Ultracold Gases far from Equilibrium

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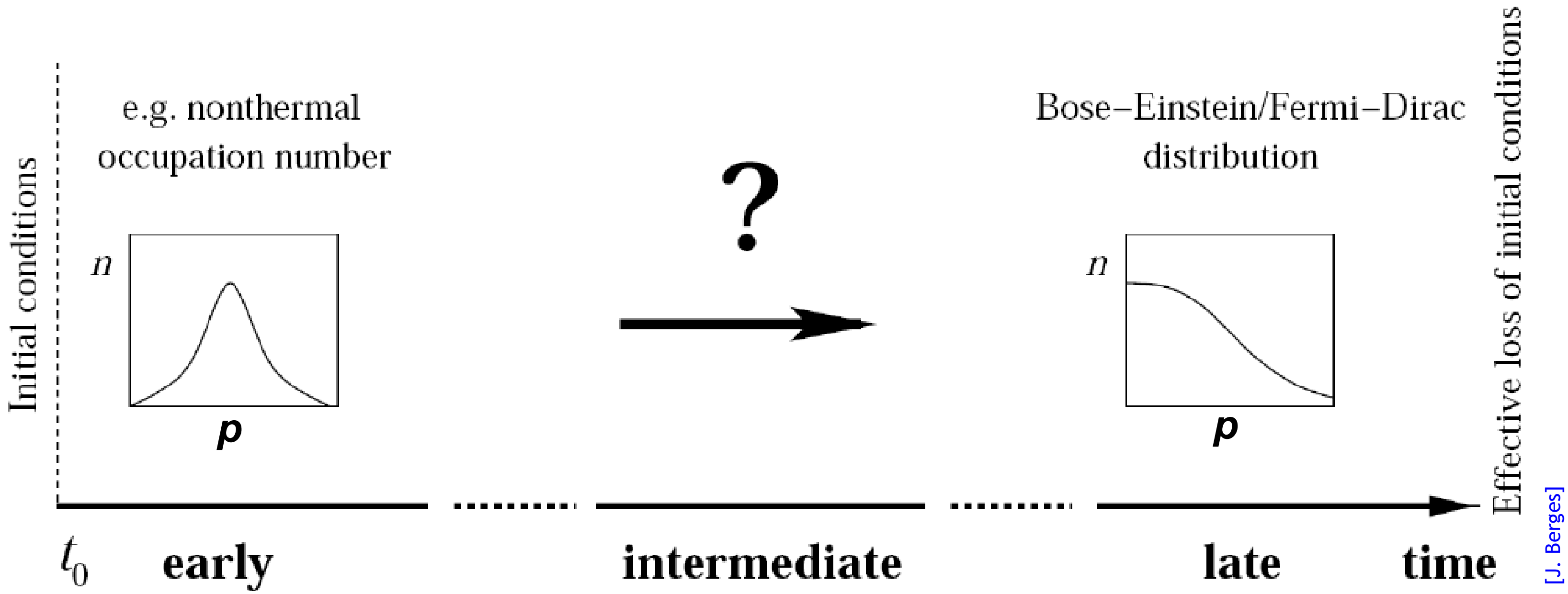
Preface

Far-from-equilibrium dynamics

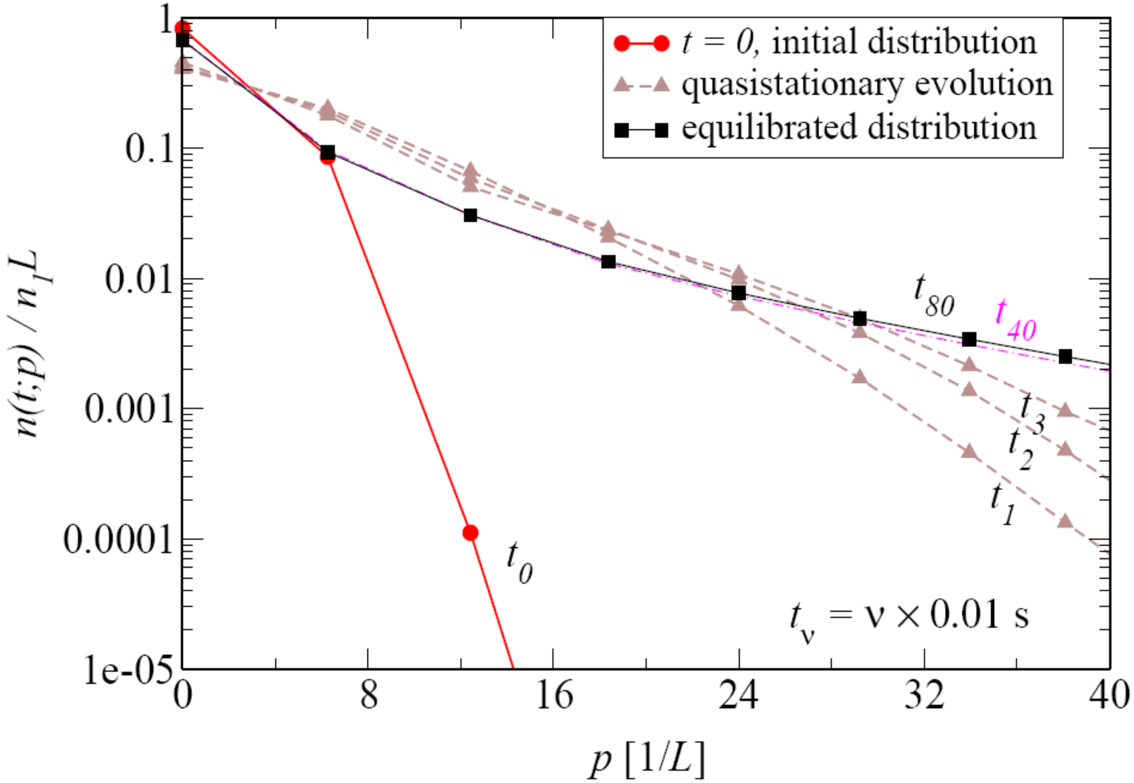


Thermal equilibrium: Loss of information about prior evolution.

Only a few conserved quantities persist.

Equilibration of a 1D Bose gas

Momentum distribution for different times:

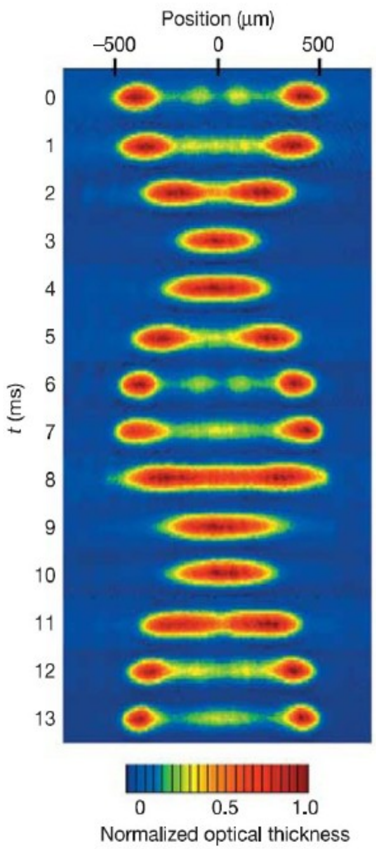


initial state: ^{23}Na atoms in 1D, $n_1 = 10^7\ \mathrm{m}^{-1}$

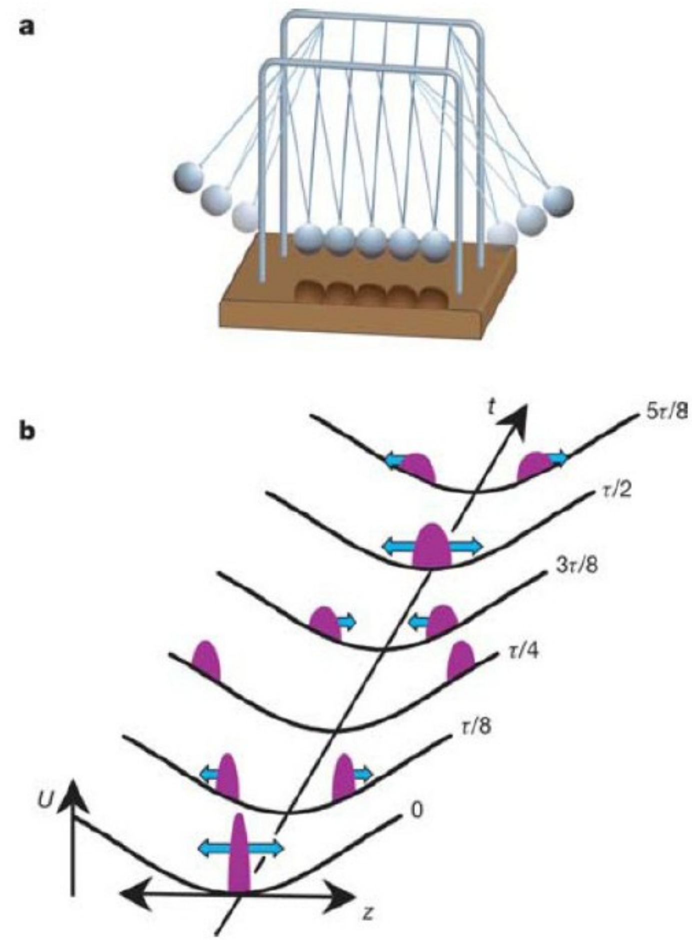
Long-time dynamics of ultracold gases

"A quantum Newton's cradle"

[T. Kinoshita et al. Nature 440 (06)]



Indication for strong suppression of damping



Effective-action for many-body dynamics

Dynamical Field Theory

We will be interested, in particular, in the explicit time dependence of the lowest-order correlation functions:

$$\phi_a(x) = \langle \Phi_a(x) \rangle \qquad \text{(mean field)}$$

$$G_{ab}(x,y) = \langle \mathcal{T} \Phi_a(x) \Phi_b(y) \rangle_{\mathrm{c}} \qquad \text{(density matrix, 2-point function, propagator)}$$

connected, i.e., $= \langle \mathcal{T} \Phi_a \Phi_b \rangle - \phi_a \phi_b$

where $x = (\mathbf{x}, t)$

Path Integral Approach

Classical dynamics of φ from $\delta S[\varphi] = 0$.

Path Integral Approach

QM transition amplitude:

$$\langle t_{fin} | t_{ini} \rangle = \int \mathcal{D}\varphi \; e^{iS[\varphi]/\hbar}$$

$$\mathcal{D}\varphi = \prod_{x=x_{ini}}^{x_{fin}} \mathrm{d}\varphi(x)$$

Classical dynamics of φ from $\delta S[\varphi] = 0$.

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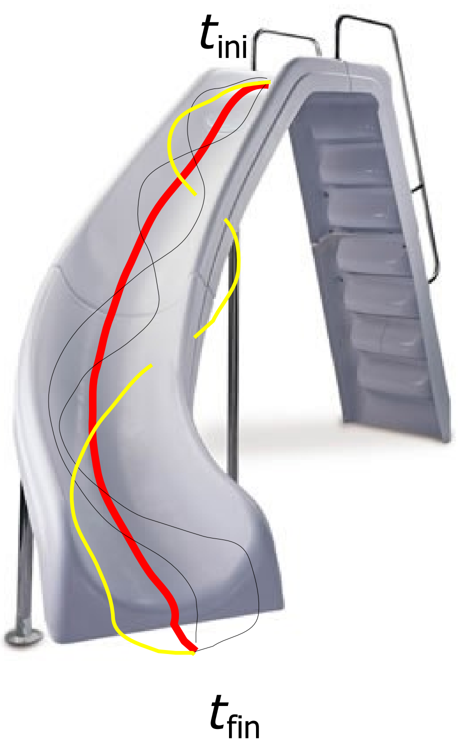
Path Integral Approach...
...in QFT:

Generating functional:



$$Z[J] = \int \mathcal{D}\varphi \; e^{iS[\varphi]/\hbar - i\int J\varphi}$$

$$\phi = i\frac{\delta \ln Z}{\delta J}\bigg|_{J=0} = Z^{-1}\int \mathcal{D}\varphi \; \varphi \, e^{iS[\varphi]/\hbar}$$

Classical dynamics of φ from $\delta S[\varphi] = 0$.

Effective Action

Generating functional:

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Classical dynamics of φ from $\delta S[\varphi] = 0$.

Quantum dynamics of ϕ from variation of an **effective action,** $\delta\Gamma[\phi]/\delta\phi = -J$:

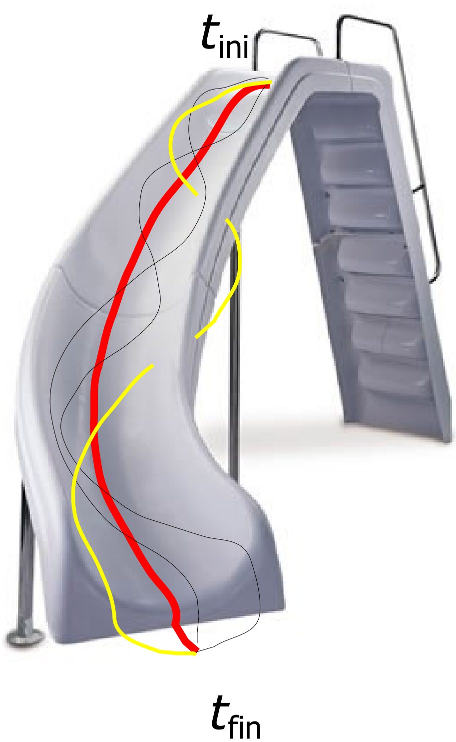
$$Z[J] = \int \mathcal{D}\varphi\; \delta[\varphi - \phi]\, e^{i\Gamma[\varphi]/\hbar - i\int J\varphi}$$

$$\Gamma[\phi] = -i\hbar \ln Z[J] - \int J\phi$$

Legendre transform

Effective Action

Generating functional:



$$Z[J] \;=\; \int \mathcal{D}\varphi \; e^{\,i\,S[\varphi]/\hbar \,-\, i\int J\varphi}$$

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Classical dynamics of φ from $\delta S[\varphi] \;=\; 0.$

Quantum dynamics of ϕ from variation of an **effective action**, $\delta\Gamma[\phi]/\delta\phi \;=\; -J$:

$$\begin{aligned} Z[J] \;&=\; \int \mathcal{D}\varphi\;\delta[\varphi-\phi]\,e^{\,i\,\Gamma[\varphi]/\hbar \,-\, i\int J\varphi} \\ \Gamma[\phi] \;&=\; -\,i\,\hbar\,\ln Z[J] \;-\; \int J\,\phi \\ &=\; S[\phi] \;-\, i/2\;\, \mathrm{Tr}\ln G \;+\; \ldots \end{aligned}$$

Gaussian integration

2PI Effective Action (Φ-Functional)

[Luttinger, Ward (60); Baym (62); Cornwall, Jackiw, Tomboulis (74)]

- Double **Legendre transform**:

$$\Gamma[\phi, G] = -i \ln Z[J, K] - \phi_i J_i - \frac{1}{2}(\phi_i \phi_j + G_{ij}) K_{ij},$$

$$-i \left.\frac{\delta \ln Z[J, K]}{\delta J_i}\right|_{J=K\equiv 0} = \phi_i = \langle \hat{\Phi}_i \rangle,$$

$$-2i \left.\frac{\delta \ln Z[J, K]}{\delta K_{ij}}\right|_{J=K\equiv 0} = \phi_i \phi_j + G_{ij} = \langle \mathcal{T} \hat{\Phi}_i \hat{\Phi}_j \rangle.$$

2PI Effective Action (Φ-Functional)

[Luttinger, Ward (60); Baym (62); Cornwall, Jackiw, Tomboulis (74)]

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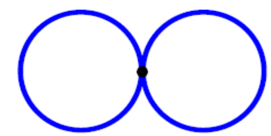
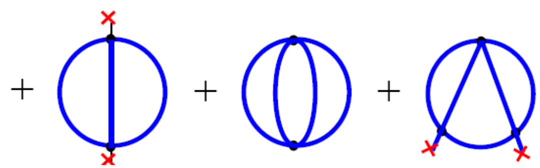
$$\Gamma[\phi, G] = S[\phi] + \tfrac{i}{2}\mathrm{Tr}(\ln G^{-1} + G_0^{-1}(\phi) G) +$$ 



Only 2-particle irreducible (2PI) loop diagrams

Conservation laws fulfilled

2PI Effective Action (Φ-Functional)

[Luttinger, Ward (60); Baym (62); Cornwall, Jackiw, Tomboulis (74)]

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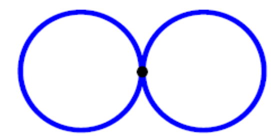
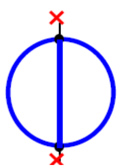
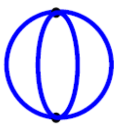
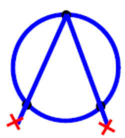
$$\Gamma[\phi, G] = S[\phi] + \tfrac{i}{2}\mathrm{Tr}(\ln G^{-1} + G_0^{-1}(\phi) G) +$$ 

$+$  $+$  $+$ 

- Dynamic equations: [closed for Gaussian initial conditions (only ϕ, G $\neq$ 0 @ t = 0)]

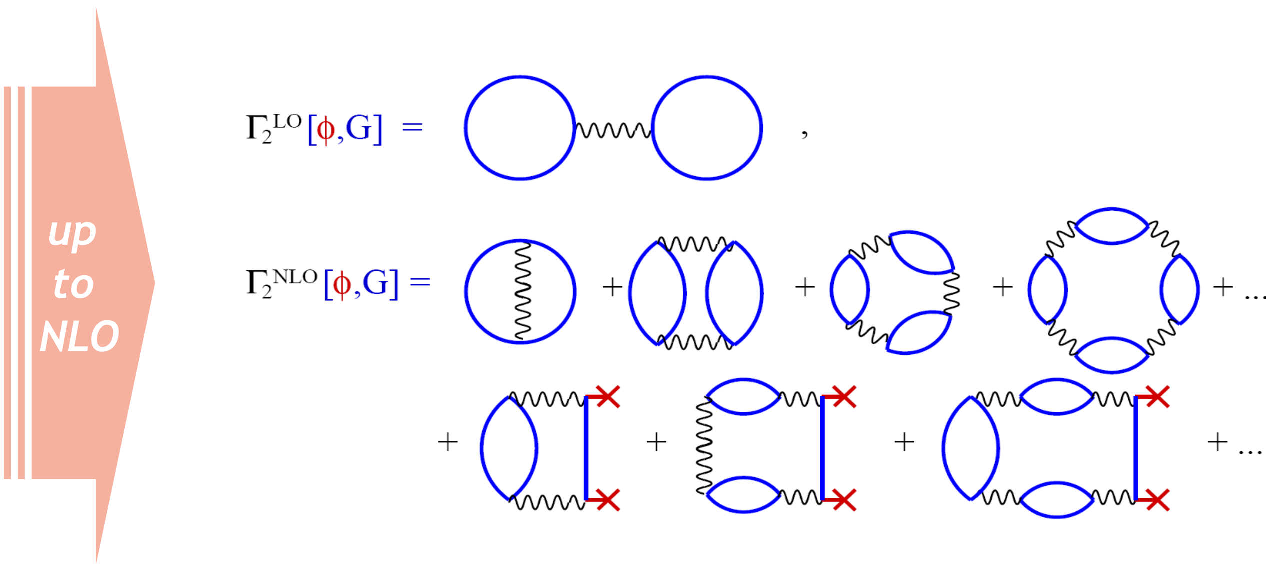
(@ J = 0, K = 0)

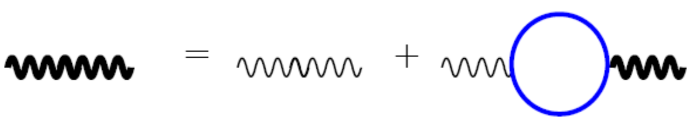
$$\frac{\delta\Gamma[\phi, G]}{\delta\phi_x} = 0, \qquad \frac{\delta\Gamma[\phi, G]}{\delta G(x, y)} = 0$$

Conservation laws fulfilled

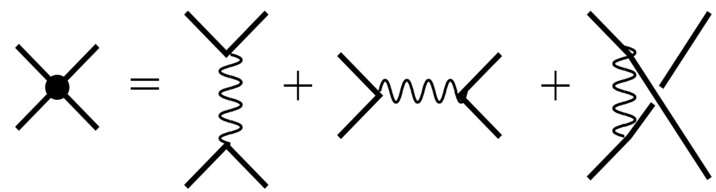
2PI $1/\mathcal{N}$ Expansion

[Berges, NPA **699** (02) 847; Aarts, Ahrensmaier, Baier, Berges, & Serreau, PRD **66** (02) 45008]





Vertex resummation



bare vertex

[Also: Mihaila, Dawson & Cooper (01); Cooper, Dawson & Mihaila (03); Berges & Serreau (03); Berges, Borsanyi & Serreau (03); Berges, Borsanyi & Wetterich (04); Alford, Berges & Cheyne (04); Arrizabalaga, Smit & Tranberg (04); ...; Rey, Hu, Calzetta (04); Baier & Stockamp (04); TG, Berges, Schmidt & Seco (05); ...]

Equilibration of a 1D Bose gas

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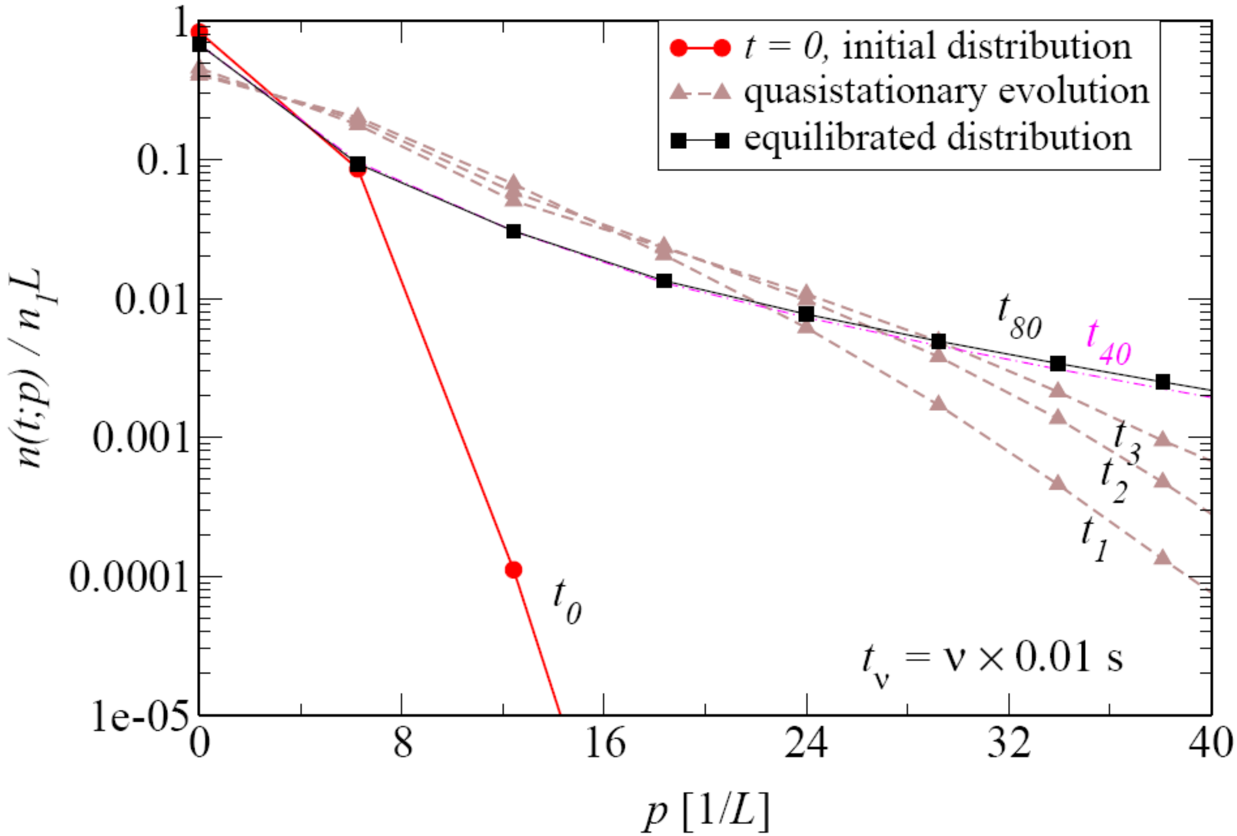
Momentum distribution for different times:

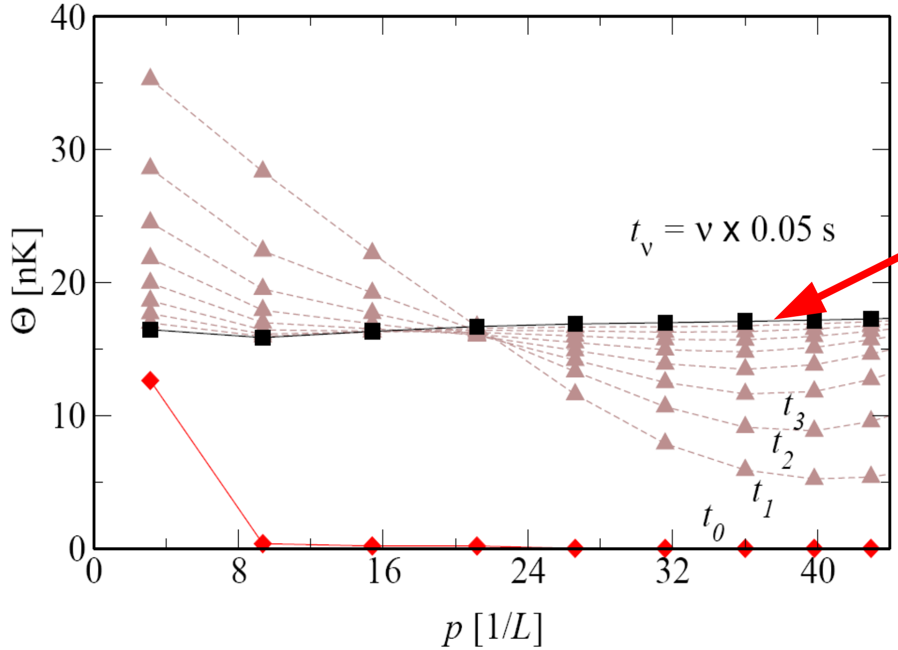


[TG, J. Berges, M. Seco & M.G.Schmidt, PRA **72** (05); J. Berges & TG, PRA **76** (07)]

Temperature appears

[also: Berges & Cox (01), Berges (01), Berges, Borsanyi, & Serreau (03), Berges, Borsanyi, & Wetterich (04)]

'Temperature' parameter $\Theta(p)$ at t_n



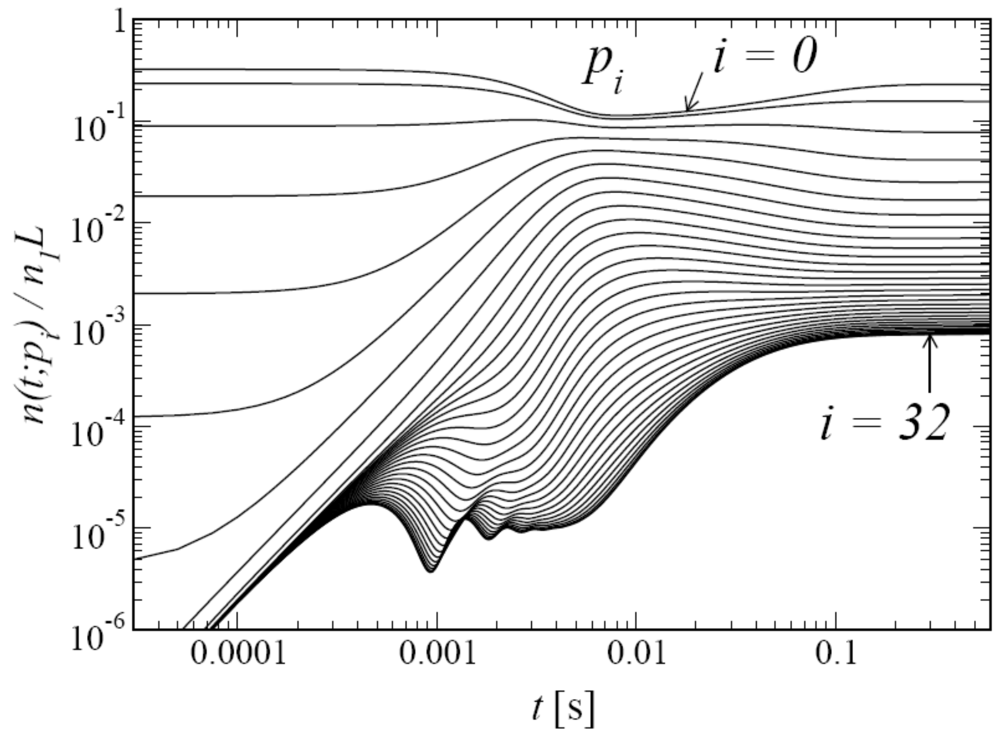
flat = temperature

[J. Berges & TG, PRA **76** (07)]

$$n(t;p) = \frac{1}{e^{\frac{1}{k_B \Theta(p)}\left(\frac{p^2}{2m} - \mu\right)} - 1}$$

Far-from-equilibrium evolution

Time evolution of mode occupation no^s:



initial state:

- ^{23}Na atoms in 1D, $n_1 = 10^7$ m^{-1}
- interaction parameter $\gamma = \lambda m/(\hbar^2 n_1) = 7.5 \cdot 10^{-4}$
- Gaussian momentum distribution

[J. Berges & TG, PRA **76** (07)]

Far-from-equilibrium evolution

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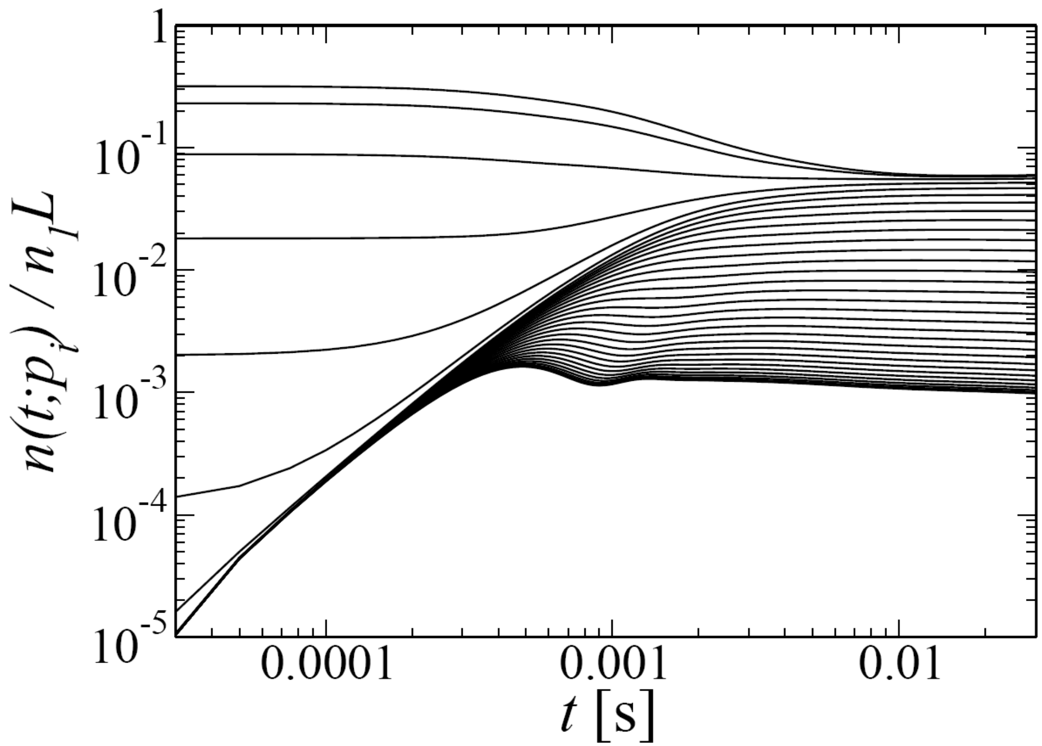


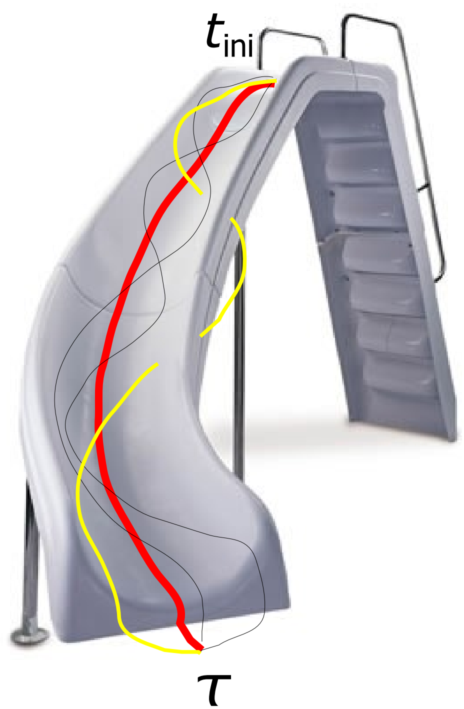
initial state:

- ^{23}Na atoms in 1D, $n_1 = 10^5\,\mathrm{m}^{-1}$
- interaction parameter $\gamma = \lambda m/(\hbar^2 n_1) = 15$
- Gaussian momentum distribution

Functional RG approach

RG approach
to far-from-equilibrium dynamics

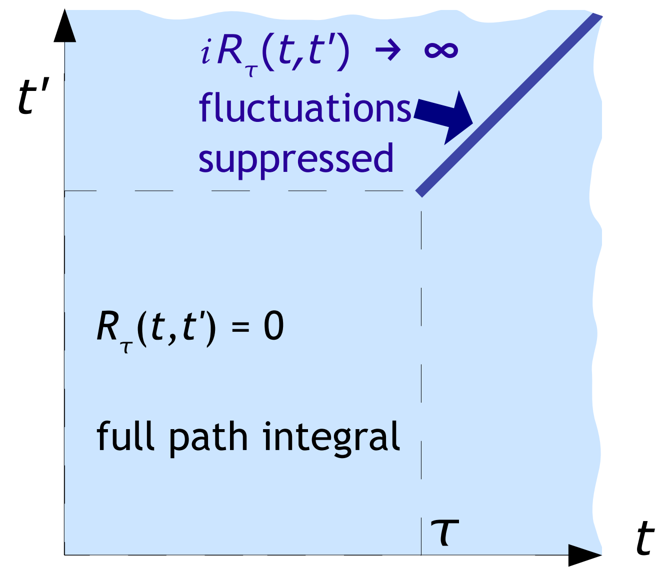
[TG & J.M. Pawlowski, cond-mat/0710.4627]



shift max. time τ
and watch Z_τ and Γ_τ evolve

$$Z[J;\rho_0] = \int \mathcal{D}\varphi\, \rho_0 \exp\left\{ iS[\varphi] + i\int_{x,\mathcal{C}} J_a \varphi_a \right\}$$

$$Z_\tau = \exp\left\{ i \int_{x,y;\mathcal{C}} \frac{\delta}{\delta J_a(x)} R_{\tau,ab}(x,y) \frac{\delta}{\delta J_b(y)} \right\} Z$$



Functional RG equation [C. Wetterich (93)]:

$$\partial_\tau \Gamma_\tau = \frac{i}{2} \int_{\mathcal{C}} \left[\frac{1}{\Gamma_\tau^{(2)} + R_\tau} \right]_{ab} \partial_\tau R_{\tau,ab}$$

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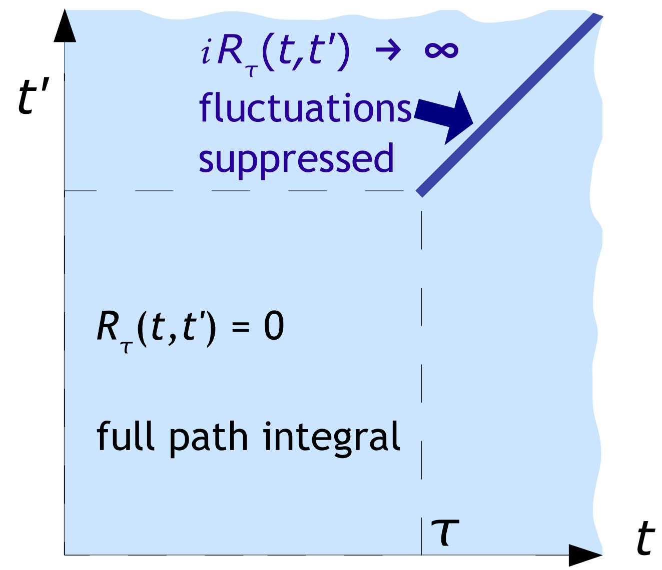

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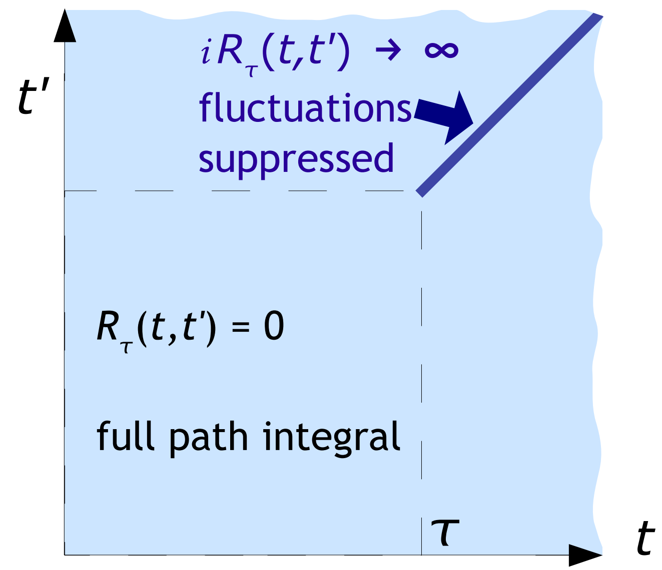

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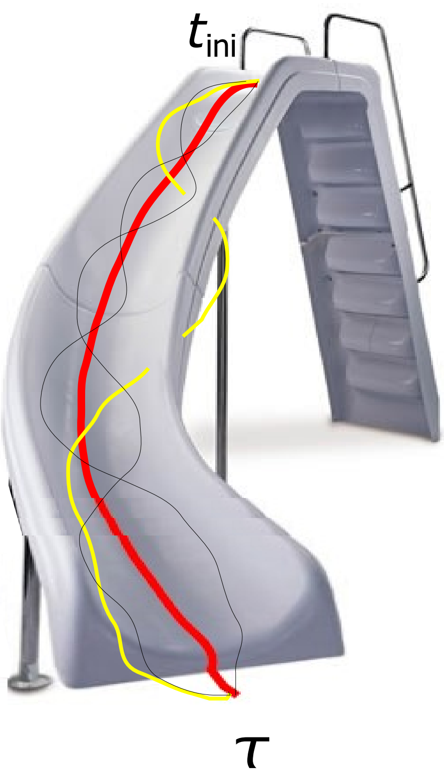

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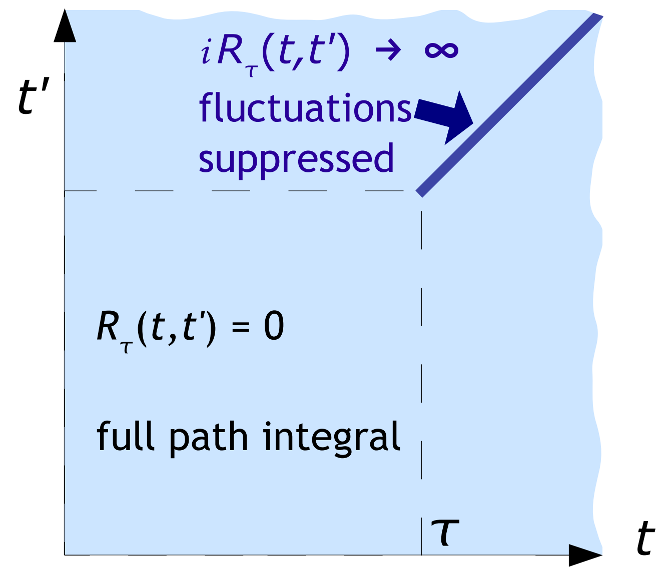

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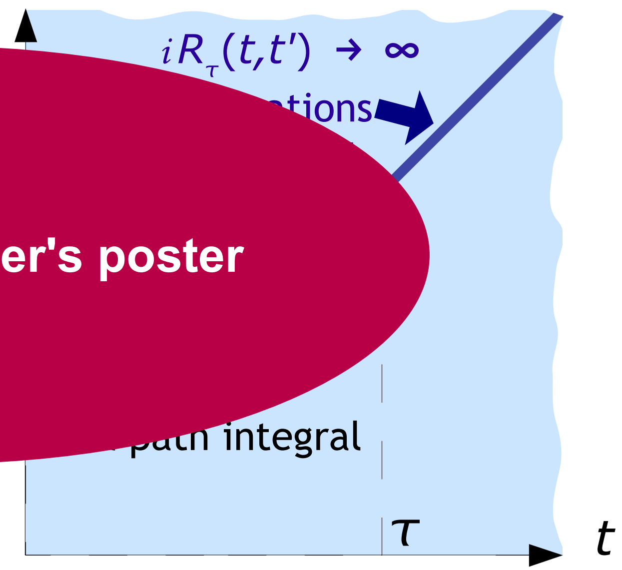

See Stefan Kessler's poster

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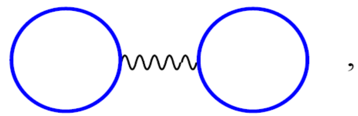
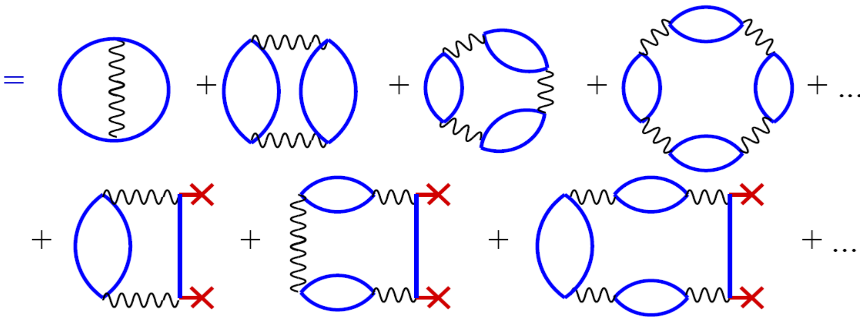
2PI $1/\mathcal{N}$ Expansion

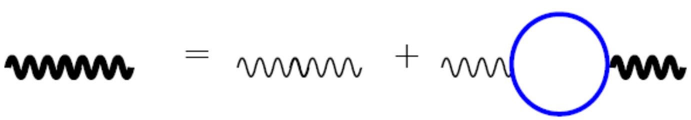
[Berges, NPA **699** (02) 847; Aarts, Ahrensmaier, Baier, Berges, & Serreau, PRD **66** (02) 45008]

$\Gamma_2^{\mathrm{LO}}[\phi,G]$ =

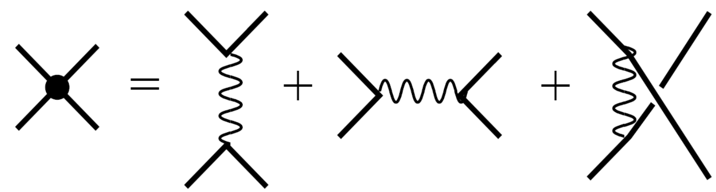


$\Gamma_2^{\mathrm{NLO}}[\phi,G]$ =





Vertex resummation



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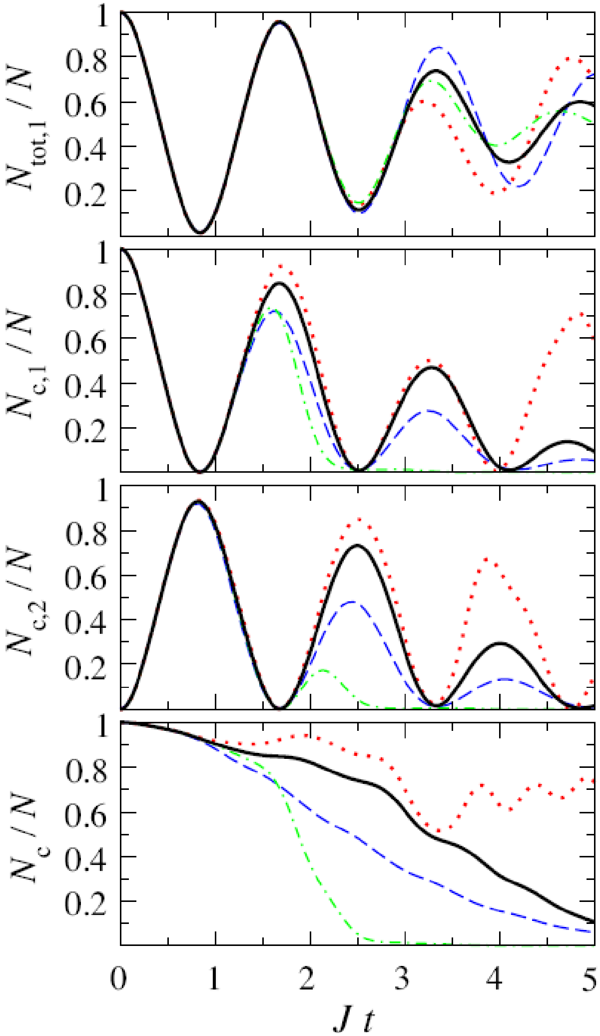
[Also: Mihaila, Dawson & Cooper (01); Cooper, Dawson & Mihaila (03); Berges & Serreau (03); Berges, Borsanyi & Serreau (03); Berges, Borsanyi & Wetterich (04); Alford, Berges & Cheyne (04); Arrizabalaga, Smit & Tranberg (04); ...; Non-relativistic: Rey, Hu, Calzetta (04); Baier & Stockamp (04); TG, Berges, Schmidt & Seco (05); ...]

Benchmarks & applications

Dynamics in lattice potentials

Benchmark tests

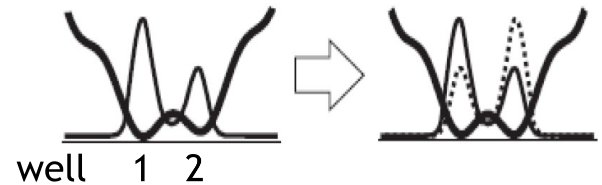


total atom # in well 1

condensate well 1

condensate well 2

total condensate frac.



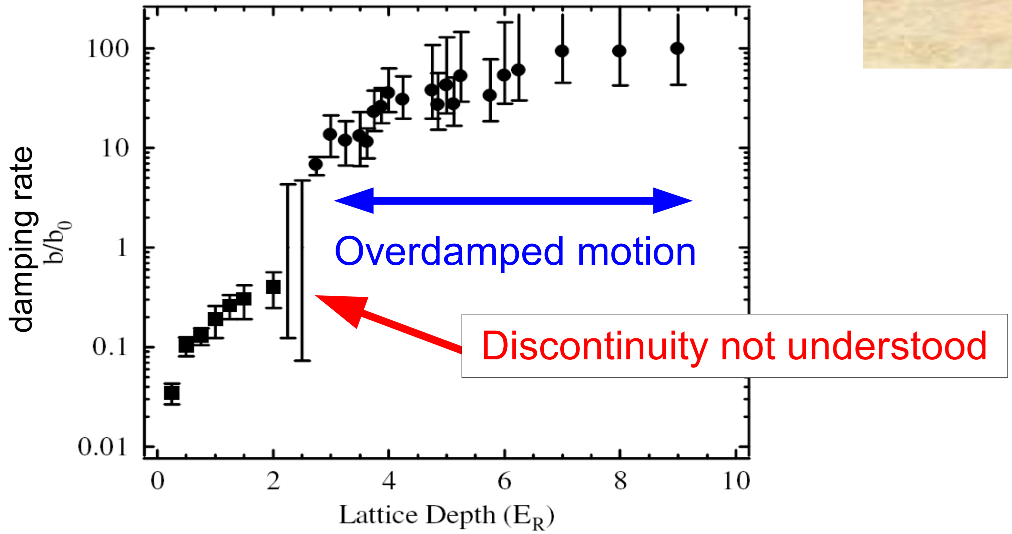
— 2PI $1/\mathcal{N}$
- - exact
· · · HFB
· – · 2^{nd} O. $1/\mathcal{N}$

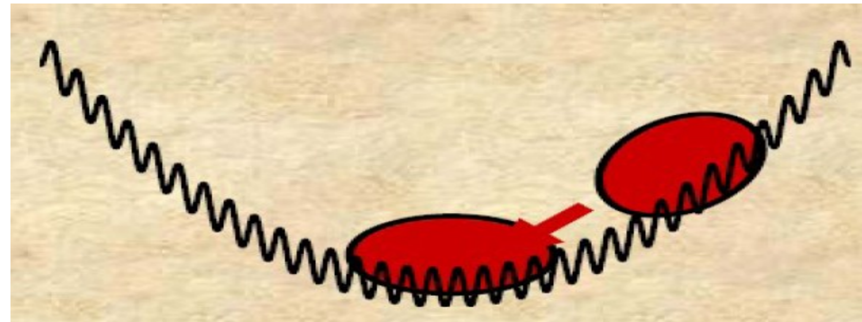
[Temme, TG (06)]

Nonequilibrium dynamics in lattices

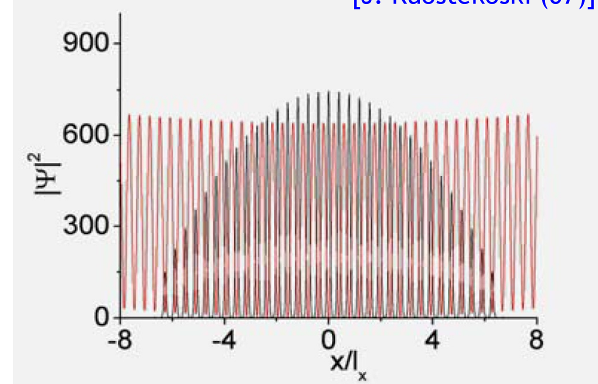
Dipole oscillations in lattices:
Damping rates?

[Expt. @ NIST: Fertig et al. PRL94 (05)]

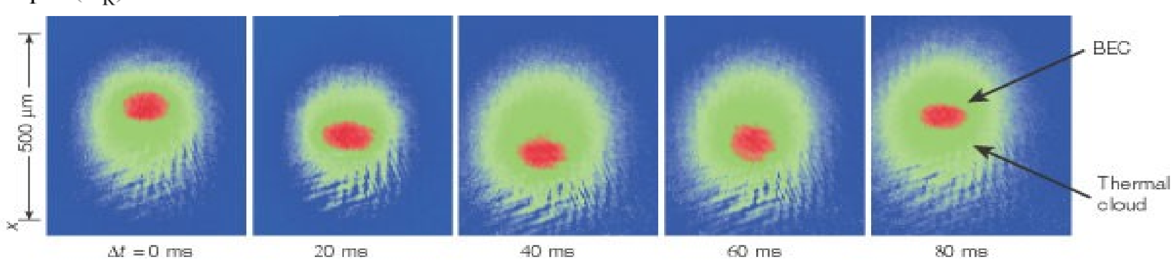




[J. Ruostekoski (07)]



[Anglin & Ketterle (02)]



Fermions

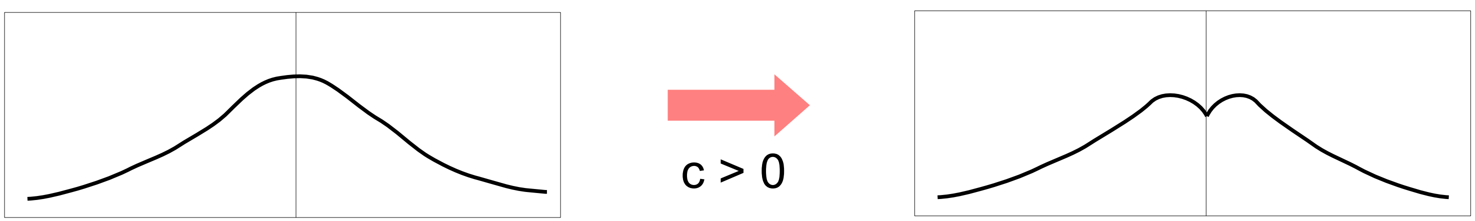
- 2PI dynamics: See Matthias Kronenwett's poster

Exactly solvable model

Lieb-Liniger gas in 1D: [PR **130**, 1605 & 1616 (63)]

$$i\frac{\partial \psi_B}{\partial t} = -\sum_{i=1}^{N}\frac{\partial^2 \psi_B}{\partial x_i^2} + \sum_{1\le i<j\le N} 2c\,\delta(x_i - x_j)\psi_B$$

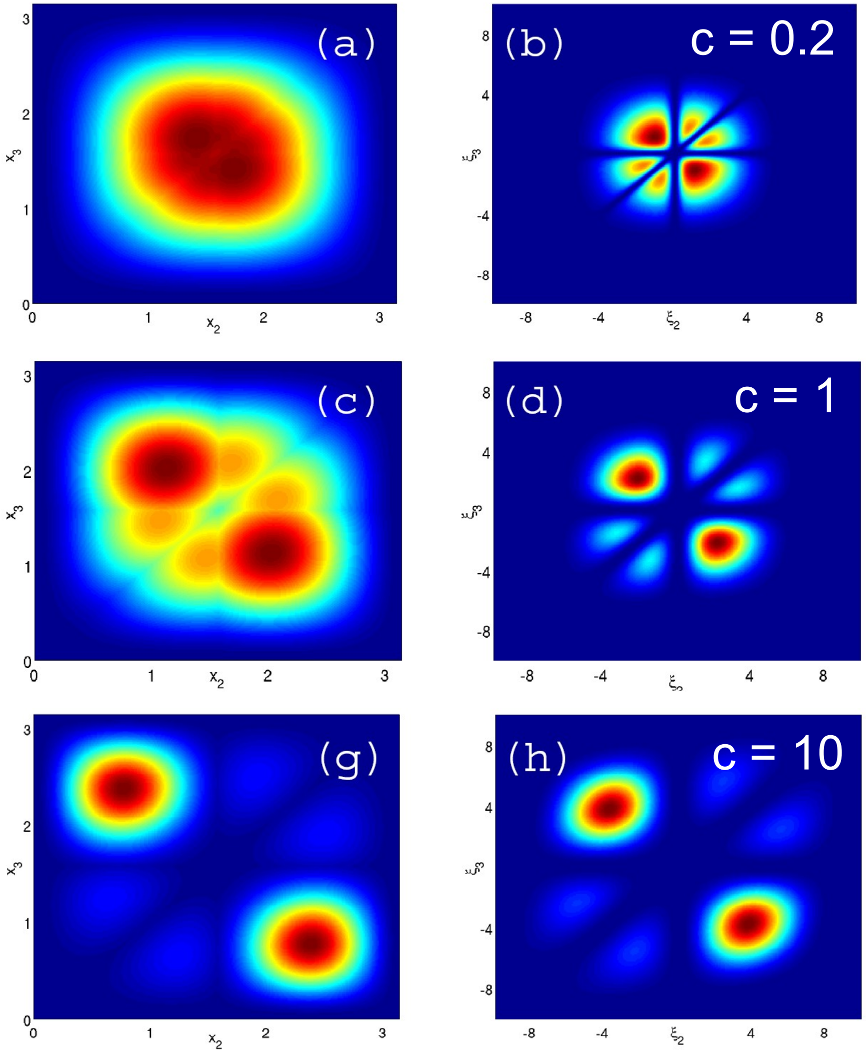
Solution via Bethe-Ansatz with cusp conditions @ particle contact hyperplanes



$x_i = x_j$ Dirichlet-von Neumann

Dynamics: H. Buljan, R. Pezer, & TG, PRL **100** (2008)

Free 1D expansion: Exact results

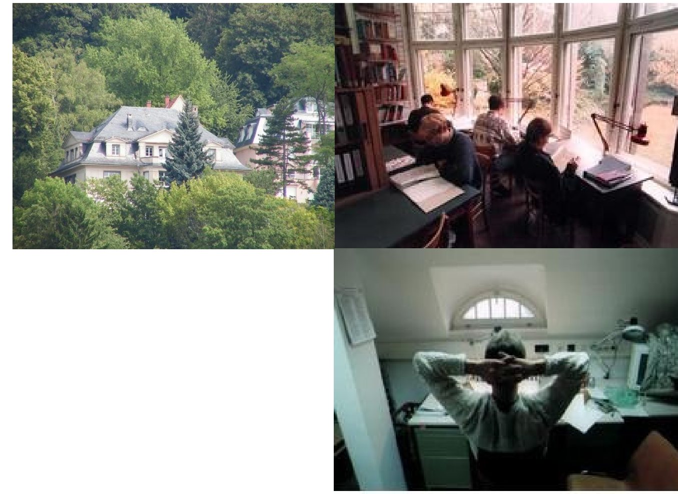


$$i\frac{\partial\psi_B}{\partial t} = -\sum_{i=1}^{N}\frac{\partial^2\psi_B}{\partial x_i^2} + \sum_{1\le i<j\le N} 2c\,\delta(x_i - x_j)\psi_B$$

$$\left|\psi_B(0, x_2, x_3, t)\right|^2$$

[H. Buljan, R. Pezer, & TG, PRL **100** (2008)
D. Jukic, H. Buljan, R. Pezer, & TG, cond-mat/0804.2580]

Thanks & credits to...



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