

Aspects of Strong QCD

Christian S. Fischer

TU Darmstadt

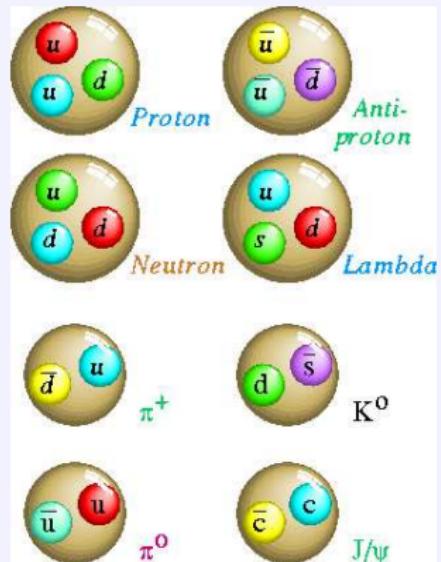
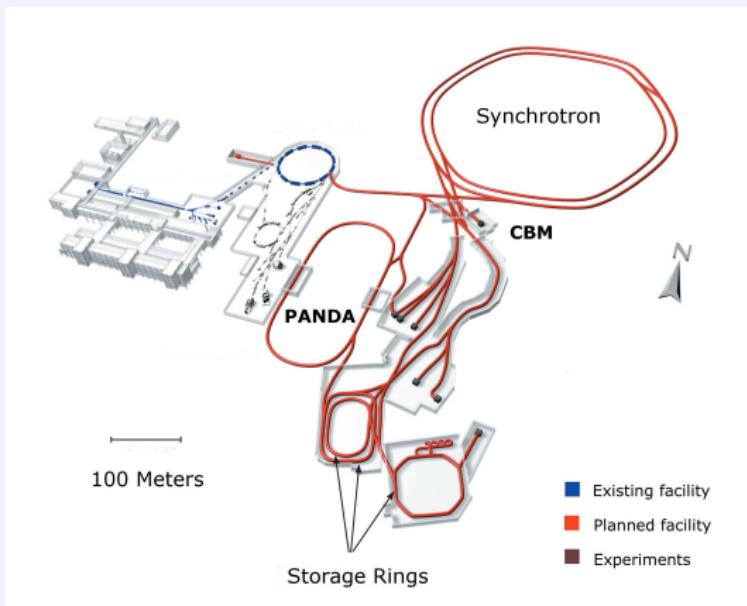
Helmholtz Young Investigator Group "Nonperturbative Phenomena in QCD"



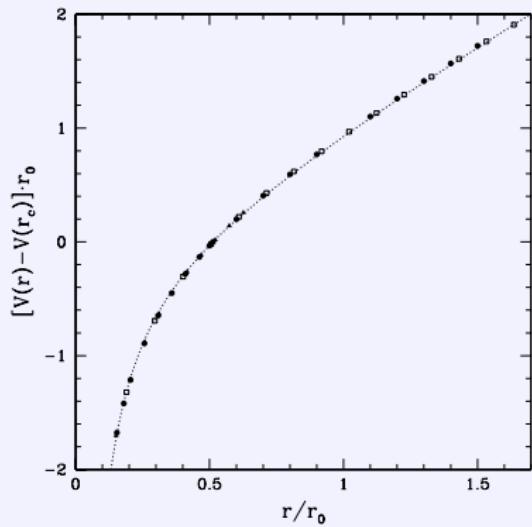
- 1 Introduction
- 2 Infrared properties of $SU(N)$ Yang-Mills theory
- 3 Dynamical chiral symmetry breaking: Quarks and gluons
- 4 Chiral phase transition (preliminary)

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Motivation I: FAIR: PANDA and CBM



Motivation II: Confinement

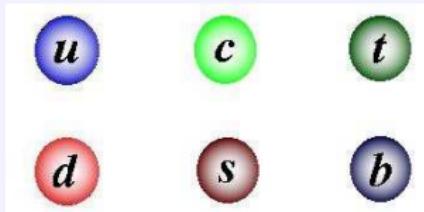


S. Necco and R. Sommer, Nucl. Phys. B **622** (2002) 328

- Linear rising potential:
 $V(r) \sim r$
- Quark-Antiquark system cannot be split!
'Quark-Confinement'
- in addition:
Gluon-Confinement

What are the driving mechanisms?

Motivation III: Dynamical mass generation

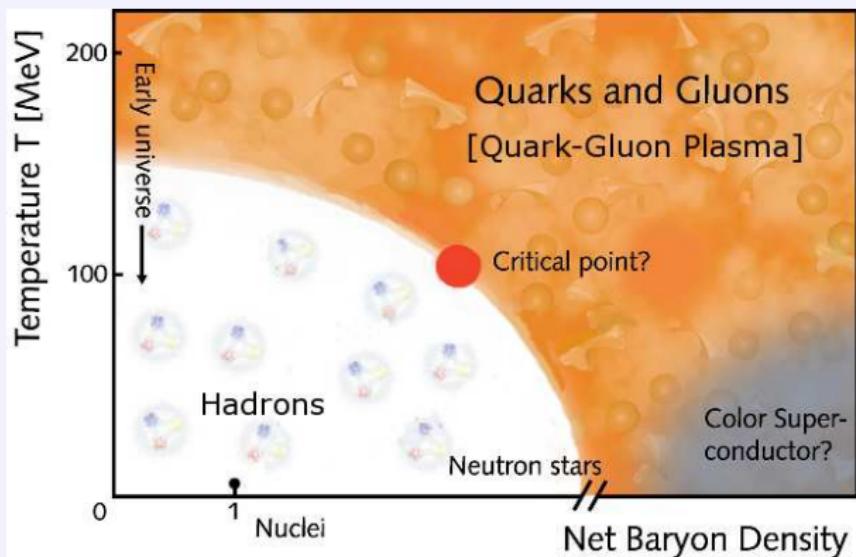


Quark mass generated by **weak** and **strong** interaction
(QCD: explicit vs. dynamical breaking of chiral symmetry)

	u	d	s	c	b	t
M_{weak} [MeV]	3	5	100	1300	4000	175000
M_{strong} [MeV]	400	400	400	400	400	400
M_{tot} [MeV]	400	400	500	1700	4400	175000

M_{strong} : Nonperturbative effect!

Motivation IV: Chiral phase transition



Chiral limit ($M_{\text{weak}} = 0$):

- order parameter: chiral condensate
- Goldstone bosons: pseudoscalar mesons (π, K, η)

QCD in covariant gauge

quarks, gluons and ghosts:

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \exp \left\{ - \int d^4x \left(\bar{\Psi} (i\not{D} - m) \Psi \right. \right.$$
$$\left. \left. - \frac{1}{4} (F_{\mu\nu}^a)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

$$S_{QCD} = \int d^4x \quad \left(\begin{array}{c} \text{---} \\ \longrightarrow \end{array} \right)^{-1} + \begin{array}{c} \text{---} \\ \text{---} \end{array} \bullet \begin{array}{c} \text{---} \\ \longrightarrow \end{array} + \begin{array}{c} \text{---} \\ \text{---} \end{array} \bullet \begin{array}{c} \text{---} \\ \longrightarrow \end{array}^{-1} + \begin{array}{c} \text{---} \\ \text{---} \end{array} \bullet \begin{array}{c} \text{---} \\ \longrightarrow \end{array} +$$
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QCD in covariant gauge

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$$\left. \left. - \frac{1}{4} (\not{F}_{\mu\nu}^a)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

Propagators in momentum space:



$$D_{\mu\nu}^{\text{Gluon}}(p) = \frac{\mathbf{Z}(p^2)}{p^2} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right)$$



$$D^{\text{Geist}}(p) = -\frac{\mathbf{G}(p^2)}{p^2}$$



$$S^{\text{Quark}}(p) = \frac{\mathbf{Z}_f(p^2)}{-ip + M(p^2)}$$

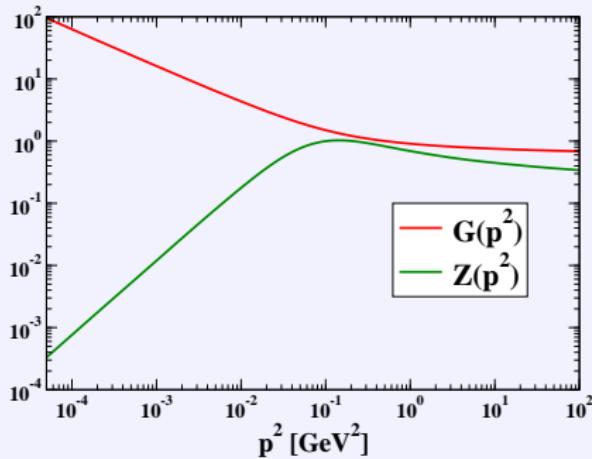
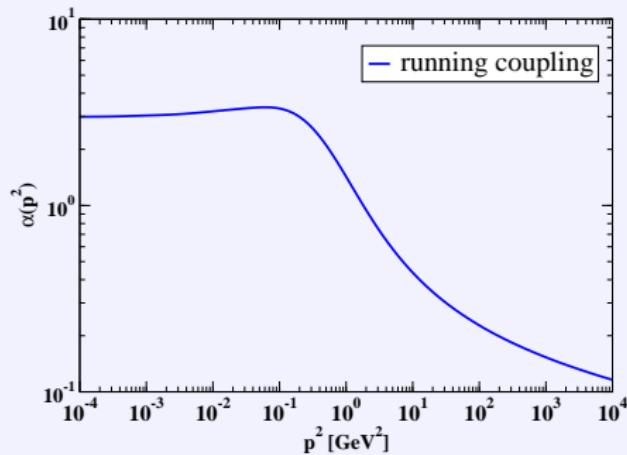
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Dyson-Schwinger equations (DSEs)

$$\begin{aligned} -1 &= \text{---} - \frac{1}{2} \text{---} \\ &\quad - \frac{1}{2} \text{---} - \frac{1}{6} \text{---} \\ &\quad - \frac{1}{2} \text{---} + \text{---} \\ -1 &= \text{---} - \end{aligned}$$

The diagrammatic equations represent Dyson-Schwinger equations. The left side of each equation is a bare vertex (a shaded circle). The right side consists of a sum of terms. Each term is a loop diagram with a shaded circle at one vertex and a white circle at another. The loops are composed of wavy lines (fermions) and straight lines (gluons). The coefficients in front of the terms are rational numbers: -1 , $-\frac{1}{2}$, $-\frac{1}{6}$, $-\frac{1}{2}$, and -1 .

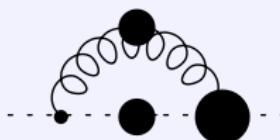
Ghost, Glue and Coupling



- dynamically generated scale
- fixed point of coupling $\alpha(p^2) = g^2/(4\pi)Z(p^2)G(p^2)$
- power laws in dressing functions

Infrared Structure of YM-theory I

- Example: Ghost-propagator

$$\begin{array}{c} -1 \\ \text{---} \bullet \text{---} \end{array} = \begin{array}{c} -1 \\ \text{---} \rightarrow \text{---} \end{array} - \begin{array}{c} \text{---} \bullet \text{---} \\ \text{---} \bullet \text{---} \end{array}$$
A Feynman diagram representing the ghost propagator. It consists of a horizontal dashed line with two solid black circles representing vertices. Above the left vertex is the number '-1'. Above the right vertex is the number '-1'. An arrow points from the left vertex to the right vertex, indicating the direction of flow. A minus sign '-' is placed to the right of the arrowed line. To the right of the minus sign is another horizontal dashed line with two solid black circles at its ends. The top circle has a small loop attached to it.

- Selfconsistency:

$$G(p^2) \sim (p^2)^{-\kappa}, \quad Z(p^2) \sim (p^2)^{2\kappa}, \quad \Gamma^{2,1}(p^2) \sim \text{const.}$$

L. v. Smekal, A. Hauck, R. Alkofer, Phys. Rev. Lett. **79** (1997) 3591

C. Lerche, L. v. Smekal, Phys. Rev. D **65** (2002) 125006.

D. Zwanziger Phys. Rev. D **65** (2002) 094039.

Infrared Structure of YM-theory II

Scaling analysis: All external scales $p^2 \ll \Lambda_{QCD}$

Dressing function with n ghost and m gluon legs

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n/2-m)\kappa}$$

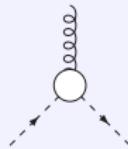
- solves whole tower of DSEs and STIs
- unique scaling solution
- κ not fixed by scaling analysis, but $\kappa > 0$

$$\left. \begin{array}{l} \text{Ghost : } G(p^2) \stackrel{p^2 \rightarrow 0}{=} \infty \\ \text{Gluon : } Z(p^2)/p^2 \stackrel{p^2 \rightarrow 0}{=} 0 \end{array} \right\} \begin{array}{l} \triangleright \text{Kugo Ojima confinement szenario supported!} \\ \triangleright \text{Gluon confinement via positivity violation} \end{array}$$

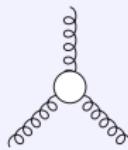
R. Alkofer, C. F., F. Llanes-Estrada, Phys. Lett. B **611** (2005) C.F. and J. M. Pawłowski, Phys. Rev. D **75** (2007) 025012.



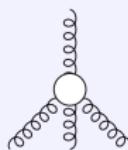
Consequence: Running coupling for $p^2 \rightarrow 0$



$$\alpha^{gg}(p^2) = \frac{g^2}{4\pi} G^2(p^2) Z(p^2) = const_{gg} \approx 2.97$$



$$\alpha^{3g}(p^2) = \frac{g^2}{4\pi} [\Gamma^{3g}(p^2)]^2 Z^3(p^2) = const_{3g}$$



$$\alpha^{4g}(p^2) = \frac{g^2}{4\pi} [\Gamma^{4g}(p^2)] Z^2(p^2) = const_{4g} \approx 0.0029$$

Qualitatively universal fixed point!

C. Lerche and L. v. Smekal, Phys. Rev. D **65** (2002) 125006

R. Alkofer, C.F., F. Llanes-Estrada, Phys.Lett.B611 (2005) 279-288.

C. Kellermann and C.F., PRD **78**,(2008) 025015.

Christian S. Fischer (TU Darmstadt)

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Masses of quarks in QCD



- **External** quark-masses:

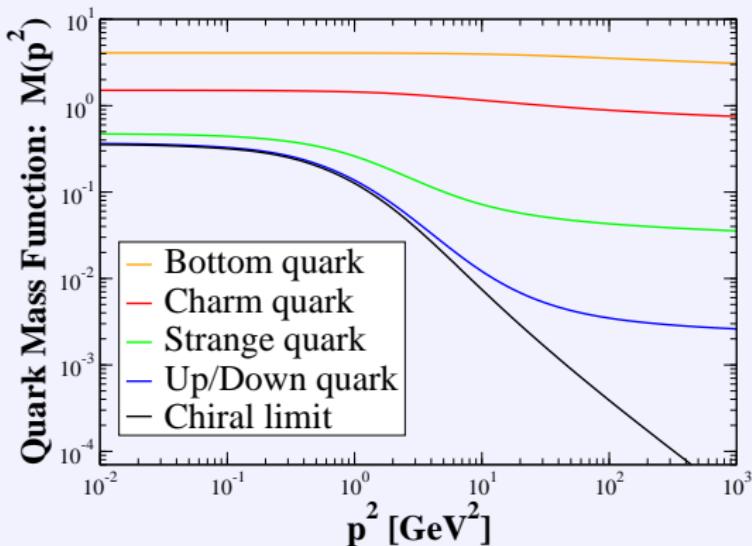
$$M_{\text{weak}} = M_{u,d,s,c,b,t} \neq 0$$

- **Dynamical** quark-masses:

$$M_{\text{strong}}(p^2) \neq 0 \quad \text{Non-perturbative effect!}$$



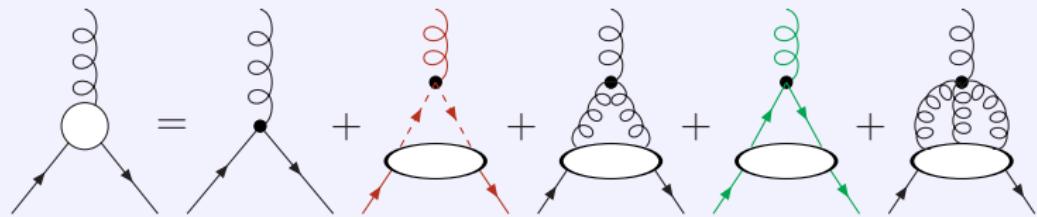
Massen der Quarks: Numerische Lösung der DSE



- $M(p^2)$: momentum dependent!
- Dynamical masses
 $M_{\text{strong}}(0) \approx 350 \text{ MeV}$
- Flavour dependence because of M_{weak}

C. F., J. Phys. G **32** (2006) R253

Quark-gluon vertex



Ghost diagram: Infrared leading

- infrared slavery (quark-gluon coupling)
- generates linear rising quark-antiquark potential
- generation of topological mass of η' ($U_A(1)$ -problem)

$$\alpha^{qg}(p^2) \sim \frac{1}{p^2}$$

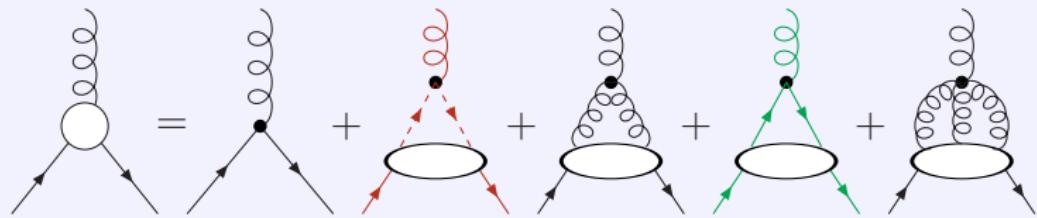
$$V(r) \sim r$$

θ	m_η [MeV]	$m_{\eta'}$ [MeV]
-23.2	480	905

R. Alkofer, C.F., F. Llanes-Estrada, Kai Schwenzer, Annals of Physics in press, arXiv:0804.3042 [hep-ph]

R. Alkofer, C.F., R. Williams, arXiv:0804.3478 [hep-ph]

Quark-gluon vertex



- Quark diagram: Pion cloud effects

$$\text{---} \bullet = \text{---} - \text{---} - \text{---}_{YM} - \text{---}^{\pi}$$

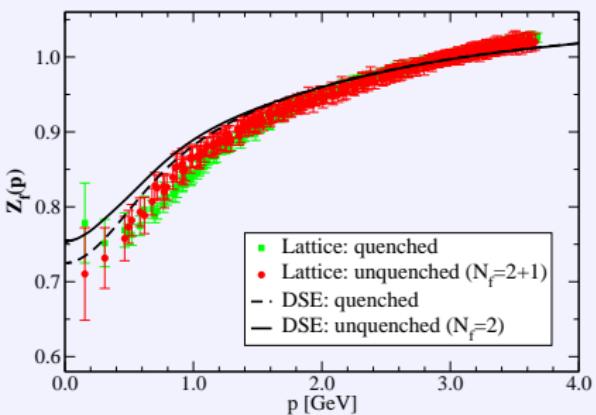
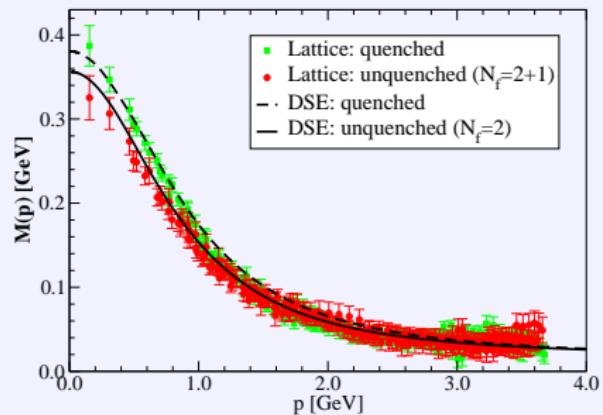
A diagrammatic equation showing the quark propagator (a horizontal line with a black dot) equal to its bare value minus a loop correction. The loop correction consists of two parts: a quark loop labeled YM and a pion loop labeled π .

- Large pion cloud effects in quark propagator
- Large pion cloud effects in light meson spectrum

C.F., D. Nickel and R. Williams, arXiv:0807.3486 [hep-ph]

C.F. and R. Williams, PRD in press, arXiv:0808.3372 [hep-ph]

Pion cloud effects in the quark propagator



CF, D. Nickel and R. Williams, arXiv:0807.3486 [hep-ph]

- Unquenching effects of similar size as lattice

P. O. Bowman, et al. Phys. Rev. D **71** (2005) 054507

Pion cloud effects in light mesons I

$$\begin{array}{ccccccc} -1 & & -1 & & YM & & \pi \\ \text{---} \bullet \text{---} & = & \text{---} \rightarrow \text{---} & - & \text{---} \bullet \text{---} \bullet \text{---} \circ \text{---} & - & \text{---} \bullet \text{---} \bullet \text{---} \circ \text{---} \\ \\ \pi, \dots & = & \pi, \dots & + & \pi, \dots & + & \updownarrow \end{array}$$

The diagram illustrates the renormalization of a bare quark loop. The first row shows the subtraction of a bare quark loop from a dressed quark loop. The bare quark loop is labeled -1 . The dressed quark loop is labeled -1 and contains a gluon loop labeled YM . The second row shows the renormalization of a bare pion loop. The bare pion loop is labeled π . It is equated to a dressed pion loop plus a counterterm. The dressed pion loop has a gluon loop attached to it. The counterterm consists of a gluon loop with a pion line attached to one of its vertices.

- Axial Ward-Takahashi identity satisfied

Pion cloud effects in light mesons II

	wo π	with π	Experiment
M_π	125	138 [†]	138
f_π	102	93 [†] (91)	93
M_σ	638	485	400 – 1200
M_ρ	795	703	776
f_ρ	138	159 (165)	162
M_{a_1}	941	873	1230
M_{b_1}	879	806	1230

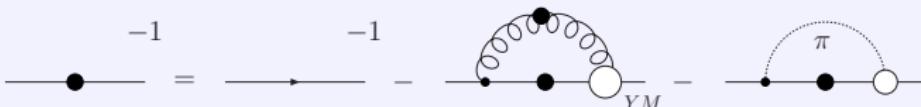
- Effects of order of 100 MeV, similar extrapolated lattice results
- Yang-Mills part of quark-gluon vertex still too simple!

Überblick

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Finite Temperature

work in progress with J. Müller and D. Nickel



- Quark propagator:

$$S(p_{\omega_n})^{-1} = (i \vec{\gamma} \cdot \vec{p} \textcolor{red}{A}(p_{\omega_n}) + i \gamma_4 \omega_n \textcolor{red}{C}(p_{\omega_n}) + \textcolor{red}{B}(p_{\omega_n}))$$

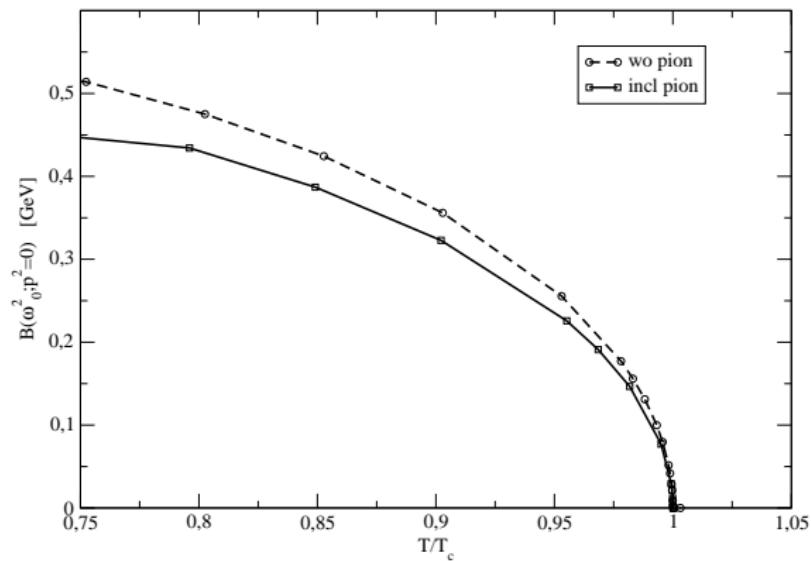
- "frozen gluon approximation"

- Order parameters:

- $\langle \bar{\psi} \psi \rangle_\mu = -Z_m(\mu) \sum_{p_\omega}^\Lambda \text{Tr}_{c,f,D}(S(p_\omega; \mu) Z_2(\mu))$

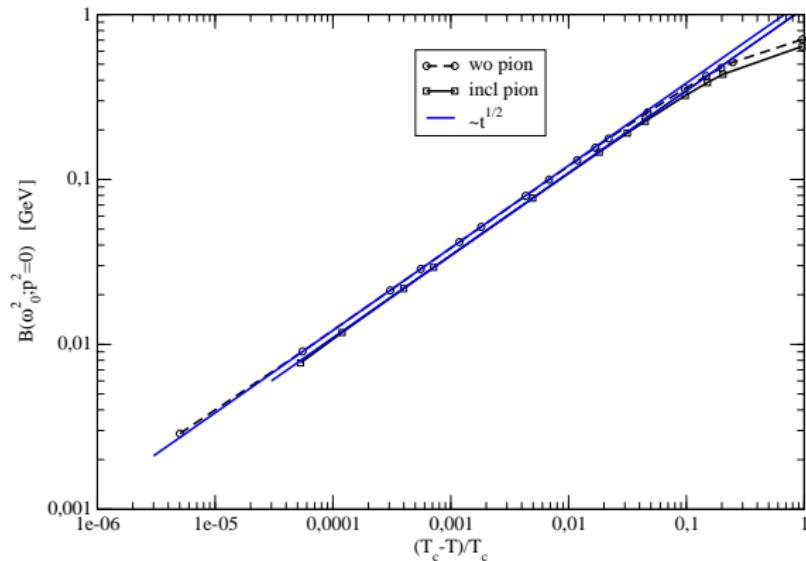
- $\chi_B := B(\omega_0, \vec{p}^2 = 0)$

Results: χ_B



- wo pion: $T_c \sim 200$ MeV
- with pion: $T_c \sim 190$ MeV

Results: χ_B



- mean field critical exponents

Towards quark spectral functions

Quark spectral functions above T_c

- QGP near phase transition → strongly interacting system
- Extraction of quark spectral functions from DSE's via MEM is possible

[D. Nickel, Annals Phys. 322 (2006)]

- Explore quasi-particle properties
 - Is the quasi-particle picture valid in this regime ?
 - How do bosonic modes affect the quark spectrum ?

[M. Kitazawa, T. Kunihiro, K. Mitsutani, PRD 77 (2008)]

[M. Harada, Y. Nemoto, PRD 78 (2008)]

[F. Karsch, M. Kitazawa, PRL B 658 (2007)]

Summary

Landau gauge Yang-Mills theory:

- scaling solution: 1PI-function with n ghost and m gluon legs:

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n/2-m)\kappa}$$

- Infrared fixed point in running coupling

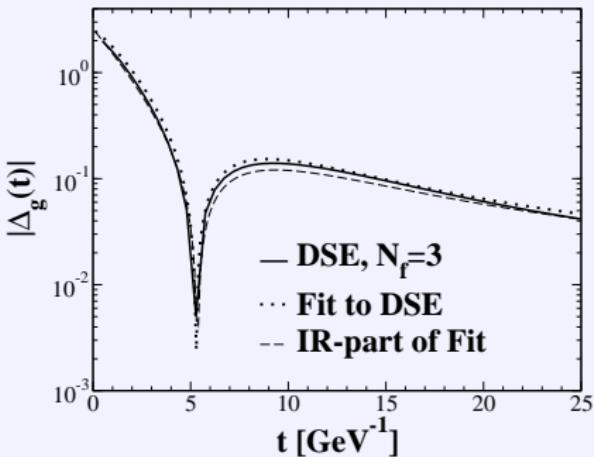
Unquenched Landau gauge QCD:

- Infrared singular quark-gluon vertex
→ solves $U_A(1)$ -problem
- Unquenching effects in light meson spectrum: large!
- Quarks in QGP: work in progress

Gluon confinement

► $\int \frac{d^4x}{(2\pi)^4} \Delta g(x^2) = \left(\frac{Z(p^2)}{p^2} \right) \Big|_{p^2=0} = \left(\frac{(p^2)^{2\kappa}}{p^2} \right) \Big|_{p^2=0} = 0 \quad \text{for } \kappa > 1/2$

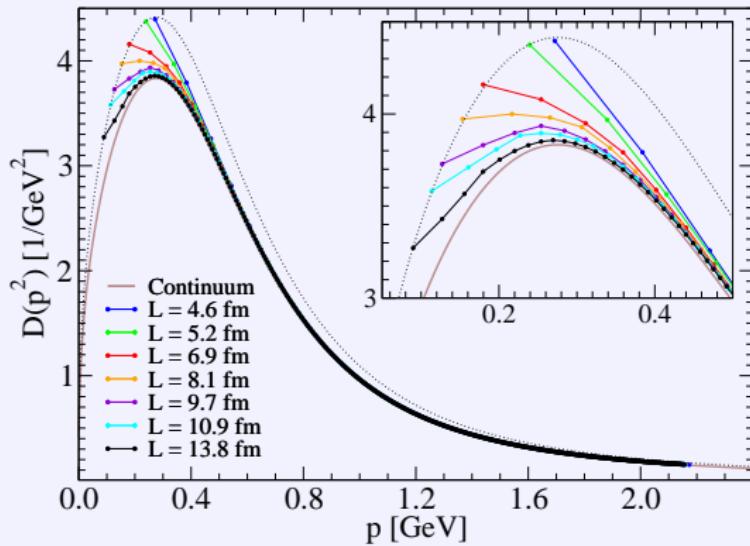
$$\Delta g(t) := \int d^3x \int \frac{d^4p}{(2\pi)^4} e^{i(tp_4 + \vec{p}\vec{x})} \frac{Z(p^2)}{p^2}$$



► Violation of positivity \Rightarrow Signal for **confined gluons**

R. Alkofer, W. Detmold, C. F., P. Maris, Phys. Rev. D **70** (2004) 014014

DSEs on a torus: volume effects

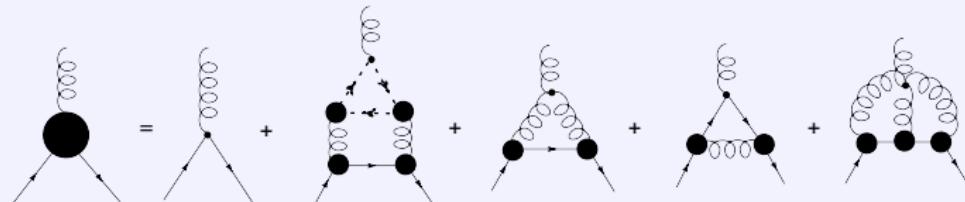


- Contemporary lattices large enough to exclude large volume effects

C.F., A. Maas, J. M. Pawłowski and L. von Smekal, Annals Phys. **322**, 2916 (2007)

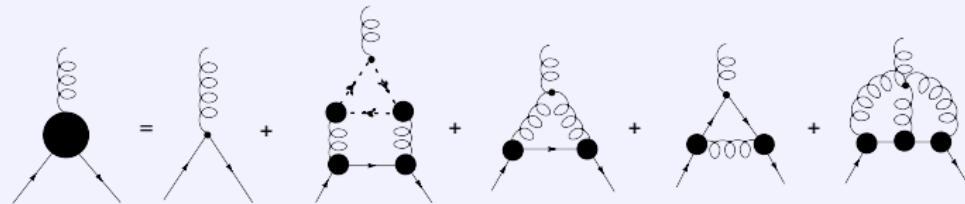
Infrared Structure of QCD II

- Quark-gluon vertex: **lowest order** in skeleton expansion



Infrared Structure of QCD II

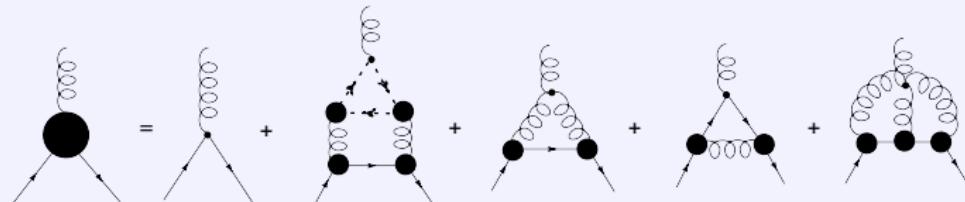
- Quark-gluon vertex: **lowest order** in skeleton expansion



$$S(p) = i\cancel{p} \frac{Z_f}{p^2 + M^2} + \frac{Z_f M}{p^2 + M^2}$$

Infrared Structure of QCD II

- Quark-gluon vertex: **lowest order** in skeleton expansion

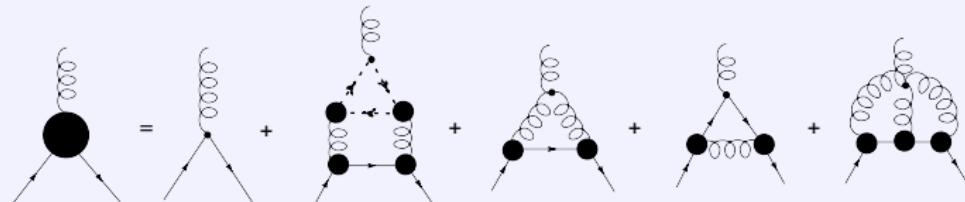


$$S(p) = i\cancel{p} \frac{Z_f}{p^2 + M^2} + \frac{Z_f M}{p^2 + M^2}$$

$$\Gamma_\mu = ig \sum_{i=1}^4 \lambda_i G_\mu^i : \quad G_\mu^1 = \gamma_\mu, \quad G_\mu^2 = \hat{p}_\mu, \quad G_\mu^3 = \hat{p} \hat{p}_\mu, \quad G_\mu^4 = \hat{p} \gamma_\mu$$

Infrared Structure of QCD II

- Quark-gluon vertex: **lowest order** in skeleton expansion

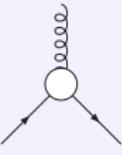


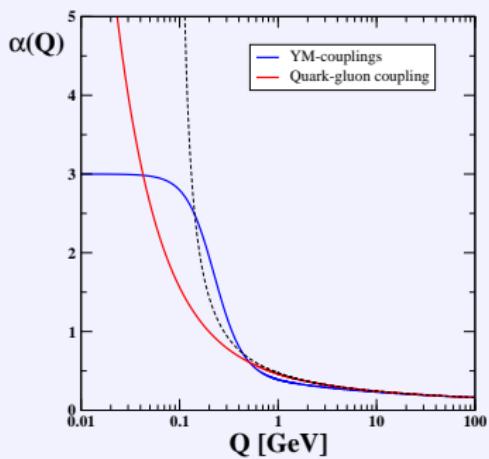
$$S(p) = i\cancel{p} \frac{Z_f}{p^2 + M^2} + \frac{Z_f M}{p^2 + M^2}$$

$$\Gamma_\mu = ig \sum_{i=1}^4 \lambda_i G_\mu^i : \quad G_\mu^1 = \gamma_\mu, \quad G_\mu^2 = \hat{p}_\mu, \quad G_\mu^3 = \hat{p} \hat{p}_\mu, \quad G_\mu^4 = \hat{p} \gamma_\mu$$

$$\lambda_{1,2,3,4} \sim (p^2)^{-1/2-\kappa} \qquad \leftrightarrow \qquad \lambda_{1,3} \sim (p^2)^{-\kappa}$$

Running Coupling: IR-slavery

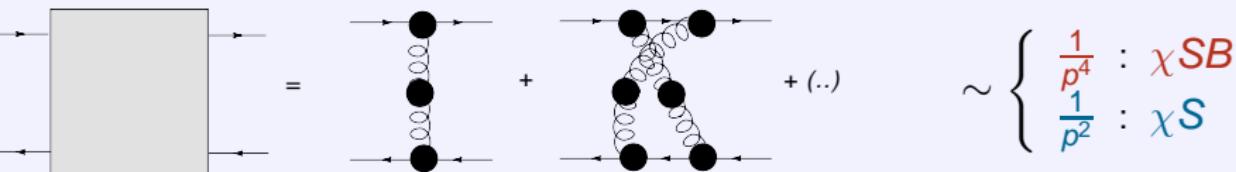

$$\alpha^{qg}(p^2) = \alpha_\mu [\Gamma^{qg}(p^2)]^2 [Z_f(p^2)]^2 Z(p^2) \sim \begin{cases} \frac{1}{p^2} : \chi SB \\ const : \chi S \end{cases}$$



R. Alkofer, C. F., F. Llanes-Estrada, MPL A **23**, 1105 (2008).

The quark-antiquark potential

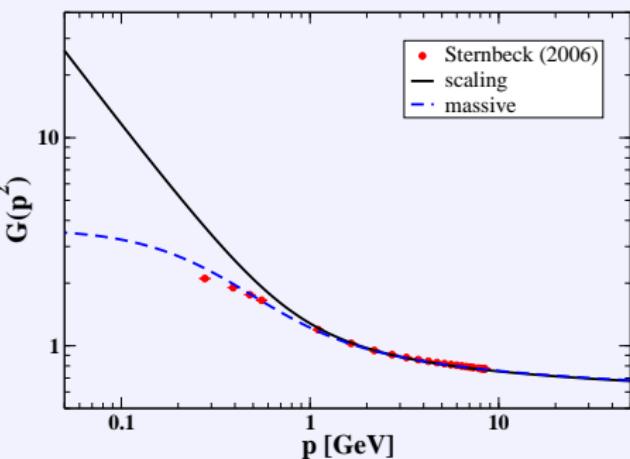
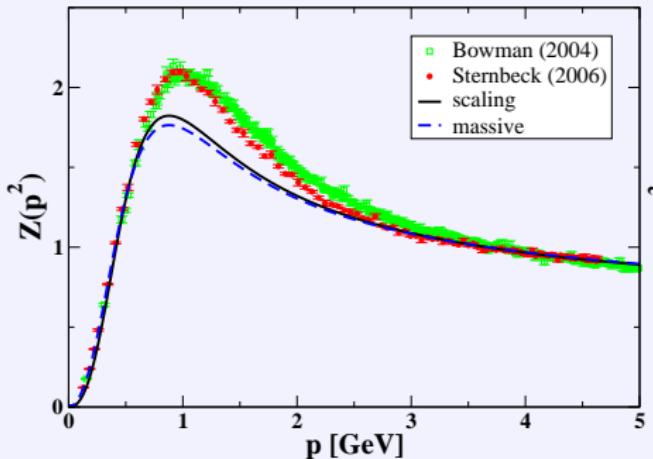
- quenched QCD



$$V(r) = \frac{1}{(2\pi)^3} \int d^3 p e^{i\mathbf{p}\mathbf{r}} \boxed{\quad} \sim \begin{cases} |r| : D\chi SB \\ \frac{1}{|r|} : \chi S \end{cases}$$

Quark confinement $\leftrightarrow \chi SB$

DSEs vs Lattice I



- $p^2 \approx 1 \text{ GeV}$: Systematic improvement possible for **DSEs**
- Deep infrared: Subtle problems for **Lattice**