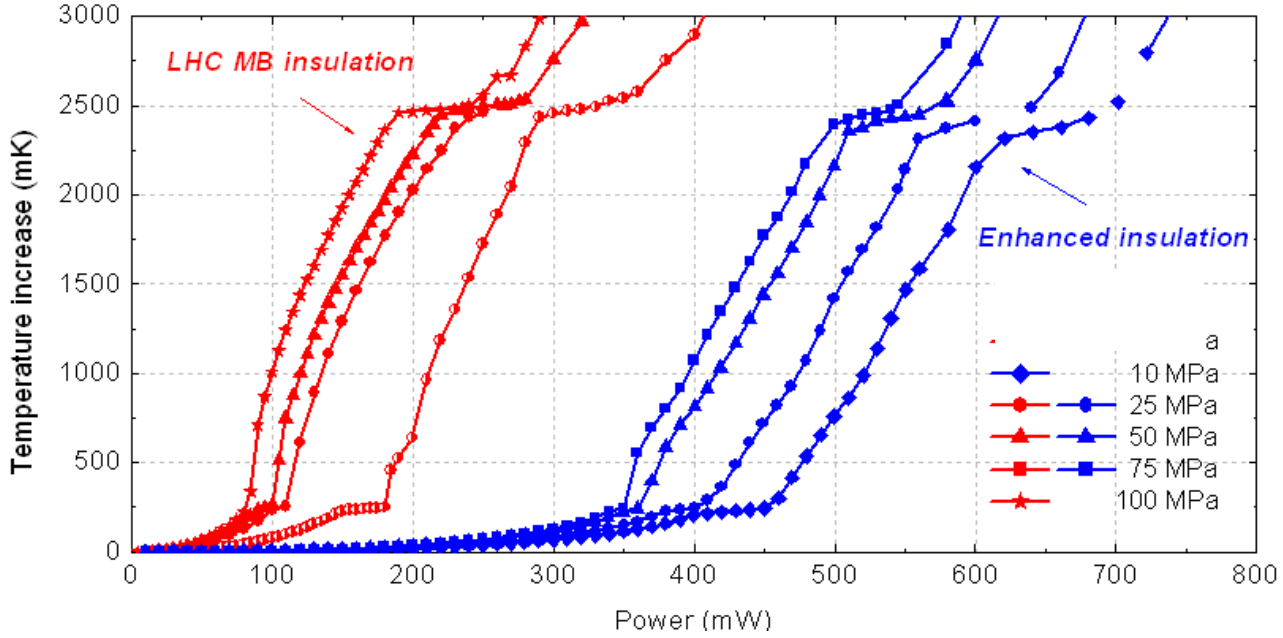
Protection against particle debris is the single most serious issue of the upgrade.

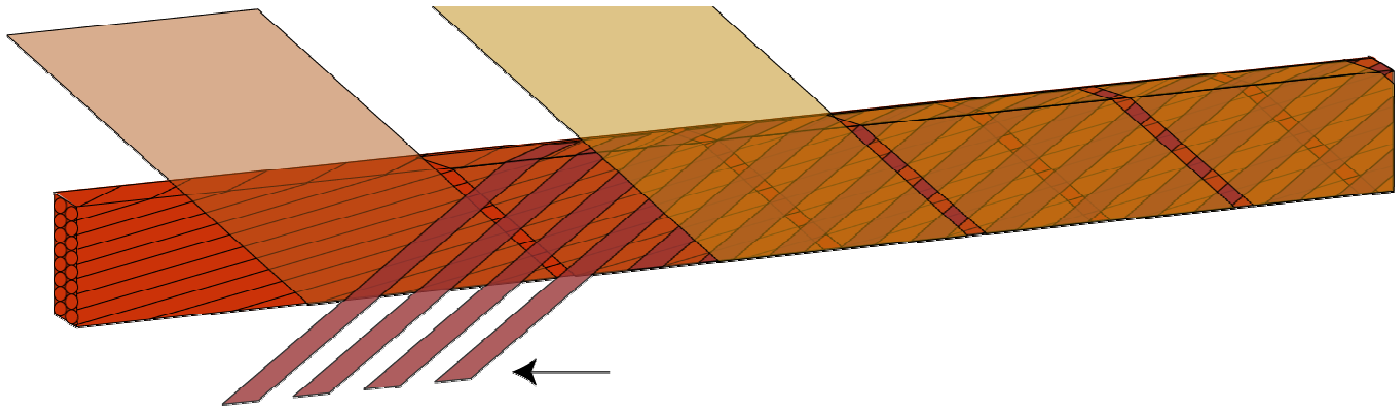
- Energy deposition in the coils and magnet lifetime.
- Equipment protection around the beamline (TAS, TAN).
- Protection of electronic equipment in underground areas.
- Maintenance and interventions …



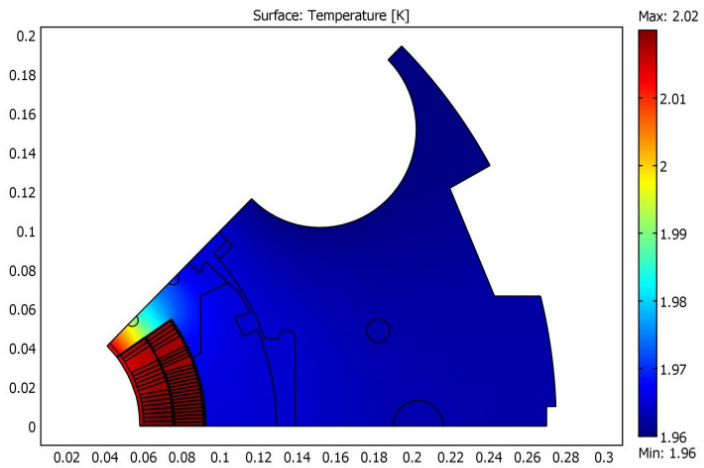
Average dose 1.5 MGy/100 fb^{-1}

Heat Transfer up to T λ

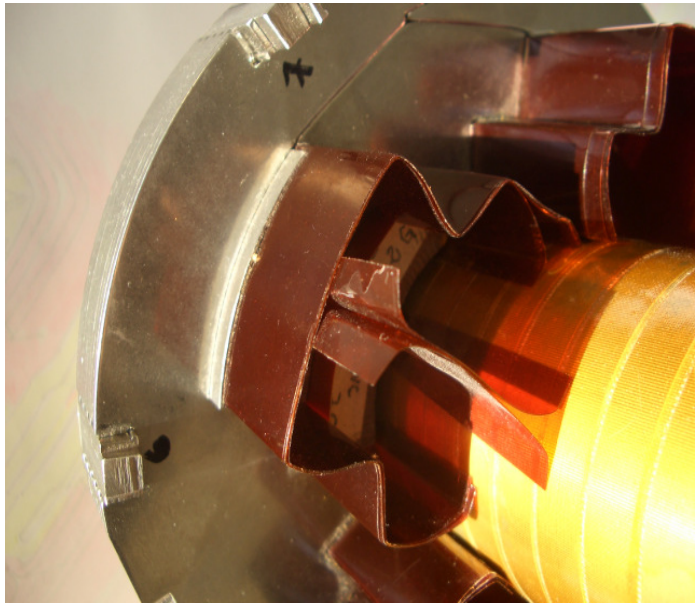


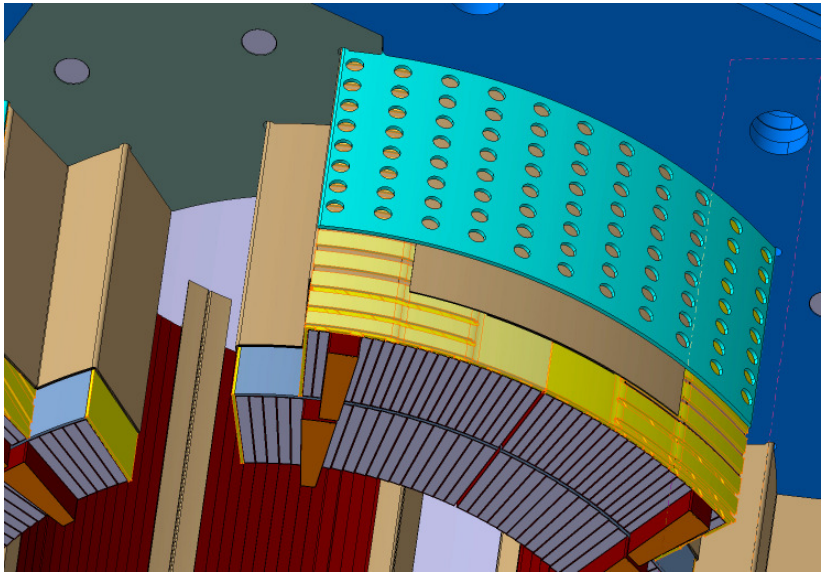


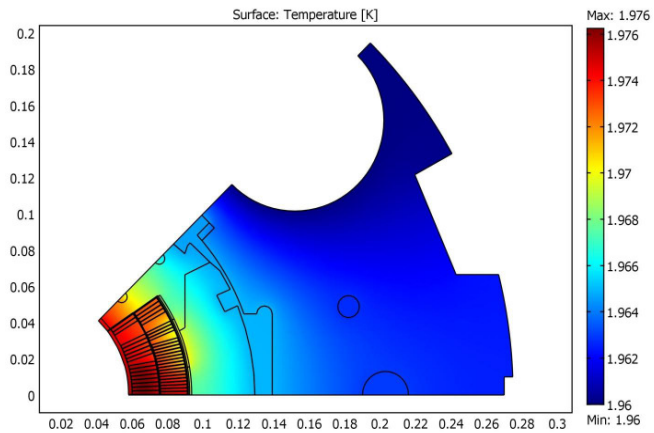
Extraction of the Steady-State Heat-Load 1



Conventional ground insulation



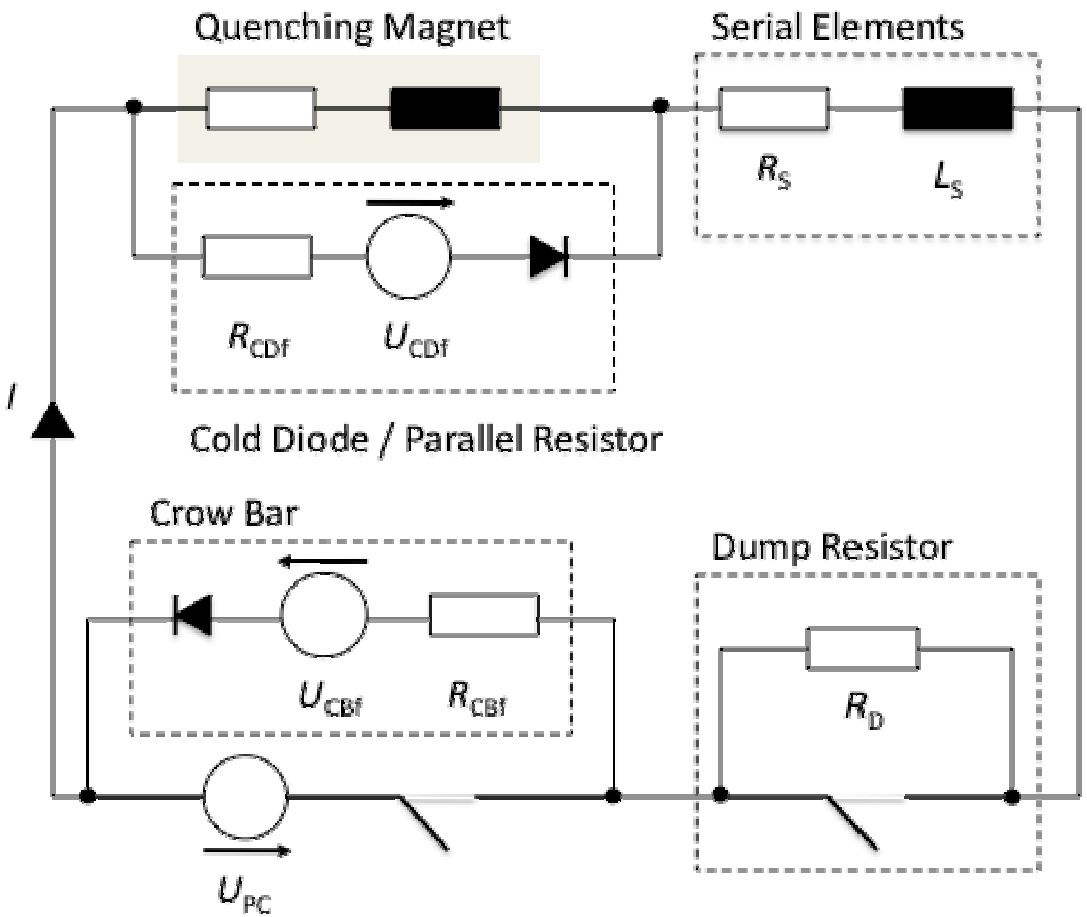




Open KCS ground insulation

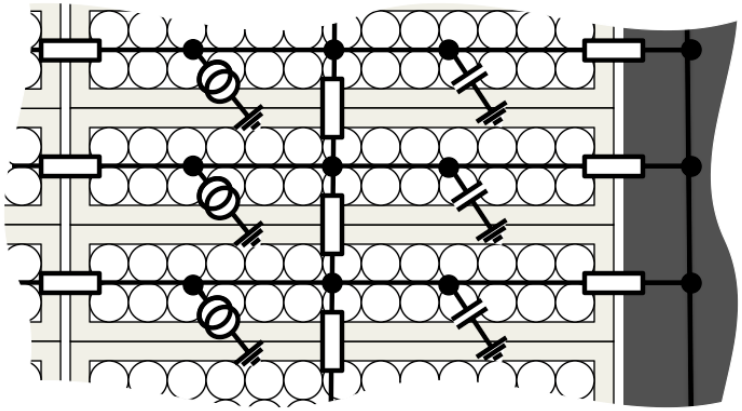
Electrical (External) Network

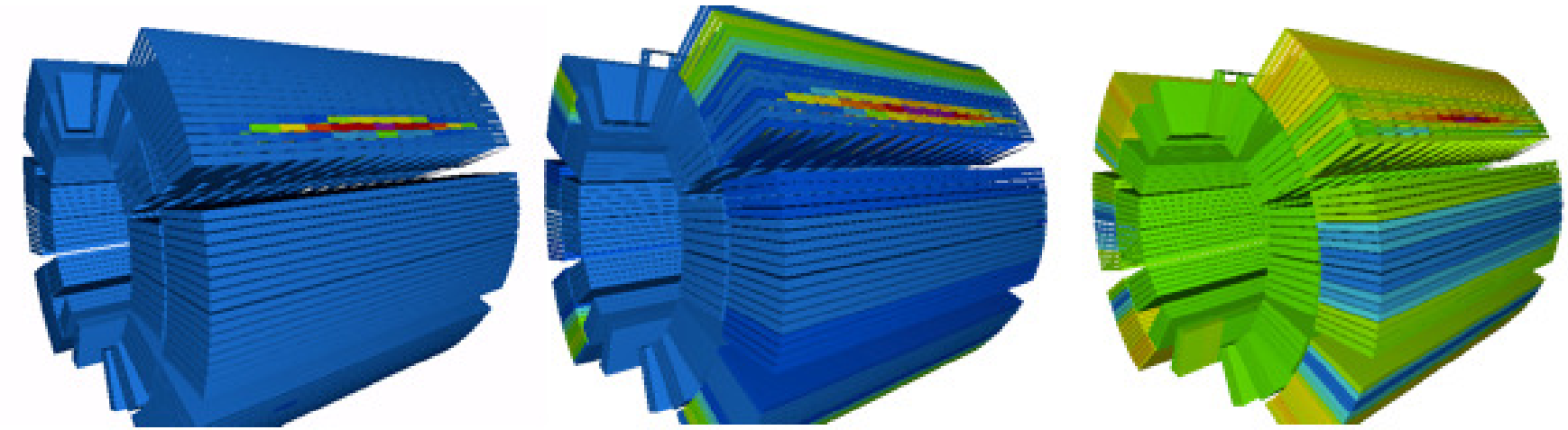
$$L_{\mathrm{d}}(B)\frac{\mathrm{d}I}{\mathrm{d}t} = U_{\mathrm{Diode}} - (R_{\mathrm{Q}}(B,T) + R_{\mathrm{P}}(t))I$$



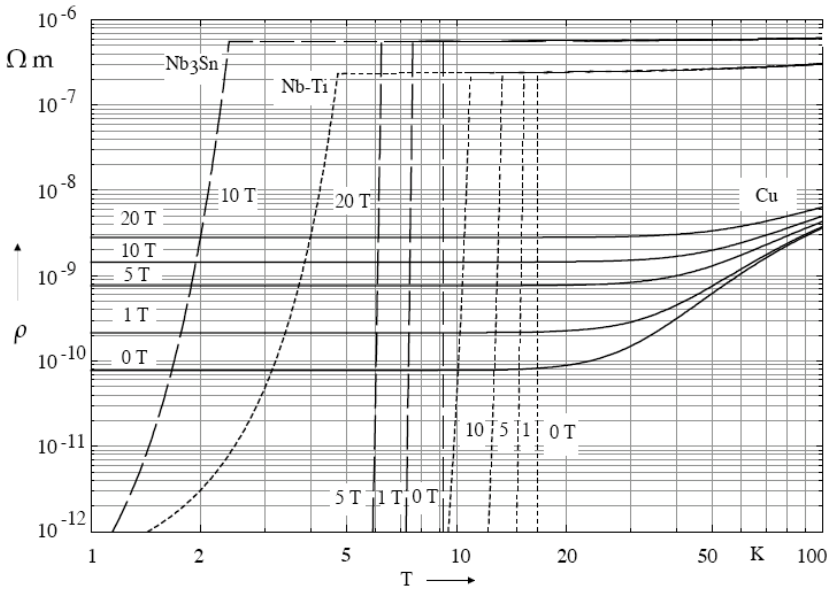
Thermal Model

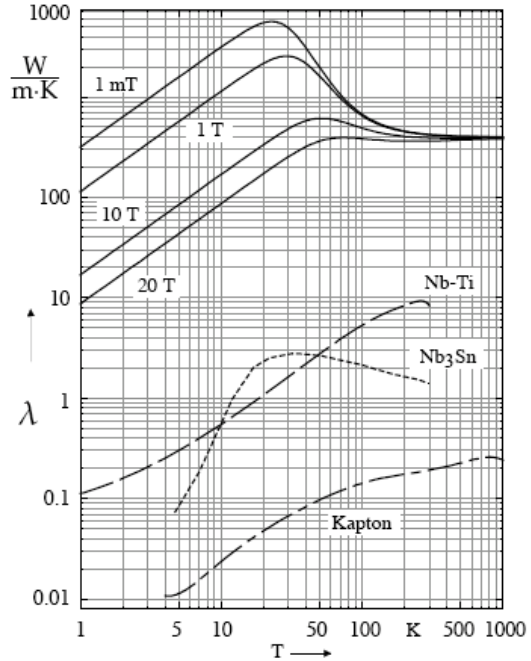
$$c(T)\frac{\mathrm{d}T}{\mathrm{d}t} - p - \vec{\nabla}\cdot\left(\kappa(T,B)\vec{\nabla}T\right)$$

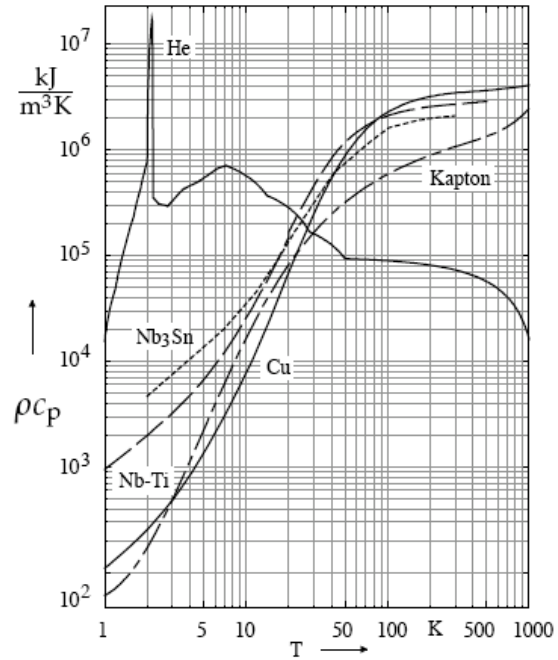




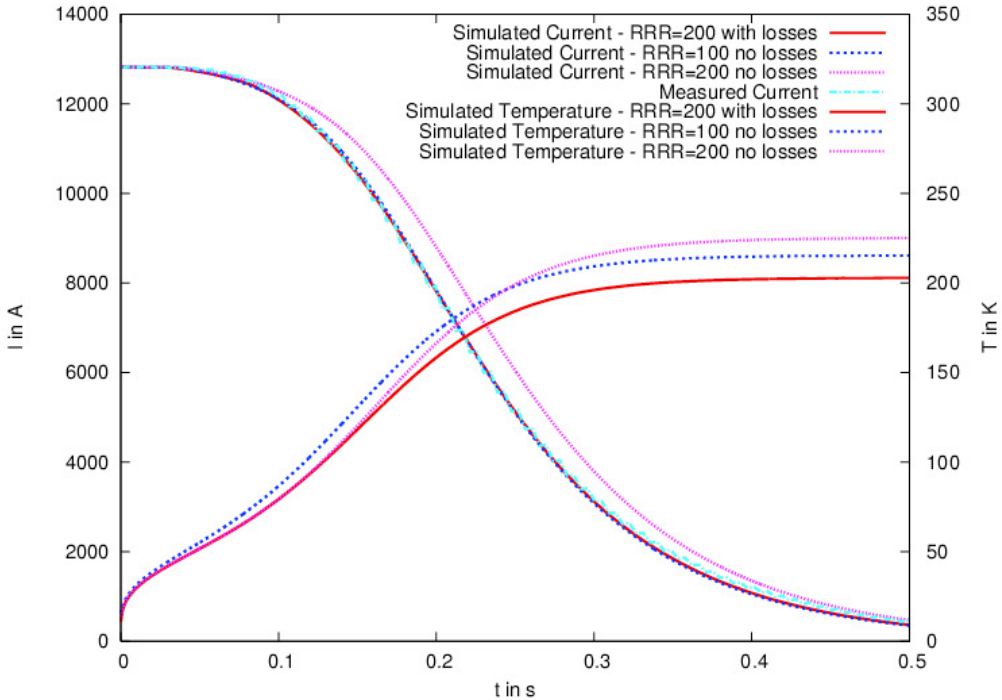
Material Parameters







Validation



Empirical parameters:

- RRR
- Ra/Rc
- IFCC effective res.
- heat conductivity
- heat capacity

➔ Different families of parameters yield exactly the same observable I(t).

➔ More than one solution exists.

➔ Great care must be taken to model

- all relevant phenomena,
- using realistic material parameters.

Quench Simulation

The challenge of quench simulation:

Model **all relevant physical phenomena** with **adequate accuracy** so that we can be confident to **simulate internal states** of a quenching magnet and understand its behavior.

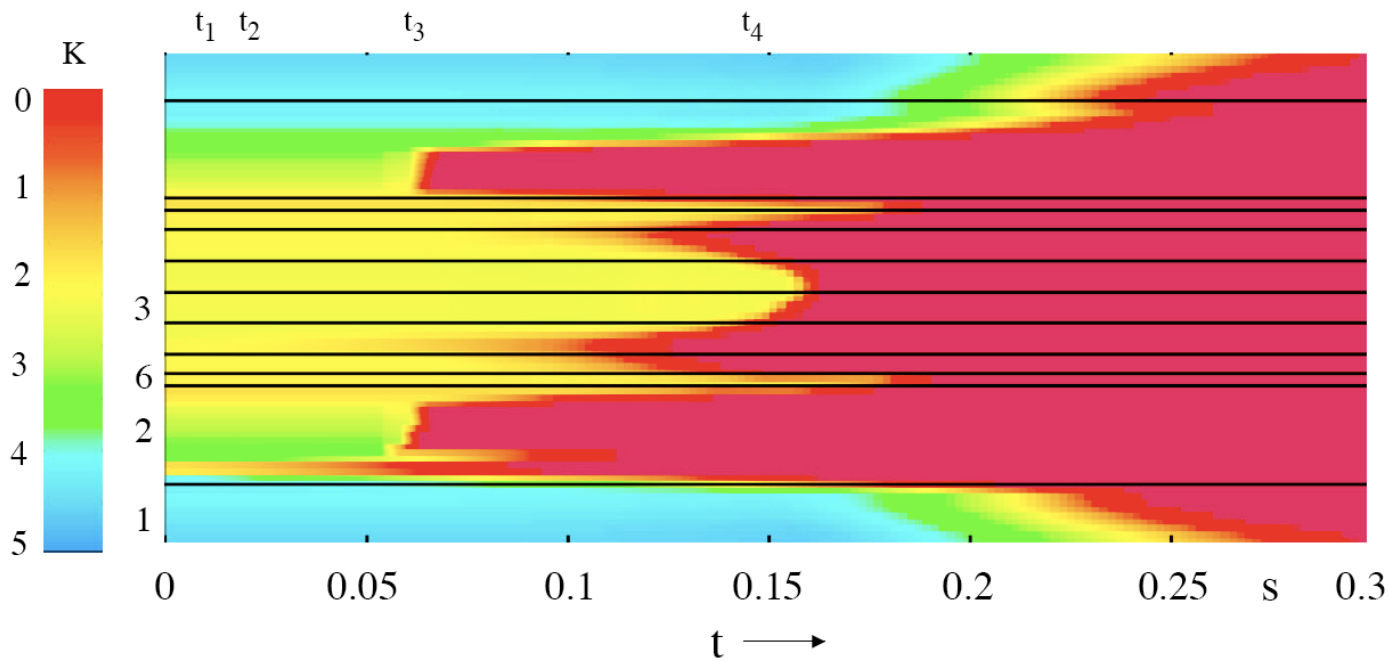
Validation:

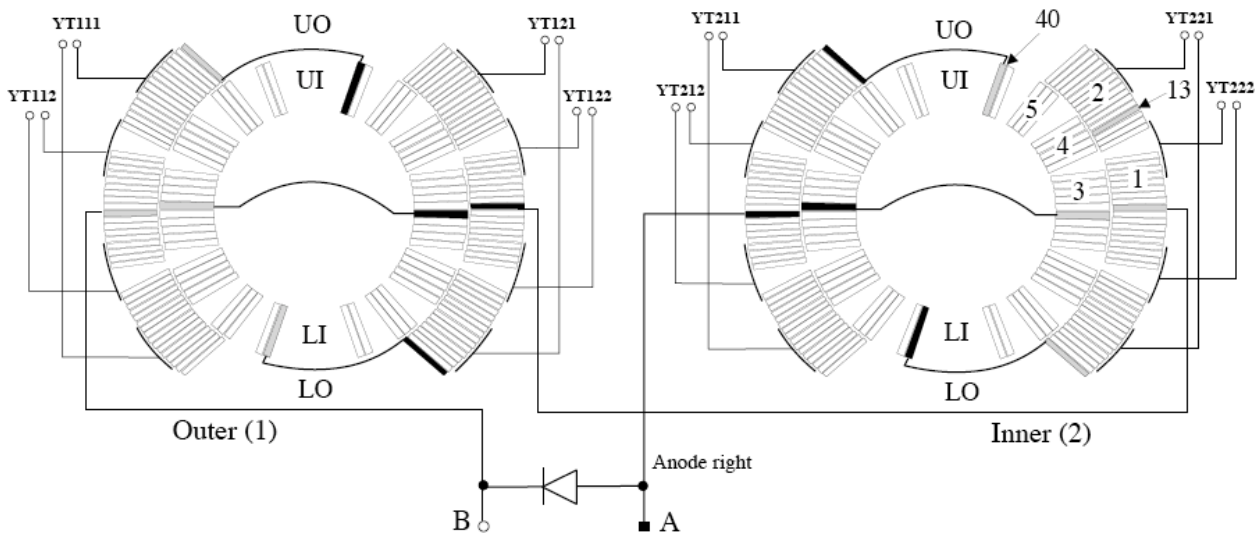
Measured quantities can be **reproduced** with all **material- and model-parameters within their range of uncertainty,**

Extrapolation and Introspection

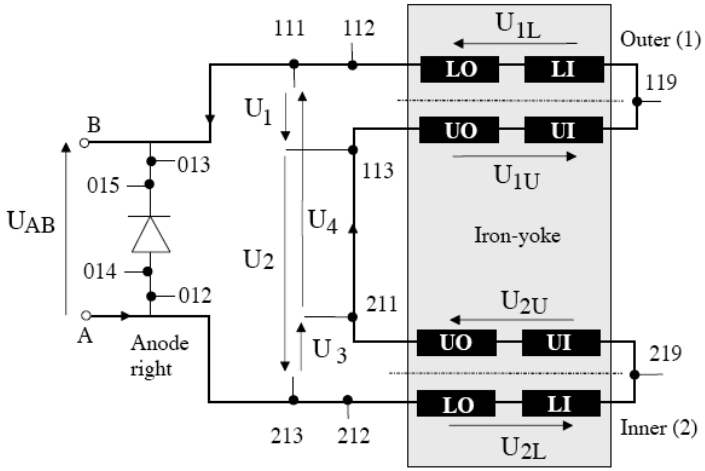
If the above criteria are reached, extrapolated results will match measurements **without adaptation of material- and model parameters**. It is then also possible to **simulate internal states** of the magnet that escape measurements.

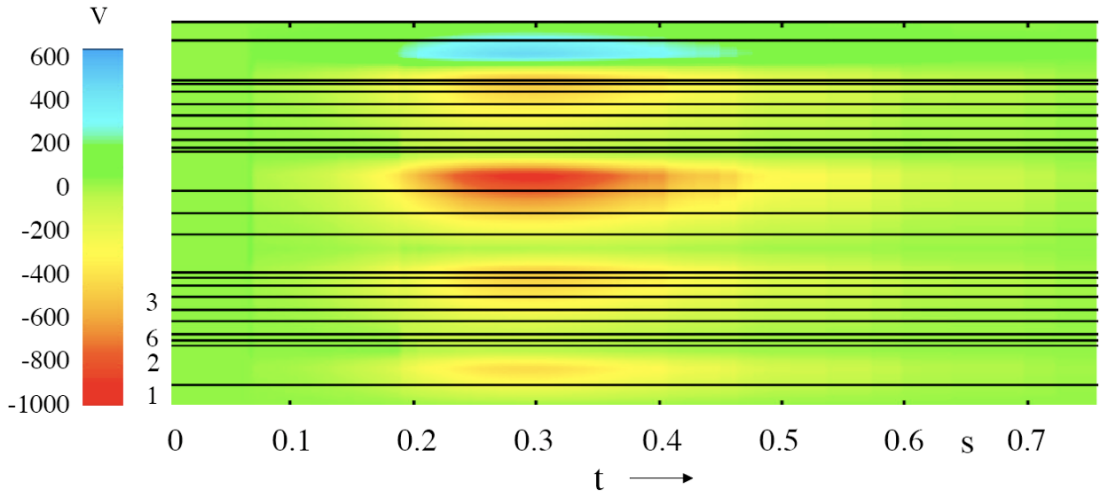
Introspection (Quench Margin)

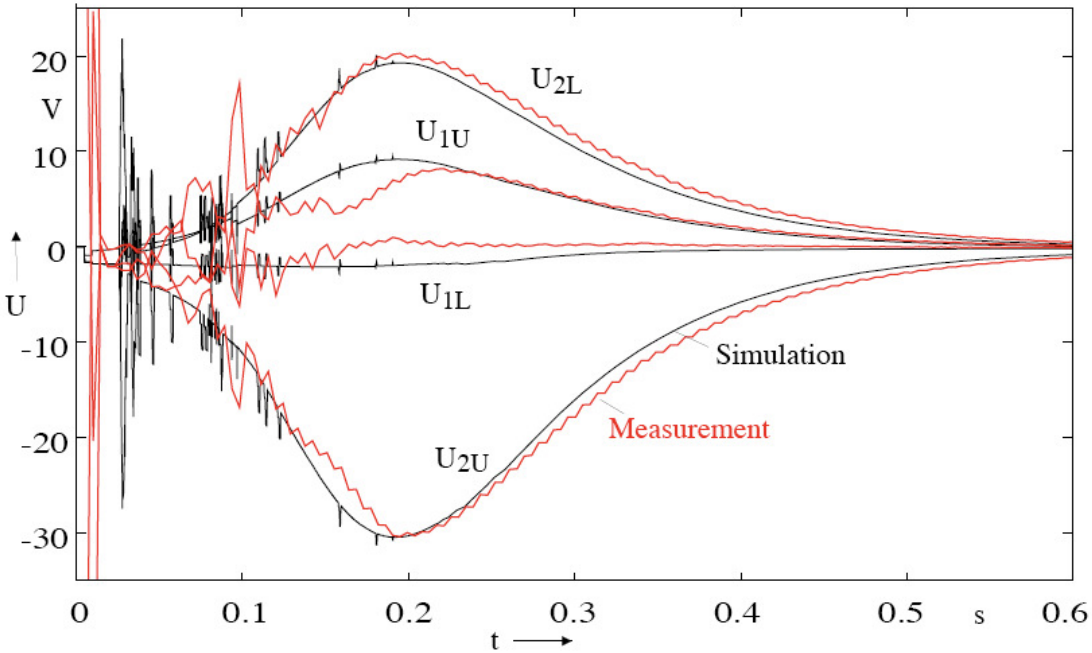




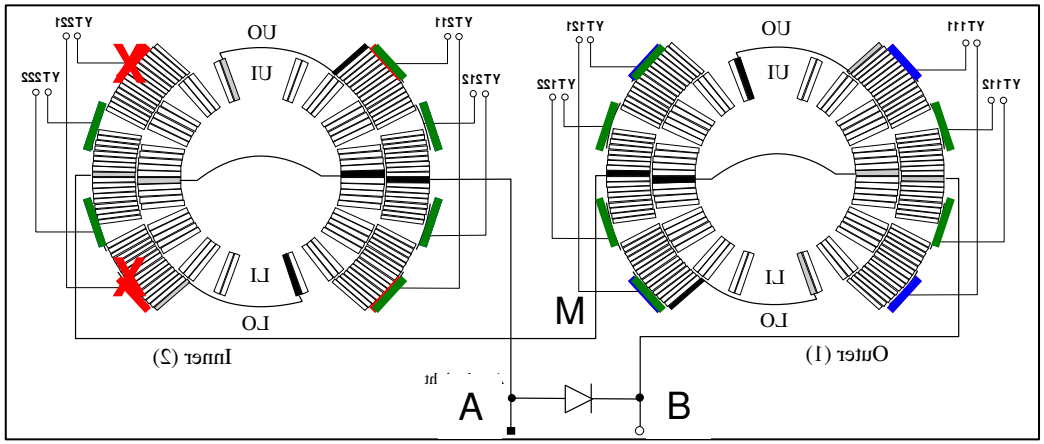
Introspection (Voltage Ripples)

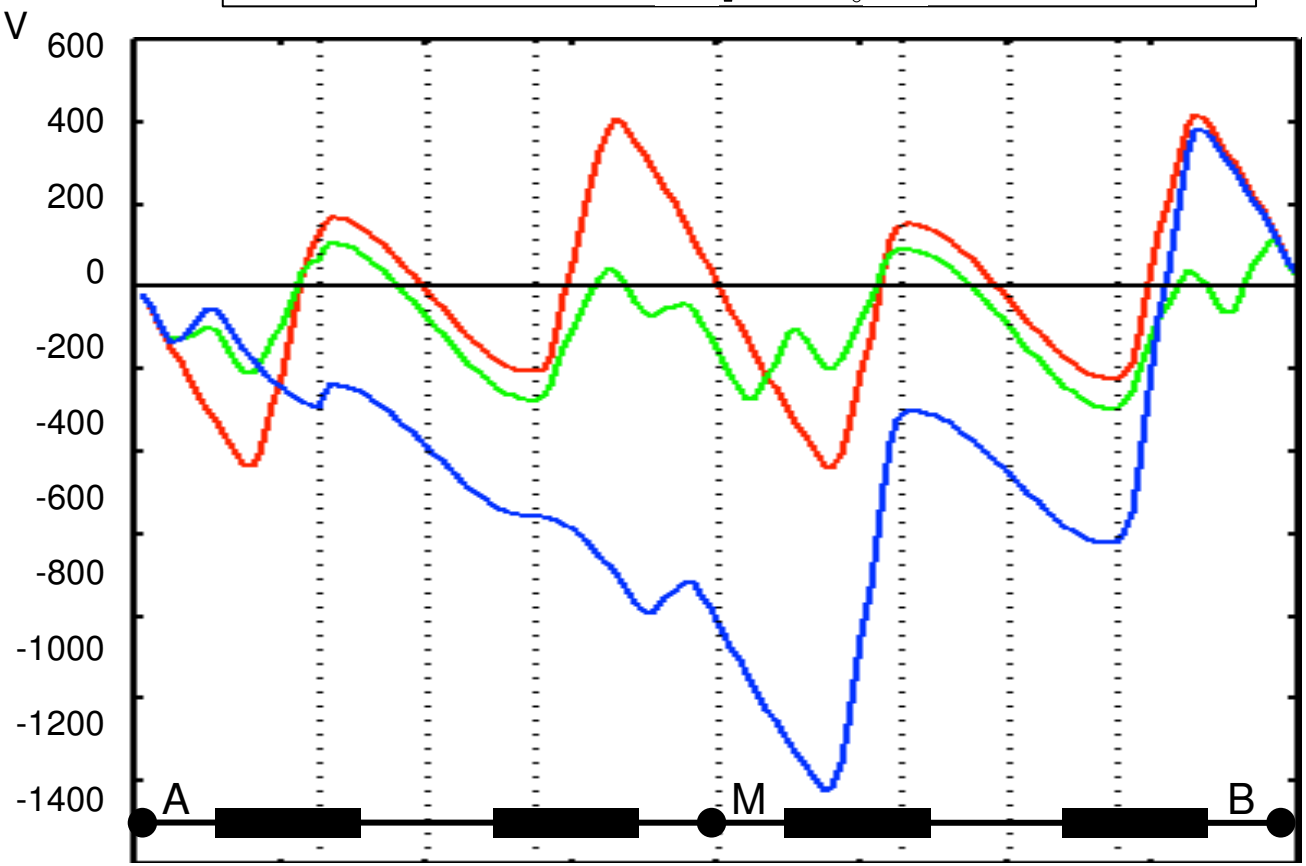






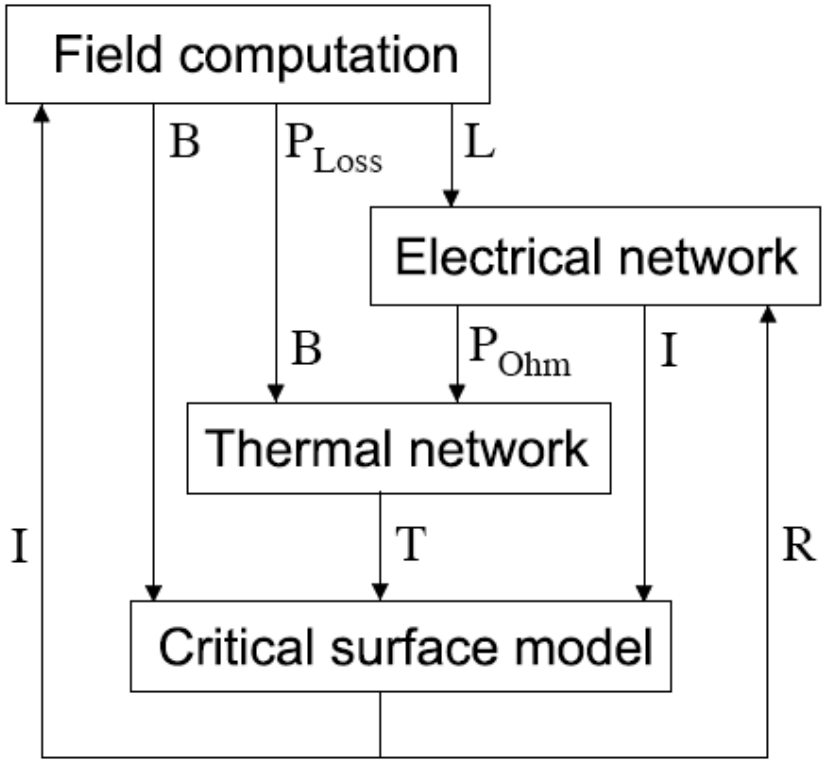
Defect on Quench Heater Circuit





Challenges

- Multiscale
  - Filaments 6 μ
  - Strands 1 mm
  - Cable 0.1 m
  - Magnet 10 m
  - String 3.2 km
- Multiphysics
- The smallest time constant determines the Runge-Kutta step



- More advanced methods require C++
  - But problem to find resources