# Fixed Field Alternating Gradient Accelerators (FFAG)

Shinji Machida
ASTeC STFC Rutherford Appleton Laboratory
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# Contents to be covered

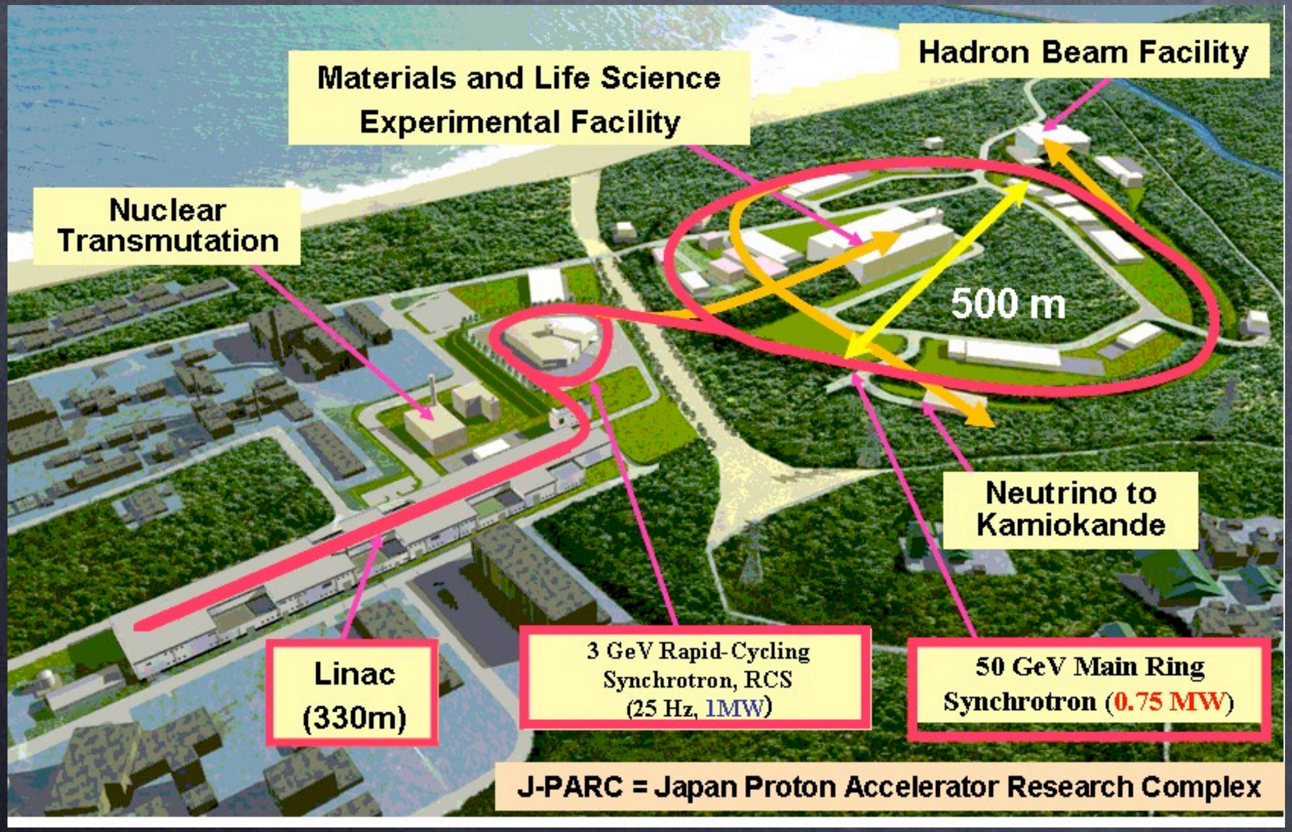
- Introduction
  - Future directions of (hadron) accelerators
- FFAG without constraint of tune
  - non-scaling type
- Scaling FFAG and "tune stabilised" FFAG

# Future directions of (hadron) accelerators

# High beam power

- Neutron, Muon source
  - Synchrotron based facility: ISIS, J-Parc
  - Linac based facility: SNS, ESS
- Accelerator driven system (ADS)

J-Parc

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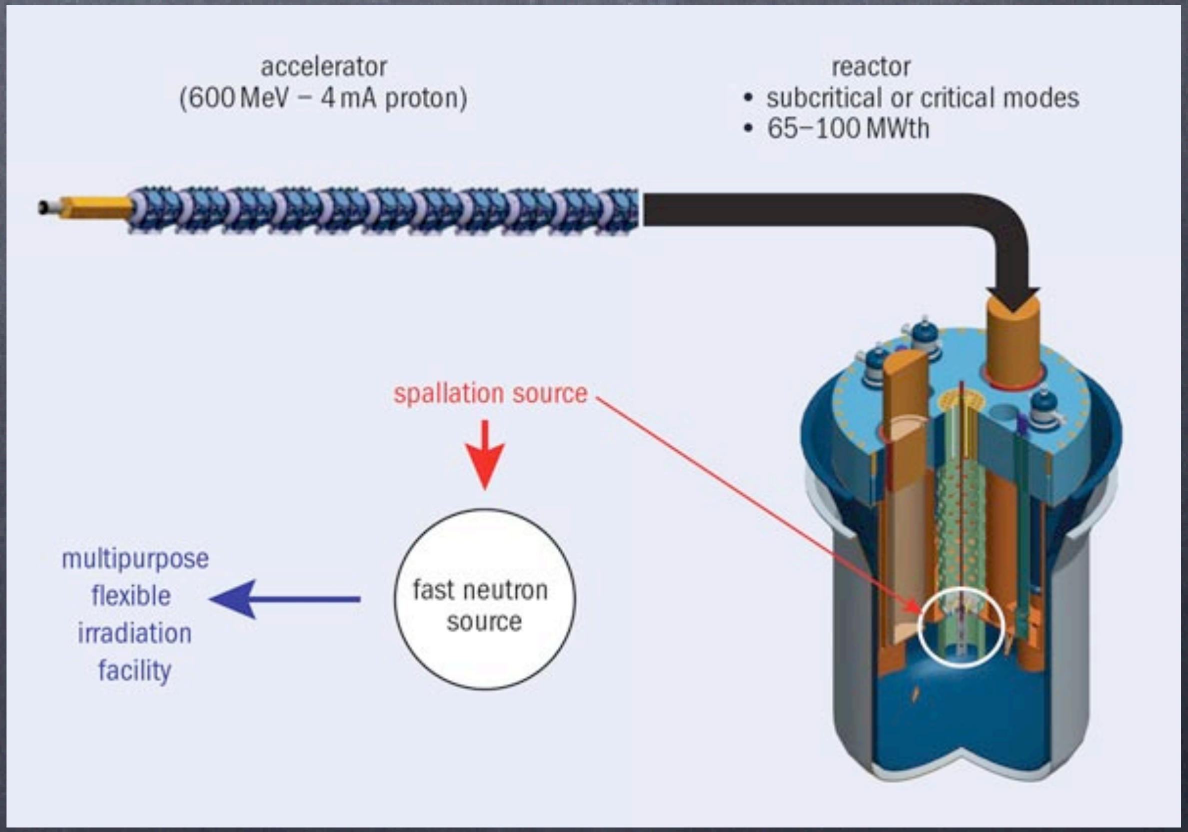
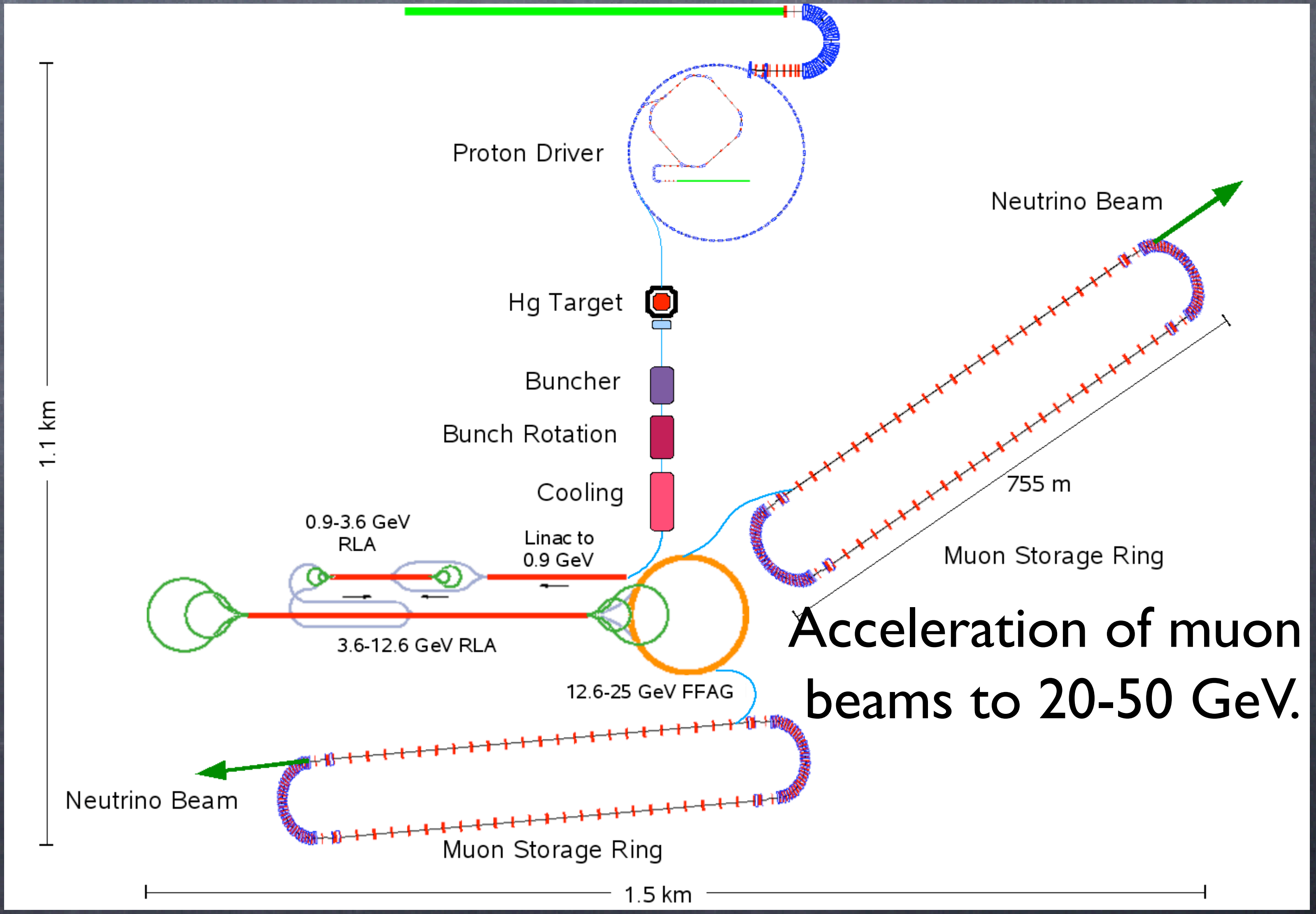


# Acceleration of short lived particles

- Neutrino factory, Muon collider
  - Linac or re-circulating linac to accelerate muons
- Accelerator of unstable nuclei

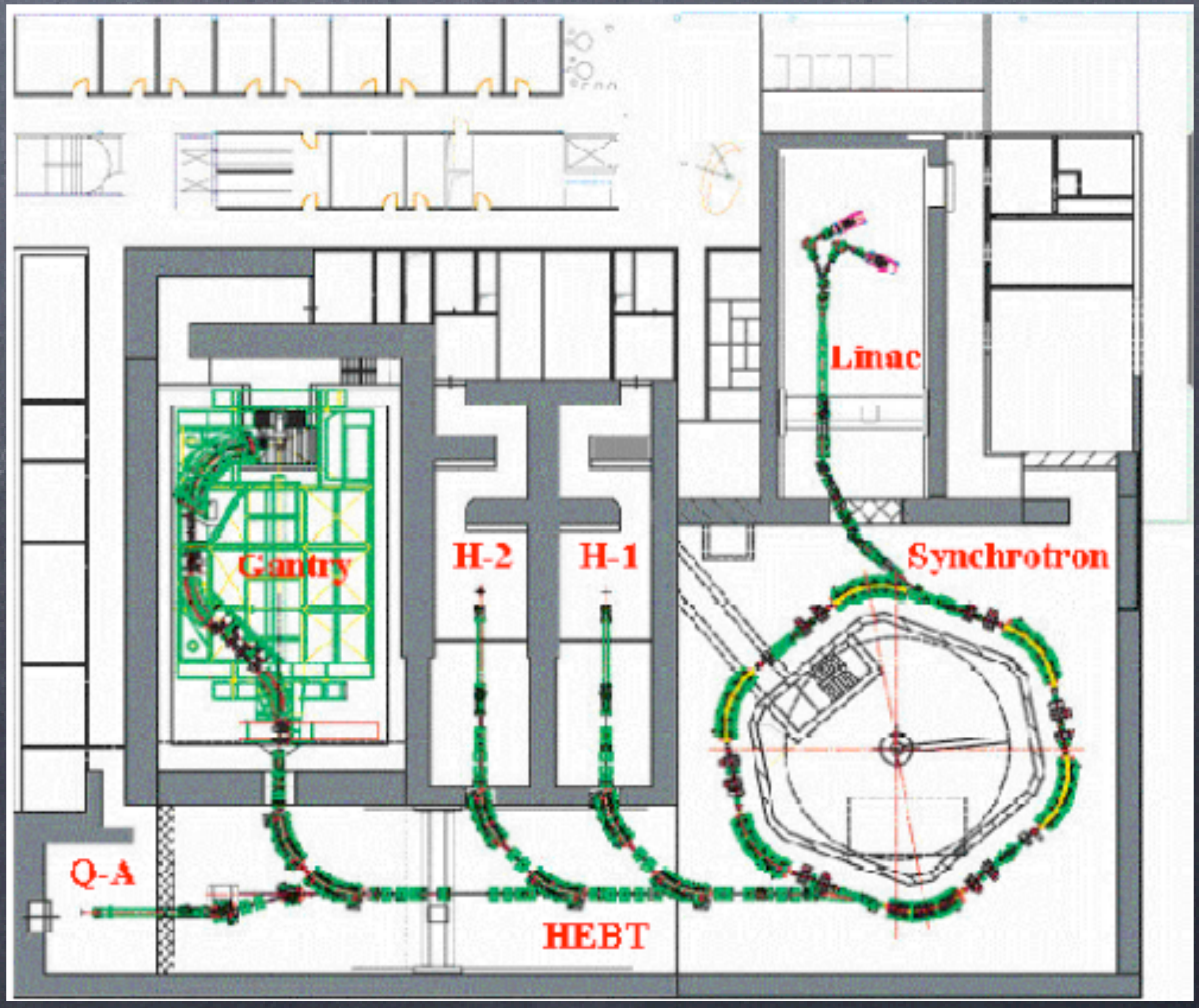
Neutrino factor (original layout)



# Accelerator for many other applications

- Particle therapy
  - Cyclotron based: proton therapy
  - Synchrotron based: proton and heavy ion therapy
- Industrial use
  - sterilisation
  - cargo scan for security

Particle therapy



# Questions

- Are the accelerators available today good enough?
- Any other options without making a huge jump?

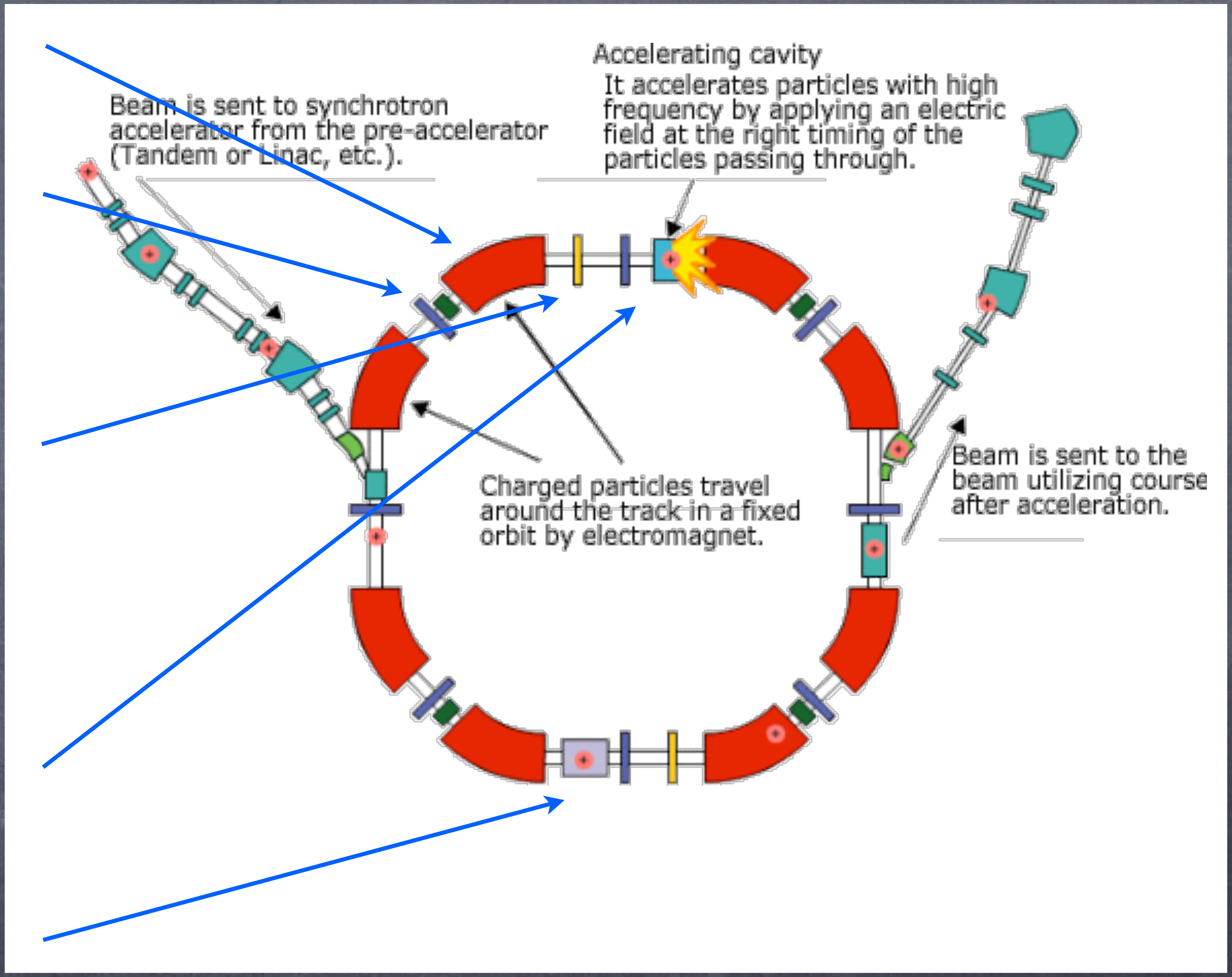
# Fixed Field Alternating Gradient (FFAG)?

# FFAG without constraint of tune

## non-scaling machine

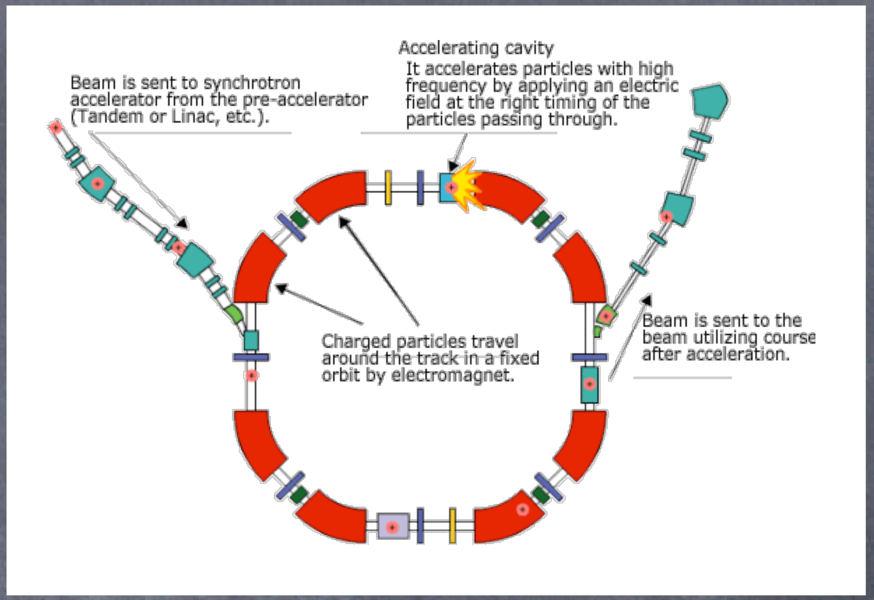
# Synchrotron (1)

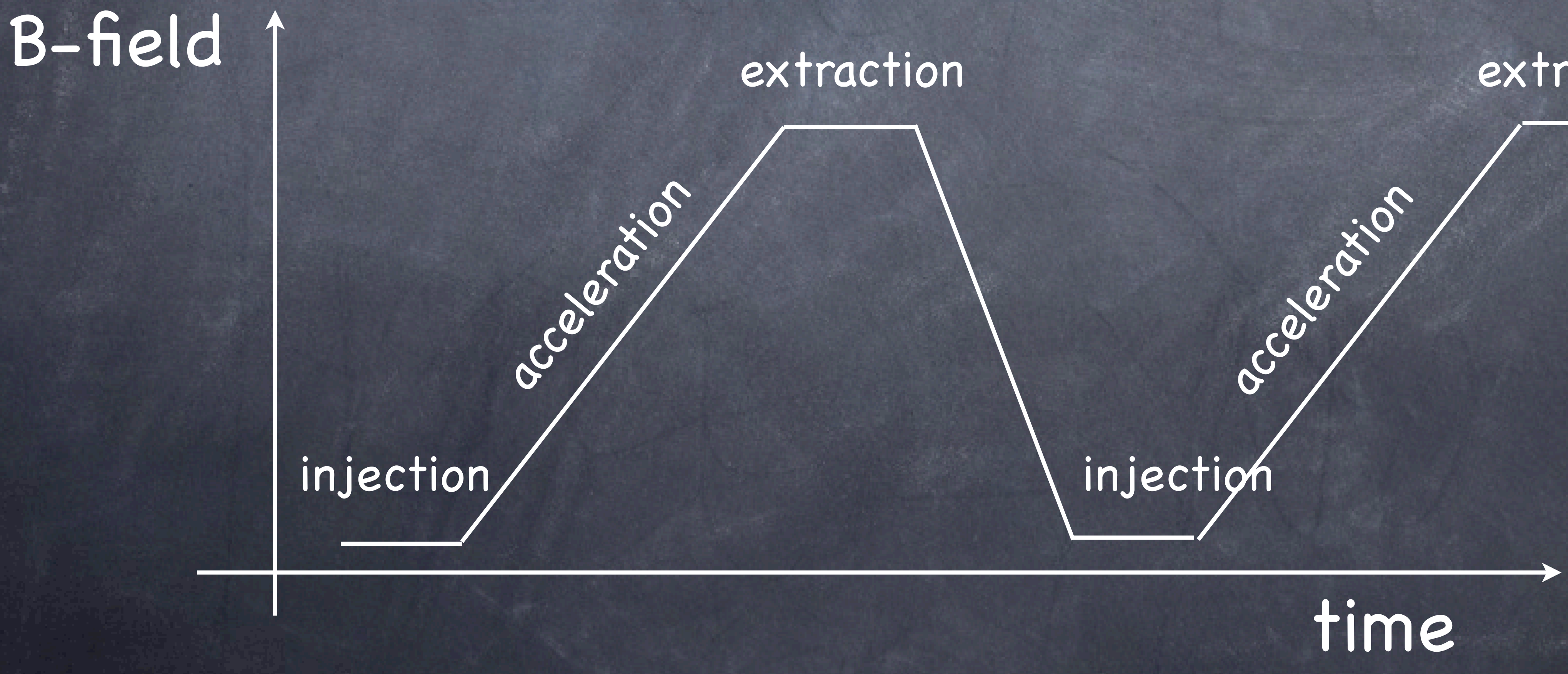
- Bending magnet
- Quadrupole magnet
- Multipole magnet for special purpose
- rf cavity
- Diagnostic



# Synchrotron (2)

- When a beam is accelerated at rf cavity, magnetic field is increased to keep the orbital shape constant.



# Synchrotron (3)

- Circulating radius (orbit) becomes constant.

$$\theta = \frac{B(t)L}{p(t)/e}$$

- Transverse focal length (optics) becomes constant.

$$\frac{1}{f} = \frac{(dB(t)/dx)L}{p(t)/e}$$

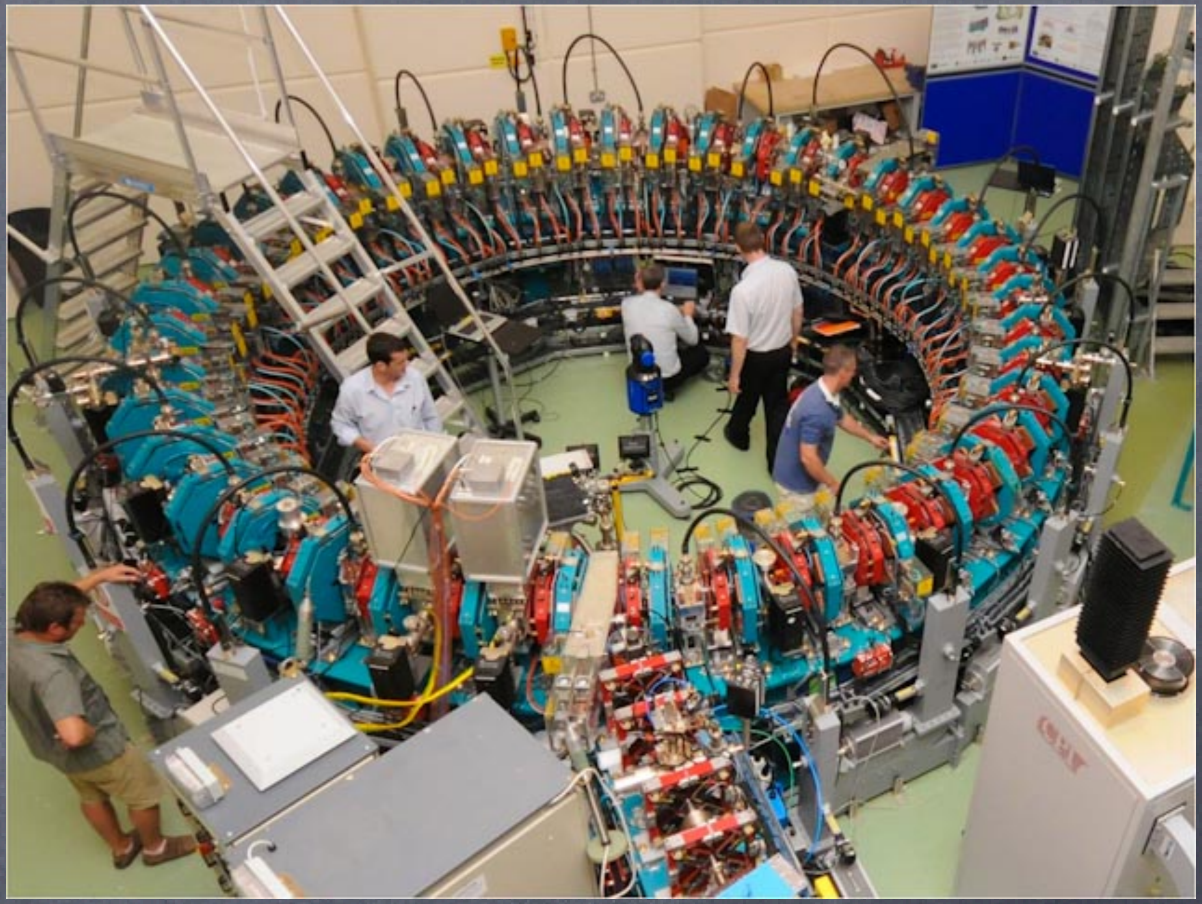
> small aperture magnets.

# Have you heard EMMA?

the first non-scaling FFAG
"Electron Model for Many Applications"

# EMMA lattice (1)

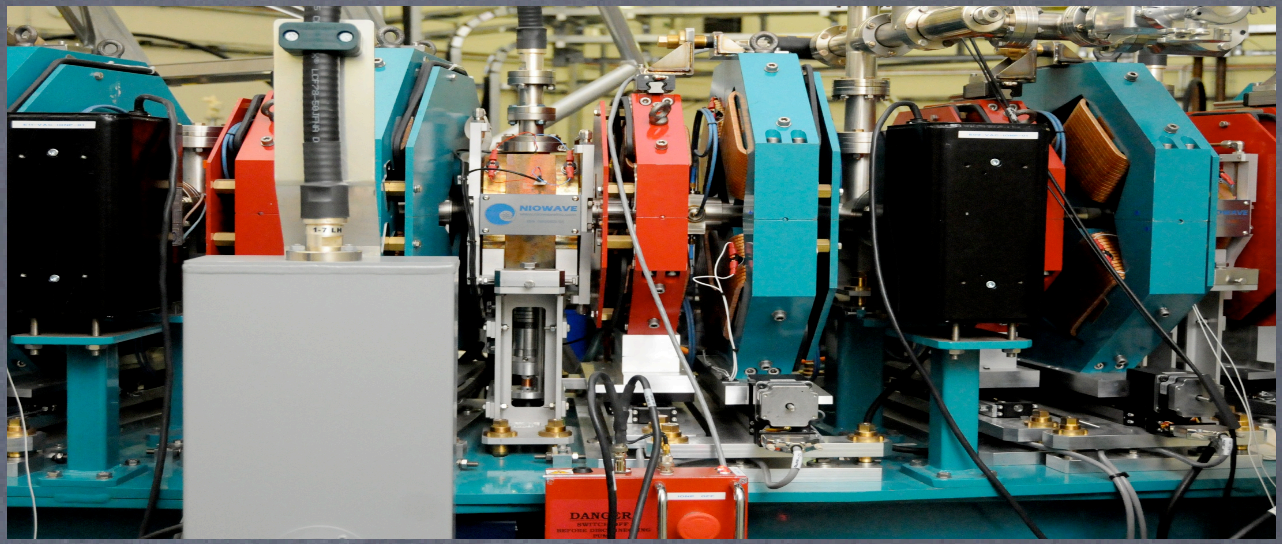
- EMMA at Daresbury ->
- Lattice with very regular 42 doublet cell.

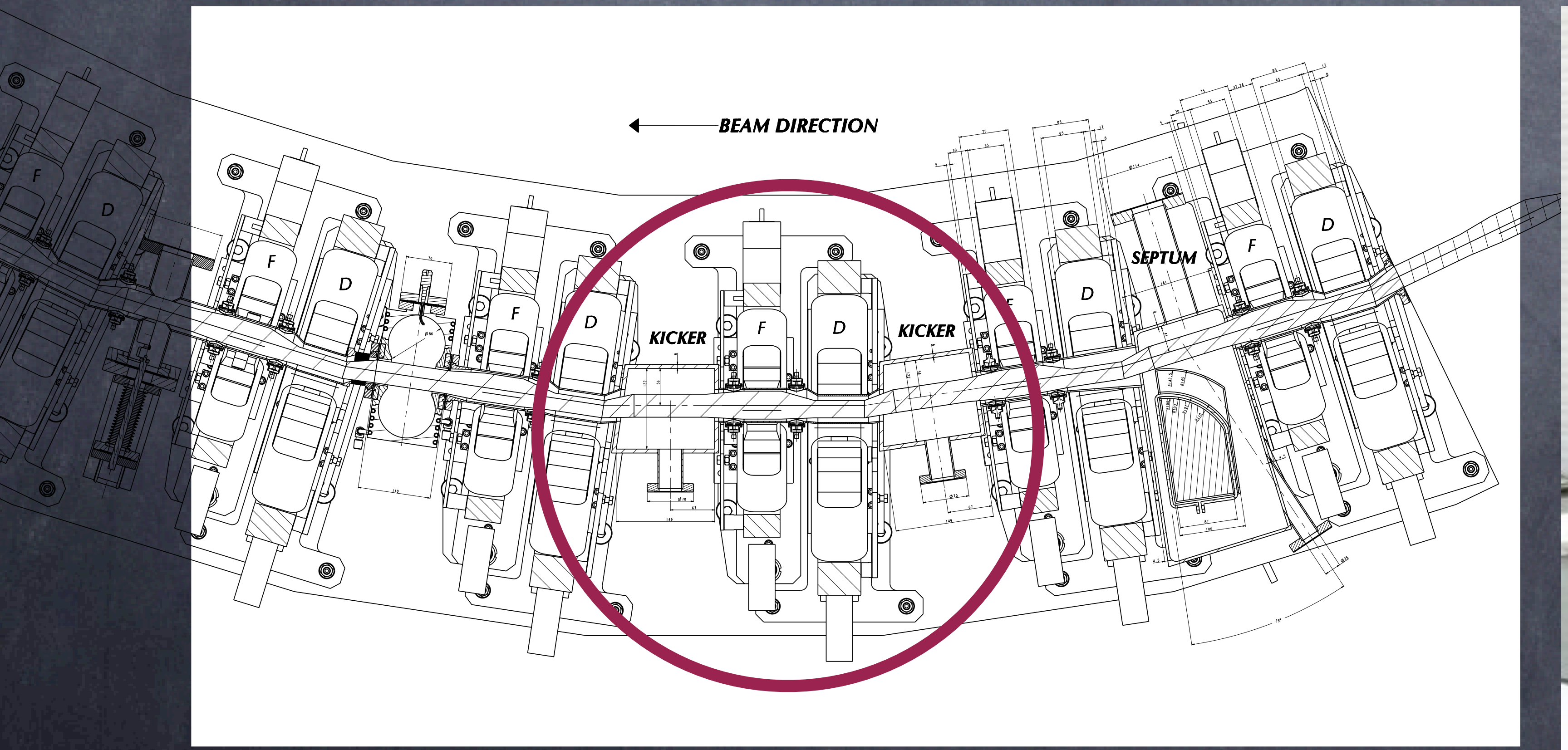


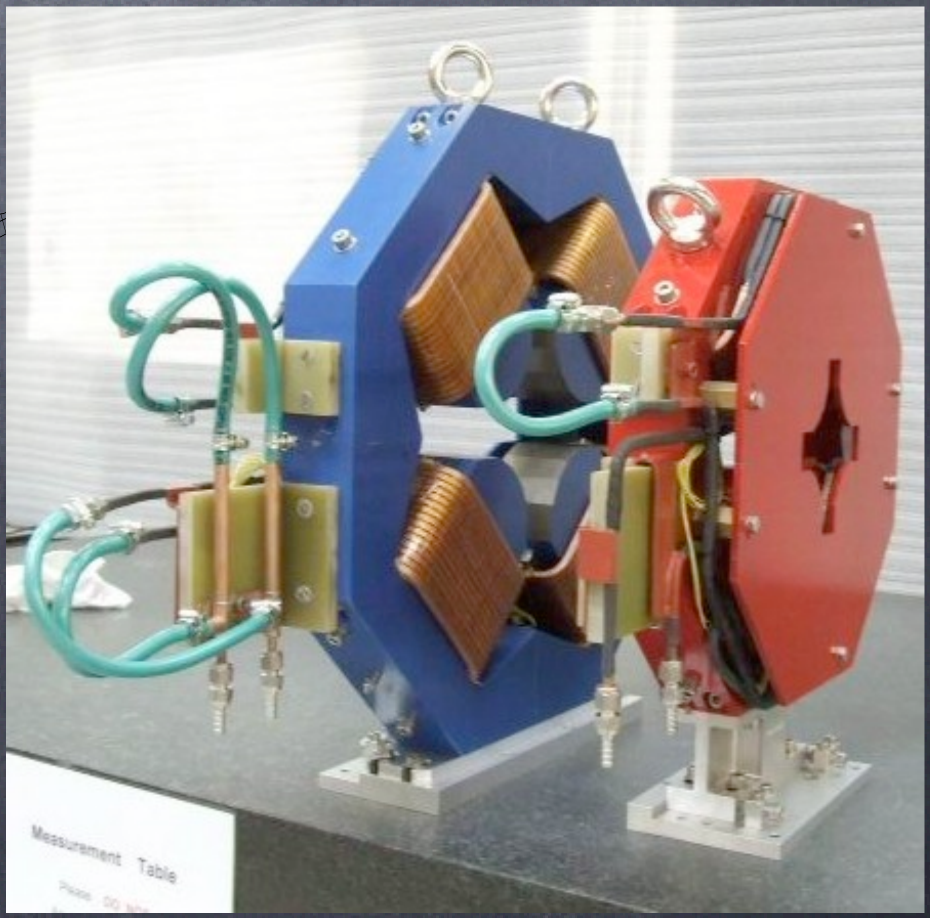
- Diamond light source lattice as a comparison.

# EMMA lattice (2)

- Quadrupole only.
- Radially shifted quadrupole to create bending component.

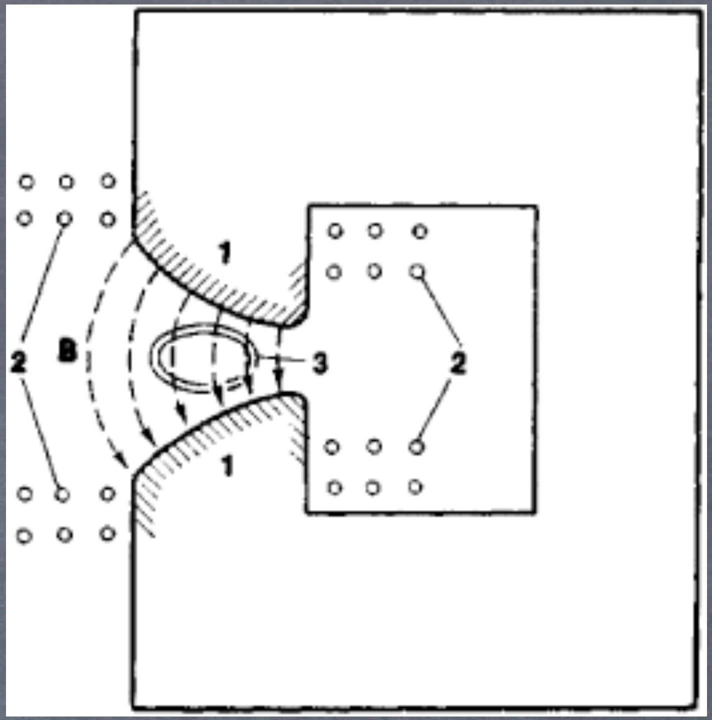




# EMMA lattice (3)

- Synchrotron magnets are used to be all “combined function” type.
  - BNL AGS, CERN PS, etc.



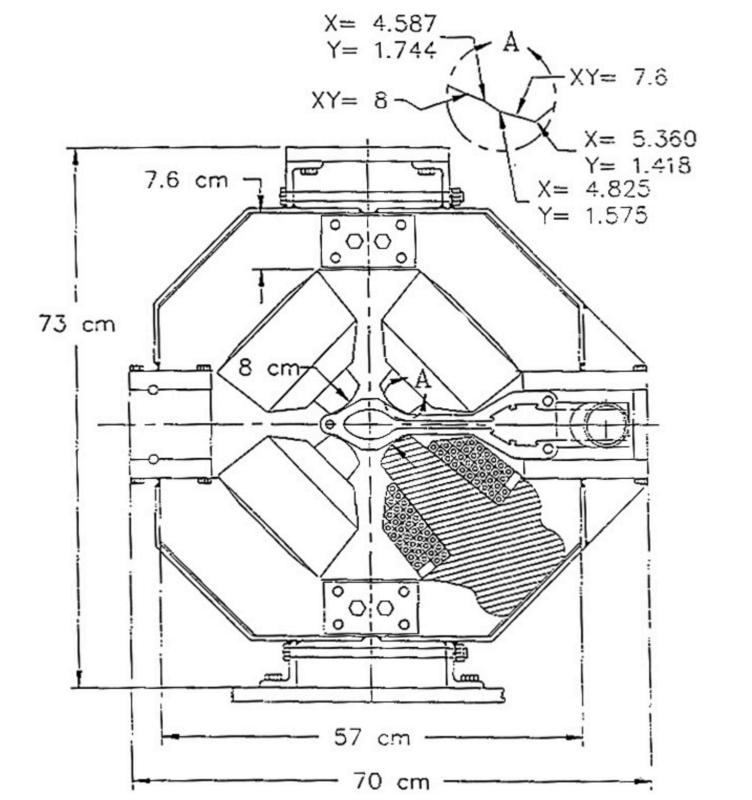



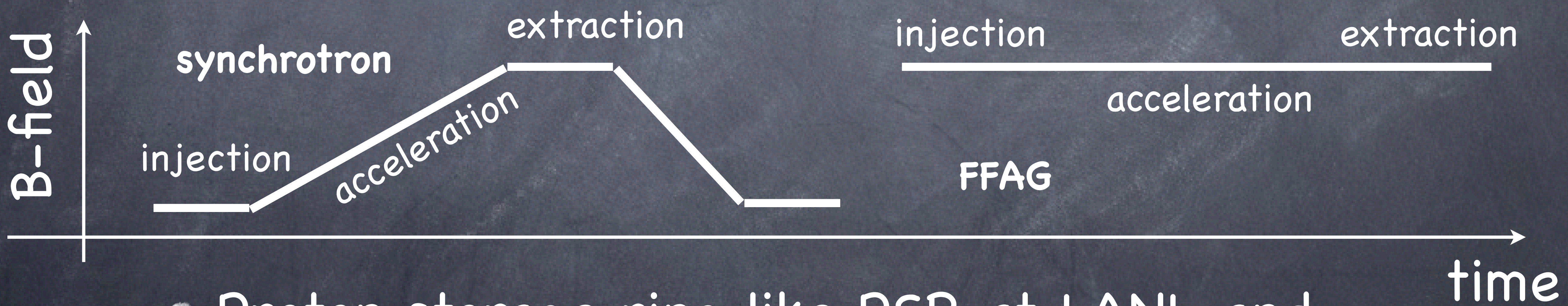

**Figure 1.3.1.1.2-010 Cross Section of the Storage Ring Quadrupole Magnet**

- “Separated function” magnets (separate bending and quadrupole magnets) were later invented.

SNS ring

# EMMA lattice (3)

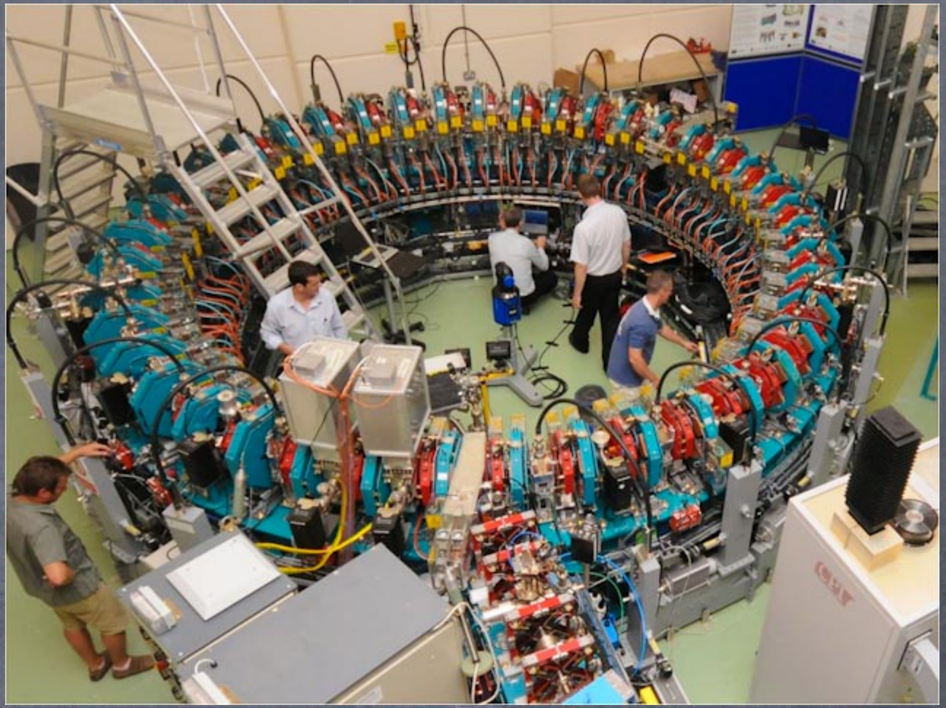
- What is special in EMMA ?
- The only difference from a synchrotron is
  - Magnetic field does not ramp while a beam is accelerated, i.e. fixed field lattice.

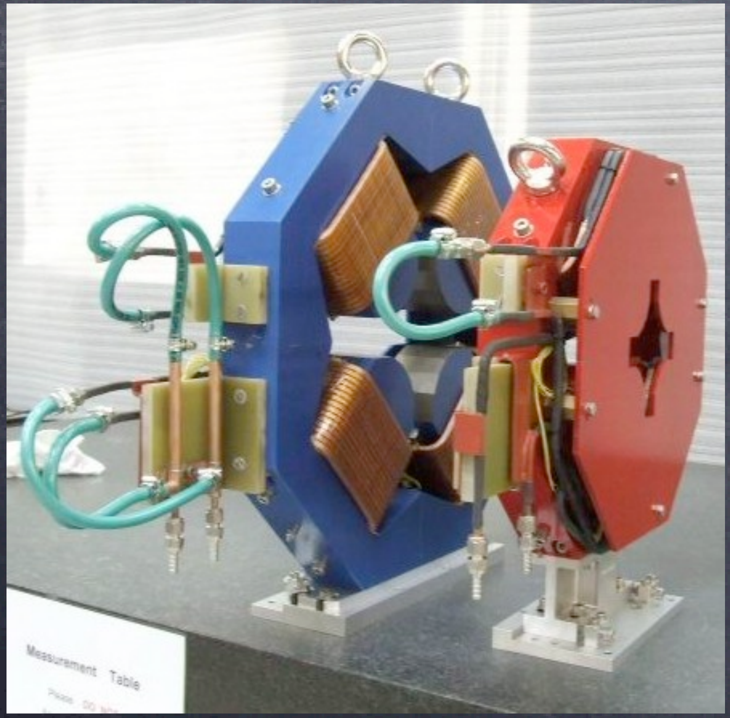



- Proton storage ring like PSR at LANL and SNS at ORNL uses fixed field, but does not accelerate a beam.

# EMMA lattice (4)

- EMMA is one family of Fixed Field Alternating Gradient (FFAG) accelerator.






- This family of FFAG, in particular, uses the same type of lattice magnets and has very similar optical property as an ordinary synchrotron.

Operation with fixed field magnets.

# Advantage to use fixed field magnet (1)

- Hardware point of view
- Operation point of view
- Beam dynamics point of view

# Advantage to use fixed field magnet (1) hardware point of view

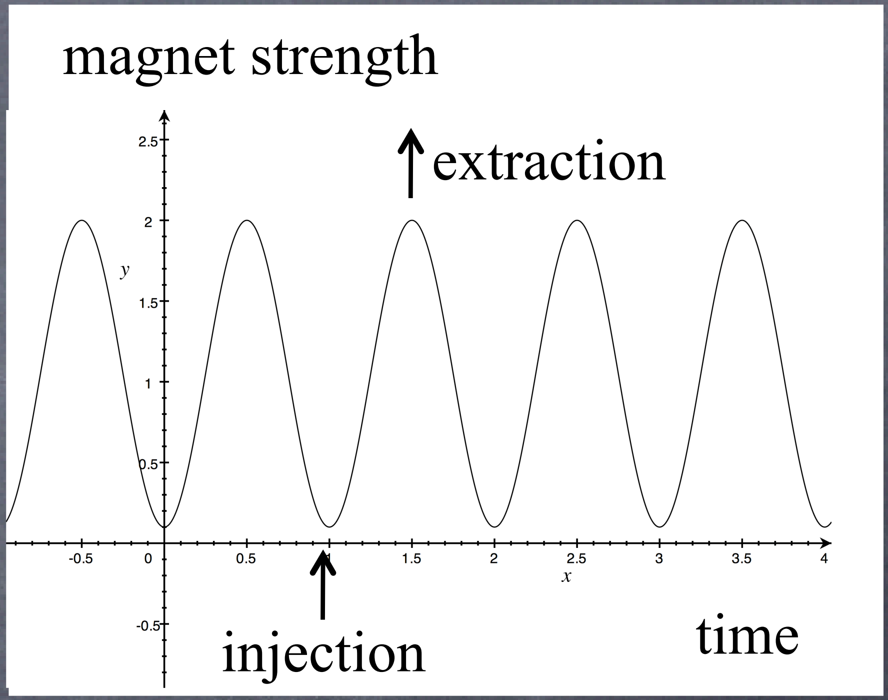
- Power supply is simpler and cheaper.
  - No programming and feedback.
- No eddy current in magnet or on vacuum chamber.

$$V_{ind} = -\frac{dB(t)}{dt}$$

- No synchronisation loop between magnets and rf.
- More reliability compared with machine with pulsed magnets.

# Advantage to use fixed field magnet (2) operational point of view

- In a synchrotron, time to complete one acceleration cycle is determined by ramping of magnets.
  - ISIS operates with 50 Hz.
  - 10 ms or ~10,000 turns.




- In FFAG, it is possible to accelerate a beam very fast. Only limited by available rf voltage (power).
  - ~10 turns (for muon acceleration).

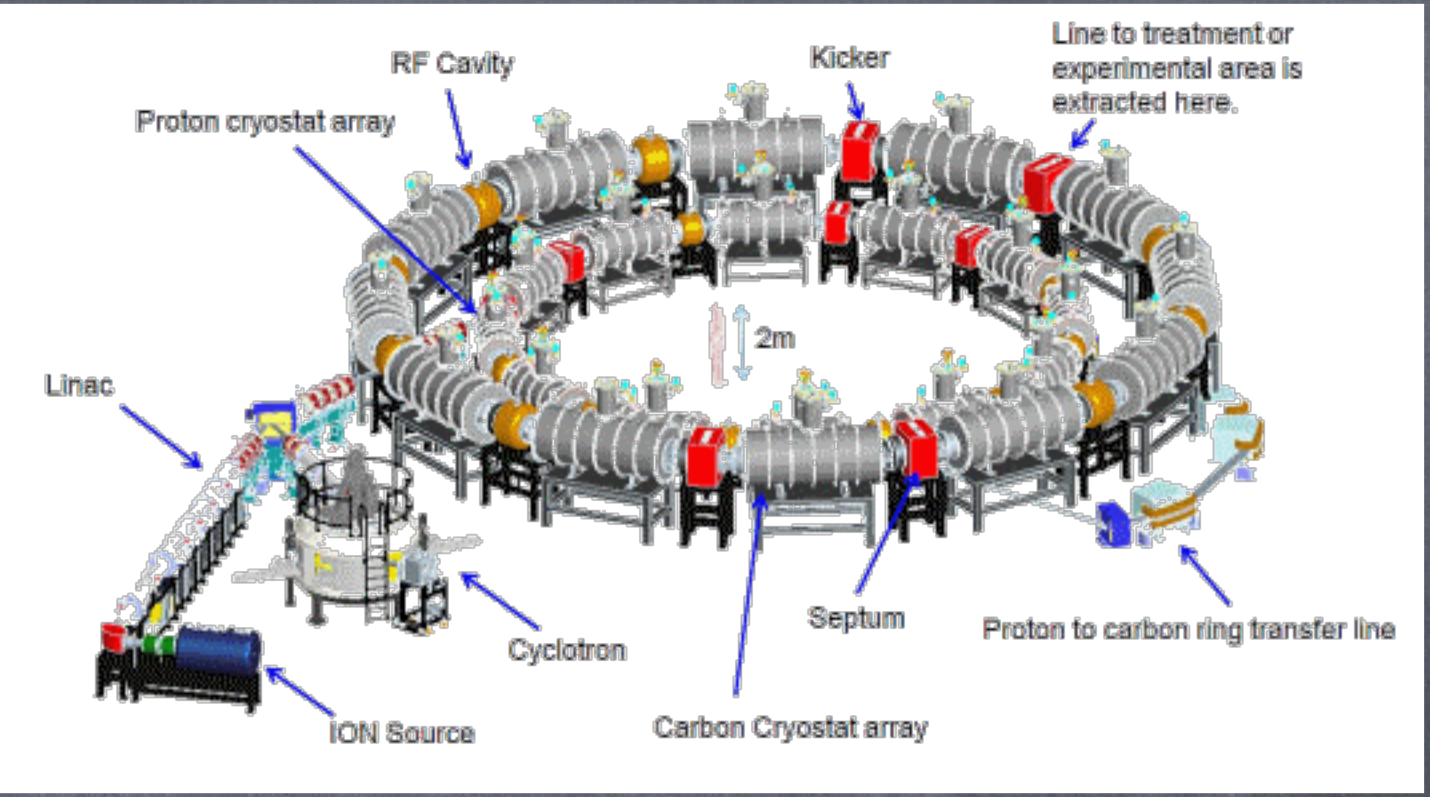
# Advantage to use fixed field magnet (3) beam dynamics point of view

- Higher repetition makes the high average current.
  - Beam power = Energy x NofP x Rep.
- High "Rep." can reduce "NofP" to get the same beam power. Space charge effect is a single bunch effect and proportional to "NofP" only.

$$\Delta Q = -\frac{r_p n_t}{2\pi\beta^2\gamma^3\varepsilon_t B_f}$$

# Advantage to use fixed field magnet (4) altogether

PAMELA




- Compact
- Cheap
- Easy to handle
- Accelerators for medical and security use.
  - spot scanning

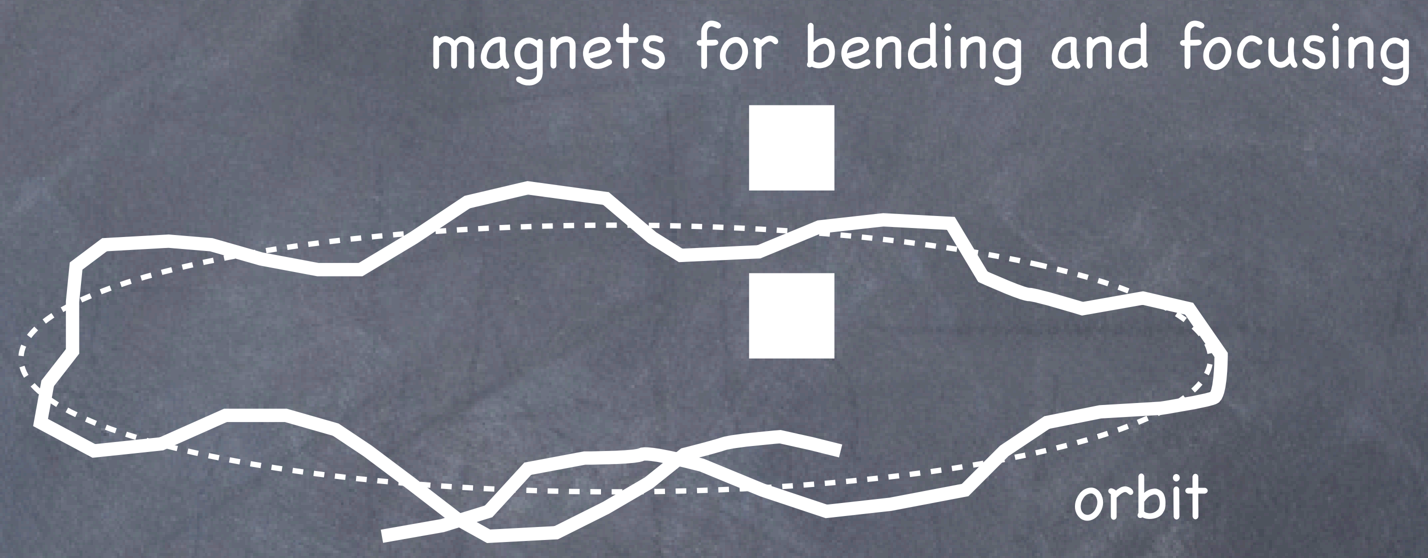
- Muon acceleration.
- Accelerator of short lived particles.
- High power accelerators for n, m and ADSR.

# Summary so far (1)

- FFAG accelerators are similar to synchrotron, but operated with fixed field magnets, which give many advantages.
- What is the disadvantages?

# Three basic parameters of circular accelerators (1)

- Revolution time
  - rf frequency
- Circulating radius
  - vacuum chamber size
- Transverse focal length or effective strength
  - beam dynamics



# Three basic parameters of circular accelerators (2)

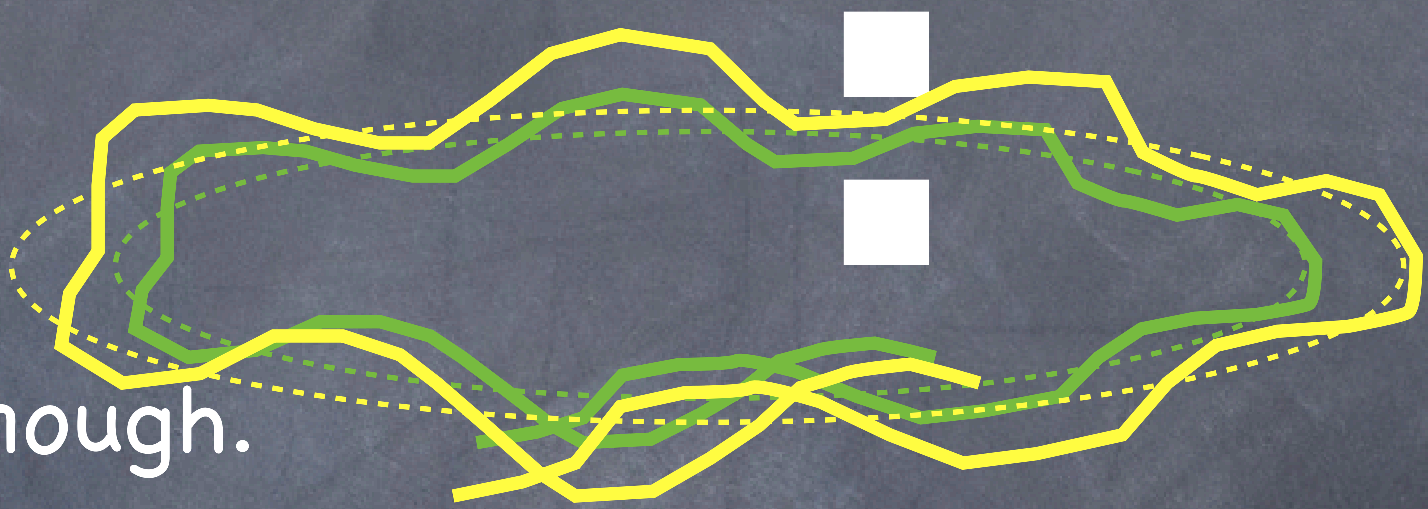
| | Cyclotron | Synchrotron | FFAG |
|---|---|---|---|
| Revolution time | constant<br>fixed frequency rf | varies<br>const for relativistic beams | varies |
| Circulating radius | varies | constant | varies |
| Transverse focusing | varies | constant | varies |

FFAG seems most ambitious or crude approach?

# Dispersion function (1)

- Change of circulating radius can be small with small dispersion function.

$$\Delta R = D(dp/p)$$



- How small is small enough.
  - dp/p ~ +/-33% (factor 2) to +/-50% (factor 3).
  - To make the orbit shift within a few cm, dispersion function should be less than 10 cm.
    - D = dR/(dp/p)
  - In an ordinary synchrotron, it is  > 1 m.

# Dispersion function (2)

- H-function defined as

$$H = X_d^2 + P_d^2$$

$$X_d = D/\sqrt{\beta_x} = \sqrt{2J_d}\cos\phi_d$$

$$P_d = (\alpha_x D + \beta_x D')/\sqrt{\beta_x} = -\sqrt{2J_d}\sin\phi_d$$

- Without bending, $J_d$ is invariant and $\phi_d$ is identical to the betatron phase advance.

- With bending (thin lens), $J_d$ changes.

$$\Delta X_d = 0$$

$$\Delta P_d = \sqrt{\beta_x}\Delta D = \sqrt{\beta_x}\theta$$

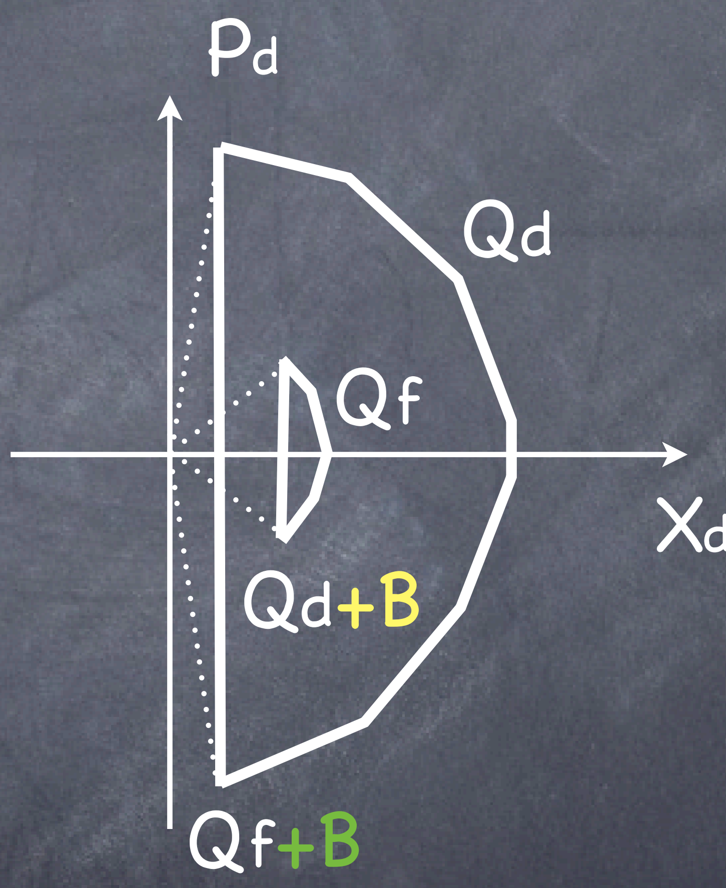
# Dispersion function (3)

- Trick is to bend where the horizontal beta is small.

$$\Delta P_d = \sqrt{\beta_x}\Delta D = \sqrt{\beta_x}\theta$$

- Remember beta_Qf > beta_Qd and phase advance is proportional to 1/beta.

$$\phi = \int \frac{ds}{\beta_x}$$




- Bend in Qd gives small ΔPd and phase advance in Qf is small.
- Bend in Qf gives large ΔPd and phase advance in Qd is large.

# Focusing force (1)

- Focusing due to quadrupole magnets.
  - Example: FODO lattice.

$$\sin\frac{\phi}{2} = \frac{L}{2f} \qquad \frac{1}{f} = \frac{(dB/dx)L_m}{B\rho}$$

$\phi$ : phase advance, $L$ : half cell, $L_m$ : quad length, $B\rho$ : rigidity

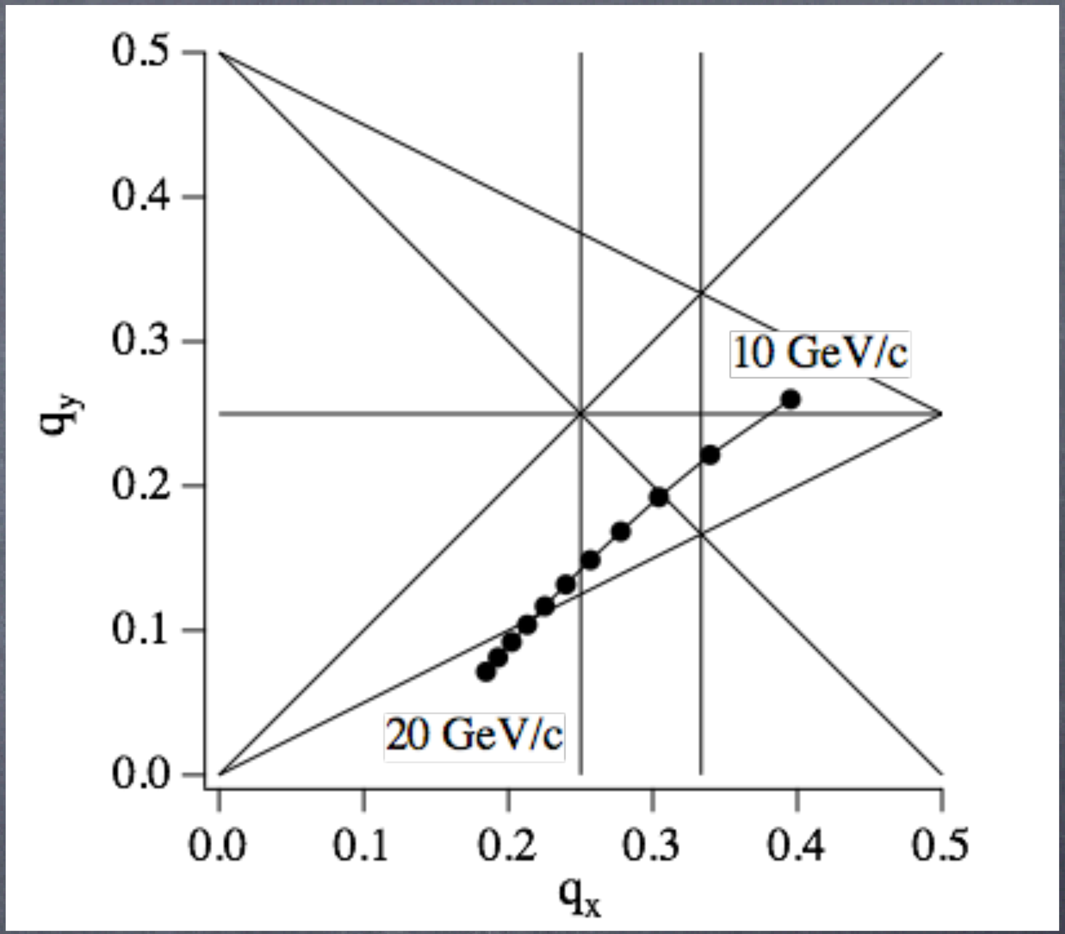
  - Beta function becomes large with high momentum.

$$\beta_F = \frac{2L[1+\sin(\phi/2)]}{\sin\phi}$$

$\beta_F$ : beta function

# Focusing force (2)

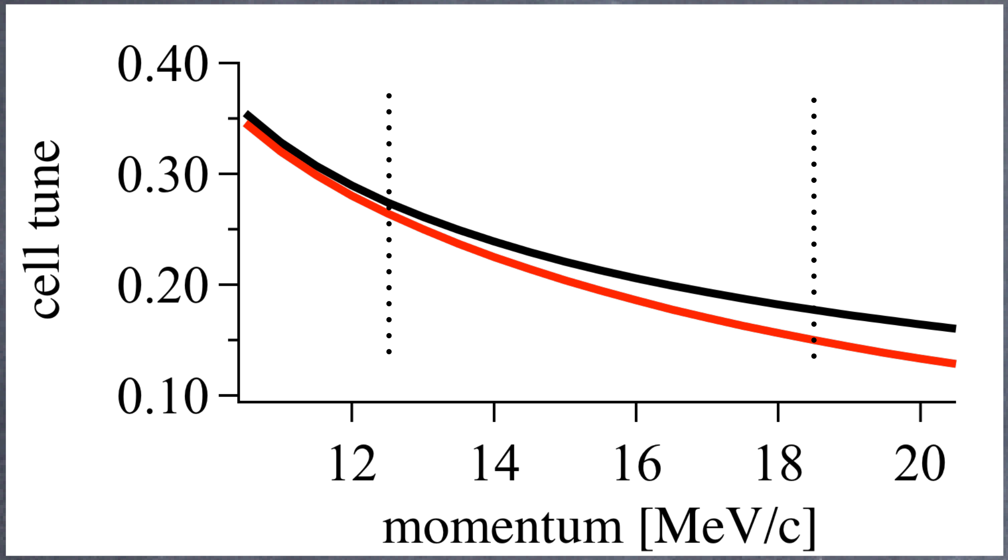
- In a synchrotron, tune is fixed and far from resonance lines.



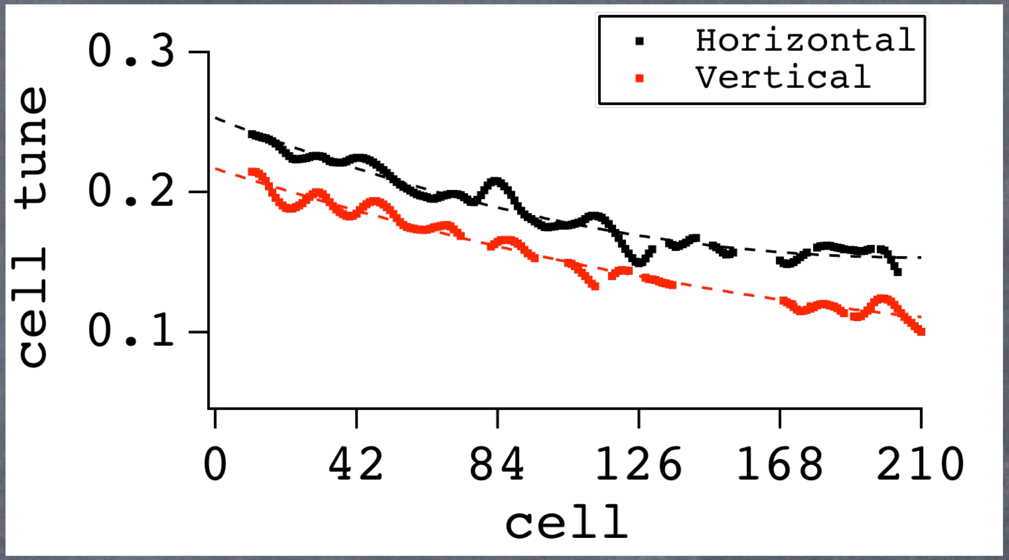

- In a FFAG, tune moves through acceleration.
- The number of resonance lines is minimised by high periodic lattice.
- How harmful to go through "resonances" at rational tunes?
  - Depends on other parameters.

# EMMA results (1)

- When a beam is accelerated, tune decreases.

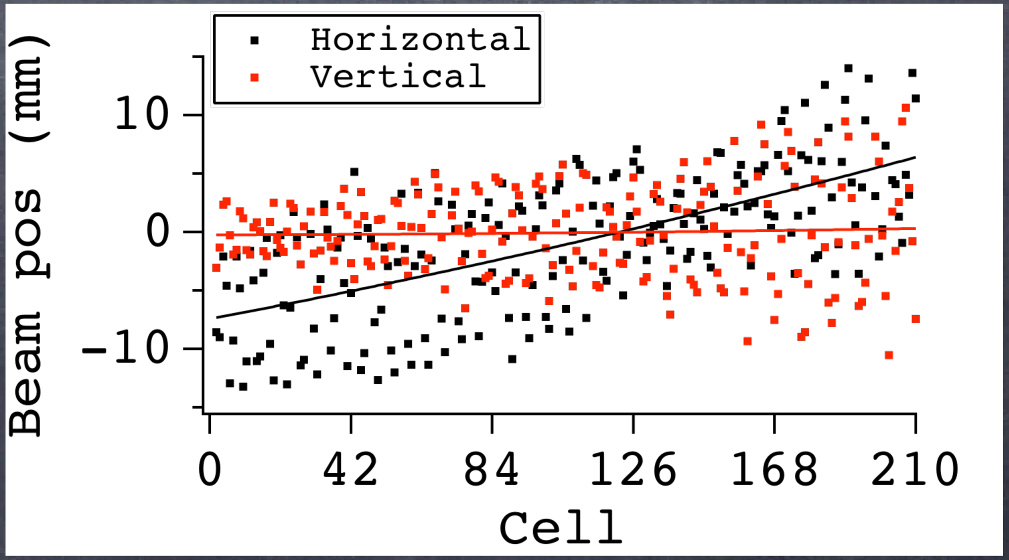



simulation



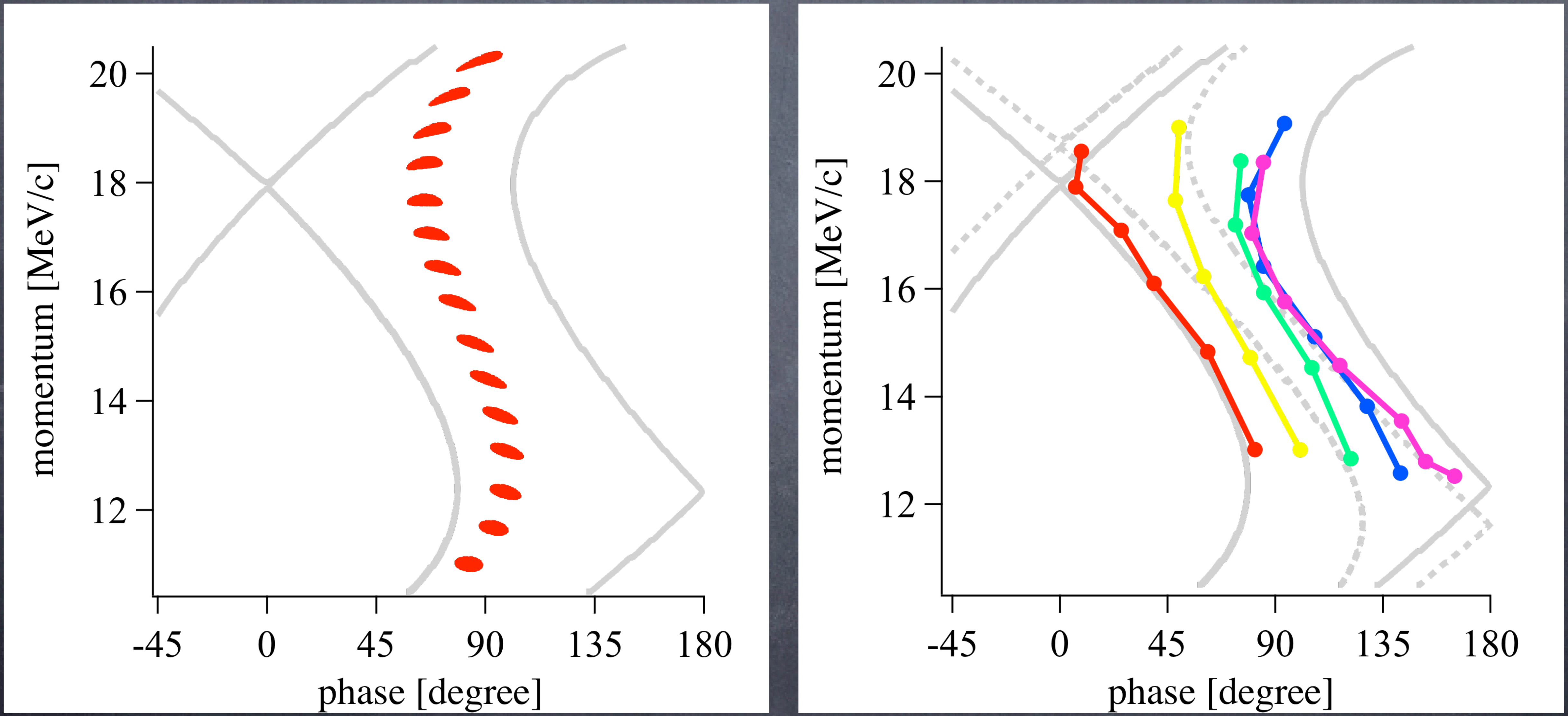

experiment (12.5 to 18.5 MeV/c )

- When a beam is accelerated, orbit expands horizontally, but not in vertical.



# EMMA results (2)

- Reconstruction of longitudinal phase space.
  - “Serpentine channel acceleration”

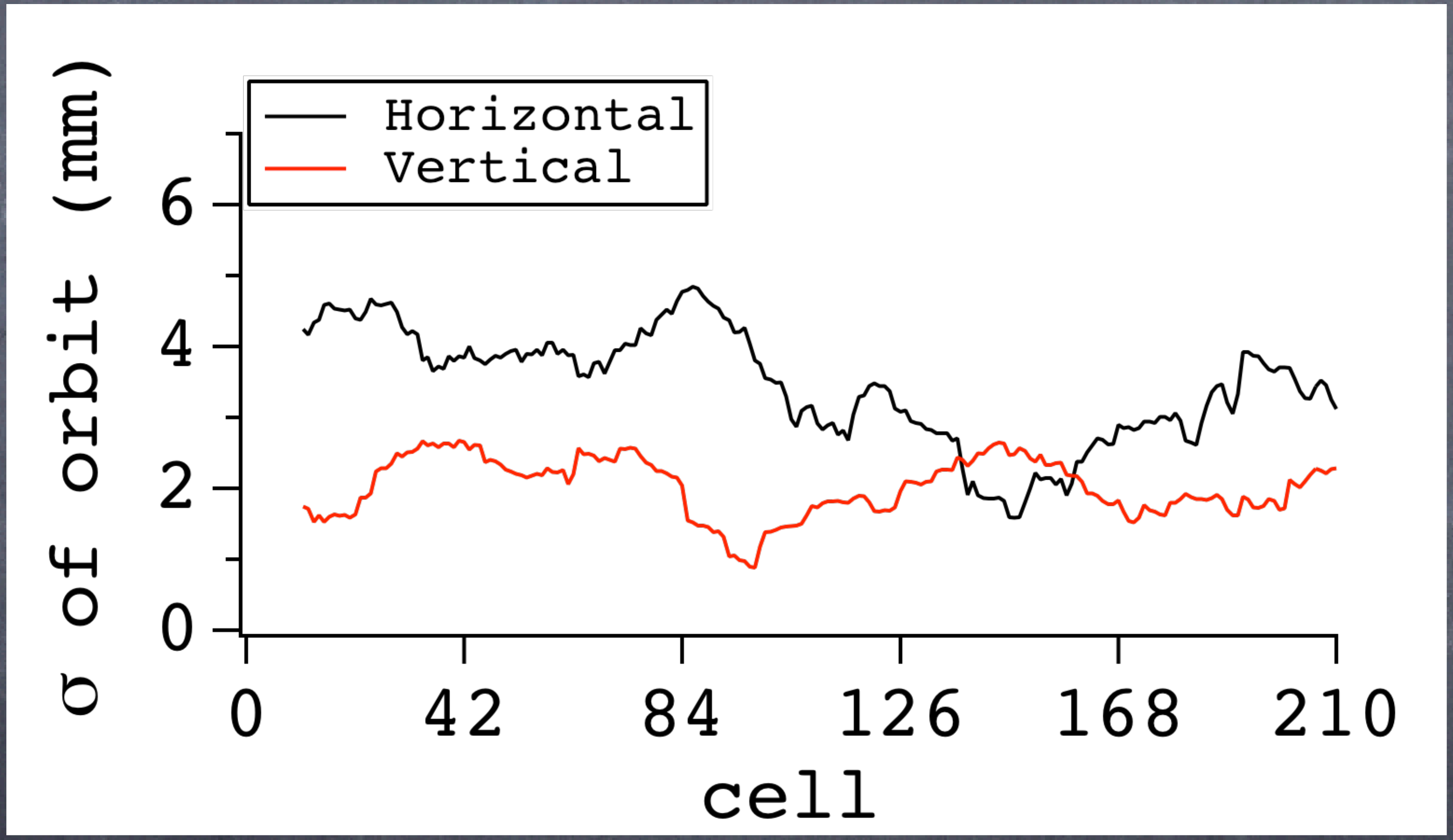



simulation

experiment (12.5 to 18.5 MeV/c )

# EMMA results (3)

- No blowup of beam size despite crossing of resonances.




experiment (12.5 to 18.5 MeV/c )

Ref: S. Machida et al., Nature Physics 8, 243 (2012)

# Summary so far (2)

- FFAG accelerators are similar to synchrotron, but operated with fixed field magnets, which give many advantages.
- Specific optics and orbit design for FFAG overcomes potential disadvantages as being fixed field operations.

# FFAG family

- Non-scaling FFAG like EMMA is in fact a new comer in the FFAG family.
- Original FFAG design has orbit shift, but tune is fixed by "scaling" principle, invented in 1950s.

# FFAG with constant tune

scaling machine

# Three parameters of circular accelerators (3)

| | Cyclotron | Synchrotron | FFAG |
|---|---|---|---|
| Revolution time | constant | varies (const for relativistic) | varies but small range |
| Circulating radius | varies | constant | varies but small range |
| Transverse focusing | varies | constant | varies |

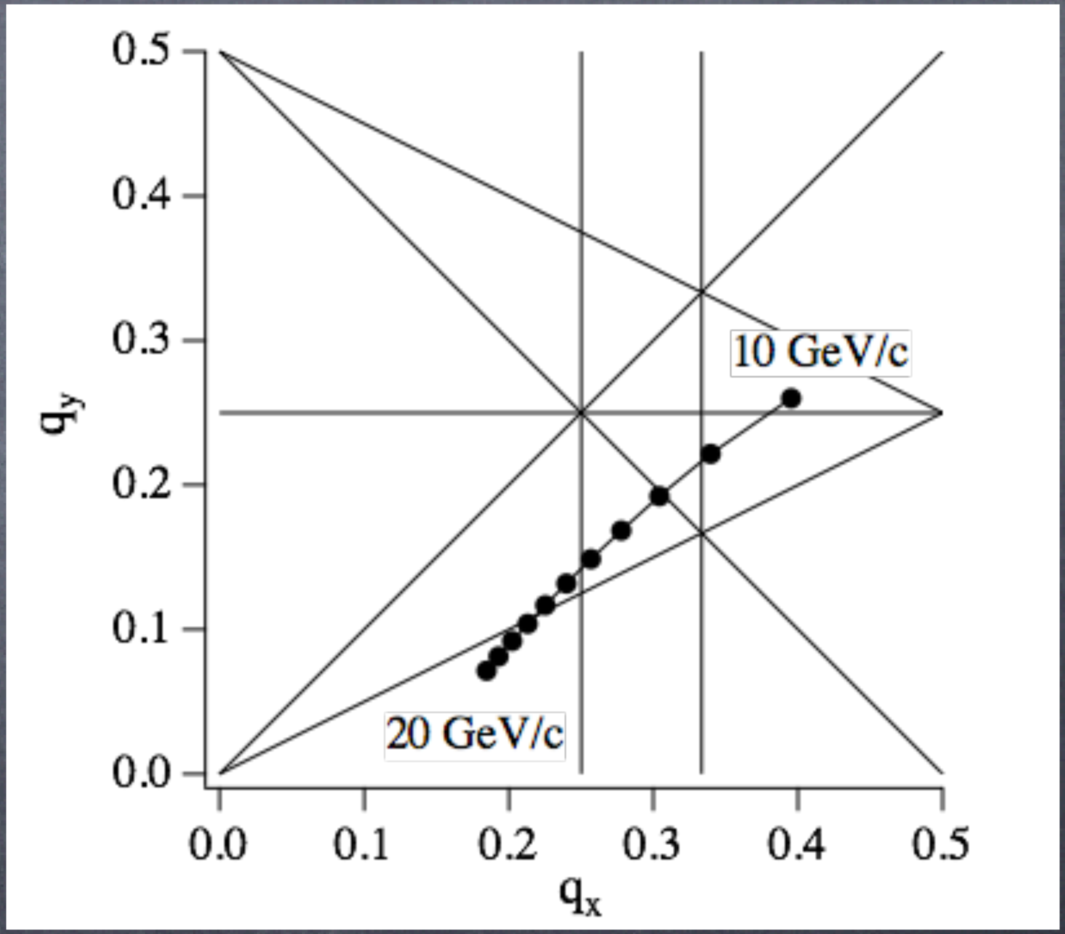
FFAG seems most ambitious or crude approach?

# Focusing force

- For muon acceleration, a beam goes around only ~10 turns.

- Large tune shift during acceleration is very scary for other applications.

- Larger resonant build up may happen.

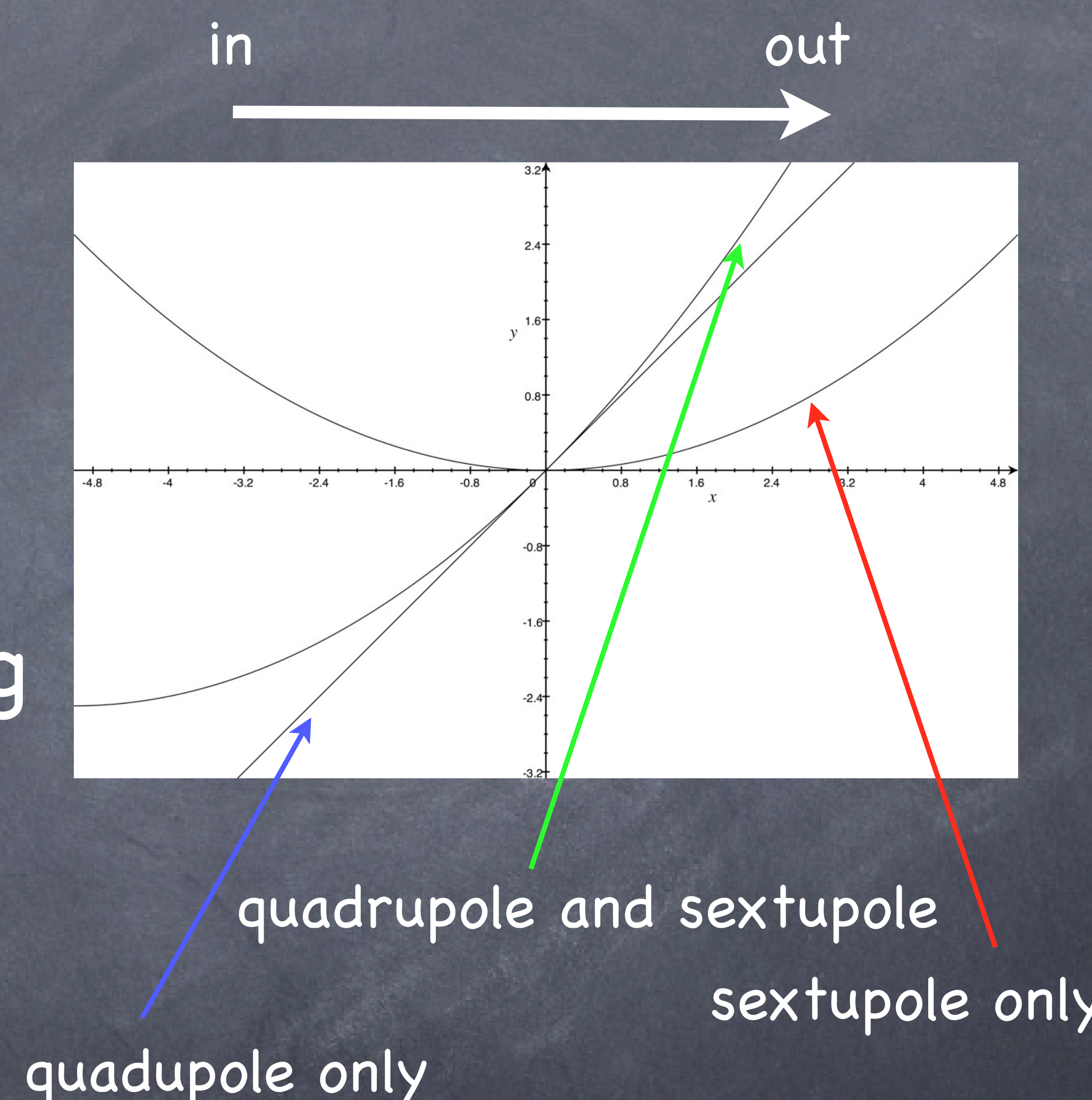


# Chromaticity correction (1)

- In an ordinary synchrotron, small shift of tune due to off-momentum particles (< 1%) could be a big problem.
- Sextupole for chromaticity correction is a common practice to compensate it.

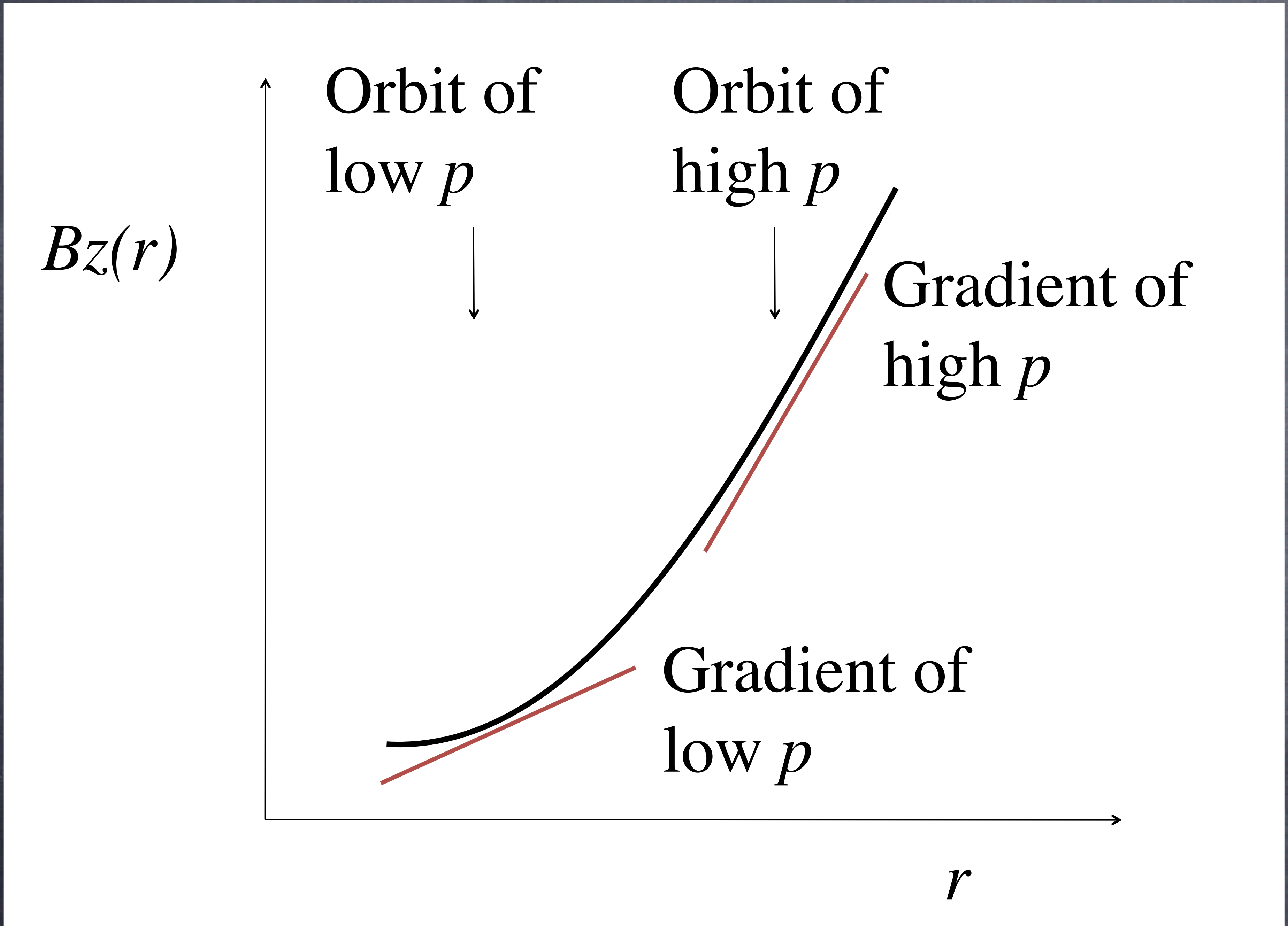
# Chromaticity correction (2)

- Off-momentum particle circulates outer orbit.
  - make a magnet which has gradient depending on its radial position.




- Sextupole (x^2) is enough to correct for small dp/p (< 1%).

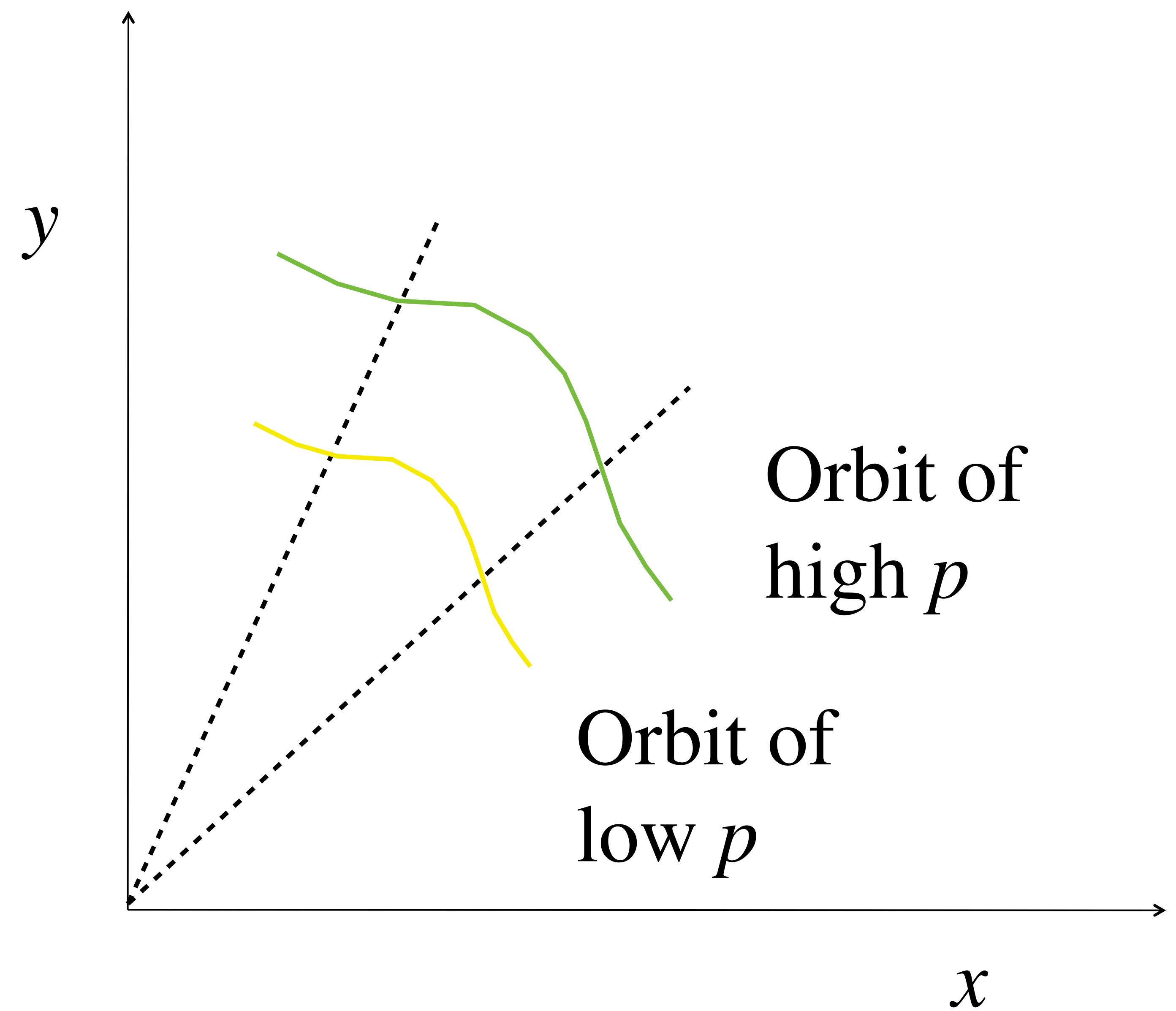
# Cardinal condition of scaling FFAG (1) "constancy of field index"




$$\left.\frac{\partial k}{\partial p}\right|_{\vartheta = const.} = 0$$

$$k = \frac{r}{B}\left(\frac{\partial B}{\partial r}\right)$$

# Cardinal condition of scaling FFAG (2) "geometrical similarity"




$$\left.\frac{\partial}{\partial p}\left(\frac{\rho}{\rho_0}\right)\right|_{\vartheta = const.} = 0$$

$\rho_0$ : average curvature

$\rho$ : local curvature

$\vartheta$ : generalised azimuth

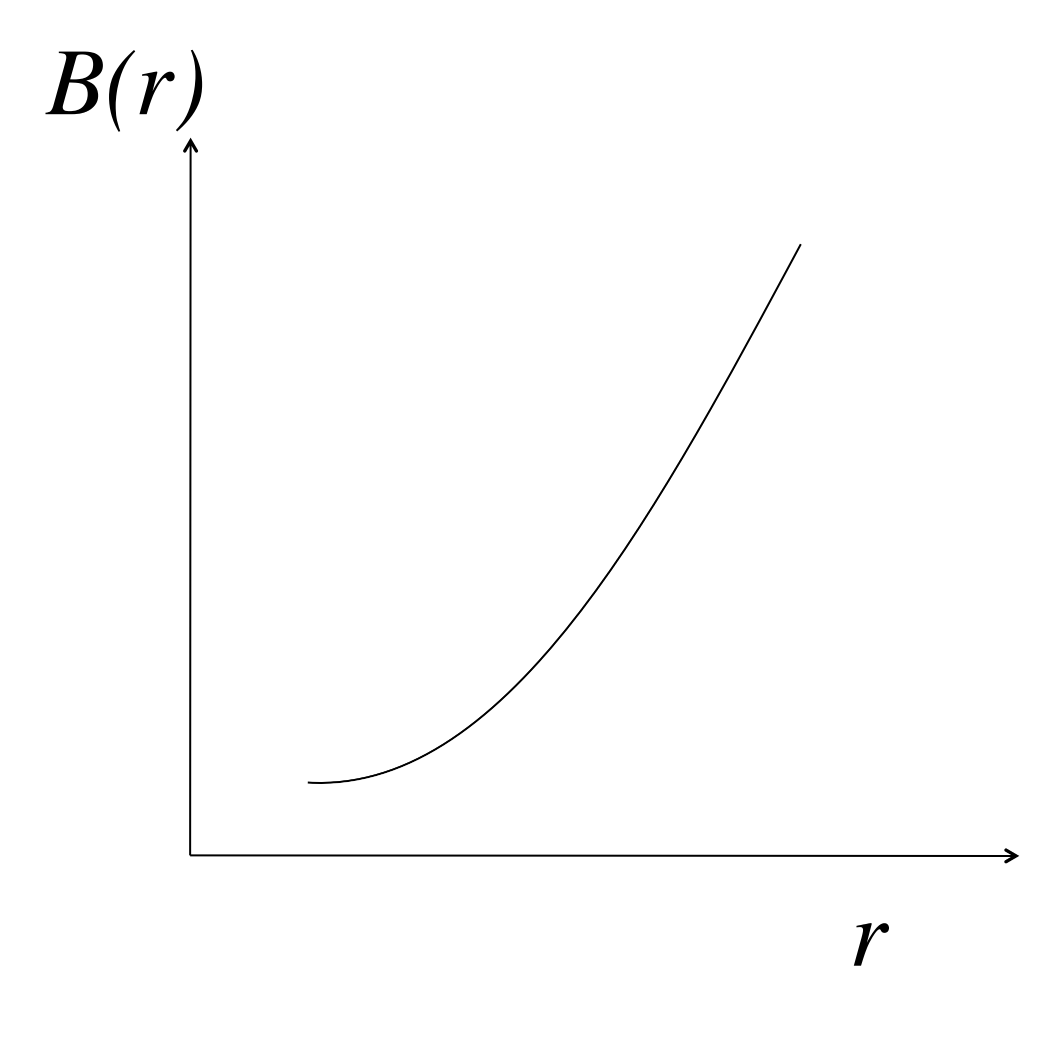
# Field profile in radial direction

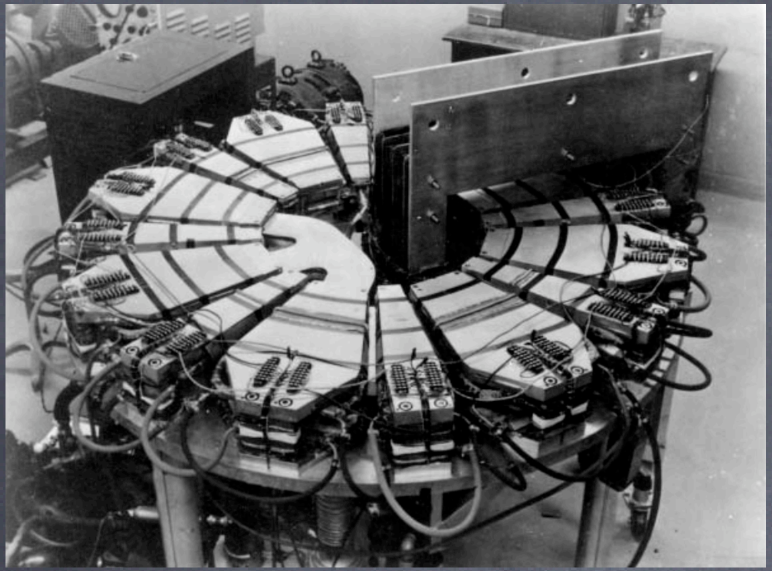
- If the field profile has the following shape, cardinal conditions are satisfied.

$$B(r,\theta) = B_0\left(\frac{r}{r_0}\right)^{k} F(\vartheta)$$

- Field shape is determined by focusing condition, not by isochronous condition.

$$\omega = \frac{eB}{m\gamma} \neq \text{constant}$$





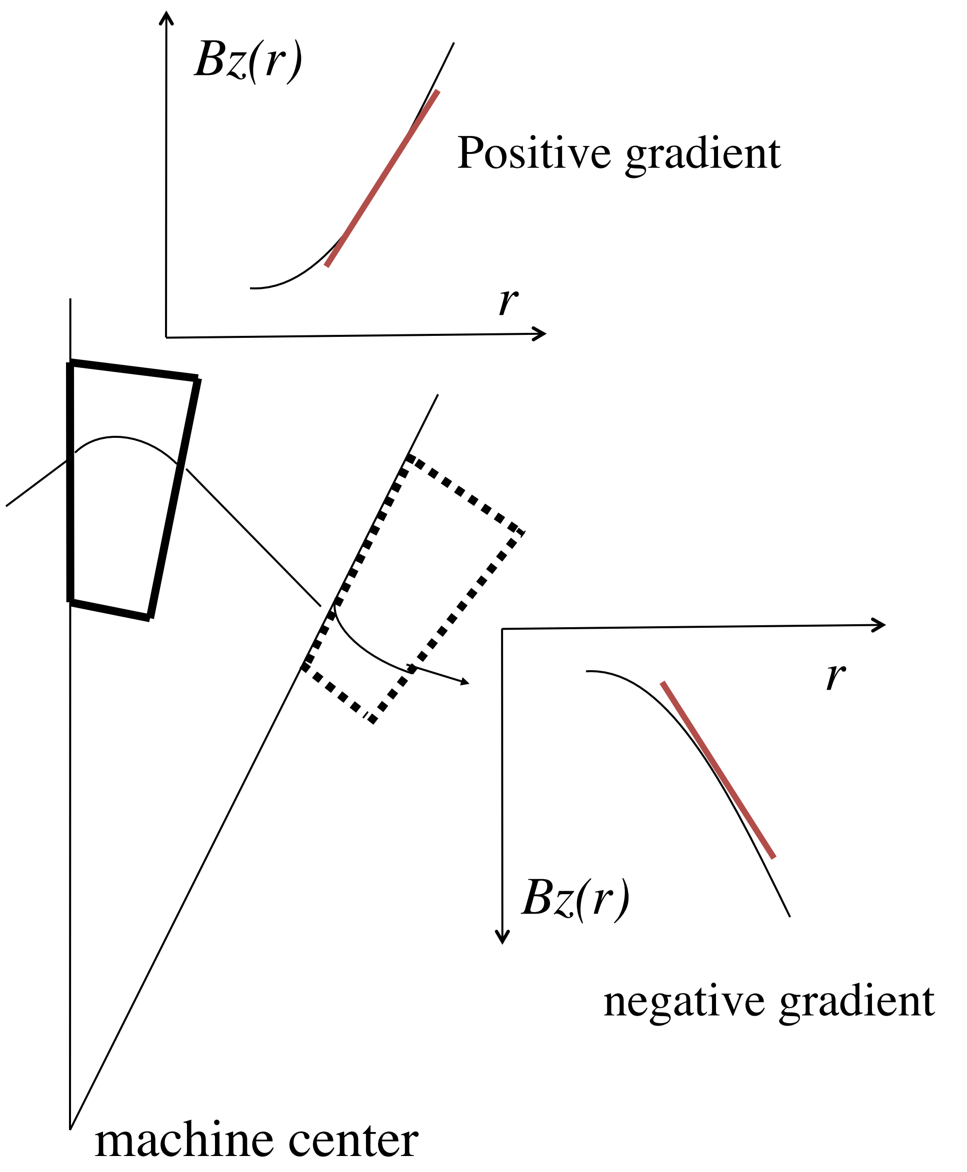
# Radial sector

$$B(r,\theta) = B_0\left(\frac{r}{r_0}\right)^k F(\vartheta)$$

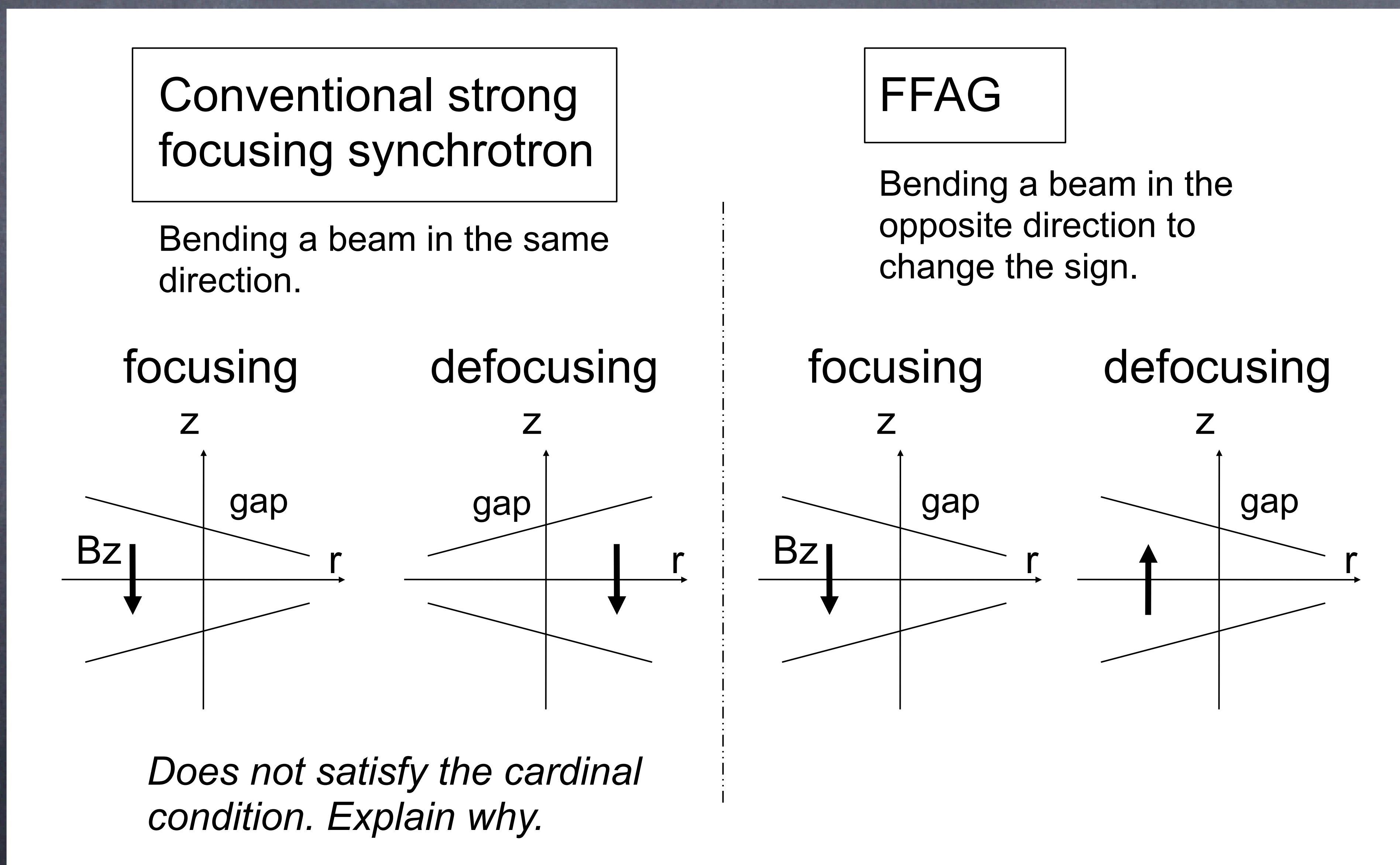
$$F(\vartheta) = F(\theta)$$

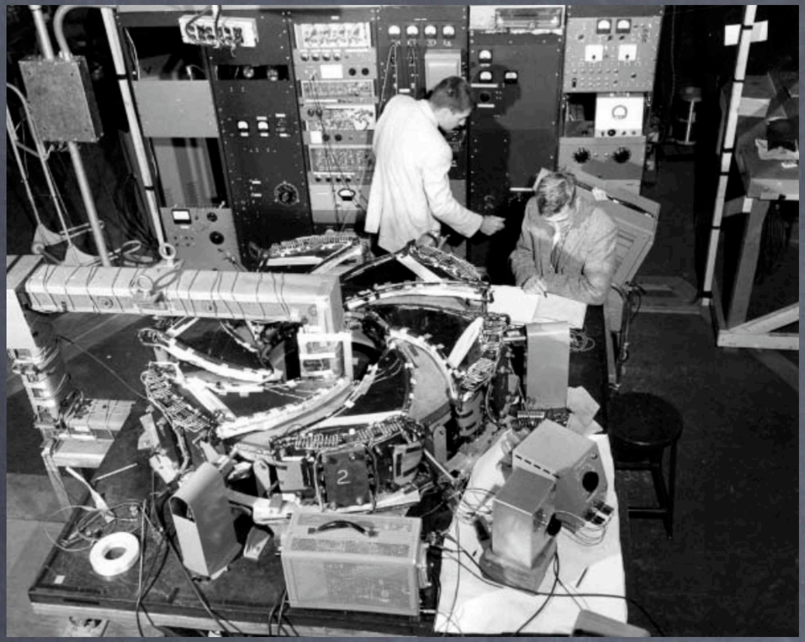
Radial sector type

Alternating magnet has opposite sign of bending.



# AG focusing in synchrotron and FFAG



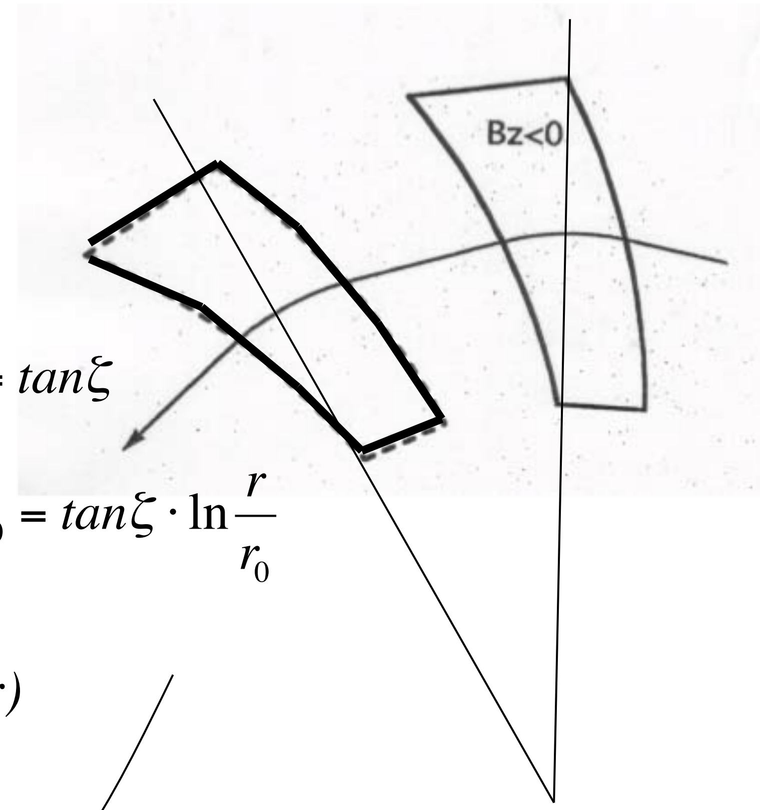


# Spiral sector

$$B(r,\theta) = B_0\left(\frac{r}{r_0}\right)^k F(\vartheta)$$

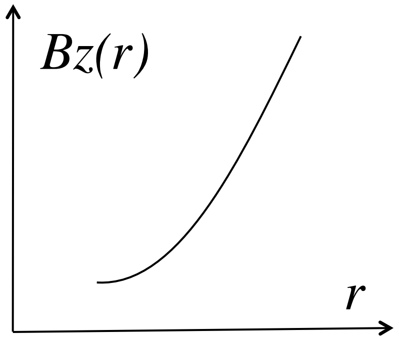
$$\boxed{F(\vartheta) = F\left(\theta - tan\zeta \cdot \ln\frac{r}{r_0}\right)}$$

Spiral sector type




$$\frac{rd\theta}{dr} = tan\zeta$$

$$\theta - \theta_0 = tan\zeta \cdot \ln\frac{r}{r_0}$$




Spiral angle gives strong edge focusing.

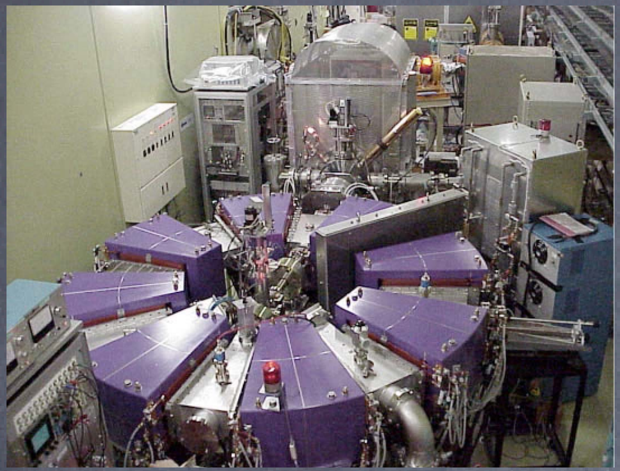
$$\therefore \Delta p_z = \frac{e}{v_x}\int_{-\infty}^{\infty}\left(-v_y B_x\right)dx = -eB_{z0}\tan\zeta \cdot z$$

# Three variables of circular accelerators (3)

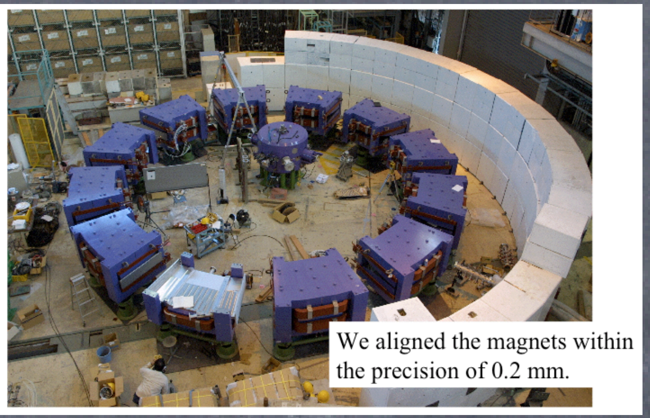
| | Cyclotron | Synchrotron | non-scaling FFAG | scaling FFAG |
|---|---|---|---|---|
| Revolution time | constant | varies (const for relativistic) | varies but small range | varies |
| Circulating radius | varies | constant | varies but small range | varies |
| Transverse focusing | varies | constant | varies | constant |

# Re-birth of FFAG for the last few years

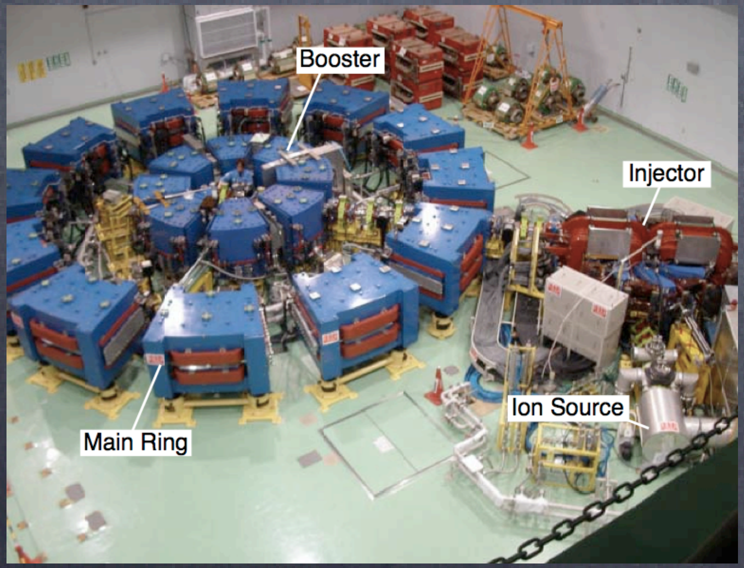
- 2000s



Proof of principle (proton acceleration with rf cavity) machine was constructed in 1999 and demonstrate rapid acceleration with 1 kHz.




Scale up version of PoP FFAG was constructed as a prototype of proton therapy machine.

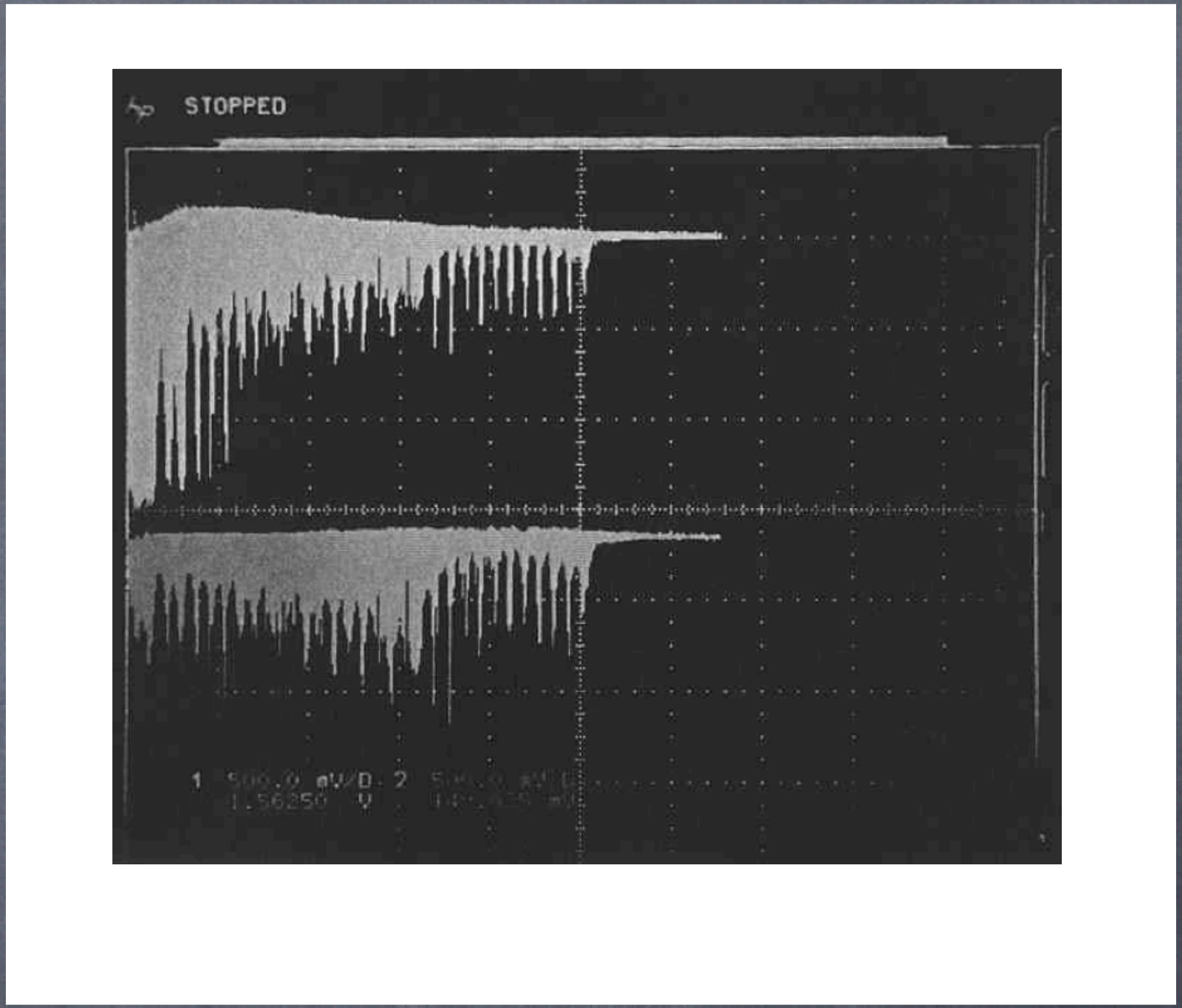


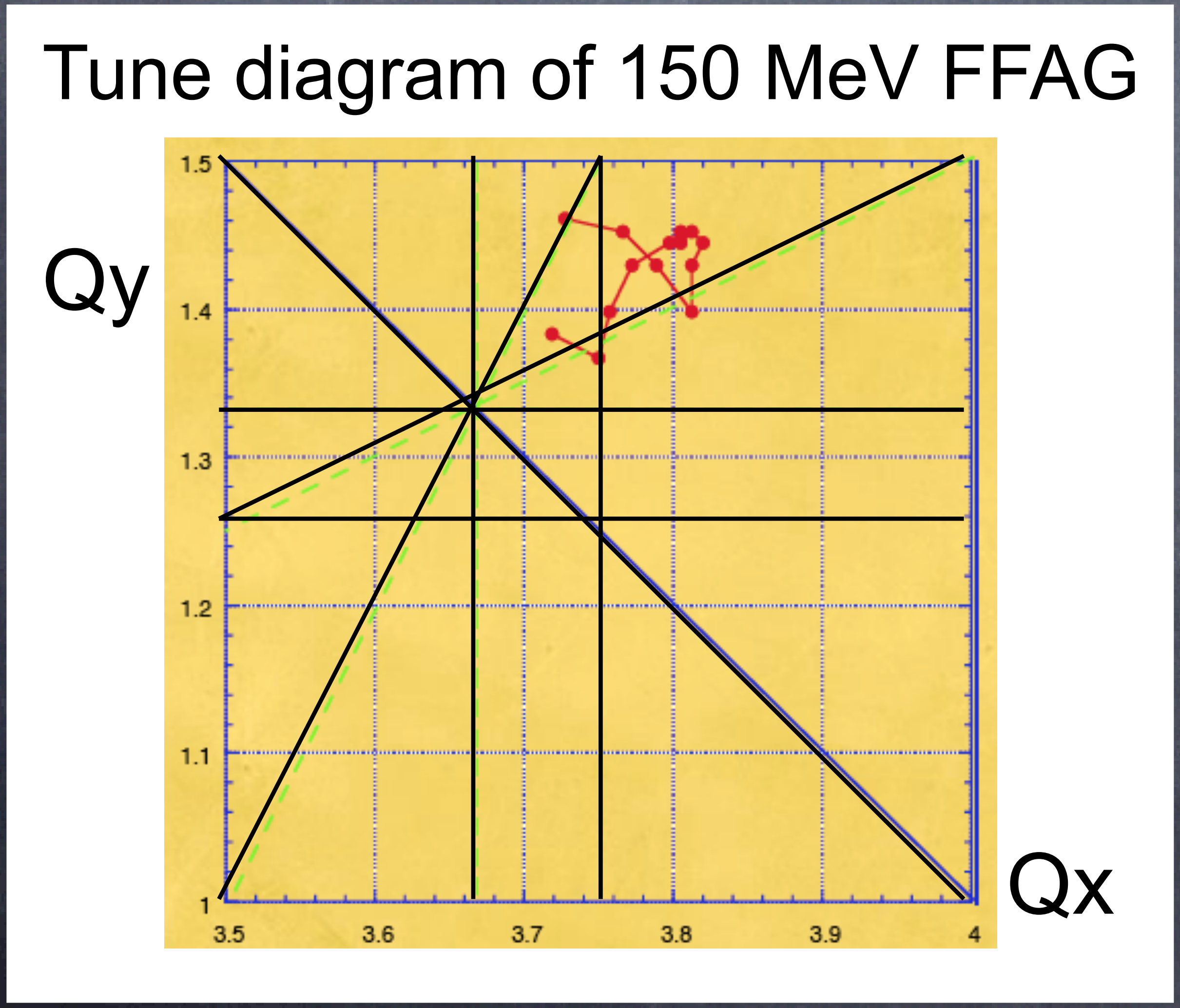

3 stage FFAG for ADSR
- 2.5 MeV spiral (ion beta) FFAG with induction cores
- 25 MeV radial (booster) FFAG with RF and flat gap
- 150 MeV radial (main) FFAG with RF and tapered gap

*Beam power is still moderate.*

# Scaling FFAG results

- Demonstration of 1 ms acceleration (1 kHz equivalent).




- Tune excursion in 150 MeV FFAG.

# Summary so far (3)

- FFAG accelerators are similar to synchrotron, but operated with fixed field magnets, which give many advantages.
- Specific optics and orbit design for FFAG overcomes potential disadvantages as being fixed field operations.
- Specific shape of field make the transverse tune constant.

# Tune stabilised FFAG

- EMMA is a linear non-scaling FFAG.
- Original FFAG follows the scaling design.
- It is possible to fix the tune without following scaling principle.
  - Tune stabilised FFAG.
  - or Nonlinear non-scaling FFAG.

# Scaling FFAG vs. synchro-cyclotron

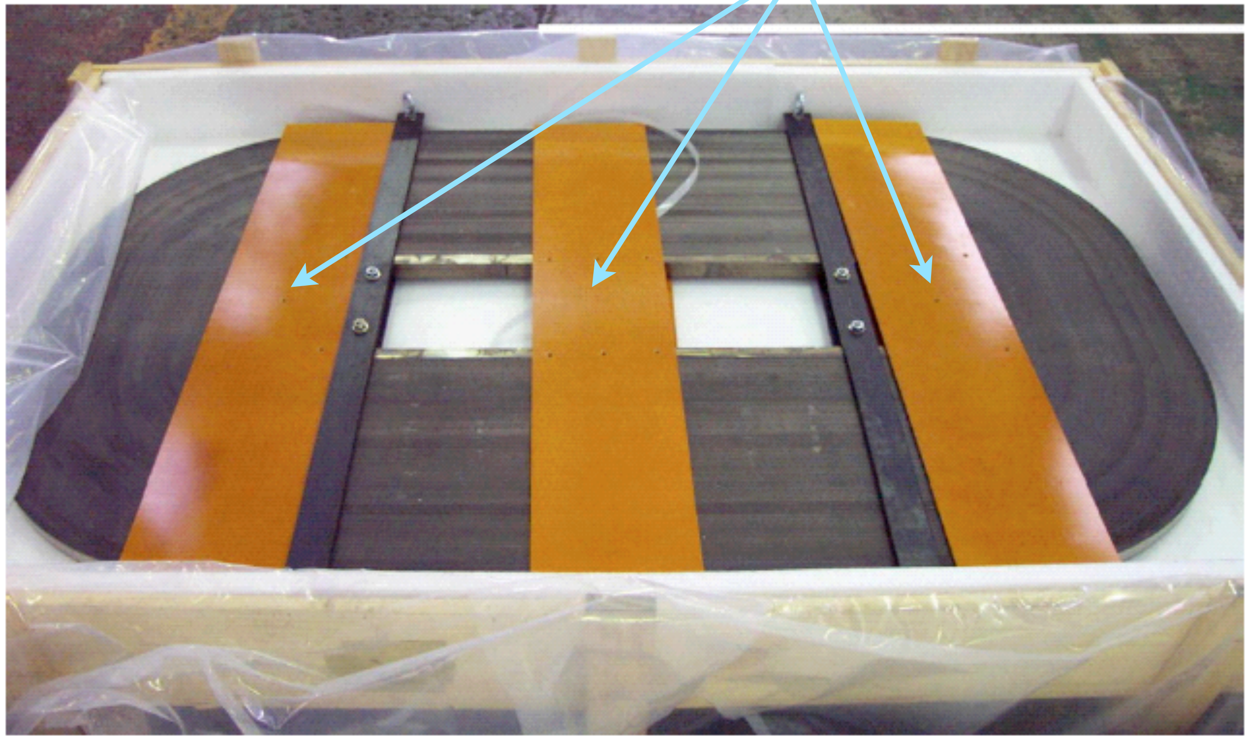
- What is the difference between FFAG and synchro-cyclotron?
  - very similar
  - strong transverse focusing (alternating gradient)
  - transverse tune is constant
  - orbit does not move much
- probably there is no point to make a synchro-cyclotron now, it should be a FFAG.

# One slide on hardware

- Magnetic Alloy (MA) cavity was the most crucial hardware part of recent scaling FFAG development.

- High shunt impedance for quick acceleration.
- Low Q for quick change of frequency.
- Arbitrary shape by thin tapes.

These plates should be taken off before the installation.



MA core for 150MeV FFAG
1.7m x 0.985m x 30mm

# Summary

- FFAG accelerators are similar to synchrotron, but operated with fixed field magnets, which give many advantages.
- Specific optics and orbit design for FFAG overcomes potential disadvantages as being fixed field operations.
- Specific shape of field make the transverse tune constant.
- So far, it seems promising idea. However, we need to demonstrate high beam power operation.

Thank you for your attention.

Thank you

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