Cold Copper Structures and the Anomalous Skin Effect

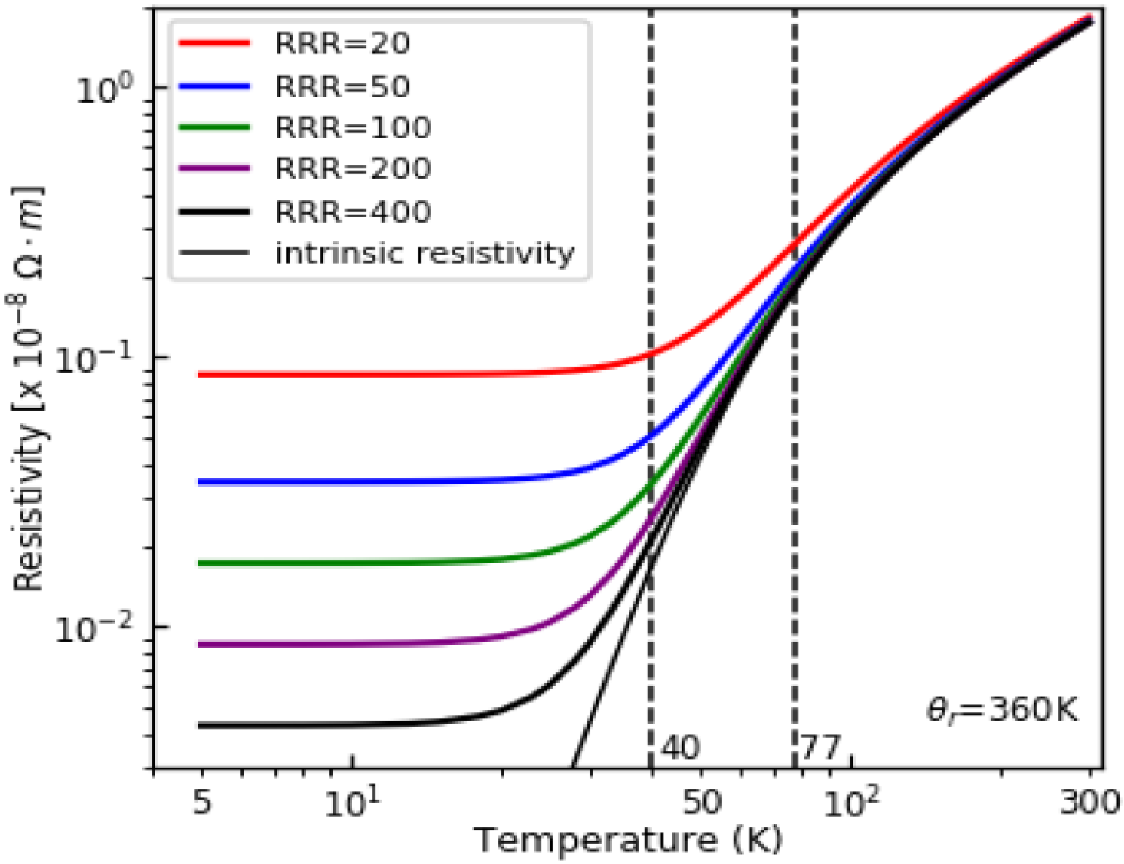
Winterseminar St. Michael, 25.02. – 02.03.24

U. Ratzinger

Motivation

- Copper can show an enormous resistivity reduction at low temperatures

Specify the RRR – value when buying the copper!



Resistivity $\varrho / \Omega m$

Conductivity $\kappa / (\Omega m)^{-1} = 1/\varrho$

$$RRR \equiv \frac{\varrho(293K)}{\varrho(4.2K)}$$

Motivation

- The Q – value of a cavity depends on $\varrho^{-1/2}$ due to the normal skin effect

$$Q = g \cdot \frac{V}{A \cdot \delta}; \quad P_{loss} \sim \frac{1}{Q}$$

$g \equiv$ geometric factor

$$\delta = \sqrt{\frac{2 \cdot \varrho}{\omega \cdot \mu_0}} = \sqrt{\frac{2}{\omega \cdot \mu_0 \cdot \sigma}}$$

$\delta \equiv$ skin depth

Maximum possible gain in Q by cooling (at normal skin effect):

$$G_{max} = \sqrt{RRR}$$

Motivation

- Let us assume a copper cavity with $RRR = 400$

  $G_{max} = \sqrt{RRR}$=20

  at operation temperature $\mathrm{T} \leq 20K$

  That would be very attractive for pulsed beam operation:

  Instead of 1 MW a cavity would need only 50 kW to get

  the same acceleration voltage.

Motivation

Cooling efficiency at temperature T=20K:

$$\eta_{cryo}(T) \cong \frac{T}{293 - T} \cdot 0.3 = 0.022.$$

At a typical synchrotron injector duty factor of

$$\eta_{duty} = 1 \cdot 10^{-3}$$

the grid power needed for cooling away the dynamic losses is about

$$P_{cool} \cong \frac{P_{cavity} \cdot \eta_{duty}}{\eta_{cryo}(T)} = \frac{5 \cdot 10^{4} \cdot 1 \cdot 10^{-3}}{0.022} = 2.3\text{kW}.$$

Motivation

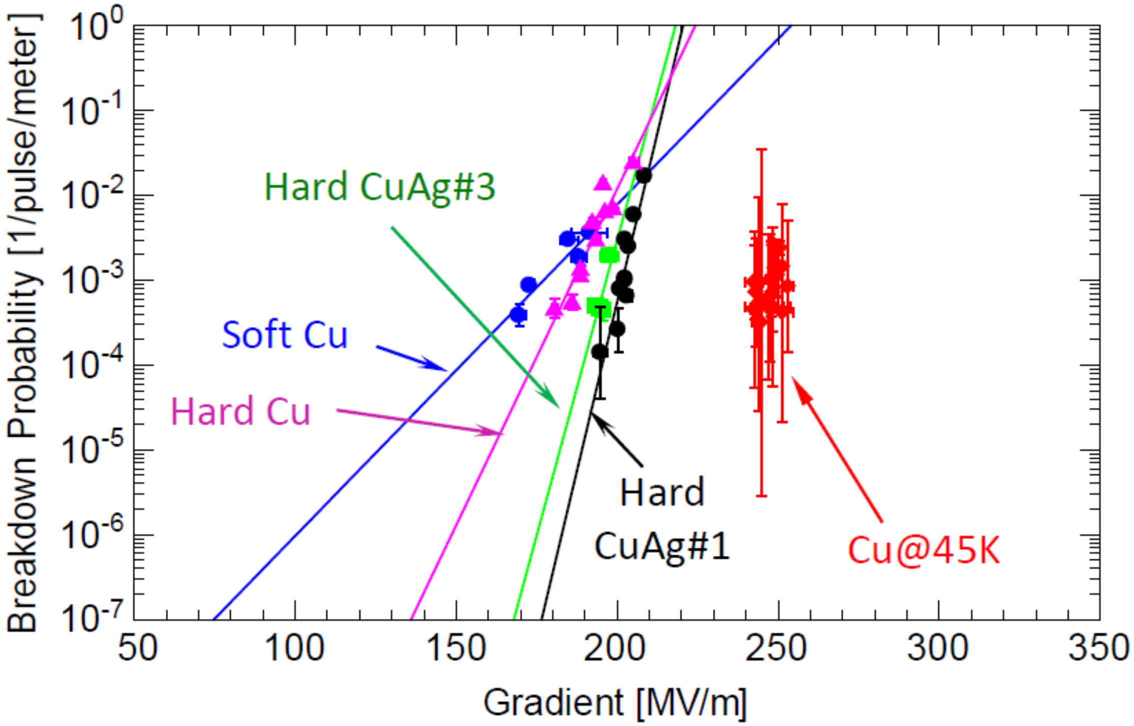
- Measurements from CERN and SLAC tell us

  Higher operable fields in „cool copper cavities“

  M. Jacewicz et al., PHYS. REV. APPLIED **14,** 061002 (2020)

  M. Nasr et al., PHYS. REV. AB 24, 093201(2021)

  A.D. Cahill et al., PHYS. REV. AB 21, 102002 (2018)



$\Delta t \cong 200ns$

$f \cong 11GHz$

V. Dolgashev et al., SLAC, HG Workshop 2017, Spain

Motivation Ion Acceleration

Maximum effective voltage gains of 10.7 MV/m along a 202 MHz IH cavity were demonstrated successfully at room temperature: 1.3 MW, 1.54 m tank length, 1.87 – 3.05 MeV/u.

That is an RF power of 840 kW/m.

Operation at 20 K operation and 0.1% duty factor

$$E_{gain} = 10\ MV/m$$

Grid power for cooling: $P_{cool} \cong \frac{4.2 \cdot 10^{4} \cdot 1 \cdot 10^{-3}}{0.022} = 1.9\mathrm{kW/m}$

RF power at 20 K operation: about 42 kW/m + beam power.

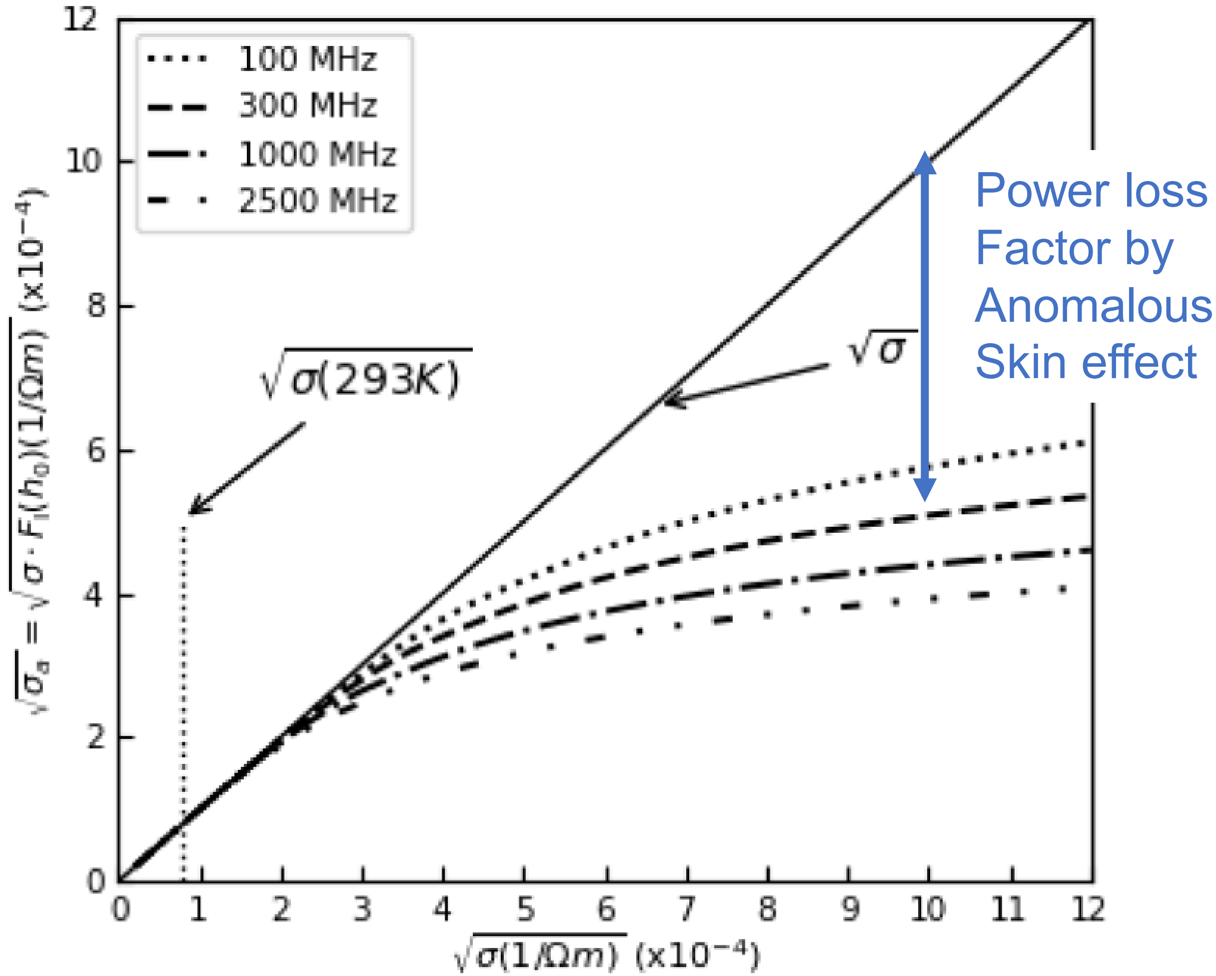
Would be great!!!

Motivation

- However: The anomalous skin effect reduces the gain in RF conductivity!!!

That means, the gain factor $G_{max} = \sqrt{RRR}$ is too optimistic in many cases

That is the reason why the anomalous skin effect has to be studied carefully



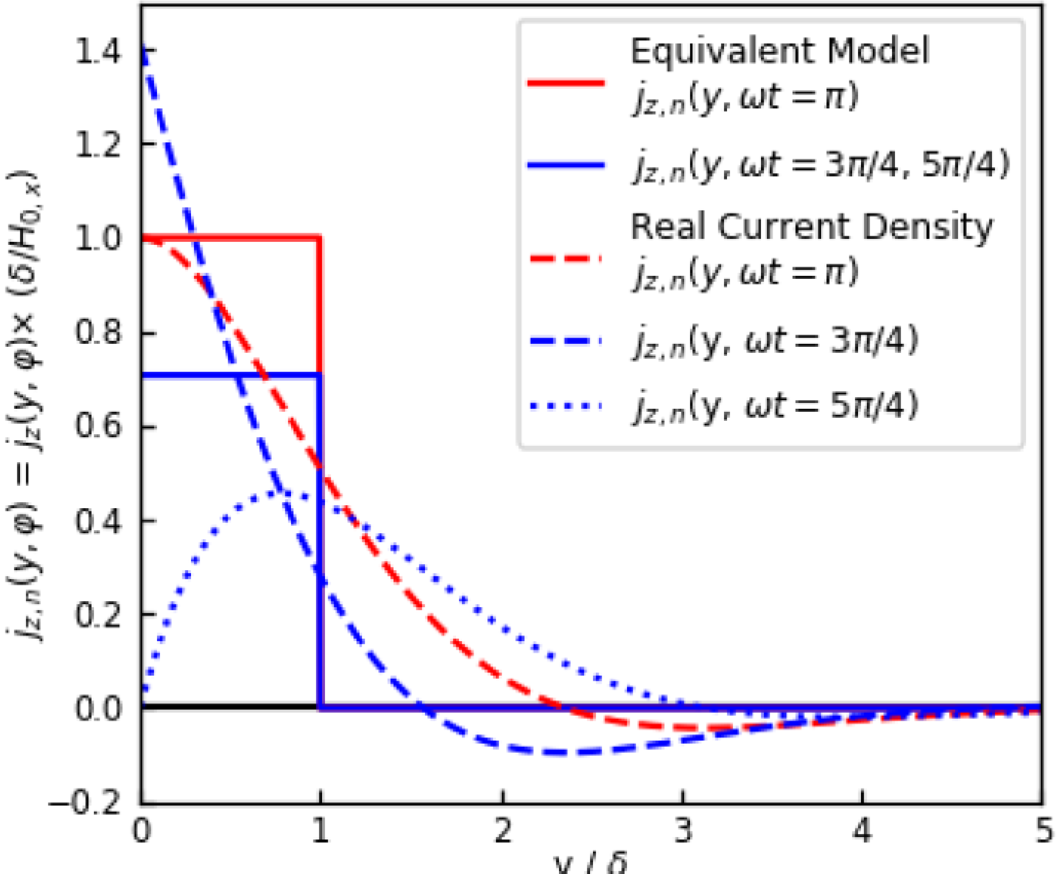
Relevant features of the free electrons in copper

$$\sigma(T) = \frac{n_e e^2 \tau(T)}{m}$$

$$l = v_F \cdot \tau$$

Fermi – energy: E_F/eV	7.0	
Fermi – velocity $v_F/m/s$	$1.57 \cdot 10^6$	
Free electron density n_e/m^{-3}	$8.5 \cdot 10^{28}$	
Electron mass mc^2/keV	511	
Parameters at 293 K		Parameters at 15K , RRR=300
Spec. electr. resistivity $\varrho_n/\Omega m$	$1.7 \cdot 10^{-8}$	$5.65 \cdot 10^{-11}$
El. Conductivity $\sigma_n/S/m$	$5.9 \cdot 10^7$	$1.77 \cdot 10^{10}$
e^-- ion - collision time τ/s	$2.46 \cdot 10^{-14}$	$7.38 \cdot 10^{-12}$
Free electron path $l/\mu m$	0.039	11.7

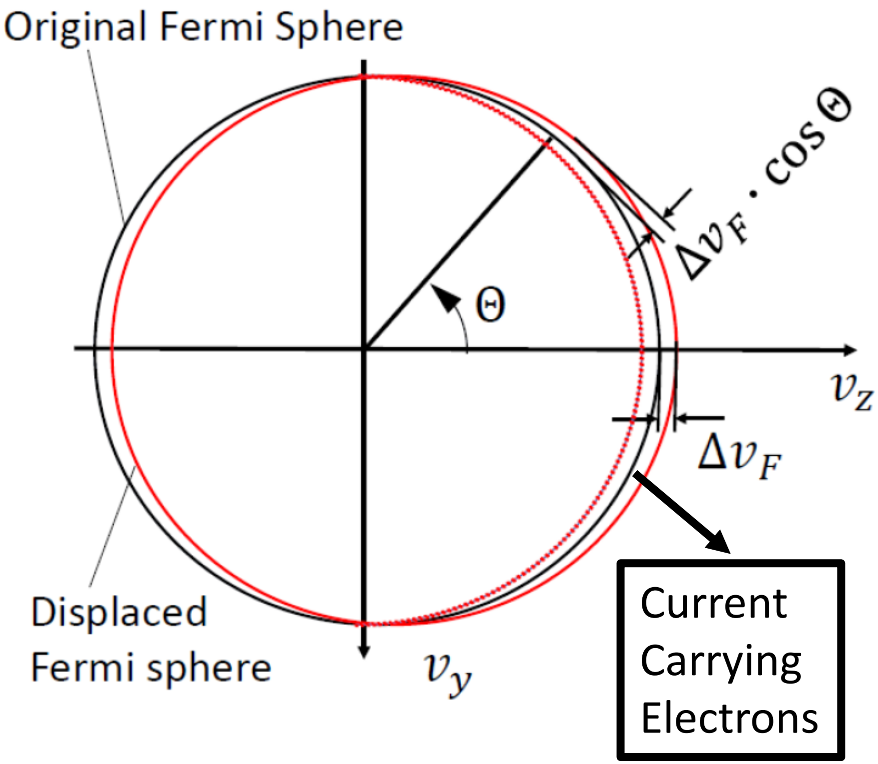
Relevant features of the free electrons in copper

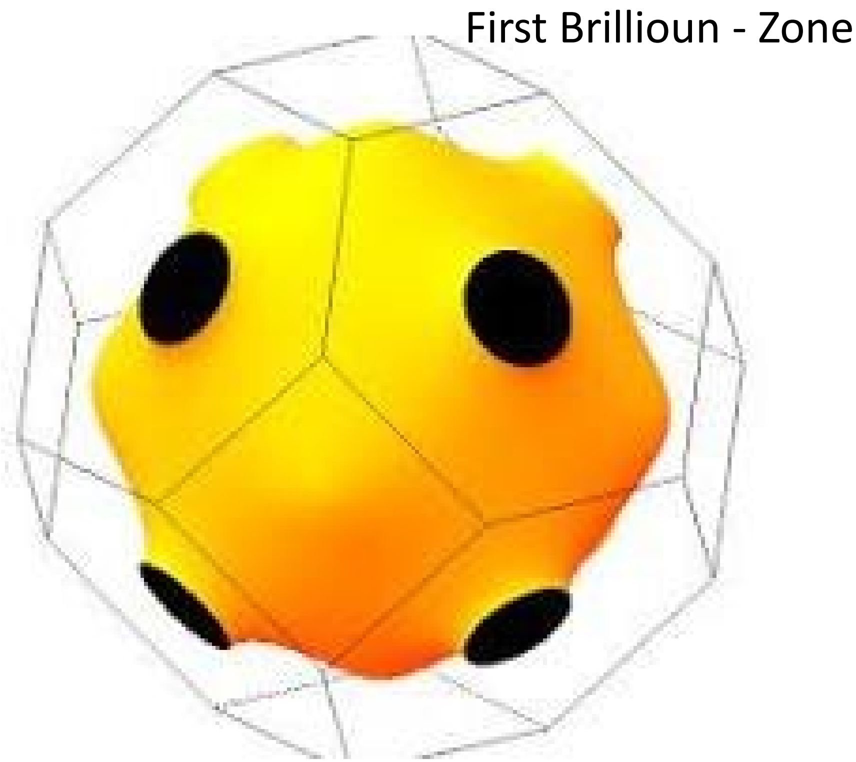


$f\,/MHz$	$\delta(293K)/\mu m$
50	9.28
100	6.56
200	4.64
300	3.79
400	3.28
500	2.94
600	2.68
700	2.48
800	2.32

Current density distribution at different phase angles across the skin sheath

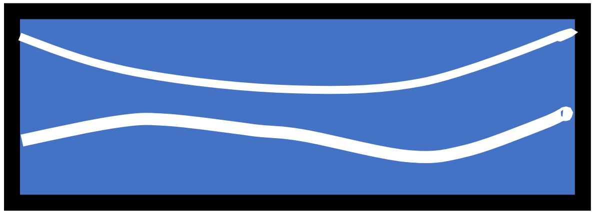
The Free Electron Model





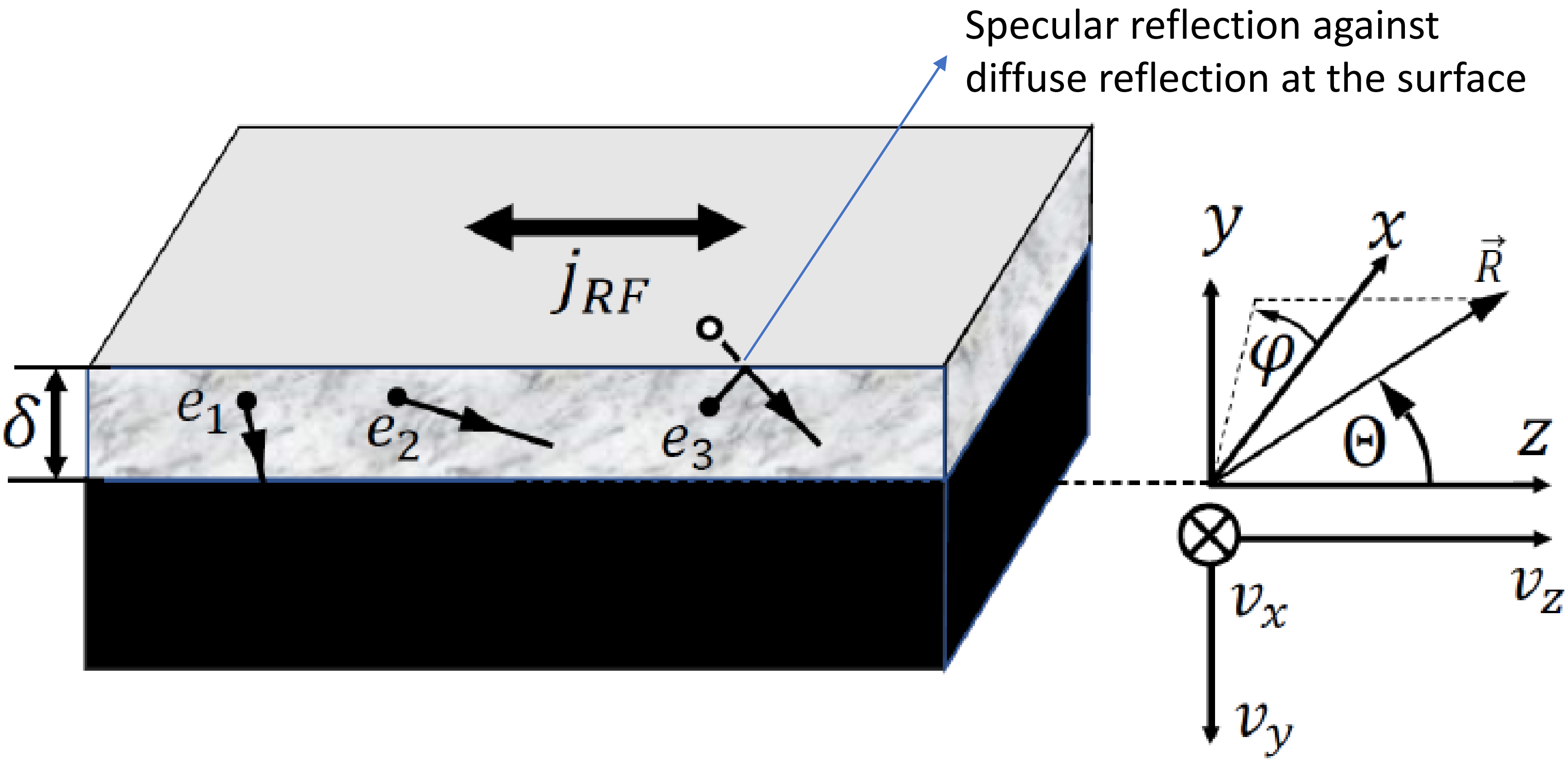
Source: https://www.hzdr.de/projects/fermisur/cu.html

Free electron velocity distribution
at a certain density in local space



Real Cu Fermi surface:
Reaching locally the Brillioun-Zone,
else corresponding well to the
free electron model

The Anomalous Skin Effect



Paths of current carrying electrons and coordinate system

Task: How to calculate the reduced RF conductivity within δ ?

The Anomalous Skin Effect

The Diffusion Model from Pippard, Reuters, Sondheimer, Chambers from the years 1947 – 1952 explained the anomalous skin effect - up to optical frequencies. It is based on small angle scattering of electrons, and a diffusion model with using the Boltzmann Kinetic equation + Maxwell equations!

$$\frac{\partial f}{\partial t} - \frac{2\pi\epsilon}{h}\left(\mathscr{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right).\mathrm{grad}_{\mathbf{k}} f + \mathbf{v}.\mathrm{grad}_{\mathbf{r}} f = -\frac{f-f_0}{\tau},$$

$$f = f_0 + f_1(\mathbf{v}, z), \qquad f_0 = 1/\{e^{(E-\zeta)/kT} + 1\}$$

The surface impedance for a wave directed normal to the metal surface is calculated

The Anomalous Skin Effect

Speculation: The diffusion model seems too optimistic – at least at frequencies relevant for ion accelerators (below 2 GHz).

This was our motivation to develop a model for these lower frequencies. The result is the Geometric Model

(**arXiv:2211.00135v2 [physics.app-ph] 30 Nov 2023)**

It calculates the surface resistance only and is focused on applications like cavities and waveguides

The Anomalous Skin Effect below 2.5 GHz

Electron Equation of motion at a given collision time τ :

$$m\dot{v}_z + m\frac{v_z}{\tau} = e \cdot E_z$$

$$m \cdot \left(\frac{1}{\tau} + i\omega\right) v_z = e \cdot E_z$$

$$j_z = -n_e \cdot e \cdot v_z = -n_e \cdot e \cdot \Delta v_F = \sigma \cdot E_z$$

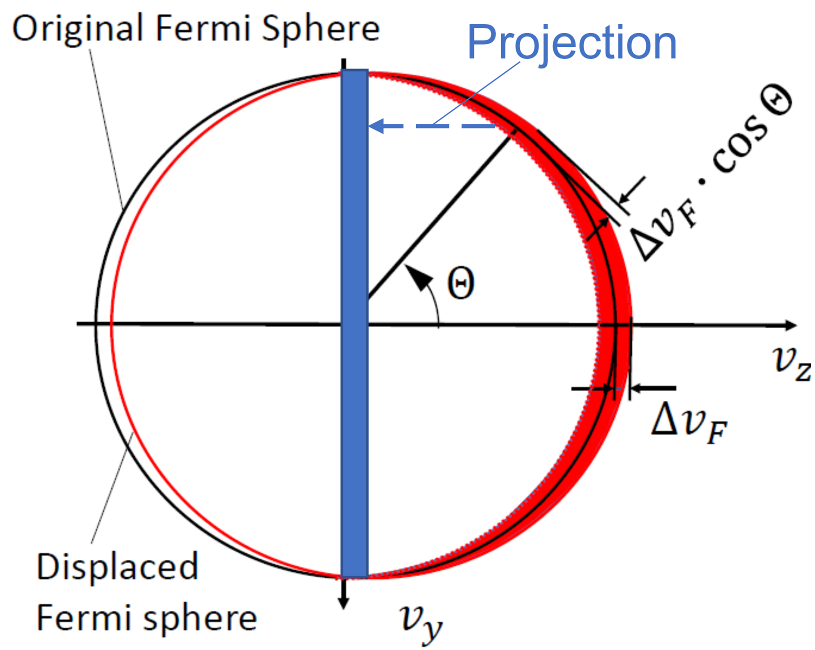
$$\sigma = \frac{n_e \cdot e^2 \cdot \tau \cdot (1 - i\omega\tau)}{m \cdot (1 + \omega^2\tau^2)} \rightarrow \frac{n_e \cdot e^2 \cdot \tau}{m} \quad \text{for } f \le 2.5\ GHz$$

Starting point is the DC conductivity in good approximation

For $RRR \le 300,\ \tau(4.2K) = 7.38 \cdot 10^{-12} s;$

Geometric Model

Task: Calculation of the lost electron fraction in every velocity space angle within the red coloured half sphere at the Fermi surface and for every position y within the skin layer.



$$dn_c(\Omega) \sim 2\Delta v_F \cdot v_F^2 \cdot \cos\Theta \cdot \sin\Theta \cdot d\theta d\varphi$$

$$v_z = v_F \cdot \cos\Theta$$

$$j_{z,tot} \sim e \cdot 2\Delta v_F \cdot v_F^3 \cdot \int_0^{\frac{\pi}{2}} \int_0^{2\pi} cos^2\Theta \sin\Theta \, d\varphi d\Theta = \frac{4\pi}{3} \cdot \mathrm{e} \cdot \Delta v_F \cdot v_F^3$$

Simplifikation:
Projection into xy-plane:

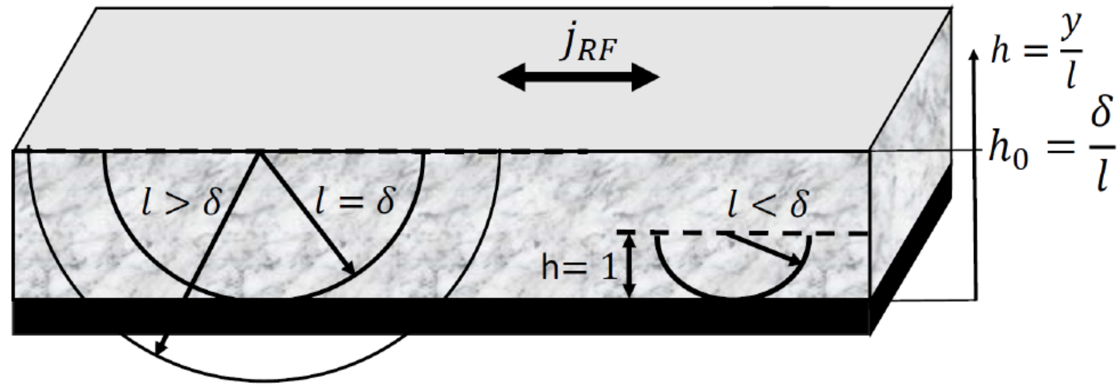
$$j_{z,tot} \sim e \cdot 2\Delta v_F \cdot v_F^3 \cdot \int_{-1}^{1} \int_{-(1-w^2)^{1/2}}^{(1-w^2)^{1/2}} (1 - u^2 - w^2)^{1/2} \, du dw.$$

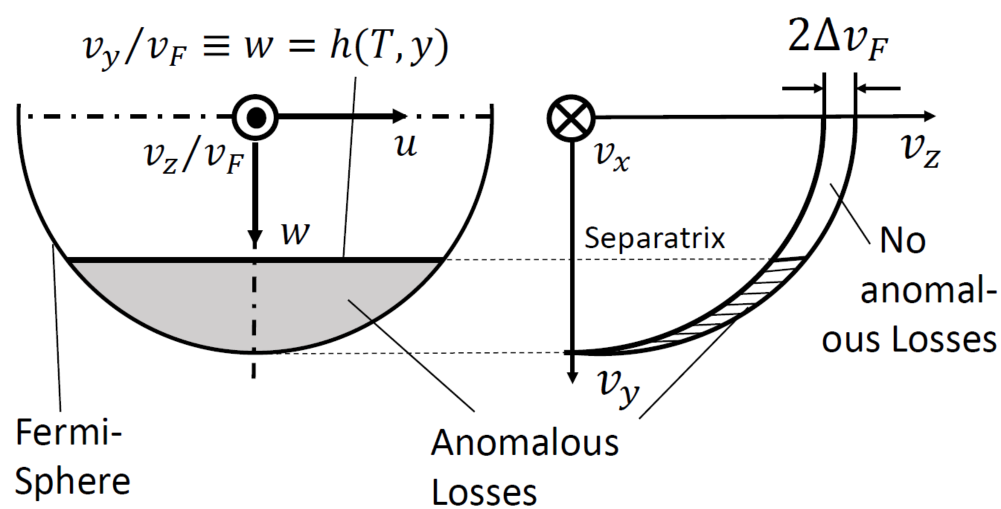
$$u = v_x / v_F \, , w = v_y / v_F$$

Fraction of lost electrons with positive velocity

The loss condition for an RF current carrying electron at y is

$$y < v_y \cdot \tau(T),$$





$$w > 1.53 \cdot 10^{15} \cdot \frac{y}{\sigma} \ ;$$

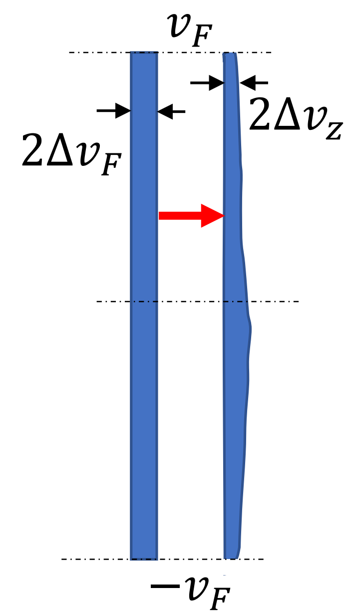
$0 < y < \delta$ (diffuse refl.),

$0 < y < 2\delta$ (specular refl.)

Velocity Space angle dependent losses in plane y

Within a given loss angle around Θ, φ, the loss time at y is given by

$$\Delta t = \frac{y}{v_y} = \frac{y}{v_F \cdot \sin\Theta \sin\varphi} = \frac{h \cdot \tau}{\sin\Theta \sin\varphi} = \frac{h \cdot \tau}{w}$$



Including this loss mechanism in the electron eq. of motion one gets

$$m\dot{v}_z + m\frac{v_z}{\tau} + m\frac{v_z}{\Delta t} = -e \cdot E_z$$

and in equilibrium $$m \cdot \frac{v_z}{\tau'} = -e \cdot E_z$$

with $$\tau' = \frac{\tau \cdot \Delta t}{\tau + \Delta t} = \tau \frac{h}{w+h}$$

$$h(T, y) \equiv h = \frac{y}{v_F \cdot \tau(T)} = \frac{y}{l(T)}; \qquad w = \frac{v_y}{v_F}.$$

Anomalous losses in plane y

This results in a reduced local conducting electron sheath thickness in velocity phase space:

$$2\Delta v_z'(\mathrm{h},\mathrm{w}) = 2\Delta v_F \cdot \frac{\tau'}{\tau} = 2\Delta v_F \frac{h}{w+h}, \qquad \text{if} \quad h(T,y) < 1$$

The resulting conductivity at the layer $h = y/l$ within the skin sheath is calculated now by replacing the constant Δv_F by $\Delta v_z'(\mathrm{w})$ within the range $h \le w \le 1, h < 1$, and adding the loss free part in the velocity range $0 \le w < h,\ h < 1$.

$$j_{z,1}'(h) \sim e \cdot 2\Delta v_F \cdot v_F^3 \cdot \int_h^1 \int_{-(1-w^2)^{1/2}}^{(1-w^2)^{1/2}} (1-u^2-w^2)^{1/2} \cdot \frac{h}{w+h}\, du dw.$$

Anomalous losses

$$j_{z,2}'(h) \sim e \cdot 2\Delta v_F \cdot v_F^3 \cdot \int_0^h \int_{-(1-w^2)^{1/2}}^{(1-w^2)^{1/2}} (1-u^2-w^2)^{1/2}\, du dw$$

Loss free

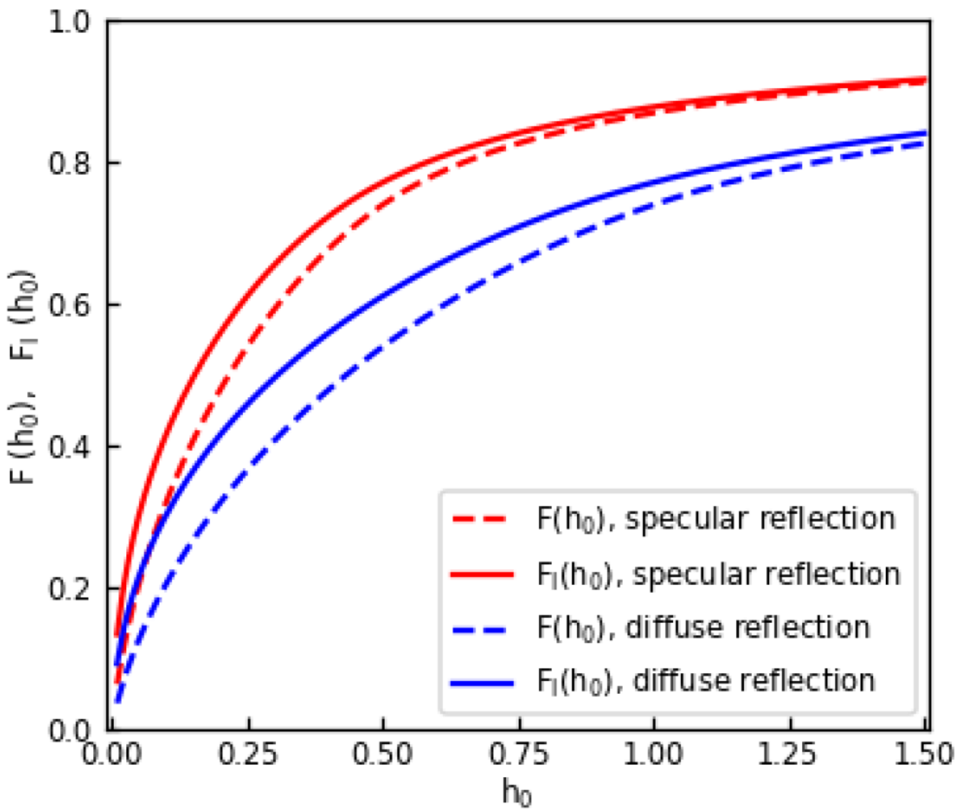
$$j_{z,3}'(h) \sim e \cdot 2\Delta v_F \cdot v_F^3 \cdot \int_0^1 \int_{-(1-w^2)^{1/2}}^{(1-w^2)^{1/2}} (1-u^2-w^2)^{1/2}\, du dw = \frac{2}{3}\pi e \Delta v_F v_F^3 .$$

Loss free

Anomalous losses within the skin sheath

- To get all losses within the skin sheath an integration along the y – axis is performed from $h = y/l = 0$ to $h = h_0 \equiv \delta_c/l$.
- The velocity space with negative v_y is included
- Specular and diffuse reflection at the surface are calculated separately
- The classical skin depth δ_c is iteratively changed into δ_a

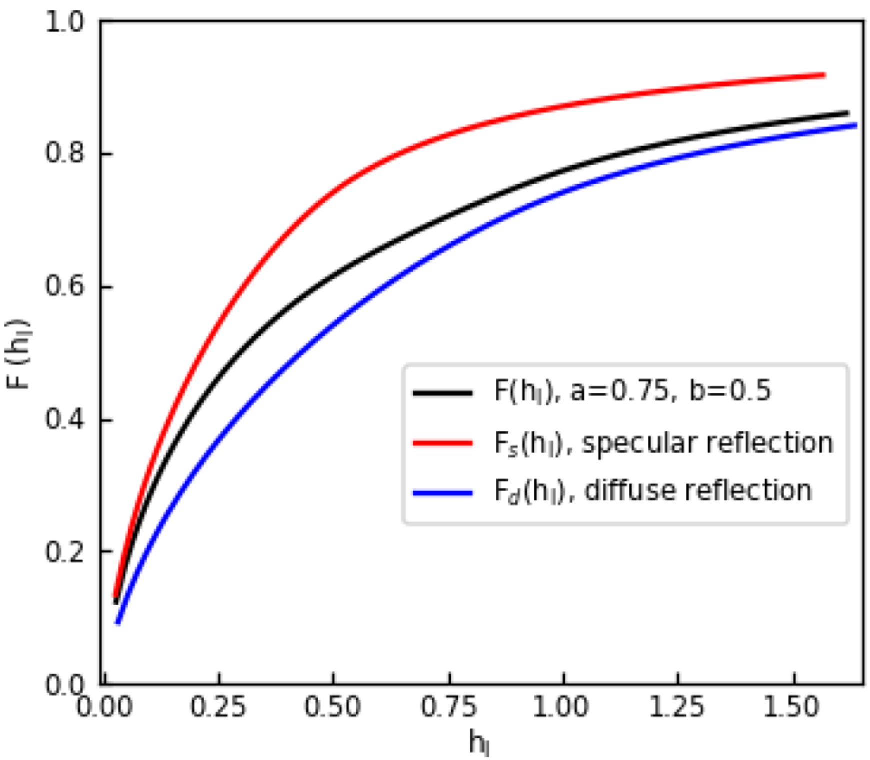
Resulting reduction in RF conductivity

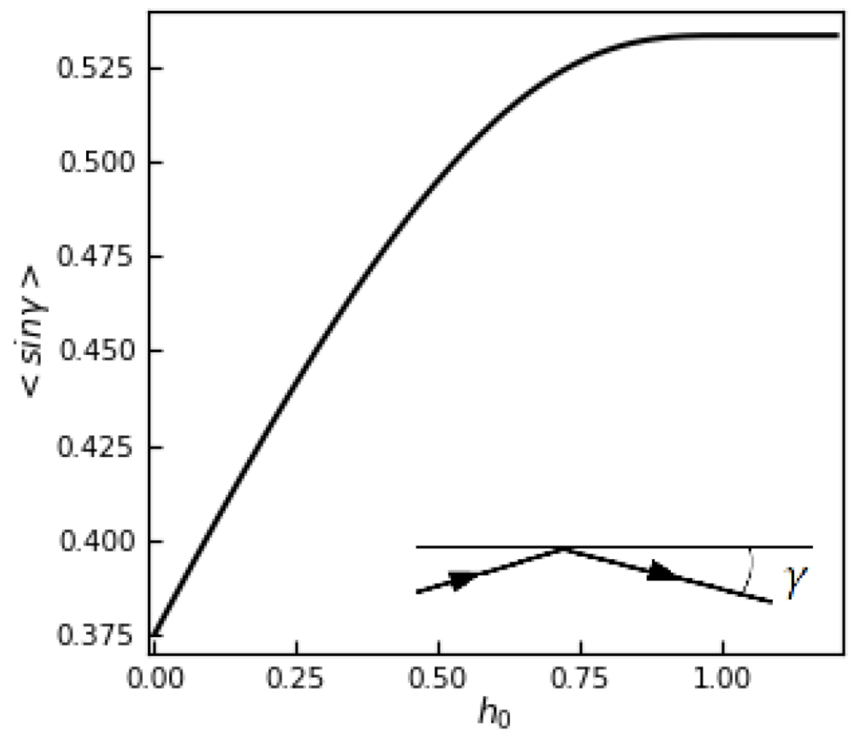


$$F_l(h_0) = \sigma_a(T)/\sigma(T)$$

$$h_0 = \delta_c/l$$

Fit to experimental data by a combination of „specular“ and „diffuse“ reflection



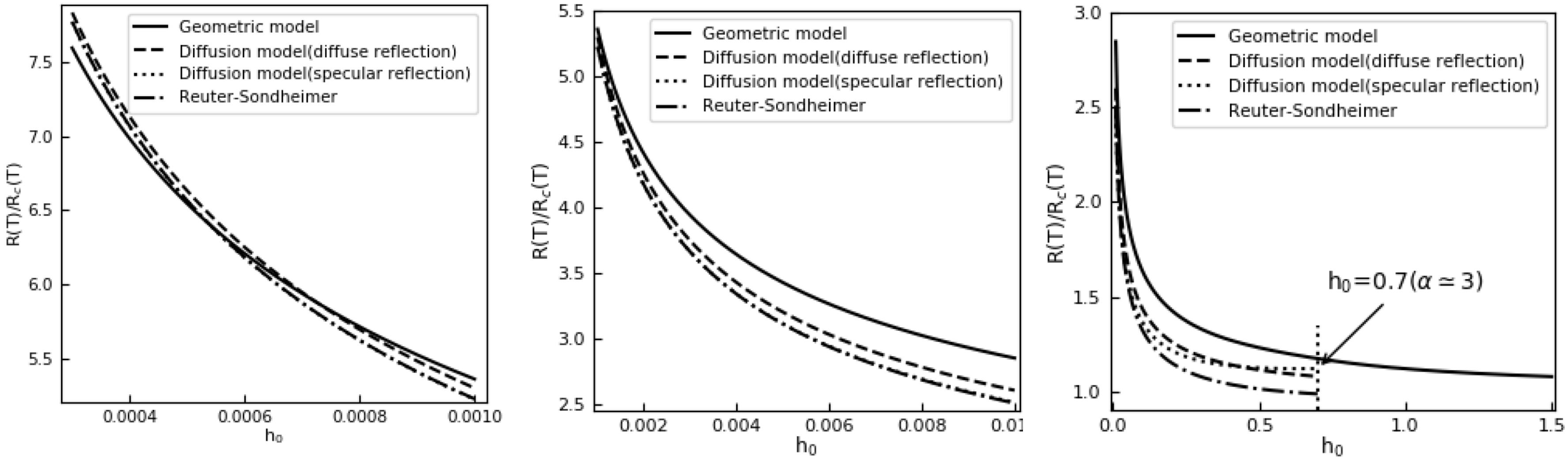


$h_l = \delta_a / l$

$h_0 = \delta_c / l$

At low temperatures the averaged reflection angle γ gets smaller, motivating an increased probability for specular reflection.

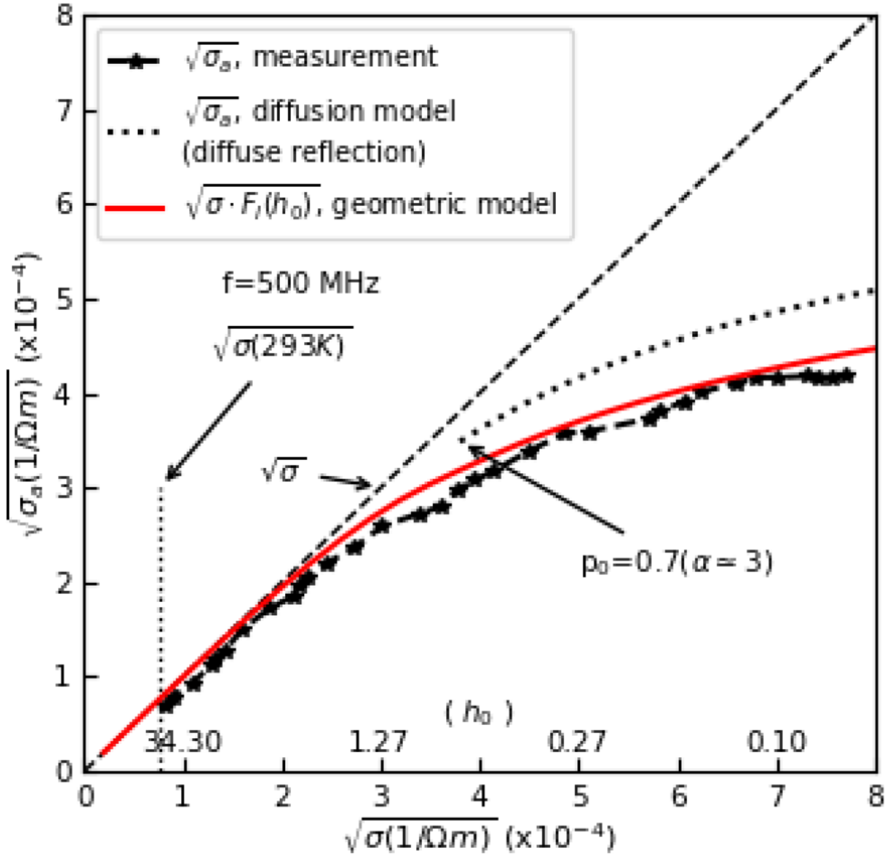
Surface Resistance – a comparison of models

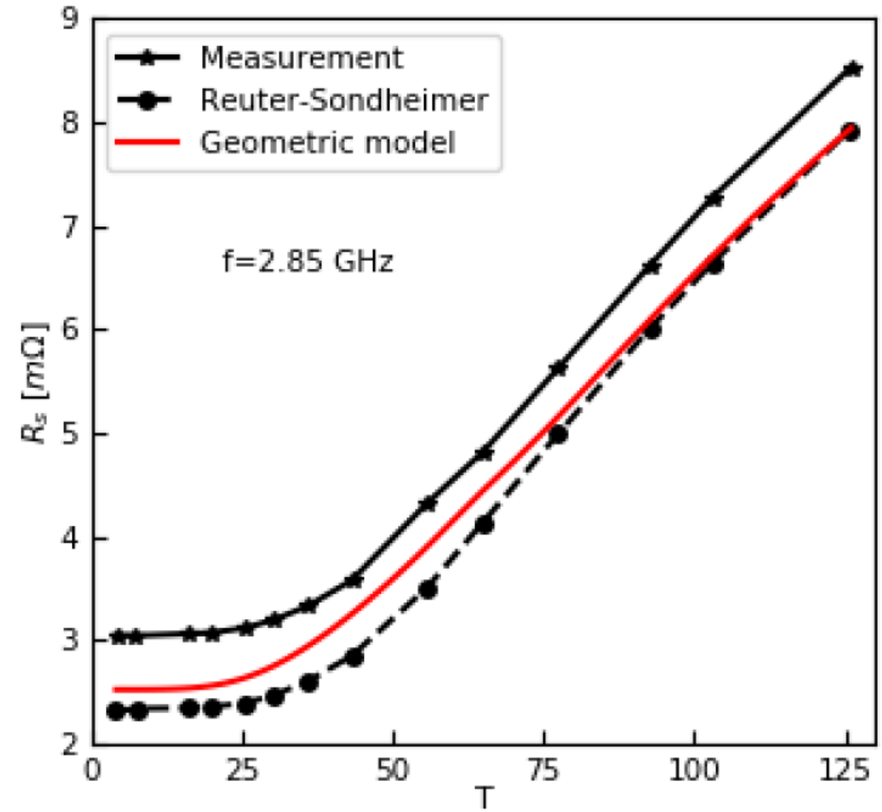


$$R(T)/\Omega = \frac{\varrho_a(T)}{\delta_a(T)} = \sqrt{\frac{\omega \cdot \mu_0}{2\sigma_a(T)}};$$

$$R(T)/\Omega = \sqrt{\frac{\omega \cdot \mu_0}{2 \cdot \sigma(T) \cdot F_l\,(h_0)}} = R_c(T) \cdot F_l\,(h_0)^{-1/2}$$

Comparison to measurement





W. Weingarten, CERN
Parliclc World, Vol I. No 4 p 93—]03. 1990

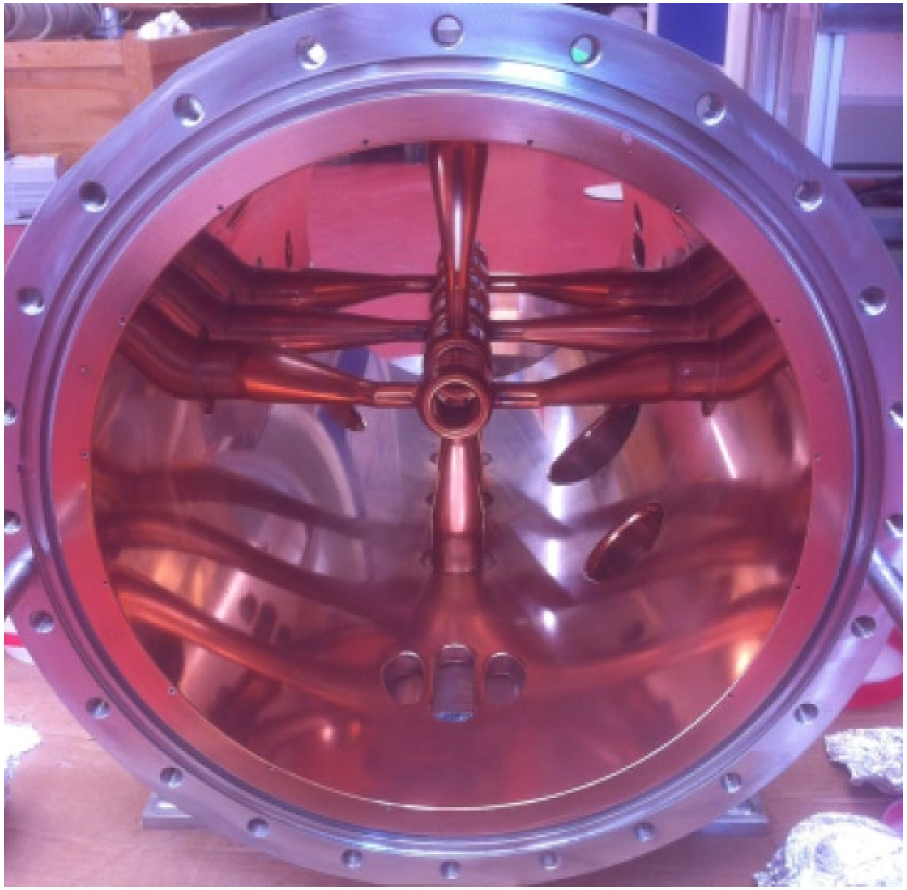
A. D. Cahill, thesis University of California, Los Angeles, 2017)

Suited cold copper structures

- Simple DTL‘s with no electromagnetic quadrupoles
- Small transverse dimensions
- High voltage gain per meter
- High shunt impedance

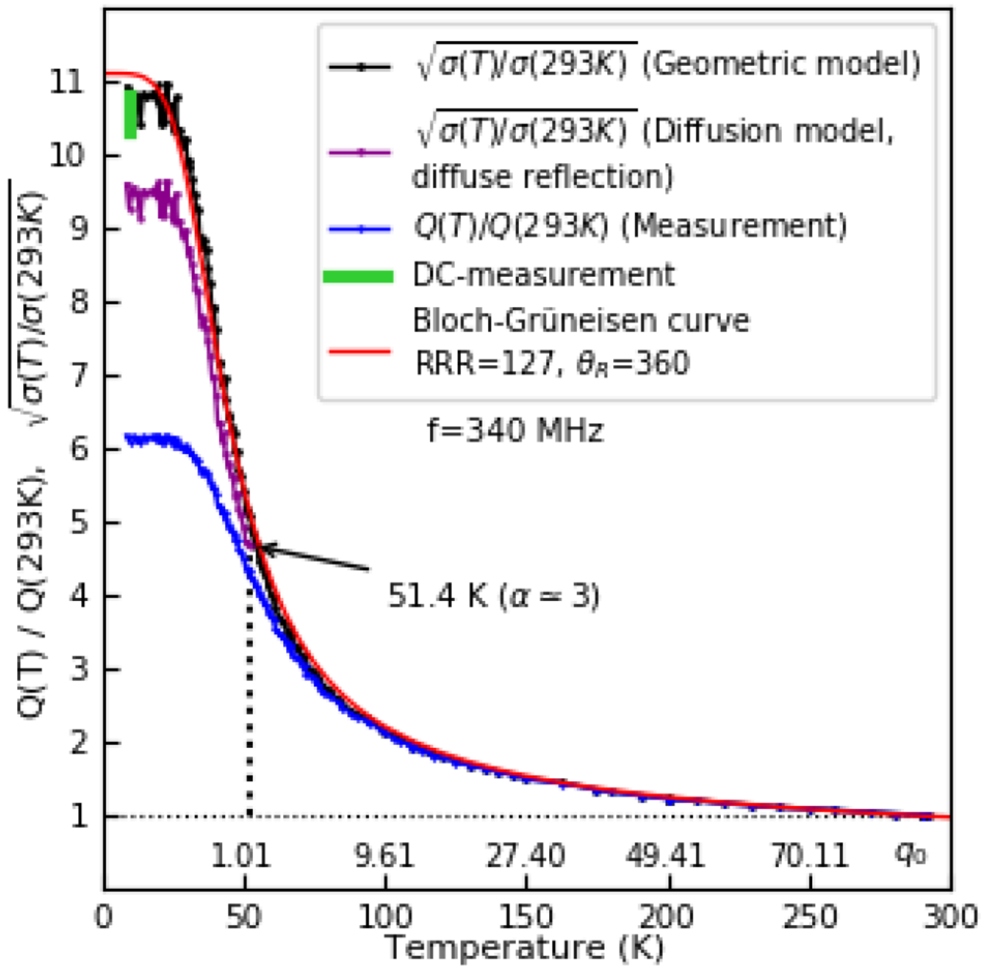
325 MHz CH power test cavity for GSI klystron test bench, ready for testing

Made from bulk copper, it would be a candidate for cryogenic operation



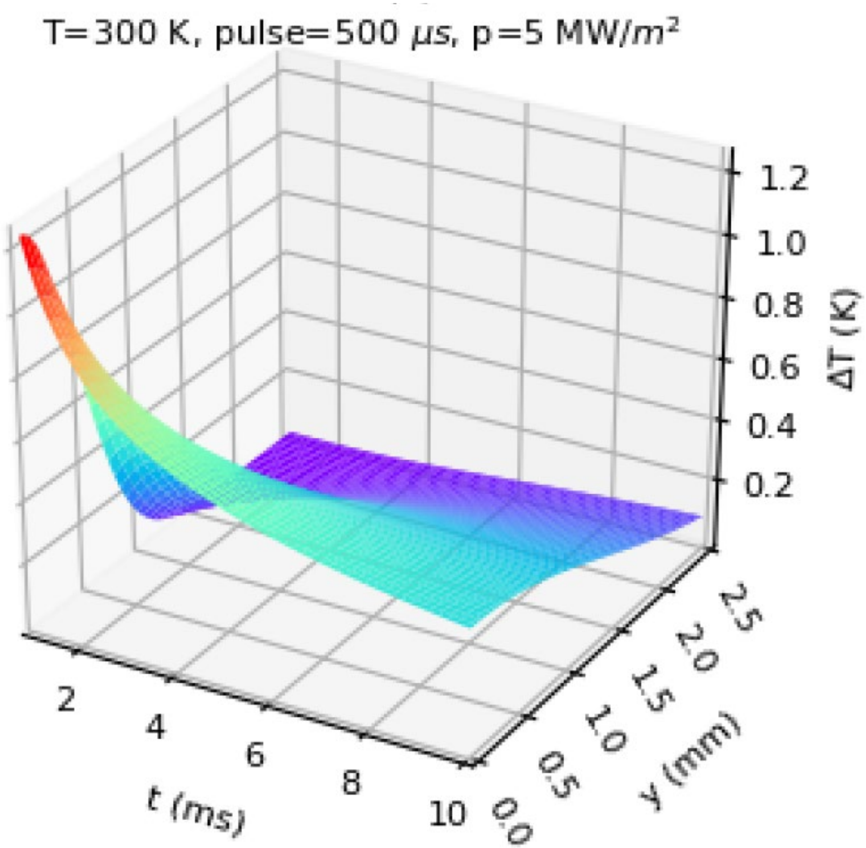
Measurements at IAP at 340 MHz

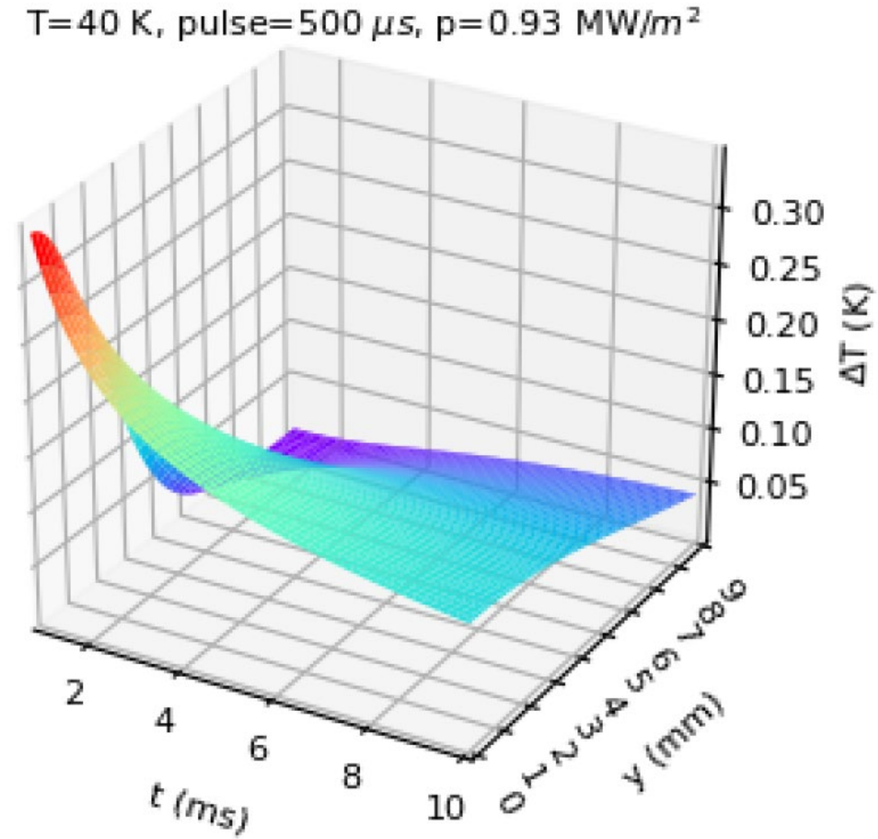
Bulk copper, coaxial quarter wave
Cavity, matt copper plated



Huifang Wang, Dr. Thesis 2023, IAP

Thermal situation, cooling



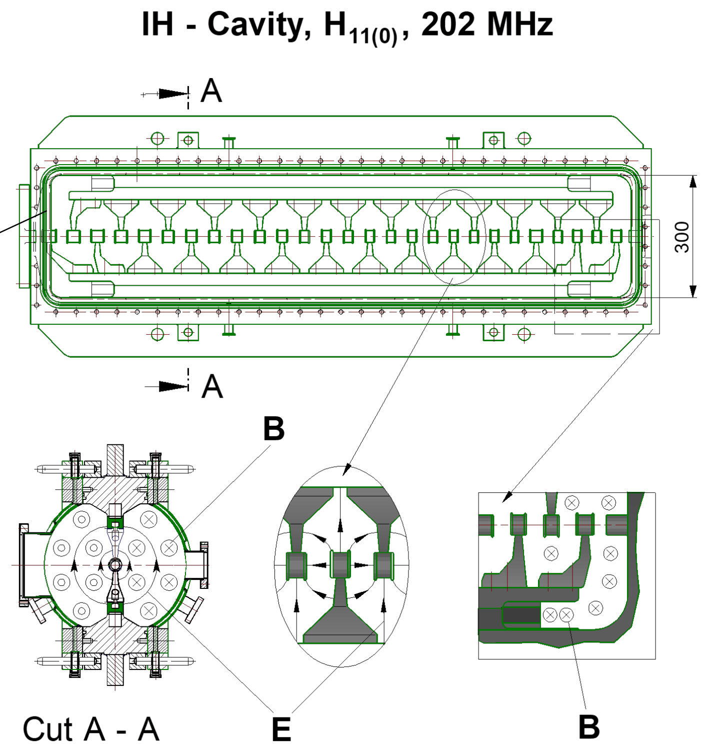


$$\Delta T = \frac{W_0}{A\rho c \cdot \sqrt{\pi \alpha t}} \cdot e^{-y^2/4\alpha \cdot t}$$

W_0/J, $c/kJ/kg \cdot K$, $\alpha/m^2/s$, $\rho/kg/m^3$, A/m^2.

Formula from: J. P. Holman. Souvik Bhattacharyya. Heat Transfer. 10th edition. Mc Graw Hill. 2011

Back to the IH-cavity example with anomalous skin effect



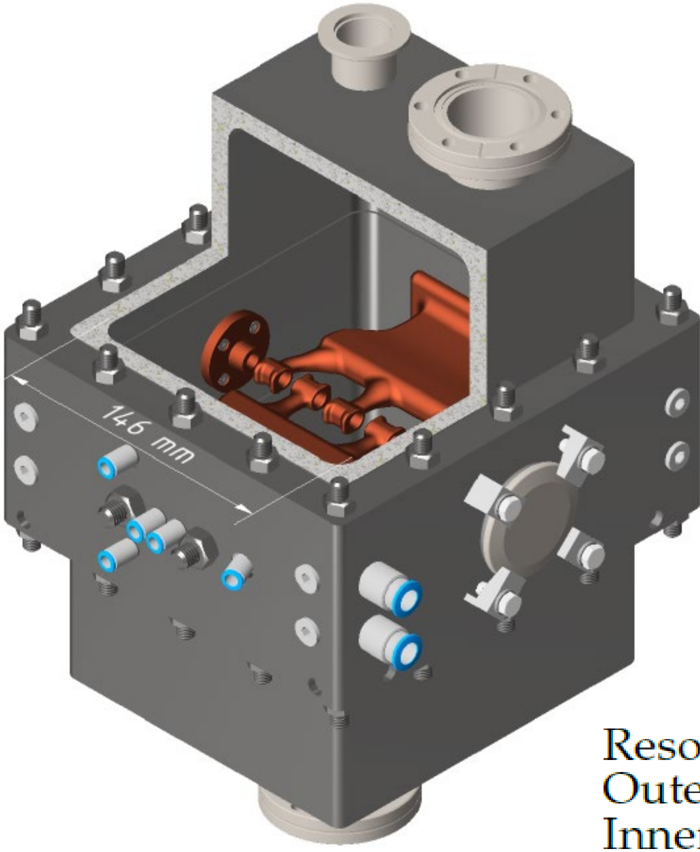
CERN IH2, 1.54m long, tested up to 10 MV/m

$P_{RF}(300K) = 840\frac{kW}{m}$; 0.1% duty factor

$P_{RF}(40K) \cong 168\frac{kW}{m};\ \ \eta_{cryo}(T) \cong \frac{40}{293-40}\cdot 0.3 = 0.047;\ \ P_{cool} \cong \frac{1.68\cdot10^5\cdot1\cdot10^{-3}}{0.047} =$ 3.6kW/m.

(Without anomalous skin effect: Operation at 20K, P(20 K)≅ $42\frac{kW}{m};\ \ P_{cool} \cong 1.9\text{kW/m}$)

Small Sizes are attractive for cold structures



Hähnel et al., Instruments **2023**, 7, 22. https://doi.org/10.3390/instruments7030022

Resonance Frequency: f_{res}	433.632 MHz
Outer Dimensions (w/l/h):	221 mm/206 mm/261
Inner Length: L_{inner}	146 mm
Period Length ($\beta\lambda/2$): L_p	19 mm–24 mm
Reference Particle:	proton
Initial Energy: W_{in}	1.4 MeV
Final Energy: W_f	2.4 MeV
Simulation Results (ideal model)	**Value**
Unloaded Q-factor: Q_0	8601
RF losses for 1 MV: $P_{loss,1MV}$	24.8 kW
Effective Shunt Impedance: Z_{eff}	287.1 MΩ/m

Conclusions

- A geometric model is a useful tool to estimate the anomalous surface resistivity for copper surfaces with known RRR value of the surface material
- H – type structures seem suited to realize cold copper structures for ion acceleration
- The reduction factor in RF power at 40K operation is about a factor 5 for RRR=200 material
- Duty factors around one permille seem feasible.
- Voltage gains up to 15 MV/m might be achieved