High Intensity Effects in Linacs

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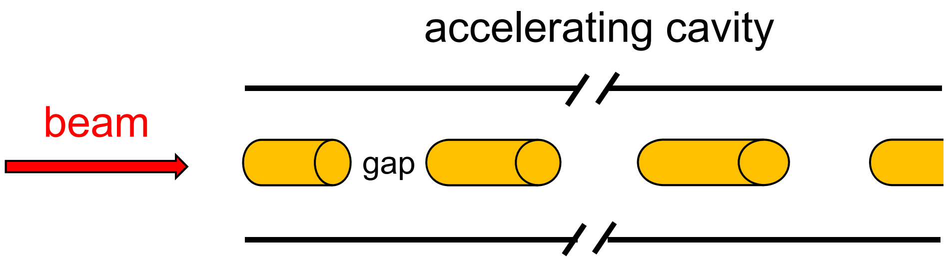
<u>Outline</u>

- Beam Loading
- Space Charge Field, Tune Shift
- Emittance Growth, Matching
- Resonances & Instabilities
- Particle-Particle close Encounters
- *Coherent Radiation*

Beam Loading



- to provide design accelerating field inside cavity, a fixed amount of rf-power P_{cav} is required
- P_{cav} depends on cavity geometry, material, and field oscillation mode
- P_{cav} dissipated by finite resistance of the cavity material
- power transferred to beam called beam load

$$P_b := \Delta E_u \cdot A \cdot \dot{N} = \frac{\Delta E_u \cdot A \cdot I}{qe}$$

- beam load can considerable exceed P_{cav}
- beam load should be provided fastly, i.e. while beam fills cavity

$$\tau_{fill} \sim n_{gaps} \cdot \tau_{rf} = \frac{n_{gaps}}{f_{rf}}$$

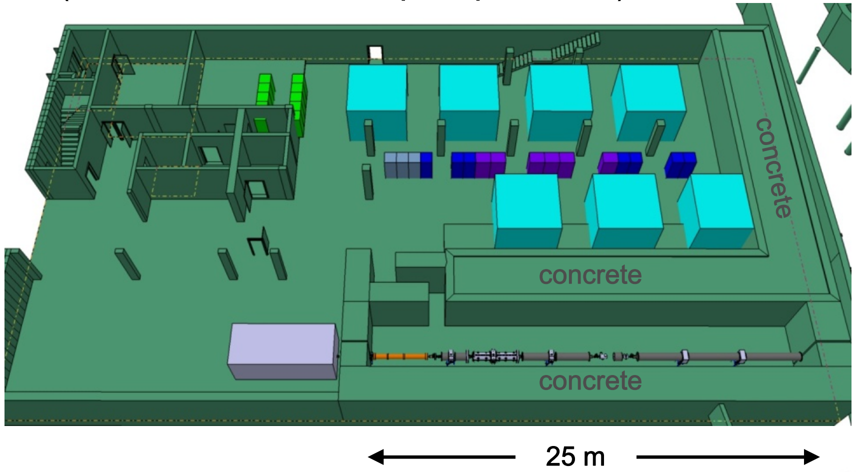
Beam Loading Examples

	GSI Alvarez Cavity I	FAIR Proton Linac CH-Cavity II
Ion	$^{40}Ar^{10+}$	Protons
Energy Gain per Nucleon	2.21 MeV/u	12.5 MeV
Beam Current	10 mA	70 mA
Beam Loading	88 kW	870 kW
P_b/P_{cav}	0.18	1
n_{gaps}	62	27
T_{rf}	9.2 ns	3.1 ns
T_{fill}/T_{rf}	62	13.5

- rf-power sources are major cost contribution for linacs
- linac project cost are sensitive to beam loading
- generally rf-controls work slower, ≈ 1500 ns ≈ 300 T_{fill}
- bunches within this time are accelerated less → lost or appropriatly cut away

More Beam → More Losses → More Cost

- more ions in the machine lead to higher losses
- lost ions hit the equipment and might cause damage
- machine needs protection, i.e. increasing demands on:
  - diagnostics
  - controls
  - euipment close to beam (cooling, radiation hard, etc ...)
- lost ions cause radiation: gammas, neutrons, ...
- persons must be protected from that, i.e.
  - machine inside tunnel from shielding material (concrete, stainless, paraphine, ...)
  - access rules & surveillance for tunnel



main loss driver is electromagnetic particle-particle interaction

Space Charge Force within Coasting Beam

from Maxwell eqs,
Gauss & Stokes:

$$E_r(r) = E(r) = \frac{I}{2\pi\epsilon_o R^2 \beta c} r$$ repulsive

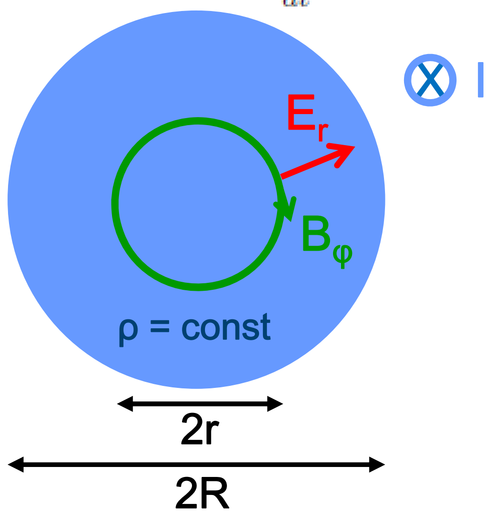
$$B_\varphi(r) = B(r) = \frac{\mu_o I}{2\pi R^2} r$$ attractive

$$r'' = \frac{1}{\beta^2 c^2} \ddot{\vec{r}} = \frac{eq}{A m_o \gamma \beta^2 c^2} \left[\vec{E} - \vec{v} \times \vec{B}\right]$$

$$r'' = \frac{eqI}{2\pi\epsilon_o A m_o (\beta\gamma c)^3 R^2} r =: \frac{P}{R^2} r$$

Perveance $P := \frac{eqI}{2\pi\epsilon_o A m (\beta\gamma c)^3}$

$$I = \frac{dQ}{dt} = \rho\pi R^2 \beta c$$



- net force is defocusing
- force decreases with energy: β→1: r“ = 0
- ρ = const: force is linear, acts like defocusing quadrupole

Space Charge Tune Shift

space charge adds to Hill's Equ.

$$x'' + \left[\kappa_o(s) - \frac{P}{R^2}\right] x := x'' + \kappa(s) x = 0$$

phase advance σ of oscillating x, called „tune"

$$\sigma = \sqrt{\kappa}$$

tune shift from space charge

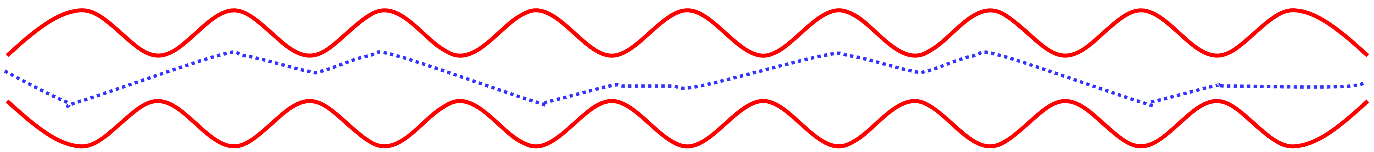
$$\sigma^2 = \sigma_o^2 - \frac{P}{R^2} =: \sigma_o^2 - \Delta\sigma^2$$

inhomogeneous beam

$$\rho = \rho(r) \longrightarrow \sigma(r) \longrightarrow \Delta\sigma(r) \longrightarrow \kappa(r)$$

- non-constant focusing κ → rms-emittance not preserved, but it growths !!!
- larger emittance → larger beam size → more losses
- space charge emittance growth can be minimized by „envelope-matching"

Matched Envelope

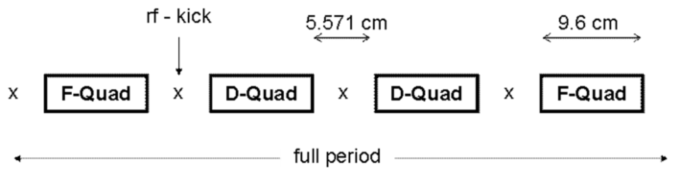


$\sigma_{part} < \sigma_{env} = 360°$

- “matched" beam: periodicity of envelope reflects periodicity of the lattice
- matched beam is in equilibrium with its environment
- sum of total energy of beam particles is minimized (free energy is zero)

Matched Envelope (GSI Linac)

Alvarez DTL has periodic lattice

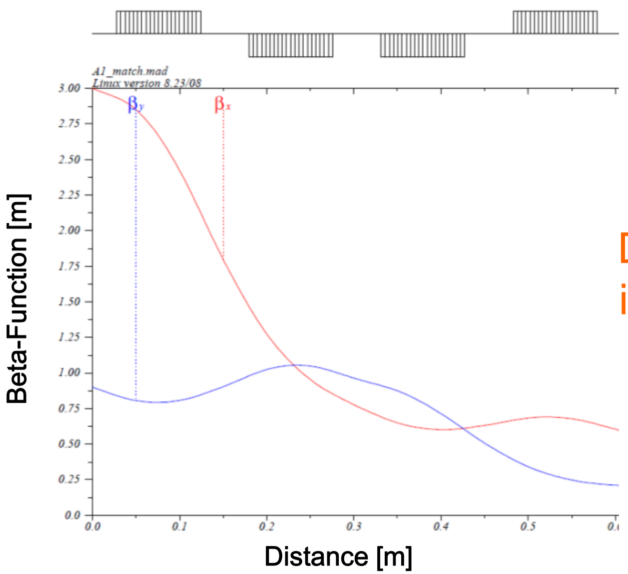


B´ = 36.187 (T/m) * MAZ / 8.5

transv. rf-kick = 0.1109 1/m

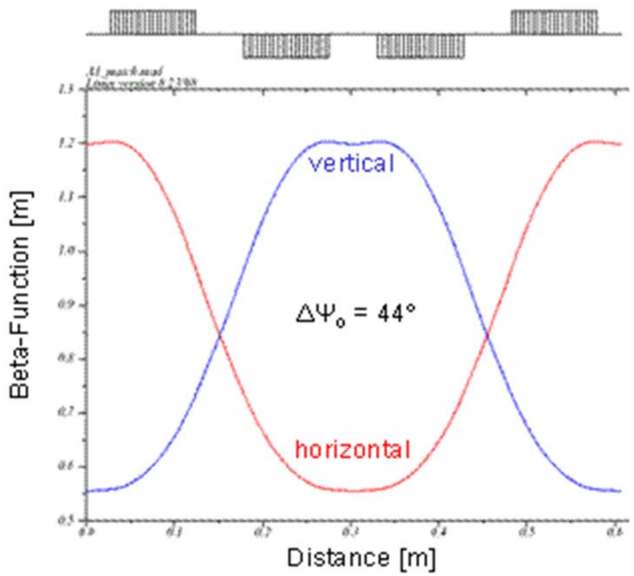
long. rf-kick = -0.2218 1/m

generally, envelope has an asymmetric, non-periodic shape

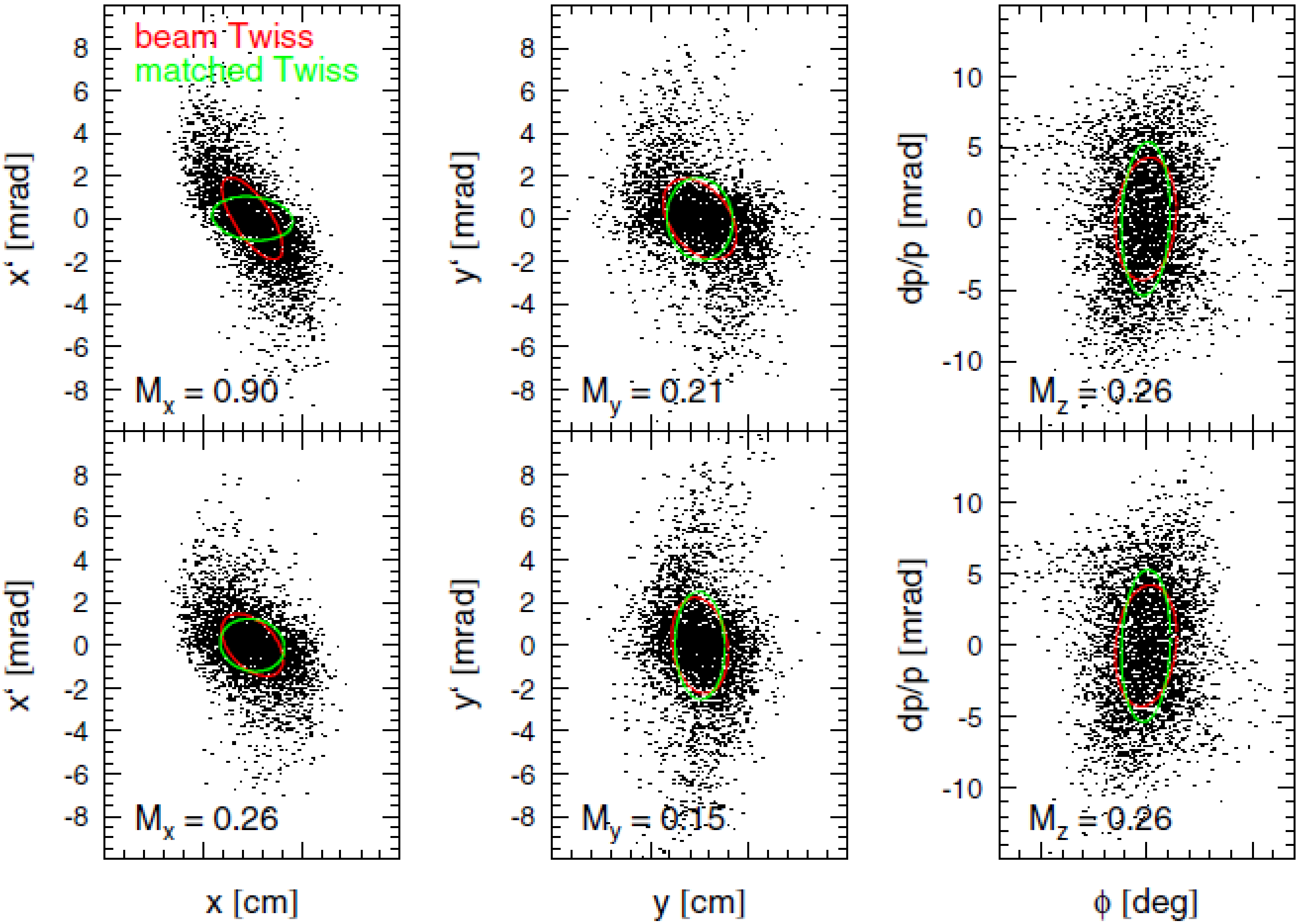


DTL-envelope depends on injection, i.e. "matching" to periodic lattice

lattice has one symmetric, periodic solution of envelope (matched)



Mismatch Definition

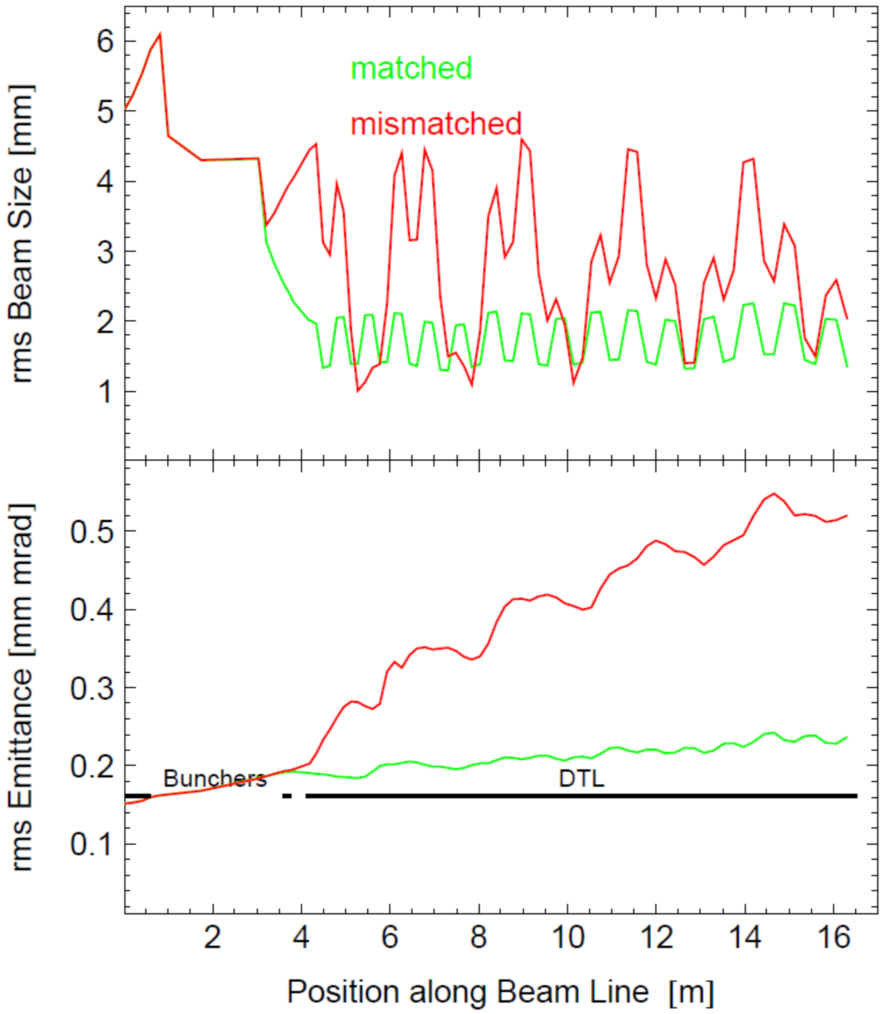


$$M = \left[1 + \frac{\Delta + \sqrt{\Delta(\Delta + 4)}}{2}\right]^{1/2} - 1$$

$$\Delta = (\Delta\alpha)^2 - \Delta\beta\Delta\gamma,$$

Matched Envelope → Minimized Emittance Growth

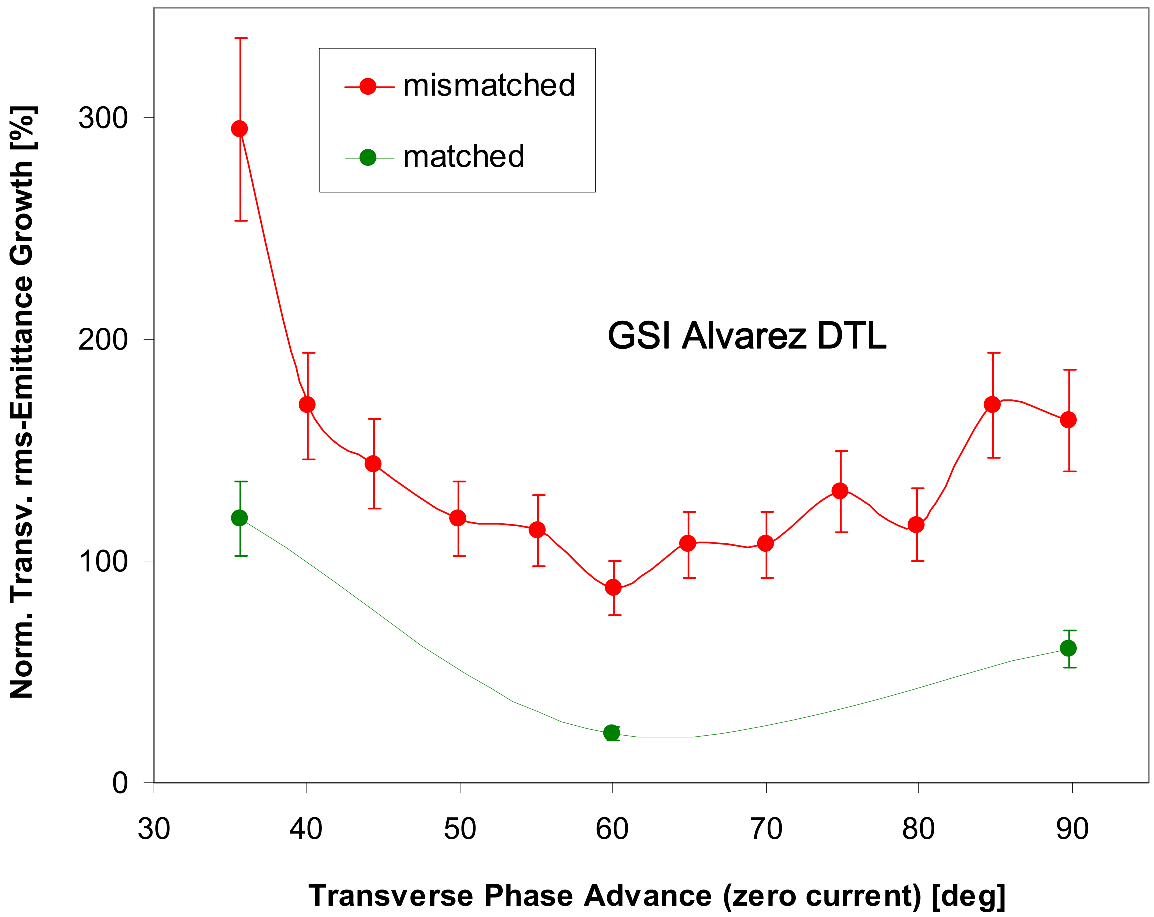
Simulation



Mismatched Beams:

- might give also 100% transmission
- might deliver "nice" beam profiles
- increase emittance strongly
- manifest as beam losses later
- cannot be detected directly

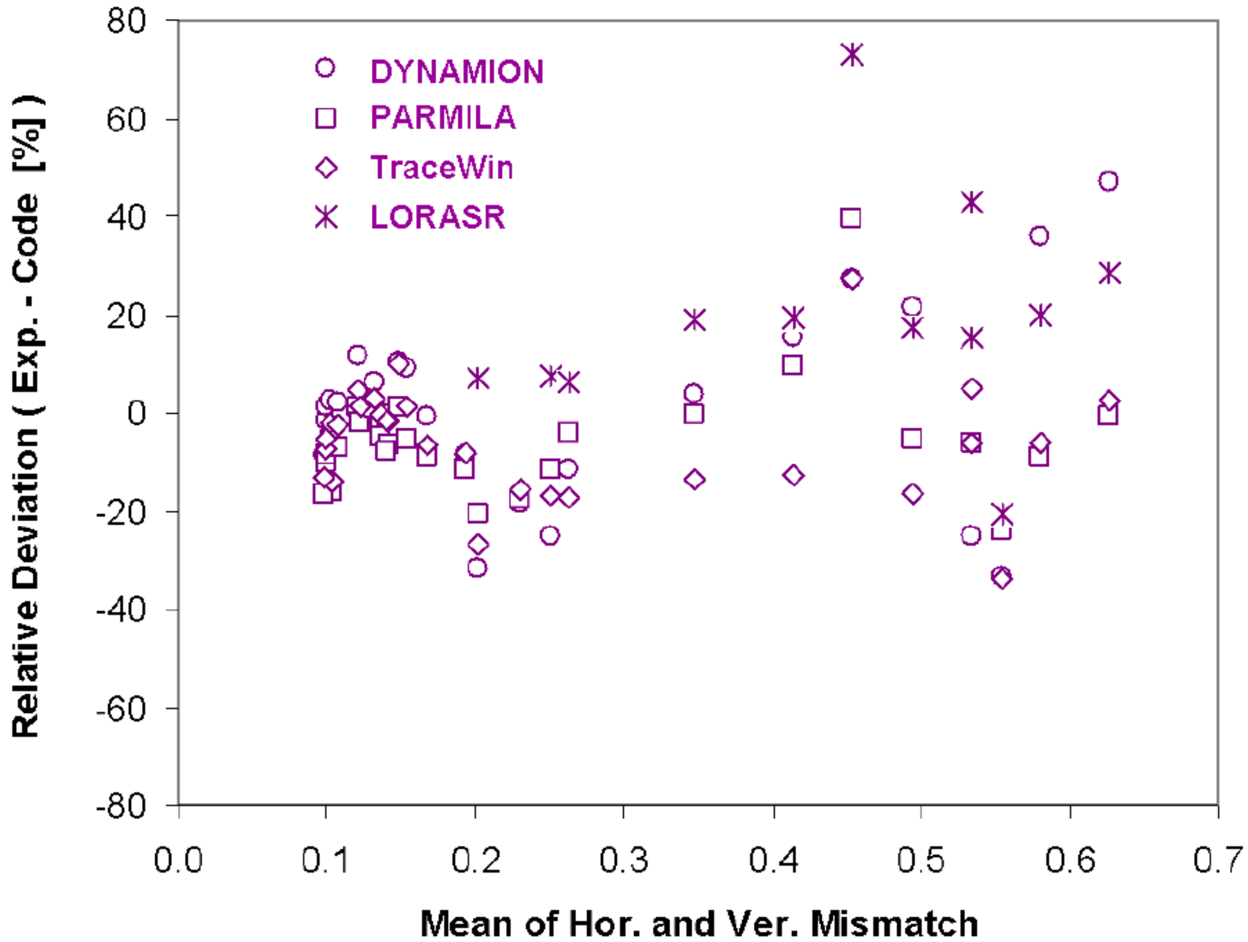
Measured emittance growth along the DTL for (mis)matched envelopes



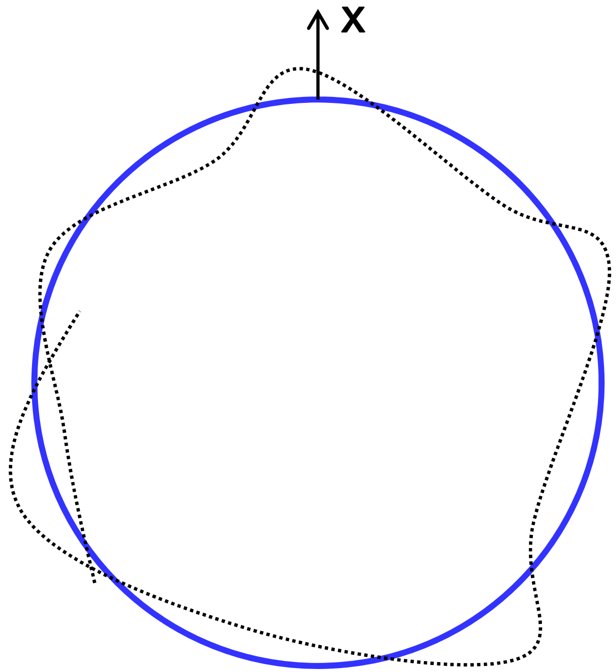
Beam matching successfully demonstrated, PRST-AB 11, 094201 (2008)

Matched Envelope → Higher Simulation Reliability

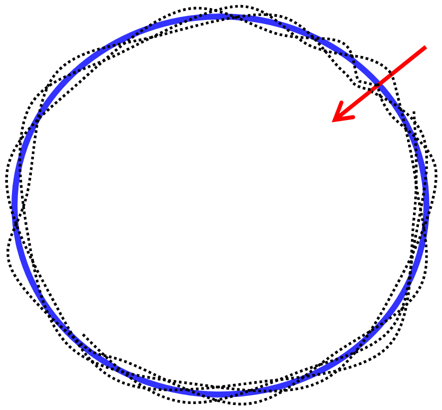
Comparison of measured & simulated rms-emittance growth along a drift tube linac



- Circular machines have intrinsically periodic focusing lattices
- Single particles do quasi-periodic oscillations around design orbit
- No perturbations: $x'' + \sigma^2 x = 0$
- σ is given by lattice, i.e. drifts, dipoles, and quadrupoles



Resonances (in Circular Accelerators)



- Perturbation generally from errors in single devices (magnets)
- Each particle passes many times the perturbing device
- Perturbation kicks single particle
- Same perturbation is applied periodically to each particle: $x'' + \sigma^2 x = \underline{a \cdot x^n \cdot e^{i\sigma_p s}}$
  - n=0, pert ~ x^0, dipolar
  - n=1, pert ~ x^1, quadropolar
  - n=2, pert ~ x^2, sextupolar
  - n=3, pert ~ x^3, octupolar
  - ...

Resonances (in Circular Accelerators)

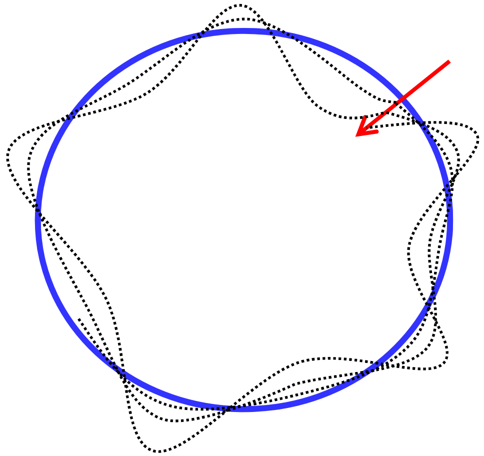
- Perturbations drive resonances and cause beam loss if σ is chosen badly

- Suppose perturbation is weak → solution oscillates with un-perturbed σ: $x = e^{-i\sigma s}$

- Plug into perturbed oscillator equ.: $x'' + \sigma^2 x = a \cdot e^{-in\sigma s} \cdot e^{i\sigma_p s} = a \cdot e^{i(-n\sigma + \sigma_p)s}$

- Effective frequency of perturbation = $-n\sigma + \sigma_p$

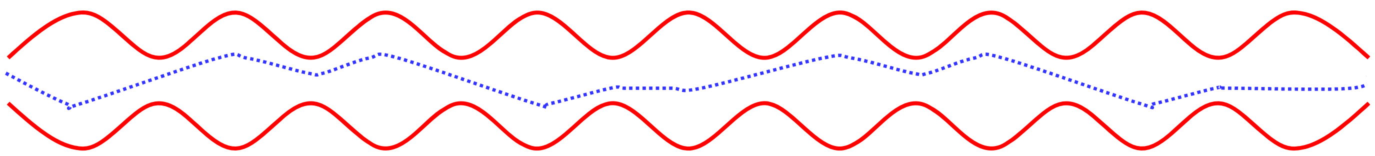
- Resonance: pertubation at unperturbed frequency, i.e. $-n\sigma + \sigma_p = \sigma$



resonance at $\sigma = \dfrac{\sigma_p}{n+1}$

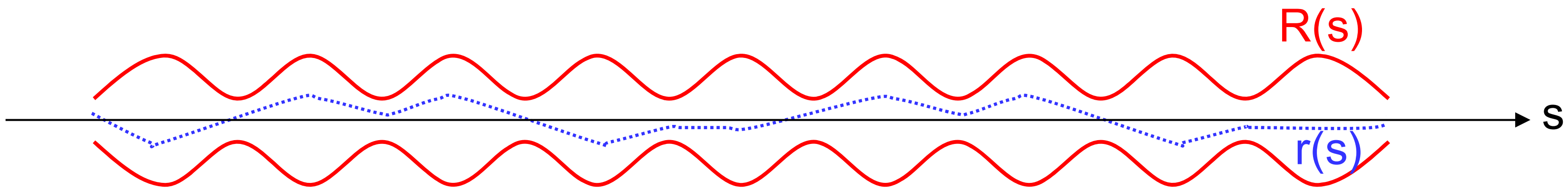
Resonances in a Linear Accelerator

- Each device is seen by particle just once
- Single devices cannot cause resonant perturbation
- Q–diagrams are not used in linac design
- High beam current :
  - space charge (sc) of beam acts on each single particle
  - sc force acts always on particle
  - sc force depends on beam dimensions
  - periodic change of beam dimensions (envelope) → periodic sc force on particle



$\sigma_{part} < \sigma_{env} = 360°$

Model for Resonance



• assumption of a periodically breathing beam envelope with phase advance σ_{env}

• envelope has radial symmetry

• bunch is homogeneously charged along s, length = (βλ)/6 = 60°

• single particle experiences :

  • constant external focusing with σ_o from magnets

  • electric field of breathing envelope

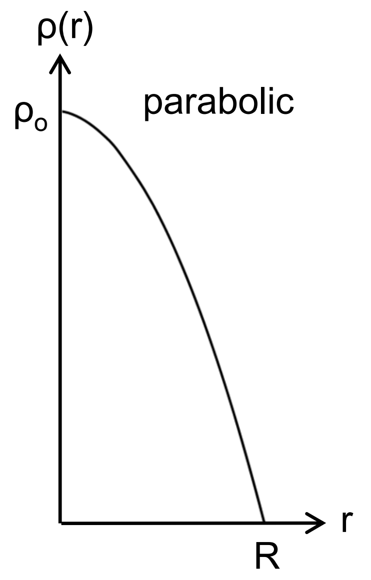
Model for Resonance

beam charge density depends on radius r :

$$\rho(r) = \rho_o(s) \cdot \left[1 - \frac{r^2}{R(s)^2} + O(r)\right]$$

$r^{\geq 4}$ neglected

breathing with σ_{env}



creating a field :

$$E_r = \frac{6 \cdot I}{\pi \epsilon_o \cdot R(s)^2 \beta c}\left[r - \frac{r^3}{2R(s)^2}\right], \quad r \leq R(s)$$

octupolar field component (r^3)

single particle motion driven by two components (β << 1, self-magn. field neglected):

$$r'' = -\sigma_o^2 r + \frac{e \cdot q}{A \cdot m_u} \cdot E_r$$

external focusing

space charge field from beam (perturbation)

$$r'' + \left[\sigma_o^2 - \frac{eq}{Am_u} \cdot \frac{6 \cdot I}{\pi\epsilon_o \cdot R(s)^2(\beta c)^3\gamma}\right] r = -\frac{eq}{Am_u} \cdot \frac{3 \cdot I}{\pi\epsilon_o R(s)^4 (\beta c)^3\gamma} \cdot r^3$$

tune depression from repulsive space charge

quasi-oscillates with envelope frequency

$$r'' + \left[\sigma_o^2 - \Delta\sigma^2\right] r = a \cdot r^3 \cdot e^{i\sigma_{env} s}$$

$$r'' + \sigma^2 r = a \cdot r^3 \cdot e^{i\sigma_{env} s}$$

depressed phase advance

Single Particle Motion

$$r'' + \sigma^2 r = a \cdot r^3 \cdot e^{i\sigma_{env} s}$$ depressed phase advance

Ansatz : $r = e^{-i\sigma s}$

"New" oscillator equation :

$$r'' + \sigma^2 r = a \cdot e^{i(\sigma_{env} - 3\sigma)s}$$ frequency of effective perturbation

Resonance condition:

$$\sigma_{env} - 3\sigma = \sigma$$

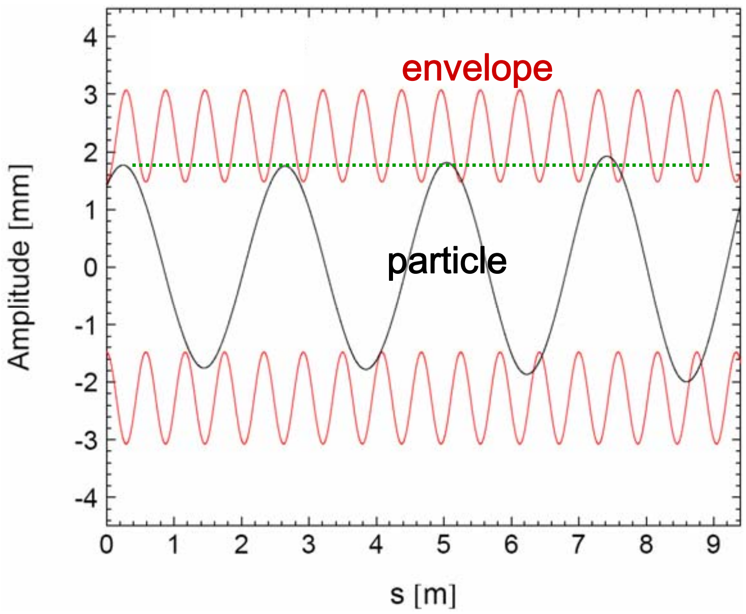
resonance condition : $\sigma_{env} = 4\sigma$

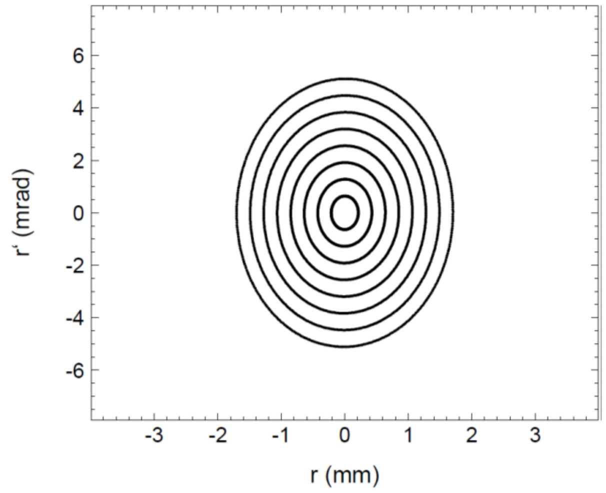
envelope oscillates 4 times faster than single particle

σ_{env} = 360° → σ = 90°

4th order resonance occurs at σ = 90°, i.e. $\sigma_o \geq 90°$

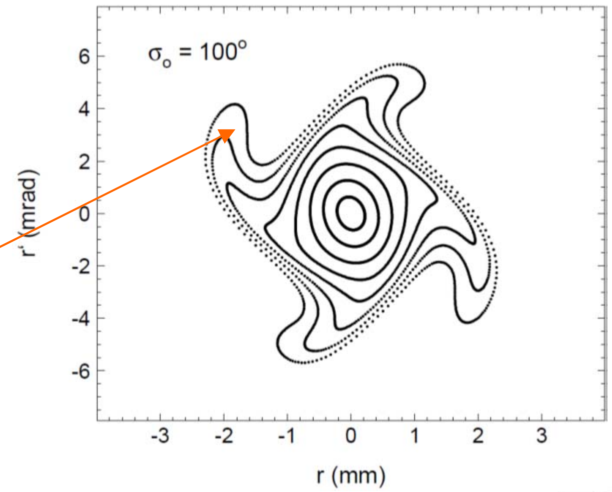
Numerical Integration of Diff. Equation





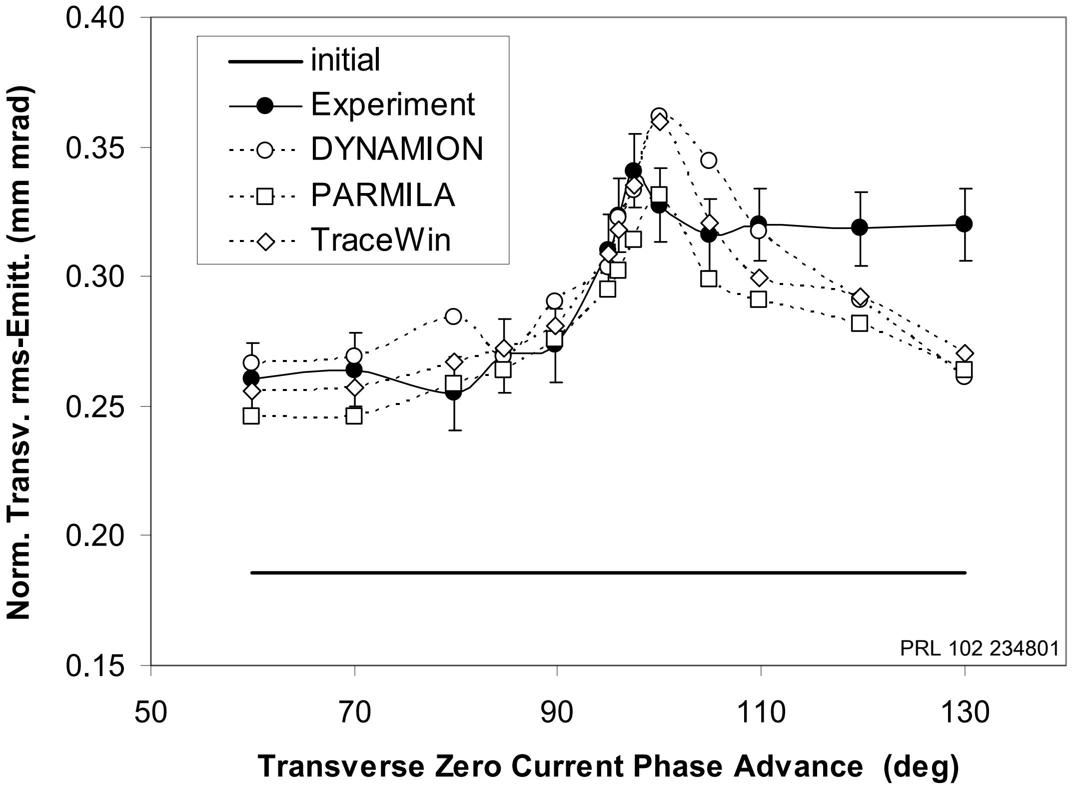
σ_oscillation (envelope) = 4 * σ_oscillation (particle)

→ resonant excitation of single particles



4 wings: characteristic feature of 4^{th} order resonance

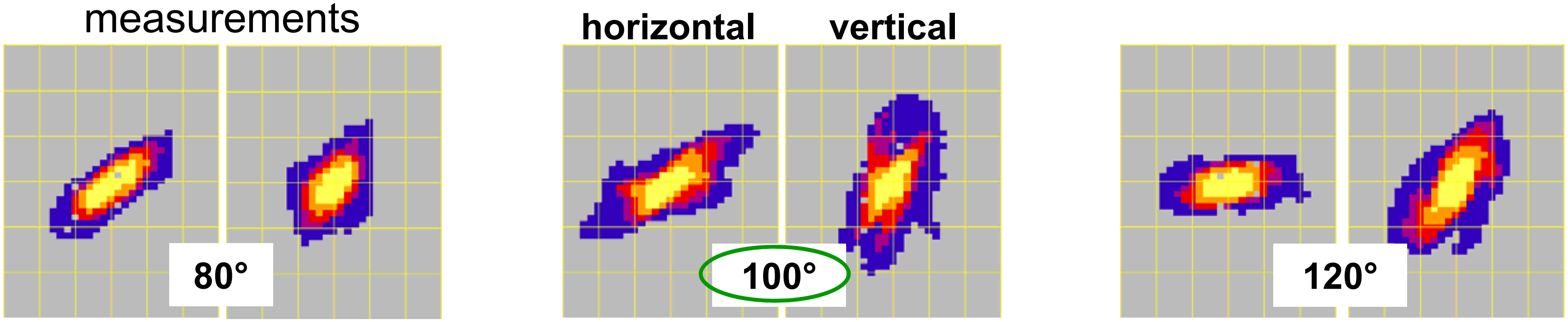
Measurements: DTL Exit rms Emittance vs. σ_o

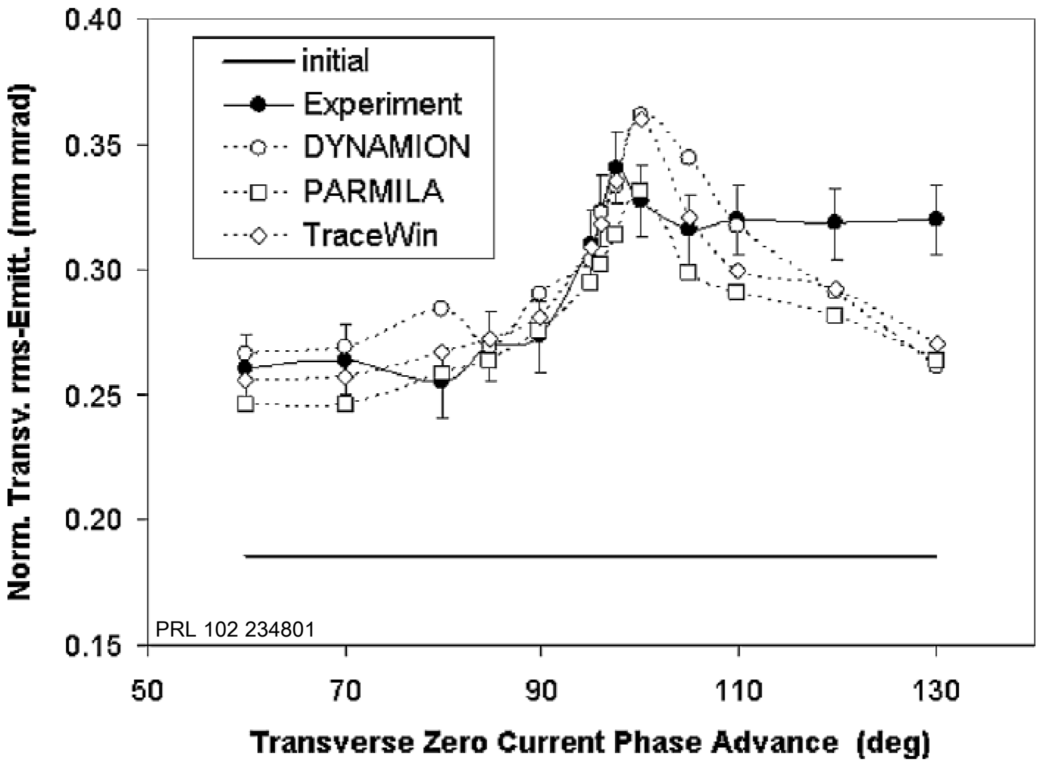


- strong growth approaching $\sigma_o \approx 100°$
- tune depression: $\sigma_o \approx 100° \rightarrow \sigma \approx 90° = 360° / 4$
- good agreement with three simulation codes
- strong hint for space charge driven 4th order resonance

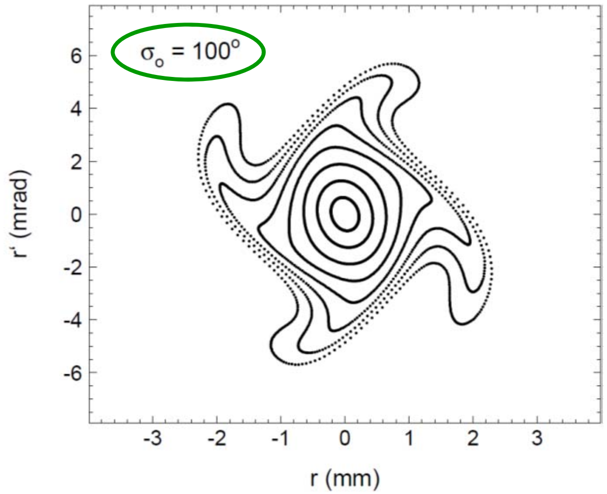
Proof for 4th Order Resonance in the UNILAC

measurements





4 wings were observed



4^{th} Order Space Charge Resonance in a Linac

- predicted by D. Jeon (SNS/ORNL) → PRST-AB **12**, 054204 (2009)
- measured first time in GSI UNILAC
- justifies golden design rule of avoiding $\sigma_o > 90°$ in the design of linac lattices
- rule originally to avoid so-called „envelope instability“, which
  - assumes a homogeneously charged beam: δ = const
  - states that mismatch results in exp. envelope growth at $\sigma_o >= 90°$
  - but :
    - beams with δ = const (to my knowledge) have never been seen
    - this instability is much weaker than resonant emittance growth

Envelope Instability ?

M. Reiser, *Theory and Design of Charged Particle Beams*

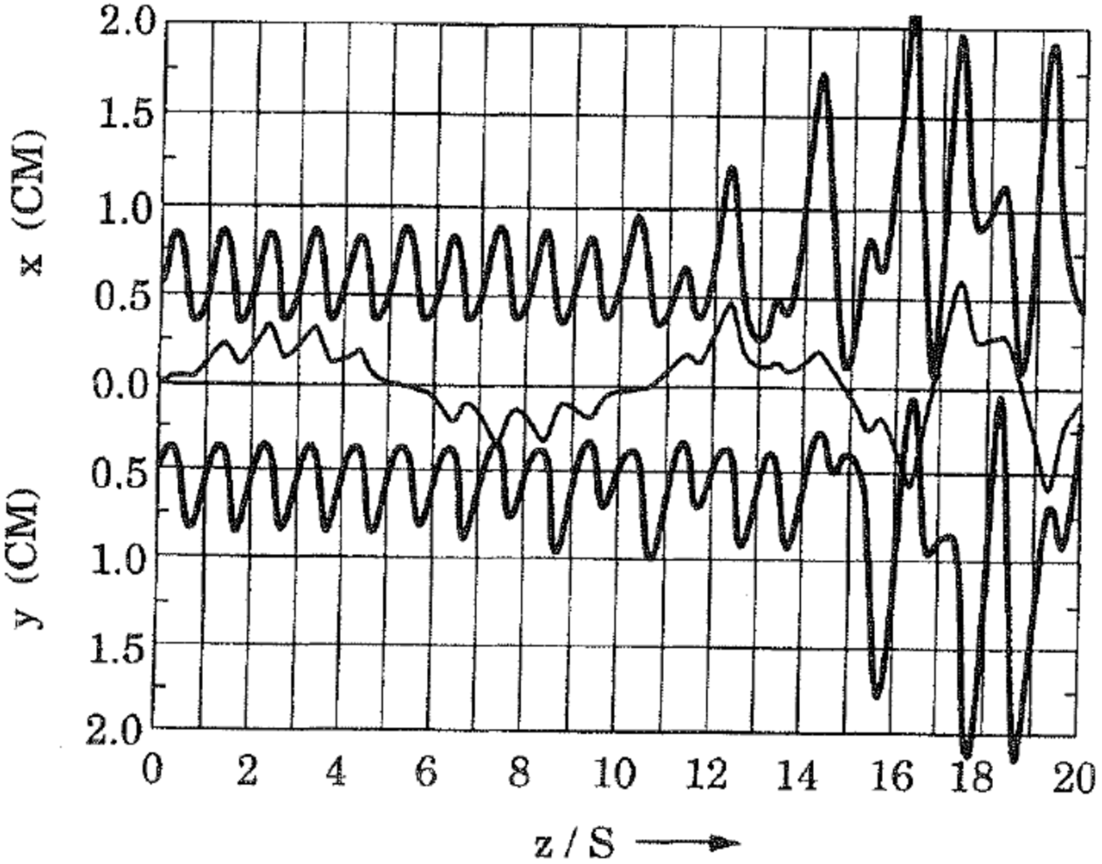
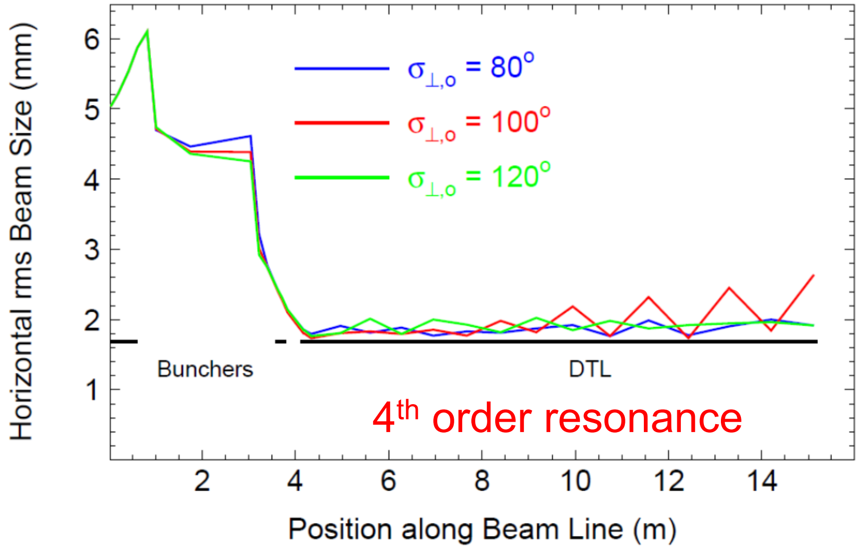


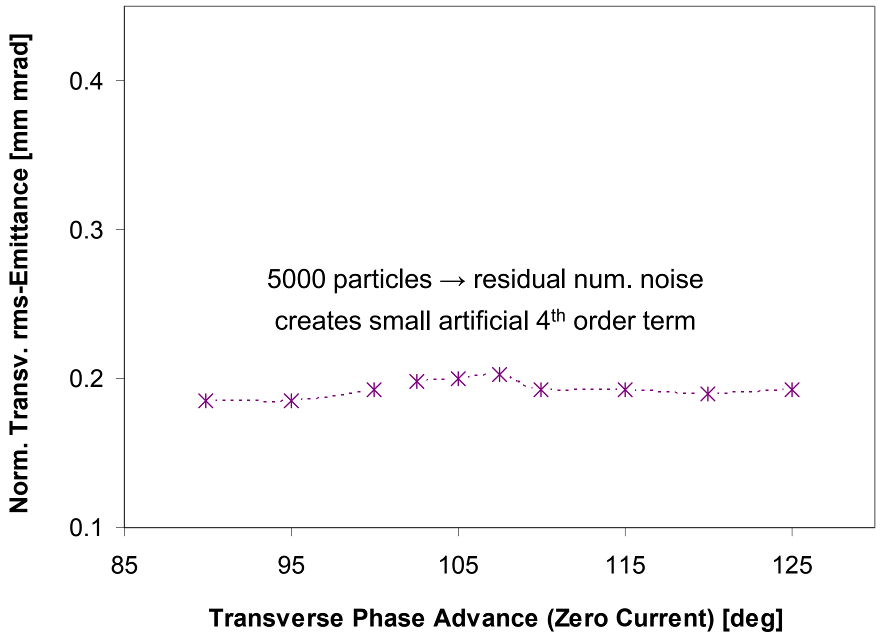
Figure 4.17. Quadrupole channel, slightly mismatched beam ($\sigma_0 = 120°$, $\sigma = 35°$). (From Reference 12.) J. Struckmeier and M. Reiser, Part. Accel. 14, 227 (1984)

Envelope Instability ?

simulated envelopes
→ no instability at $\sigma_o > 90$



simulated rms emit. growth
δ=const beam, no 4th order term → no growth



DTL too short and/or mismatch too small
for envelope instability growth

Parametric Resonances from Inter-Plane Coupling

- previous resonances were purely transverse → occur also in coasting (dc) beams
- linacs use bunched beams → space charge forces couple long. & transv. planes
- coupling might trigger energy transfer from "cold" plane to "hot" plane
- beam temperature $T_\perp \sim < r' >^2 \sim \frac{\epsilon_\perp}{< \beta_\perp >}$
- phase advance $\sigma_\perp = \frac{1}{L}\int_0^L \frac{ds}{\beta_\perp}$ $\sigma_\perp \sim \frac{1}{< \beta_\perp >}$ → $T_\perp \sim \epsilon_\perp \cdot \sigma_\perp$
- heat (emittance) transfer, if $T_\perp \neq T_{||}$
- modern high current linacs: $\sigma_{||} \approx \sigma_\perp, \quad \epsilon_{||} > \epsilon_\perp \longrightarrow T_{||} > T_\perp$

Emittance transfer from longitudinal to transverse plane is expected

Parametric Resonances from Inter-Plane Coupling

- early 80's: I. Hofmann investigated (theoretically) beams with homogeneous charge density
- homogeneous beams have linear sc forces only → no emittance-growth or -exchange
- introduction of density perturbation → how does perturbation evolve?

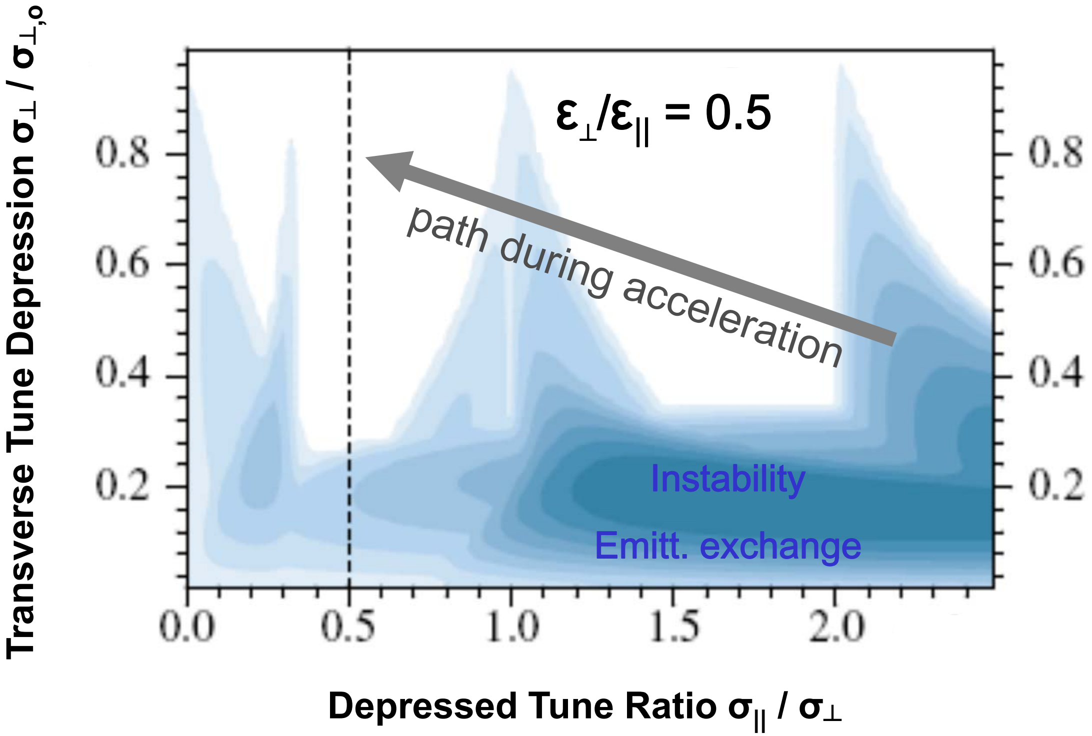
  - exponential increase, i.e. emittance transfer ?

  - re-distribution to homogeneous density, i.e. no transfer ?

- result:
  - transfer occurs just in vicinity of $\frac{\sigma_{||}}{\sigma_{\perp}} = \frac{m}{n}$, except $\frac{\epsilon_{\perp}}{\epsilon_{||}} = \frac{m}{n}$, i.e. $T_{||} = T_{\perp}$
  - away from these regions no emittance transfer even at $T_{\perp} \neq T_{||}$
  - m=n is strongest resonance

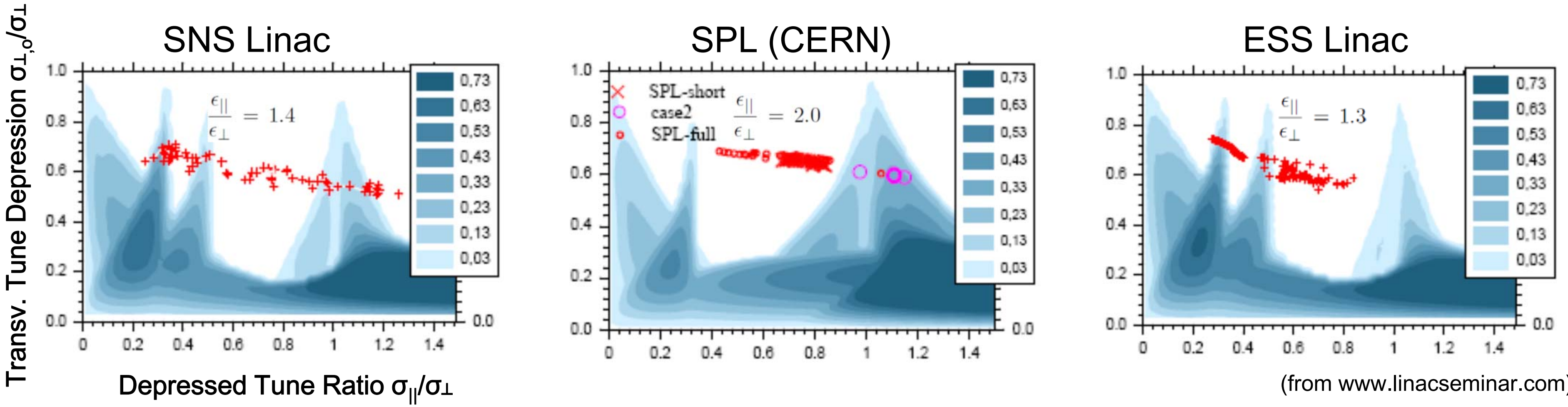
detailed description in PRE **57**, 4713 (1998)

Hofmann's Stability Charts



- charts plot regions where emittance transfer is expected
- charts depend just on long./transv. emittance ratio
- transfer at $T_{||} = T_{\perp}$ strongly suppressed

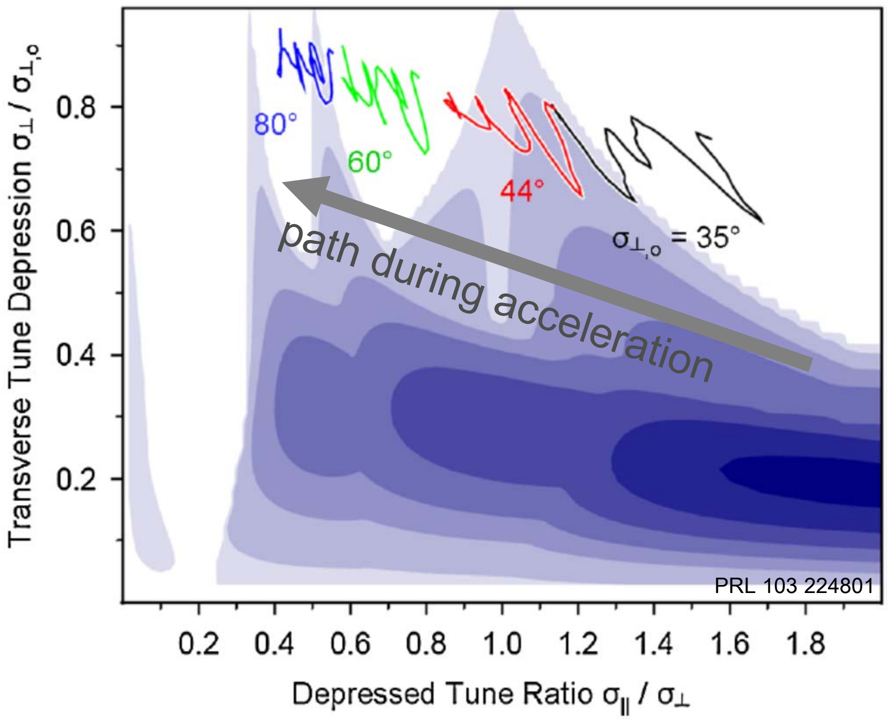
Parametric Resonances from Inter-Plane Coupling

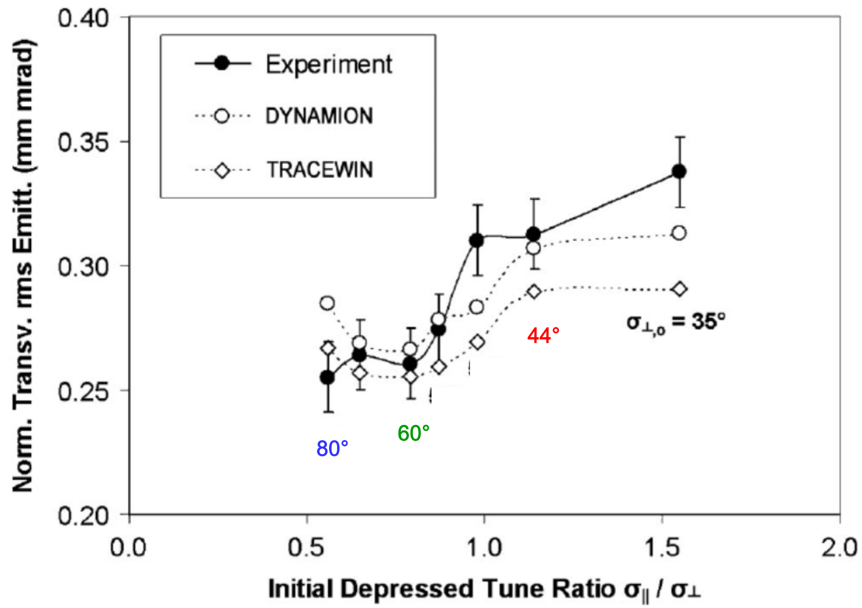


(from www.linacseminar.com)

- Hofmann charts: well excepted linac design tool
- simulations: just $\sigma_{||} \approx \sigma_{\perp}$ harmful to machine performance
- no experimental verification
- experiment done at GSI UNILAC, first DTL tank

Experimental Evidence for Parametric Resonance at the UNILAC





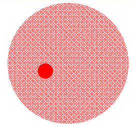
- tune ratio approaches 1.0 → increased transv. growth measured
- result in good agreement with simulations

Ordered according to amount of growth (strong to low):

- Envelope mismatch
- Transverse Resonance
- Parametric Resonance
- Envelope instability

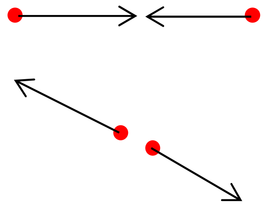
Close Particle-Particle Encounters

- space charge (simulation) smears out other particles to a continuum



- space charge omits granular nature of beam

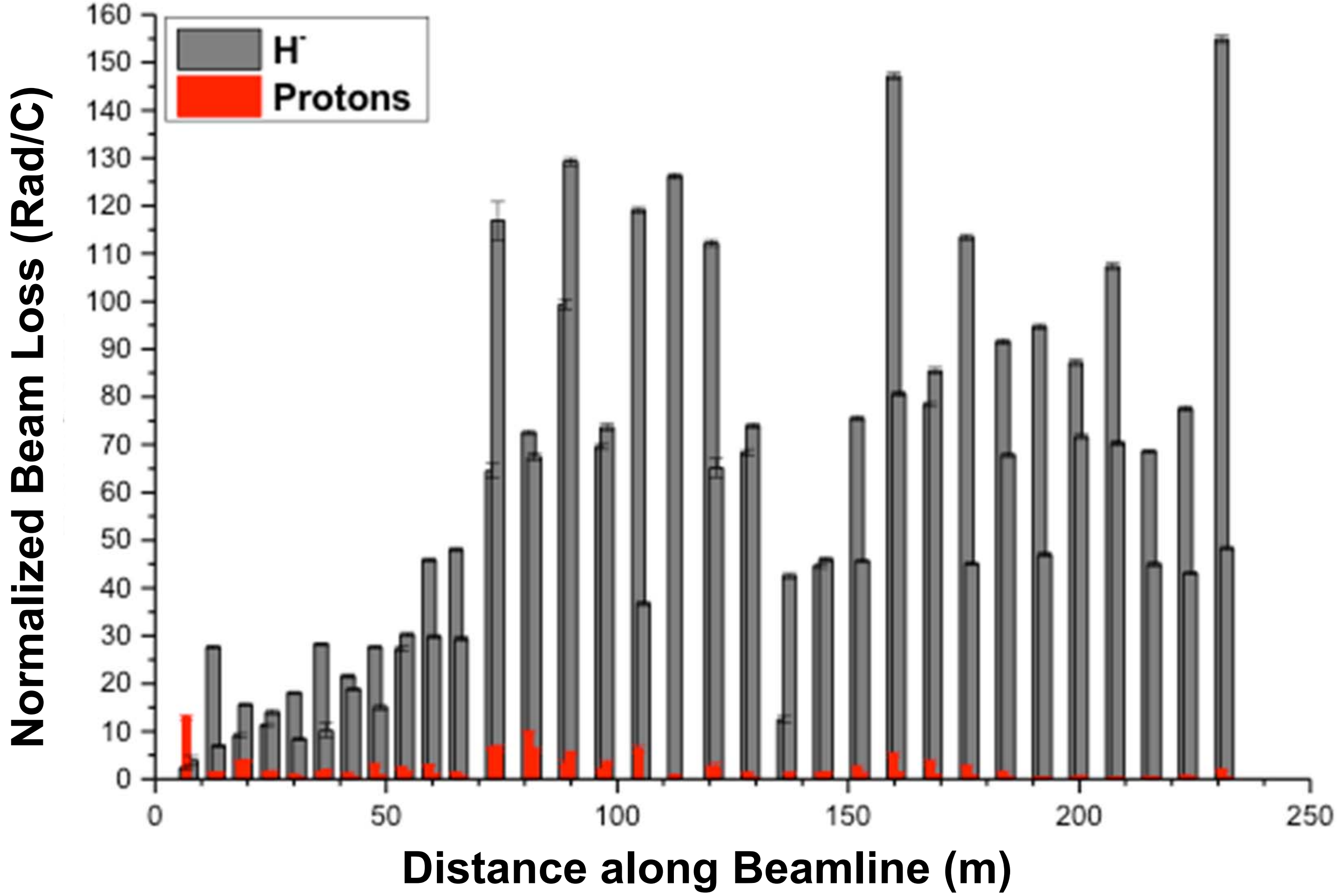
- its forces do not have „spikes“
- but spikes exist and:
  - occur at close encounters between to colliding particles
  - may kick single particles strongly
  - may kick them out of machine acceptance → losses



- ring machines: repeated collision between ions lead to emittance growth, called „intra-beam“ scattering. Not considered so far in linac design/operation
- linac: collisions between ions may change charge state of ions called „intra-beam stripping (IBS)“ → ions lost
- IBS causes losses in the recently commissioned Spallation Neutron Source (SNS) linac

Intra-Beam Stripping within H^- Beam

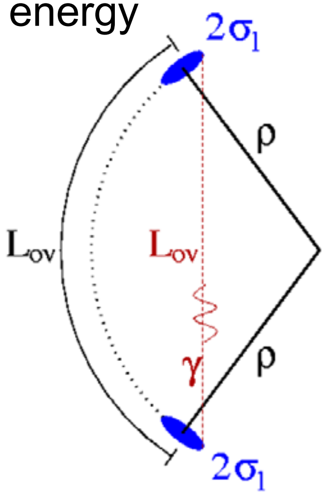
Measurements at SNS :



presented by J. Galambos (ORNL) at Linac2012 at Tel Aviv, Israel, and PRL 108, 114 801, (2012)

Coherent Synchrotron Radiation (CSR)

- relativistic particles in a bending dipole radiate (synchrotron radiation)
- $P_{rad} \sim q^2,\ P_{rad,inc} \sim Nq^2,\ N =$ particles per bunch
- dense & short bunch:
  - radiation from tail hits head
  - radiation interacts with bunch → induces additional radiation
  - radiation gets coherent
  - bunch radiates like single particle with charge (Nq)
  - $P_{rad,coh} \sim (Nq)^2$
  - might reach MW level and lowers beam energy
  - reduces beam quality



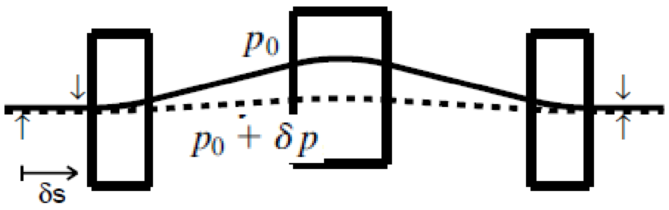
Overtaking Condition :

$$L_{ov} = \sqrt[3]{48\ \sigma_l\ \rho^2}$$

$$\Delta P_{coh} = 0.028\ N^2 \frac{c\ e^2}{\varepsilon_0\ \rho^{2/3}\ \sigma_l^{4/3}}$$

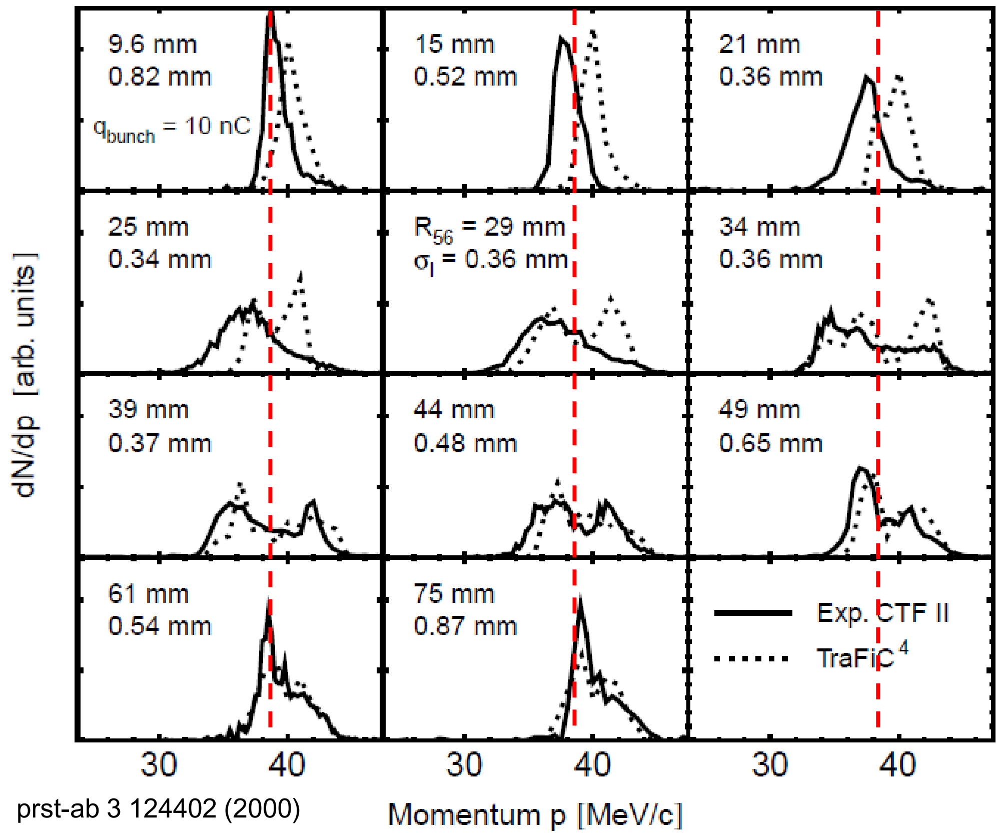
CSR: Effect on Beam Energy Spectrum

magnetic bunch compressor :



- compresses & bends bunches
- $R_{56} := \frac{\delta s}{\delta p}$
- given by bending angle

momentum spectrum vs. bending angle (bunch length)



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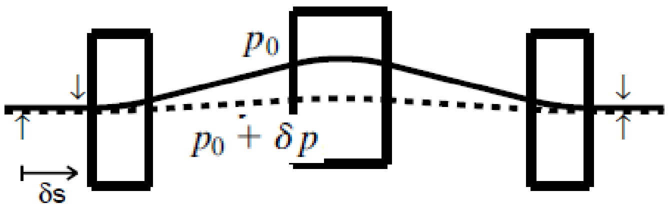
CSR effect on momentum spectrum:

- momentum spread increase
- shift to lower momenta

CSR: Effect on Beam Quality

magnetic bunch compressor :



- compresses & bends bunches
- $R_{56} := \frac{\delta s}{\delta p}$
- given by bending angle

- opposite to space charge, coherent radiation increases with energy
- causes beam energy loss
- can cause considerable beam quality degradation
- one limiting factor in X-FEL performance

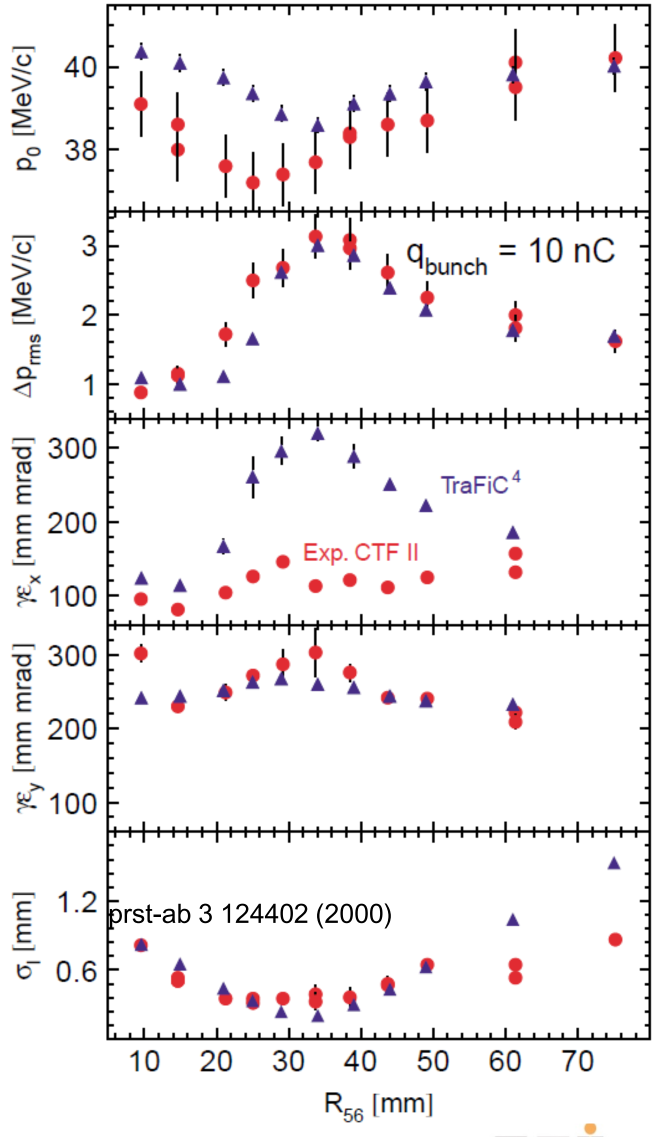
mean momentum

momentum spread

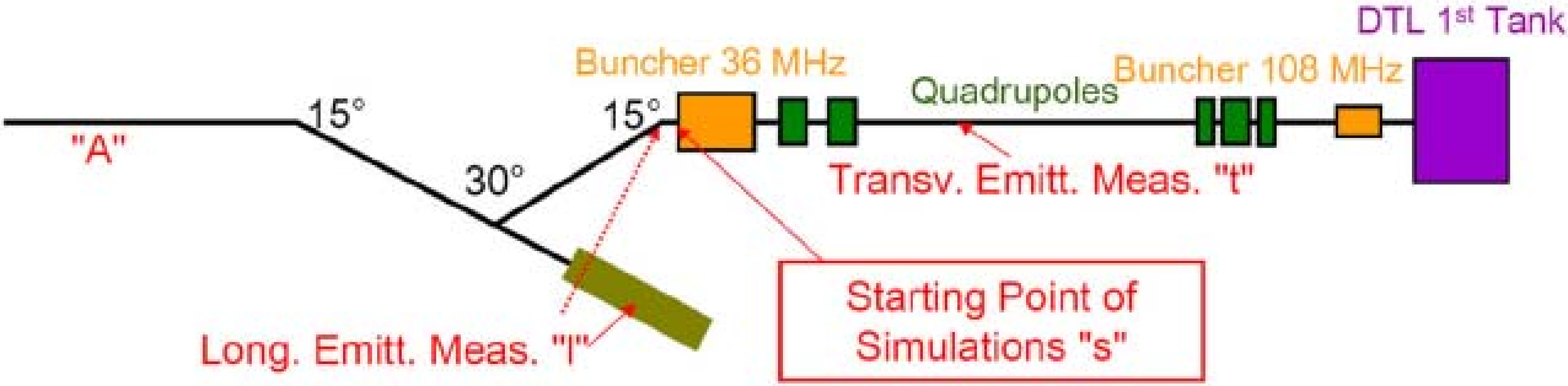
hor emittance

ver. emittance

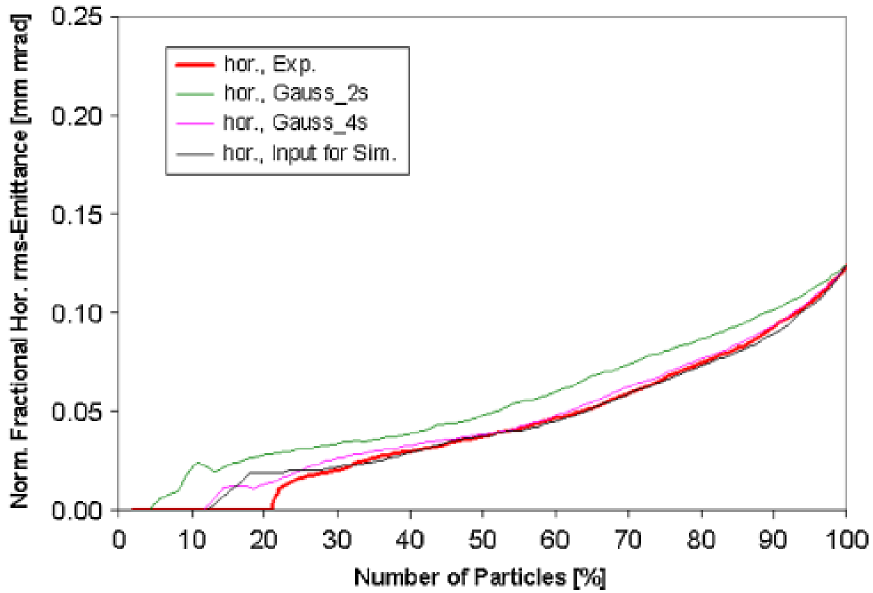
bunch length

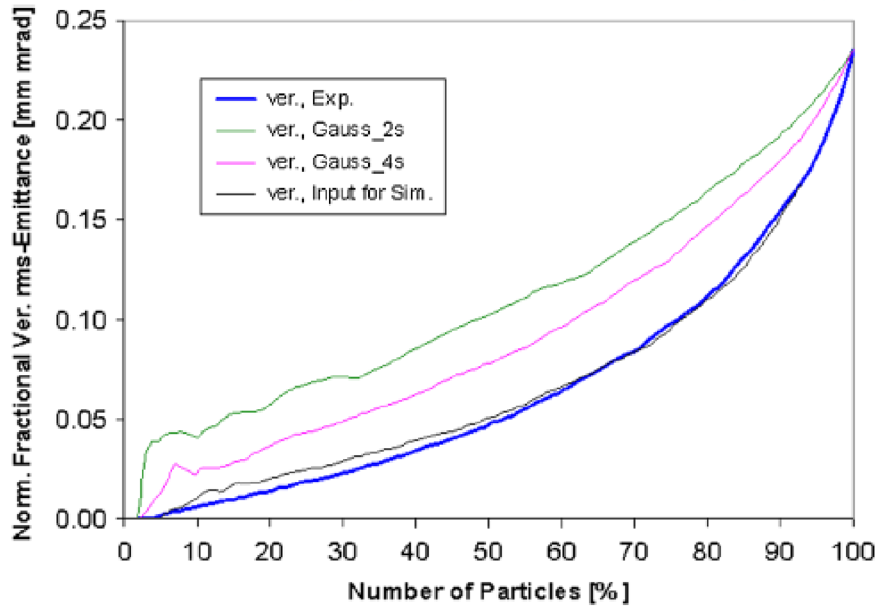


Reconstruction of initial Twiss Parameters



Emittance Growth depends on Distribution





$$\tilde{R}^2 = X^2 + X'^2 + Y^{1.2} + Y'^{1.2} + \Phi^2 + (\delta P/P)^2$$

and

$$f(\tilde{R}) = \frac{a}{2.5 \times 10^{-4} + \tilde{R}^{10}}, \qquad \tilde{R} \leq 1$$

$$f(\tilde{R}) = 0, \qquad \tilde{R} > 1,$$