Aspects of Strong QCD

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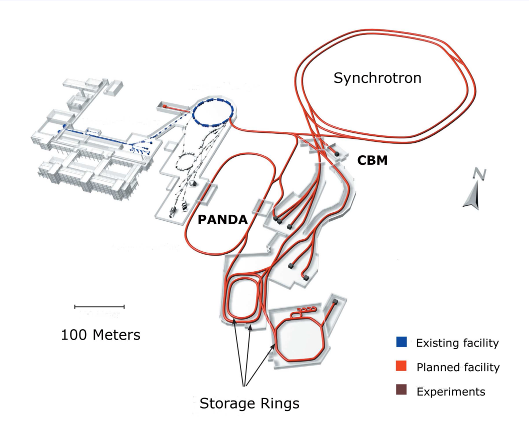
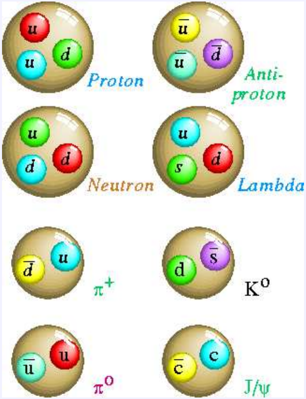
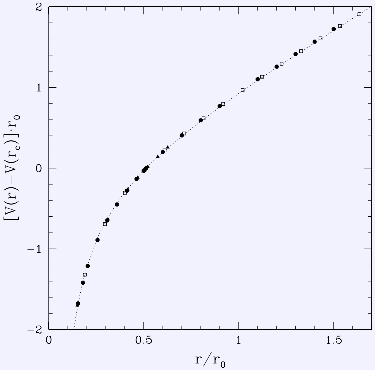
Motivation I: FAIR: PANDA and CBM





Motivation II: Confinement

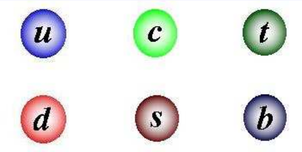


S. Necco and R. Sommer, Nucl. Phys. B **622** (2002) 328

- Linear rising potential: $V(r) \sim r$
- Quark-Antiquark system cannot be split! 'Quark-Confinement'
- in addition: Gluon-Confinement

What are the driving mechanisms?

Motivation III: Dynamical mass generation

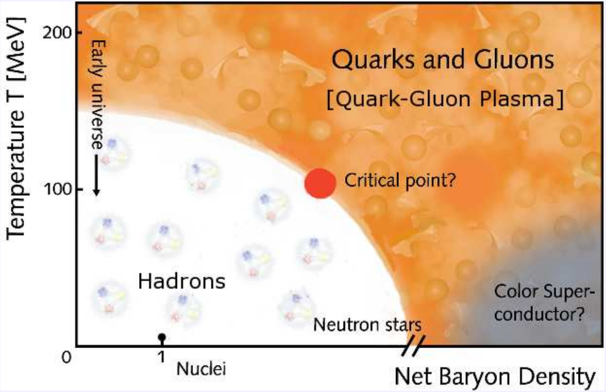


Quark mass generated by weak and strong interaction
(QCD: explicit vs. dynamical breaking of chiral symmetry)

	u	d	s	c	b	t
M_{weak} [MeV]	3	5	100	1300	4000	175000
M_{strong} [MeV]	400	400	400	400	400	400
M_{tot} [MeV]	400	400	500	1700	4400	175000

M_{strong}: **Nonperturbative effect!**

Motivation IV: Chiral phase transition



Chiral limit ($M_{weak} = 0$):

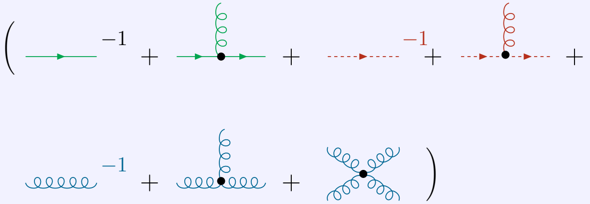
- order parameter: chiral condensate
- Goldstone bosons: pseudoscalar mesons (π, K, η)

QCD in covariant gauge

quarks, gluons and ghosts:

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c]\ \exp\left\{ -\int d^4x \left(\overline{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

$$S_{QCD} = \int d^4x$$



QCD in covariant gauge

quarks, gluons and ghosts:

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \; \exp\left\{ - \int d^4x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

Propagators in momentum space:

$$D^{\text{Gluon}}_{\mu\nu}(p) = \frac{\mathbf{Z}(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$D^{\text{Geist}}(p) = -\frac{\mathbf{G}(p^2)}{p^2}$$

$$S^{\text{Quark}}(p) = \frac{Z_f(p^2)}{-i\not{p} + M(p^2)}$$

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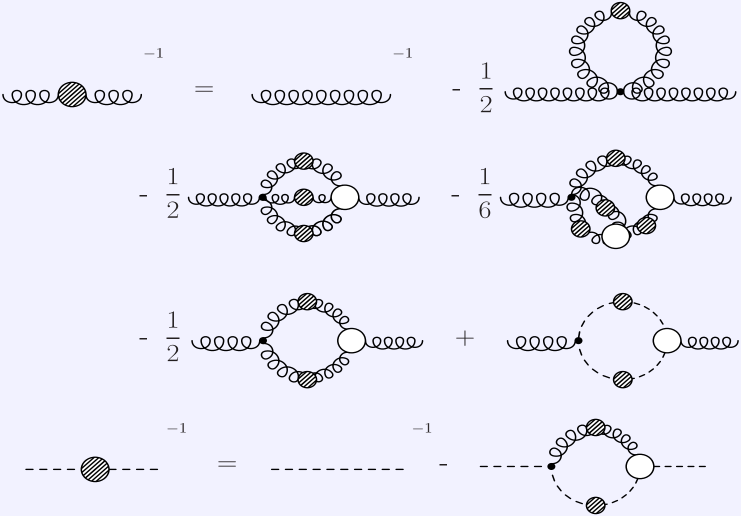
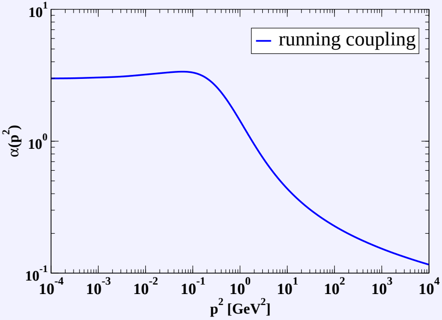
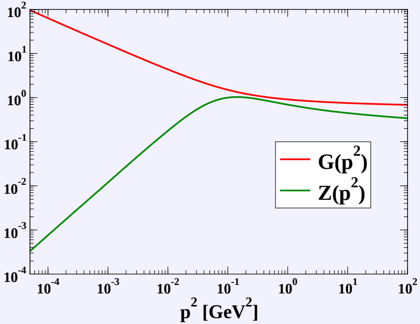
Dyson-Schwinger equations (DSEs)



Ghost, Glue and Coupling



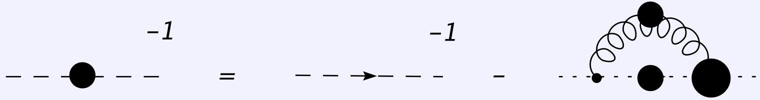


- dynamically generated scale
- fixed point of coupling $\alpha(p^2) = g^2/(4\pi)Z(p^2)G(p^2)$
- power laws in dressing functions

CF and Alkofer, PLB 536 (2002) 177.

Infrared Structure of YM-theory I

- Example: Ghost-propagator



- Selfconsistency:

$$G(p^2) \sim (p^2)^{-\kappa}\,, \quad Z(p^2) \sim (p^2)^{2\kappa}\,, \quad \Gamma^{2,1}(p^2) \sim const.$$

L. v. Smekal, A. Hauck, R. Alkofer, Phys. Rev. Lett. **79** (1997) 3591

C. Lerche, L. v. Smekal, Phys. Rev. D **65** (2002) 125006.

D. Zwanziger Phys. Rev. D **65** (2002) 094039.

Infrared Structure of YM-theory II

Scaling analysis: All external scales $p^2 << \Lambda_{QCD}$

Dressing function with *n* ghost and *m* gluon legs

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n/2-m)\kappa}$$

- solves whole tower of DSEs and STIs
- unique scaling solution
- κ not fixed by scaling analysis, but $\kappa > 0$

Ghost : $G(p^2) \overset{p^2 \to 0}{=} \infty$

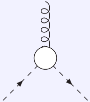
Gluon : $Z(p^2)/p^2 \overset{p^2 \to 0}{=} 0$

▷ Kugo Ojima confinement szenario supported!

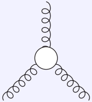
▷ Gluon confinement via positivity violation

R. Alkofer, C. F., F. Llanes-Estrada, Phys. Lett. B **611** (2005) C.F. and J. M. Pawlowski, Phys. Rev. D **75** (2007) 025012.

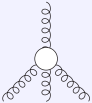
Consequence: Running coupling for $p^2 \longrightarrow 0$



$$\alpha^{gg}(p^2) \quad = \quad \frac{g^2}{4\pi} G^2(p^2) Z(p^2) = const_{gg} \approx 2.97$$



$$\alpha^{3g}(p^2) \quad = \quad \frac{g^2}{4\pi} [\Gamma^{3g}(p^2)]^2 Z^3(p^2) = const_{3g}$$



$$\alpha^{4g}(p^2) \quad = \quad \frac{g^2}{4\pi} [\Gamma^{4g}(p^2)] Z^2(p^2) = const_{4g} \approx 0.0029$$

Qualitatively universal fixed point!

C. Lerche and L. v. Smekal, Phys. Rev. D **65** (2002) 125006

R. Alkofer, C.F., F. Llanes-Estrada, Phys.Lett.B611 (2005) 279-288.

C. Kellermann and C.F., PRD **78**,(2008) 025015.

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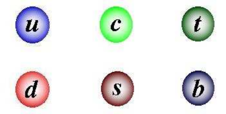
Masses of quarks in QCD

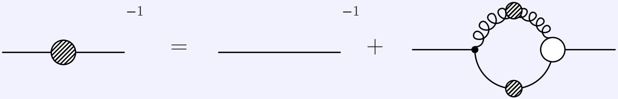


- **External** quark-masses:

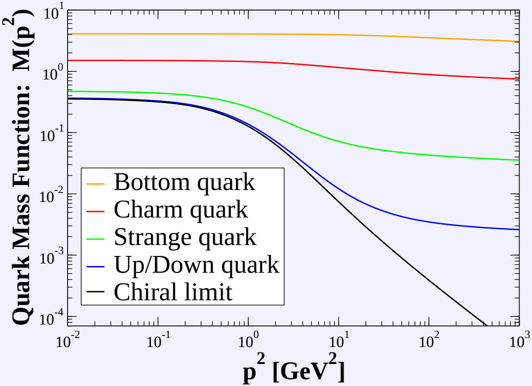
$$M_{weak} = M_{u,d,s,c,b,t} \neq 0$$

- **Dynamical** quark-masses:

$M_{strong}(p^2) \neq 0$ Non-perturbative effect!



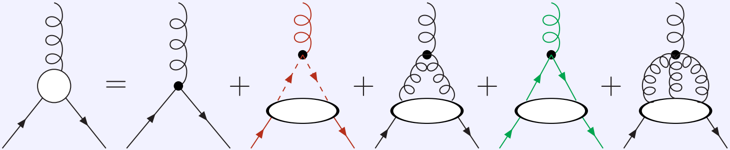
Massen der Quarks: Numerische Lösung der DSE



- $M(p^2)$: momentum dependent!
- Dynamical masses $M_{strong}(0) \approx 350 MeV$
- Flavour dependence because of M_{weak}

C. F., J. Phys. G **32** (2006) R253

Quark-gluon vertex



Ghost diagram: Infrared leading

- infrared slavery (quark-gluon coupling)

$$\alpha^{qg}(p^2) \sim \frac{1}{p^2}$$

- generates linear rising quark-antiquark potential

$$V(r) \sim r$$

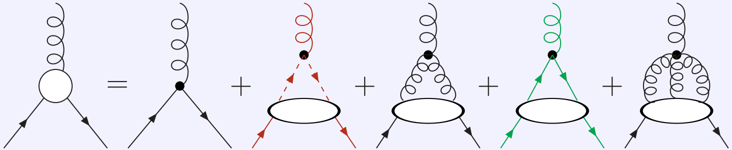
- generation of topological mass of η' ($U_A(1)$-problem)

θ	m_η [MeV]	$m_{\eta'}$ [MeV]
-23.2	480	905

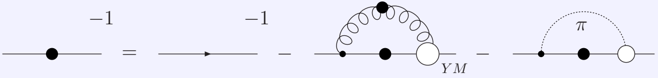
R. Alkofer, C.F., F. Llanes-Estrada, Kai Schwenzer, Annals of Physics in press, arXiv:0804.3042 [hep-ph]

R. Alkofer, C.F, R. Williams, arXiv:0804.3478 [hep-ph]

Quark-gluon vertex



- Quark diagram: Pion cloud effects

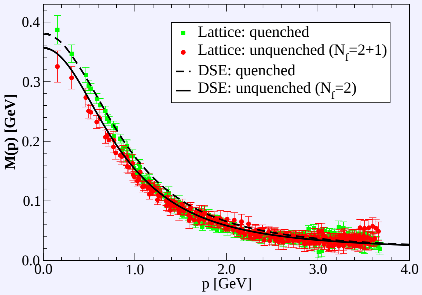


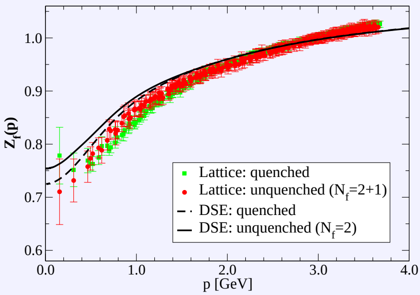
  - Large pion cloud effects in quark propagator
  - Large pion cloud effects in light meson spectrum

C.F., D. Nickel and R. Williams, arXiv:0807.3486 [hep-ph]

C.F. and R. Williams, PRD in press, arXiv:0808.3372 [hep-ph]

Pion cloud effects in the quark propagator



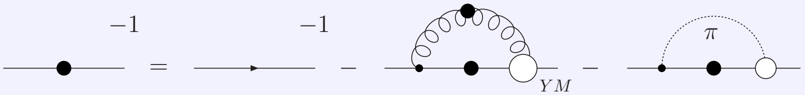


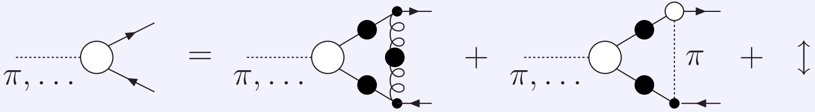
CF, D. Nickel and R. Williams, arXiv:0807.3486 [hep-ph]

- Unquenching effects of similar size as lattice

P. O. Bowman, et al. Phys. Rev. D **71** (2005) 054507

Pion cloud effects in light mesons I





- Axial Ward-Takahashi identity satisfied

Pion cloud effects in light mesons II

	wo π	with π	Experiment
M_π	125	$138^\dagger$	138
f_π	102	$93^\dagger$	93
		(91)	
M_σ	638	485	400 – 1200
M_ρ	795	703	776
f_ρ	138	159	162
		(165)	
M_{a_1}	941	873	1230
M_{b_1}	879	806	1230

- Effects of order of 100 MeV, similar extrapolated lattice results
- Yang-Mills part of quark-gluon vertex still too simple!

CF and R. Williams, PRD in press, arXiv:0808.3372 [hep-ph]

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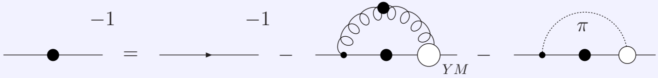
Finite Temperature

work in progress with J. Müller and D. Nickel

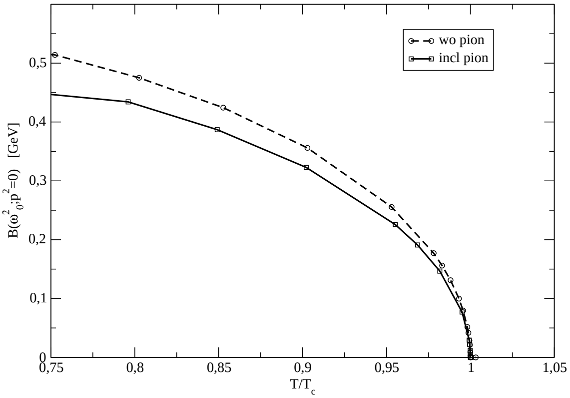


- Quark propagator:

$$S(p_{\omega_n})^{-1} = (i\,\vec{\gamma}\cdot\vec{p}\,A(p_{\omega_n}) + i\,\gamma_4\omega_n\,C(p_{\omega_n}) + B(p_{\omega_n}))$$

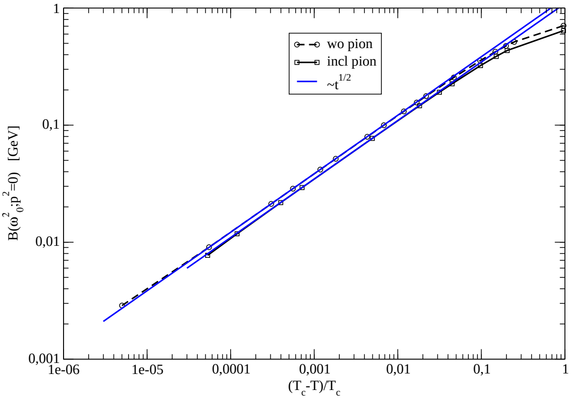
- "frozen gluon approximation"
- Order parameters:
  - $\langle\bar{\psi}\psi\rangle_\mu = -Z_m(\mu)\sum_{p_\omega}^{\Lambda}\mathrm{Tr}_{c,f,D}(S(p_\omega;\mu)Z_2(\mu))$
  - $\chi_B := B(\omega_0, \vec{p}^2 = 0)$

Results: χ_B



- wo pion: $T_c \sim 200\,\text{MeV}$
- with pion: $T_c \sim 190\,\text{MeV}$

Results: χ_B



- mean field critical exponents

Towards quark spectral functions

Quark spectral functions above T_c

- QGP near phase transition $\rightarrow$ strongly interacting system
- Extraction of quark spectral functions from DSE's via MEM is possible

  [D. Nickel, Annals Phys. 322 (2006)]

- Explore quasi-particle properties
  - Is the quasi-particle picture valid in this regime ?
  - How do bosonic modes affect the quark spectrum ?

  [M. Kitazawa, T. Kunihiro, K. Mitsutani, PRD 77 (2008)]
  [M. Harada, Y. Nemoto, PRD 78 (2008)]

  [F. Karsch, M. Kitazawa, PRL B 658 (2007)]

Summary

Landau gauge Yang-Mills theory:

- scaling solution: 1PI-function with *n* ghost and *m* gluon legs:

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n/2-m)\kappa}$$

- Infrared fixed point in running coupling

Unquenched Landau gauge QCD:

- Infrared singular quark-gluon vertex
  $\rightarrow$ solves $U_A(1)$-problem

- Unquenching effects in light meson spectrum: large!

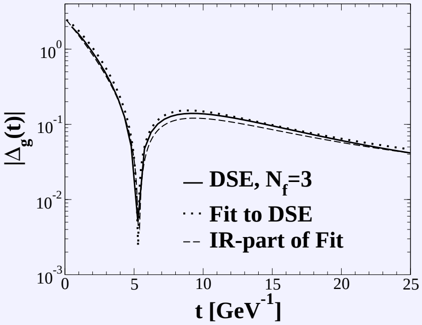
- Quarks in QGP: work in progress

Gluon confinement

- $$\int \frac{d^4x}{(2\pi)^4} \Delta_g(x^2) = \left(\frac{Z(p^2)}{p^2}\right)\Bigg|_{p^2=0} = \left(\frac{(p^2)^{2\kappa}}{p^2}\right)\Bigg|_{p^2=0} = 0 \quad \text{for } \kappa > 1/2$$

$\Delta_g(t) :=$

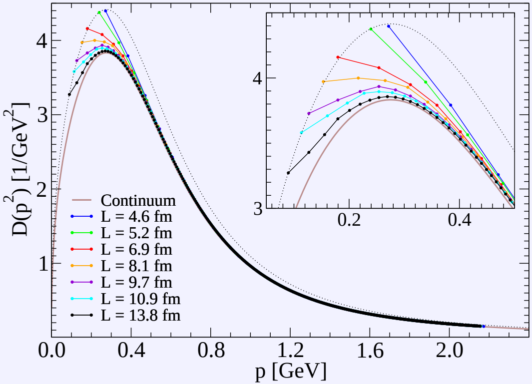
$$\int d^3x \int \frac{d^4p}{(2\pi)^4} e^{i(tp_4+\vec{p}\vec{x})} \frac{Z(p^2)}{p^2}$$



- Violation of positivity $\Rightarrow$ Signal for **confined gluons**

R. Alkofer, W. Detmold, C. F., P. Maris, Phys. Rev. D **70** (2004) 014014

DSEs on a torus: volume effects

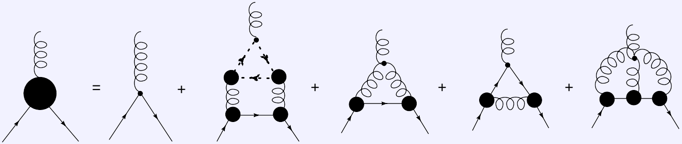


- Contemporary lattices large enough to exclude large volume effects

C.F., A. Maas, J. M. Pawlowski and L. von Smekal, Annals Phys. **322**, 2916 (2007)

Infrared Structure of QCD II

- Quark-gluon vertex: **lowest order** in skeleton expansion



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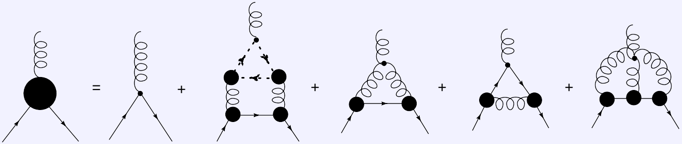
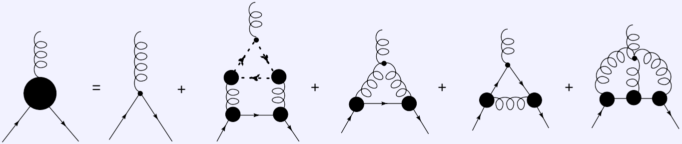


$$S(p) = i\not{p}\,\frac{Z_f}{p^2+M^2} + \frac{Z_f M}{p^2+M^2}$$

Infrared Structure of QCD II

- Quark-gluon vertex: **lowest order** in skeleton expansion

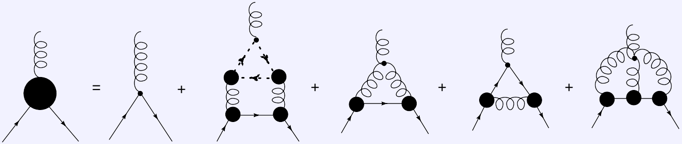


$$S(p) = i\not{p}\,\frac{Z_f}{p^2+M^2} + \frac{Z_f M}{p^2+M^2}$$

$$\Gamma_\mu = ig\sum_{i=1}^{4}\lambda_i G^i_\mu \;: \quad G^1_\mu = \gamma_\mu\,,\;\; G^2_\mu = \hat{p}_\mu\,,\;\; G^3_\mu = \hat{\not{p}}\,\hat{p}_\mu\,,\;\; G^4_\mu = \hat{\not{p}}\,\gamma_\mu$$

Infrared Structure of QCD II

- Quark-gluon vertex: **lowest order** in skeleton expansion

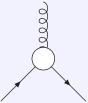


$$S(p) = i\not p \frac{Z_f}{p^2 + M^2} + \frac{Z_f M}{p^2 + M^2}$$

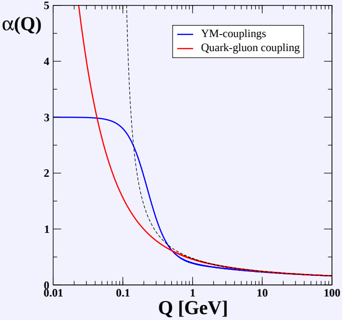
$$\Gamma_\mu = ig \sum_{i=1}^{4} \lambda_i G^i_\mu \;:\quad G^1_\mu = \gamma_\mu\,,\;\; G^2_\mu = \hat{p}_\mu\,,\;\; G^3_\mu = \hat{\not p}\,\hat{p}_\mu\,,\;\; G^4_\mu = \hat{\not p}\,\gamma_\mu$$

$$\lambda_{1,2,3,4} \sim (p^2)^{-1/2-\kappa} \qquad \leftrightarrow \qquad \lambda_{1,3} \sim (p^2)^{-\kappa}$$

Running Coupling: IR-slavery



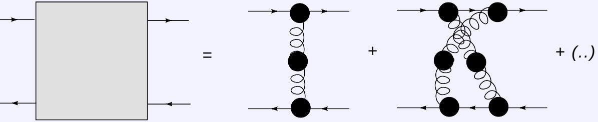
$$\alpha^{qg}(p^2) = \alpha_\mu \, [\Gamma^{qg}(p^2)]^2 \, [Z_f(p^2)]^2 \, Z(p^2) \sim \begin{cases} \frac{1}{p^2} : \chi SB \\ const : \chi S \end{cases}$$



R. Alkofer, C. F., F. Llanes-Estrada, MPL A **23**, 1105 (2008).

The quark-antiquark potential

- quenched QCD

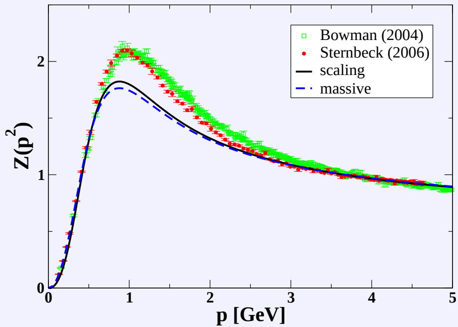


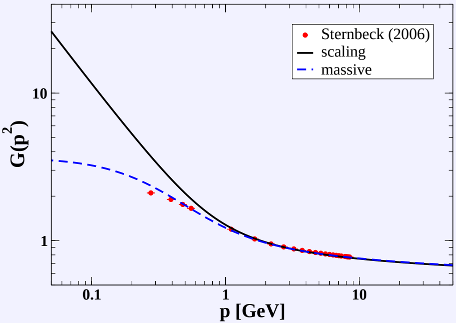
$$\sim \begin{cases} \frac{1}{p^4} : \chi SB \\ \frac{1}{p^2} : \chi S \end{cases}$$

$$V(\mathbf{r}) = \frac{1}{(2\pi)^3} \int d^3p \; e^{i\mathbf{pr}} \, \square \quad \sim \begin{cases} |r| : D\chi SB \\ \frac{1}{|r|} : \chi S \end{cases}$$

Quark confinement $\leftrightarrow \chi SB$

DSEs vs Lattice I





- $p^2 \approx 1 GeV$: Systematic improvement possible for **DSEs**
- Deep infrared: Subtle problems for **Lattice**