Beam instabilities in ion beams

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Instabilities in ion beams

- Beam instabilities are driven by the **electromagnetic interaction with the accelerator environment**.

- Above a certain **intensity threshold** the beam's oscillation amplitude increases exponentially and the beam is either **lost at the wall** (transverse instabilities) or the **emittance increases**.

- Presently, instabilities are one of the **main beam quality and intensity limitation** in synchrotrons and storage rings for protons and ions !

- Finding **cures for instabilities** is one of the major challenges in beam physics and accelerator technology for **future machines**.

This presentation will focus on (transverse) instabilities in synchrotrons and storage rings

Presentations by G. Arduini (LHC), J. Stadlmann (synchrotrons),
G. Franchetti (beam dynamics)

Literature

Books:
- Alex Chao, *Physics of collective beam instabilities in high energy accelerators* (1993) -> online
- B. Zotter, S. Kheifets, *Impedances and wakes in high-energy accelerators* (1998)
- K.Y. Ng, *Physics of intensity dependent beam instabilities* (2006)

CERN Lectures (CAS): A. Hofmann , J.L. Laclare, K. Schindl,…

Other lectures:
- B. Zotter, F. Sacherer
  Transverse instabilities of relativistic particle beams in accelerators and storage rings (1976)
-J. Gareyte, Transverse mode coupling instabilities (2001)
-……

See also:
- IPAC and ICFA-HB confererence proceedings (JACoW)
- Phys. Rev. ST-AB and Nucl. Inst. Meth. A

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Instability mechanism

1. Electromagnetic (EM) interaction of the (perturbed) beam with its surroundings
-> Frequency dependent **impedance or wake fields**

2. **Coherent beam oscillations**: Coherent tunes Q and spectra f(Q)

3. Beam instabilities:
Imaginary part of the **coherent tune shift dQ** gives the growth rate τ^{-1}

4. **Cures:** Impedance reduction, increase the tune spread, feedback systems

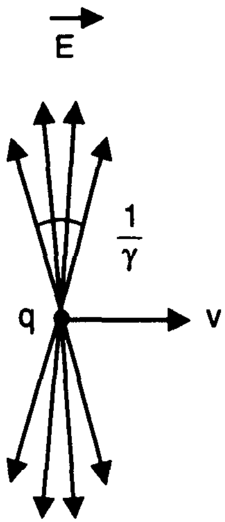
Impedance reduction $\rightarrow$ impedance: **Z**

Counter measures

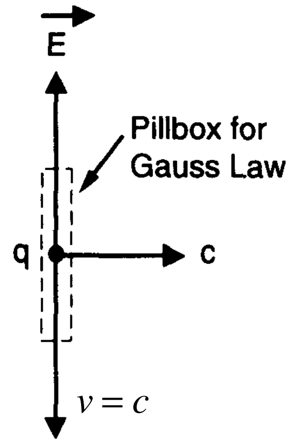
$\updownarrow$ EM interaction

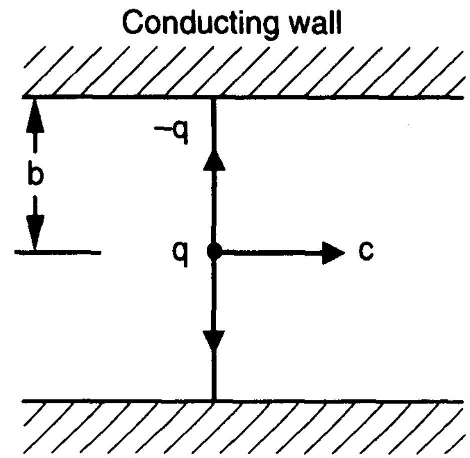
Landau damping, Feedback $\leftrightarrow$ oscillation modes**: Re Q** $\xrightarrow{\text{above threshold}}$ Instability**: Im Q**

Fields of a moving charge in vacuum



pancaked field





Electric field:

$$\vec{E} = \frac{q\vec{r}}{4\pi\varepsilon_0\gamma^2\left(z^2 + r^2/\gamma^2\right)^{3/2}}$$

$$E_r = \frac{q\lambda(s,t)}{2\pi\varepsilon_0 r}$$

$$B_\phi = \frac{q\mu_0\lambda(s,t)}{2\pi r}$$

Line density:

$$\lambda(s,t) = \frac{dN}{ds} = \delta(s - ct)$$

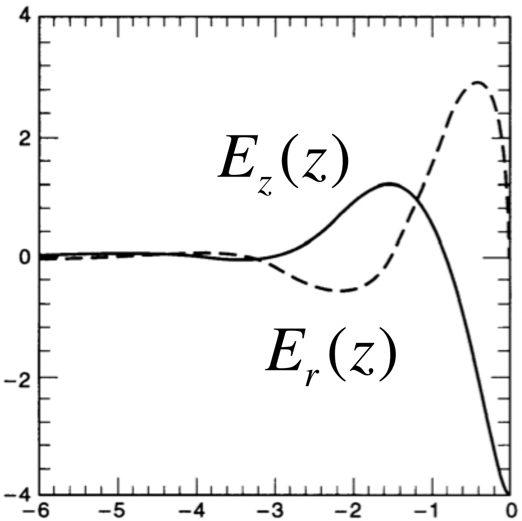
Image charges/currents follow the particle

A.Chao (1993)

Fields of a moving charge in a pipe

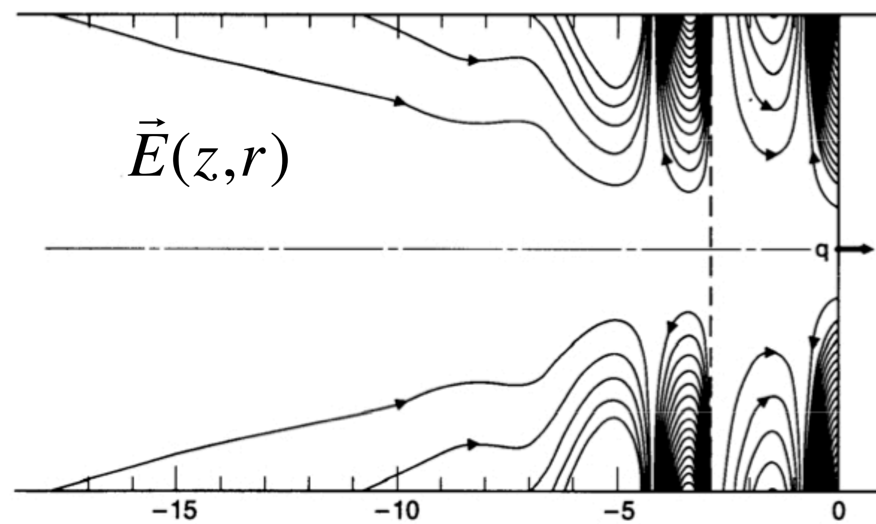
finite wall conductivity: σ_w

Longitudinal wake field



z

Wake electric field lines



z

Fields of/in a bunch: $$E_z(z) = -\frac{1}{\beta_0 c}\int_z^{\infty} I_b(z')W_z(z-z')dz'$$

A.Chao (1993)

Wake fields and impedances

current modulation

$$I(z,t) = \hat{I} e^{i(kz-\omega t)}$$

longitudinal impedance

$$Z_{\|}(\omega) = -\frac{1}{\hat{I}} \int_{z=0}^{L} E_z(z) e^{-i\omega z/c}\, dz \quad [\Omega]$$

induced voltage:

$$V_z(z,t) = -I(z,t) Z_{\|}(\omega)$$

beam offset modulation:

$$\bar{x}(z,t) = \hat{x} e^{i(kz-\omega t)}$$

transverse impedance:

$$Z_x(\omega) = \frac{-i}{q\beta_0 I\hat{x}} \int_0^L F_x e^{-i\omega z/c} dz \quad [\Omega / \mathrm{m}]$$

deflecting force:

$$F_x = q\left(E_x + \left[\vec{v}_0 \times \vec{B}\right]_x\right)$$

beam pipe

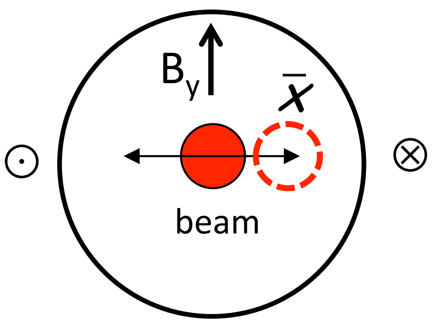
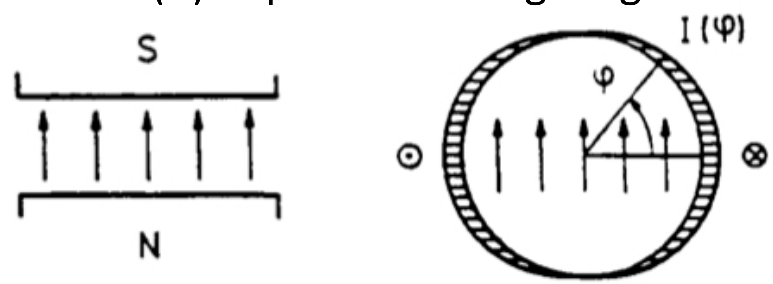


image currents

cos(ϑ)-superconducting magnet



surface image current: $j_s(\phi) = j_s(0)\cos(\phi)$

Impedance of a resistive beam pipe

Longitudinal

$$\oint \vec{E}\cdot d\vec{s} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{A} \;\Rightarrow\; E_z = E_w - \frac{g}{4\pi\varepsilon_0\beta_0 c\gamma^2}\frac{\partial I}{\partial z}$$

(Faraday's law)

$$Z_\parallel^I(\omega) = \frac{i\omega g Z_0}{2\omega_0\beta\gamma^2}$$

$$Z_0 = \left(\varepsilon_0 c\right)^{-1} = 377\ \Omega$$

$$E_w = -\frac{1}{2\pi R}Z_\parallel^R I \;\Rightarrow\; Z_\parallel^R = \frac{R}{\sigma_w db}$$

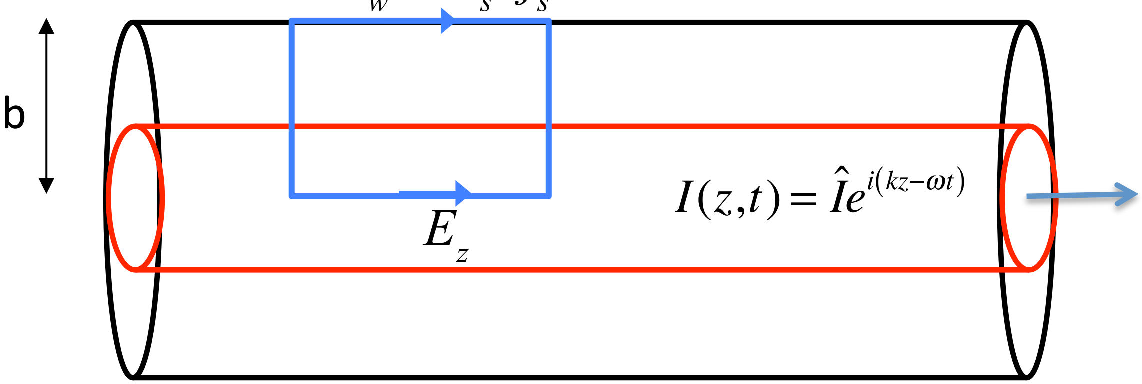
(low frequency resistive wall impedance)

$$d \lesssim \delta_w \qquad \delta_w = \sqrt{\frac{2}{\mu_0\sigma_w\omega}}$$

(skin depth)

surface wall conductivity: σ_s wall thickness: d

$$I_w \approx -I \qquad E_w = \sigma_s^{-1} j_s \qquad j_s \approx -\frac{I}{2\pi bd}$$



$$Z_\parallel(\omega) = Z_\parallel^R + iZ_\parallel^I(\omega)$$

$$Z_\parallel^R = \frac{R}{\sigma_w db}$$

(resistive wall impedance)

$$Z_\parallel^I(\omega) = \frac{i\omega g Z_0}{2\omega_0\beta\gamma^2}$$

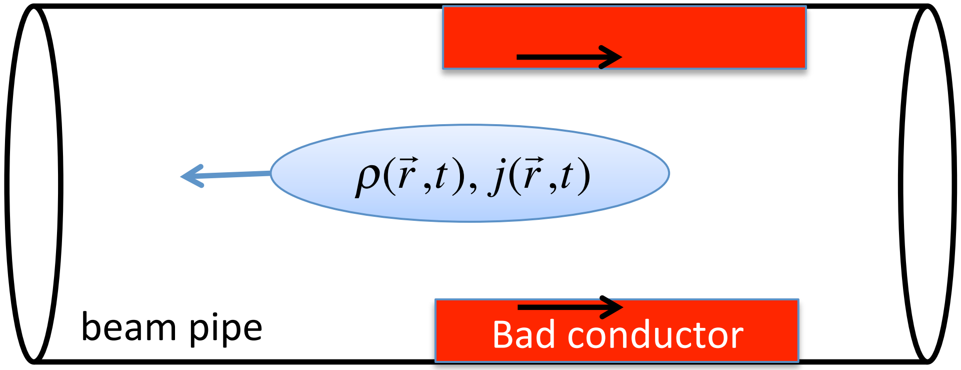
(space charge impedance)

Application: Energy loss and heat load

Energy loss of a bunch:

$$\frac{dW}{ds} = -q\int \lambda_b(z)E_z(z)dz$$

$$\lambda_b(z) = \frac{N_i}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$



Energy loss per turn (see e.g. A. Chao):

$$\Delta W_b = -\frac{q^2}{R}\int_{-\infty}^{\infty}|\lambda(\omega)|^2\ \mathrm{Re}Z^{\parallel}(\omega)d\omega$$

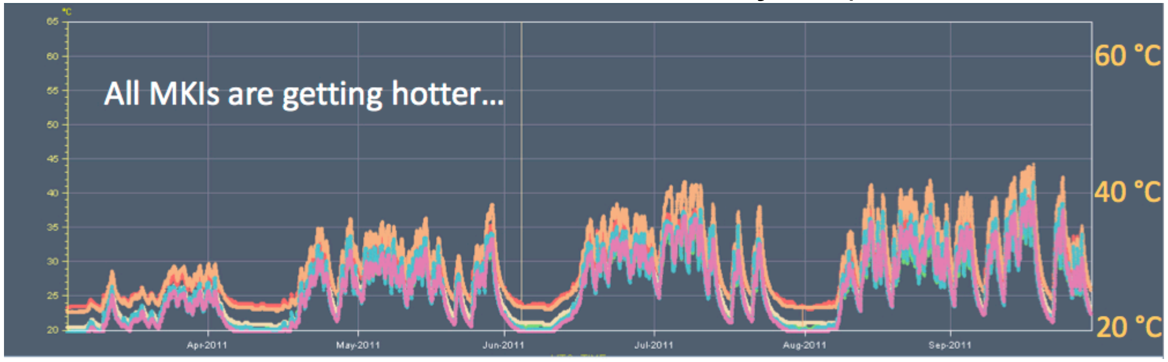
$$\lambda_b(\omega) = N_b\exp\left(-\frac{\omega^2\sigma_z^2}{2\beta_0^2c^2}\right)$$

Heat load: $P = \frac{c}{l_{bb}}\frac{dW}{ds}\quad [\mathrm{W/m}]$

Example: (Thin) resistive wall

$$Z_{\parallel}^{R} = \frac{R}{\sigma_w db} \quad \Rightarrow \Delta W_b \propto \frac{q^2N_b^2}{\sigma_z^{3/2}}\frac{1}{\sigma_w bd}$$

LHC injection kicker (ferrite loaded) heat up and one needs to wait hours that it cools down to reinject (B. Salvant, 2012)



-> CERN Courier, 2012

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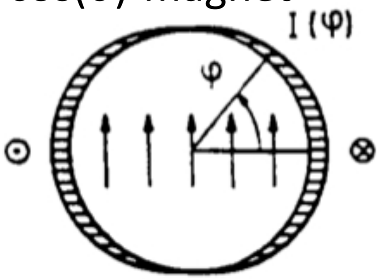
Transverse impedance of cylindrical beam pipe

beam with a horizontal offset in a pipe

cos(ϑ)-magnet



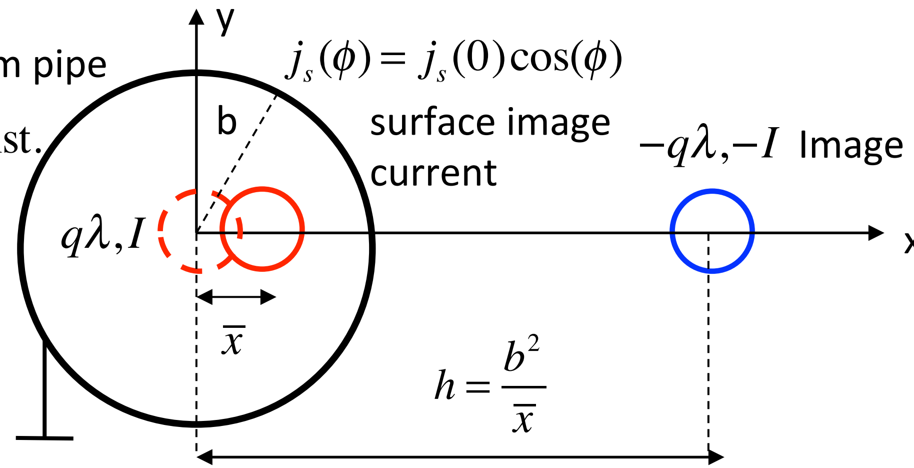


Image fields at the center of the beam:

$$E_x=\frac{q\lambda}{2\pi\varepsilon_0 h} \qquad B_y=\frac{\mu_0 I}{2\pi h}$$

Force on the beam center:

$$\Rightarrow \quad \bar{F}_x=q(E_x-v_0B_y)=\frac{qE_x}{\gamma^2}=\frac{q^2\lambda\bar{x}}{2\pi\varepsilon_0\gamma^2b^2}$$

$$\Rightarrow \quad Z_x=-i\frac{Z_0}{2\pi(\beta_0\gamma_0)^2b^2} \qquad Z_0=\left(\varepsilon_0 c\right)^{-1}=377\ \Omega$$

(imaginary wall impedance)

Transverse impedance of a resistive beam pipe

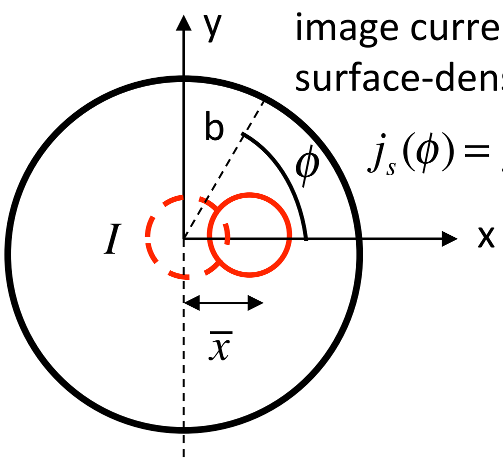


image current surface-density:

$$j_s(\phi) = j_s(0)\cos(\phi)$$

beam offset:

$$\bar{x}(t) = \hat{x}\exp(-i\omega t)$$

image surface current density:

$$j_s = \hat{j}_s \exp(-i\omega t) \qquad j_s(0) \approx \frac{I\bar{x}}{\pi b^2}$$

longitudinal electric field:

$$E_z = \hat{E}_z \exp(-i\omega t) \qquad E_z = \sigma_s^{-1} j_s \qquad \sigma_s = \sigma_w d$$

(surface conductivity)

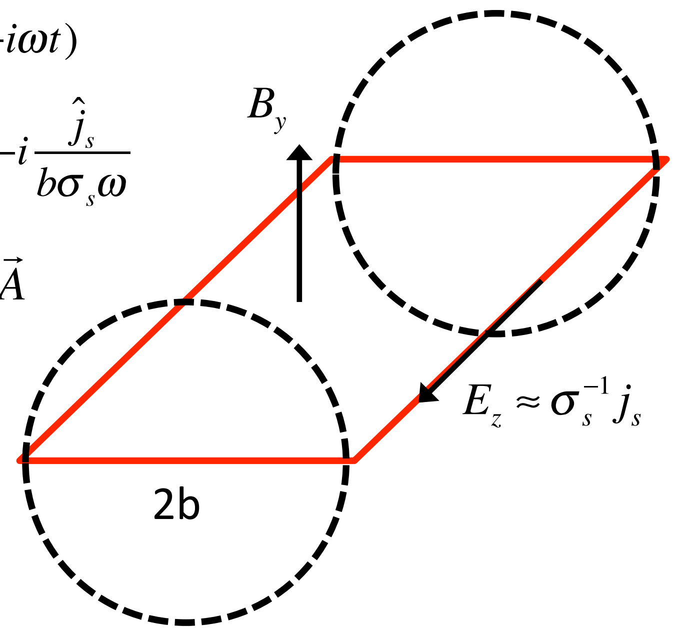
Vertical B-field:

$$B_y = \hat{B}_y \exp(-i\omega t)$$

$$\hat{B}_y = -i\frac{\hat{E}_z}{\omega b} = -i\frac{\hat{j}_s}{b\sigma_s \omega}$$

$$\oint \vec{E}\cdot d\vec{s} = -\int \dot{\vec{B}}\cdot d\vec{A}$$

(Faraday)



$$Z_x(\omega) = \frac{-i}{\beta_0 I\hat{x}}\left(\hat{E}_x - v_0\hat{B}_y\right)L \quad \Rightarrow$$

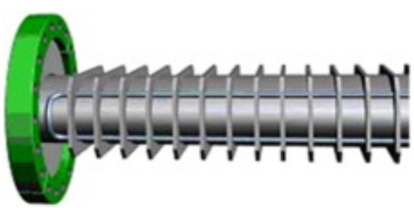
$$Z_x(\omega) = \frac{2cR}{b^3 d\sigma_w \omega}, \quad d \ll \delta_w$$

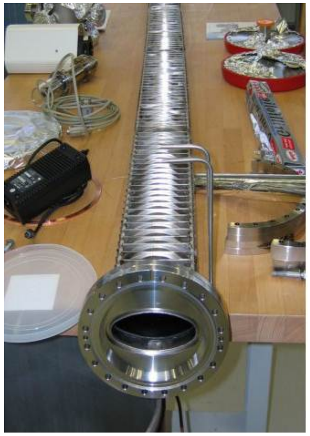
(resistive wall impedance)

$$\delta_w = \sqrt{\frac{2}{\mu_0 \sigma_w \omega}} \qquad \text{(skin depth)}$$

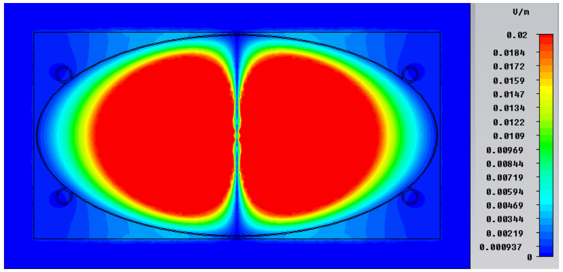
Example: The thin SIS-100 beam pipe

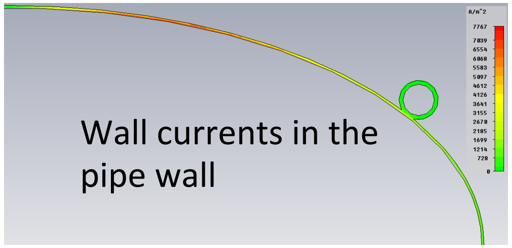
SIS-100 thin (**0.3 mm**) stainless steel beam pipe with cooling tubes attached.





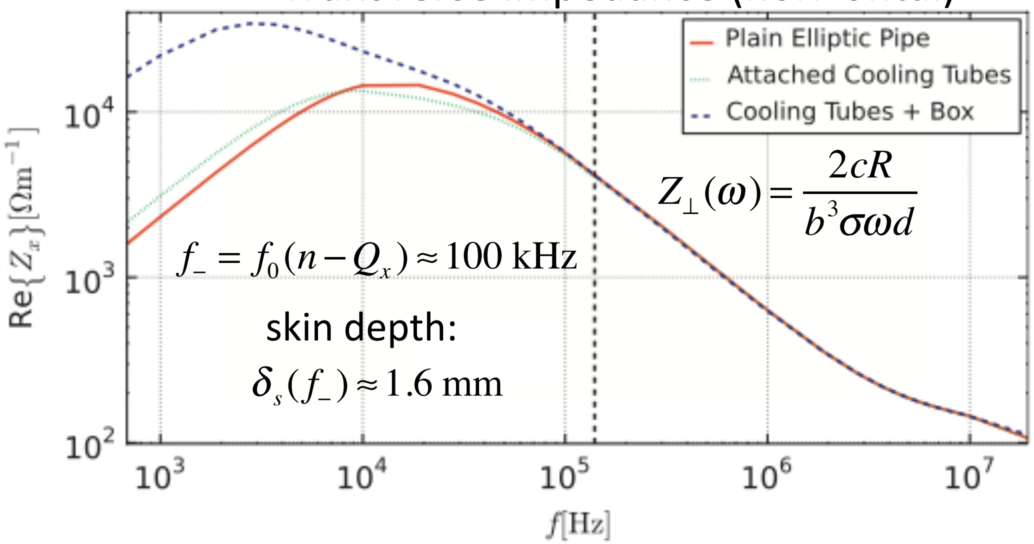
Longitudinal electric field in the SIS100 pipe structure resulting from a dipolar excitation (≈ 150 kHz).





- still mechanically robust (+ rips)
- for 4 T/s: tolerable heating + field distortion
- rf shielding of EM fields (> 50 kHz)
- problem: large resistive impedance !

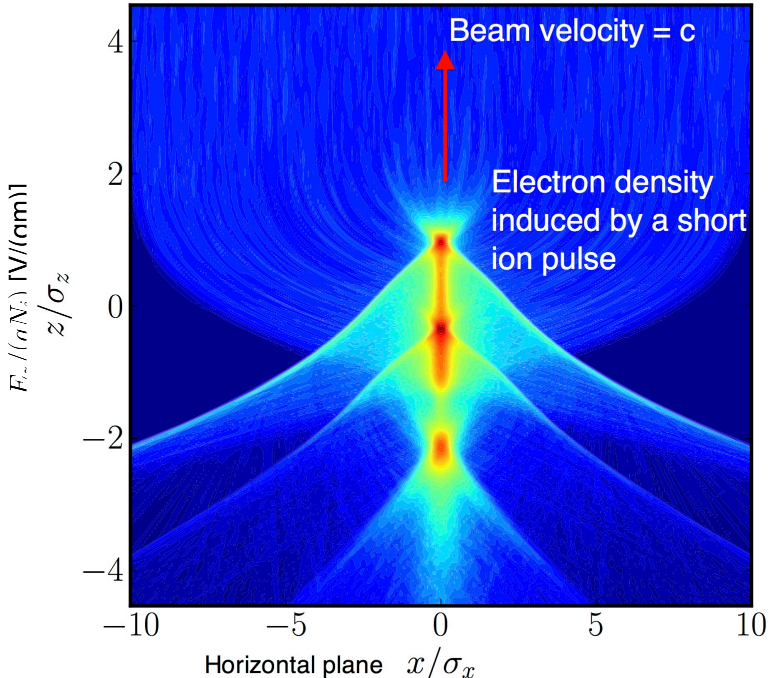
Transverse impedance (horizontal)

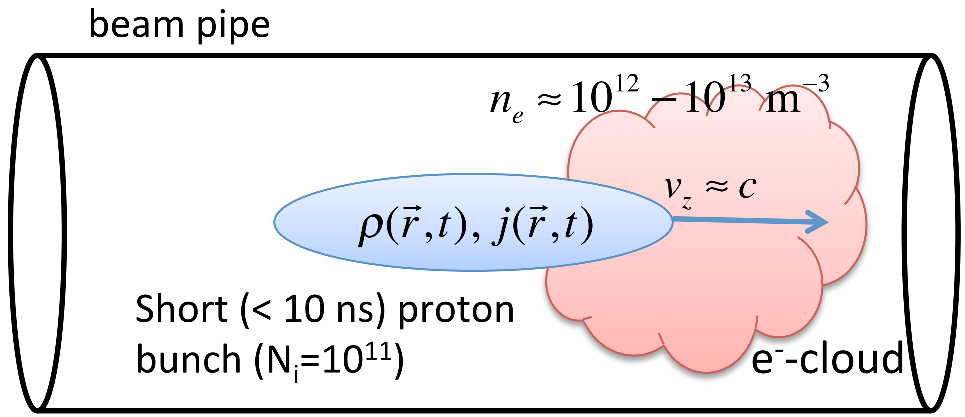


U. Niedermayer, O. Boine-F., NIM A 2012

- In the frequency range of interest outside structures do not contribute to the impedance.
- **The thin resistive beam pipe is the major source for the expected head-tail instabilities in SIS-100**.

Field of research: Wake field of an electron cloud

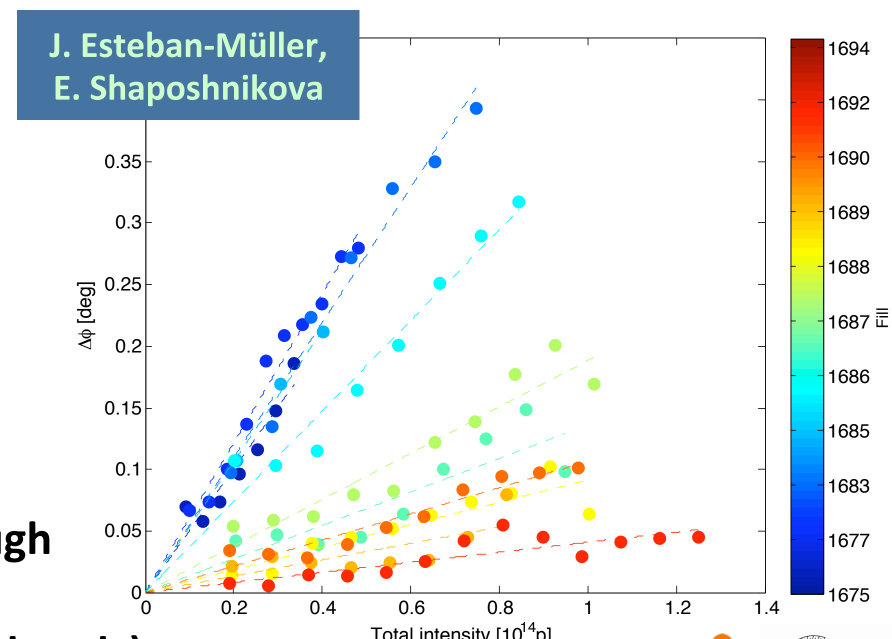




in an electron cloud:

$$\frac{dW}{ds} \approx \frac{4\pi}{\epsilon_0} q^2 N_b^2 n_e r_e \ln\left(\frac{b}{a}\right)$$

-> observed rf phase shift in LHC.



The e-cloud cannot strictly be described through a longitudinal impedance !

-> depends on beam parameters (e.g. bunched or dc)

Field of research: Wake field of an electron cloud

Transverse wake can be approximated as a broad band oscillator:

$$F_x(z) \approx c\frac{R_s}{Q}\sin\left(\frac{\omega_e}{c}z\right) \qquad \frac{R_s}{Q} \propto n_e$$

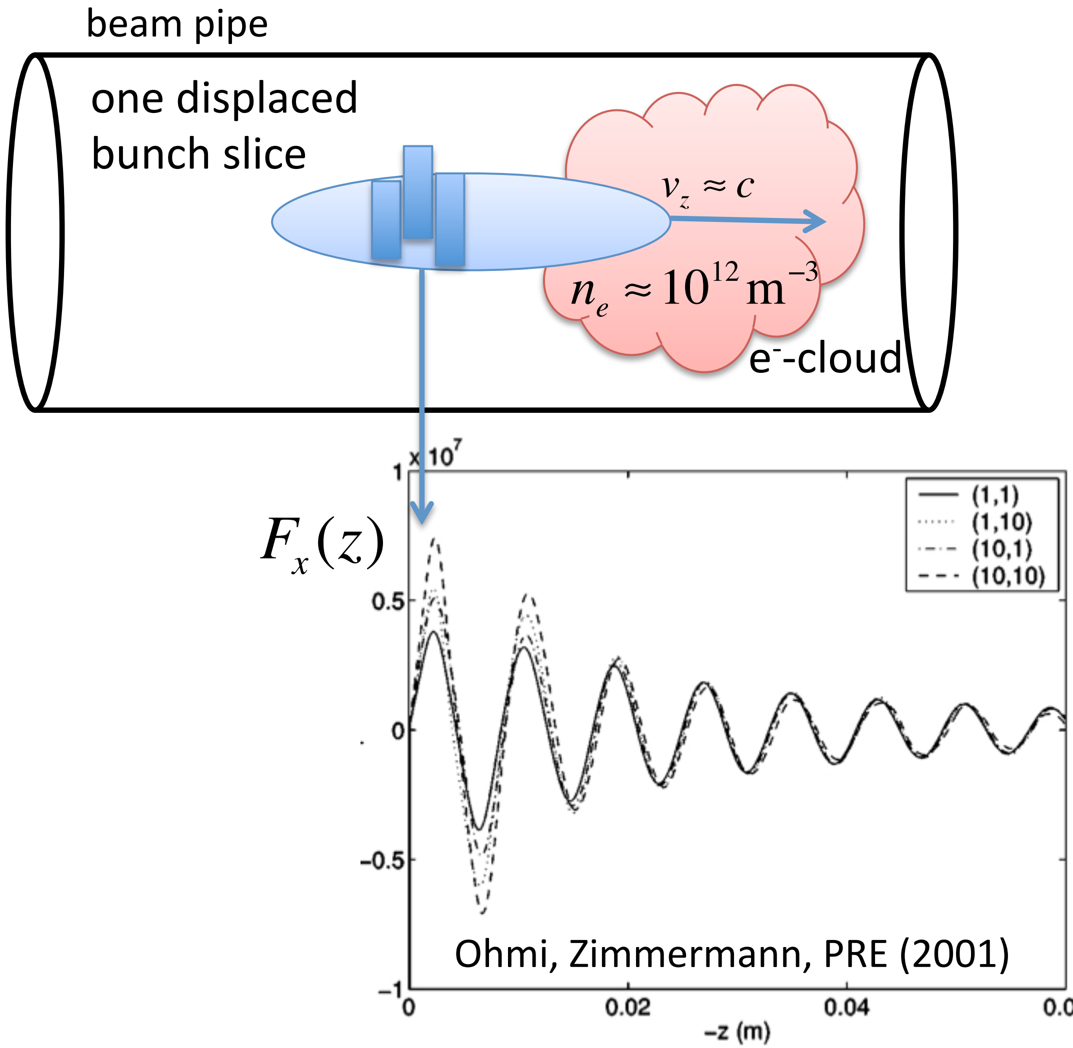
electron oscillation frequency:

$$f_e \approx 100 - 300\ \mathrm{MHz}$$

Transverse impedance:

$$Z_x(\omega) \approx \frac{c}{\omega}\frac{R_s}{1 + iQ\left(\frac{\omega_e}{\omega} - \frac{\omega}{\omega_e}\right)}$$

-> Transverse beam instabilities



Transverse beam oscillation spectra

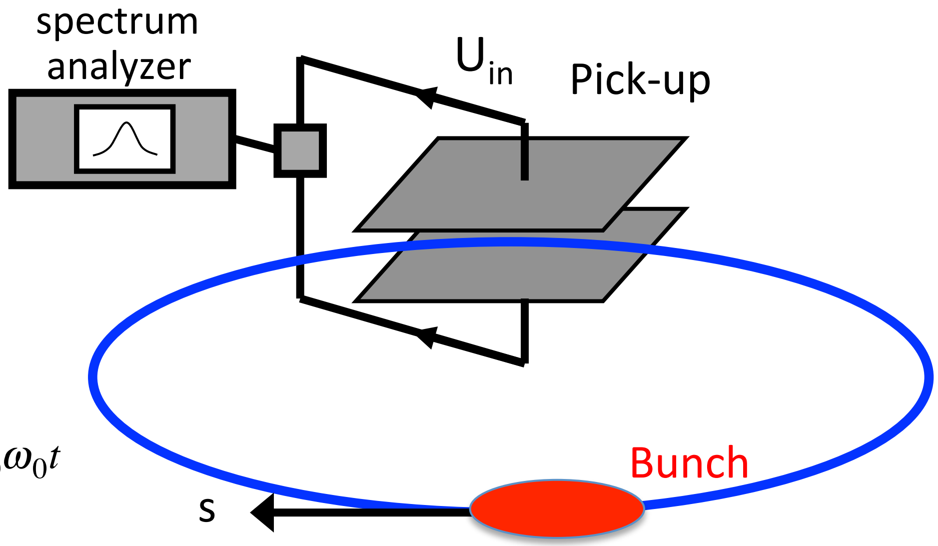
Dipole moment: $\Delta(t) = I(t)\overline{x}(t)$

$\overline{x}(t)$: transverse beam offset

Difference signal U_Δ : beam 'offset' fluctuations

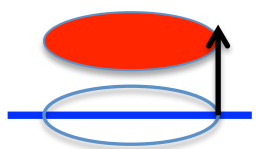
Transverse oscillation spectrum:

$$\Delta(t) \xrightarrow{FFT} \Delta(f)$$



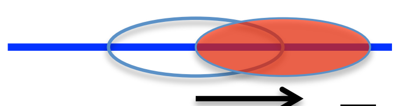
$s = \beta_0 c(\tau - \tau_0) = z - z_0$ (longitudinal position)

Transverse (rigid) bunch oscillation:



$$\overline{x}(t) \sim e^{iQ_0\omega_0 t}$$

Longitudinal (rigid) binch oscillation:



$$\overline{\tau}(t) \sim e^{i\omega_0 Q_s t}$$

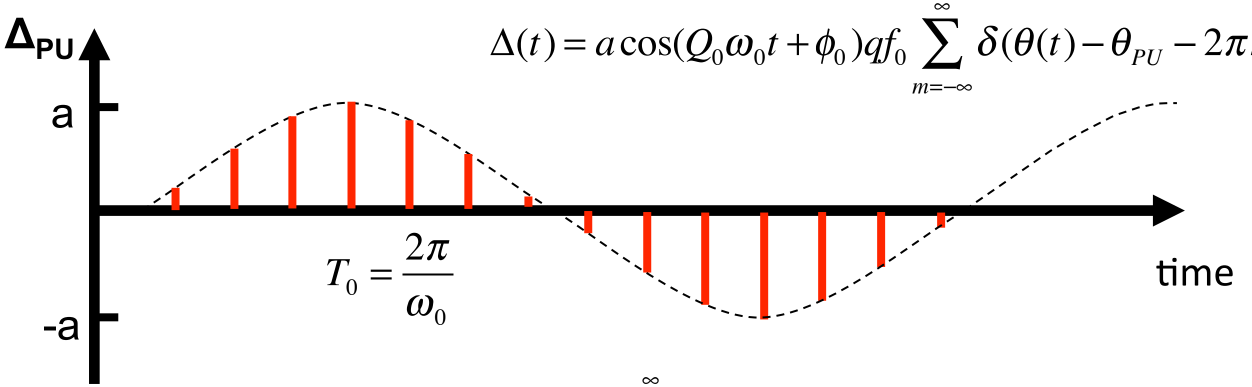
Bunch current profile:

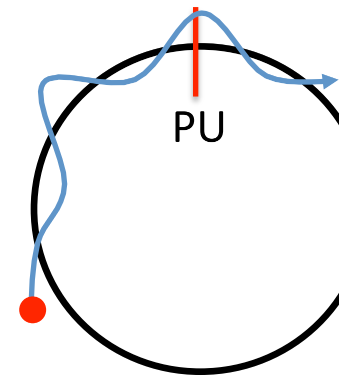
$$I(t) = I_p \exp\left(-\frac{(\tau - \overline{\tau})^2}{2\sigma^2}\right)$$

Single particle oscillation spectrum

Betatron oscillation: $x(t)=a\cos(Q_0\omega_0 t+\phi_0)$ Dipole moment: $\Delta(t) = x(t)\, I(t)$

$$\Delta(t)=a\cos(Q_0\omega_0 t+\phi_0)qf_0\sum_{m=-\infty}^{\infty}\delta(\theta(t)-\theta_{PU}-2\pi m)$$

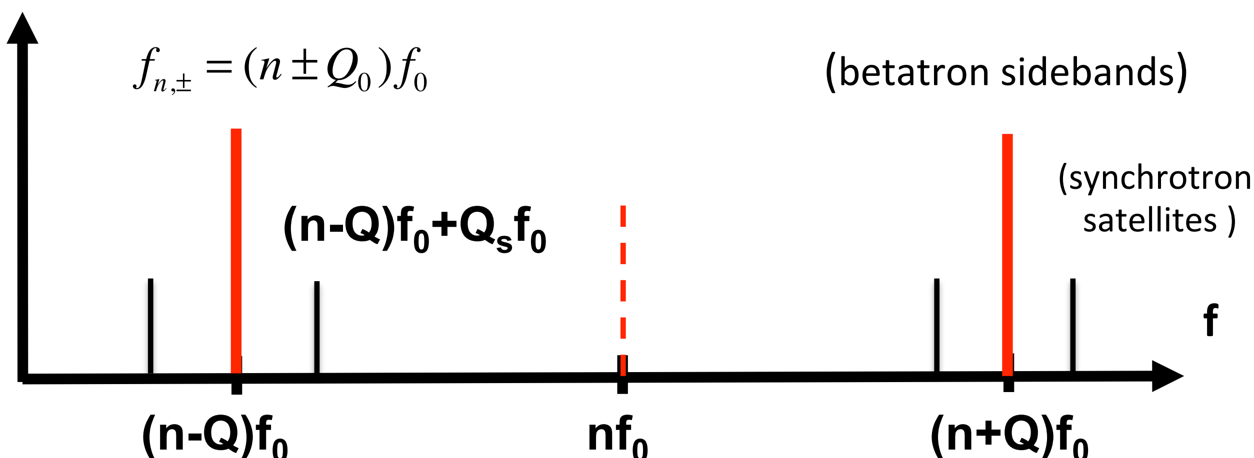




Frequency domain: $\Delta(t)=qaf_0\Re\sum_{n=-\infty}^{\infty}\exp\left[-i(n+Q_0)\omega_0 t+\phi_0\right]$

Looking at positive frequencies only:

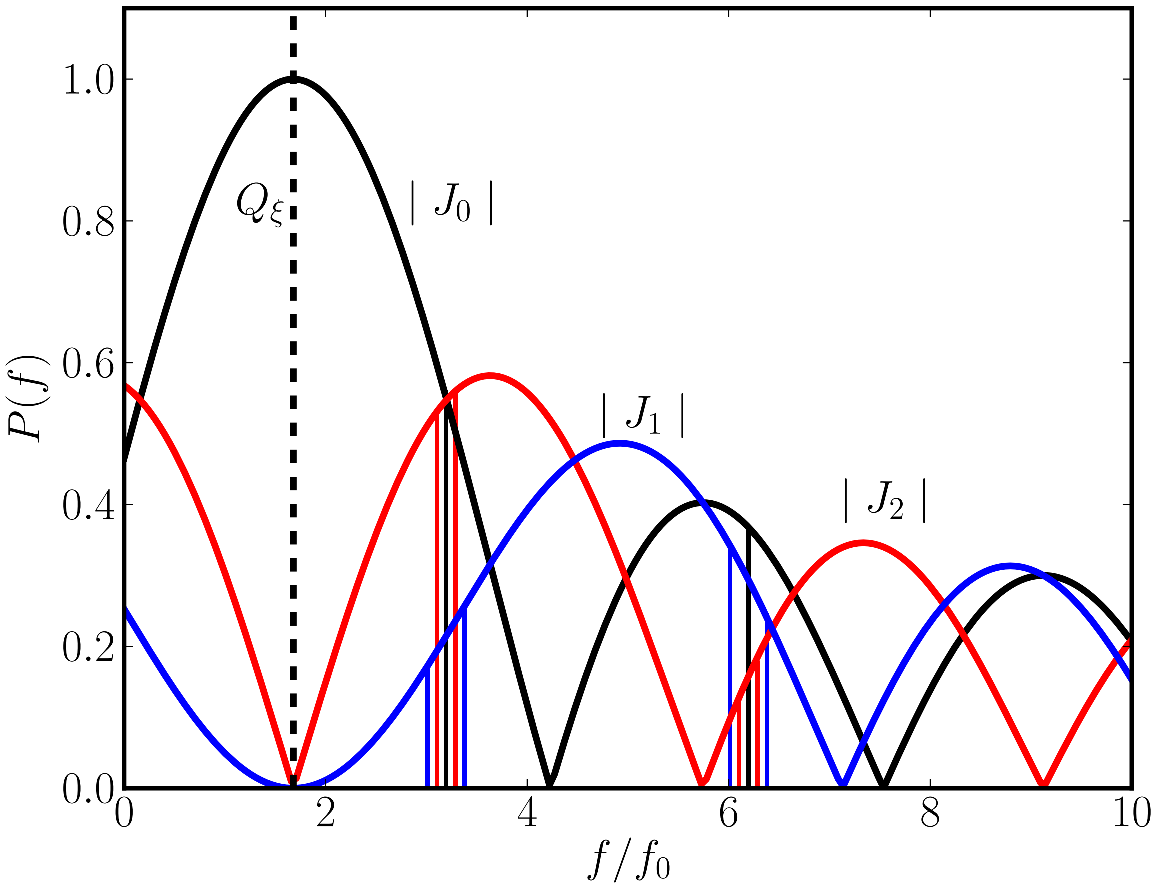
$f_{n,\pm}=(n\pm Q_0)f_0$ (betatron sidebands)



Only positive lines can be seen by the spectrum analyzer.

Single particle power spectrum

$$h(f) = |\Delta_k|^2 \sim \left| J_k\left[((n \pm Q_0) - Q_\xi)\omega_0 \hat{\tau}\right] \right|$$



Chromatic tune shift:

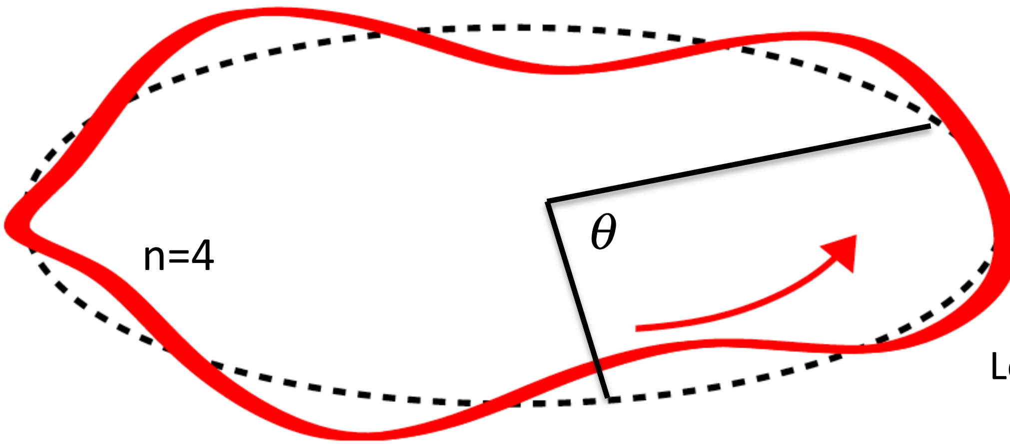
$$Q_\xi = \xi / \eta_0$$

Chromaticity:

$$\xi = \frac{\Delta Q}{\Delta p / p_0}$$

Coasting beam: transverse spectrum

Betatron oscillation of one beam slice: $\bar{x}(\theta_0, t) = \hat{x}e^{\pm i\omega_0 Q_0 t}$



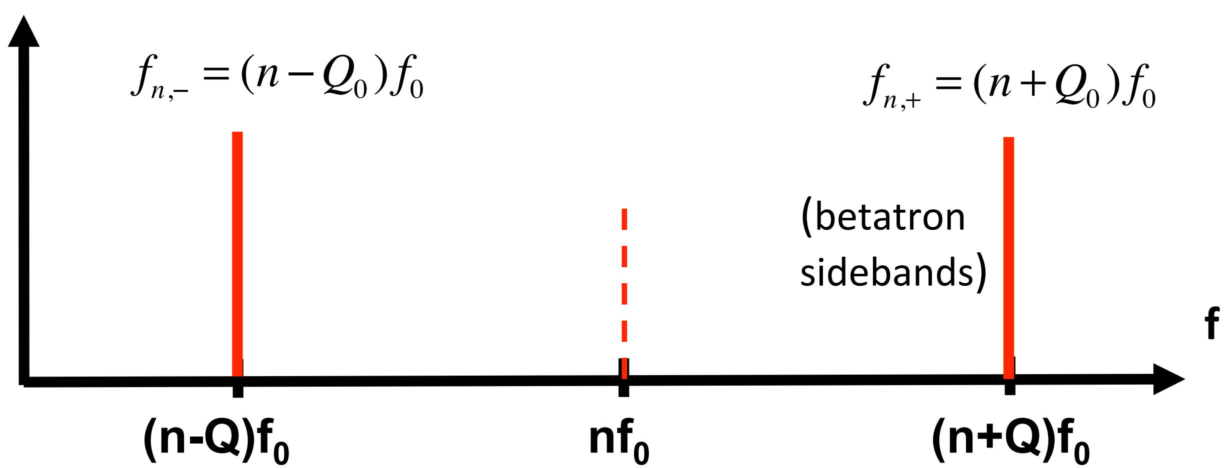
Seen by a stationary observer (pick-up):

$$\bar{x}(\theta, t) = \hat{x}e^{i(n\theta - \omega t)}$$

fast/slow mode: $\omega_{n,\pm} = (n \pm Q_0)\omega_0$

Lowest mode in SIS-18 ($Q_{0y} = 3.24$):

$$f_{\min,-} = (4 - Q_{y0})f_0 \approx 0.4 f_0 \approx 80 \text{ kHz}$$



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Beam oscillations with impedances

coasting beam

single particle betatron oscillations: $x'' + \frac{Q_0^2}{R^2}x = \frac{F_x}{\gamma_0 m v_0^2}$

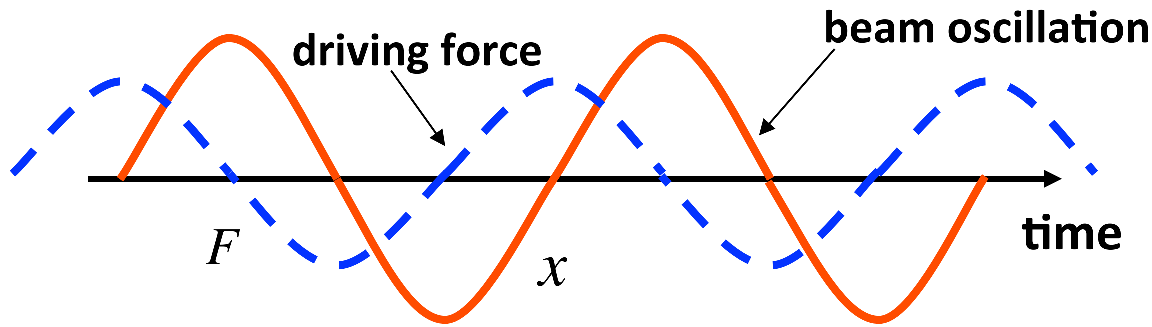
horizontal divergence: $x' = \frac{dx}{ds}$ path length: $s = \beta_0 ct$

beam offset: $\bar{x}(s) = \int x\rho_b(x,y,s)dxdy$

$$\bar{x}'' + \frac{Q_0^2}{R^2}\bar{x} = \frac{\bar{F}_x}{\gamma_0 m v_0^2} = i\underbrace{\frac{qIZ_x}{\beta_0 E_0}}_{\frac{2Q_0\Delta Q_x^c}{R^2}}\bar{x}$$

$$Z_x(\omega) = \frac{-i}{q\beta_0 I\bar{x}}\bar{F}_x$$

$$\Rightarrow \quad \bar{x}(\theta_0) = \hat{x}e^{i\omega_0(Q_0+\Delta Q_x^c)t}$$



In order to increase the amplitude of a driven oscillator the driving force must be ahead (in phase) of the motion.

Anyone who has pushed a child on a swing will know this.....

Coherent tune shift: $\Delta Q_x^c = -i\frac{qIR^2 Z_x}{2Q_0\beta_0 E_0}$

Complex impedance: $Z_x = Z_x^R + iZ_x^I$

Coherent tune shift: $\Delta Q_R^c = \omega_0 \operatorname{Re}\left(\Delta Q_x^c\right)$

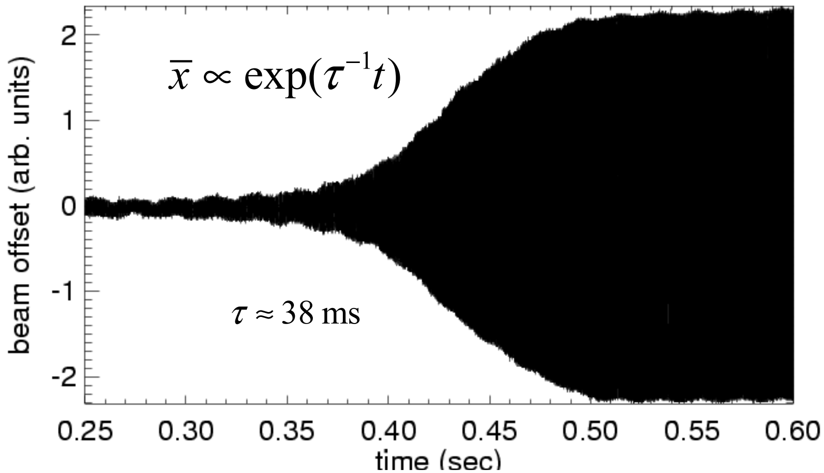
Growth rate: $\tau_I^{-1} = \omega_0 \operatorname{Im}\left(\Delta Q_x^c\right)$

$$\Rightarrow \quad \bar{x}(\theta_0) = e^{-\tau_I^{-1}t}e^{\pm i\omega_0(Q_0+\Delta Q_R^c)t}$$

Resistive wall instability

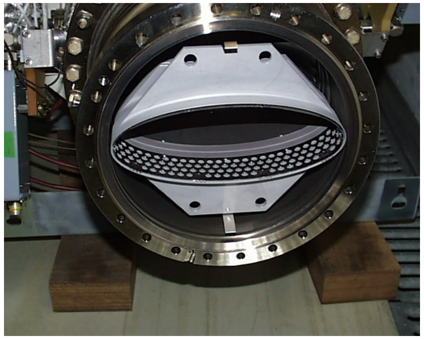
coasting Ar^{18+} beam in SIS 18 at injection (11.4 MeV/u)

Beam offset vs. time observed in SIS 18



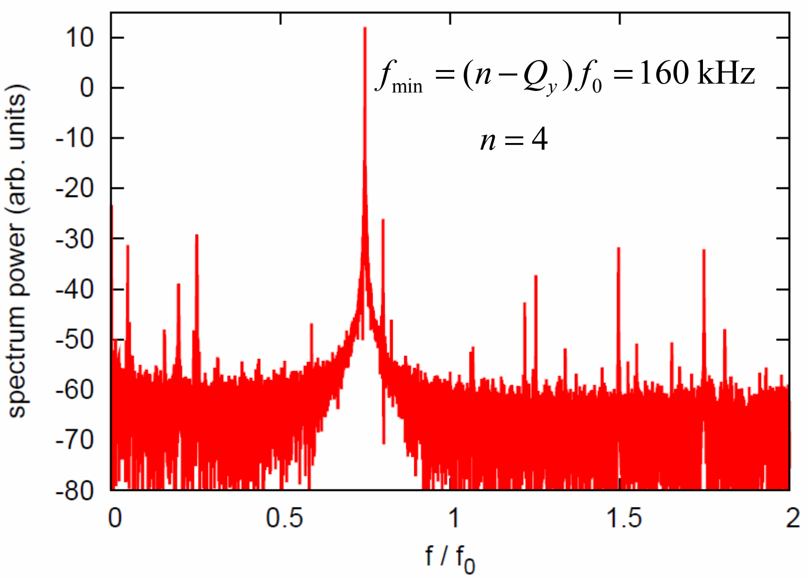
$$Z_x^R(\omega) = \frac{c}{\pi b^3 d \sigma_w \omega}$$

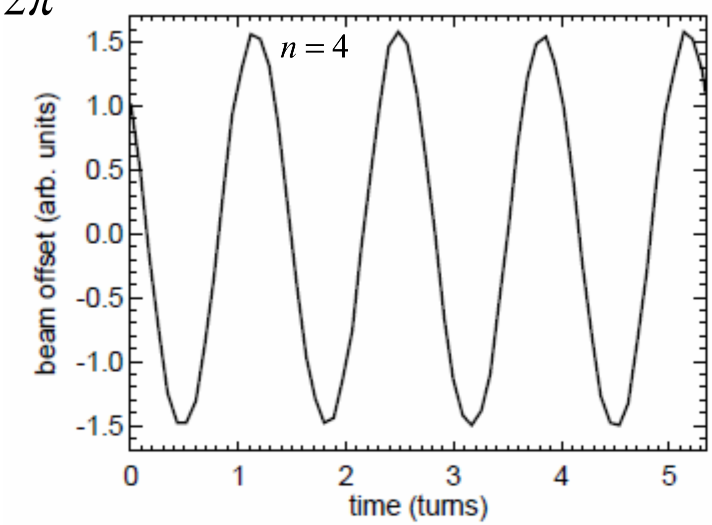
(transverse resistive wall impedance)



$$\tau^{-1} = \omega_0 \mathrm{Im}\left(\Delta Q_x^{coh}\right) = \frac{qIR^2c}{2\pi Q \beta_0 E_0 b^3 d \sigma_w \omega}$$

$$\frac{\omega}{2\pi} = f_{\min,-} = (4 - Q_{y0}) f_0 \approx 160 \text{ kHz}$$





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Landau damping
coasting beam

A. Hofmann, Landau damping, CAS 1995

slow mode/betatron line:

$$\omega_n = (n-Q)\omega_0$$

momentum dependence:

$$\frac{d\omega_n}{(dp/p)} = (n-Q)\frac{d\omega_0}{(dp/p)} + \frac{dQ}{(dp/p)}\omega_0$$

$$\xi = \frac{\Delta Q}{\Delta p/p_0} \qquad \frac{\Delta\omega}{\omega_0} = -\eta_0 \frac{\Delta p}{p_0}$$

(chromaticity) (frequency slip)

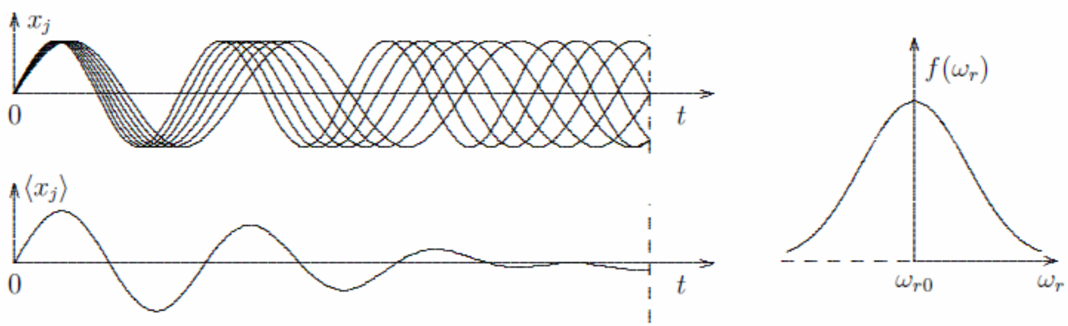
Frequency/tune spread:

$$\Rightarrow \quad \delta\omega_n = (\xi - \eta_0(n-Q))\omega_0 \frac{\Delta p}{p}$$

Damping:

$$\tau_n^{-1} \approx \delta\omega_n, \quad \bar{x}(t) = \hat{x}\exp\left(i\omega_n t - \delta\omega_n t\right)$$

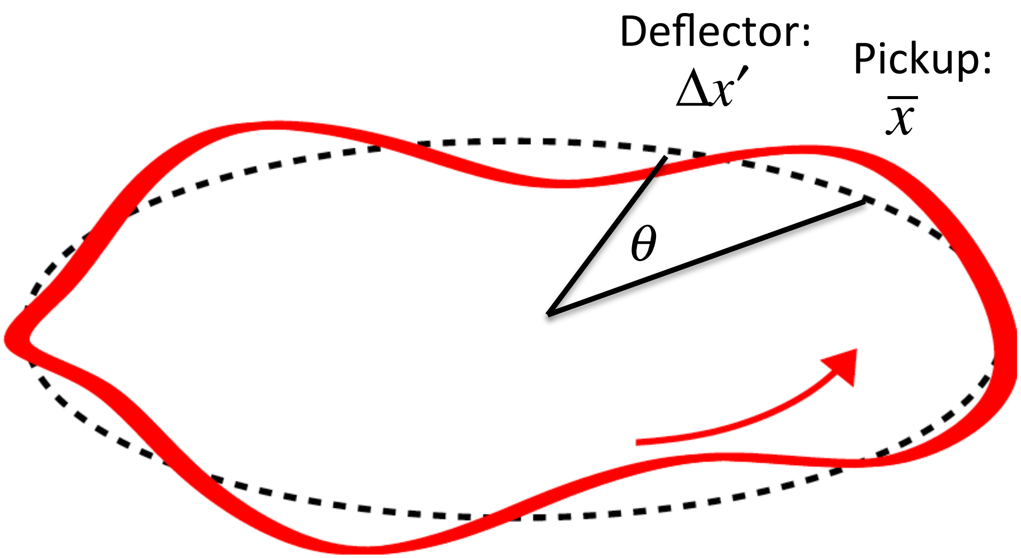
- Landau damping vs. phase mixing
- Landau damping causes emittance growth
- Landau damping can be reversed: Echoes
- Octupoles: increase Landau damping



Beam stability: $\tau_n^{-1} \gtrsim \tau_I^{-1}$ or $\delta Q_n \gtrsim \left|\Delta Q^c\right|$

Instability threshold !

Feedback systems (transverse dampers)



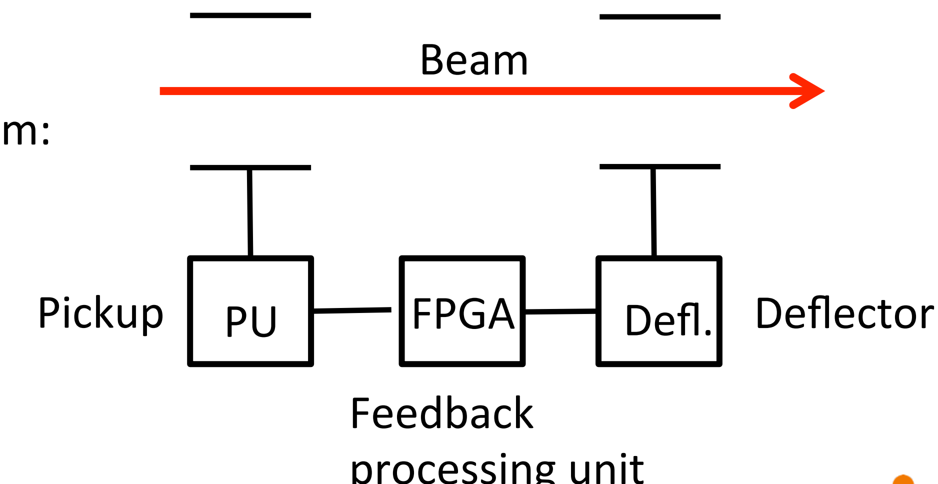
$$\bar{x}'' + \frac{Q_0^2}{R^2}\bar{x} = \Delta x' \delta(s - s_k)$$

Normalized gain:

$$G = \frac{R\Delta x'}{Q_0 \bar{x}} \approx 2T_0 \tau_I^{-1}$$

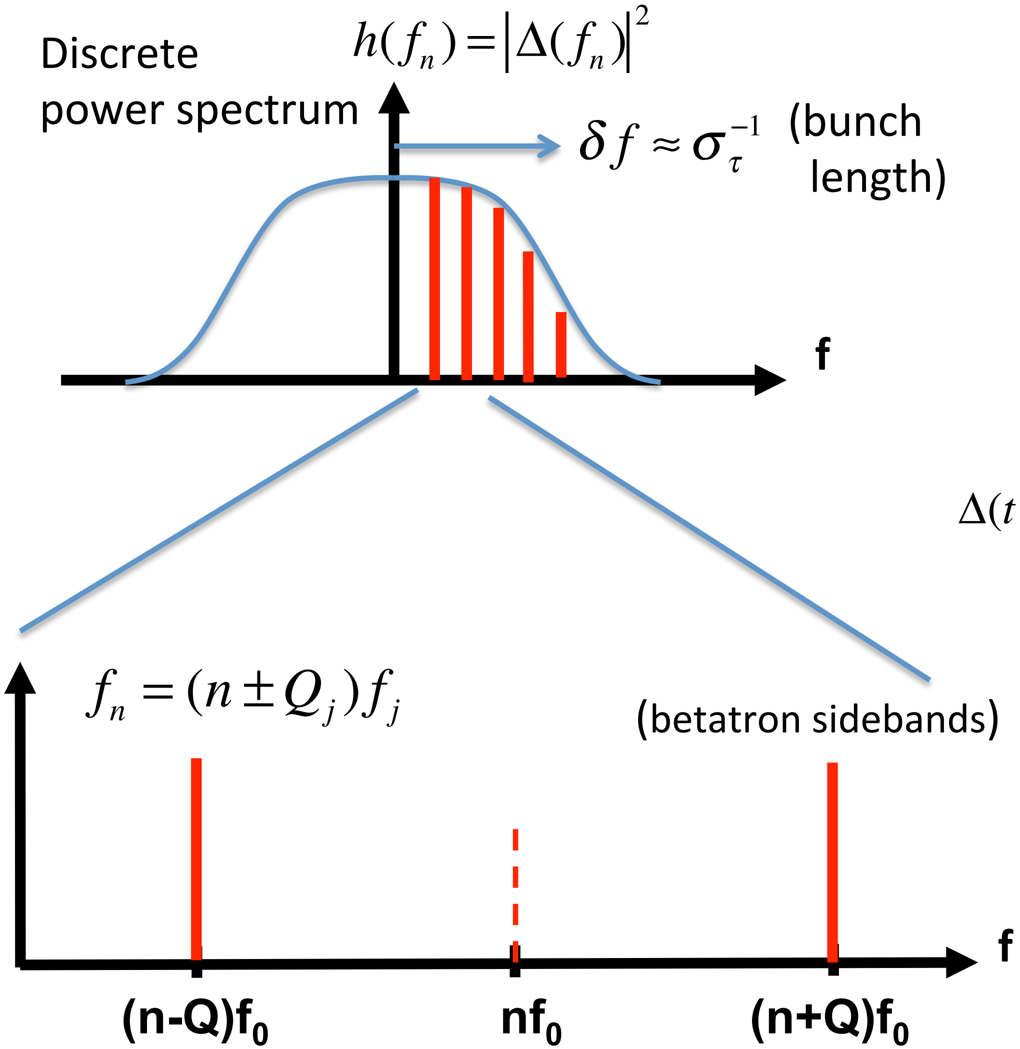
Frequency range: 10 kHz-50 MHz

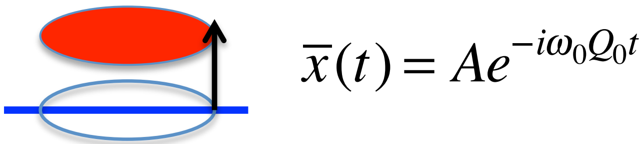
(Digital) transverse feedback system:



Poster: A. Alhumaidi

Transverse rigid bunch oscillation





Dipole signal: Standing wave pattern.

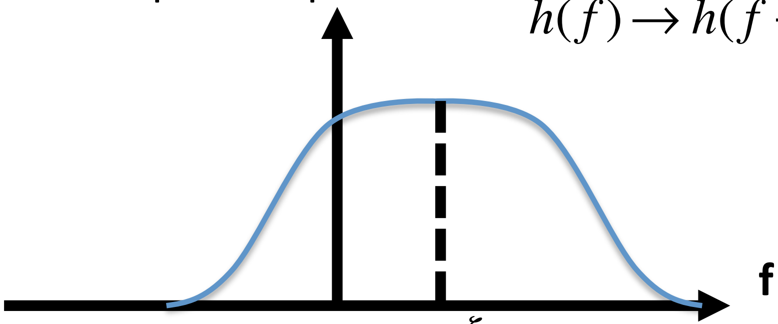
$$\Delta(t) = \quad I_b(\tau) \quad \times Ae^{i\omega_0 Q_0 t}$$

Transverse rigid bunch oscillation
with chromaticity

Zotter, Sacherer (1976)

Shifted power spectrum

$$h(f) \rightarrow h(f - f_\xi)$$

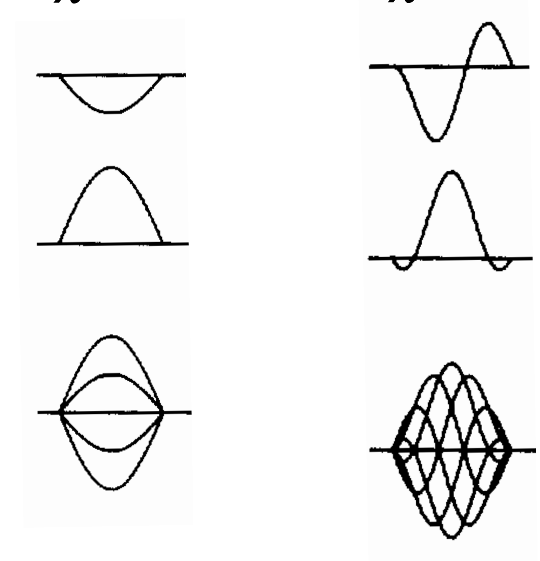


$$f_\xi = \frac{\xi}{\eta_0} f_0$$

$$\bar{x}(t) = A e^{i\chi \frac{\tau}{\tau_b}} e^{-i\omega_0 Q_0 t}$$

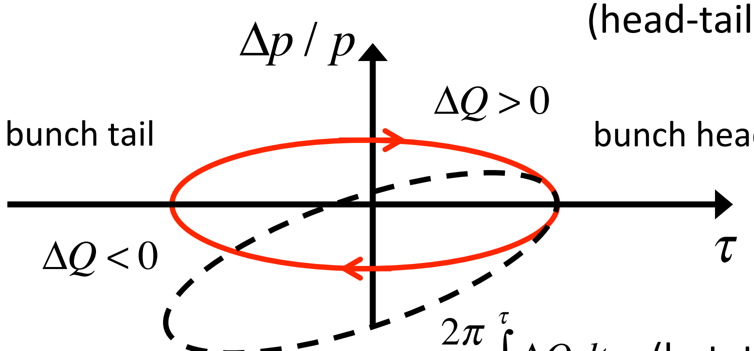
Traveling wave pattern !

$\chi = 0$ $\quad\quad$ $\chi \neq 0$



$$\Delta Q = \xi \frac{\Delta p}{p} \qquad \frac{\Delta p}{p} = -\frac{\dot{\tau}}{\eta_0} \qquad \chi = \frac{2\pi}{T_0} \int_0^{\tau_b} \Delta Q \, dt = \frac{\xi}{\eta_0} \omega_0 \tau_b$$

(head-tail phase shift)

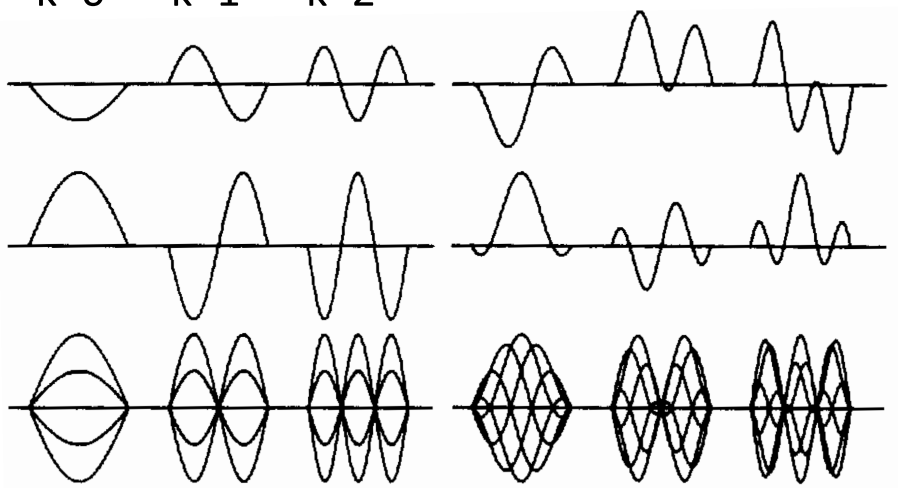


$$\frac{2\pi}{T_0} \int_0^{\tau} \Delta Q \, dt$$ (betatron phase shift)

Head-tail oscillations

Zotter, Sacherer (1976)

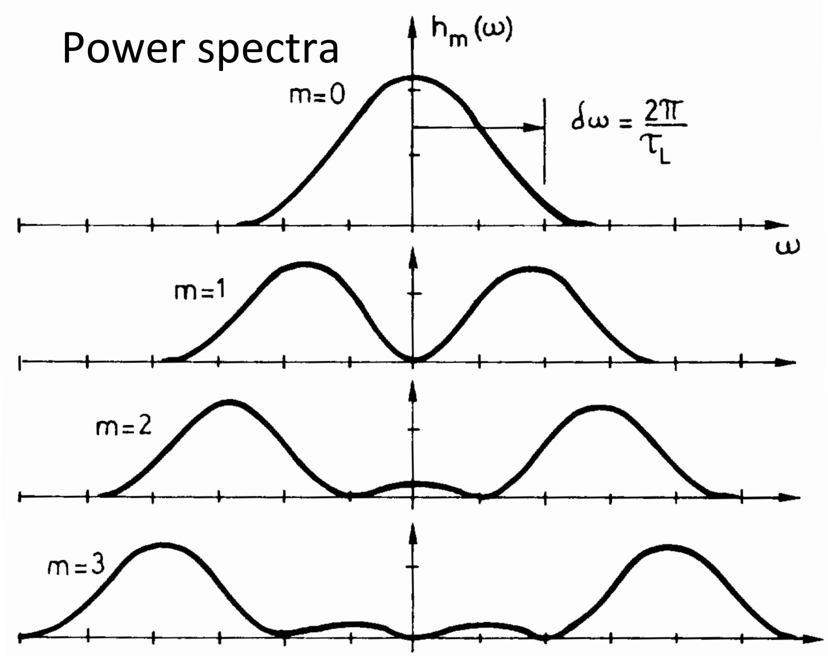
k=0 k=1 k=2



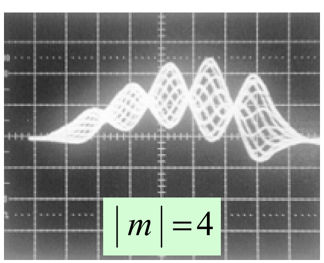
$\chi = 0$ $\chi \neq 0$

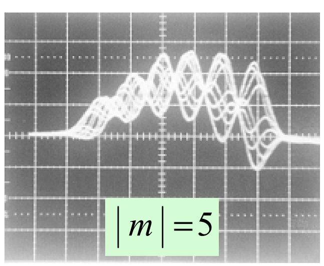
$$\times\, e^{i\omega_0 Q_k t} \qquad Q_k = Q_0 + kQ_s$$

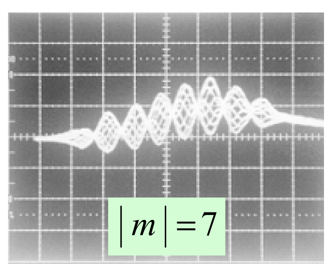
$$\Delta(\tau,t) = I(\tau)\bar{x}_k(\tau,t) \propto \cos(k\pi\tau/\tau_b)\, e^{-i\chi\tau/\tau_b} e^{-iQ_k t}$$



Unstable head-tail oscillations in the CERN PS







E.Metral et al. (2005)

Head tail instability

coasting beam spectrum / coherent tune shift:

Zotter, Sacherer (1976)

$$\Delta(\omega) = I_0 \bar{x}(\omega) = \delta(\omega - \omega_n) \qquad \Delta Q_x^c = -i\frac{qIR^2}{2Q_0\beta_0 E_0} Z_x(\omega_n)$$

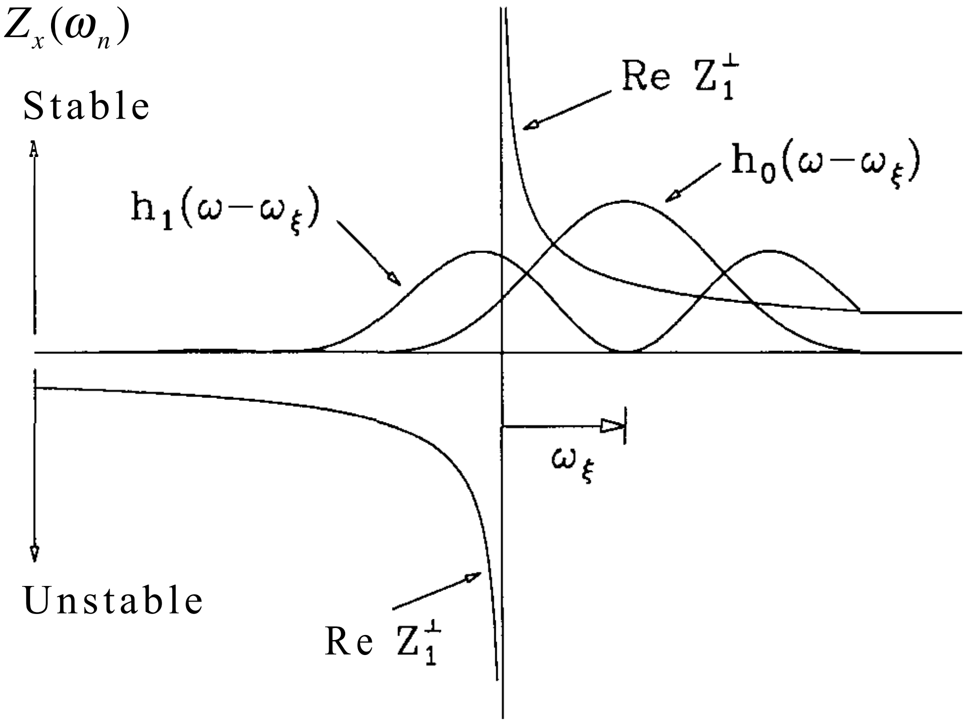
Head-tail tune shift:

$$\Delta Q_x^c \propto \sum \left|\Delta(\omega)\right|^2 Z(\omega) \qquad \omega_\xi = \frac{\xi}{\eta_0}\omega_0$$

$$\tau_k^{-1} = -\frac{1}{1+k}\frac{qI}{2Q_0\beta_0 E_0 l_b}\frac{\sum h(\omega-\omega_\xi)\,\mathrm{Re}\,Z(\omega)}{\sum h(\omega-\omega_\xi)}$$

for a smooth impedance:

$$\tau_0^{-1} = -\frac{qI}{2Q_0\beta_0 E_0 l_b}\mathrm{Re}\,Z_x(\omega_\xi)$$



Positive/negative chromaticity above/below transition shifts all modes towards the positive frequency side. Mode k = 0 becomes stable, but mode k = 1 may be unstable because it samples more negative Z than positive. **No head-tail instability for compensated chromaticity.**

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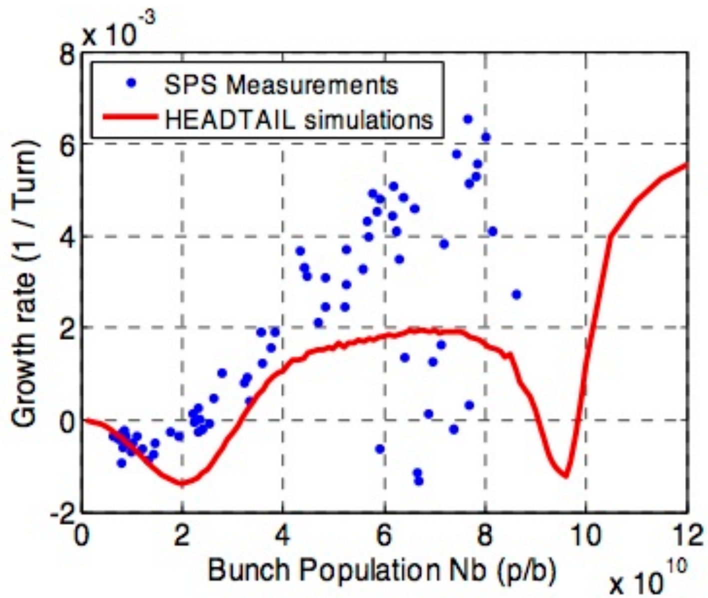
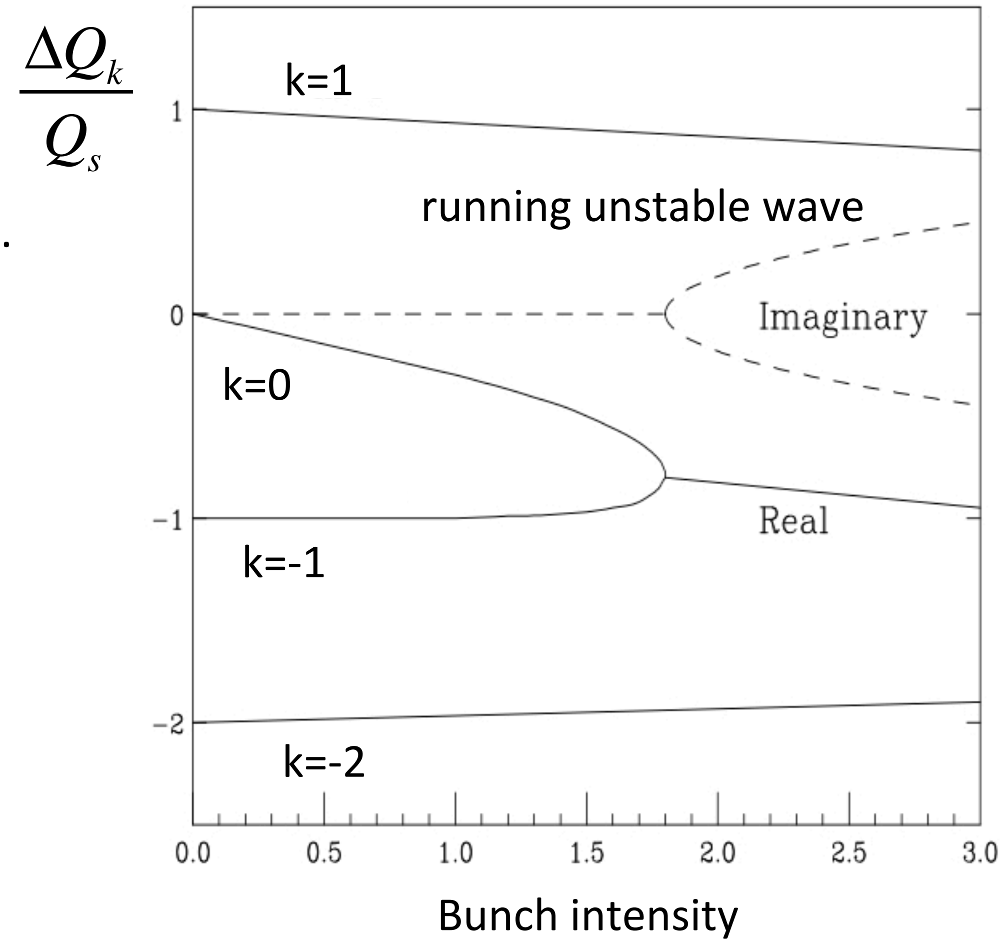
Transverse Mode-Coupling Instability (TMCI)

Well known for electron ring machines.

Observed in the CERN SPS with protons.

Caused by the kicker impedance or e-clouds.





B. Salvant et. al. 2008

-> **field of study:** feedback for TMCI

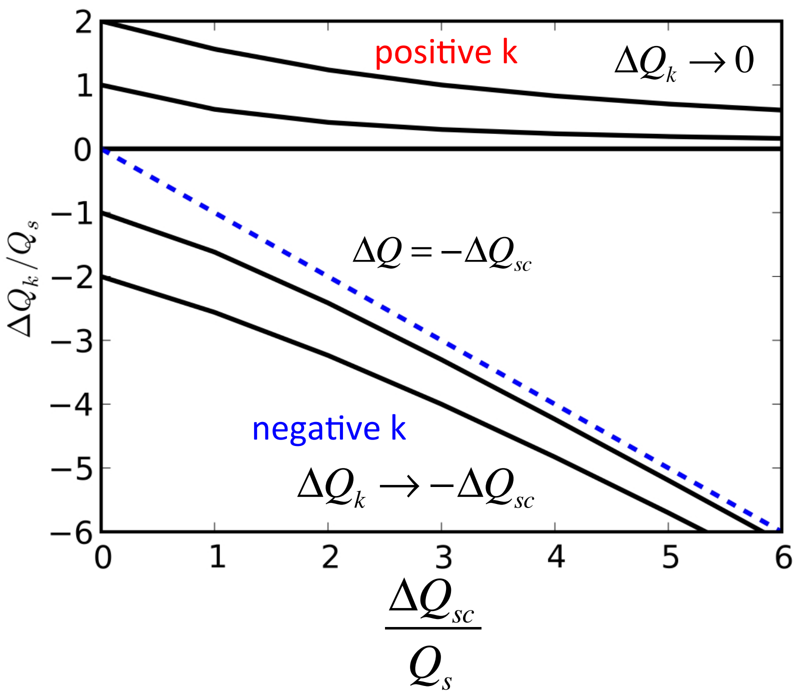
Field of study: role of space charge

Incoherent space charge tune shift:

$$\Delta Q_x^{sc} \propto -\frac{q}{m}\frac{I_p R}{\varepsilon_x \beta_0^2 \gamma_0^3}$$

Coasting beams:

- Loss of Landau damping.
- Increases the Landau damping caused by octupole
- Does not change the coherent tune !



Bunched beams:

- Does change the coherent tunes !
- Causes 'intrinsic' Landau damping.
- Suppresses TMCI.

head-tail tune shifts:

$$\Delta Q_k = -\frac{\Delta Q_{sc}}{2} \pm \sqrt{(\Delta Q_{sc}/2)^2 + (kQ_s)^2}$$

$$\Delta Q^{sc} = 0: \quad \Delta Q_k = kQ_s$$

Blaskiewicz, Phys. Rev. ST Accel. Beams (1998)
Boine-F., Kornilov, Phys. Rev. ST Accel. Beams (2009), Burov (2009), Balbekov (2009)

Summary and Open questions

Impedance reduction → impedance: **Z**

Counter measures

EM interaction ↕

Feedback, Landau damping ↔ oscillation modes**: Re Q** $\xrightarrow{\text{above threshold}}$ Instability**: Im Q**

Fields of study (selection):

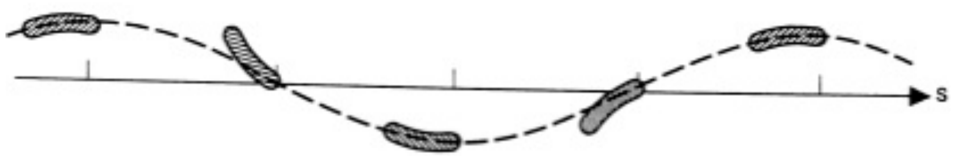
- Damping of fast, high-frequency or multiple mode instabilities ?

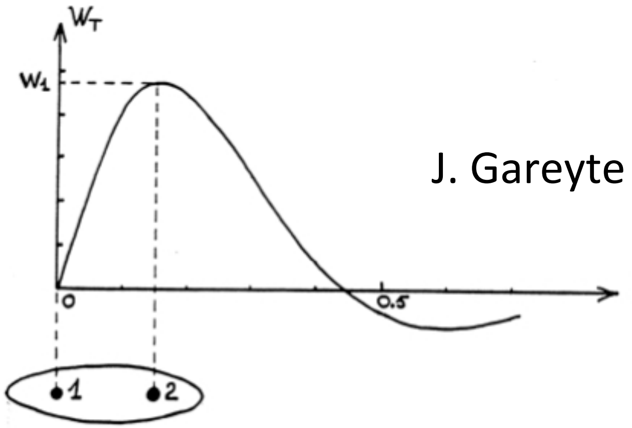
- Possible cure: High gain, broadband feedback (> 100 MHz) to resolve the internal head-tail bunch offset.

  C. Rivetta et al., BROAD-BAND TRANSVERSE FEEDBACK AGAINST E-CLOUD OR TMCI, Proc. ICFA-HB2012

- Interplay of different coherent and incoherent effects (-> strange new instabilities):
  Space charge, beam-beam, impedances, octupoles, feedback, beam cooling
  -> Large scale simulation studies, experimental validation.

Beam breakup instability





J. Gareyte (2000)

The instability (in rings) is usually suppressed by the synchrotron oscillations.

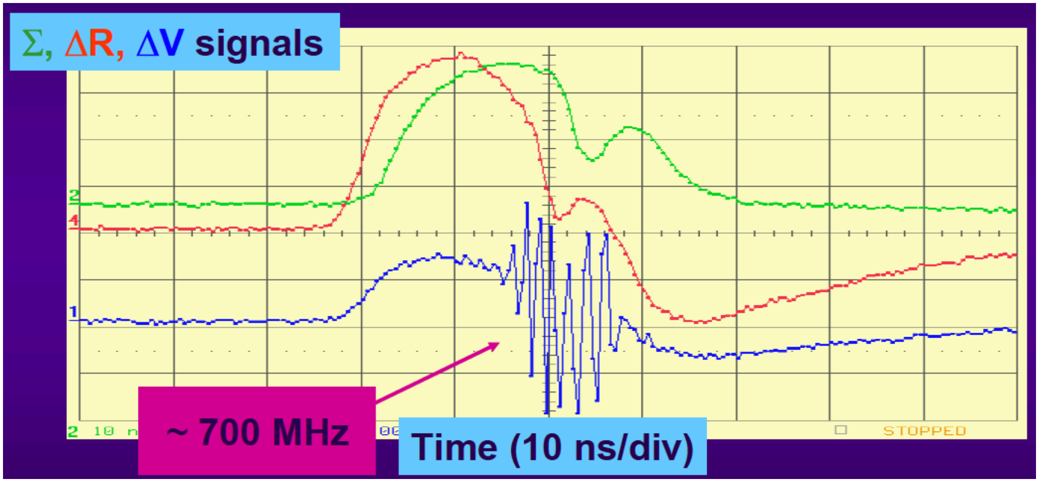
$$x_1(t) = A\cos(Q_0\omega_0 t)$$

$$\ddot{x}_2 + (Q_0\omega_0)^2 x_2 = \alpha x_1(t)$$

$$\alpha = \frac{q^2 N W_1}{2m\gamma_0}$$

$$x_2(t) \approx A\frac{\alpha}{2Q_0\omega_0} t \sin(\omega_0 Q_0 t)$$

Observation in the CERN PS near transition



R. Cappi et al. (2000)